

Chapter II, Section I:

Three Assumptions

In Chapter I, I specified a logic of probable inference, and demonstrated its application to a single illustrative inductive situation.

I now put forward three propositions upon the truth of which the succeeding discussion will be based. Each assumes that an induction is a decision made by a machine in an inductive situation to act as if certain relevant propositions were true and the remainder false, based upon its calculation of the respective probabilities of the relevant propositions with respect to the evidence. I shall not argue for the truth of the following propositions, but rather seek to make their meaning and consequences clear.

- I. The most accurate induction which a human organism could make in a given inductive situation is that which a Θ - machine having an identical program would have made in the same situation, assuming that if

(a) $\neg \Theta \vdash (p \rightarrow q) > \frac{1}{2}$; and

(b) $\Theta(p)$; then

(c) Θ believes q (i.e. is prepared to act as if q were true).

- II. The essential activity of a human organism is making accurate inductions.

- III. A work of art is an inductive game which exercises the deepest

habitual responses of the organism.

Consider firstly proposition I: In Chapter I, I defined 'probability' with respect to a set of Θ -machines, each member of which possessed at any moment perfect knowledge of the set of propositions upon which it was programmed to act; i.e. each member of which possessed at any moment total self-awareness and total memory. Human organisms, in contrast, have neither. At any moment, the vast bulk of the past experiences of any human organism are subsumed under habitual responses and forgotten. (See page 50 below.) Indeed, perhaps the most basic propositions upon which the organism is programmed to act were never experienced as true at all, but were rather inherited as part of its instinctual genetic make-up. The set of human organisms, hence, is not a subset of the set of Θ -machines.

It seems to me, however, that were Θ -machines to be so constructed that if

- (a) $\neg \Theta \vdash (p \rightarrow q) > \frac{1}{2}$; and
- (b) $\Theta(p)$; then
- (c) Θ believes q (i.e. is prepared to act as if q were true),

and were human organisms to have total self-awareness and total memory, the latter would be a subset of the former and make inductions in accordance with the specifications of Chapter I.

(I shall also assume without argument that applying the results of Chapter I to more complex inductive situations than that illustrated in

Chapter I, Section II, would lead to the following general result:

Given an S -membered sample of which t are B , the probability that the next object sampled is B is a function of t and an inverse function of S .)

Consider now **proposition II**: A human organism at each moment of its existence encounters complex temporal events, some of which are conducive to the furthering of its well-being and some of which are not. To insure its self-preservation, the organism must seek the former and avoid the latter, and do so with maximum efficiency (since its resources are limited).

Achieving the above goal with maximum efficiency, however, requires that the inductive range of the conscious thought of the organism be kept to a minimum, for, if the limited though superior discriminating powers of the logical and memory faculties of the intellect are to be used to greatest effect, they must be concentrated upon those aspects of the complex temporal events encountered which are least redundant to the previous experiences of the organism. At each moment of its existence, therefore, the organism is not only involved in consciously predicting aspects of the future consequences of present situations, but is simultaneously involved in the task of converting conscious patterns of inference which have proven successful in the past into unconscious habits of reaction by which to predict such consequences in the future without conscious effort, thereby freeing its intellect for concentration on subtler predictive aspects, and hence increasing its organismic efficiency.

Habits, therefore, are responses which have become automatic.

Their very automaticity signifies the organism's fullest confidence in their ability to function efficiently to preserve its well-being. It is not surprising, therefore, that the momentary failure of an habitual response should effect a more pervasive physiological shock to the organism than the momentary failure of a conscious and hence non-habitual choice, for the effectiveness of its most trusted mechanism for self-preservation has been questioned. Such a pervasive shock causes a physiological reaction within the organism which is called an emotion.

Emotional responses, however, fall along a continuum which divides into two distinct halves, for a momentary failure of the organism's habitual response structure to anticipate accurately may either result in the expected danger to the well-being of the organism which, as the emotional reaction indicates, it fears, or it may result unexpectedly in its increased well-being. If the former, the emotions of anger, fear, terror, etc., occur. If the latter, the emotions of relief, joy, gaiety, etc., occur.

If the emotional response of the organism is itself so pervasive as to threaten the well-being of the organism, the physiological mechanism of the organism asserts control, shutting off momentarily the organism's capacity for motivated activity in a flush of tears and helplessness. And if even this proves insufficient, the physiological mechanism of the organism perpetuates its incapacity for habitual reaction to the point of denying (to various degrees) the efficacy of perception itself.

The essential features of a human organism's existence, therefore, are dependent upon its capacity to make accurate habitual inductions.

Consider lastly proposition III: By the phrase 'inductive game', I mean that a work of art is a tool whereby a human organism is able to make habitual inductions as if its well-being depended upon their accuracy, without an actual threat being present. That is, the organism consciously or unconsciously imagines such a threat, and exercises its habitual responses to avoid it. The emotions which result from the thwarting of habitual inductions made in response to a work of art, therefore, have a peculiar detachment and gentleness, indicative of the fact that the habits have been exercised in the context of an imaginary threat. Note, however: they are genuine emotional reactions to the thwarting of genuine habitual responses; only the threat to the organism's well-being which evokes the response is imaginary.

For example, I recall vividly finishing a first reading of Tolstoy's Anna Karenina with tears in my eyes, knowing well that the rare presence of tears indicated neither that I had encountered nor escaped a threat to my well-being, but rather that, having been led by Tolstoy's words thru a long and subtle sequence of hoping, caring, guessing, being proven wrong and being proven right, I had emerged from an experience in which my most deeply conditioned responses had been exercised. Long dormant habitual reactions had been tested, re-assessed, and refined. And I had emerged as a generally more sensitive person (i.e. as a more subtly-skillful inductive organism), even though the experience had presented no actual threats to my well-being but only imaginary ones.

Human organisms, of course, engage in a diversity of other inductive games (eg. tennis, golf, football, billiards, chess, bridge, etc.).

Works of art differ from the above, it seems to me, only in that

- (a) the pervasiveness of the habitual reactions which they exercise is much greater; and
- (b) the exercise is finer (i.e. more subtle).

The experience one has in playing a game of chess or tennis, therefore, is not qualitatively dissimilar from one's experience in listening to Bach or reading Proust, but rather quantitatively so. In the former, one's inductions are often consciously made and unique to the immediate situation; in the latter, they are unconscious and general.

The difference can be acutely felt, however, for whereas a second or third viewing of a videotape of a basketball game, or the replaying of a chess game, destroys the original thrill of accomplishment, the second or third re-reading of The Magic Mountain, or a re-screening of The Island, deepens the aesthetic impact. In the former cases, since the original inductions were largely conscious and momentary, the organism's remembrance of the previously experienced events is sufficient to void the necessity of it reacting with inductive skill to their re-appearance. In the latter cases, however, since the original inductions involved deeply-rooted habitual mechanisms which do not rapidly change their character thru light and transient experiences, the organism's conscious remembrance of the previously experienced events is largely irrelevant to their effect at their reappearance upon these mechanisms, which (having, in effect, no memory) are inductively re-tested thereby.

Such differences, however, ought not to blind the reader to the essential inductive function of both sporting games and works of art. For upon this similarity hangs the structural tale which I shall tell in Section II.

Chapter II, Section II:

Part I: Meditations on a Golfe Course

How does a great work of art differ structurally from a mediocre one? If the reader will recall that a work of art, roughly, is a deeper and more finely wrought sporting game, it may shock him less when I now propose that we look to the sporting world for evidence 'writ large', as it were. For example, how does a great golf course differ structurally from a mediocre one?

Every golfer in the course of a round is involved in a sequence of situations each of which forces him to make choices such as

- (a) Ought I to go over the water, or to the side?;
- (b) Which club should I choose?;
- (c) Ought I to move my rear foot inward?;
- (d) Ought I to pitch-and-run, or wedge?;
- (e) Can I drive past that bunker?; etc.

His evaluation of the situation facing him, dependent for its accuracy upon the strength of his concentration now and in the past, and hence on the strength of his memory, leads him to disregard certain factors as irrelevant to the success of his forthcoming shot, and to weigh the relative importance of the remainder. Simply put (if I consider any set of habits directed toward the achieving of a conscious goal as a skill), on the basis of his skill the golfer finds a theme, and on the basis of this theme he makes an induction and acts upon it. By acting upon the induction, he assumes as an hypothesis the relevance to the

situation at hand of the theme chosen on the basis of his skill, and puts it to the test. The accuracy or inaccuracy of the induction depends upon the relevance of the chosen theme. If he guesses inaccurately, he may yet succeed in making the shot; if he guesses accurately, he may yet fail. But if the golf course is well-designed, by the end of the round the consistently inaccurate guesser ought to find his frequency of success to be low, while the consistently accurate guesser finds his to be high. At best, therefore, a golf course ought

- (a) to reward more accurate inductions and penalize less accurate ones.

But there is a second factor to be considered. A golfer may choose not to play at all, if he finds the challenge of a course insufficient (i.e. if he finds that the range of his present skills are not being exercised). Or, put another way, unlike an encountered slice of life from which, dull or not, the organism can turn away only with major psychological damage to itself, a golf course which does not successively challenge the gamut of a golfer's inductive skills, but rather engages redundantly only a small portion of them, will soon be rejected by the golfer for greener pastures, since, by concentrating on the limited range of skills which it does challenge, he will soon exhaust the course's ability to exercise and refine them. At best, therefore, a golf course ought to present as few potential situations as possible in which generally redundant inductions will be

accurate; or, simply, it ought at best

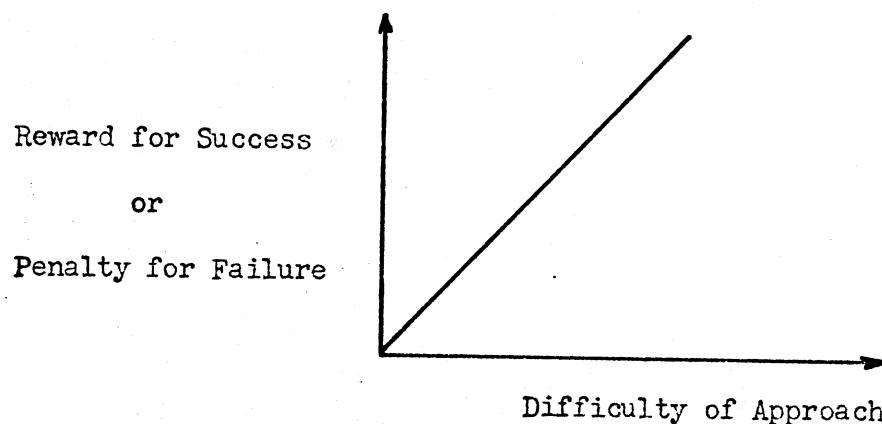
(b) to reward few redundant inductions.

The reader will notice that conditions (a) and (b) say nothing of the relative skillfulness of the golfers playing the course, holding true for highly-skillful players as well as beginners. At best, therefore, a golf course ought

(c) to satisfy conditions (a) and (b) without respect to the relative skillfulness of the players involved.

(There is, of course, a threshold-level of skillfulness below which one could hardly fall and still be said to be 'playing golf' (i.e. planning one's moves); no golf course can be expected to meet this challenge better than another course, for, indeed, there is no challenge. Similarly, there is an upper threshold-level of skillfulness against which the defences of any golf course would be ineffective. But given a workable range of skills, condition (c) holds.)

A great golf course, therefore, differs from a mediocre one to the extent that it more efficiently succeeds in satisfying conditions (a), (b), and (c). But structurally what does this entail? It entails, in particular, that each hole be strategically approachable in a variety of ways, ranging from safe to dangerous, with the rewards for success and the penalties for failure varying directly with the degree of danger; i.e.,



For example, the constructive musings of a golf course architect designing a hole might go as follows: 'If I place the green directly behind the pond, it leaves the moderately skillful golfer no choice but to go for a long shot over the water. If, on the other hand, I half-hide the green to the right, back of the pond, it leaves the left open for the moderately skillful golfer to avoid the water. But then, since the left route is no further from the green than the pond route, the better golfer will not risk the water either, for there is nothing in his favor if he does so. So let's dogleg the green back to the right behind the pond, forcing the better golfer to shoot over the pond or lose a stroke. But that puts the green in Fairway 7. Well, then, why not place a grove of trees to the left of ...'. Etc.

Condition (b) entails, furthermore, that the structure of the course be such that the inductive positions in which a player finds himself differ sufficiently from preceding ones that some recently unexercised facet of his skill must be engaged to enable him to make the induction accurately. Condition (b), hence, has structural implications both for the design of a single hole and for the design of

the course as a whole. For example, the musings of a course architect plotting the general layout of a course might go as follows: 'If I make Number 4 a short hole, stopping before the trees, that makes two short holes in a row with little difference between them. On the other hand, if I continue Number 4 straight beyond the trees but stop at the road, the moderately skillful golfer will have to hit two consecutive long irons on similar terrain. If, instead, I dogleg Number 4 to the left shortly beyond the trees, it leaves the moderately skillful golfer with a short iron for his second shot, and gives the better golfer a direct chance of hooking over the trees with a long wood. But, damn, Number 5 was to have been a short dogleg left; and if we change 5, then the whole structure of 6 thru 10 will be messed-up. What if we elevate the Number 4 tee, and ... '. Etc.

(I have been discussing the structural differences between a great golf course and a mediocre one. But this is not quite the distinction between a well-designed golf course and ^apoorly-designed one (though a great course is well-designed and a mediocre course is poorly designed). A golf course might well satisfy conditions (a) and (b) but not (c); that is, it might reward accurate non-redundant inductions and penalize inaccurate ones for a very skillful golfer, but be so difficult to play as to be unworthy of the efforts of a moderately skillful golfer. Such a course, though finely designed, would lack the general attractiveness which I consider part of the meaning of the word 'great'.)

Since conditions (a), (b), and (c) say nothing of the depth of

the mechanisms of the organism involved in making the inductions, nor of the particular nature of the skills involved, they are directly applicable to works of art. A great work of art, thus, differs structurally from a mediocre one to the extent that it succeeds more efficiently in rewarding non-redundant accurate inductions and penalizing inaccurate ones, without respect to the relative skillfulness of the players involved.

A player of a work of art, of course, is usually involved in less strenuous physiological activity than a golfer, often that of perceptual attention alone; but the skill involved remains a function of concentration and memory. Success in the refinement of a golfing skill is quickly and clearly indicated by a decreasing score; success in the refinement of a basic habitual response of an organism, in contrast, often has no external manifestation (and sometimes no internal manifestation, either).

But given the greater depth and pervasiveness of the habitual reactions being exercised by a work of art, the structural conditions necessary for a great work of art are identical to those of a great golf course, though the results are subtler. Hence, the great artist is not (and ought not to think of himself as) performing a function qualitatively distinct from that of the great golf course architect. Each operates with different raw materials, different tools with which to achieve the final object, and often an incidentally different awareness of their function; but, given the degree of difference in the depth of the relevant responses to be exercised and the consequent

degree of difference in subtlety of approach, the structural conditions for their success are identical.

Chapter II, Section II:

Part II: The 19th Hole

Let us now examine conditions (a), (b), and (c) to determine with greater precision what they entail structurally in a work of art.

Firstly, what constitutes a reward for an accurate induction, or a penalty for an inaccurate induction, to an organism perceiving a work of art? On a golf course, to reward an accurate induction is to put the player in such a position that he can proceed along the course with less physiological effort than had he guessed inaccurately; to penalize an inaccurate induction, on the contrary, is to put the player in such a position that he can proceed along the course only with greater physiological effort than had he guessed accurately. Similarly, to reward an accurate induction by a player perceiving a work of art is to put the player in such a position that he can proceed to the next element of the work with less physiological effort than had he guessed inaccurately; to penalize an inaccurate induction, on the contrary, is to put the player in such a position that he can proceed to the next element of the work only with a greater physiological effort than had he guessed accurately. On a golf course, a penalty requires the player to take a larger number of strokes or play shots which are more difficult than those which would have been necessary had he guessed accurately in the first place. In perceiving a work of art, a penalty requires the player to shift from the habitual response which he had erroneously thought adequate to the induction to a more refined response which will enable him, upon reassessment of that which he has

already experienced, to proceed to the next element of the work with understanding.

To the player of a work of art, therefore, being in a position of perfect understanding is equivalent inductively to a golfer having a 'perfect lie': i.e., each indicates that the player is in inductive control of the game and faces no immediate threat to his inductive well-being. In extreme contrast, being in a position of complete bewilderment is equivalent to having an 'impossible lie': i.e., each indicates that the player has lost inductive control of the game and faces imminent danger of being unable to continue at all. Between the two extremes, of course, lies a continuum of intermediate positions requiring progressively greater physiological effort on the player's part if he is to finish the game at par level.¹

Condition (a) structurally entails, therefore, that the successive elements of a work of art be understandable in terms of the preceding elements, and be relevant to the understanding of the succeeding ones. Initially, the wise player of a work of art makes inductions which are very general, for he has been given little information upon

1. Note that I have not disregarded here my earlier contention that (eg.) a highly skillful golfer in the course of a round may, and usually does, expend greater physiological effort than a mediocre player, for the conscious goals which he sets himself usually differ from those chosen by a wise mediocre player and require for their achievement the accurate functioning of many skills not shared by the latter.

The more highly skillful player would exert less effort than a mediocre player only with respect to the achievement of a goal common to both players (whether it be (eg.) the successful completion of a particular shot on a golf course, or the achievement of a particular level of understanding of a work of art).

which to base them. As his experience of the work progresses, however, and the amount of given information increases, the inductions, if accurate, ought to become more exact. Condition (a), therefore, entails that, at best, each experienced element of a work of art

(a') be consistent with those habitual inductions accurately based upon the previously experienced elements; and

(b') be necessary, at some succeeding point in the experience of the work, if an accurate habitual induction is to be made.

If the reader will interpret the phrase 'be necessary' in (b') to mean 'cannot be disregarded', he will note that condition (b) has been subsumed already under (b'). A redundant element in a work of art, although consistent with those inductions accurately based upon the previous information, could not satisfy (b'), for it could add no information at any succeeding point in the work not contained already in that earlier element of the work with respect to which it is redundant. (Note carefully, however, that redundancy is not necessarily equivalent to identity. Except for position, two elements of a work of art may be identical and yet each be necessary to the accuracy of a later induction dependent upon the number of instances of this element. And the second of two non-identical elements of a work of art may indeed be redundant, if it adds nothing to the accuracy of any later induction to which the first is not relevant, and yet adds nothing beyond the accuracy already provided by the first to those later inductions to

which the first is relevant.)

Conditions (a) and (b), therefore, entail (a') and (b') when structurally applied to works of art.

Consider thirdly condition (c). The player of a work of art, at any moment of his experience, is constructing an inductive proof; i.e. he is making an induction based upon his memory of the previous information and the subtlety of his habitual responses. The latter is not a function of the work of art, but rather of the player's skillfulness. The former is a function of both, however, for given the particular degree of skillfulness in concentration and memory of the player, the amount of previous information he has retained is a function of its mode of presentation (i.e. a function of the structure of the work of art).

As a simple illustration (noting that valid deductions may be construed as accurate inductions having probability equal to one), imagine scanning a valid deductive proof of m lines, the m^{th} of which I shall call 'the conclusion'. Since the proof is valid, the first $m-1$ lines of the proof, conjoined in any order, imply the conclusion. But, as the reader is no doubt aware, the ease with which one could later recall the proof depends essentially upon the order in which the first $m-1$ elements were presented and experienced. If the order of the first $m-1$ elements were irrespective of their mutual implications, even the most intuitively skillful deductive logician would be hard-pressed to recall the proof, for its inference structure would be hidden.. If, in contrast, they were ordered such that, for each ele-

ment e_i of the $m-1$,

- (1) if e_i is implied by the conjunction of no fewer than i other elements of the $m-1$, then e_i succeeds each of the i in the order, immediately succeeding one of them; and
- (2) for each e_j and e_k of the i elements in (1), if the fewest number of elements of the $m-1$ which, conjoined, imply e_j is greater than the fewest number of elements of the $m-1$ which, conjoined, imply e_k , then e_j precedes e_k in the order;

a deductive logician of given skill could most easily remember the proof, since its inference structure would be most transparent.

The satisfaction of conditions (1) and (2), therefore, would entail roughly that a valid deductive proof be presented as simply as possible, given its elements (i.e. that it enable the player to experience the proof with the minimum of concentration, given his skillfulness). Structurally, it would entail that each element of a valid deductive proof occur no sooner than needed in the proof. Hence, this structural condition would be sufficient to guarantee the satisfaction of condition (c) for valid deductive proofs.

Since valid deductions are a particular sort of accurate induction, however, it may be shown by a strictly analogous though more general argument that a structural condition sufficient to guarantee the satisfaction of condition (c) for accurate inductive proofs is

that each element of the proof occur no sooner than needed in the proof (i.e. that each element of the proof be experienced by the player at that temporal moment in his experience of the proof which occurs as shortly as possible before the moment at which the element is logically needed in his experience of the proof). In particular, therefore, with respect to the elements of a work of art satisfying conditions (a') and (b'), it is structurally sufficient to guarantee the satisfaction of condition (c) that, at best, each element of the work of art

(c') be experienced as shortly as possible before the
'succeeding point' of (b').

The greatness of a work of art, therefore, is not to be found in the complexity of its inference structure, for this would mitigate against its being a valuable exercise for those of lesser skill. Rather, it is to be found in the subtlety and richness of the structured elements (i.e. in their ability to reinforce simultaneously accurate inductions of varying degrees of skill).

A further consequence of condition (c') is that a complex inductive puzzle is not equivalent to a work of art. (Eg., Being coarsely particular, all great novels are not detective stories.) Puzzles are constructed by rearranging the premises of a proof to make its inference structure more complex. But notice: an organism facing a puzzling situation has difficulty in deciding how to proceed; i.e., he is facing a situation to which, by definition, his most pervasive habitual responses, those most deeply reinforced thru frequent and successful use,

are inapplicable. (Otherwise it would not be puzzling.) Since, however, it is in the exercise of these latter responses that the organism's experience of a work of art consists, it follows that puzzles are gist for the conscious intellectual probe, not the unconscious habitual response. Or, put another way, a puzzle, once solved, ceases to puzzle, for once the patterns of inference have been laid bare, the initial source of its fascination has vanished. But prior familiarity with the inference patterns of a work of art not only does not lessen its fascination, but indeed increases it, for a player's conscious expectations serve to delineate more accurately the goals of the habitual responses called forth, and hence make possible their subsequent increased refinement and sensitivity. Indeed, the presence of puzzles within a work of art may well serve to defeat its purpose by making it impossible for the player ever to delineate with great accuracy the goals of the habitual responses called forth, hence limiting their possible refinement and increased sensitivity. (Consider again the example of a golf course, in which the ultimate testing of a golfer's skill comes when he is thoroughly familiar with the hazards of the course, and not when the hazards remain unexpected. A course consisting of shifting and cleverly hidden hazards with devious clues to their presence, though perhaps a fascinating puzzle, would be of slight use to the highly skillful golfer wishing to exercise and refine his skills.) Condition (c'), hence, quite properly entails that the structural conditions necessary for a complex puzzle are contrary to those necessary for a work of art.

In summary, then, the three general structural conditions necessary for a great work of art are that, at best, each experienced element of the work

- (a') be consistent with those habitual inductions accurately based upon the previously experienced elements;
- (b') be necessary, at some succeeding point in the experience of the work, if an accurate habitual induction is to be made; and
- (c') be experienced as shortly as possible before the 'succeeding point' of (b').

I shall now discuss the structural implications of the above three conditions with respect to the traditional descriptive canons of narrative and non-narrative works of art.

Chapter II, Section III:

Narrative Works of Art

Part I: Traditional Narrative Structures

Artists have been at work for thousands of years. It is not surprising, therefore, that descriptive canons of artistic design have been distilled from their products and instilled in our culture. It is surprising, however, that though these canons have proven to be descriptively true, they have proven in larger part to be compositionally useless.

The reasons are twofold: (1) most of the canons, though true of the work of art as a finished object, entail nothing with respect to the function of each element of the object in the causing and sustaining of the aesthetic experience; hence, they are of no use to the artist who, of necessity, begins with the task of finding and combining elements for which no finished object as yet exists; and (2), most of the remaining canons mistake symptoms of the aesthetic experience (eg. emotional reactions, etc.) for the experience itself, and thus suggest compositional procedures which structurally apply to the symptoms but not to the elements of the work which cause them; hence, they are of no use to the artist who, of necessity, begins with elements and not the symptoms which they, in combination, may produce.

Since the traditional canons generally have proven descriptively true of finished works of art, however, it is necessary now for me to show that the results of applying (a'), (b'), and (c') in practice would lead to the construction of objects which, in general, conform to the canons. It would be fruitless, of course, to be exhaustive; but

to the core of the task I now turn. (For brevity's sake, I shall hereafter refer to conditions (a'), (b'), and (c') as 'the Trinity'.)

The members of the Trinity are applicable to any work of art. They appear to differ, therefore, from the traditional descriptive canons of the various arts, most of which (from a cursory glance at their unique vocabularies) appear to differ qualitatively. When the latter are interpreted with respect to the Trinity, however, they are found to differ quantitatively if at all (i.e. to differ with respect to greater or lesser subtlety in their range of application). Although I shall begin, therefore, by discussing the fundamental categories by which narrative works of art have traditionally been described, the reader ought to keep in mind that the distinctions are due to structural features which, as we shall note later, are largely common to non-narrative works of art as well.

The experience of a narrative work of art has been described with general accuracy for centuries as consisting of three parts:

- (1) an Exposition;
- (2) a Development; and
- (3) an Obligatory Scene-Climax-Denouement;

in roughly that order. Let us consider each of the parts individually.

For example, what is it that is being exposed in part (1), the Exposition, of a drama? The expectations of a playgoer at any moment of his experience of a drama are inductively determined by his current understanding of the motivations of the characters and their existential (i.e. situational) context. The Exposition, therefore, is that

succeeding members. And, moreover, it is a principal character (indeed, the principal character, the protagonist), a theme upon whose recognition and increased understanding by the player depend his accurate understanding of the bulk of the remainder of the elements of the drama. (In contrast, a secondary character is a theme upon whose recognition and increased understanding by the playgoer depend his accurate understanding of little of the remainder of the elements of the drama, given that its momentary effect upon the principal characters has been accurately noted. Hence, given the restrictions on space and time in a theatrical experience, secondary characters are usually stereotypical, i.e., dramatic themes whose inductive effect is immediately understandable by the playgoer without time-consuming exposition. Such stereotyping is less necessary in a feature-length novel, for example, where the player's time of experience is must less constricted.)

Imagine, then, a narrative work of art based upon a set of principal characters and conforming to the Trinity. Since an accurate understanding of the principal characters is necessary to an accurate understanding of the bulk of the remaining elements of the work (for, otherwise, they would not be principal characters), satisfaction of the Trinity would require that the principal characters be experienced and adequately understood by the player of the work of art prior to his experience of the bulk of the remaining elements of the work. Hence, a narrative work of art conforming to the Trinity would entail structurally that the earliest part of the player's experience be an exposition of the principal characters, in accordance with the traditional descriptive canons.

If the reader, with me, will now skip momentarily over the traditional second part of the drama, the Development, I should like firstly to discuss the third part, the Obligatory scene-Climax, Denouement. Does the structure of a work of art satisfying the Trinity entail that a player's experience of the work contain a climax in the traditional sense?

Since condition (b') entails that each experienced element of a work of art be necessary to the player at some succeeding point in his experience of the work if he is to make an accurate induction, it follows, assuming that a work of art is finite, that at least one element of the work must be an inductively necessary condition of each of the preceding elements: namely, the last element experienced. But since condition (c') entails that less complex inductions be experienced before more complex ones, in effect it structurally prohibits anti-climaxes. The satisfaction of the Trinity by any work of art, therefore, entails structurally that a player's experience of the work, given inductive accuracy, will contain a single climax at or near the end of his experience. Hence, in particular, satisfaction of the Trinity by a narrative work of art would be a sufficient condition for its having a climax in the sense and order of the traditional descriptive canons.

Returning now to the traditional second part of the drama, the Development, the question becomes: Since works of art satisfying the Trinity entail that the experience of the player begin with an exposition of the principal themes and conclude with a climax, and hence entail that the middle part of his experience develop inductively

from one to the other, does the structure of the middle part of his experience conform to the traditional canons of the Development?

What is it that is developed in the second part, the Development, of a drama? The usual answer is 'the plot'. What, then, is 'the plot', and how and why is it developed?

As usually understood, the word 'plot' refers to an outline of the dramatic action. The dramatic action at any moment in a playgoer's experience is inductively determined by his current understanding of the principal characters and their existential context. Imagine, now, a drama whose principal themes are characters and which satisfies the Trinity. Since the bulk of the elements of the drama are determined by the activities of the principal characters (or they wouldn't be principal characters), their existential context at any moment in a playgoer's experience of the drama has been largely determined by their previous activities. Since, therefore, the members of the Trinity entail that the structural development of the drama follow the development of the playgoer's understanding of the principal characters, it follows that the structure of a drama whose principal themes are characters and which satisfies the Trinity will also conform to the traditional canons of the plot.

In general, therefore, narrative works of art structured in accordance with the Trinity would conform to the traditional tri-partite descriptive canons of narrative art.

Chapter II, Section III:

Narrative Works of Art

Part II: Tragedy and Comedy

Besides having a tri-partite structure, some great narrative works of art have proven to be accurately classifiable under the traditional descriptive categories of Tragedy and Comedy. What, then, are the structural implications of the Trinity for these works?

I indicated above that a principal character of a drama was a principal theme, because the playgoer's accurate understanding of the bulk of the remaining elements of the drama depended upon his recognition and increased understanding of the character. But notice: I did not say that the principal themes of a drama had to be principal characters (or even characters in any sense). What, then, would a great drama be like structurally whose principal themes were not principal characters?

If we reconsider in greater detail what a great drama would be like structurally whose principal theme is a single principal character, the protagonist, the contrast will serve us well. It was noted above that the climax of such a drama would be felt to be inevitable by the playgoer whose successive inductions had been accurately founded upon his increasing understanding of the protagonist. But condition (c') entails that the actions of the protagonist must be simple enough to exercise the deepest habitual responses of even the most inductively coarse playgoer, while simultaneously being subtle enough to exercise the deepest habitual responses of even the most refined playgoer. If

the reader will try to imagine a sequence of actions by a protagonist satisfying both extremes of inductive sophistication, he will readily note that the candidates are scarce rather than plentiful.

By common consent, however, there is a single sort of sequence of activities which has proven to satisfy both extremes most naturally: namely, a sequence of activities culminating in the imminent death of the protagonist. Since death and its causes are part of the environment of every playgoer, such a sequence provides ample material upon which to structure the exercise of the deepest responses of the most inductively coarse playgoer. On the other hand, since death is the ultimate defeat for an organism, an organism faced with death has no other choice but to summon every inductive resource of its being to the battle; hence, the successive actions of a protagonist culminating in his death provide the richest material upon which to structure the exercise of the deepest responses of the most inductively refined playgoer.

Notice, furthermore, that since, by the meaning of 'protagonist', the climax of the drama must be inductively determined by the previous actions of the protagonist, this entails that the protagonist must be, in effect, the principal cause of his own death. But since death is the ultimate inductive defeat for an organism, the self-caused death of the protagonist entails that his character contain a crucial inductive flaw; i.e. that there exist in his character some hinderance (whether due to pride (eg. Oedipus Rex), moral indeciciveness (eg. Hamlet), etc.) which prevents him from confronting with inductive accu-

racy some aspect of his experience crucial to his survival.

The above argument could be expanded, but needlessly, for the consequent should now be apparent to the reader: namely, a great drama whose principal theme is a principal character conforms most naturally to the traditional descriptive and narrative canons of the Tragedy.

Returning now to the question 'What would a great drama be like structurally whose principal themes were not principal characters?', the reader will recall that I earlier indicated that a playgoer's understanding of the action at any moment in his experience of a great work of art is inductively determined by his current understanding of the characters and their existential context; and that, if the principal themes of the drama are principal characters, then the existential context is inductively determined by them.

The existential context at any moment in a playgoer's experience of a drama is that dramatic situation in which the characters exist, and against which the playgoer judges the inductive credibility of their actions. But, conversely, the credibility of the existential context is measured against the actions of the characters. Neither can be defined apart from the other, and needn't be (any more than one can define the phrase 'subject of a sentence' without first understanding to some degree the meaning of the phrase 'predicate of a sentence', since, to be a subject, a noun must have a predicate). What must be noted, however, is that in some dramas one aspect may take precedence over the other, in the sense that it may come to serve as the playgoer's principal measuring stick of credibility rather than being measured it-

itself (i.e. it may come to be assumed to be more simply credible, and hence the remaining aspect is measured in terms of it rather than the converse). In a drama having a protagonist, the protagonist is the dramatic object whose transformations are constantly being measured and re-assessed, for the playgoer's understanding of the bulk of the remaining elements of the drama depends upon his accurate inductive assessment of them. If the protagonist is a character, then the credibility and direction of its activities are measured against its existential context, which must be assumed to be credible; if the protagonist is an existential context, then the credible course of its development is measured against the activities of the characters, which must be assumed to be credible. It is necessary, therefore, in a drama containing a protagonist, that the credibility of the remaining aspect be capable of being easily assumed by the playgoer (i.e. that the remaining aspect wear its credibility on its sleeve, as it were), for otherwise the tool of measurement necessary to the accurate understanding of the drama by the playgoer would be absent. (For this reason, therefore, the basic existential contexts of the great Tragedies are notably simple and obvious; i.e. structurally melodramatic.)

Imagine, then, in contrast to the structure of the Tragedy, a drama in which the principal theme is a single existential context rather than a principal character; i.e. imagine that the protagonist of a drama, rather than being a character, is a developing situation. Given that the drama were great, the playgoer's understanding of the action at any moment in his experience of the drama would then be inductively

determined by his previous understanding of the situation, with his current understanding of the actions of the characters being inductively determined by it also. But this entails, simply put, that the characters in the drama be stereotypical. For if the playgoer's understanding of the action at any moment of his experience of the drama inductively depends solely upon his understanding of this situation, it cannot depend upon a refinement of his understanding of any character; hence, his understanding of the motivations of each character must have remained largely unchanged from the beginning of his experience of the drama. But, since the accuracy of his measurement of the credibility and direction of the developing protagonal situation depends upon the assumed credibility of the characters, the range of their motivations and hence possible reactions to the developing situation must be extremely narrow and simple, and obvious from the beginning of the drama; i.e. they must be stereotypical.

But condition (c') entails that the transformations of the protagonist (i.e. the situation) must be simple enough to exercise the deepest habitual responses of even the most inductively coarse playgoer, while simultaneously being subtle enough to exercise the deepest habitual responses of even the most refined playgoer. If the reader will try to imagine a sequence of transformations of a protagonal situation satisfying both extremes of inductive sophistication, he will readily note again that the candidates are scarce rather than plentiful.

By common consent, however, there is a single sort of sequence of transformations of a protagonal situation which has proven to satisfy

both extremes most naturally: namely, a sequence of transformations of the situation culminating in the disintegration of the situation. Just as the disintegration (i.e. death) of a protagonal character indicates that the drama has reached a point at which the protagonist cannot continue to act in that situation with inductive credibility, as judged by the playgoer against the scale of credibility assumed in the dramatic situation, without either losing his measured credibility as a character or else destroying the assumed credibility of the situation, so the disintegration of a protagonal situation indicates that the drama has reached a point at which the situation can no longer develop in a credible way, as judged by the playgoer against the scale of credibility assumed in the activities of the stereotypical characters, without either losing its measured credibility as a viable existential context or else destroying the assumed credibility of the stereotypical characters.

Since the range of motivations, and hence activities, of each stereotypical character is extremely narrow, in a complex situation such characters essentially seem to react rather than act. The playgoer's assumption of the credibility of such a character would vanish, hence, should its identity as an essentially reacting being disappear. How is it possible, therefore, for the successive reactions of such characters within a developing dramatic situation to culminate in a point of inductive frustration for each of them, while being assumed in each instance by the playgoer to have been founded upon an accurate induction? The answer is that, in some situations, an induction may

be inappropriate though not inaccurate.

Imagine, for example, a simple minded dramatic character, having unknowingly wandered one night into a fireworks warehouse, striking a match thereby to see his watch and tell the time. Given his simple mindedness, the choice might well be the most accurate induction he could make; given the situation, however, his choice is singularly inappropriate. (Given, furthermore, that he is stereotypical, he can't learn much from the experience; though, given his continued existence as a character, he will surely escape relatively unscathed.)

The above argument could be expanded, and examples proliferated, but needlessly, for the consequent should now be apparent to the reader: namely, a great drama whose principal theme is a protagonal situation conforms most naturally to the traditional descriptive and narrative canons of the Comedy.

In a great work of narrative art, thus, Tragedy results from a structural emphasis upon characters as principal themes measured against melodramatic existential situations, while Comedy results from a structural emphasis upon existential contexts measured against the reactions of stereotypical characters. And since, by the meaning of 'protagonist', one cannot have two protagonists within a single work of art, one cannot have a great work of art having both a character and an existential context as protagonists; hence, one cannot have a great work of art which is simultaneously a Tragedy and a Comedy. (On the other hand, if one's characters are stereotypical and their existential context is melodramatic, one can have a proper Melodrama — which is not

a great work of art by traditional criteria, and indeed obviously cannot satisfy the Trinity.)

Chapter II, Section IV:Non-narrative Works of ArtPart I: The Sonata Form

I have been discussing the structural features of great narrative works of art (i.e. those great works of art whose principal themes are manifested in temporal order, and which, for inductive purposes, resemble human beings and their existential situations). I now wish to discuss the structural features of great non-narrative works of art (i.e. those great works of art whose principal themes, whether manifested in temporal order or not, do not, for inductive purposes, resemble human beings or their existential situations). Resemblance, of course, is a matter of degree, and I do not wish to deny that the inductive experience of many players of abstract works of art is conditioned by their having found resemblances to human events in the shapes, colors, and sounds. For clarity's sake, however, I shall assume for the remainder of this Section that the works of art of which I speak are strictly non-narrative, and hence that their principal structural features owe nothing to their anthropomorphisation by the player.

I shall begin by discussing those non-narrative works of art whose structural features depend principally upon the temporal ordering of the manifestations of their principal themes (eg. aural music and its visual analogue), for the traditional descriptive canons of these arts, where such exist, have much in common with the descriptive canons of narrative art previously discussed. I shall then discuss briefly those non-narrative works of art whose structural features depend

principally upon the non-temporal ordering of the manifestations of their principal themes, as exemplified in the art of painting. (I shall not discuss works of art whose elements require for their recognition that the player use sensory organs other than his eyes and his ears.)

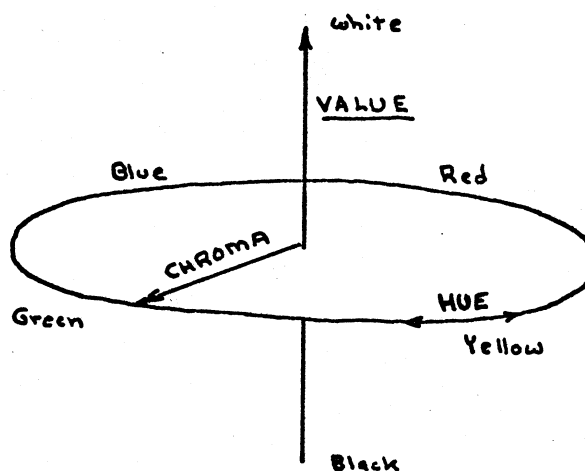
Imagine a temporally ordered theme each element of which is simultaneously ordered with respect to each other element in the theme by an intrinsic non-temporal relation. (Eg. Imagine a sequence of sounds each of which is not only later than, but also louder than, the previous sounds in the sequence.) This theme, then, possesses intrinsically the elements of a more general type of temporally ordered theme: namely, that theme which consists of the temporally ordered sequence of intervals between the temporally ordered elements of the former theme.

In this sense, the sounds or colors which characterize the elements of the themes of non-narrative though temporally ordered works of art possess an intrinsic order which provides the basis for an essential thematic dimension other than that due to the temporal order of the elements themselves. To understand the structural features of such works of art, therefore, it is necessary that the reader be familiar with the dimensions of these intrinsic orders. Although I shall not duplicate here what I have written elsewhere at length on this matter (Cameron, pp. 28-47), a brief resume of my results is necessary for the purposes of further discussion:

A. Colors:

As the work of Munsell, Ostwald, and others have shown, all

perceived colors can be taken as points in an ordered 3-dimensional color solid having value as its vertical dimension, chroma as its radial dimension, and hue as its circumferential dimension.

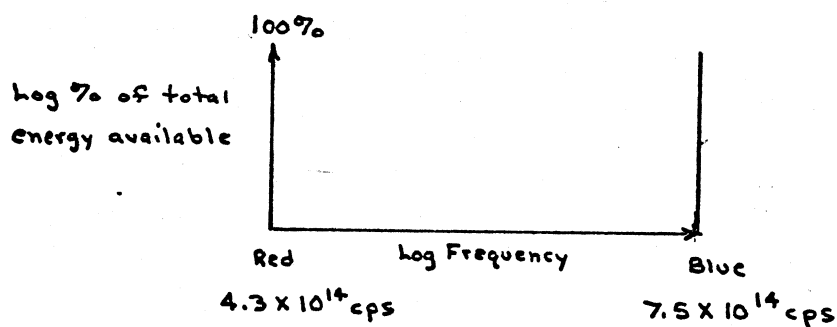


Hue is that quality of perceptual difference in color points which we usually designate by calling one 'red' rather than 'yellow' or 'green', etc. Chroma is that quality of perceptual difference in color points which we usually designate by calling one 'purer' or 'more saturated' than the other. (As is evident from the diagram, chroma is a measure of a color point's distance from the achromatic axis of the greys; i.e. a measure of its chromaticity or relative chromatic-ness.) Value is that quality of perceptual difference in color points which we usually designate by calling one 'brighter' or 'less dark' than the other (in Ostwald's phrase, a measure of how much 'black' or 'white' a color point contains).

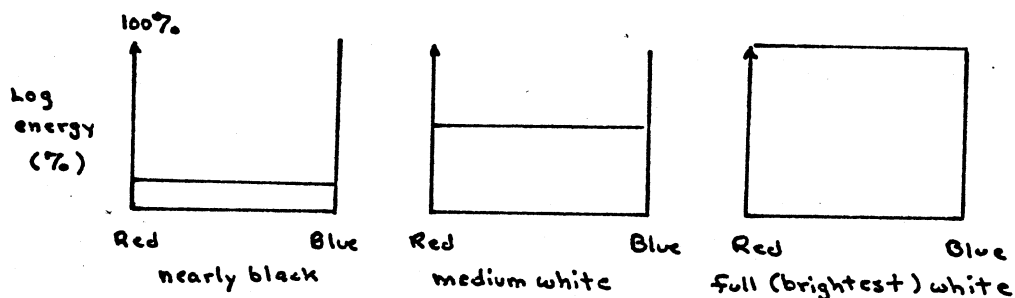
The perceptual color solid is intrinsically ordered in the following sense: The hues located diametric to each other in the solid are complementary in that, when mixed in proper proportions on a Maxwell color wheel rotating rapidly, they blend perceptually

to achromatic grey. Given any complementary pair of hues as measuring points, it is possible to determine perceptually whether or not two color points experienced at random occur on the same or opposite sides of the hue circle as divided by the complementary pair, to determine to which member of the complementary pair each color point is closest, and lastly, if both are on the same side, to determine which is closer to either member of the complementary pair. Similarly, given any plane perpendicular to the value axis as reference, it is possible to determine perceptually whether the value planes of two color points experienced at random are on the same or opposite sides of the value space as divided by the reference plane. Similarly, given any cylinder of points having equal chroma as reference, it is possible to determine perceptually whether the chroma cylinders of two color points experienced at random are on the same or opposite sides (i.e. inside or outside) of the chroma space as divided by the reference cylinder. (I am, of course, assuming that all differences given above do not fall beyond the threshold levels of perceptual discrimination.)

Assuming that the dimensions of the color solid are strictly in accordance with the Weber-Fechner law of sensation (which implies that perceptual differences are logarithmic functions of the differences in physical stimuli), the physical basis for the dimensions of the color solid is as follows: Imagine the usual diagram of the electro-magnetic spectrum for visible light (with the horizontal axis converted from the metric of wavelength to that of frequency).

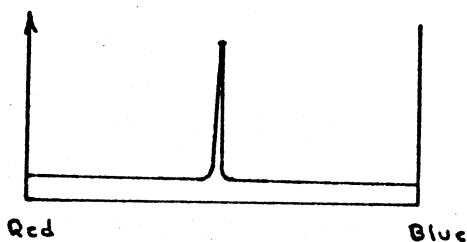


When statistically equal amounts of energy are present at all frequencies, the resulting light is seen as achromatic. Thus, diagrammatically, an energy line parallel to the frequency axis indicates an achromatic perceptual stimulus.

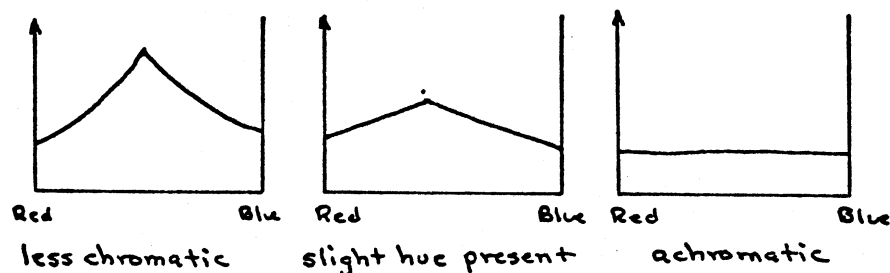


The relative height of an energy line parallel to the frequency axis, therefore, indicates roughly the same perceptual situation as is measured on the value scale of the perceptual color solid.

As the spectrum color at each frequency is, by definition, the most saturated color possible at that frequency, a single vertical energy line at any given frequency would represent a source of the most saturated color possible at that frequency.

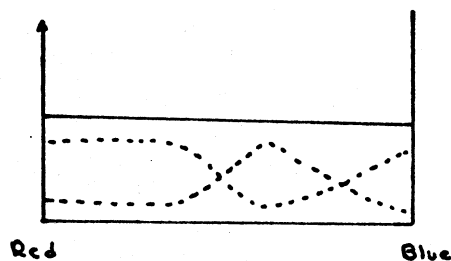


As the energy line which peaks in the vicinity of any given frequency becomes less vertical and more horizontal, the color at that frequency becomes correspondingly less chromatic (i.e. more achromatic).



Thus, the relative restrictedness of the range of peak energy indicates roughly the same perceptual situation as is measured on the chroma scale of the perceptual color solid.

The relative position of the energy peak (or combination of peaks) with respect to the frequency axis, of course, indicates roughly the same perceptual situation as is measured on the hue scale of the perceptual color solid, complementary hues being those whose energy lines, when added together, form a composite energy line parallel to the frequency axis indicating an achromatic perceptual stimulus.

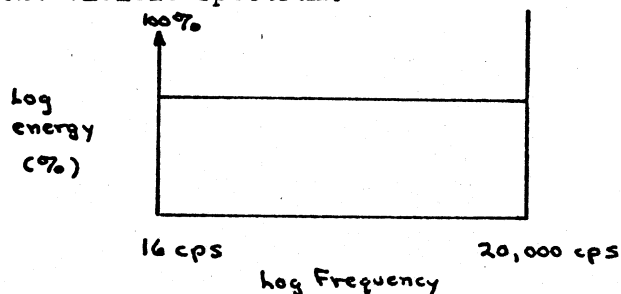


B. Sounds:

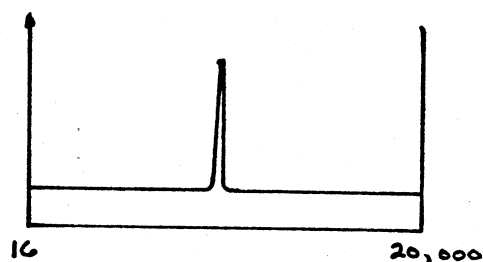
Assuming the Weber-Fechner law of sensation, all perceived sounds can also be taken as points in an ordered sound solid whose dimensions are logarithmic functions of the physical stimuli. Since the notion of a sound solid has never gained much headway as a compositional tool (for reasons which will become apparent below), I shall firstly describe the physical stimulus, and then secondly describe the perceptual correlates.

When the sound spectrum is contrasted with the spectrum of light, one obvious difference is apparent. The range of possible frequencies in the spectrum of light is a factor of less than 2 times the lowest perceivable frequency (7.5×10^{14} is equal to 4.3×10^{14} times less than 2), while that of sound is more than 2^{10} times the lowest perceivable frequency (20,000 is equal to 16 times more than 2^{10}). This difference is of crucial perceptual importance, and I shall speak more of it below. In other respects, however, the analysis of the physical stimuli are identical in both cases.

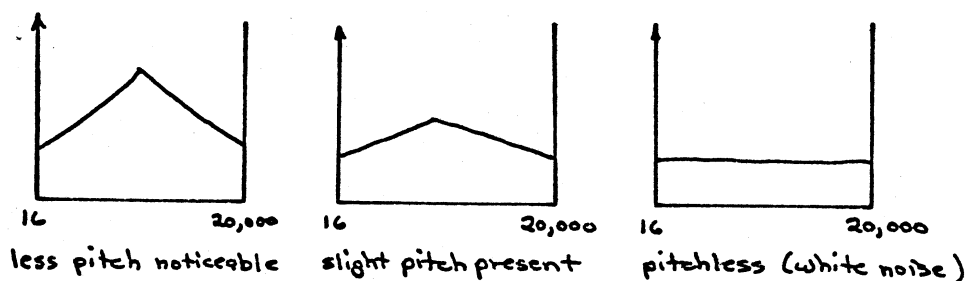
Whenever nearly equal amounts of energy are present statistically at all frequencies, the resulting sound is called 'white noise', and is formally equivalent to the achromatic (i.e. white) light of the visible spectrum.



As the spectrum sound at each frequency is, by definition, the purest possible sound at frequency, a single vertical energy line at any given frequency would represent a source of the purest sound possible at that frequency.



As the energy peak in the vicinity of any given frequency becomes less vertical and more horizontal, the sound at that frequency becomes correspondingly less pure (i.e. more like 'white noise').



The relative position of the energy peak (or combination of peaks) to the frequency axis is also determinable.

Thus, the physical stimulus of perceptual sound provides the basis for at least a three-fold metrical variety of perception strictly analogous to that which was specified for the color solid. And, indeed, corresponding to the scales of hue, value, and chroma in the color solid, there would exist the scales of pitch, loudness, and purity (or clarity of pitch) in the mythical sound solid.

Sound, however, possesses another scale in virtue of the range.

of its frequency spectrum which light cannot possess. It was mentioned earlier that the range between the lowest and highest frequencies of sound varies by a factor of more than 2^{10} . That is, as I proceed from the lowest to the highest frequency along the spectrum, I encounter at least 100 frequencies which are integral multiples of the lowest frequency (i.e. 16 c.p.s. times 2, times 3, times 4, etc.), many which are integral multiples of the second lowest frequency (i.e. 17 times 2, times 3, etc.), and so on. As the greater of any one of two of these frequencies contains the lesser as an integral factor, I ought not to be surprised that there exists in the physical stimulus of sound the functional basis for an additional perceptual metrical scale corresponding to nothing in the physical stimulus of light or the perceptual color solid. That is, since no frequency in the spectrum of light can be an integral factor of any other, but many of the frequencies in the spectrum of sound are integral factors of each other, it is not surprising that the latter are perceived to be functionally related in a manner in which the former could not be. Such sets of integrally-multiple frequencies, logarithmically construed, constitute the tonal families of pitch, and are the basis, as Helmholtz has shown, for the sensation of tonality (roughly, the 'C'-ness of a pitch, or its 'A#'-ness).

The hearing of sounds, consequently, unlike the seeing of colors, involves perceiving in a dimension of the sound space which has no analogous metrical scale in the color solid. The following correspondences hold for both perceptual colors and

sounds:

<u>Color solid</u>	<u>Sound solid</u>
Hue scale	Pitch scale
Value scale	Loudness scale
Chroma scale	Purity scale (i.e. clarity of pitch; lack of noise)

But for the tonality scale in the sound solid, there is no analogous scale in the color solid:

..... Tonality scale

The perceptual sound solid, therefore, is 4-dimensional, unlike the 3-dimensional color solid, and does not thus lend itself easily to conceptualization as a visual model. For this reason, and because musical composers until recently have been relatively uninterested in the 'purity' scale of perceptual sound (except as conceived pragmatically under the blurry notion of the respective 'timbres' of the various musical instruments), the task of manifesting (eg. on magnetic tape) a precise perceptual sound solid has justly proven of little interest. But the dimensions of the solid are perceptually accurate (though, as the reader will note, the dimensions of neither the color solid nor the sound solid are metrically independent; eg. to have hue, a color must have some degree of saturation, and to have tonality, a sound must have pitch).

The principal themes of great non-narrative though temporally ordered works of art, therefore, have thematic dimensions other than

the temporal which simultaneously exercise the inductive facilities of the player. Given that the essential ordering dimension of the principal themes is temporal, however, as in the musical arts, the reader ought to expect the traditional descriptive canons of these works of art to resemble closely the traditional canons of the narrative arts previously discussed. And, indeed, they do.

I shall take the traditional descriptive canons of the musical arts as my examples, since, until quite recently, the tools were unavailable for the composing of non-narrative though temporally ordered works of visual art, and hence no traditional descriptive canons have been derived as yet for the latter.

With respect to the possible sounds of the sound solid, the range of the existent traditional descriptive canons of musical art is greatly restricted, for, until recently, the only tools available to the composer of musical works for producing and sustaining sounds were instruments restricted to relatively pure pitches and a narrow range of timbres. Neglecting the latter, therefore, the elements of the principal themes of musical works of art (and the consequent intervalic themes dependent upon them) have been largely determined by their pitch and their loudness, the latter often involving an intricate pattern of attack and release due to the physical requirements of the act of playing the instrument producing the sound. Such themes, thus, have two primary inductive aspects, the melodic and the rhythmic, and have usually been called 'melodies'.

The human organism has the ability to perceive simultaneously two or more distinct sounds (within perceptual limits). Traditionally, two

or more distinct pitched sounds heard simultaneously have been called 'chords'. The player of a musical work of art, therefore, who experiences the successive elements of two or more melodies simultaneously, is experiencing a sequence of chords called a 'chord progression', whose tonal and rhythmic inductive aspects have traditionally been said to be harmonic.

Many compositional techniques have been derived for combining melodies into chord progressions which preserve the inductive characteristics of each. Such techniques are said to be contrapuntal, and, depending upon whether the melodic or harmonic aspect is emphasized, divide into techniques of melodic exposition (eg. the baroque techniques of melodic imitation, augmentation, diminution, inversion, etc.) or harmonic modulation (eg. the techniques of root inversion, augmented intervals, proscribing parallel octaves and fifths, etc.). In some instances, these techniques have been integrated into general patterns of contrapuntal structure which have proven useful in structuring large compositions (eg. the techniques of writing to a cantus firmus, the fugue, the madrigal, hymn cadences, etc.).

But specific techniques aside, given a player experiencing a great musical work of art, we have a situation precisely analogous to that discussed earlier for great narrative works of art: namely, the expectations of the player at any moment of his experience of the music are inductively determined by his current understanding of the melodies and their harmonic context. By arguments which strictly parallel those of pages 76 - 80 above, therefore, it follows that a musical work of art which conforms to the Trinity would entail structurally that

- (A) the earliest part of the player's experience of the music be an exposition of its principal themes;
- (B) the player's experience of the music, given inductive accuracy, contain a single climax at or near the end of his experience; and
- (C) the remainder of the player's experience be a development of his understanding of the principal themes.

The reader ought not to be surprised, therefore, that the most widely applicable traditional descriptive canon of musical works of art is the sonata form, which consists of

- (1) an Exposition;
- (2) a Development; and
- (3) a Recapitulation (with or without coda);

in that order; for the formal similarity of the sonata form to the basic tri-partite narrative form is not structurally accidental: both are a consequence of the primacy of the temporal order of the experienced elements and their satisfaction of the Trinity.¹

1. As the musically literate reader will note, most musical forms supposedly distinct from the sonata form (eg. variations on a theme, the classical suite, etc.) are most accurately construed as a development in greater detail of some aspect of it. A set of variations on a theme, for example, is an exercise in exposition. (And, as previously noted, the so-called contrapuntal forms (eg. fugue, madrigal, etc.) are not forms at all, but techniques of exposition and development.) It would, therefore, be most difficult to underestimate the general pervasiveness of the sonata form in its various aspects to musical form in general.

The Recapitulation of a musical work of art in sonata form may seem to imply redundancy, contrary to (b'), since it entails structurally that a former manifestation of a principal theme be repeated. But the reader ought to note that, whereas the playwright may safely assume that the bulk of the daily experiences of each playgoer have re-inforced deep habitual responses which he will find relevant to assessing accurately the natural events (i.e. themes) of the narrative, the composer of music can make no such assumption, for the abstract nature of his themes guarantees that the bulk of the daily experiences of each member of his audience have been irrelevant to reinforcing the deep habitual responses necessary to assess accurately the themes of the music. The structure of each piece of music, therefore, must build up from scratch, as it were, the context necessary for the development of habitual reactions to its themes, chiefly thru repetition of them. Hence, in a great musical work of art, as Tovey notes, even if the Recapitulation

"...is full and has a deceptive appearance of regularity...in reality it is anything but mechanical. It is just that kind of difference by which stereoscopic pictures produce the effect of binocular vision. In the light of the recapitulation the listener finds that those points which were superficial in the exposition have now become solid."

(Tovey, p. 215)

Simply put, the repetition of a theme in the Recapitulation is as necessary to the climax of a great piece of music in sonata form as its first manifestation.

Although the tri-partite canons of narrative art are mirrored in

the sonata form of the non-narrative but temporally ordered arts, the more general narrative categories of Tragedy and Comedy are not; and the reason has already been given in the preceding paragraph. The reader will recall that Tragedies and Comedies require that the player bring with him to his experience of the narrative work of art a set of relevant habitual responses deeply conditioned by his past experiences of everyday life, so that in the former case he may be free to focus his attention on successively developing his understanding of the principal characters while accurately assessing their successive existential situations with little effort, and in the latter case be free to focus his attention on successively developing his understanding of the principal existential situations while accurately assessing the stereotypical reactions of the characters to them with little effort. Since, however, a great work of musical art cannot presuppose such a set of relevant habitual responses, but must rather contextually develop habitual responses both to its melodies and its harmonic themes, it is not structurally free to concentrate on either for a sustained length of time. (Or, put another way, the context of habitual responses built up within a musical work of art is so thematically integrated and fragile, in comparison to the thematic strength of narrative habitual responses, that sustained structural concentration on either aspect would weaken the other so much that its habitual context would be destroyed.) The reason, therefore, why there have been (eg.) no great comic musical works of art is not structurally accidental, but rather a consequence of their intrinsic abstractness. (Note that, since I am

speaking only of non-representational, and hence non-programmatical, musical works of art, I am not asserting either that there exist no great programmatical musical works of art which are comic (consider, on the contrary, Mozart's The Magic Flute, Dukas's The Sorcerer's Apprentice, or the Frère Jacques theme in Mahler's 1st Symphony), or that non-representational musical works of art cannot contain comical elements thru devices of orchestration, dynamics, etc. (consider, on the contrary, the namesake chord in Haydn's Surprise Symphony, or any rapid duet between a tuba and a bassoon). Rather, I am asserting that no great non-representational musical work of art could be consistently comical (and, hence, comic), for a consistently comical non-representational musical work of art could be achieved only by vitiating precisely those structural conditions necessary to its greatness, as argued above.)

I mentioned above that there are no traditional descriptive canons of non-narrative though temporally ordered visual works of art, because the inception of their production has been too recent. To the extent that such works have been composed, however, they seem most naturally to conform to the general canons of musical structure, as the reader might expect from the primacy of their temporal ordering. Such works of art, however, being visual, differ from musical works of art in the non-temporal dimensions of their ordering. The order dimensions for colors were given earlier, but more must be said, since an essential order dimension of a visual work of art has not been discussed: namely, the spatial. To make the point with least complication, I turn now to

a brief discussion of the structure of those non-narrative visual works of art whose elements are not temporally differentiated, as exemplified in the art of painting.

Chapter II, Section IV:

Non-narrative Works of Art

Part II: Paintings

Paintings have been distinguished traditionally

- (a) by the sort of physical objects depicted (eg. landscape, seascape, still life, madonna, portrait, etc.);
- (b) by the geographical location at which they, or their stylistic predecessors, were produced (eg. of the Dutch school, of the German school, etc.);
- (c) by the emotional or conceptual reactions caused by them or experienced by the artists painting them, by gross structural features, or both (eg. impressionistic, expressionistic, dadaist, surrealist, cubistic, rococco, pointillistic, etc.); and, of course,
- (d) by the name of the painter (eg. a Van Gogh, a Picasso, etc.).

But nowhere in our culture do there exist generally applicable traditional descriptive canons of the structures of great paintings, akin (eg.) to the sonata form in music. It is difficult to say of two great paintings, for example, that they have an essential structural identity in a sense visually analogous to that in which it can truthfully be said of Oedipus Rex and Hamlet, or of any two pianoforte sonatas by Beethoven and Schubert, that they have an essential structural identity; for our culture, and its languages, lack the categories nec-

essary to making such an assertion. The reason our culture lacks such categories, I suspect, is because, unlike the works previously discussed in which there was a predominant single structural dimension, the temporal, there is no single predominant dimension of our experience upon which the effective structure of paintings depend; rather, paintings are essentially multi-dimensional in their effect. Since our languages, the tools by which our culture transmits its useful categories, are themselves discursive, however, it is extremely difficult to describe therein even the simplest multi-dimensional forms with ease and yet precision. But the structure of even the simplest of our great paintings involves forms of vastly greater complexity. Due to the inefficiency of our languages in describing such complex forms to any useful effect, therefore, such descriptive categories have not arisen.

I cannot argue, therefore, as in previous discussions, that paintings which conform to the Trinity will conform also to the traditional descriptive canons of the art, since the latter do not exist. Rather, I shall attempt to indicate briefly and in a general way how a painting can be said to conform to the Trinity, and hence in what sense, it seems to me, the experience of a great painting is an inductive exercise comparable to those discussed above. Again: I shall not argue for the accuracy of the following observations and prescriptions in any way. My confidence in their usefulness arises solely from my own experience in composing and perceiving visual works of art by myself and others.

Simply put, the experiencing of a painting, like that of a narrative or musical work of art, consists in the experiencing of an expo-

sition and development of principal themes, and a climax. The difference is that any two of the elements of the former can be experienced simultaneously, while most of the elements of the latter can be experienced only successively.

The visual field of a human perceiver is bounded by the limits of his peripheral vision, and consists at each moment of a continuous 2-dimensional spatial distribution of color points (taking the latter to be the smallest colored spatial areas of that field which fall within the limits of spatial discrimination of the organism).¹ Besides being spatially distinct, however, two color points, as previously indicated, can also differ in each of three colormetric dimensions: hue, value, and chroma. The visual perceptual skills of a human perceiver, therefore, encompass his ability to compare accurately color points in virtue of their metrical differences in each of the above dimensions.

Imagine now a stationary visual field (i.e. a visual field in which the color points do not change their relative spatial or colormetrical positions). It seems to me, without further ado, that each color point in the visual field is most accurately perceived to be spatially surrounded by a color field which is such that, were a second color point to be introduced into that field, an attractive force would be exerted on the latter along the line joining the two which is inversely proportional to the product of their colormetrical differences and

1. I assume that the human organism's perception of spatial depth is a physiological construct based upon 2-dimensional visual data.

to the square of their spatial distance apart¹ (assuming that the two color points, if colormetrically identical, have the product of their colormetrical differences defined to be $= 1$). There is a single spatial point at which the vector potential of the field is strongest with respect to any other color point: namely, that spatial point occupied by the color point itself. (The analogy to the Newtonian model of gravitational forces should be apparent to the reader.)

A painting is a 2-dimensional stationary continuous subspace of a visual field. As such, it consists of a 2-dimensional stationary continuous spatial distribution of color points, each of which is most accurately perceived to be spatially surrounded by a color field. The perceiver may concentrate on these color points singly or, by shifting the focus and increasing the lateral movements of his eyes, in groups. Groups of color points, however, are themselves most accurately perceived to be spatially surrounded by a color field which is the vector sum of the color fields surrounding its members. And since each group is finite, there is a single spatial point at which the vector potential of the field is strongest with respect to any color point. A painting, thus, being a finite group of color points, is surrounded by a complex color field which has a single spatial point at which the vector potential of the field is strongest. The latter is the climax of the painting.

A player experiencing a painting is searching inductively for the

1. If the reader should wish to know the arguments I have put forward at length elsewhere for this particular proposition, he ought to see Cameron, pp. 73-88 .

climax. Or, more precisely, a player experiencing a painting is engaged in the task of determining the position of the climax with ever greater precision by making successive inductions accurately based upon his increasing awareness of the structural subtleties of the color field of the painting. A great painting, hence, cannot wear its climax on its sleeve, as it were. On the other hand, since a great painting must be such that players of every range of visual perceptual skill will be exercised by it, structurally this entails that its coarser perceptual features ought to reward the accurate player with a general sense of the position of the climax, while requiring that its more refined features be understood before a more precise sense of the position of the climax is forthcoming. Hence, if the reader will consider the coarser structural features of a painting to be its principal themes, and its progressively more refined structural features (being subsets of those sets of color points constituting the former) to be successive developments of them, the experiencing of a painting by a player of given inductive skill consists of an exposition and development culminating in successively closer approximations to the climax.

It is in this sense, it seems to me, that the Trinity is applicable to non-narrative non-temporally ordered works of art. (The reader ought to recall again, however, that I have been speaking of non-representational works of art; for there are complexities to the inductions made by a player experiencing the smile in da Vinci's Mona Lisa or Goya's War Sketches, for example, which go beyond the narrower (and, for me, more puzzling) structural concerns of abstract painting discussed above.)

Conclusion

If a great work of art is an inductive game, as I contend, then many of the notoriously vague and puzzling, though admittedly true, propositions over which aestheticians have spoken at length since Aristotle take on definite meaning:

- I. A great work of art arouses the deepest emotions of the spectator, and the resulting experience may even be cathartic; and yet, at the same time, the object itself must remain at a proper psychological distance from the spectator, or the effect is compromised.

Since a great work of art exercises the most firmly conditioned habitual responses of the organism, the emotional reactions which result are indeed pervasive. But a conscious sense of distance is necessary simultaneously to assure the organism that, indeed, it is an exercise going on, and hence that it is existentially safe for it to allow its deepest responses to be tested therein.

- II. Each element of a great work of art seems to be structurally inevitable and essential (i.e. seems to fit 'just right'); and yet, at the same time, each element seems often to be intrinsically ambiguous.

Since each element of a great work of art must be necessary to the inductive efficacy of the experience being had by the player, it will necessarily be felt to have been inevitable, once experienced, and will

necessarily prove to have been essential. But since a great work of art must exercise the inductive capacities of each player regardless of his inductive skillfulness, there can be no exclusively correct way of interpreting each element, for the correct interpretation will depend upon the inductive skills of the player making the induction.

III. There is an aura of universality which permeates a player's experience of a great of art; and yet, at the same time, the experience is felt to be both natural and intimate.

Since the essential activity of any human organism is to make accurate inductions, a player's experience of a great work of art, which exercises its deepest inductive responses, affects the essential and hence universal activity of it being a human organism. But since the activity is essential, it is therefore the most natural and intimate activity in which the player could engage.

But, more importantly, if a great work of art is an inductive game, as delineated in the preceding Chapters of this essay, this fact has structural implications which are general use to an artist engaged in the chores of composition - a feature uncommon to general aesthetic speculations.

Imagine, for example, the dramatist mentioned in the Preface to this essay who, having attempted to fit a few sketched scenes for an unfinished play into rough structural order, senses that one of the scenes in that context 'doesn't work'. Given that a work of art is an inductive exercise, the meaning of the phrase to the dramatist is pre-

cise: namely, as judged by the dramatist, the habitual expectations of a player having experienced that scene in that context would not be those which the dramatist wishes him to have. But the dramatist is then faced with a set of structural choices: eg.,

- (1) Rewrite the scene so that it will serve to exercise the desired habitual responses; or
- (2) Rewrite the preceding scenes so that, in context with that scene, the desired habitual responses will be exercised; or
- (3) Rewrite the succeeding scenes to conform to the habitual responses which would, in fact, be exercised; or... Etc.

The choices he makes, of course, will depend upon how important he judges the various inductive aspects of his scenes in hand to be. The point to notice, however, is that the dramatist, being no longer puzzled by the question 'Why doesn't it work?', is free to engage in choosing among the various answers to the structural question 'How ought I to make it work most efficiently?' - which is a question upon which he can bring to bear all the technical resources of his craft and skill.

In summary and in general, therefore, if this essay has been successful in specifying how a great work of art functions as an inductive exercise, the artist has been given a unique intellectual tool by which to direct the technical resources of his skill at each step of the compositional process - a gift, it seems to me, of extraordinary value.

Bibliography

1. Aristotle, Basic Works of Aristotle (ed. by R. McKeon), Random House, New York, 1941.
2. Arnheim, Rudolph, [1], Art and Visual Perception, University of California Press, Berkeley, 1969.
3. _____, [2], Film as Art, University of California Press, Berkeley, 1964.
4. Ashby, W. R., An Introduction to Cybernetics, John Wiley & Sons, New York, 1966.
5. Bethers, Ray, The Language of Paintings: Form & Content, Pitman Publishing Co., New York, 1963.
6. Birkhoff, G. D., Aesthetic Measure, Harvard University Press, Cambridge, Massachusetts, 1933.
7. Boring, E. G., and S. S. Stevens, "The Nature of Tonal Brightness", Proceedings of the National Academy of Sciences, (abbreviated hereafter as PNAS), Vol. #22, 1936.
8. Braithwaite, R. B., Scientific Explanation, Cambridge at the University Press, Cambridge, 1957.
9. Bullough, Edward, Aesthetics: Lectures and Essays (ed. Elizabeth Wilkinson), Stanford University Press, Stanford, California, 1957.
10. Cameron, Evan, "On Mathematics, Music, and Film", Cinema Studies, Vol. #3 (Spring, 1970), the Experiment Press, Bridgewater, Massachusetts, 1970.
11. Carnap, Rudolph, [1], The Logical Foundations of Probability (2nd Edition), University of Chicago Press, Chicago, 1962.
12. _____, [2], The Logical Syntax of Language, Little, Adams, & Co., Paterson, New Jersey, 1959.
13. _____, [3], "On Inductive Logic", Philosophy of Science, Vol. 12, #2, pp. 72-97. (Reprinted in Probability, Confirmation, and Simplicity, op. cit., pp. 35-61.)
14. Collingwood, R. G., The Principles of Art, Oxford at the Clarendon Press, Oxford, 1938.
15. Color as Seen and Photographed, Kodak Data Book E-74, Eastman Kodak Co., Rochester, New York, 1962.

16. Color Chemistry; Color as Light; Color in Use: Three Monographs on Color, The Research Laboratories of the International Printing Ink Corporation, New York, 1935.
17. Cooper, Grosvenor, and Leonard Meyer, The Rhythmic Structure of Music, University of Chicago Press, Chicago, 1966.
18. Copland, Aaron, What to Listen for in Music, A Mentor Book, The New American Library, New York, 1953.
19. The Creative Process (ed. by Brewster Ghiselin), A Mentor Book, The New American Library, New York, 1953.
20. Culver, Charles, Musical Acoustics (4th Edition), McGraw-Hill Book Company Inc., New York, 1956.
21. Dallin, Leon, Techniques of Twentieth Century Composition (2nd Edition), William C. Brown Company, Dubuque, Iowa, 1964.
22. Day, John Patrick, Inductive Probability, The Humanities Press, New York, 1961.
23. Dewey, John, Art as Experience, Capricorn Books, New York, 1958.
24. Eisenstein, Sergei, Film Form & The Film Sense (trans. by Jay Leyda), Meridian Books, World Publishing Co., Cleveland, Ohio, 1963.
25. Evans, Ralph M., An Introduction to Color, John Wiley and Sons, New York, 1948.
26. Findlay, Francis, Chrono-Rhythmics: Studies in Rhythm, Leeds Music Corporation, New York, 1939.
27. Forster, E. M., Aspects of the Novel, Harcourt, Brace & World, Inc., New York, 1954.
28. Freeman, Harold, Introduction to Statistical Inference, Addison-Wesley Publishing Co., Reading, Massachusetts, 1963.
29. Gombrich, E. H., Meditations on a Hobby Horse and Other Essays on the Theory of Art, Phaidon, New York, 1963.
30. Good, I. J., Probability and the Weighing of Evidence, Charles . Griffin and Company Ltd., London, 1950.
31. Goodman, Nelson, [1] , Fact, Fiction, and Forecast (2nd Edition), The Bobbs-Merrill Company, Inc., Indianapolis, 1965.
32. _____, [2] , The Structure of Appearance, The Bobbs-Merrill and Co., Inc., New York, 1966.

33. Groves Dictionary of Music and Musicians (3rd Edition), Vol. I-VI, MacMillan, New York, 1946.
34. Hambridge, Jay, The Elements of Dynamic Symmetry, Dover Publications, New York, 1967.
35. Handbook of Experimental Psychology (ed. by S. S. Stevens), John Wiley & Sons, Inc., New York, 1965.
36. Helmholtz, Hermann, On the Sensations of Tone, Dover Publications, New York, 1954.
37. Hogarth, William, The Analysis of Beauty (ed. by J. Burke), Oxford at the Clarendon Press, Oxford, 1955.
38. Jacobson, Egbert, Basic Color: an Interpretation of the Ostwald Color System, Paul Theobald, Chicago, 1948.
39. Jeffreys, Harold, Theory of Probability (3rd Edition), Oxford at the Clarendon Press, Oxford, 1961.
40. Judd, Deane B., Color in Business, Science, and Industry, John Wiley & Sons, Inc., New York, 1952.
41. Kant, Immanuel, Critique of Judgment, (trans. by J. C. Meredith), Oxford at the Clarendon Press, Oxford, 1911.
42. Kemeny, John and Laurie Snell, Finite Markov Chains, D. Van Nostrand Company, New York, 1960.
43. _____ and Laurie Snell and Gerald Thompson, Introduction to Finite Mathematics (2nd Edition), Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1966.
44. Keynes, J. M., A Treatise on Probability, Macmillan & Co., Ltd., London, 1929.
45. Kneale, William, Probability and Induction, Oxford at the Clarendon Press, Oxford, 1963.
46. Koestler, Arthur, The Act of Creation, The Macmillan Company, New York, 1964.
47. Kracauer, Siegfried, Theory of Film, A Galaxy Book, Oxford University Press, New York, 1965.
48. Kyburg, Henry E., Probability and the Logic of Rational Belief, Wesleyan University Press, Middletown, Connecticut, 1961.
49. Land, E. H., [1], "Color Vision and the Natural Image: Part I", PNAS, Vol. 45, #1, January, 1959.

50. _____, [2], "Color Vision and the Natural Image: Part II",
PNAS, Vol. 45, #4, April, 1959.
51. _____, [3], "Experiments in Color Vision", Scientific American, May, 1965.
52. _____, [4], "The Retinex", American Scientist, Vol. 52,
#2, June, 1964.
53. Langer, Susanne K., ^[1]Feeling & Form, The Scribner Library, Charles
Scribner & Sons, New York, 1953.
54. _____, [2], Philosophy in a New Key, A Mentor Book,
The New American Library, New York, 1962.
55. Laplace, Pierre Simon, Marquis de, A Philosophical Essay on Pro-
babilities (trans. by Frederick Truscott and Frederick
Emory), Dover Publications, Inc., New York, 1951.
56. Lawson, J. H., [1], Film: the Creative Process, Hill & Wang,
New York, 1964.
57. _____, [2], Theory and Technique of Playwriting, Hill
& Wang, New York, 1965.
58. Lovelock, William, Form in Brief (revised edition), A. Hammond &
Co., London, 1954.
59. Maerz. A., and Mr. Rea Paul, A Dictionary of Color, McGraw-Hill
Book Company, Inc., New York, 1950.
60. Meyer, Leonard B., Emotion and Meaning in Music, University of
Chicago Press, Chicago, 1965.
61. Mitchell, W. J., Elementary Harmony (2nd Edition), Prentice-Hall
Inc., Englewood Cliffs, New Jersey, 1961.
62. Moles, Abraham, Information Theory and Esthetic Perception (trans.
by Joel E. Cohen), University of Illinois Press, Urbana &
London, 1966.
63. Nagel, Ernest, Principles of the Theory of Probability (11th Im-
pression), International Encyclopedia of Unified Science,
Vol. I, #6, University of Chicago Press, Chicago, 1965.
64. Pearce, Charles W., Student's Counterpoint, G. Schirmer Inc., New
York, 1926.
65. Pierce, J. R., Symbols, Signals, and Noise, Harper Torchbooks,
Harper & Row, New York, 1961.

66. Prall, D. W., [1], Aesthetic Analysis, Thomas Y. Crowell Company, New York, 1936.
67. _____, [2], Aesthetic Judgment, Thomas Y. Crowell Company, New York, 1929.
68. Probability, Confirmation and Simplicity: Readings in the Philosophy of Inductive Logic, (ed. by Marguerite Foster and Michael Martin), The Odyssey Press, Inc., New York, 1966.
69. Quine, W. V. O., [1], Mathematical Logic (revised edition), Harper Torchbook, Harper & Row, New York, 1962.
70. _____, [2], Methods of Logic (revised edition), Holt, Rinehart & Winston, Inc., New York, 1959.
71. _____, [3], Set Theory and its Logic, Belknap Press, Harvard University, Cambridge, Massachusetts, 1963.
72. Raisbeck, Gordan, Information Theory, Massachusetts Institute of Technology Press (MIT), Cambridge, Massachusetts, 1966.
73. Reflections on Art, (ed. by Susanne K. Langer), A Galaxy Book, Oxford University Press, New York, 1961.
74. Resnick, Robert., and David Halliday, Physics for Students of Science and Engineering, Vol. I & II, John Wiley and Sons, Inc., New York, 1960.
75. Rimington, A. W., Color-Music: The Art of Mobile Color, Hutchinson & Company, London, 1912.
76. Salzer, Felix, Structural Hearing: Tonal Coherence in Music, Vol. I & II, Dover Publications, Inc., New York, 1962.
77. de Saumarez, Maurice, Basic Design: the Dynamics of Visual Form, Reinhold Publishing, Co., New York, 1964.
78. Schillinger, Joseph, The Schillinger System of Musical Composition, Vol. I & II, Carl Fischer, Inc., New York, 1946.
79. The Science of Color, Committee on Colorimetry, Optical Society of America, Thomas Y. Crowell, Co., New York, 1953.
80. Scott, Theodore G., Basic Computer Programming, Doubleday & Company, Garden City, New Jersey, 1962.
81. Shannon, Claude E. and Warren Weaver, The Mathematical Theory of Communication, University of Illinois Press, Urbana, Illinois, 1964.

82. Skeyrms, Brian, Choice and Chance: an Introduction to Inductive Logic, Dickenson Publishing Company, Inc., Belmont, California, 1966.
83. Smith, David E., A Source Book in Mathematics, Vol. I & II, Dover Publications, New York, 1959.
84. Springer, Clifford and Robert Herlihy, Robert Mall, and Robert Beggs, Probabilistic Models, Richard D. Irwin, Inc., Homewood, Illinois, 1968.
85. Stevens, S. S., [1], "A Scale for the Measurement of a Psychological Magnitude: Loudness", Psychological Review, Vol. 43, 1936.
86. _____, [2], "The Volume and Intensity of Tones", American Journal of Psychology, Vol. 46, 1934.
87. _____, and E. B. Newman, "The Localization of Pure Tones", PNAS, Vol. 20, 1934.
88. Stewart, D. J., Automaton Theory and Learning Systems, Thompson Book Company, Washington, D. C., 1967.
89. Sze, Mai-mai, The Way of Chinese Painting, Vintage Books, New York, 1959.
90. Tarski, Alfred., [1] Introduction to Logic and the Methodology of the Deductive Sciences, A Galaxy Book, Oxford University Press, New York, 1965.
91. _____, [2], Logic, Semantics, Meta-Mathematics, Oxford at the Clarendon Press, Oxford, 1956.
92. Taylor, J. F. A., Design & Expression in the Visual Arts, Dover Publications, Inc., New York, 1964.
93. Thomas, G. J., "Volume and Loudness of Noise", American Journal of Psychology, Vol. 65, 1952.
94. Toch, Ernst, The Shaping Forces in Music, Criterion Music Corporation, R.K.O. Building, New York, 1948.
95. Todhunter, Isaac, A History of the Mathematical Theory of Probability, MacMillan & Company, Cambridge & London, 1865.
96. Tovey, Donald Francis, The Forms of Music, Meridan Books, World Publishing Company, Cleveland, Ohio, 1966.
97. Wald, Abraham, [1], On the Principles of Statistical Inference, Notre Dame Mathematical Lectures #1, Notre Dame, Indiana, 1942.

98. _____, [2], Sequential Analysis, John Wiley & Sons, Inc.,
New York, 1947.
99. _____, [3], Statistical Decision Functions, John Wiley
& Sons, Inc., New York, 1950.
100. Weyl, Hermann, Philosophy of Mathematics and Natural Science,
Atheneum, New York, 1963.
101. Wiener, Norbert, Cybernetics (2nd Edition), MIT Press, Cambridge,
Massachusetts, 1961.
102. Williams, Donald, The Ground of Induction, Russell and Russell,
Inc., New York, 1963.
103. von Wright, Georg Henrik, Logical Studies, The Humanities Press,
Inc., New York, 1957.
104. Young, Hugh D., Statistical Treatment of Experimental Data, Mc-
Graw-Hill Paperbacks, McGraw-Hill Book Company, Inc., New
York, 1962.
-