HEXACHORDS RECONSIDERED
TOWARDS A REVISED COMPOSITIONAL PARADIGM

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ABSTRACT

This dissertation demonstrates, through a number of original works, the depth and variety offered by the exclusive use of hexachords as a generative compositional resource. Part One provides an overview of hexachords, outlines the connection between hexachords and twelve-tone music, reviews specific techniques, and discusses how these techniques were used by prominent twentieth century composers. Although a great deal of western art music has been created using these techniques, exclusive composition with six-note collections in equal tempered systems is rare. Further, much of the literature devoted to hexachords is concerned solely with their use in twelve-tone music. Interest in the hexachord has long been overwhelmed by the preeminence of twelve-tone music. Part Two introduces eighteen hexachordal compositions. Various techniques developed for use in twelve-tone music are adapted and employed. My research seeks to expand the understanding and use of hexachords as a compositional resource apart from serialism. Another component of this research, and one that sets it apart further from existing work in this area is my interest in different musical genres – in addition to exploring hexachordal composition as a resource within western art music, this dissertation presents a distinctly rare investigation into the use of hexachords in jazz composition and improvisation.

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GLOSSARY AND TERMS

The following outlines my use of terms, techniques, and naming conventions.

- **Atonal:** Music that does not suggest a home key. Harmonic motion is usually not derived from movement through the cycle of fifths. Atonal music is not necessarily serial or twelve-tone music but is rather a broad definition that *may* include those genres.[1]
- **Classical:** When using this term, I am referring to *all* western art music, including baroque, classical, romantic, modern, twentieth-century, contemporary, etc.
- **Functional Harmony:** In this dissertation "functional harmony" will refer to harmonic movement associated with tonal music, the baroque and classical periods, movement through the cycle of fifths, a key center, and cadential resolution.
- **Inv0**: the intervallic inversion of **P0**
- **Inv1**, **Inv2, etc.**: the transposed intervallic inversions of **P0**
- **Integer 0** will = C unless otherwise noted.
- **Jazz:** For the most part, in my work, the harmonic and melodic tradition that is the foundation of jazz has been abandoned. However, many of these pieces do contain elements that still align with this genre – instrumentation, rhythmic groove, and improvisation. Thus, for convenience, and to distinguish these pieces from my other hexachord works, I will refer to these as jazz compositions.
- **Prime Form:** the most compact version of a set, rendered as integers and transposed such that the first integer = 0

[1] I say "*may* include these genres" because it is possible to create "tonal" serial or twelve-tone music.

- **P0**: the prime form of the chosen hexachord. Unless otherwise noted Rahn's set-class listings are used. See Rahn 1980,142-3 for complete list.
- **P1, P2, etc.**: transpositions of the chosen hexachord, **P1** = **P0** transposed up by 1 semitone etc.
- **Serialism:** In this dissertation, unless otherwise noted, serialism is synonymous with twelve-tone music.
- **Tonal:**[2] In general, music that suggests a home key. Harmonic movement usually involves departures from and returns to this key center. In this sense, many of my pieces, created with atonal techniques, may sound tonal.
- **Twelve-tone/Twelve-tone Music:** Music generated from the transformations and manipulations of an ordered collection (row) of all twelve tones of the chromatic scale – the row contains twelve discrete tones, no repeats.
- **Z-mate**: Hexachord pairs that complete the aggregate always have the same interval content. If the second hexachord is not related to the first hexachord by transposition, it is referred to as the z-mate of the first. More information is provided on pages 16-17.
- Note that I am using **unordered** collections in my work. The notes of the chosen hexachord may be used in any order and repeated as desired. In most cases all six pitch-classes of the current transposition or inversion of the hexachord are used.
- Unless otherwise noted, the term **inversion** is used to refer to the collection of pitch-classes that are produced when the intervals between the pitch-classes of a given

[2] **Tonal** and **Atonal** are awkward terms. Is not any music that contains tones "tonal", and would not atonal music be bereft of tones? Further, "tonal" is often used to refer to music that suggests a resolution of some kind, and may be a subjective assessment, just as "atonal" is sometimes used as a pejorative for any passage disagreeable to the listener.

hexachord are inverted. Although the interval content remains the same, inversion usually produces a hexachord with a different collection of pitch-classes. For more information see Rahn 1980,45.

- When using **integer notation**, 10 is represented as *t* and 11 as *e*.

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- When discussing tonal chord progressions, the name of the chord will be expressed in full when referred to individually (F minor triad), and abbreviated when referred to as part of a chord progression (Fm, Fm/Eb, Fm/D).

Terms Adapted from Robert Morris (1979,458-60):

- **Interval Class (ic):** (Morris hyphenates the term pitch-class but not interval class) A set of all intervals that differ by multiples of 12 semi-tones and/or are complementary with respect to the octave. There are six interval classes. Interval class 1 contains all minor 2nds, major 7ths, diminished octaves, augmented octaves, minor 9ths etc.; interval class 2 contains all major 2nds, dim 3rds, augmented 6ths, minor 7ths, major 9ths etc.; This protocol continues up to interval class 6, which represents all tritones, augmented 4ths or diminished 5ths.
- **Interval Class Vector:** A listing of the number of interval classes of each kind found in a particular set. Because there are six possible interval classes, each interval class vector will contain six elements regardless of the cardinality of the set. The minor triad (3-11) [037], for example, has an ic vector of [001110], while the hexachord (6-1) [012345] has an ic vector of [543210].
- **Invariance:** A set is invariant if it remains unchanged after transformation by transposition, inversion, transposed-inversion or multiplication.
- **Invariance Vector**: Displays the amount of invariance a set possesses when operations

of transposition, inversion and multiplication are performed. Invariance vectors for hexachords display information concerning the hexachord and its complement. For examples and more detail see the section on Pre-composition beginning on page 54.

- **Pitch-class (pc):** A set of all pitches that are enharmonically identical and/or related by any number of octaves. There are twelve pcs numbered 0-11 (0-e). Unless otherwise noted pitch-class 0 contains all C s as well as all B#s and all D double-flats, pitch-class 1 contains all C#s and all Dbs, and so on.
- **Pitch-Class Set:** An unordered collection of pitch-classes (distinct integers) with no duplicates. All possible pitch-class sets were first categorized by Allen Forte (1973). In this dissertation I use the categorizations made by John Rahn (1980). The numbers that identify each set, for example 6-25 or 7-35 etc., are referred to as Forte numbers, despite the fact that they are accessed from Rahn's list.
- **Set-Class:** A collection of sets related to one another by transposition or inversion.

PART ONE

Introduction: Why Hexachords?

From the beginning of my graduate studies I have been investigating alternate harmonic environments derived from the twelve notes of the equal tempered scale. My intent with this process is to explore methods of composition that abandon the familiar directives of functional harmony. Research for my master's thesis, concerned with symmetrical scales, led to an interest in six-note collections – also known as hexatonic scales or hexachords. Composing with, and understanding the nature of, hexachords has become the focus of my current research.

Contemporary western art music is often identified by how much it abandons or adheres to the long-established harmonic paradigm dictated by one seven-note collection: (0, 2, 4, 5, 7, 9, e) – commonly known as the major scale. The classical modes, the minor scales, key signatures, triadic harmony, movement through the cycle of fifths, cadences, harmonic and melodic architecture, the resolution of the tritone – are all derived from, related to, or controlled in some way by this one seven-note collection. Many composers have sought ways of breaking free from or altering the nature of this established system of tonal organization.

In the early twentieth century serialism, in the form of twelve-tone technique, dramatically altered how music is understood and created. Established and employed by Arnold Schoenberg and the Second Viennese School, the twelve-tone language was continuously refashioned and reinterpreted, not only by the original practitioners, but also by subsequent generations of composers.

As composers began working with this new musical language, they became aware of the importance of the hexachord. A hexachord may be defined as a collection of any six discrete pitch-classes selected from the twelve notes of the equal tempered chromatic scale. A defining

and important characteristic of hexachords, and one that has played an important role in twelve-tone composition, is that the first six notes (hexachord) of any twelve-tone row will always have the same interval content as the remaining complementary second hexachord, or as expressed by John Rahn's first hexachordal theorem, "The unordered pc (pitch-class) interval content of any two complementary hexachords is identical" (Rahn 1980,105).

As defined by Allen Forte (1973) and John Rahn (1980), there are fifty different six-note collections. Each hexachord presents a harmonic palette unique to that collection. Some, like hexachord 6-32 (0,2,4,5,7,9) present a familiar sound very close to a major scale. Still others, like the hexachord 6-35 (0,2,4,6,8,10) – the "whole tone" scale – have become familiar adjuncts to functional harmony. Many offer what a listener steeped in tonality might call, exotic or unusual collections. Nonetheless, effectual composition with hexachords, regardless of how, or if, tonality is referenced, benefits greatly from an understanding of the techniques developed by twelve-tone (serial) composers. To become compositionally fluent with six-note collections a composer must be fluent with twelve-note collections, regardless of where a composer's interests lie.

However, while twelve-tone music is inextricably linked to the hexachord, hexachordal technique and composition can take place without serialism. Twelve-tone technique became a dominant compositional focus of the twentieth century. Hexachordal technique, which evolved concurrently and propelled much of the creation of twelve-tone music, remains largely ignored as an independent compositional resource.

This research demonstrates, through a number of original works, the great variety of harmonic and melodic environments offered by the exclusive use of hexachords as a generative compositional resource. Various techniques, developed for use in twelve-tone music, are adapted

and employed. Among these techniques are the use of invariant hexachords and transposed rotational arrays, the combination of various transformations of the hexachord and the use of the complementary hexachord (z-mate). All of these techniques are discussed in greater detail in part two of this dissertation.

Another component of this research, and one that sets it further apart from existing work is my interest in different musical genres. In addition to exploring hexachordal composition as a resource within western art music, this dissertation presents a distinctly rare investigation into the use of hexachords in jazz composition and improvisation.

Although a great deal of western art music has been created using these techniques, their use in this context is rare, simply because exclusive composition with six-note collections in equal-tempered systems is rare. Interest in the hexachord has long been overwhelmed by the preeminence of twelve-tone music. Much of the literature devoted to hexachords is concerned solely with their use in twelve-tone music. My research seeks to expand the understanding and use of hexachords as a compositional resource apart from serialism.

I come to this research as a mature graduate student with decades of practical experience. As a professional composer I have produced, composed, and recorded over a thousand scores for television, radio and film. This background, which demands a high level of technical facility and creative eclecticism, adds a unique enhanced perspective to my research.

Hexachord Overview

It is evident that any six discrete notes chosen from the twelve notes of the chromatic scale (a hexachord), will exclude another six notes (another hexachord). From the beginning of twelve-tone composition, Arnold Schoenberg, Josef Mathias Hauer, and others realized the

importance of six-note collections. Although it was Milton Babbitt who years later was able to prove the *hexachord theorem*, both Schoenberg and Hauer were aware, to varying degrees, that when a twelve-note collection was divided into two hexachords - the hexachords were related. Put simply, the hexachord theorem states that any two hexachords that together make up the aggregate will have the identical interval content. Hauer divided twelve-note collections into hexachord pairs called tropes and categorized these tropes by how they replicated themselves when operations of transposition and inversion were performed. According to Milton Babbitt (1987, 61-68), Schoenberg, was not aware that *all* hexachord pairs that form an aggregate have the same interval content, preferring instead to seek out specific six-note collections for his compositions that displayed invariance on inversion and transposition.

George Rochberg, in an article titled "The Harmonic Tendency of the Hexachord" (1959), observes that while twelve-tone composition is usually thought to be "arbitrary and fanciful," there exists within the technique "a new language of chords with its own inherent laws of structure, the chief one being that of symmetry" (209). I would suggest that symmetry is provided by the identical interval content shared by the two hexachords that make up a twelve-tone row.

The exclusive use of hexachords as a compositional resource apart from twelve-tone music is scarce. As noted earlier, much of the literature concerning hexachordal theory, technique or aesthetics is offered in the context of twelve-tone music. Any interest, therefore, in hexachordal technique or aesthetics is inextricably linked to twelve-tone music. For this reason, a great deal of the discussion in this paper is concerned with serial technique and aesthetics, referencing hexachords when applicable.

Today, while almost all composers of western art music are familiar with serial technique,

few indulge. Most are unaware of the myriad permutations and variations that evolved over the last century. Although, as John Covach observes, twelve-tone music is "one of the cardinal markers of musical modernism" and plays "a central role in understanding twentieth-century music and culture," it is generally tucked beneath the surface of contemporary musical experience, rising occasionally as something startling, innovative and unique. (2002, 603)

I will begin with an overview of serial and hexachordal aesthetics, examining some of the challenges facing composers and listeners now and over the last century. This will be followed by an exploration of the various ways that serial and hexachordal techniques have evolved, an investigation of extended and alternate techniques, a look at how specific composers adapt the techniques, and a consideration of attempts to fuse these techniques with jazz.

Aesthetic Overview

It is common to think of twelve-tone music as a prescribed method, strict and ordered. From the beginning, however, a great variety of applications, interpretations and adjustments have occurred. Straus (2005, 217) reminds us that "[t]welve-tone music is not a uniform or monolithic enterprise" but rather "... a world of musical possibilities, and within that world, each composer has discovered or created a country or province with its own distinct landscape." Contrary to popular belief, of the Second Viennese School triumvirate, only Webern follows the strict rules of ordering and row use. Berg frequently employs tonal elements and uses different rows in the same composition while Schoenberg's practice "... can be said to conform to his rule of the unique series only if the term 'series' or 'row' is replaced by 'set' and if the latter is considered only partially defined by the serial ordering." (Headlam et al 2013, 7)

Serialism and atonality were for Schoenberg deeply connected to, and an expression of, the

Viennese classical tradition. In his essay *Schoenberg and the Canon*, Christopher Hailey notes that many *fin de siècle* Viennese composers were also teachers dedicated to "a legacy of craft". The work of Schoenberg and others from that time and place represents "a revitalized dialogue with that legacy and a process of self-discovery within that culture." (Brand et al 1997,166-7) While Schoenberg (1975, 216) outlines that twelve-tone music "grew out of a necessity," the result of a century-long progression away from tonality, Dalhaus (1987,64) argues that the idea of a singular historical sequence is misleading. The correct state of serialism, or any music, cannot be seen as the result of a simple lineage. "Real history consists of histories in the plural: of events and chains of events which, at times autonomous and at times interwoven, emanate from a whole range of heterogeneous origins and lead to diverging results."

Babbitt (1987, 16-17) observes that the character of each twelve-tone work is defined by its particular ordering of notes[3] Although it shares the principles of transformation (transposition, inversion etc.) with other twelve-tone compositions, "Twelve-tone music is much more contextual and much less communal than ...tonal music." Tonal composition maintains specific "dependencies and contingencies" common to *all* tonal music (17). Conversely, each twelve-tone row possesses a singular harmonic and melodic paradigm. Composition is directed by the singular characteristics of a specific collection rather than the "universal principles" that guide tonal music (23-24).

Although the twelve-tone row serves as a source of thematic material, its most important feature, and that which distinguishes it in a profound way from tonal music, is its uniqueness. Straus (2005, 182), like Babbitt, reminds us that tonal music is relatively communal. Countless composers and musicians have explored the major and minor landscape. By contrast, each of the

[3] However, not all twelve-tone compositions are created with *strictly* ordered rows.

479,001,600 possible twelve-tone rows contains a distinct set of relations: “from the immediate surface to the deepest structural level, the series shapes the music.”

When each piece is created from a different ordering of pitch-classes and when the methods of deriving material and permutations for each composition are quite different, the stylistic reference points necessary for a listener are blurred. As Moore (1995, 91) observes, “The piece becomes hermetic, in that the processes which give it substance are not transferable to the experience of other pieces.” For many works, the genesis technique or creation method, although probably not audible to the listener, becomes part of a “rhetorical overlay” which compensates for the lack of stylistic similitude; “...it is the rhetoric which forms the substance of the communication.”

Dahlhaus (1987,74-75) argues that in Schoenberg’s case, critics were too concerned with how his music was created, rather than *what* was created. His atonal work was criticized for “violating aesthetic norms;” his twelve-tone music was labeled the work of a “musical mathematician or engineer.” Indeed, throughout the history of twelve-tone music, the discourse is often concerned with how a particular piece was created rather than the nature and quality of the music itself.

Babbitt, further, laments the fact that the term *twelve-tone system* has come to suggest the individual notes rather than the intervals *between* the notes. Significant is the fact that basic operations performed on a row do not alter these intervallic relations; they become the essence of a composition (1987, 46-48).

In the early 1920s, both Schoenberg and Josef Mathias Hauer[4] were independently developing twelve-tone techniques. Initially Schoenberg was reluctant to reveal much about his

[4] Both composers were residents of Vienna and were, for a number of years, collaborators and friends.

method of twelve-tone composition for fear of being misunderstood. Then, because of the publication in 1920 of Hauer's *Vom Wesen Musickalishe*[5] he feared being thought an imitator and made his method public in 1923. Nonetheless, he did not publish a detailed description of his technique until 1941. Schoenberg's pupil Erwin Stein published the first description in 1924. Stein presents the technique as a return to the essentials of counterpoint, a reaction against harmony, and a reversal of "the stylistic change that occurred from Bach to Mozart." The emphasis is placed on the transformation of an original idea through transposition, inversion, retrograde etc. (Covach 2002, 610).

Considering how radically different twelve-tone music was from what had come before, it is curious that Schoenberg regarded it, at that time, a mere tool and not a theory. He suggested that the composer using his method "compose as you always had done previously" and "use the same kind of themes, melodies, sound, rhythms as you used before." Perhaps this directive was intended for his own students – students that had already been steeped in his transitional atonal musical world (Schoenberg 1975, 213-49).

At the time of Schoenberg's 1933 arrival in America, many composers on this side of the Atlantic knew of his twelve-tone method and his earlier atonal works. There were, however, no recordings available in North America and certainly no performances of any of his twelve-tone compositions had taken place here. The only access to this music was through a few scores that could be studied. Krenek (1953, 514) observes that before WWII it was generally perceived that twelve-tone composition was the "private hobby of a few composers personally associated with its inventor." Nonetheless, he contends that during the war, a great number of composers took up the method and by the late 1940s twelve-tone composition had come into "widespread

[5] Schoenberg, in the referenced essay from *Style and Idea,* mistakenly refers instead to Hauer's *Vom Melos zur Pauke* which was published in 1925. (Schoenberg 2010, 213)

acceptance." After the war there was "an astonishing and widespread growth of interest" in the Second Viennese School, such that "...by the end of the 1950s there remained few composers in the USA or Europe who were not in some way influenced by the concepts of 12-note composition" (Headlam et al 2013, 21).

Stiles (1979, 95) indicates that between 1959 and 1979 there was a trend to view serialism and serial technique as being a "classical course" because of its "formalistic nature." By this time, most composers had in some way been schooled in, and most likely urged to pursue, serial technique. The push back, according to Stiles, came in the form of aleatoric music. "Serialism then was seen as not only too formal and structural, but also cold and mathematical." Stiles notes the mathematical background of mid-century serialists like Babbitt and Stockhausen. Of course, one of the reasons for the inception of Schoenberg's original technique was to introduce order to the randomness of atonality and this was done in part by incorporating the formal, structural, and mathematical techniques of transposition, inversion and retrograde.

Perception

> Tonality provides a paradigm of musical organization... attempts to discard it entirely seem only to confirm its authority over the musical ear. (Scruton 1999, 239)

Scruton likes to refer to serialism as "systematic tonality" observing that each piece of serial music, by its nature, establishes a set of unique rules that will guide that composition. This is in opposition to tonality's long familiar musical syntax. Why does he refer to twelve-tone music as a kind of tonality? In order for the listener to appreciate this music, Scruton argues, a new way of listening must be adopted that eliminates the expectations of tonality. That is, however, very difficult to do. The listener may attempt to listen with non-tonal ears but there is no assurance that the music will be heard that way. We may as listeners be able to discern that

some process is going on in serial music, but to go beyond that – to actually hear the process as "a movement of tones" – requires what Scruton calls "metaphorical organizations." The vast majority of western ears have been programmed to organize musical sounds within a tonal domain; if these patterns are not immediately perceived, we seek them out. The crux of this argument, and I believe it is a powerful one, – the reason that Scruton refers to serialism as a form of tonality – is that we cannot help but impose our tonal organizational references when we hear serial music. Citing the work of Fred Lerdahl, Scruton reminds us that the "compositional grammar" of serial music does not match the "listening grammar" (Scruton 1999, 281-6, 295).

Although serial composition offers the equality of all twelve tones, Scruton suspects that "the ghost of tonal order" is always present, negating any true endowment of tonal equality (296). Furthering his argument, he points out that many of the great works of serialism intentionally reference tonality, offering as an example the tonal tone-row from Berg's *Violin Concerto* (300).

Schoenberg and other serialists suggest that twelve-tone music is multi-dimensional. A musical idea is presented, viewed and interpreted from different perspectives. Traditional forward motion is abandoned, almost as if time has been suspended. Compositional form and structure are redefined. "In this space... there is no absolute down, no right or left, forward or backward" (Schoenberg 1975, 113). While Schoenberg thus describes how we *should* hear this music, Scruton (1999, 304) maintains that that is not how we *do* hear it: "...the grouping required by serial organization forbids the experience of musical movement, as we know it. We are to hear the music as *sounds,* rather than tones, exhibiting an acoustical, rather than a musical order. Yet this is *not* what we hear, in hearing atonal music."

Although I do agree with Scruton, I don't believe that our inability to alter our means of

perception diminishes the experience and enjoyment of this music. Krenek (1960, 321-22) contends that mood or feeling is not latent in the music, but rather a "product of the listener's auditory experience." Certainly, each listener approaches and perceives music within the context of experience. "There is no reason to assume that the nature of serial music excludes the possibility of interpreting it as some sort of communication."

Still, Scruton contends: "The possibility remains that tonal music is the only music that will ever mean anything to us, and that, if atonal music sometimes gains a hearing, it is because we can elicit within it a latent tonal order" (1999, 308). Yes, we may be consciously or unconsciously imposing a tonal order on what is heard. However, that does not mean that we are unable to appreciate or understand the work.

As noted above, serialism and atonality were, in the mind of Schoenberg, deeply connected to the Viennese classical tradition. Further, as Reinhold Brinkmann observes, Schoenberg's serialism was born from a "desperation" and "inability to change the external world" followed by a turning inward, a seeking of a safe, controlled environment which in turn produced an "artistic outburst" or *Durchbruch* – from which the work of art now presents a "utopian message" and a "spiritual truth"[6] (Brand et al 1997, 197-8).

Because little was known about the technique during the 1930s, other than the fact that all of the twelve tones were considered to be equal, many at that time thought the communality of the technique to be related to the left-leaning political ideas of the time (Babbitt 1987, 26). In response to those that referred to his twelve-tone technique as being communist or Bolshevik music, Schoenberg offered a curious opposing premise. He suggested that the basic set, or prime collection of twelve ordered tones, could be compared to a dictator "on whom all depends, who

[6] While Schoenberg's "spiritual truth" sought acceptance and comprehension, Hauer's intense and often bizarre twelve-tone journey was for the most part, introspective and exclusory.

distributes power and function to every tone ... who is responsible for all the subsequent transpositions and variations" (Schoenberg 1975, 250).

Nonetheless, in the following decades very few listeners seemed to appreciate the music produced by Schoenberg, Webern, Berg and a host of other serialists. Adorno (2006, 102), in the 1940s presents a rather pessimistic overview – this new music, basically ignored, is destined for "absolute oblivion…(no one) wants anything to do with it. It dies away unheard without an echo." In the following decade Babbitt (2003, 38) writes "...the label itself supplies a basis for automatic rejection." Commenting on the lack of interest during 1950s and 1960s in Stravinsky's late-life twelve-tone works, Straus (2001, 5) observes: "...serialism was never more than a marginal phenomenon in American and European musical life. Serial works rarely penetrated the repertoires of established performers or ensembles, hardly attracted notice in the mass media, and scarcely entered the consciousness of most musical audiences."

Dissonance and Forward Motion

Schoenberg's "emancipation of the dissonance" places dissonance "on an equal footing with the sounds regarded as consonances" (1975, 260). "Emancipation meant that the forward-moving tendency of the dissonance, which made for coherence, was abolished together with the obligation to resolve" (Dahlhaus 1987,77). Does this emancipation create something of a dilemma? Scruton (1999, 301) contends that with dissonance being "emancipated" it no longer has anything to oppose and thus becomes aimless and impotent. Meanwhile, Adorno (2006, 67) argues that dissonance is the new order and it is essentially fine on its own: "The new sounds (of twelve-tone music) are not the successors of the old consonance. They differ in that their unity is entirely articulated in itself."

If dissonance can stand alone and not resolve, if the forward motion created by tension and

release is abandoned, does the musical idea, the motif, or the row, in all its permutations and transformations, provide enough unity and interest? Schoenberg argued that consonance and dissonance were not opposites but degrees. While what is considered consonant or dissonant is subjective and has changed a great deal over the history of western art music, his emancipation is an attempt to eliminate any distinction between the two (Dahlhaus 1987, 120 -21).

Schoenberg acknowledges "... in a key, opposites are at work, binding together. Practically the whole thing consists exclusively of opposites, and this gives a strong effect of cohesion. To find means of replacing this is the task of *the theory of twelve-tone composition*" (Schoenberg 1975, 209). He suggests that each of the 479,001,600 possible orderings of twelve tones contains a unique set of harmonic, intervallic, and melodic possibilities that compensate for the loss of "the regularity and logic of earlier harmony" (219). The great number of possibilities notwithstanding, it follows that these possibilities compensate only if they replace the forward motion of tonal music *or* if that forward motion is no longer important. For Adorno the latter appears to be the case: "Twelve-tone technique substitutes conscious construction for 'mediation', the 'transition', and the forward drive that is implied by the leading tone" (2006, 66).

Composer, pianist, and music analyst Rudolph Reti (1978, 44) notes that using Schoenberg's technique does not necessarily preclude tonality. It is certainly possible to create tone rows that contain triads, and reference traditional harmony. "Still, the technique facilitates the attempt to exclude tonality if one so wishes. And this wish, this advice to refrain from any tonal element was strongly emphasized by Schoenberg." Reti also reminds us that musical form is closely related to, and grows from, traditional harmony. "To replace one structural force (tonality) by another (increased thematic oneness) is indeed the fundamental idea behind the

twelve-tone technique" (45).

Thus, according to Schoenberg, Reti, Adorno and others, forward motion, in terms of resolution of dissonance, is irrelevant in most serial composition. The vertical and horizontal structures however may be considered harmonic motifs, individual statements, gestures or expressions. In the same way that the mind can identify a physical object from any angle, Schoenberg attests that a musical idea may be displayed and recognized. He invokes Swedenborg's concept of heaven in which there is "no absolute down, no right or left, forward or backward" (Schoenberg 1975, 223).

The *ordering* of a twelve-tone row serves as the unifying element. Regardless of which transposition, inversion or retrograde of the row is used, a tone will maintain intervallic relationships with its neighbours – relationships that Schoenberg maintains the listener eventually recognizes. He is critical of Hauer's technique which employs pairs of unordered hexachords, arguing that it lacks the "fundamental logic" present in his own technique (Schoenberg 1975, 246-7).

As Dahlhaus (1987,164) observes "Schoenberg radically changed the concept of motif." Chords could now be considered a motivic structure; it is no longer necessary to have motion to express a motif. The motif can be embodied in a horizontal structure. "Schoenberg was occupied by his notion of the musical idea...his aim in composition was to present a single complete thought... to relate everything in a composition to one idea" (Schoenberg, Carpenter and Neff 2006, xix). Unfortunately, the music is usually not perceived this way. Writing in 1941, a frustrated Schoenberg acknowledges that most listeners and many critics are not adequately prepared to understand twelve-tone composition. He wants this music to be understood: "...artistic value demands comprehensibility, not only for intellectual but also for emotional

satisfaction" and "composition with twelve tones has no other aim than comprehensibility" (Schoenberg 1975, 215).

Narrative and Forward Motion

Form in western art music is defined in large part by harmonic functions, key areas and modulations. However, in atonal and twelve-tone composition, a key signature is no longer relevant and movement to a new key area, modulation, and return to a home key do not apply. Thus, what *needs* to be said and what *can* be said are limited (Schoenberg 1975, 217-8). This contributes to the fact that many early pieces of twelve-tone music are remarkably brief – Webern's compact *Symphony* opus 21, for example, is usually performed in less than ten minutes.

As previously noted, serial music does not usually possess the quality of forward motion; it tends to concern itself with a number of related musical events. Movement toward a conclusion or climactic event is usually not present. While there are some serial works that do achieve this forward motion, Griffiths (1985, 249) observes that these works tend to "...have generally found some alternative to diatonic harmony as fuel for continuous forward motion, or have retained sufficient diatonicism to guarantee a certain dynamism." Others have claimed this as well: the most successful twelve-tone compositions are, according to Scruton (1999, 305), pieces that serve as accessory to, or are combined with other expressive arts. Songs, drama, opera, film, any kind of narrative arc serves to provide structure and form, excusing the need for forward motion or narrative in the music. The shorter pieces of serialism are short perhaps because they lack a narrative. The longer pieces tend to combine with other art forms or incorporate elements of tonality. Alban Berg combines serialism with opera in *Lulu,* which runs over 90 minutes in

length, and with tonal elements in both the *Lyric Suite* and the *Violin Concerto,* each close to 30 minutes long. Schoenberg was also willing to combine serialism with an imported narrative. He completed the first ever twelve-tone opera *Von Heute auf Morgen* in 1929 (a comedy!) but was unable to finish the considerably more somber *Moses and Aaron*.

Atonal Set Theory

As John Rahn (1980, 1) points out, atonal music refers to any western art music that is not tonal. Twelve-tone music, integral serialism, and other variations of its technique should all be considered as subcategories of atonal music. Created as a tool for the analysis of all atonal music, pitch-class set theory serves as a useful aid to serial and hexachordal composition. Requisite is the recognition of all notes as pitch *classes* rather than pitches, granting octave equivalence, and enabling the conversion of all pitch-classes to integers. Integers provide "a more neutral way of notating pitch-classes and the relations among them" (Wuorinen 1979, 81). Texts by Rahn, Straus, Wuorinen and Forte all outline approaches to numerical notation in various levels of detail.

Conversion of a collection to integers usually includes rendering these integers in prime form.[7] Prime form facilitates quick recognition of pitch-class collections that are related by transposition and inversion. As stated previously, music theorists Allen Forte and John Rahn have each identified fifty different prime form six-note collections – hexachords.[8]

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[7] These conversions may be facilitated by software that will quickly identify the set-class, prime form, invariance vectors etc. of a discrete collection, containing between 2 and 10 pitches. (There is only one prime form of the set-classes of cardinality 1,11 and 12). All integer calculations use modulo 12.

[8] However, two different algorithms exist for creating the prime form of pitch-class sets. Alan Forte's method favours making smaller numbers smaller - compacting towards the left – while Rahn's method prefers making the larger numbers smaller – compacting towards the right. In most cases they agree but there are two six-note sets (6-29 and 6-31) that differ.

While all hexachord pairs that form the aggregate have the same interval content, they are not always related by transposition. If this is the case the two complementary hexachords are identified as *z-mates.* For example the hexachord that forms the *complement* of the first six notes of the chromatic scale, identified as hexachord 6-1, or integers [0,1,2,3,4,5] is simply a transposition of the original hexachord - whereas the hexachord 6-45 [0,2,3,4,6,9] is complemented by its z-mate 6-23: a transposition of 6-45 will not complete the aggregate.

Set-class tables are especially useful when working with hexachords, usually providing the prime form, z-mate, interval vector, and invariance vectors, as well as identifying sets that are related through operations of multiplication. For example, the six- note collection [C, Eb, E, G, Ab, A], will convert to the integers [0,3,4,7,8, 9]. The prime form is **[**0,1,2,5,6,9]. Forte number 6-44 identifies this collection. The set-class table tells us that the hexachord 6-19[9] will complete the aggregate when combined with 6-44. The interval vector [313431] reveals the number of occurrences of each interval within the hexachord: in this case, three semi-tones, one whole tone, three minor thirds, four major thirds, three perfect fourths and one tritone.[10] The interval vector also displays the number of common tones present when the collection is transposed by each respective interval. For example, when this hexachord is transposed up a major third or down a minor sixth there will be four pitch-classes that remain the same. Invariance vectors also reveal to some degree the character of a collection, identifying how often the original hexachord will be replicated through transposition, inversion and multiplication.

[9] Completing the aggregate of 6-44, 6-19 is now the z-mate and may be identified as 6-z19.

[10] Because of inversion equivalence and octave equivalence it is not necessary to include any intervals beyond a tritone in an interval vector. A major seventh interval is the same as a semi-tone, a major ninth interval is the same as a major second and so on.

Basic Operations

In pre-composition, rendering the chosen twelve-tone row or hexachord in a twelve-tone matrix displays all transpositions, inversions, retrogrades, and retrograde-inversions of the original row, or of the original hexachord and its complement. For many, these basic contrapuntal operations, borrowed from the music of the late baroque, form the building blocks of serial composition.

When a major scale is transposed to another key all of the intervals present in the original scale remain, but the collection of notes changes, sometimes by just one note, sometimes by all seven. When a twelve-tone row is transposed, all the intervallic relationships also remain intact, but all twelve pitch-classes remain in the row. In tonal music inversion takes place within the confines of tonality - inverting a motif, for example, will have different results depending on the tonality. However, in post-tonal music, inversion is what Perle (1996, 5) calls "literal" – it is not altered by tonal parameters.

Ordering

Why must the tones of a twelve-tone row be ordered? When ordered, intervallic relationships between the notes are preserved. A row is a series of intervals; the spaces/intervals between the notes, rather than the notes themselves, are important. The character, sound, unity and cohesion of a row are determined by these intervallic relationships. Without ordering, all twelve-tone rows would offer the same random selections from the chromatic scale.

However, if a row is divided into two paired hexachords, and the integrity of each hexachord is maintained (i.e. their content is kept separate), intervallic relationships are maintained in a different manner. The interval content of the two hexachords will be identical *and* the interval content of each hexachord will remain constant regardless of the ordering. As

ordering within each hexachord now becomes extraneous, the number of possible hexachord pairs that complete the aggregate, fifty, is considerably less than the number of possible ordered twelve-tone rows - hundreds of millions. Of course, manipulating and recognizing the qualities of hexachords does not preclude the use of ordered rows and ordered hexachords.

With twelve-tone technique, each composition has a collection of relationships unique to the row being used. Dividing the ordered row into smaller discrete segments, and changing the ordering within those segments, despite the fact that this alters the intervallic relationships within the row, is common practice. Further, Charles Wuorinen (1979, 19) encourages composers to occasionally step back from strict ordering by, for example, breaking the row into four unordered three-note segments. The pitch-classes may be reordered but remain within the segment. A relaxing of the original ordering is achieved while still maintaining ordering on a higher level.[11]

Perle (1991, 94) observes that Schoenberg was not averse to varying the ordering of hexachordal segments within the larger structure of an ordered twelve-tone row, providing examples from Opus 41 *Ode to Napoleon* and "Tranzcence" from Opus 24 *Serenade*. This proclivity is also affirmed by Babbitt (2003, 42): "...his (Schoenberg's) increased preoccupation with the hexachord as an independent unit led to his using it often without regard to ordering."

According to Moore, this varied approach to ordering, this great difference in how permutations are formed, reveals one of the "profound contradictions" at the heart of serialism. Moore's 1995 article argues that there is "no single identification of the practices of serialism." Instead there exists a variety of opposing presentations, all "... integral to the method which

[11] Jazz musicians working with twelve-tone technique often segment a row into trichords or build rows from invariant trichords. When improvising or composing, these three note cells are often re-ordered - see discussion later in this paper on the work of John O'Gallagher and Bruce Arnold.

Schoenberg invented"[12] (84, 89).

Hexachords and Hauer

Josef Mathias Hauer (1883-1959), the self-proclaimed inventor of twelve-tone music, lived in Vienna, and was, until a falling out in the 1920s, a friend of Schoenberg. While Hauer's compositions are generally considered to be mediocre and eccentric, he was the first to segment the aggregate into two discrete hexachord pairs. Hauer meticulously identified forty-four pairs, which he called *tropes*. "His concept of hexachordal tropes was a remarkable anticipation of subsequent developments in 12-note composition and theory" (Headlam et al 2013, 2).

A trope is *not* a hexatonic scale, nor is it a chord or a pitch-class set, or an interval class set. A trope *is* a "framework of contextual interval relations" – relationships *within* each half and *between* each half of the trope. Its main purpose is for composition. Within each trope is a unique collection of intervals and a unique symmetry between the two hexachords. The great advantage of the trope is that it provides the composer a quick understanding of the intervallic relationships and symmetry inherent in any row created from that particular trope (Sedivy, 83-5).

A trope consists of two hexachords. The aggregate of the two six-note collections contains all twelve notes of the chromatic scale. The interval content of one half of the trope will always be the same as the complementary remaining half. Changing the order of tones within one half of the trope will not affect the interval content. Similar to the inverting of a triad, the sequencing of intervals changes, but all of the intervals remain unchanged. Exchanging tones between the halves *will* change the trope to another trope. These hexachord pairings were classified by their transpositional invariance. From these tropes any of the possible 479,001,600 twelve-tone rows

[12] Moore's article provides examples from the work of Babbitt and Roberto Gerhard. The pairing of unordered hexachords in the work of Gerhard is perhaps more a reflection of Hauer's technique than that of Schoenberg.

can be formed by transposition, re-ordering, inversion, retrograde etc. (Gustafson, 23-4; Whittall 2008, 24).

Reviewing a rare performance of Hauer's music in 1979, the *New Yorker's* Nicholas Kenyon commented on Hauer's tropes and the importance of the combinations of notes – the intervallic relationships within each trope:

> What Hauer created, then, is something like a new set of scales or modes. As in the familiar major or minor modes, it is the particular combination of notes which gives each trope its character (Kenyon 1979, 185).

In *The Harmonic Tendency of the Hexachord,* an article from 1959, George Rochberg recognises these intervallic relationships and the important role they play in non-tonal music:

> What we have discussed strongly suggests that equal tempered chromaticism, organized hexachordally, incorporates in an organic way fundamental tonal functions having to do with the connection between intervals while discarding the exhausted machinery of historical tonality (Rochberg, 223).

From the trope Hauer creates a twelve-tone row that serves three functions: it provides a harmonic resource, a structural foundation, and presents (or approximates) all of the notes of the overtone series, which in turn, for Hauer, represent the constancy, and "totality of the cosmos" (Reinhardt, 6).

Other techniques introduced by Hauer include:

- *Kontinuum* – a composition device that distributes an ordered row though a series of predetermined harmonic positions, changing one note at a time
- *Row Splicing* – a method used to form extended rows by combining different transformations of a twelve-tone series
- *Rotation* (non-transposing) of ordered hexachords. Although a distinction is often made between the Second Viennese School and Hauer in relation to the use of ordered hexachords, it should be made clear that Hauer uses ordered hexachords with r*otation,*

ordered twelve-tone rows with *row splicing* and *kontinuum*, and un-ordered hexachords when working with *tropes*[13] (Covach 2002, 607; Sedivy 2011)

In the early 1920s Hauer and Schoenberg were intending to collaborate on a book outlining the twelve-tone methods of each composer. However, at some point Hauer began accusing Schoenberg of stealing his ideas and the two parted ways. In 1937 Hauer had a rubber stamp made which, for the rest of his life, he applied to his correspondence and scores (Gustafson 1979, 21):

Josef Matthias Hauer –The spiritual father and (in spite of many imitators!) still the only master and connoisseur of twelve-note music. (translated from German)

Hexachords and Schoenberg

From the beginning of Schoenberg's explorations of twelve-tone technique, he was composing with hexachords. In the fourth movement of *Five Pieces for Piano* op. 23, Lefkowitz (1997, 309-10) points out that most of the material is derived from three complementary hexachord pairs; 6-44/ 6-z19, 6-14/6-14 (self-complementary) and 6-10/ 6-z39. This piece may reveal Schoenberg's first use of "a systematic approach" to hexachordal pitch content and "is a clear foreshadowing of his use of hexachordal combinatoriality in the later serial works."

Schoenberg, especially in his later twelve-tone work, relies on hexachordal properties of combinatoriality and invariance, creating rows that provide inversionally related hexachords. As Babbitt (1987, 14) relates: "For him, (Schoenberg) the way to begin a piece was to find a pair of hexachords that not only would combine to form an aggregate but were also inversionally

[13] Much more information about the work and curious spiritual explorations of Hauer is revealed in John Covach's PhD dissertation (Covach 1990) and the work of Dominik Sedivy (Sedivy 2011).

related." Schoenberg was probably unaware that *any* two hexachords that produce the aggregate share the same interval content, only seeing these qualities in the special hexachords he preferred to work with. Nonetheless, Babbitt makes it clear that Schoenberg's twelve-tone pre-composition process included specific hexachord selection and that the composition itself is guided by hexachord combinatoriality (15, 61-68).

It is interesting to note that after 1928 Schoenberg used combinatorial hexachords almost exclusively as a basis for composition – a choice that greatly limited the number of available hexachords and rows (Headlam et al 2013, 13).

Rotation

As noted above, Hauer employed hexachord rotation. Ernst Krenek advanced the technique in the 1940s by the introduction of transposition. First used in his *Lamentatio Jeremiae Prophetae,* each rotation of the hexachord is transposed such that each subsequent rotation begins with the same pitch-class. This results in a rendering of the five "modes" of the hexachord (analogous in tonal music to beginning each of the classical modes on the same note).

The examples below compare the rotation and transposed rotation of hexachord 6-13. Note that regular rotation produces five re-orderings of one hexachord that include only the original six pitch-classes. Transposed rotation produces five transformations of the original hexachord. Each rotation begins with the same pitch-class and includes a variety of pitch-classes not contained in the original hexachord. In this case all twelve pitch-classes are rendered, yet each hexachord possesses the same interval content.

Figure 1: Rotations

Rotation of Hexachord 6-13 [0,1,3,4,6,7]
0,1,3,4,6,7
1,3,4,6,7,0
3,4,6,7,0,1
4,6,7,0,1,3
6,7,0,1,3,4
7,0,1,3,4,6

Transposed Rotation of Hexachord 6-13
0,1,3,4,6,7
0,2,3,5,6,e
0,1,3,4,9,t
0,2,3,8,9,e
0,1,6,7,9,t
0,5,6,8,9,e

Another way of visualizing transposed rotation may be realized if we place the hexachord into a matrix. Each row in the example below contains an unordered version of each transposed rotation.

Figure 2: Rotation Matrix

	I_0	I_1	I_3	I_4	I_6	I_7
P_0	0	1	3	4	6	7
P_{11}	e	0	2	3	5	6
P_9	9	t	0	1	3	4
P_8	8	9	e	0	2	3
P_6	6	7	9	t	0	1
P_5	5	6	8	9	e	0

For Krenek, rotation is a process that maintains the integrity of the twelve-tone row while excluding the requirement to always employ a complete iteration of the row. Clare Hogan, writing in 1982, suggests that Krenek regards this technique as a common ground between serialism and tonal harmony (Hogan 1982, 23).

Verticals

Verticals, simultaneities, or chords present a problem for a composer wishing to maintain the distinct identity of an ordered twelve-tone row. When realized as a vertical structure, any ordering of the notes established in the row is abandoned. Consequently, many composers seek vertical creation techniques that suggest some kind of logic related to the row ordering, but not always derived specifically *from* the row ordering - consider Perle's *cycled sets,* or Hauer's *kontinuum*. In his 1991 book, *Serial Composition and Atonality,* George Perle notes several distinct ways that verticals are presented in the works of Schoenberg, Webern and Berg:

- Verticalization of adjacent tones in the row
- Verticalization of non-adjacent tones in the row
- Invariant formations – verticals constructed from invariant row elements that occur within the set-complex
- Segmentation – creating verticals from hexachords (maintaining the integrity of the two hexachord pairs provides a level of unity which is not altered by internal ordering within the hexachord – considered thusly, there is no ordering to compromise)
- Simultaneous use of different transformations of the row (Perle 1991, 87-8) – Stravinsky's serial choral work reveals the augmentation of similar techniques
- Creating verticals from simple segmentation of the row – for example dividing the row into three tetrachords
- Creating verticals from hexachordal rotational arrays – using both horizontal and vertical slices of the hexachord array
- Creating verticals by slicing simultaneous renderings of **P**, **I**, **R** and **IR** of the row (Straus 2001,148,150,168-9)

Straus (1999, 73) offers an example of this last technique from the conclusion of the second part of *Movements* measures 62-67. Here the row is realized horizontally while each pitch- class of the row is supported by vertical elements derived from the **I**, **R** and **IR**. "Along with the verticals of rotational arrays, these four-part arrays are Stravinsky's most striking innovation, his most original contribution to twelve-tone theory" (Straus 2001, 170).

The Appeal of Webern

Hermann Danuser notes that as serialism evolved Schoenberg was in a sense abandoned. Two serialist streams seemed to emerge – those that aligned with "... the expansion of his twelve-tone technique via Webern" and an alignment with Berg by composers who "strove to reconcile the serial technique with tonality and expression."

In his 2010 book on the twelve-tone music of Luigi Dallapiccola, Brian Alegant compares the late work of Webern and Schoenberg. He observes that Webern prefers sparse orchestration, linear presentation of the row, symmetrical rows and derived aggregates. Schoenberg's twelve-tone work is often dense, sometimes uses unordered hexachords, tends towards semi-combinatorial rows and at times employs cross partitions. Webern maintains the integrity of the row while Schoenberg is more concerned with forming aggregates (Alegant 2010, 9-10). "[But] Webern, no less than Schoenberg, saw twelve-tone composition as a solution to the problem of writing extended music in the total chromatic, and his works show an extraordinary sensitivity to the possibilities of the twelve-tone system for embodying the formal strategies of earlier music..." (Andrew Mead 1993,173-4).

Webern is able to integrate twelve-tone technique with traditional forms, allowing a reanimation of "the formal process of his tonal predecessors." Curiously, while Webern is lauded for this integration, Paul Griffiths (1985, 45-46) points out that in Europe after WWII,

Schoenberg was disparaged for maintaining classical forms. Whether they aligned with Berg or Webern, by the early 1950s composers on both sides of the Atlantic were testing the limits of the technique (Brand et al 1997,185).

Wolpe, Gerhard, and Finney

German composer Stefan Wolpe, before immigrating to the United States in 1938, was exposed to the twelve-tone work of Hauer, as well as that of the Second Viennese School. Although Wolpe employed elements of Schoenberg's methods in his own work, he was particularly drawn to Hauer's use of unordered hexachords. Austin Clarkson suggests that Wolpe's work fuses a number of techniques:

> In a series of pieces composed in 1935-36, Wolpe worked with derived sets and hexachordal combinatoriality in a way that George Perle later formalized as 'twelve- tone tonality'. Wolpe's *Passacaglia* 1936 epitomizes a synthesis of Hauer, Schoenberg and Webern while incorporating constructive and expressive principles learned at the Bauhaus. (Clarkson 1999, 386)

Wolpe had a subtle influence on American jazz and film music, teaching his methods to Eddie Sauter, Gil Evans, George Russell, Elmer Bernstein, and many others (391-99).

Roberto Gerhard, who studied with Schoenberg, originally preferred working with unordered hexachord pairs, considering the strict ordering of tone rows to be too limiting. In the early 1950s, however, he further enhanced his technique, applying serial elements to structural elements. A unique interpretation of *integral serialism* is applied to the length of individual notes, the use of transpositions of the row, and the length of these transpositions. "The series, therefore, has the ability to govern microcosmic and macrocosmic parameters of a composition" (Sproston 2009, 22-3).

The American composer and educator Ross Lee Finney (1906-1997) often used hexachordal segmentation and employed twelve-tone technique to serve a tonal ideology

(Onderdonk 1968,125-6). A student of Alban Berg in the 1930s, Finney considered that the "Equality of all tones is to me one of the catastrophes of 12-tone composition…" and that "some notes… should be more equal than others" (Peacock and Finney 1991,7). Finney approached twelve-tone and hexachordal technique on his own terms, exclaiming in an interview that the use of combinatoriality was too "complex". When he returned to work on his 1984 opera *Weep Torn Land,* after a break to work on another piece, he admits to finishing the opera with the "wrong" hexachord. When he realized his mistake, he decided that he liked the outcome and let it be (10). While Finney worked with hexachords a great deal, it is curious that he often uses the term "*complementarity"* to refer to a musical idea that may have two different functions or meanings – easily confused with the term *complementary* (Finney and Labenski 2004, 27).

Hexachords and Stravinsky

In the early 1950s Stravinsky gradually embraces twelve-tone technique.[14] Straus (2001, 23) reveals that Stravinsky was probably more influenced by Webern than by Schoenberg. Characteristically, Stravinsky manages to transform the ideas of the former: "Webern's entangled, opaque chromaticism with its insistence on the primacy of the twelve-tone aggregate, is washed away in favor of a crystalline transparent diatonicism, replete with constant pitch repetitions."

Stravinsky's first use of a complete twelve-tone row occurs with *Agon* (1954-57). His first completely twelve-tone work, *Threni,* is presented in 1958, followed by works containing different levels of hexachord rotation, and eventually the use of more than one row in a composition – *Movements* (1958-59) and *Requiem Canticles* (1966). Beginning with *Movements*

[14] Despite being one of the great composers of the twentieth century, Stravinsky's twelve-tone offerings suffered much the same neglect as serial works by other composers. (Straus 2001, 5)

Stravinsky begins a systematic use of hexachords and rotational arrays. He employs rotation to develop his own serial voice, adapting Krenek's method to his own creative needs (Hogan 1982, 28). As Straus (2005, 231) points out, Stravinsky's late-life twelve-tone composition is concerned with rotational arrays and thus to a large extent is hexachord-based.

Composer Ernst Krenek introduces Stravinsky to both the use of the inversion of the retrograde[15] (as opposed to the more common use of the retrograde of the inversion) and transposed hexachord rotation, which, starting with *Movements*, becomes "the basis for all Stravinsky's major compositions" (Straus 2001, 28). Further, Stravinsky expands Krenek's original idea by employing the verticals as well as the horizontals produced by a rotated hexachordal array (32). Stravinsky favours hexachord manipulation through arrays rather than the use of entire rows, often creating "vast melodic expanses" by cycling through numerous rotations of an array (103-06).

In the pre-composition outline for *Movements,* Stravinsky divides the chosen row into two hexachords. Each hexachord is rotated five times, and then the original hexachord pair is rotated and *transposed* five times, producing a total of 24 hexachords. In measures 13-17 of the first movement, the flute solo is created primarily from various three-note segments excerpted from these hexachords. "Serial techniques in the flute solo are like weaving techniques in tapestry, for beneath the surface of every tapestry lie hundreds of strands... carefully concealed from our view, ...creating the intended design on the visible surface" (Rust 1994, 65). Babbitt (1964, 48) calls Stravinsky's attraction to the hexachordal structures in his later pieces "a profound connection to the world of intervallic composition."

It is important to note that while intervallic unity is maintained through all horizontal

[15] The inversion of the retrograde is merely a transposition of retrograde of the inversion.

cycles of a hexachordal array, this is not the case with verticals extracted from the transposed and rotated array. However, as shown later in my composition *Memory Dust*, complete hexachordal verticals may be extracted from the twelve-tone matrix before rotation. While I am using unordered hexachords in my compositions, I am drawing my hexachords from the ordered sets provided in the matrix. Thus, I am able to extract full hexachord verticals. Stravinsky, on the other hand, is composing with ordered sets, but re-ordering them – rotating them – in pre-composition. The verticals of the transposed rotations are not hexachords. For example, if six-note verticals are extracted from a transposed rotation of hexachord 6-7 [3,4,10,8,9,2] (the version of the 6-7 hexachord used by Stravinsky in *Movements*), the result is a variety of non-hexachordal pitch-class sets with duplicated pitch-classes: unison set, 4-20, 5-27, 4-3, 5-27, and 4-20. These results are calculated from the *gamma* hexachord displayed in Rust (1994, 64).

Babbitt

In America after WWII, along with Babbitt and Perle, other composers like Elliot Carter, Roger Sessions, and Aaron Copland came to embrace serialism, adapting and reinterpreting the technique to suit specific needs. Of note is the amount of influence that Perle, and especially Babbitt had on American twelve-tone music at this time (Griffiths 1985,43). Straus notes that Babbitt's technique draws from both Webern and Schoenberg: "Babbitt's arrays are concerned with trichordal derivation and with trichordal and hexachordal combinatoriality ... For Babbitt the trichord array was a theoretically fertile intersection of Schoenbergian combinatoriality and Webern derivation" (1999, 72). His work in the post-war years, according to Griffiths (94), presents a clear, precise technique: "Serial structuring of this thoroughness was certainly not to be found among the works of European composers." American theorists who studied with, were greatly influenced by, and /or expanded the work of Babbitt are John Rahn, David Lewin,

Donald Martino, Andrew Mead, Robert Morris and Allen Forte (Covach 2002, 624).

Perle

George Perle, before thoroughly understanding Schoenberg's technique – in large part because of the paucity of scores, texts or general information about dodecaphonic composition before WWII – believed the forty-eight versions of a twelve-tone row, as revealed through transposition and inversion in a matrix, to be the primary expression a row. He considered vertical/horizontal associations to be valid source material, developing his own technique, which he continued to employ long after realizing his misinterpretation of the original (Perle 1996, xi). Perle's book *Twelve Tone Tonality* is not a description of Schoenberg's technique, but rather a presentation of his own adaptation of the technique. The book "...sets forth a system of twelve-tone relationships comparable to but for the most part entirely different from the familiar axioms of Schoenberg's and Hauer's twelve-tone systems" (Devoto 1978, 294).

Martha MacLean (117-18) points out three problems that Perle sees in Schoenberg's technique. First, vertical structures (chords) destroy the order of a row. Second, "unrestricted latitude in the intervallic structure" eliminates any command over the harmonic material.[16] Finally, vertical structures created from the row have no defined relationships with other simultaneities – while in tonal music triadic structures refer to and interact with each other in long established patterns. To address these perceived weaknesses Perle's *cycled sets* create what he calls "tonic functions." He has created his own version of tonality from a reinterpretation of twelve-tone technique. Unfortunately, his presentation is often difficult to decipher. As MacLean points out, Perle's book suffers from "unclear presentation", incomplete theoretical foundations",

[16] This is not quite clear in MacLean's article. Is Perle troubled by the ability of the composer to freely select the interval content of the row?

"historical misconceptions" and numerous ambiguous definitions (123).

Composing with More Than One Hexachord

While each of the compositions created for this research investigates the generative properties of just one hexachord, the use of multiple hexachords within a single composition is certainly nothing unusual. When working with more than one hexachord it is useful to know how closely related the interval content of the collections are. If we compare two different hexachords, we see that there can be at most four common interval numbers. In other words, no more than four intervals can occur the same number of times in each hexachord. With four common interval occurrences, two hexachords are considered to be maximally related.[17] Let us consider the maximally related hexachords 6-32 [024579] and 6-33 [023579]. While the pitch-class sets only differ by one integer, the interval vectors are [143250] and [143241] respectively.[18] As can be seen, both of these hexachords contain one semitone, four whole-tones, three minor thirds, and two major thirds, but differ in the number of perfect fourths and tritones.

Using two maximally related hexachords may expand the harmonic vocabulary while maintaining unity through the four common interval groups. I have found a tendency for the individual hexachord collections to be diluted and/or to appear as one larger collection. The combination of 6-32 and a transposition of 6-33, for instance, will produce a major scale. While this in itself is not necessarily a negative aspect, I prefer, at this stage of my engagement, the

[17] Robert Morris provides a method of determining similarity between pitch-class sets and has created a similarity index, **SIM,** to represent these similarities. Additional information can be found in "A Similarity Index for Pitch-Class Sets" (Morris,1979).

[18] There does not appear to be a consistent bracketing convention for interval vectors. While pitch-class sets are almost always bracketed as [xxxxxx], and invariance vectors as <xxxxxxxx> I have seen ivs bracketed both ways. I am using the [xxxxxx] convention in my work. If in doubt as to the character of the bracketed material, remember that hexachord pitch-class sets will be comprised of six different integers rising from lowest to highest, interval vectors will often have replicating integers/numbers and these integer/numbers will never be greater than the integer/number 6.

clarity and the challenges of creating with one hexachord (with the occasional addition of that hexachord's z-mate).

Nonetheless, a masterful example of multiple hexachord use can be found in the fourth movement of Schoenberg's Opus 23. David Lefkowitz observes that this movement employs five different hexachords in three different groupings: 6-44 [012569] and 6-19 [013478], which are z-mates (same interval content); 6-39 [023458] and 6-10 [013457], which are also z-mates; and 6-14 [013458], which has no z-mate. Although Schoenberg assigns different roles to the three groupings, there remains a great deal of ambiguity in the piece, in part, due to the similarity of the hexachords. Additionally, the combined hexachord collections are subsets of larger heptatonic collections. 7-37 [0134578] for instance contains 6-10 [013457], 6-14 [013458], and 6-19 [013478] (Lefkowitz 1997, 310,334). (6-14 and 6-19 are maximally related.) It is worth noting that this piece was created just at the time Schoenberg was formalizing his own ideas about twelve-tone music, and at the time his friend (soon to be ex-friend) Josef Matthias Hauer, who had already published his own writings about twelve-tone music, was working with hexachord pairs. Clearly both composers realized that the hexachord was an integral part of their explorations.

Integral Serialism

Despite the burgeoning interest in twelve-tone music that followed WWII, Boulez, Stockhausen, and other young composers already considered Schoenberg's method something that reinforced "outdated forms of tradition, the outworn aesthetics of German Romanticism." A strong desire to expand and improve upon the past led to the expansion of serial technique beyond what Stiles (1979, 97) refers to as "monodimensional serialism". By the late 1940s,

composers on both sides of the Atlantic were applying what came to be known as total or *integral serialism* to various elements of music.

When a twelve-tone row is considered as a sequence of integers, the numeric values may be extracted from the row and used to control other elements of the music. What to control, and how to apply that control, are choices each composer makes. Rhythm, timbre, dynamics, instrumentation – any aspect of the performance – may now be manipulated and guided by a series of numbers (Griffiths 1985, 13, 34-37).

As the controlled aspects of the music expand, the possibility of accurate performance becomes more challenging. In 1953 Krenek (525) contends that while Boulez's approach to integrated serialism is new and exciting, it contains "a degree of rhythmic complexity that almost defies description as well as accurate execution." It appears, according to Stiles, that by pushing the boundaries of what was playable, Boulez anticipated performers would add the element of chance by their imprecise performance (Stiles, 98).

While integral serialism exhibits a great deal of control, the choice of *what* to control and *how* to control it is, according to some critics, arbitrary. When applying serialism to parameters other than pitch "...these numbers detach themselves from the objects with which they were associated (the tone row) and take on a life of their own in the various operations performed" (Krenek 1960, 219). Although present, does this subtle underlying organic connection become irrelevant? Perhaps its usefulness evaporates at a certain level of complexity. Further, Krenek argues that integral serialism stifles creativity – the more parameters controlled by serial applications, the less chance or creative inspiration is present. The only creative decisions made with fully integrated serialism are in the creation of the row and the choice of how the parameters are applied. After that, any unpredictable or serendipitous results are pre-determined.

"The unexpected happens by necessity. The surprise is built in" (229).

Indeed, the variety and complexity of the transformations and derivations performed on a row could all but render the original ordering meaningless. Discussing Boulez's *Le Marteau sans maître* Griffiths observes:

> [T]he apparatus of total serialism remains... but as a serial construction the work is to a large degree impenetrable. The techniques are so various and versatile that a pitch series can give rise to almost any configuration. (Griffiths 1985, 89-90)

Griffiths (1985,77) points out that because of the complexity of integral serialism, many young mid-century composers were attracted to electronic media that afforded much more precision in controlling rhythm, pitch volume, dynamics etc. At that time, this included an early version of the programmable synthesizer. "Oscillators and noise generators provide the raw materials which the composer, giving the synthesizer its instructions on a punched paper roll, can obtain at will with a high degree of control over pitch, timbre and volume" (158-59.)

By the early 1960s the "anything goes" attitude that had guided and occupied most European composers gave way to frustration in at least some of the musical elite. As Griffiths (138) observes, despite some promising efforts "... by 1962... it was plain that the possibility of a common language based on the principles of serialism was becoming ever more remote." Some composers, in reaction to the overwhelming challenge (and perhaps the absurdity) of total integrated serialism appear to have moved in the opposite direction. Consider, for example, Ligeti's *Lux Aeterna,* (1968) which blurs both harmony and rhythm. "Ligeti, Xanakis, Kagel, Bussotti and others – were opening up terrain which could neither be accommodated within serialism nor ignored" (143). In addition, Danuser observes that in the 1970s a younger generation of composers "weary of the obligation imposed on them by the avant-garde" found a return to Schoenberg's "artistic morality" more appealing. (Brand et al 1997,185).

Other Extended Techniques

When a row is converted to integers, operations of *multiplication* can be applied to generate new material. However, only multiplication by 1, 11, 5, or 7 will produce a twelve-note row without pitch-class duplications. Multiplication by 1 replicates the original row, by 11 produces the inversion of the original row, by 5 yields a cycle of fourths transformation (converts a chromatic row into the cycle of fourths) and by 7 produces a cycle of fifths transformation (converts a chromatic row into the cycle of fifths). Multiplication by 1 and 11 will preserve the interval content of a row; multiplication by 5 or 7 produces a different row with different interval content. While the practicality of this may seem trivial, multiplication *does* produce invariance – every hexachord when multiplied by 5 or 7, maps into another hexachord or itself. For many composers, this discrete, often concealed, connection of source material is crucial, providing order, and structure (Rahn 1980, 54-55).

A different method of multiplication, notably used by Pierre Boulez, may be applied to smaller groups of pitch-class sets. *Simple multiplication* of a triad by a dyad, for example, is achieved by adding, not multiplying, the set-class numbers of each segment, or as Straus explains, simply by transposing the defined segments of a row to the pitches of another segment. Variations of this technique, *Compound* and *complex multiplication,* produce sets even further removed from the ordering and interval relationships of the original row. Creating, manipulating, and recognizing these techniques bespeaks the skills of a mathematician as much as that of a composer. Stephen Heinemann, for instance, defines *complex multiplication* as "...transposition of the simple multiplicative product by the ordered pitch-class interval from the transpositional-determining constant of the pitch-class of the multiplicand" (Heinemann 1998, 87).

Derivation may be used to generate a complete row from (usually) three-note pitch-class

sets. Various transpositions and/or inversions of a collection are selected such that the aggregate is formed. In Webern's *Concerto for Nine Instruments* Opus 24 a row is generated from four trichords chosen from **P, RI, R** and **I** forms of set-class 014. This produces in Webern's work an "intervallic economy" and ensures that the structures that created the series become the "…basic musical motif of the composition" (Straus 2005, 249-50). Babbitt also favours this technique in his earlier works (1947-1960).

Melodic and harmonic material may be derived from *cross partitions* – created by dividing a twelve-tone row into smaller groups. As described by Brian Alegant "…a cross partition arranges the pitch-classes of an aggregate (or a row) into a rectangular design. Typically, the vertical columns… are derived from the source row's segments, whereas the horizontal rows… contain non-adjacent elements of the source row." Varieties of cross partitions may be identified by integers with exponents. The integer designating the number of vertical elements and the exponent revealing the number of horizontal elements (Alegant, 2010, 20).

A 4^3 partition of the row 0,1,2,3,4,5,6,7,8,9,t,e could be formed as such:

0, 3, 6

9 ,1, 4

7, t, 2

5, 8, e

While a 2^6 partition of the same row could be presented as:

0, 6, 1, 7, 2, 8

3, 9, 4, t, 5, e

Variants may be created by reordering each vertical. Notice that a 2^6 partition is a division of the row into two hexachords.

While invariance denotes the replication of a segment, *combinatoriality* is concerned with

the completion of the aggregate. The two concepts are of course related, for when a segment is invariant the possibility exists that the aggregate may be completed by combining a segment of the original row, often a hexachord, with a segment of a transformation of the row, often another hexachord. By creating twelve-tone rows that contain semi-combinatorial hexachords a composer may assemble the 48 versions of the row into groups that contain the same hexachordal content. “For Schoenberg in particular, these twelve-tone areas are something like keys in a tonal piece. Large scale motion in his music often involves movement from area to area” (Straus 2005, 227). As discussed above, additional invariant material may be created by operations of multiplication.

Displaying hexachordal technique within twelve-tone technique, Alegant (2010, 52) reveals the use of hexachord content as a determining structural compositional device in Dallapiccola’s *Cinque canti* (1956). By creating a row that is **RI** invariant using two 6-30 hexachords “...multiple labels exist for each row... every **P**-x is equal to **RI**(x+9) and by extension every **I**-x is identical to **R**(x+3).” Note that the complement of this hexachord is also a 6-30 hexachord, and that Dallapiccola’s chosen row is a transposed and reordered version of the prime form of the hexachord – the prime form of 6-30 is [013679], while **P0** for Dallapiccola is [0e5862].

The row contains a great deal of invariance. The two hexachords that form **P0** are replicated in seven other transformations within the set-complex – a total of eight instances. Thus the 48 iterations of the row are “reduced” to 6 groups of identical, but unordered hexachords.

In the following example the matrix displays the two 6-30 hexachords paired in **P0** as [0e5862] and [7314t9]. The instances of [0e5862] are bracketed. If the rows/hexachords are ordered, each **P** row may be used in retrograde, and each **I** column may be used in retrograde

inversion. Therefore, all pitch-classes of this version of the hexachord, [0e5862], are present in **P0, P6, I9**, and **I3** as well as in **R0, R6, RI9,** and **RI3**.

Figure 3: 6-30 Matrix

	I_0	I_{11}	I_5	I_8	I_6	I_2	I_7	I_3	I_1	I_4	I_{10}	I_9	
P_0	0	e	5	8	6	2	7	3	1	4	t	9	R_0
P_1	1	0	6	9	7	3	8	4	2	5	e	t	R_1
P_7	7	6	0	3	1	9	2	t	8	e	5	4	R_7
P_4	4	3	9	0	t	6	e	7	5	8	2	1	R_4
P_6	6	5	e	2	0	8	1	9	7	t	4	3	R_6
P_{10}	t	9	3	6	4	0	5	1	e	2	8	7	R_{10}
P_5	5	4	t	1	e	7	0	8	6	9	3	2	R_5
P_9	9	8	2	5	3	e	4	0	t	1	7	6	R_9
P_{11}	e	t	4	7	5	1	6	2	0	3	9	8	R_{11}
P_8	8	7	1	4	2	t	3	e	9	0	6	5	R_8
P_2	2	1	7	t	8	4	9	5	3	6	0	e	R_2
P_3	3	2	8	e	9	5	t	6	4	7	1	0	R_3
	RI_0	RI_{11}	RI_5	RI_8	RI_6	RI_2	RI_7	RI_3	RI_1	RI_4	RI_{10}	RI_9	

The invariance vector for this hexachord is <20020220>. The first position in the vector displays the number of times the chosen hexachord is invariant through transposition – this number includes the original statement of the hexachord, hence the first number in the vector will always be at least 1. In this case [0e5862] occurs twice as **P0** and **P6**. The second position of the vector presents the number of times the hexachord occurs in the inversions of the original hexachord – in this case 0. The fifth position shows the number of times the original hexachord is invariant in the complement of the transpositions of the original hexachord - 0, and the sixth position displays the number of times the original hexachord occurs in the complement of the inversions of the original complementary hexachord– twice as **RI9** and **RI3.** The third, fourth seventh and eighth positions show invariance through multiplication.

Most of these extended techniques may be difficult or even impossible for the listener to detect. Wuorinen observes that the use of combinatoriality in a serial composition may be desirable but doubts whether such completion of the aggregate is always perceived. Nonetheless, he acknowledges: "Just because perception is not directly involved here does not mean that a composer who employs these relations will not be constructively disciplined by using them... they may be of enormous structural and methodological significance" (Wuorinen 1979, 125-8). At times these techniques are *intentionally* concealed. Stravinsky challenged anyone to discover his pre-composition method for the flute solo in *Movements*. Indeed, the variety and complexity of the transformations performed on a row may be invisible and perhaps meaningless to anyone but the composer.

The Third Steam Movement and Jazz

Scruton (1999, 239) observes that "the search for an alternative musical order defines the work of the composer in our time, and sets the agenda for every creative gesture." The quest for something new, rare, challenging, and unique is of course at the heart of the creative spirit, and many adventurous composers of jazz have been enticed by the possibilities of serialism.

Without doubt, there has long been an exchange of ideas between jazz and classical music. However, when classical composers borrowed from jazz in the early twentieth century, they merely integrated some of the "external trappings." Most treated jazz as folk or ethnic music, fancifully borrowing gestures and elements that appealed, without really understanding the music. Stravinsky's first exposure to jazz was through sheet music; he began using the syncopated rhythms of the genre before hearing the music played by jazz musicians. For most of these classical composers, attempts to combine the two genres failed to capture the "essential spirit of jazz" (Banks 1970, 596).

Composer and author Gunther Schuller coined the term *third stream* in 1957 to identify a genre of music that fused jazz with western art music. Third stream placed an emphasis on jazz improvisation and, for Schuller, serialism. His original definition stressed that this hybrid was separate from both jazz *and* classical music. It was a distinct musical genre that should be judged on its own.

> The standards of jazz, per se, are not applicable to third stream…. The third stream work does not to wish to be heard as jazz alone…third stream music must be born out of respect for and full dedication to *both* the musics it attempts to fuse. (Schuller 1986, 116)

Modern Jazz Concert, a recording showcasing the third stream movement was released in 1958 featuring jazz composers Jimmy Giuffre, Charles Mingus and George Russell, along with classical composers Harold Shapero, Milton Babbitt and Schuller. While Schuller and Babbitt manage to fuse the two genres with some success, the other contributing composers offer works that merely fluctuate between jazz and "classical" sections. The critic Richard Monaco observes:

> The intellectual demands on the composer of 'serious' music and the composer/performer of jazz are such that it would be a rare musician, indeed, who had the skill and experience in both kinds of music which is necessary to write completely convincing music. (Monaco 1968, 803)

An additional challenge facing *third stream* was the availability of musicians that could play the music. While Schuller's 1960 recording *Abstraction* features an elite ensemble including Jim Hall on guitar, Scott Lafaro on bass and Ornette Coleman on saxophone, very few jazz and classical players at that time had the skills necessary to function at a high level in *both* jazz and classical performance.

The third stream movement, as originally conceived by Schuller, never gained much acceptance. Audiences were often confused or challenged by the fusion of serial music and improvisation. Few composers could write it, almost no one could perform it, there was very

little audience for it, and most critics disliked it.[19]

Joyner, who claims that third stream existed from about 1945 until 1965, argues that it failed as a movement because of restrictions placed upon improvisers and its lack of swing. This judgement reveals another challenge. It is rare to find someone who can leave behind his or her classical or jazz bias when judging this music. From the classical perspective, jazz is too often restricted by conventional twelve, sixteen or thirty-two bar structures, and melodic phrases usually fall within two, four or eight bar groupings. From the jazz point of view, classical music, especially twentieth century classical music can be dense, dissonant, and rhythmically un-cohesive, (it doesn't groove). Moreover, there is little room for personal expression or divergence from a strictly prescribed set of directions. Third stream, for the jazz world, presents a threat to the idea of swing; it limits improvisation and is overly concerned with composition and form. Additionally, classical musicians, unfamiliar with the rhythmic subtlety of jazz, impede the groove (Joyner 2000,73-86)

In his 1973 book *Modern Twelve-Tone Technique,* the Canadian jazz educator Gordon Delamont presents basic serial techniques for the jazz arranger and composer. Delamont recognizes twelve tone writing as "the most important compositional development of the 20th century." In his book he presents Schoenberg's method but reminds us that "...it is *only* a technique which, like all others, should be regarded as a servant of the composer and not his master" (Delamont 1973, iii, 26).

Offering a completely different way of considering twelve tones in 1959, George Russell's acclaimed but confusing *Lydian Chromatic Concept for Tonal Organization* is in some ways an

[19] The term *third stream* is now often used to refer to any combination of jazz and classical music. Curiously, Schuller and Ran Blake, while running the third stream program at the New England Conservatory in the 1970s, changed the definition of third stream to include fusions of *all musical genres*. (Blake 1976, 30-32, Blake 1981, 13)

emancipation of the chromatic scale. All twelve tones create a "tonal universe" within any tonal center. Each mode of the chromatic scale becomes a key unto itself. The concept is, according to Russell, "...free of rules, laws and biases. There are no wrong notes or progressions" (Russell 2001, Technical Appendix B).

With the book *Twelve-Tone Improvisation,* John O'Gallagher presents trichord cells with the same interval content as an improvisation resource for jazz musicians. In the manner of Webern and Babbitt, trichords are used to generate complete twelve-tone rows. O'Gallagher offers a method in which the trichords and rows are practiced and incorporated as part of the improviser's language. He contends that this improvisation method can be applied to twelve-tone, atonal, and tonal music. O'Gallagher's 2012 recording *The Anton Webern Project* demonstrates, through his reinterpretation of numerous works by Webern, his use of this technique (O'Gallagher 2013, 8). Of striking similarity is *My Music: Explorations In The Application of 12 Tone Techniques To Jazz Composition And Improvisation*, a method book presented in 2003 by educator, composer and guitarist Bruce Arnold.[20] Here as well, four trichords of the same set class are used to build a twelve-tone row, and these three-note cells provide material for improvisation and composition (Arnold 2003).

Jerry Bergonzi's *Hexatonics* (2006) presents an improvisation method based upon what appear to be hexatonic scales (hexachords). He offers a technique that pairs various triads, creates patterns from the combinations, and offers these patterns as an improvisational resource – a resource that may be applied to tonal chord progressions. Although some of the triad pairs do form six-note collections,[21] many of the combinations, because of common tones between triad

[20] Despite the lengthy title, the book is surprisingly short, offering only a few cursory explanations of his method.

[21] Combining a C major triad with each of the eleven other major triads yields just three hexachords: C/B and C/Db = 6-19, C/Bb and C/D = 6-33, C/F# =6-30. All other major chord combinations, because of common tones, yield five note collections. Bergonzi similarly combines each major triad with eleven minor triads, yielding a combination

pairs, form five-note or four-note collections. Because the method is concerned with common tonal triads, only a limited number of the fifty possible hexachords are called upon. The book, while offering colourful dynamic patterns for the adventurous improviser, does not provide further guidance in hexachordal technique.

Some contemporary improvisers and composers appear to have arrived quite independently at that their own twelve-tone method. For example, Ron Jarzombeck, a "technical metal" composer/guitarist, creates patterns and chord progressions from a twelve-tone row or "key" by placing an ordered row onto a clock-face. The clock-face serves as a visual aid to facilitate various methods of segmentation. These segments are then used as riffs, chord progressions and melodic material for his compositions and improvisations (Jarzombeck n.d; *The Circle of 12 Tones* from artist's website).

Multi-Serialism

Alan Lessem notes that many mid-century North American composers, seeking instruction from Schoenberg and other European émigré composers, sought a quick introduction to twelve-tone technique. Nonetheless, Schoenberg insisted that a prerequisite to such tuition was a firm grasp of (his version of) the fundamentals of western art music – a task that most were not prepared to undertake. (Brand et al 1997, 63-64). In the 1960s, however, a number of influential Hollywood screen composers, including Alexander Courage, Earle Hagen, Quincy Jones, and Randy Newman, embraced the teachings of George Tremblay, an alternative serialist who made a point of stressing how quickly his techniques could yield results.

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of six, five, and four-note collections.

George Tremblay (1911-1982), a Canadian/American composer was in fact a student and associate of Schoenberg. He introduced an expanded and complex version of serialism that he called *multi-serialism*. Tremblay's method involves the creation of many related twelve-tone rows by an intricate method of exchanging notes between the hexachords of an original twelve-tone row. One complete cycle of this process produces 288 rows, i.e. the original row and 287 related variations. Numerous rows are used in each composition, unified by their evolution from the original row.

Tremblay presented his method as a quick and easy resource for composers in a hurry. This is, however, a bit misleading – unless a composer references the sample rows laid out in Tremblay's book, or has already prepared a different row, the mapping out of all 288 rows (the full cycle of sets from any original row) takes many hours of tedious work. Although these rows are generated by the manipulation of one original hexachord pair, the working cycle contains a great variety of different hexachords.

Tremblay's method bypasses the transformations of inversion and transposition – the immense number of derived rows replacing the need for variations of a single row. Although presented as an extension of serialism, each derived row in this technique disrupts the ordering of the original row as well as the intervallic structure of each original hexachord - the very things that provide character, identity and unity to most twelve-tone music (Tremblay 1974).

It should be noted that a significant number of Hollywood composers did pursue a more traditional approach to serialism. Although it is difficult to imagine twelve-tone music scoring light entertainment or comedy,[22] several dramatic projects from the second half of the twentieth century have scores that are entirely or partially derived from the technique. Notable examples

[22] Schoenberg's early twelve-tone comedic opera *Von Heute auf Morgen* is a rare exception, although any lightness or comedy in the performance seems impeded rather than aided by the score.

include film scores by Leonard Rosenman *The Cobweb* (1955), Benjamin Frankel *The Prisoner* (1955), *Curse Of The Werewolf* (1961), Ernest Gold *On The Beach* (1959), Jerry Goldsmith *Planet of the Apes* (1968), Jerry Fielding *The Mechanic* (1972), David Shire *Taking Of Pelham One Two Three* (1974), Michael Small *The Parallax View* (1974), and Hans Werner Henze *Muriel* (1963) and *L'amour A Mort* (1984).

Leibowitz

Although overly enthusiastic in its presentation, for many composers Leibowitz's 1947 publication *Schoenberg et son école* along with the 1949 English translation, was the first significant source of information about the techniques of the Second Viennese School. It is noteworthy that Leibowitz revels in what he sees as the lack of traditional musical structure in the work of Schoenberg and Webern. For him the pre-eminence of motivic development transplants the forward motion of tonal harmony: "*Perpetual variation* can now be realized in a more consequential way, since musical structure will no longer be fettered by the requirement of symmetrical reprises and specific tonal regions..." (Leibowitz 1979,74).

Like other European composers of the mid-century, Leibowitz is inspired by Webern: "Berg always strove to establish a connection between the discoveries of Schoenberg and the past... Webern is concerned with the *possibilities for the future* inherent in this work, and thus succeeds in *projecting* its particular and radical elements" (Leibowitz 1979, 199).

He proposes that to fully realize twelve-tone technique there must be an *"ultimate transcendence"* of the tonal system. While transcendence might suggest an extension beyond that includes some of what has come before, Leibowitz seems to advocate complete abandonment, arguing that "the spirit of variation" is impeded by "even the most 'free' and 'vague' tonal functions" (277).

A review of *Schoenberg et son école* from 1950 by Russell Hill (443-5) refers to Leibowitz's "delirious enthusiasm for twelve-tone music." Looking back from the 1970s, Paul Griffiths (1976, 741-2) is quite dismissive of Leibowitz, calling his book a "seductive manifesto" based on "shady assumptions and atrocious logic." Peter Evans, also writing in the 1970s, is more generous with the work: "...recent history may have appeared to confirm that Schoenberg is dead, Leibowitz shows how powerfully he was alive" (1971, 443-4.) John Covach (2002, 620) offers a more recent perspective: "The importance of Leibowitz's extensive writing in the mid and late 1940s is that it establishes a kind of post-WWII dodecaphonic orthodoxy, based on the Second Viennese School and focused on the primacy of pitch relations, against which composers and theorists would subsequently react."

Two Categories of Literature

There exists an immense body of literature concerned with hexachordal technique, analysis, and composition. Most of this work, however, deals only with hexachords as a component of serial composition. Nonetheless, the techniques and analyses presented in this literature provide the primary resource for my research.

Relevant literature may be generally divided into two categories. First are works which establish an overview of specific aspects of non-functional harmony in western art music. Included in these works are essential texts on pitch-class set theory, atonal composition and analysis, and twelve-tone composition and analysis. Texts include Allen Forte's *The Structure of Atonal Music* (1973), Robert Morris's *Class Notes for Atonal Music Theory* (1991), Joseph Straus's *Introduction to Post-Tonal Theory* (2005), John Rahn's *Basic Atonal Theory* (1980) and Charles Wuorinen's *Simple Composition* (1979).

Although generally acknowledged to be the originator of twelve-tone music in the early

1920s, Arnold Schoenberg never produced a book outlining his twelve-tone technique. Decades later he did provide an outline of his method in a series of essays included in *Style and idea: Selected Writings of Arnold Schoenberg* (2010). For many composers the first useful text on twelve-tone technique was René Leibowitz's *Schoenberg et son école* (1949).

In addition, theoretical and pedagogical works outside the realm of classical music offer an important backdrop. However, very little has been published on the use of twelve-tone technique in genres other than western art music. As previously noted, George Tremblay's work on multi-serialism *The Definitive Cycle Of The Twelve Tone Row* (1974), George Russell's *Lydian Chromatic Concept Of Tonal Organization* (1959), Gordon Delamont's *Modern Twelve-Tone Technique* (1973) and John O'Gallagher's *Twelve-Tone Improvisation* (2013) are among a few offerings aimed at the jazz/film composer or improviser.

Also included in this first category are works dealing with the aesthetics of twelve-tone and atonal music. Among these are Roger Scruton's *The Aesthetics Of Music* (1997), Carl Dahlhaus's *Schoenberg And The New Music: Essays* (1987), Theodor Adorno's *Philosophy Of New Music* (2006), and Paul Griffiths's *Modern Music: The Avant Garde Since 1945* (1985).

A second category of literature deals specifically with hexachords in the context of twelve-tone composition. Most of this literature is in the form of academic papers. Significant to my research are Douglas Rust's work on hexachord rotational – "Stravinsky's Twelve-Tone Loom: Composition and Pre-composition in *Movements*" (1994), Joseph Straus's "Stravinsky's 'Construction of Twelve Verticals': An Aspect of Harmony in the Serial Music" (1999), George Rochberg's "The Hexachord and Its Relation to the 12-Tone Row" (1955) and "The Harmonic Tendency of the Hexachord" (1959), and David Lefkowitz's "Listening Strategies and Hexachordal Combinatorial 'Functions' in Schoenberg's *Op. 23 No. 4*." (1997).

Also notable are works by John Covach – "The Music And Theories Of Josef Matthias Hauer" (1990) and Dominik Sedivy – *Serial Composition and Tonality: An Introduction to the Music of Hauer and Steinbauer* (2011).

Because exclusive hexachord composition is rare, works that deal specifically with hexachords as a compositional resource separate from serialism are uncommon. Nowhere in my research have I found any investigation into the use of hexachords as an exclusive compositional resource in genres other than western art music.

Many audio recordings and scores have been referenced, and studied as part of this research, and it is important to include a select few of them here. Stefan Wolpe was one of the few twentieth century composers to realize the potential of hexachords as an exclusive compositional resource – demonstrated in *Suite im Hexachord (for Oboe and Clarinet)* (1936). Transposed rotational arrays, a manipulation of hexachords introduced by Ernst Krenek are employed in the late works of Stravinsky – *Movements for Piano and Orchestra* (1960), *Requiem Canticles* (1966) and *Agon* (1957).

In addition, because hexachords are so closely related to twelve-tone music, the work of the Second Viennese School is essential to this research: the music of Webern, for its clarity *Variations for Orchestra , Op. 30* (1940), *Symphony, Op. 21* (1927-28); Schoenberg for the use of invariant hexachords – *Suite, Op. 29* (1925), *Ode to Napoleon, Op. 41* (1942); and Berg for the combination of twelve-tone technique with functional harmony – *Lyric Suite* (1926), *Violin Concerto* (1935). The works of a number of mid twentieth century composers are also relevant to this research including those of Luigi Dallapiccola – *Piccola Musica Notturna* (1954), and the unique applications of integral serialism in the music of Robert Gerhard. Because my interest extends beyond western art music, works that employ twelve-tone or hexachordal technique in

film music and jazz are also referenced. Examples include the previously noted David Shire score for *The Taking Of Pelham One Two Three* (1974), Michael Small's score for *The Parallax View* (1974), and Darcy James Argue's *Real Enemies* (2016).

PART TWO

My Compositional Practice

The techniques presented in my research are techniques that were originally created almost exclusively for use in a twelve-tone environment. Because exclusive hexachordal composition is rare, the use of these techniques in this context is similarly unique. In the following pages, adaptations of these techniques are explained and illustrated through a number of original compositions.

When I began my research, I made a decision, as part of my investigation, to compose a piece for jazz ensemble,[23] and a piece for chamber ensemble for each of the fifty possible hexachords, using only the chosen six-note collection for the entire piece. In other words, one source set is used to generate all of the pitch material for each composition.

In this dissertation, emphasis is placed on my hexachordal compositions that display the use of specific techniques, as well as hexachords that provide unusual challenges or characteristics. There are, of course, no explicit directives for composition with six-note collections. I have established in this body of work specific parameters that, for me, produce satisfying results. Unlike much twelve-tone composition, completion of the aggregate is not important, nor is employment of the complementary hexachord.

[23] As noted in my Glossary and Terms, I am reluctant to call these pieces "jazz". However, because the music is performed by a collection of instruments, and in a rhythmic manner, often associated with this genre – and usually involves improvisation – the affiliation is usually assumed by those that perform and hear this music.

The full scores for these compositions contain a great deal of information that is not relevant to our discussion. Thus, in most cases, only score excerpts that relate to a specific technique or characteristic are included. Complete audio recordings of each piece are included.[24]

Technical and Aesthetic Considerations

Many years ago, I came across a curious book of poems by Luis d'Antin Van Rooten entitled *Mots D'Heures: Gousses, Rames: The D'Antin Manuscript.* What initially appeared to be a collection of ancient French poems, was in fact a well disguised humorous homophonic rendering of *Mother Goose Rhymes,* delightfully presented such that an unaware reader might vocalize the French poem without ever realizing that he or she was at the same time reciting a nursery rhyme in English with a heavy French accent. The following example shows a few lines from one of these poems – a poem created in one language, yet also has a distinctly different meaning in another language.

Figure 4: Different Meanings

Heard as English	As Read	Heard as French
Humpty Dumpty Sat on a wall. Humpty Dumpty Had a great fall	*Un petit d'un petit* *S'étonne aux Halles* *Un petit d'un petit* *Ah! degrés te fallen* (Van Rooten,1980)	A child of a child Is surprised at the Market A child of a child Oh, degrees you needed! (Van Rooten,1980)

[24] In the case of any discrepancies between the recordings and scores, consider the scores to be correct.

With many of my hexachordal compositions a similar dualism exists. The pieces were created using serial techniques, and could certainly be analyzed, understood and critiqued from that perspective. They were also created with a distinct tonal bias, and most listeners would accept the works, despite their unusual "accent" to lie within this tonal landscape. Like Van Rooten's whimsical poems, they were created with, and have meaning in one language, yet make sense in another.

I am certainly not the first composer to explore the use of atonal techniques that may present as tonal. Ciro Scotto presents his own adaptation in his article "A Hybrid Compositional System: Pitch-Class Composition with Tonal Syntax." This system "…facilitates constructing complex and coherent musical structures, some which function in a manner similar or analogous to tonal systems (2000,171-3)." Scotto further states that his activity is by no means "…an implicit admission of failure on the part of atonal systems to create coherent and complex musical structures." He is rather seeking out of curiosity, exploring "a world that is at once familiar and alien (207)." David Lewin reverses the process in "A Formalized Theory of Generalized Tonal Functions" demonstrating that specific elements of tonal music may be applied to, and used to generate, non-tonal systems. As Lewin points out "Such systems, while shedding light on the analogous formal features of tonal theory, also suggest interesting possibilities for composition and analysis in their own right (Lewin 1982, 25)."

One of the assumed directives of twelve-tone composition is the strict ordering of notes (pitch-classes). When adhering to this rule, each rendering of the tone row, in transposed or inverted form, must maintain the prescribed sequence of notes. Consequently, the sequence of intervals is also maintained. It is this sequence of intervals, rather than the notes themselves, that provide the distinct character of each twelve-tone row. When the notes of a twelve-tone row are

unordered, the identity of the row is lost – the row is rendered meaningless. However, as noted earlier, if the two hexachords that make up the row are kept separate, a new, simpler, level of relationships is created – the notes within each hexachord may be unordered, yet the identity of each hexachord is maintained. The two hexachords that comprise the twelve-tone row have identical interval content, but not necessarily the same interval order.

Similarly, composing non-twelve-tone music with unordered hexachord collections allows a great deal of flexibility. Notes may be used in any order or repeated at any time, and some notes in the collection may be favored more than others. Despite this, the integrity and the distinct sound of the hexachord being used is maintained. Regardless of the ordering, as long as all six notes are employed, the intervallic structures that define a particular hexachord are all still present.

Many of the hexachordal pieces I have composed for jazz ensemble include improvisation. This presents special challenges for performers. More about improvisation, including technical aspects, may be found on page 114. Why would I want to emulate the harmonic world of jazz? Why not just forget about hexachords and enjoy the existing paradigm? This is perhaps, a much more complicated question than it may appear, and one that implies an even more elusive question –what is jazz? Nonetheless, we can hopefully acknowledge that jazz is a genre characterized by constant change and fusions. Yes, previous attempts to fuse jazz with serialism may not have been successful, but perhaps hexachords offer another, less austere, approach to such a combination. Additionally, composing with a familiar goal in mind, but using different building blocks produces unexpected results. Creativity is sparked, for this composer, when different rules and limitations are applied.

Being free of the constraints of functional harmony offers new challenges for the

composer. When each hexachord possesses unique harmonic and melodic possibilities, should the choices of the composer purposefully eliminate references to tonality? Many of these six-note collections contain triadic structures and/or provide intervallic sequences that also exist in major or minor scales. Frustrated by the unenthusiastic response to his twelve-tone and atonal works, Arnold Schoenberg (2010, 223) urged his audience to listen with new ears, to abandon a familiar tonal landscape that had evolved and endured for centuries. As discussed earlier, Roger Scruton (1999, 308) argues that this realignment of tonal prejudice is all but impossible for anyone born in the west to achieve. Consequently, despite best intentions and efforts, the listener will always seek a tonal center, a musical resolution. In accord with Scruton's argument, my work demonstrates that hexachords can work apart from serialism while inhabiting a unique realm that may suggest a tonal center yet is distinctly different from that of functional tonality.

Pre-composition

There are, of course, no defined rules for hexachordal composition. The steps that I am outlining are an applied method of my own design, adapted from the techniques and protocols of twelve-tone music and atonal theory. While it would be possible for a composer to simply choose any six notes from the chromatic scale and begin creating, there is an advantage gained by understanding the nature of the chosen hexachord. This is best realized by consulting the set-class tables laid out by Allen Forte or John Rahn. These tables list all possible hexachords in prime form and include each hexachord's interval vector and invariance vector and identify the complementary hexachord. It is assumed that the reader of these tables is familiar with integer notation and is able to identify the chosen hexachord.[25] (When composing I often switch

[25] There are a variety of useful online resources available that will identify a chosen collection, convert notes to integers, and allow the user to create a printable twelve-tone matrix etc.

between note names and integers when considering the hexachord I am working with.)

The harmonic language of functional harmony is concerned with the motion within one familiar seven-note collection and the excursions from and return to the chosen rendering of those seven notes – the home key. Key signatures and, for many, years of practicing innumerable variations of this collection on an instrument, reinforce this musical paradigm.

Each of the fifty different hexachords offers a unique, often unfamiliar, harmonic and intervallic articulation. Before composition begins, the chosen six-note collection is identified, the qualities of that hexachord are noted, and all transpositions and inversions are mapped onto a twelve-tone matrix. Mapping the chosen hexachord onto a twelve-tone matrix provides all transpositions and inversions, as well as all transpositions and inversions of the complementary hexachord.

Let us compare some of the characteristics of three quite different hexachords – 6-1, 6-24 and 6-35 and consider the challenges of composing with each.

Forte#	**Prime Form**	**Interval Vector**	**Invariance Vector**
6-1	[012345]	[543210]	<11001100>
6-24	[013468]	[233331]	<10000000>
6-35	[02468t]	[060603]	<66666666>

The prime form of the set-class displays the collection in its most compact form. As a compositional resource, the pitch-classes that make up the hexachord may be used in any order and in any octave.

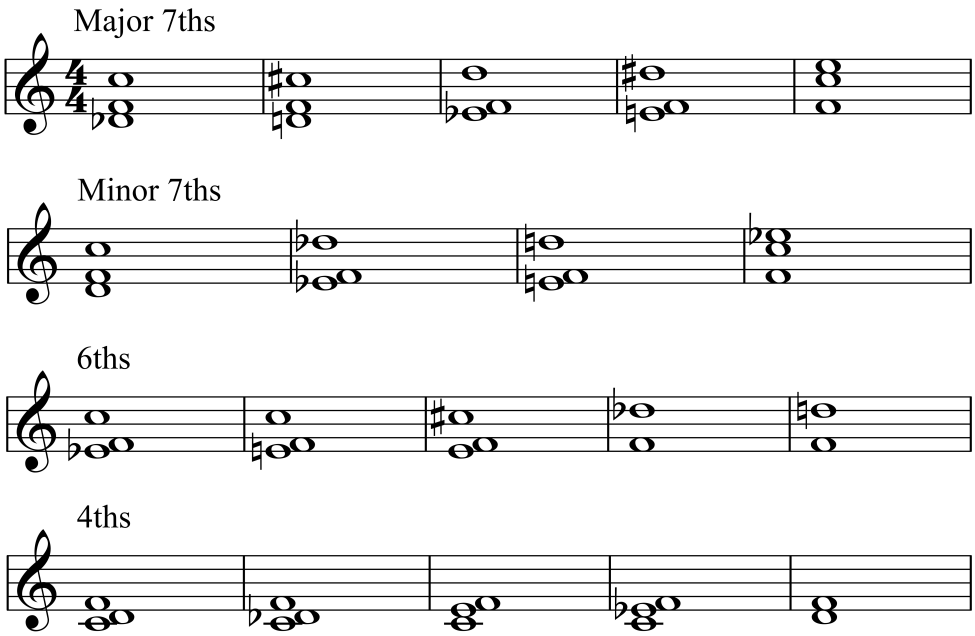
Although I am not trying to compose tonal music, it is, of course, difficult for a composer trained in western art music to abandon familiar tonal reference points when considering musical material. Through the lens of functional harmony each of these collections presents specific challenges. If we consider the interval content of each hexachord, we first see that hexachord 6-1

can easily be identified as a series of chromatic semi-tones, 6-24 appears to approximate a mode of the melodic minor scale, and 6-35 is the familiar whole-tone scale. The 6-24 collection contains at least one instance of each interval. 6-1, while containing five semi-tones, has no tritone, and 6-35 has no semi-tone, minor third or fourth intervals.

From the invariance vectors, we can see that 6-24 will not replicate itself in any transpositions or inversions. Hexachord 6-1 has a great deal of invariance – replicating itself not only by inversion but also in both a transposition and inversion of the complement. 6-35 possesses maximum invariance, the original replicating itself five times in prime form and presenting six times in inversion and six times in transposition and inversion of the complement.

Before composition begins, various harmonic and melodic resources are extracted from the chosen hexachord, creating a collection of available building blocks. Below is a partial list of vertical structures that could be built from **P0** of 6-1.

Figure 5: 6-1 Verticals



Composition

While exclusive hexachord use is closely related to twelve-tone composition and has been explored by various twentieth century composers of western art music, I have been unable to find a concentrated body of work, apart from my own research, that explores the nature of every possible six-note collection.[26]

After pre-composition, with reference materials close by, each composition is created using a digital audio workstation. Musical information is entered by a keyboard interface, and controllers that determine volume, articulations, and other musical variants. All experimentation, editing, arranging and orchestration is done in this digital environment. When required, this information is exported to scoring software. Most of the audio that accompanies this dissertation was produced in this manner.

How is harmonic and melodic interest created with just one six-note collection?
In simple functional harmony, motion is often created within a key, a journey away from and returning to a resting point – usually some combination of tonic, dominant and sub-dominant functions. Suggestions of, and full excursions to other keys bring more complexity and interest to a piece, but most often with the sense that a return will be made to the resting place established by the "home" key.

As we have seen, each hexachord contains a different set of intervals. While these intervals may occasionally suggest harmonic formations familiar to functional harmony they rarely provide, nor should they be expected to provide, the opportunity to create harmonic motion familiar to that system. Despite the absence of functional harmony, transposition and inversion provide a primary method of developing and expanding material. Further, in a number of my

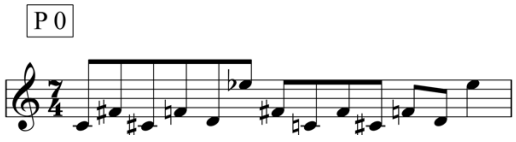
[26] While I have composed at least two pieces using each of the fifty hexachords, only selected compositions are included in this dissertation.

compositions, frequent use of these operations – changing transpositions or inversions of the hexachord every bar or two – creates harmonic motion similar to a progression of chords in tonal music. In other pieces, longer passages may remain in one transposition of the hexachord for many bars, a contrasting section established by transposition, and/or inversion.

In my work I create motion on three different levels:

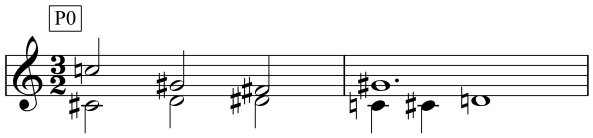
- Level 1 – simple linear motion outlining some or all intervals within the hexachord

Figure 6: Hexachord 6-3 [012356] – Level 1



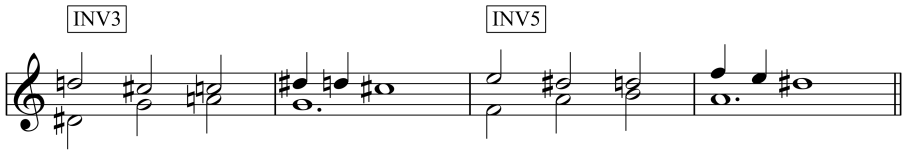
- Level 1 – simple linear motion outlining some or all intervals within the hexachord

Figure 7: Hexachord 6-41 [012368] – Level 2



- Level 3 – Level 1 and/or Level 2 structures may be moved through different transpositions and/or inversions of the hexachord to create another level of progression

Figure 8: Hexachord 6-41 [012368] – Level 2 & 3



My composition method often involves the creation and manipulation of similar structures which I think of as harmonic cells. These are usually short statements ranging in length from a few beats to a few bars. In tonal environments these cells would perhaps be sequenced through a single key or moved through a series of key centers. In hexachordal environments these cells also work well. The examples below use again the simple two-bar cell created from the hexachord 6-41 [012368] followed by the same cell transposed to **P3** and **P5**. All six notes of the collection are used.

Figure 9: Two-Bar Cell 6-41

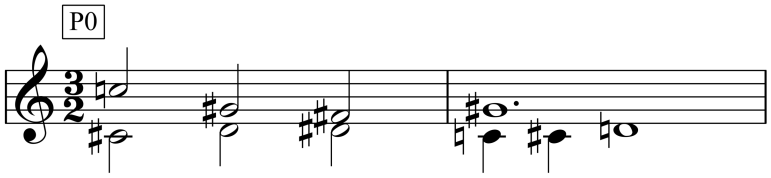
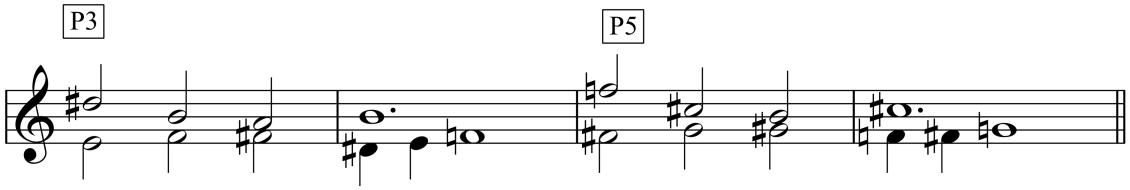
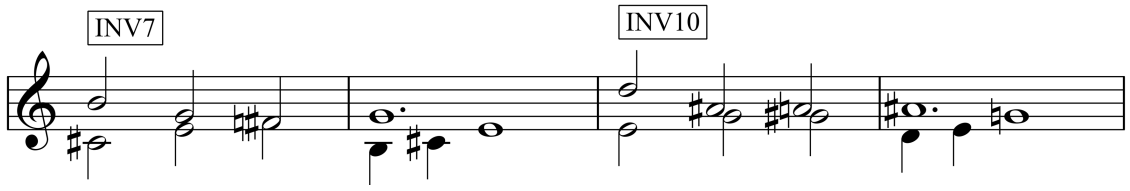


Figure 10: Two Transpositions of Cell



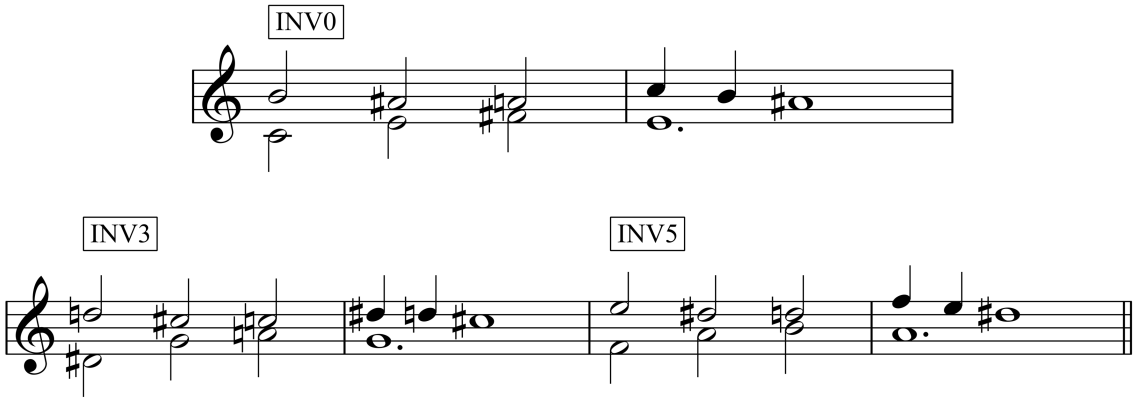
When the hexachord (not the cell) is inverted, it is possible to imitate the structure and rhythm of the cell.

Figure 11: Shape and Rhythm of Cell Applied to Inversion



Additionally, the intervals of the original cell may be inverted and, if desired, transposed.

Figure 12: Inversions of P0, P3, and P5 Cells



Motion is possible, then, within each hexachord *and* by the operations performed on the hexachord. Within each piece the transpositions and inversions of the hexachord being used are analogous to chords. The movements between these transformations are similar to chord progressions, but they are multi-dimensional. The notes within each transposition may be ordered in any way, different combinations of the six notes may be used, and because not all of the six notes need to be used at once, "sub-progressions" of chords or harmonic motion may be created within each transformation.

Of course, each hexachord offers distinct challenges and opportunities. My composition process always involves experimentation, with the hope that I may take advantage of some unique characteristic. While numerous techniques and characteristics may be revealed in each

piece, the following compositions are divided into four general categories – Invariance, Rotational Arrays, Various Techniques and Characteristics, and Improvisation.

INVARIANCE

A set is invariant if it remains unchanged after transformation by transposition, inversion, transposed-inversion or multiplication. Why is invariance an important consideration? When a hexachord is invariant, specific transformations of the hexachord are equally invariant and consequently the number of hexachords with variant pitch-class sets is reduced. There is a certain appeal and convenience provided by limiting the number of available hexachords. As discussed earlier, this quality was exploited by Schoenberg, Hauer and others. However, the greater the invariance, the less opportunity exists for harmonic variance – a sameness and lack of motion can persist.

Some hexachords that present a great deal of invariance are familiar components of western art music. 6-35 [02468t] <66666666> is familiar to most as the whole-tone scale, while 6-20 [014589] <33333333> is known to jazz musicians as the symmetrical augmented scale. Others like 6-13 [013467] <110000000>, 6-23 [023568] <11110000> and 6-30 [013679] <20020220>, while less invariant, replicate the symmetrical formations of the octatonic scale. The familiar whole-tone collection, the most invariant of all six-note collections, was probably first envisioned by equally dividing the octave, not by an investigation into six-note collections. Similarly, the six-note symmetrical augmented and the octatonic collections, are also products of different equal divisions of the equal-tempered chromatic scale. Clearly, just as these collections may be analyzed or considered in different ways, they may be derived in different ways.

The Girl With the Trees in Her Eyes

6-20 [014589]

Interval Vector [303630]

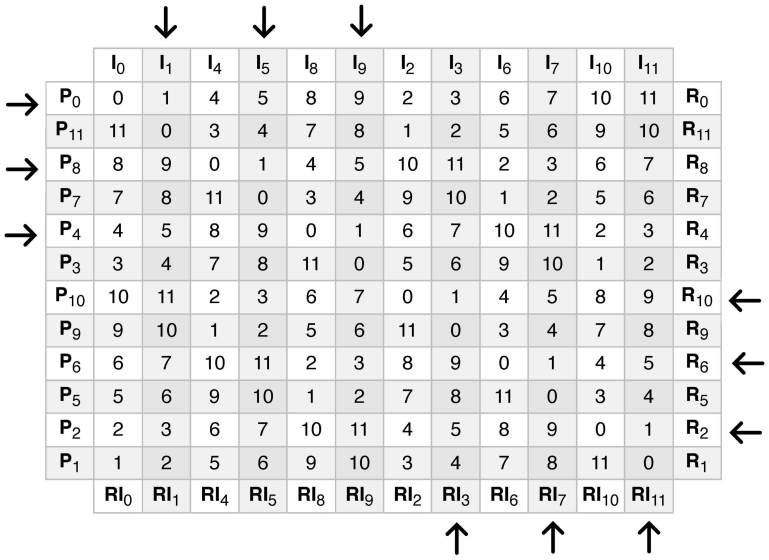
Invariance Vector <33333333>

Characteristic: Invariance

The Girl With the Trees in Her Eyes was originally composed to accompany a poem by the same name. My wish as a composer was to capture some of the wonder and mystery expressed in the poetry and enhance the recorded interpretation, using only the prescribed hexachord. While I have a great deal of experience scoring the spoken word, such limitations are both challenging and fascinating.

Within a twelve-tone matrix it can be clearly seen that each transposition or inversion of the 6-20 hexachord is replicated five times through transposition and/or inversion. The arrows in figure 13 show the original **P0** hexachord along with each additional occurrence within the matrix. The prime form of the hexachord has the same pitch content when transposed up a major third or minor sixth (Pn = Pn+4 = Pn+8). The prime form of the hexachord also has the same pitch content when transposed and inverted – up a minor second, perfect fourth, or major sixth (Pn = Inv n+1= Inv n+5= Inv n+ 9). Note that the content of the original **P0** hexachord is also replicated six times in the complement of the inversion. Thus, each hexachord occurs twelve times within each matrix and any hexachord within the matrix comprises one of only four different pitch-class collections [014589], [2367te], [e03478], or [12569t].

Figure 13: Matrix 6-20 – Occurrence of [014589] Collection



	I_0	I_1	I_4	I_5	I_8	I_9	I_2	I_3	I_6	I_7	I_{10}	I_{11}	
P_0	0	1	4	5	8	9	2	3	6	7	10	11	R_0
P_{11}	11	0	3	4	7	8	1	2	5	6	9	10	R_{11}
P_8	8	9	0	1	4	5	10	11	2	3	6	7	R_8
P_7	7	8	11	0	3	4	9	10	1	2	5	6	R_7
P_4	4	5	8	9	0	1	6	7	10	11	2	3	R_4
P_3	3	4	7	8	11	0	5	6	9	10	1	2	R_3
P_{10}	10	11	2	3	6	7	0	1	4	5	8	9	R_{10}
P_9	9	10	1	2	5	6	11	0	3	4	7	8	R_9
P_6	6	7	10	11	2	3	8	9	0	1	4	5	R_6
P_5	5	6	9	10	1	2	7	8	11	0	3	4	R_5
P_2	2	3	6	7	10	11	4	5	8	9	0	1	R_2
P_1	1	2	5	6	9	10	3	4	7	8	11	0	R_1
	RI_0	RI_1	RI_4	RI_5	RI_8	RI_9	RI_2	RI_3	RI_6	RI_7	RI_{10}	RI_{11}	

This hexachord contains a pitch-class collection that may also be identified as the symmetrical augmented scale or a truncation of Messiaen's third mode of limited transposition.[27] The 6-20 hexachord also presents the opportunity to build a number of vertical structures in thirds. Each transposition of the hexachord, and of course all of its invariant replications**,** contains three major seventh chords, and three minor-major seventh chords. When each is inverted, three different major seventh and three different minor-major seventh chords are revealed.

There is a minimum amount of harmonic movement in this piece. Each presentation of the hexachord is actually the presentation of six versions of the hexachord.

This hexachord in particular suggests tonality – no other hexachord contains as many

[27] Messiaen's third mode, (C, D, Eb, E, Gb, G, Ab, Bb, B), rendered in prime form is 9-12 [01245689t]. Compare this to 6-20 [014589].

major and minor-third intervals (the 6-35 hexachord, the whole tone collection, also has six major-third intervals but contains no minor-thirds). While tonal gravity is suggested, the piece presents as both major and minor. Further, as outlined above, because of the symmetry and invariance of the collection, each hexachord contains the same pitch-class set as five other transformations of the 6-20 hexachord.

The piece may be divided into four sections, followed by a reprise of the first:

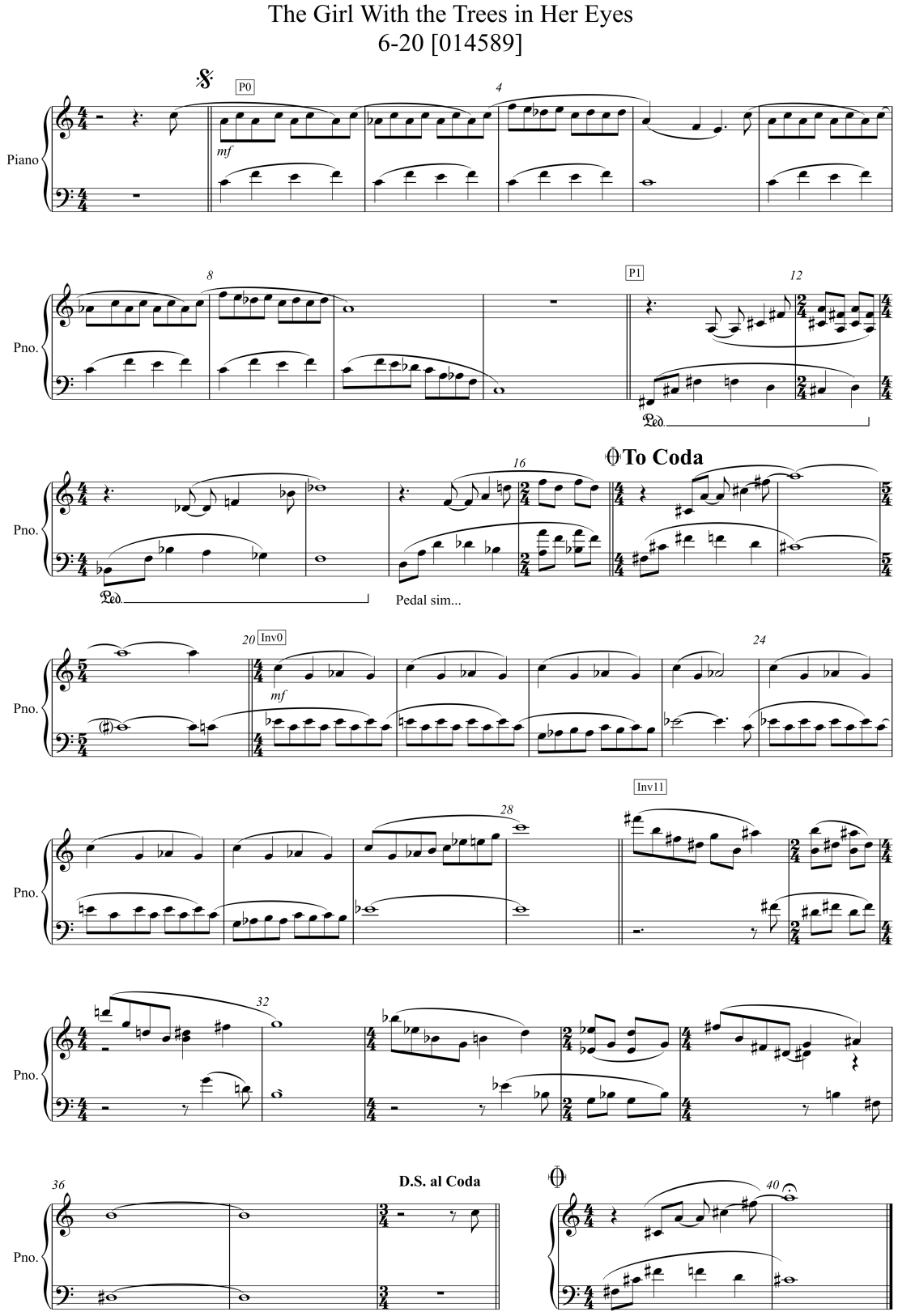
Bars 1- 10 – a repeating major to minor shift **P0**

Bars 11- 20 **–** a sequence of rising minor-major seventh chords **P1**

Bars 20 **-** 29 – an inversion of bars 1-10. **P0** inverts and becomes **Inv0**

Bars 29 -38 – a transposed-inversion of bars 11- 20 **Inv11** produces a descending sequence of major seventh chords coloured by augmented fifths

Figure 14: *The Girl With The Trees In Her Eyes* – Piano



Untitled 1

6-30 [013679]

Interval Vector [224223]

Invariance Vector <20020220>

No Z-mate

Characteristic: Invariance

Note the similarity of this collection, [013679], to the half-step/whole-step version of the octatonic (diminished) scale (8-28) [0134679t]. Both possess a great deal of invariance and music written with these collections can sound unsettled and ambiguous. I use variations of texture, frequent changes of transposition and inversion, rhythmic variation, and long melodic lines over rapidly changing accompaniment to redirect the listener from this instability.

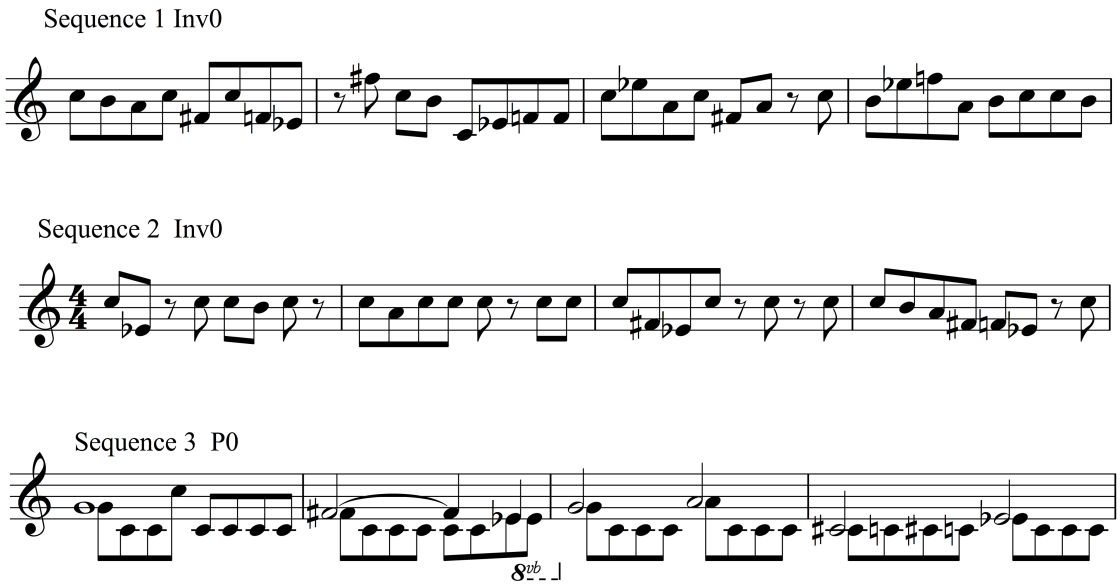
This piece comprises a series of variations built upon several programmed melodic sequences. As the piece progresses these sequences move gradually to the background, replaced in the final two variations by a driving progression of broken minor triads. The sequences were created using music software.[28]

Because each sequence may be triggered from any note of the chromatic scale and in any octave, all transpositions (sequence 3 **P0**), and all transposed-inversions (sequences 1 and 2 **Inv0**) of the hexachord are available.

All examples below show sequences that result when middle C is played on the interface keyboard.

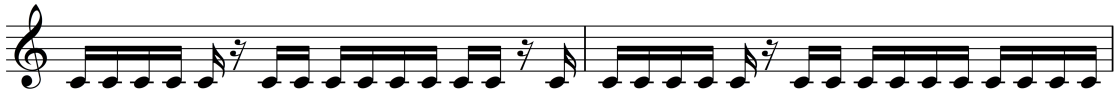
[28] With the software that I have available, I am able to generate sequences in different ways. Sequences 1, 2, and 3 were created in a step sequencer. Up to thirty-two points or steps may be assigned pitch, instrument, and duration or no sound at all. These sequences are triggered within my digital audio workstation. Other sequences in this composition were created using arpeggiators within software synthesizers *Omnisphere* and *Stylus RMX.*

Figure 15: 6-30 – Sequences



The above sequences each play a programmed sequence of notes, transposed to the note triggered on the workstation keyboard. Other sequences are programmed to play a rhythmic pattern. A note or chord sustained on the keyboard will play back one of two rhythms, either as constant eighth-notes or as shown below:

Figure 16: Rhythmic Sequence



The piece is introduced by two bars of the above rhythmic sequence on the note A, followed by three four-bar exchanges between sequence 3 playing **P9** and sequence 1 playing **Inv9**. A flugelhorn melody begins on bar 10.

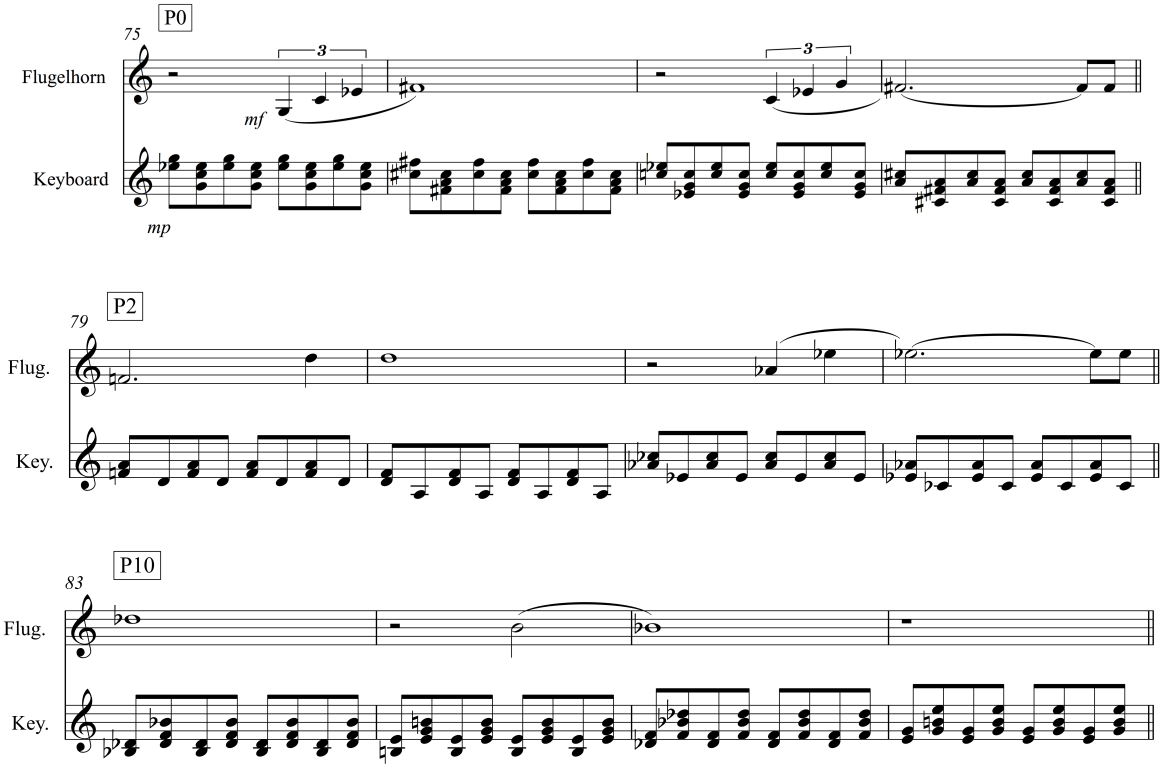
At bar 27 there is a change of texture as a string and keyboard exchange begins over rhythmic sequences and the **P9** sequence.

A darker more languid section begins at bar 43. Eight bars of **Inv1** followed by eight bars

of **Inv11** are marked by different transpositions of Inversion sequence 2. This sixteen-bar sequence is repeated with a flugelhorn melody.

A lift of energy begins at bar 75 with a progression of minor triads. Two minor triads, a tritone apart, can be extracted from the 6-30 hexachord. The excerpt below shows the progression of minor triads created from **P0**, **P2**, **P10**, and **P1**, with the flugelhorn melody. A darker version of this sixteen-bar progression follows with bass clarinet and baritone guitar. The piece concludes with a brief coda on **P0**.

Figure 17: 6-30 – Sequence of Minor Chords with Flugelhorn Melody



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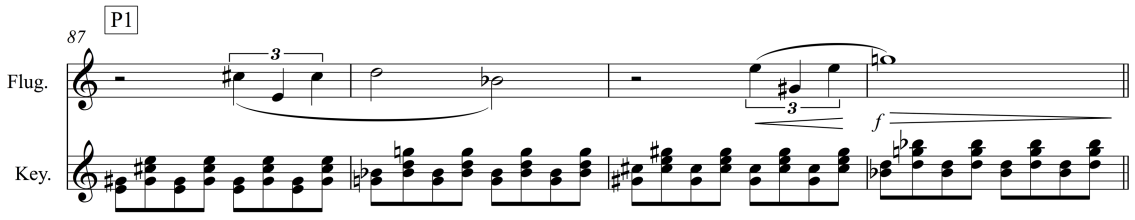


Whole-Tone Study

6-35 [02468t]

Interval Vector [060603]

Invariance Vector <66666666>

No Z-mate

Technique: Combining Hexachord with complementary hexachord

This piece, from a 2020 collection of programmatic compositions entitled "Spells and Incantations", was composed to suggest the creation and release of a magical enchantment.

The six-note collection 6-35 is commonly referred to as the whole-tone scale, although also known as Messiaen's First Mode of Limited Transposition or George Russell's Auxiliary Augmented Scale. While Debussy used this collection to great effect in such works as *La Mer* (1903) and *Voiles* (1909), the inherent ambiguity is always relieved by the addition of other harmonic material. Because of the unique nature of this hexachord, it has the maximum amount of invariance: all transpositions and inversions of **P0** replicate the pitch-class collection of **P0** *or* that of its complement. In other words, while the ordering may vary, all transformations of 6-35 are either [02468t] or [13579e].

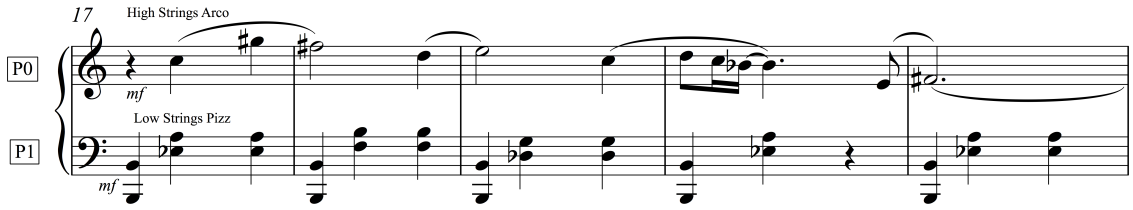
In this simple piece the two transpositions of the 6-35 hexachord are assigned to different voices of the orchestral setting – melody and accompaniment. By so doing, the limited intervallic structure inherent in the whole-tone collection is expanded, yet each voice is committed to one

six-note iteration. The piece begins with winds and low pizzicato strings outlining **P1**.

At bar 9 the upper strings begin an eight-bar melody created from **P0 –** played with the **P1** accompaniment – repeated with slight variation at bar 17.

Figure 18: 6-35 – Simultaneous Use of Two Transpositions of Hexachord

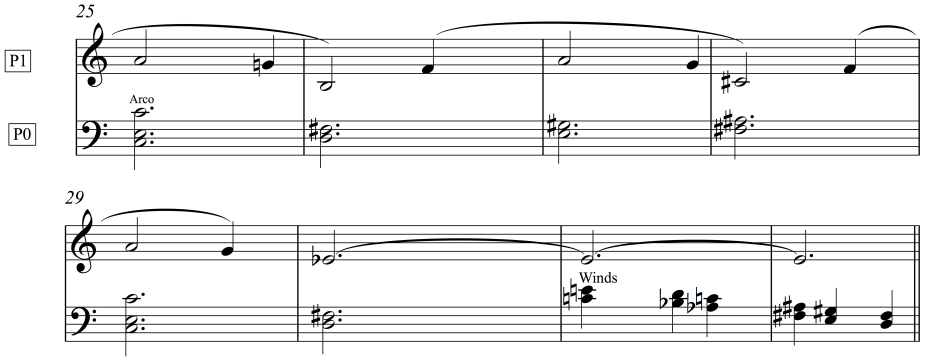
Bars 17-21



At bar 25, with a change in texture, the hexachords are exchanged: the accompaniment is now derived from **P1,** and the melody from **P0**.

Figure 19: 6-35 – Hexachords Exchanged

Bars 25-32



.

The two sections repeat with slight orchestral and ornamental variations, resolving in a brief coda derived from **P0**.

TRANSPOSED ROTATIONAL ARRAYS

> Few theorists would deny the importance of rotational arrays in the late works of Igor Stravinsky. Probably, even fewer would deny that careful study of the work of a great twentieth century composer will reveal not only the extent of his or her compositional craft, but will also indicate that craft's potential for continued use and extension by other contemporary composers. (Morris 1988, 75)

Morris observes that, despite their potential, little attention has been paid to rotational arrays. Perhaps, he suggests, they are too closely associated with Stravinsky's late style. It is "very difficult to export them out of Stravinsky's universe without losing their context and utility." (75) It is also possible to assume that there has been little exploration and extension of this device because transposed rotational arrays are so closely linked to hexachords, and because hexachords have been largely ignored apart from twelve-tone music.

When a hexachord is transposed and/or inverted, cohesion is maintained by the unchanging interval content. Transposed rotational arrays are built around a rotation point – a pitch-class common to all six hexachords in the rotation. Thus, the hexachords within the rotational array are united by both interval content and the shared rotation point. The following pieces demonstrate a number of ways that transposed rotational arrays may be used. These methods include rotation of **P0** (or a transposition of **P0)** with:

- a common pitch-class/rotation point pedaled through all six iterations
- a rotation point pedaled but not all rotations of hexachord used
- a rotation point present in all hexachords but not pedaled
- the creation of melody by extracting selected notes from underlying rotation
- the creation of an extended rotation by repeating selected rotations
- an incomplete or fragmented rotation

Additionally, as shown in the composition *Memory Dust*, rotation may be applied to all hexachords in the matrix that contain a common pitch-class including inversions and complements.

Rotation Study

6-16 [014568]

Interval Vector [322431]

Invariance Vector <10100101>

No Z-mate

Technique: expanded rotation, multiple use of rotation, extracting linear fragments of rotation to create melody and accompaniment

As noted earlier, Douglas Rust compares the serial techniques used in Stravinsky's *Movements* to that of a weaver's craft, demonstrating that artistry hidden beneath the surface can produce beauty on the surface (Rust 1994, 65). Inspired by this "hidden loom" analogy, this piece employs a number of concealed techniques that determine almost every aspect of the composition, although they are most likely not evident to the listener.

Three concealed pre-composition elements, listed below, contributed to the creation of this piece. Although not a concealed element, it is worth noting that for each repeat of the eleven-bar extended rotation, a different accompaniment was created.

1. It was my wish to compose this study using an extended transposed rotation – a rotation made longer by repeating or extending certain transpositions. After some experimentation I decided to use an eleven-bar sequence in ¾ time, constructed in the following manner: one bar each of **P0 P7 P0 P7 P0 P7 P4 P11 P6 P8 P8**. This rotation was constructed around the pitch-class 0, in this

case the note C.

2. Two melodies were extracted from this extended rotation. This was done by choosing a note or two from each transposition of the rotation and mapping these notes through the progression such that each note of melody "agreed" with the current transposition of the hexachord within the rotation. (In the example below, Melody 2 has been transposed back to a "C rotation.")

Figure 20: *Rotation Study* – Melody 1

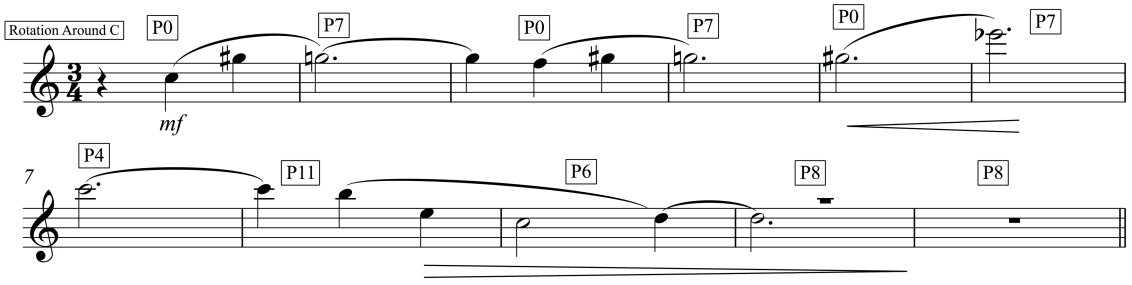
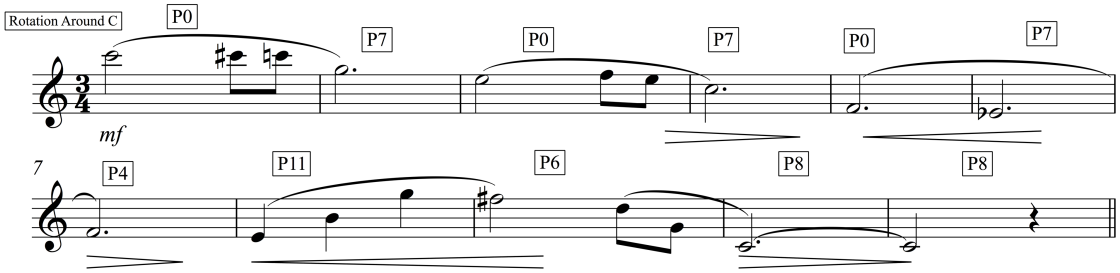


Figure 21: *Rotation Study* – Melody 2



3. A final concealment was then added. As the eleven-bar sequence repeats, it is transposed such that the center of each rotation matches one of the six notes of the original hexachord – the rotation points are C, F, Ab, Db, Gb, and E. Thus, on a macro level, as shown in the following example (figure 22), the hexachord is reproduced through transposition/rotation points.

Figure 22: *Rotation Study* – Sequence of Eleven-Bar Rotations

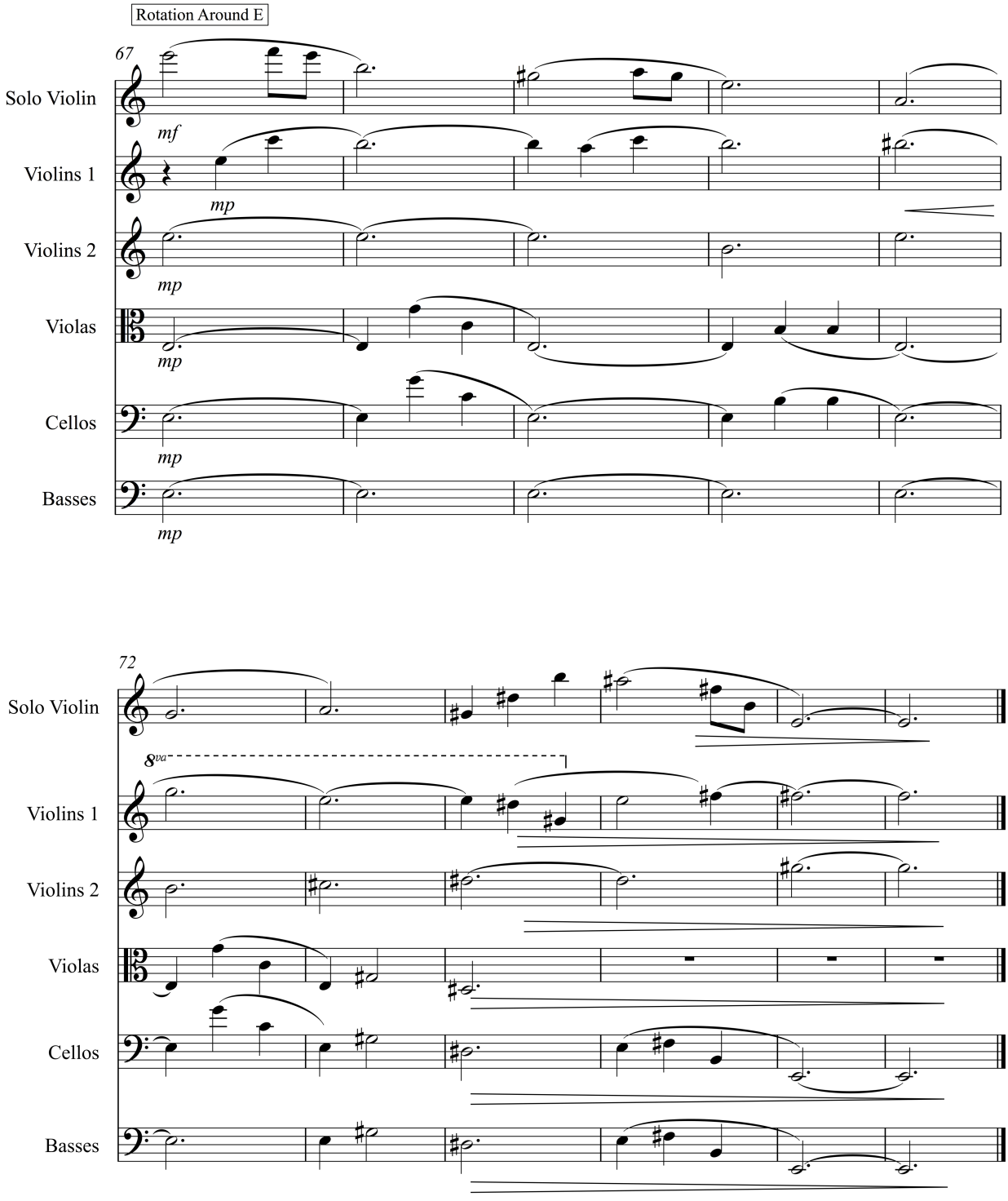
Introductory Statement of Melody 1 (extracted from rotation 1)
Rotation 1 Around C – P0 P7 P0 P7 P0 P7 P4 P11 P6 P8 P8
Rotation 2 Around F – P5 P0 P5 P0 P5 P0 P9 P4 P11 P1 P1
Rotation 3 Around Ab – P8 P3 P8 P3 P8 P3 P0 P7 P2 P4 P4
Rotation 4 Around Db – P1 P8 P1 P8 P1 P8 P5 P0 P7 P9 P9
Rotation 5 Around Gb – P6 P1 P6 P1 P6 P1 P10 P5 P0 P2 P2
Rotation 6 Around E – P4 P11 P4 P11 P4 P11 P8 P3 P10 P0 P0

Figure 23: *Rotation Study* – Sequence of Melodic Phrases

Introduction
Rotation 1 Melody 1
Rotation 2 Melody 2
Rotation 3 Melody 1 In Lower Voices
Rotation 4 Melody 2
Rotation 5 Melody 1 Middle Voices
Rotation 6 Melody 1 and 2 Combined

The final rotation (figure 24) shows the combination of the two melodic lines with accompaniment. The solo violin plays melody 2, with the first violins playing melody 1.

Figure 24: *Rotation Study* – Final Rotation of Strings Bars 67-77



Into the Night

6-19 [013478]

Interval Vector [313431]

Invariance Vector <10000010>

Z-mate 6-z44

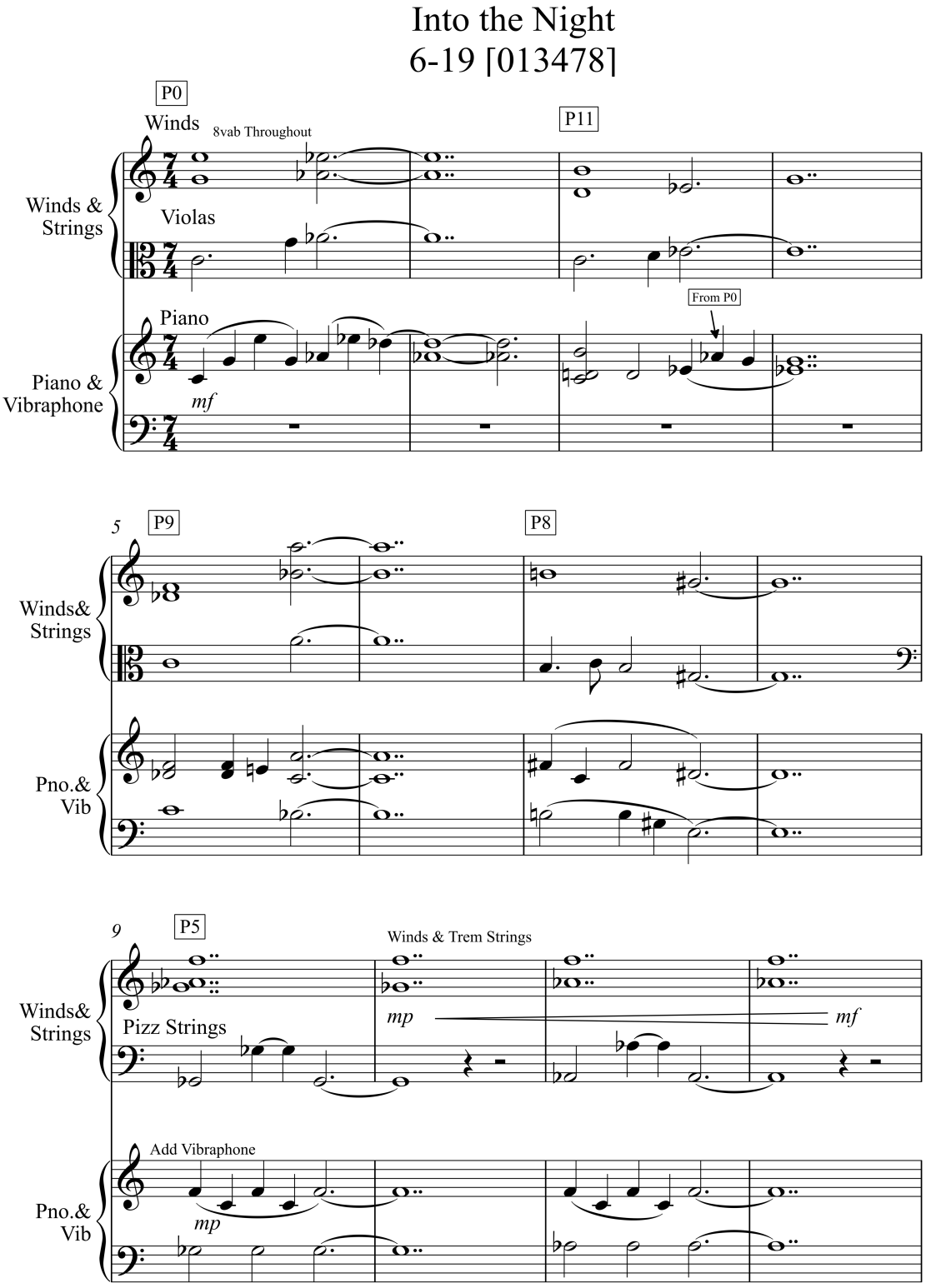
Technique: Transposed Rotation

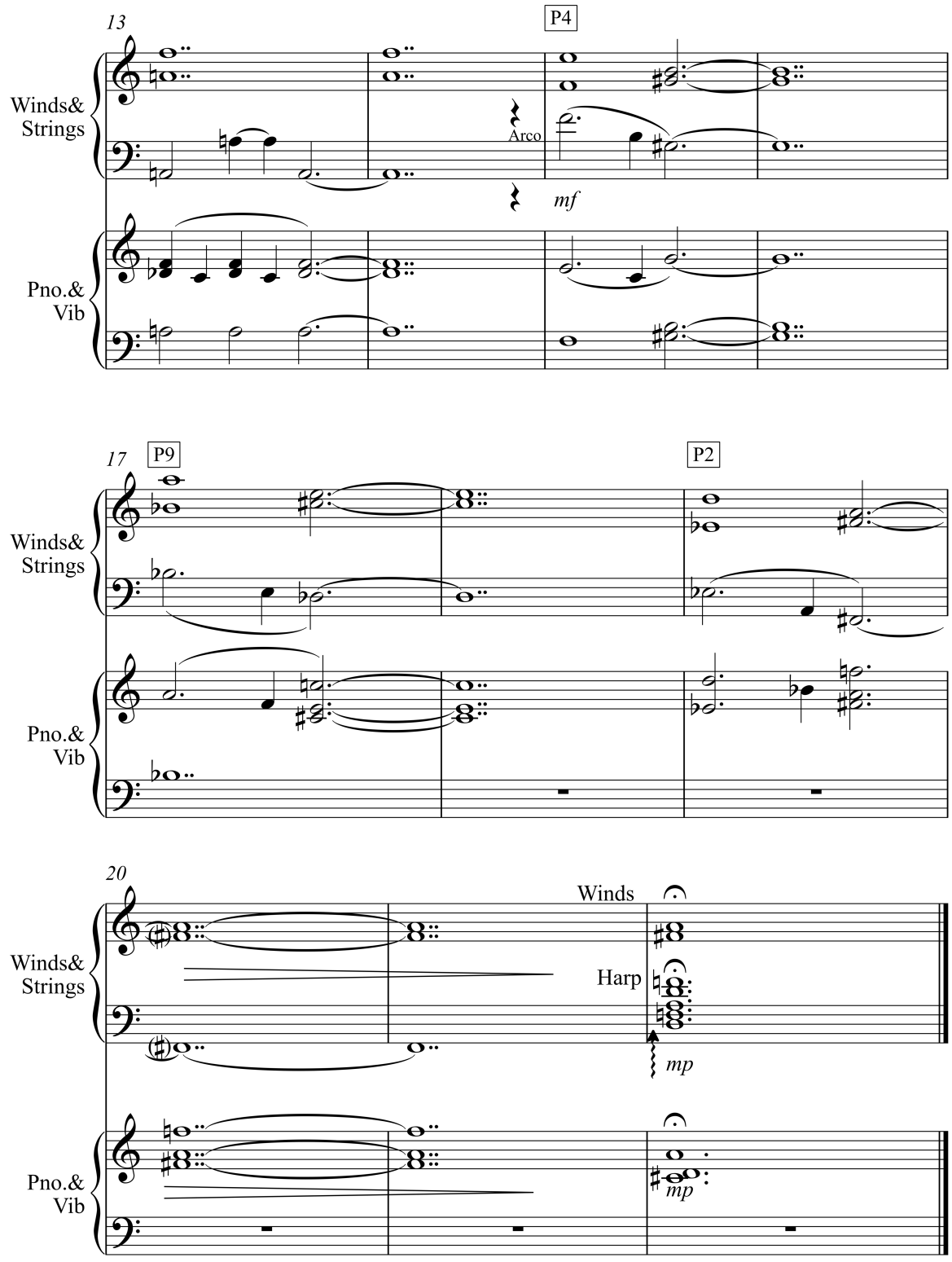
This is another short piece composed as accompaniment to a poem. A feeling of darkness, an expression of heading into the unknown, is explored by the use of transposed rotation. With the exception of the concluding **P2** hexachord, the composition is simply one extended transposed rotation.

The harp and a pianissimo sequence pedal the pitch C. A gradual rotation cycles through all hexachords that contain C. There are two bars of 7/4 for each of **P0**, **P11**, **P9**, and **P8**. **P5** is prolonged for six bars followed by **P4**, the final hexachord in the rotation. The pedal C has now faded, and a descending bass and cello line begins on this last rotation of the array repeating through two more iterations of the hexachord **P9**, previously expressed as part of the array, and **P2**.

Note the subtle use of the common rotation tone. Although my wish is to use all pitch-classes from each transformation of the hexachord used, there are times when the ear supersedes technique. In this case all hexachords are "complete" with the exception of **P11** (missing F#) and **P9** (missing D). Also note the use of Ab in bar 3 borrowed from the previous hexachord and "resolved" into the **P11** hexachord. The score reduction that follows does not include the pedal tone C.

Figure 25: *Into the Night* – Score Reduction




13
P4
Winds&
Strings
Arco
mf
Pno.&
Vib
17
P9
P2
Winds&
Strings
Pno.&
Vib
20
Winds
Winds&
Strings
Harp
mp
Pno.&
Vib
mp

Where the River Turns

6-26 [013578]

Interval Vector [232341]

Invariance Vector <11000000>

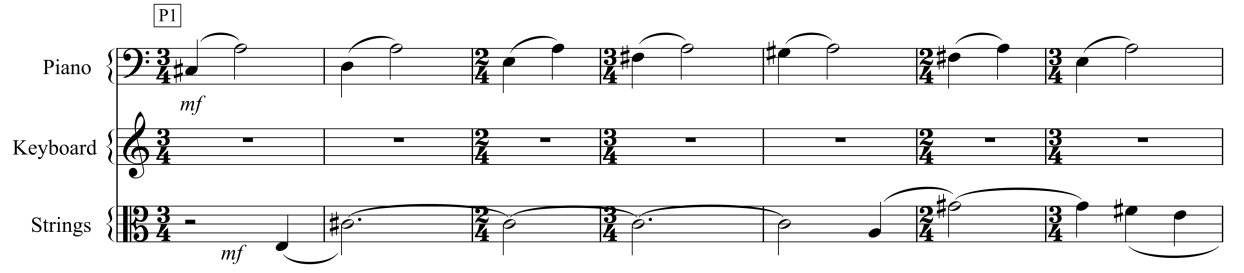
Z-mate 6-z48

Technique: Use of fragmented transposed rotation

Where the River Turns was created to underscore a piece of prose – a description of a physical journey that transforms into an introspective metaphorical one. This is represented musically through two contrasting sections, each section employing the 6-26 hexachord in a different manner.

The first section (bars 1-51) uses one transposition, **P1,** for the first twenty-seven bars, then transposes to **P8** for the remaining twenty-four bars of the section. The Interval Vector [232341] reveals that any 6-26 hexachord will have four pitch-classes in common with any transposition of that hexachord by a fourth or a fifth, therefore **P1** and **P8** are closely related. This first section contains a great deal of simple rhythmic motion. Three voices introduce different versions of a two-note motif.

Figure 26: *Where the River Turns* – Two-Note Motif in Three Voices



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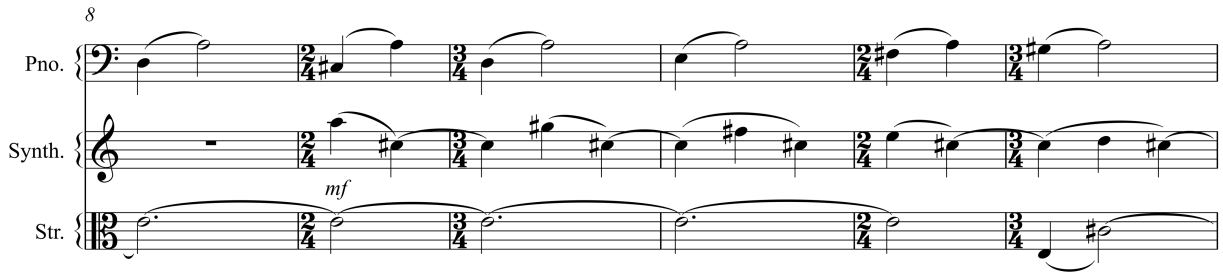
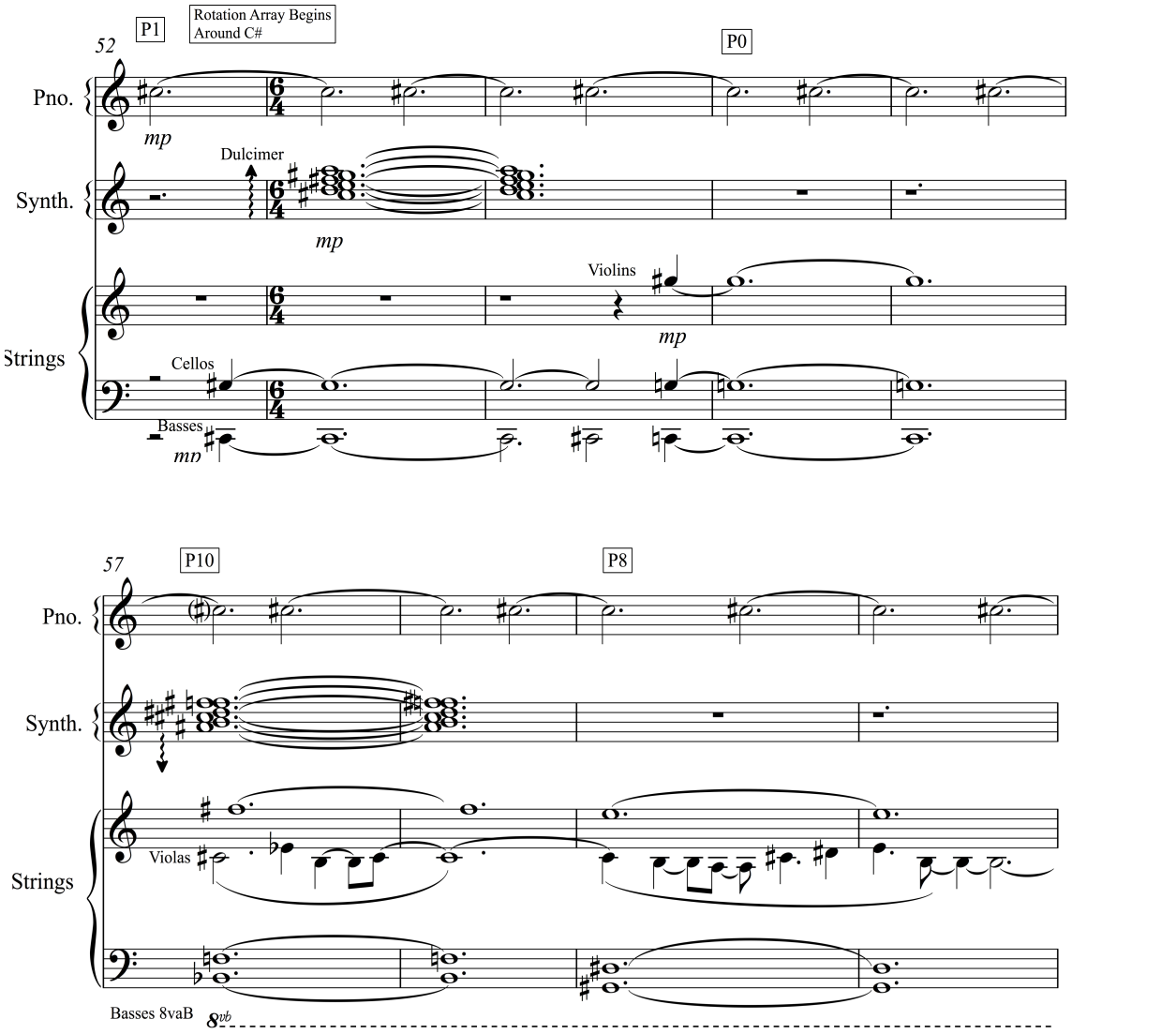


At bar 28 the vibraphone begins the two-note motif sequence transposed up a fifth to **P8**. The notes C#, E, G# and A remain constant.

A contrasting section follows. Now there is much less rhythmic motion – the repeating two note motif has given way to longer more languid string lines. The hexachord is moved by rotation through a number of transpositions. A single piano note, C#, marks the rotation point – the one note common to each rotation.

Every second rotation is marked by dulcimer-like arpeggios playing all pitch-classes of the current transposition. However, during the first and fifth rotations, not all pitch-classes are used – resulting in an incomplete or fragmented rotation. Rotation occurs through all transpositions of **P1** that contain C#. Rotation occurs in the order **P1, P0, P10, P8, P6, P5** and returning to **P1**.

Figure 27: *Where the River Turns* – Transposed Rotation



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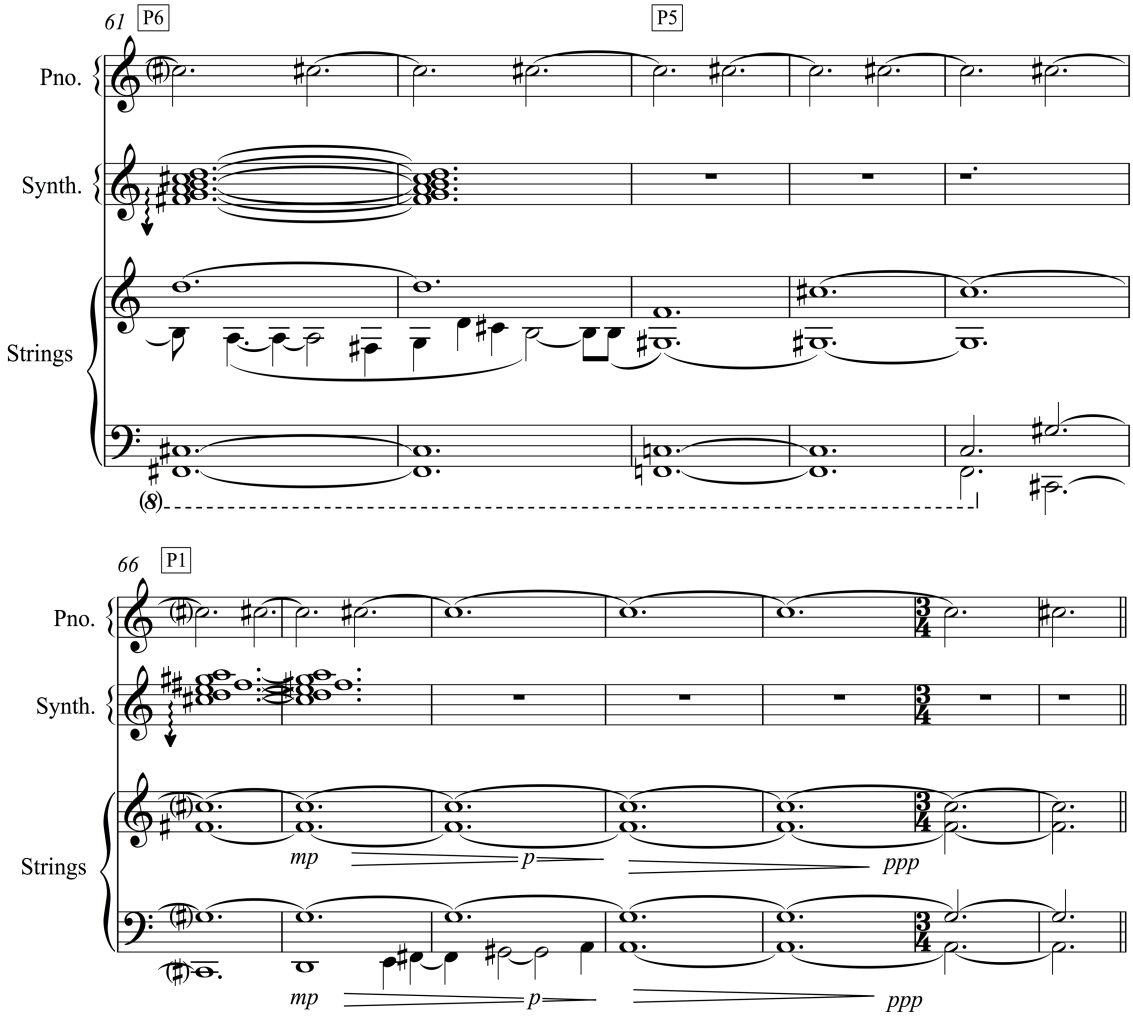


A brief coda with a transposition to **P6** reprises the repeating two-note motif. This transposition also has four pitch-classes in common with the initial **P1** transposition.

Memory Dust

6-40 [012358]

Interval Vector [333231]

Invariance Vector <10000010>

Z-mate 6-z11

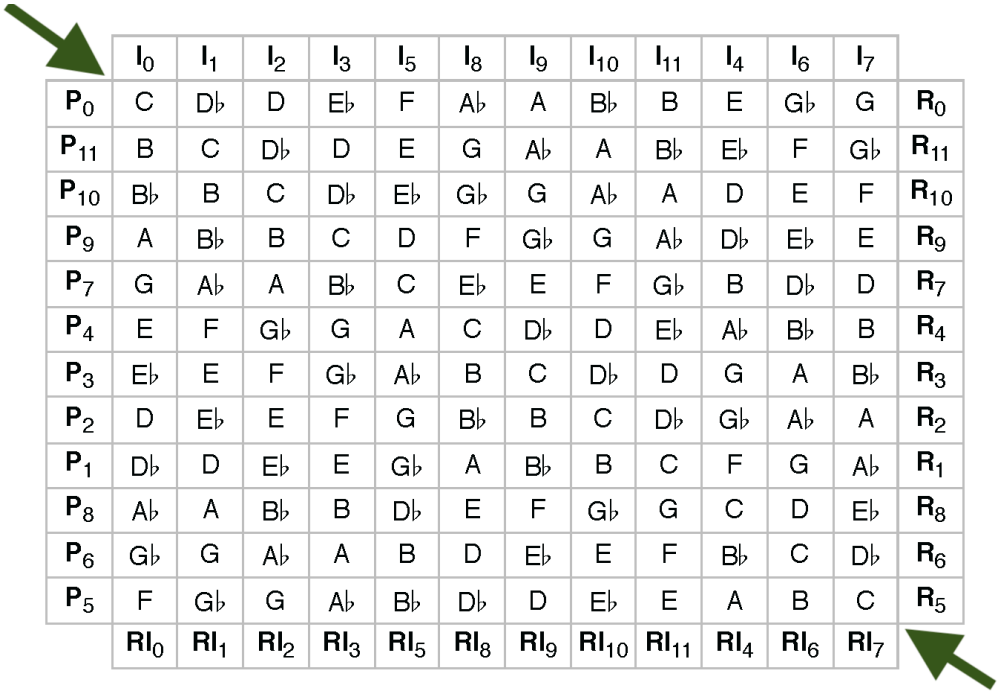
Technique: Extended Transposed Rotation

Most instances of transposed rotation that I have found in my research are applied to an ordered hexachord, often one half of a twelve-tone row. Once again, we may consider Stravinsky's *Movements*. In pre-composition the composer laid out the un-transposed *and* the transposed rotation for each hexachord – each half of the twelve-tone row. The transposed rotations are rotated around the first note of each half of the twelve-tone row (Rust 1994,64). The first hexachord rotates around Eb and the second around C. Stravinsky is using hexachord 6-7 [012678], a hexachord with a considerable amount of invariance <22222222>.

Pre-Composition for *Memory Dust*

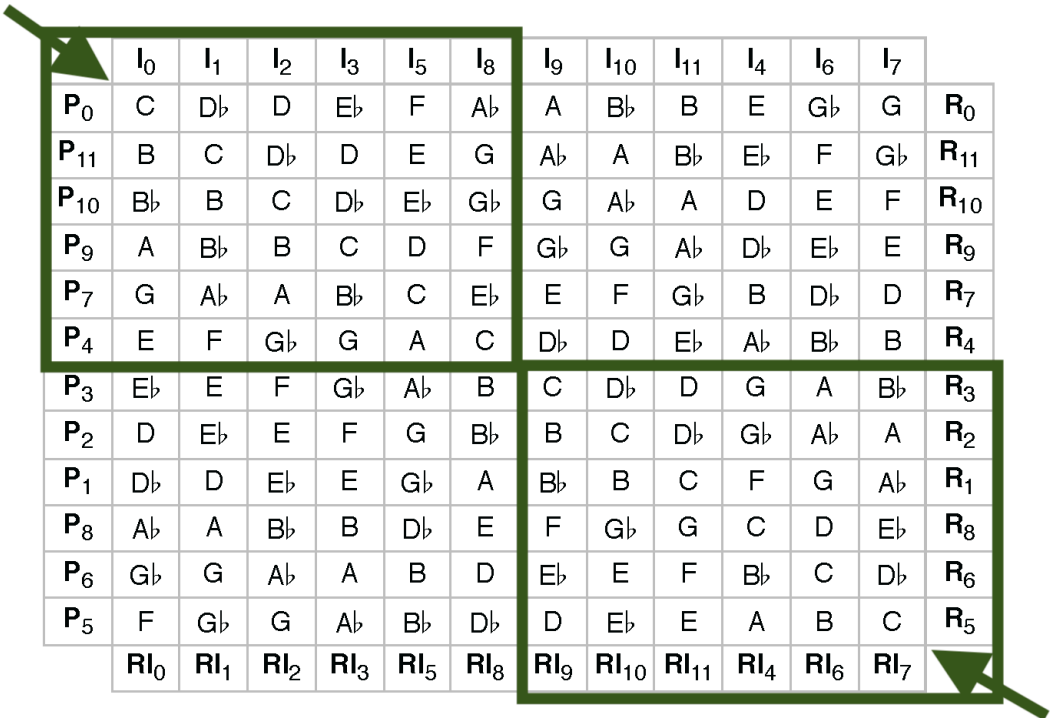
In this piece I am using an extended form of transposed rotation. The device is applied to every hexachord that contains the pitch-class 0 (in this case the note C). Thus, the original hexachord and its inversion as well as the complement of a transposed version of the hexachord and its inversion are all rotated. The application is best displayed by the matrix. Figure 28 shows the 6-40 hexachord occupying the first six positions in the **P0** row, with the complement of the hexachord 6z-11 occupying the remaining six. A diagonal through the matrix reveals all occurrences of the note C (pitch-class 0).

Figure 28: Matrix Diagonal



	I_0	I_1	I_2	I_3	I_5	I_8	I_9	I_{10}	I_{11}	I_4	I_6	I_7	
P_0	C	D♭	D	E♭	F	A♭	A	B♭	B	E	G♭	G	R_0
P_{11}	B	C	D♭	D	E	G	A♭	A	B♭	E♭	F	G♭	R_{11}
P_{10}	B♭	B	C	D♭	E♭	G♭	G	A♭	A	D	E	F	R_{10}
P_9	A	B♭	B	C	D	F	G♭	G	A♭	D♭	E♭	E	R_9
P_7	G	A♭	A	B♭	C	E♭	E	F	G♭	B	D♭	D	R_7
P_4	E	F	G♭	G	A	C	D♭	D	E♭	A♭	B♭	B	R_4
P_3	E♭	E	F	G♭	A♭	B	C	D♭	D	G	A	B♭	R_3
P_2	D	E♭	E	F	G	B♭	B	C	D♭	G♭	A♭	A	R_2
P_1	D♭	D	E♭	E	G♭	A	B♭	B	C	F	G	A♭	R_1
P_8	A♭	A	B♭	B	D♭	E	F	G♭	G	C	D	E♭	R_8
P_6	G♭	G	A♭	A	B	D	E♭	E	F	B♭	C	D♭	R_6
P_5	F	G♭	G	A♭	B♭	D♭	D	E♭	E	A	B	C	R_5
	RI_0	RI_1	RI_2	RI_3	RI_5	RI_8	RI_9	RI_{10}	RI_{11}	RI_4	RI_6	RI_7	

Figure 29: Hexachords in the Matrix containing the note C



	I_0	I_1	I_2	I_3	I_5	I_8	I_9	I_{10}	I_{11}	I_4	I_6	I_7	
P_0	C	D♭	D	E♭	F	A♭	A	B♭	B	E	G♭	G	R_0
P_{11}	B	C	D♭	D	E	G	A♭	A	B♭	E♭	F	G♭	R_{11}
P_{10}	B♭	B	C	D♭	E♭	G♭	G	A♭	A	D	E	F	R_{10}
P_9	A	B♭	B	C	D	F	G♭	G	A♭	D♭	E♭	E	R_9
P_7	G	A♭	A	B♭	C	E♭	E	F	G♭	B	D♭	D	R_7
P_4	E	F	G♭	G	A	C	D♭	D	E♭	A♭	B♭	B	R_4
P_3	E♭	E	F	G♭	A♭	B	C	D♭	D	G	A	B♭	R_3
P_2	D	E♭	E	F	G	B♭	B	C	D♭	G♭	A♭	A	R_2
P_1	D♭	D	E♭	E	G♭	A	B♭	B	C	F	G	A♭	R_1
P_8	A♭	A	B♭	B	D♭	E	F	G♭	G	C	D	E♭	R_8
P_6	G♭	G	A♭	A	B	D	E♭	E	F	B♭	C	D♭	R_6
P_5	F	G♭	G	A♭	B♭	D♭	D	E♭	E	A	B	C	R_5
	RI_0	RI_1	RI_2	RI_3	RI_5	RI_8	RI_9	RI_{10}	RI_{11}	RI_4	RI_6	RI_7	

It can be seen from figure 29 that all hexachords in both rows and columns included in the boxed areas of the matrix also contain the note C. Transposed rotations are applied to **P0, I0,** the complement of **P3,** and the complement of **I9** – identified below and in the score as **A**, **B**, **C,** and **D**.

Because transposed rotation reproduces the pitch-class collections presented by the five other hexachords in the row or column that contain the common rotation tone, the actual mechanics of rotating and transposing the pitch-classes of the hexachord are not necessary. Referring to the matrix and reordering the collections such that each begins with the rotation note C, we may produce the four groups of rotations below.

A Rotations of **P0**

P0 012358 = C, Db, D, Eb, F, Ab
P11 01247e = C, Db, D, E, G, B
P10 0136te = C, Db, Eb, Gb, Bb, B
P9 0259te = C, D, F, A, Bb, B
P7 03789t = C, Eb, G, Ab, A, Bb
P4 045679 = C, E, F, Gb, G, A

B Rotations of Complement of **P3** (6-z11)

P3 01279t = C, Db, D, G, A,Bb
P2 01689e = C, Db, Gb, Ab, A, B
P1 0578te = C, F, G, Ab, Bb, B
P8 023567 = C, D, Eb, F, Gb, G
P6 01345t = C, Db, Eb, E, F, Bb
P5 02349e = C, D, Eb, E, A, B

C Rotations of **Inv0**

Inv0 0479te = C, E, G, A, Bb, B
Inv1 0158te = C, Db, F, Ab, Bb, B
Inv2 01269e = C, D, Db, Gb, A, B
Inv3 01237t = C, Db, D, Eb, G, Bb
Inv5 023459 = C, D, Eb, E, F, A
Inv8 035678 = C, Eb, F, Gb, G, Ab

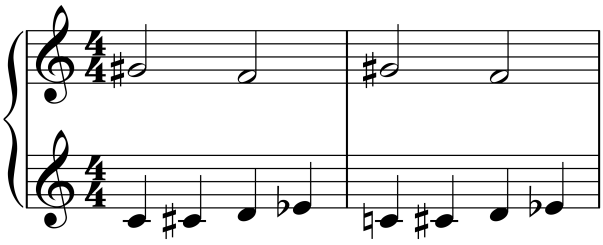
D Rotations of Complement of **Inv9**

Inv9 0235te = C, D, Eb, F Bb, B
Inv10 01346e = C, Db, Eb, E, Gb, B
Inv11 012457 = C, Db, D, E, F, G
Inv4 05679t = C, F, Gb, G, A, Bb
Inv6 02789e = C, D, G, Ab, A, B
Inv7 01389t = C, Db, Eb, Ab, A, Bb

The invariance vector for the hexachord 6-40, <10000010>, signifies that there will be no duplications of pitch-class collections through transposition or inversion. The 6-40 [012358] and the 6-z11 [012457] hexachords have different pitch-class collections, but share the same interval content, [333231].

A simple two bar cell was created using all six pitch-classes of the original hexachord – a rising figure and a descending figure (figure 30). This pattern was adapted and applied to each hexachord of each of the **A**, **B**, **C**, and **D** rotations. This sequence of patterns then served as a background template for the composition of the piece. Although the pattern is not always present in the final composition, it is guiding the harmonic, melodic, and rhythmic motion of the piece.

Figure 30: Two-Bar Cell



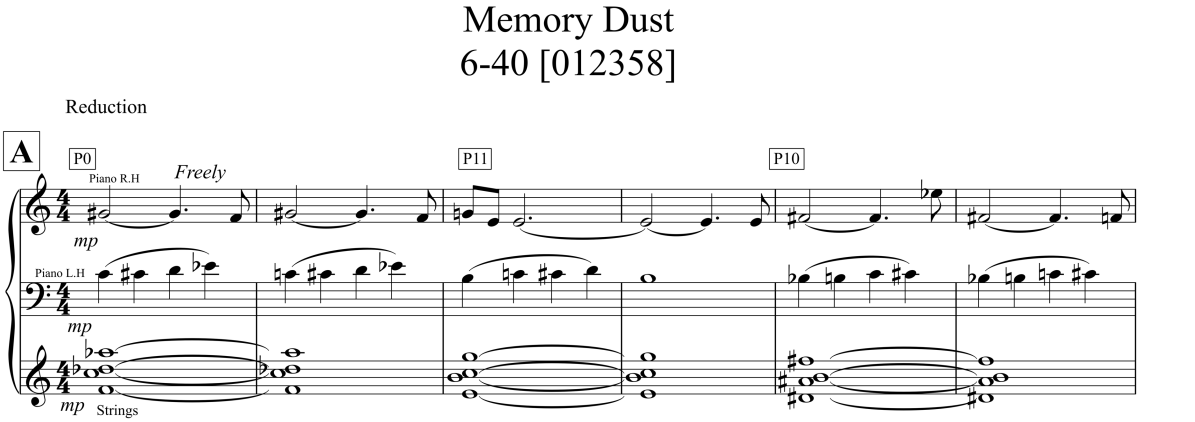
Composition

Using the template as a guide, melodic lines were created to weave through each rotation. The sequence of rotations, **A, B, C, D**, was changed to **A, A^2, C, B, C^2, D**, with different orchestrations applied to repeating rotations.

In **A,** the piano clearly outlines the two-bar cell and the first simple melodic line. The strings accompany with a four-note chord that moves in parallel motion with the rotation. This vertical structure was created by overlapping two minor 6th dyads extracted from the hexachord, although it could be regarded as a first inversion major 7th chord. The English horn and clarinet replace the piano in **A²** with the string accompaniment. There is no pedal-tone but the rotation point is present in each rotation. At **C** the piano begins a rising melodic line through the rotations of the inverted hexachord. Note the addition of one "borrowed" note in the accompaniment. At **B** the cello offers a contrasting melody and we hear, for the first time, the pedal tone. **C²** follows, with flute playing the melody. A solo viola takes the melody in **D** with second violins providing movement in sevenths.

This piece could be considered a set of variations based not on a melody but on a collection of intervals. Each hexachord in every rotation has three semi-tones, whole-tones, minor thirds and fourths as well as two major thirds and one tritone. The rotations, inversions and transpositions provide different ways of regarding the intervals.

Figure 31: *Memory Dust* – Score Reduction



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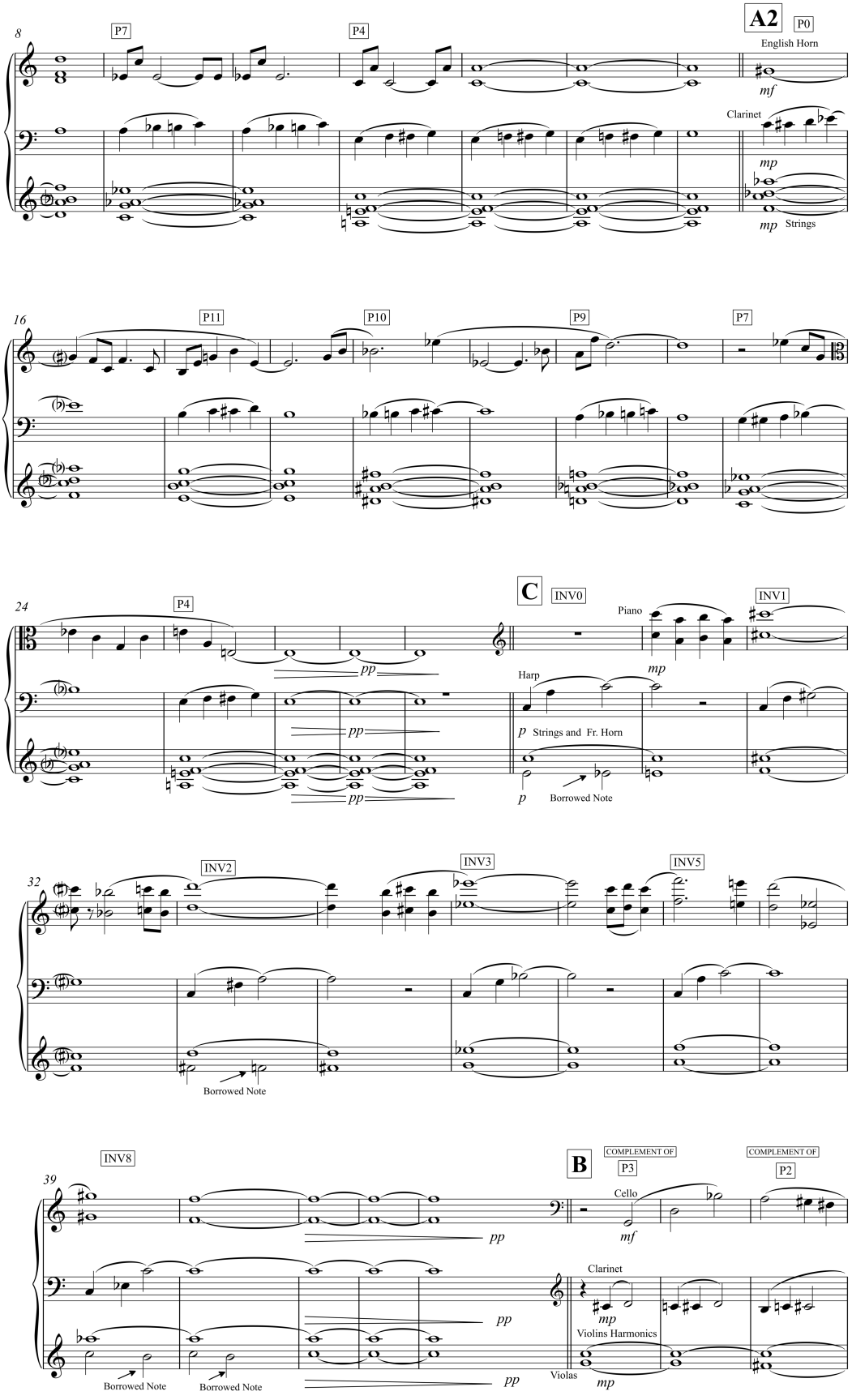
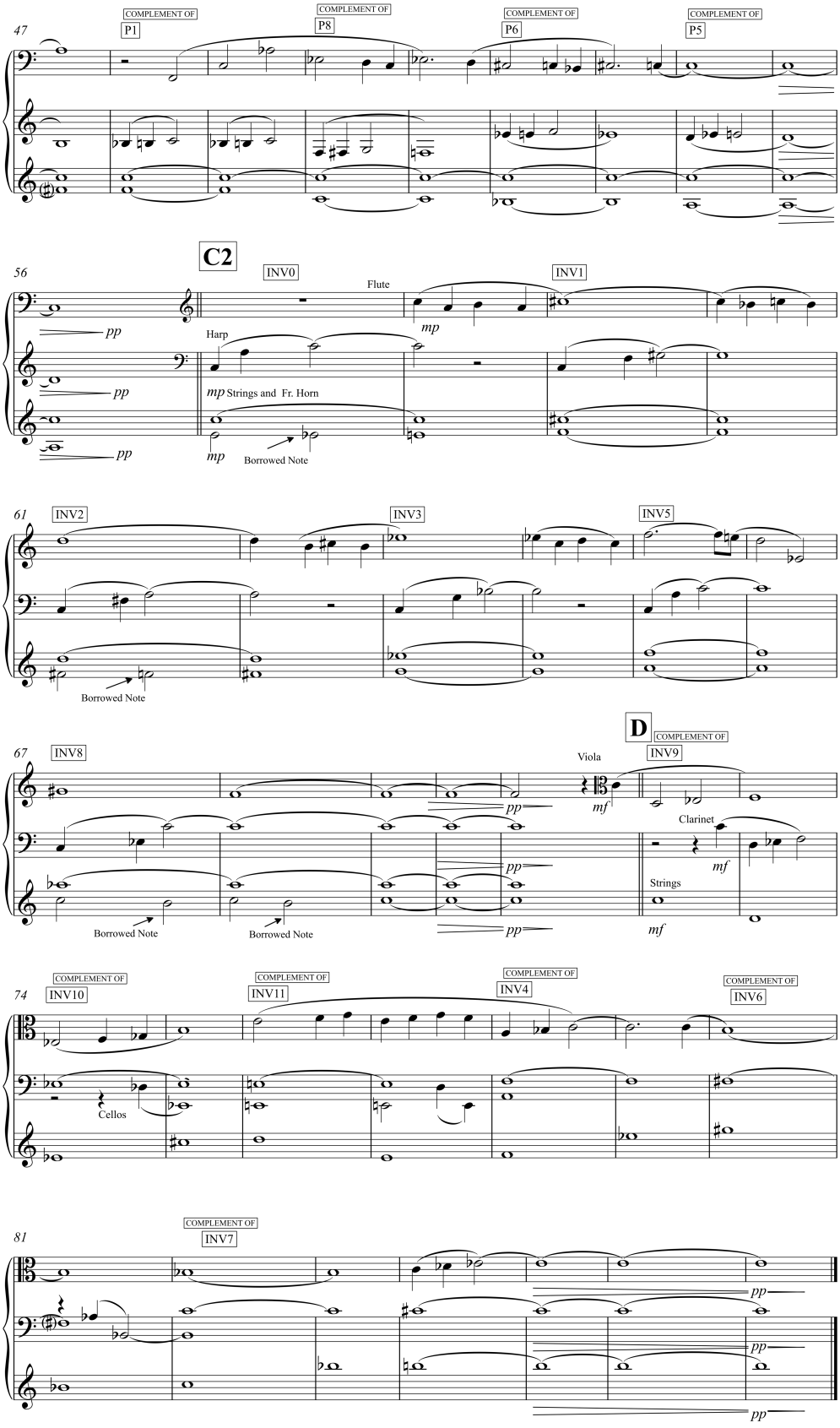

8
P7
P4
A2
P0
English Horn
mf
Clarinet
mp
mp
Strings
16
P11
P10
P9
P7
24
P4
C
INV0
Piano
INV1
pp
mp
Harp
pp
p
Strings and Fr. Horn
pp
p
Borrowed Note
32
INV2
INV3
INV5
Borrowed Note
39
INV8
B
COMPLEMENT OF
P3
COMPLEMENT OF
P2
Cello
pp
mf
Clarinet
pp
mp
Violins Harmonics
Borrowed Note
Borrowed Note
pp
Violas
mp


47
COMPLEMENT OF
P1
COMPLEMENT OF
P8
COMPLEMENT OF
P6
COMPLEMENT OF
P5
56
C2
INV0
Flute
INV1
pp
Harp
mp
pp
mp Strings and Fr. Horn
pp
mp
Borrowed Note
61
INV2
INV3
INV5
Borrowed Note
67
INV8
D
COMPLEMENT OF
INV9
Viola
pp
mf
Clarinet
pp
mf
Strings
Borrowed Note
Borrowed Note
pp
mf
74
COMPLEMENT OF
INV10
COMPLEMENT OF
INV11
COMPLEMENT OF
INV4
COMPLEMENT OF
INV6
Cellos
81
COMPLEMENT OF
INV7
pp
pp
pp

VARIOUS TECHNIQUES AND CHARACTERISTICS

In order to extend the harmonic possibilities available, these pieces incorporate the z-mate, complete the aggregate, use different transformations of hexachords at the same time and/or simulate tonal chord progressions to produce forward motion. The unique characteristics of the all tri-chord hexachord and of hexachords that resemble the classical modes are also explored.

Under the Sea

6-3 [012356]

Interval vector [433221]

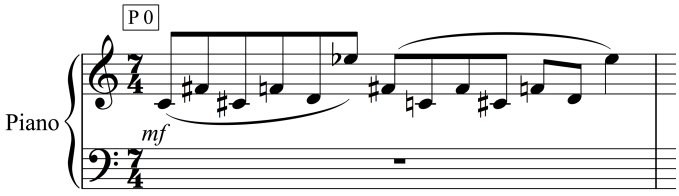
Invariance vector <10000000>

Z-mate 6-z36

Technique: Combining different transpositions and inversions of the hexachord and z-mate

In this composition various transformations of the hexachord are combined in an attempt to suggest a subaquatic world. Irregular time signatures and dissonance are offset by repeating rhythmic patterns. The piece is introduced by a piano ostinato figure outlining the 6-3 hexachord in 7/4.

Figure 32: *Under the Sea* – Piano Bar **1**



Dissonance is increased at bar 7 with the use of the clarinet using **Inv6** to harmonize the **Inv2** bassoon line. An ostinato outlining **Inv2** is played by the harp.

Figure 33: *Under the Sea* – Clarinet, Bassoon, and Harp Bars 7-9

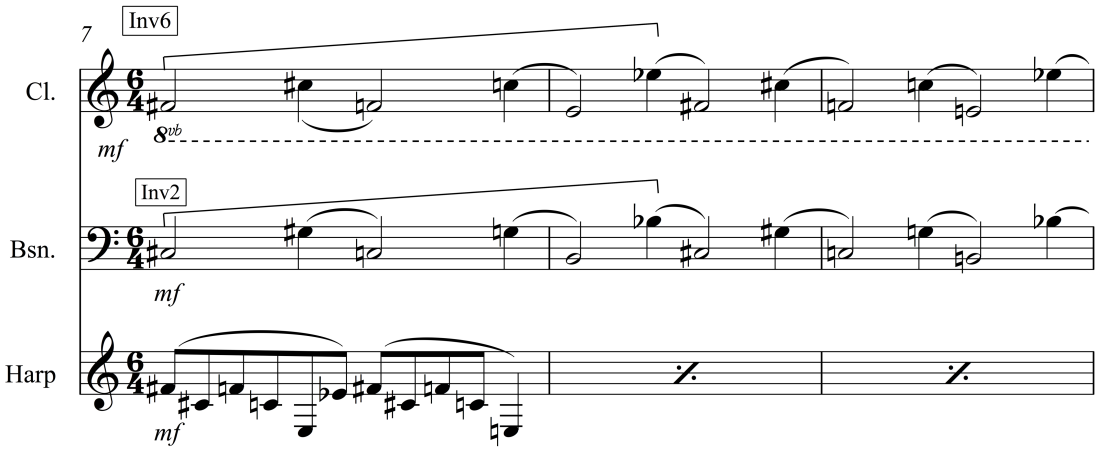
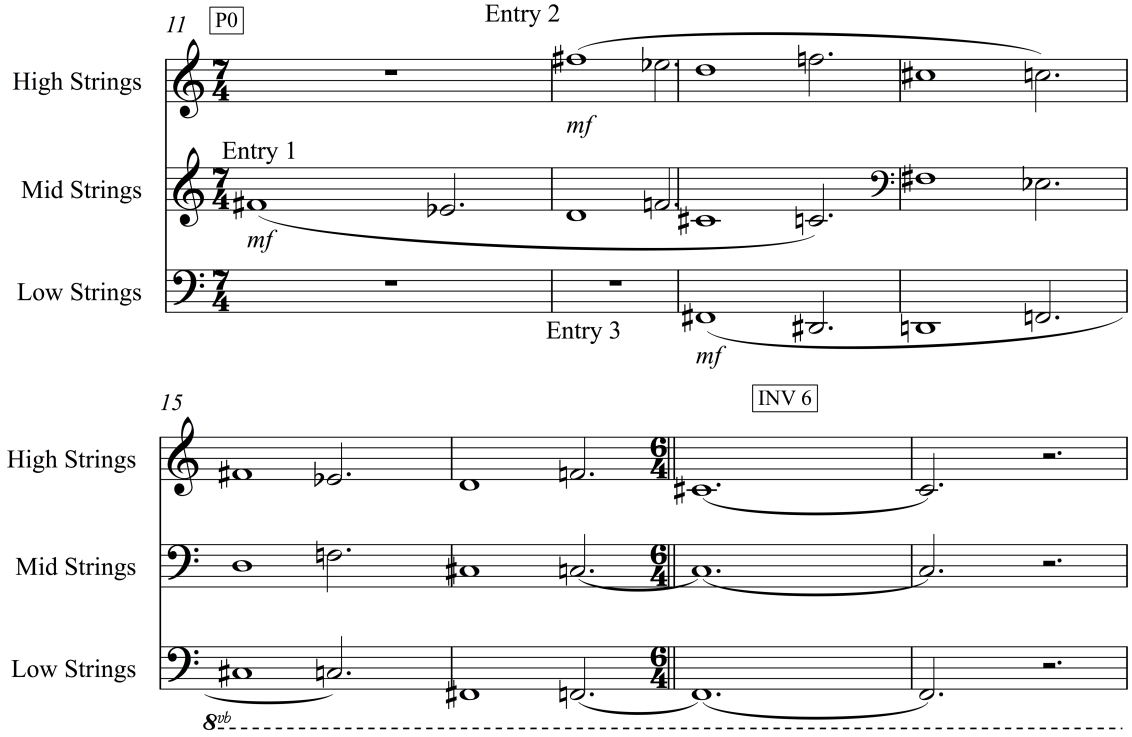


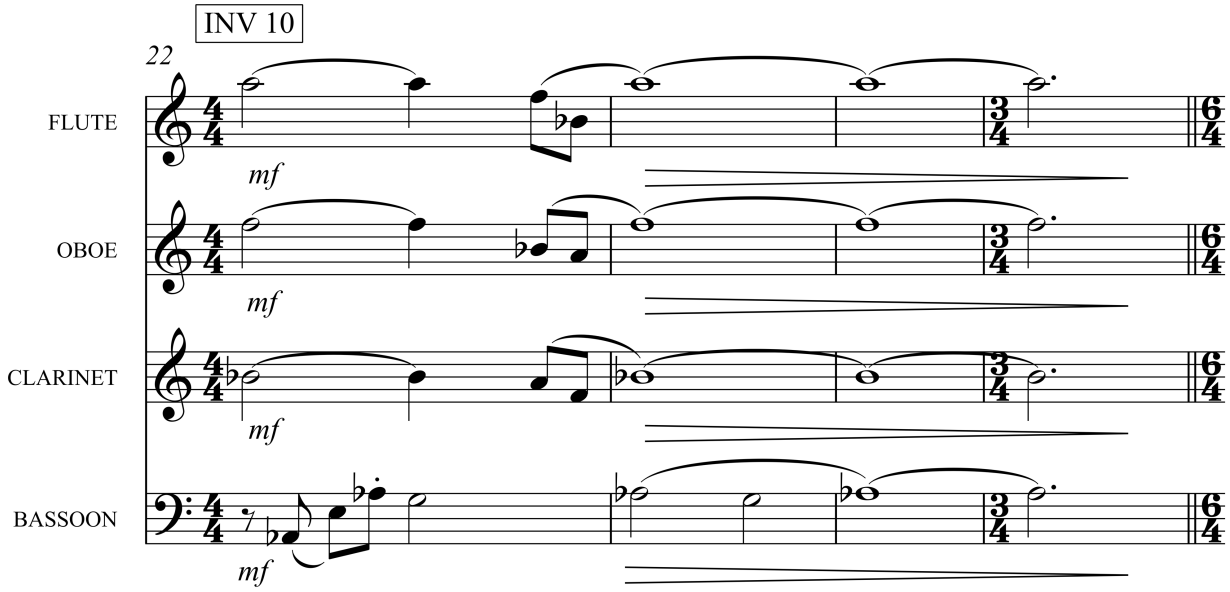
Figure 34: *Under the Sea* – Strings Canon Bars 11-19



The material introduced in the first ten measures is repeated with a different orchestration and joined by the strings playing a descending canon, an expression of **P0** (figure 34)**.**

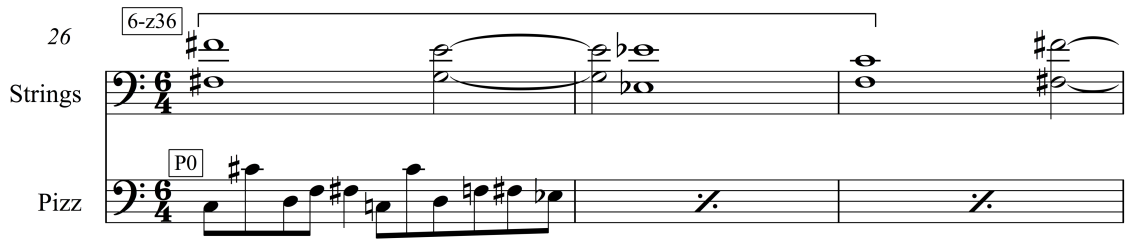
During a brief transition the bassoon plays three notes from **Inv10** while the upper winds exchange the remaining three notes.

Figure 35: *Under the Sea* – Woodwinds Transition Bars 22–25



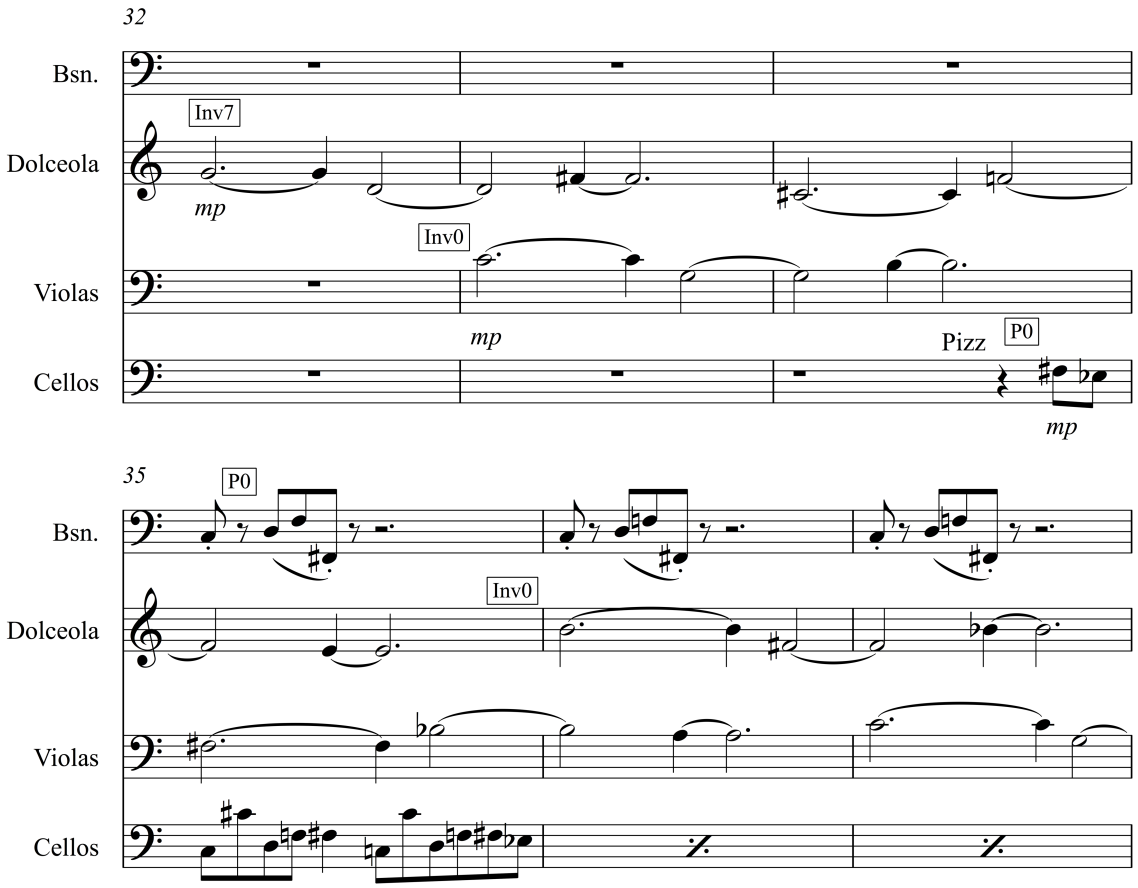
At bar 26 a pizzicato ostinato figure, **P0,** is combined with strings playing the complement of **P8** (6-z36) – C, Eb, E, F, F#, G.

Figure 36: *Under the Sea* – Strings Bars 26-28



Beginning at bar 32, three transformations of the hexachord are used. The dolceola[29] introduces a long line derived from **Inv7**, and the violas follow a bar later with the same line transposed to **Inv0.** Both are accompanied by a fragmented ostinato provided by the cellos and bassoon **P0.** The violins (not shown in example) trill from F# to G, notes common to both **P0** and **Inv7**.

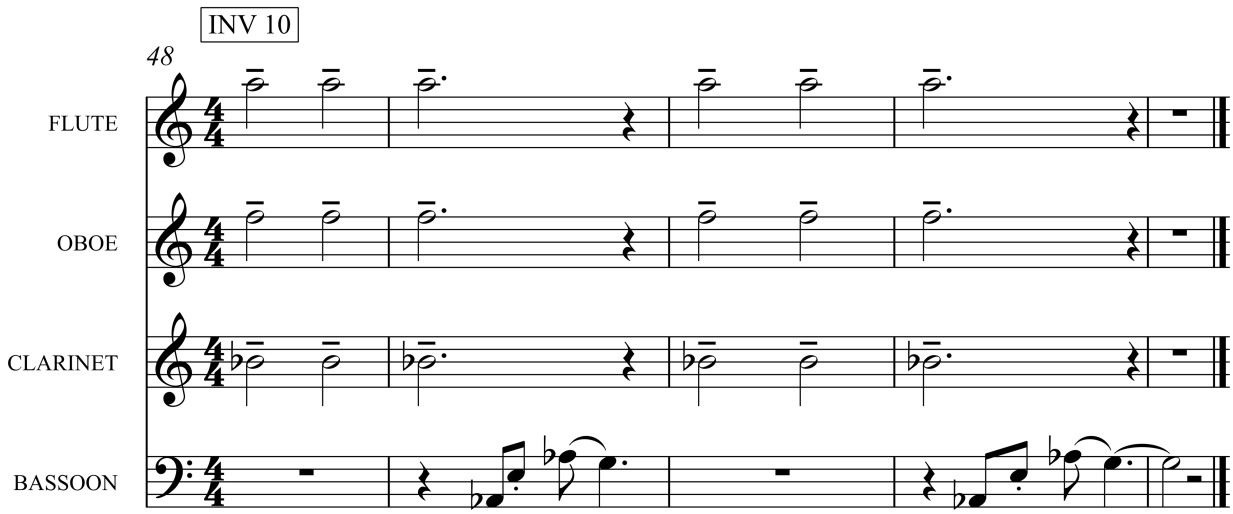
Figure 37: *Under the Sea* – Bassoon, Dolceola, Violas, and Cellos Bars 32-37



At bar 41, there is a brief reprise of the beginning material. The piece concludes with a coda using a simpler version of the earlier woodwind transition (figure 38).

[29] The dolceola is a zither with a keyboard. It was only manufactured in the United States between 1903 and 1907. I am using a software/virtual version of the instrument.

Figure 38: *Under the Sea* – Conclusion



The Mystic

6-34 [013579]

Interval Vector [142422]

Invariance Vector <10000100> (P0 maps into complement of inversion 11)

Technique: Use of different transformations of hexachord to produce harmonic motion

In *The Mystic*, a series of harmonic cells have been created. These cells create movement within the hexachord. This internal hexachordal motion is initially established by the harp (figure 39) and then joined by a different expression of the hexachord by the piano at Bar 9 (figure 40). Although the source set for this composition is a hexachord, for the first thirty-three bars of the piece, only five notes of each hexachord are used (with the exception of a passing tone in the piano part at bar 19).

Figure 39: *The Mystic* – Harp Bar 1

Expression of four notes from 6-34 Hexachord (P9)

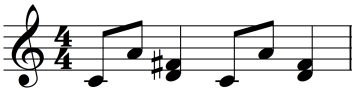
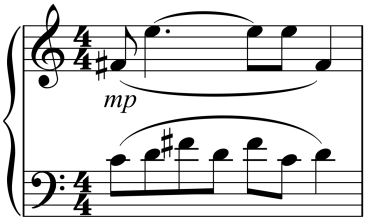


Figure 40: *The Mystic* – Piano Bar 9

Expression of four notes from 6-34 Hexachord (P9)

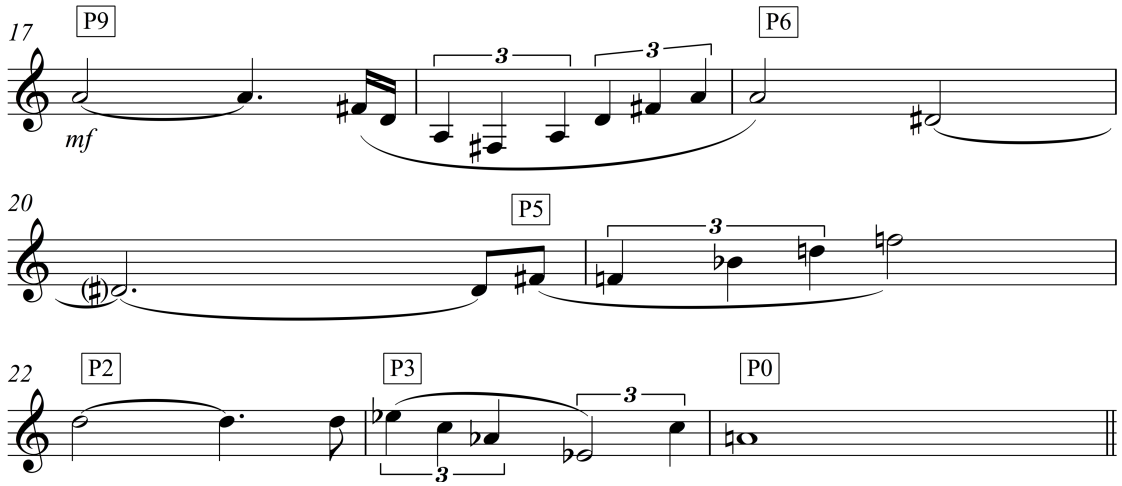


The composition is divided into three sections beginning with a repeating eight bar sequence using **P9 P6 P5 P2 P3 P0.** This sequence plays four times, initially established by harp and strings with the piano entering on the second eight-bar expression. There is motion within each hexachord while the strings create a longer flowing phrase over the eight bars from the passing transformations of the hexachord.

On the third and fourth iteration the clarinet plays a simple melody extracted from the sequence of hexachords. The following example displays the clarinet entry on the third expression of the eight-bar sequence as well as the progression of hexachord transformations.

Figure 41: *The Mystic* – Clarinet Bars 17-24

Clarinet Entry and Hexachord Sequence

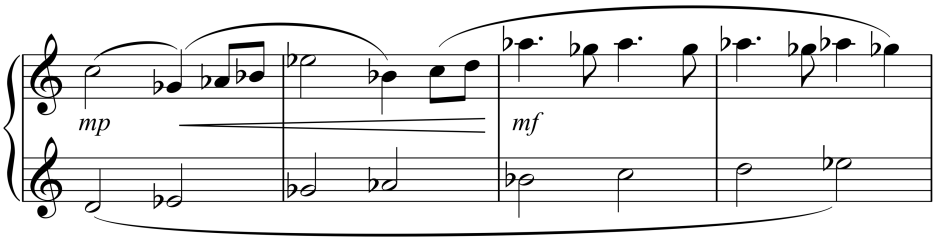


A second section begins at bar 34. This passage features strings and clarinet, and employs **Inv3** (four bars), **Inv 8** (four bars), and **Inv1** (nine bars). For the rest of the composition only inversions of the hexachord are used.

Within bar 34 the full expression of the hexachord takes place between the upper and lower voices. Up to this point only five notes of each hexachord have been employed. Additionally, the full hexachord is expressed linearly in the lower voice (bars 34-36) and the upper voice (bars 34-35) (figure 42).

Figure 42: *The Mystic* – Strings Bars 34-37

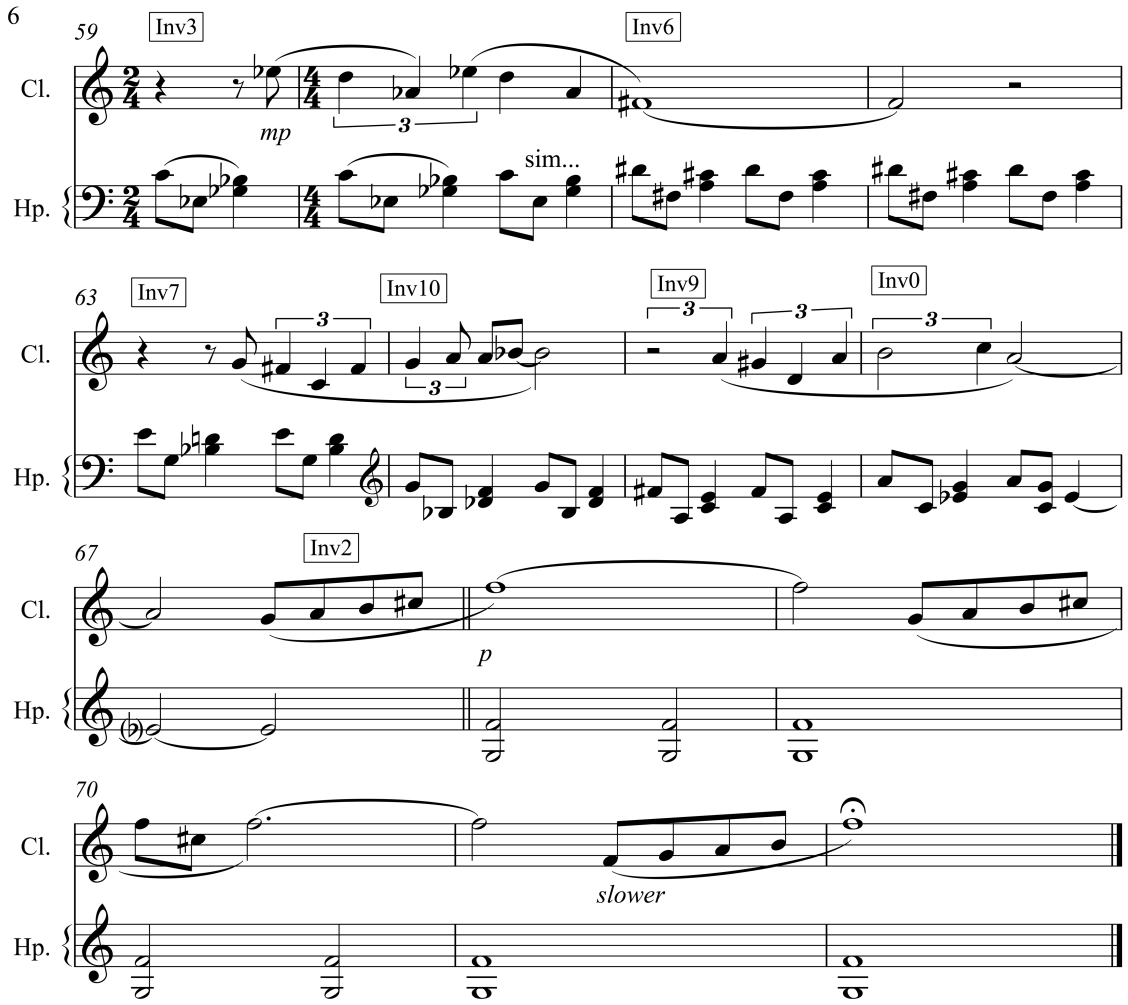
Full Expression of 6-34 Hexachord (Inv3)



A third section begins at bar 51. Here a sequence of inversions begins – **Inv3 Inv6 Inv7 Inv10 Inv9 Inv 0.** Just as the original sequence brings us from **P9** by steps – down 3, down 1, down 3, up 1, and down 3 – to **P0**, the final sequence inverts the progression, taking us from **Inv 3** – up 3, up 1, up 3, down 1, and up 3 – to **Inv 0.** A short repeating rising clarinet figure concludes the piece on **Inv 2**. Figure 43 reveals the final iteration of the inversion sequence and the conclusion of the piece.

Figure 43: *The Mystic* – Clarinet and Harp Bars 59-72

Final Sequence and Conclusion



Anima

6-25 [013568]

Interval Vector [233241]

Invariance Vector <10000000>

Z-mate 6-z47

Characteristic: Resemblance to classical mode

There are a number of hexachords that replicate structures usually associated with functional harmony. The six notes contained within the hexachord may be perceived as an incomplete mode of the major or minor scale. For example, note the associations that occur when the following hexachords are considered as ordered pitch-class sets in prime form. (As unordered sets 6-25, 6-26, and 6-32 may easily be perceived as *any* of the classical modes, albeit, missing one note.)

6-32 [024579] Ionian Mode

6-33 [023579] Melodic Minor

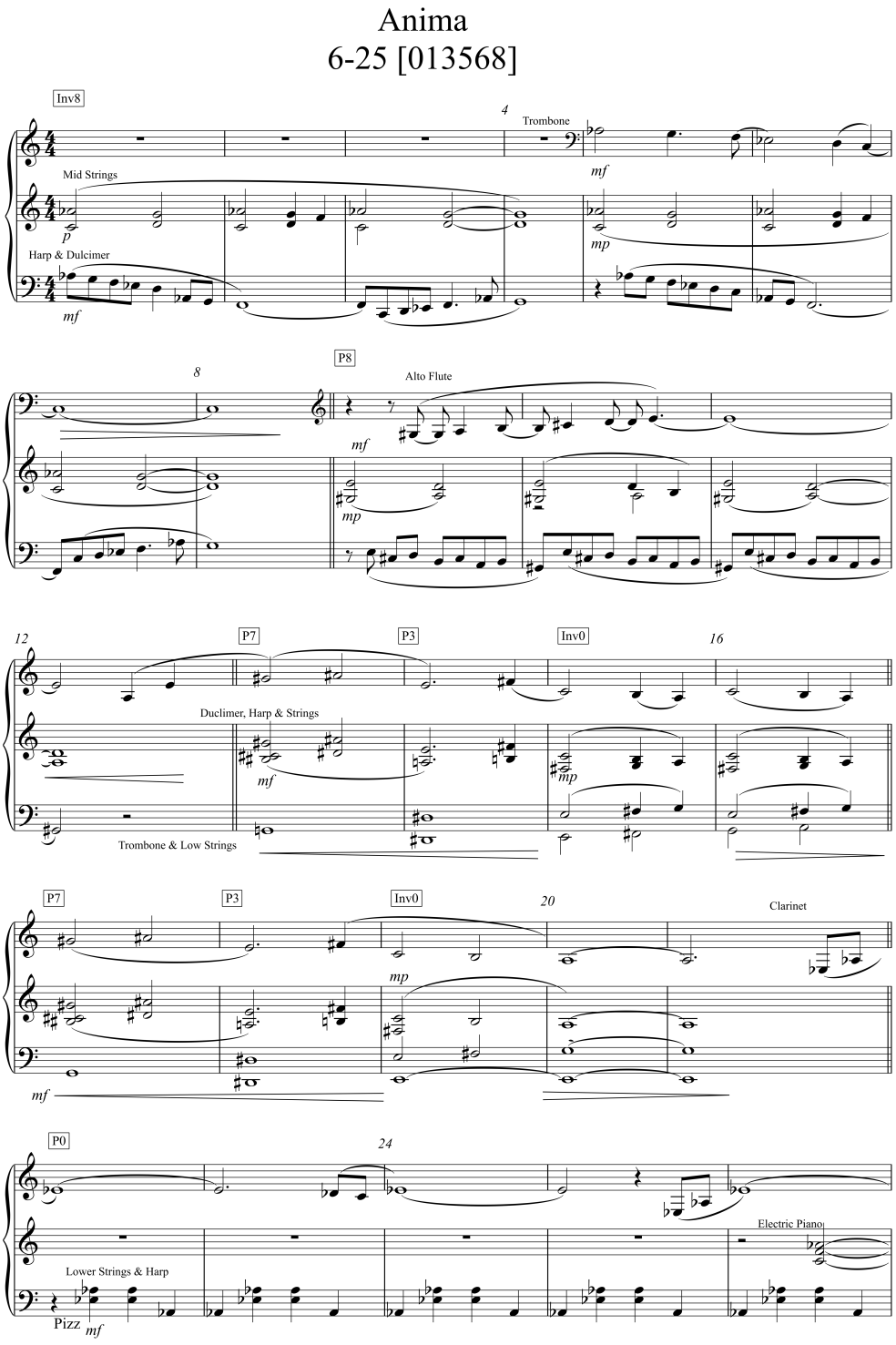
6-26 [013578] Phrygian Mode

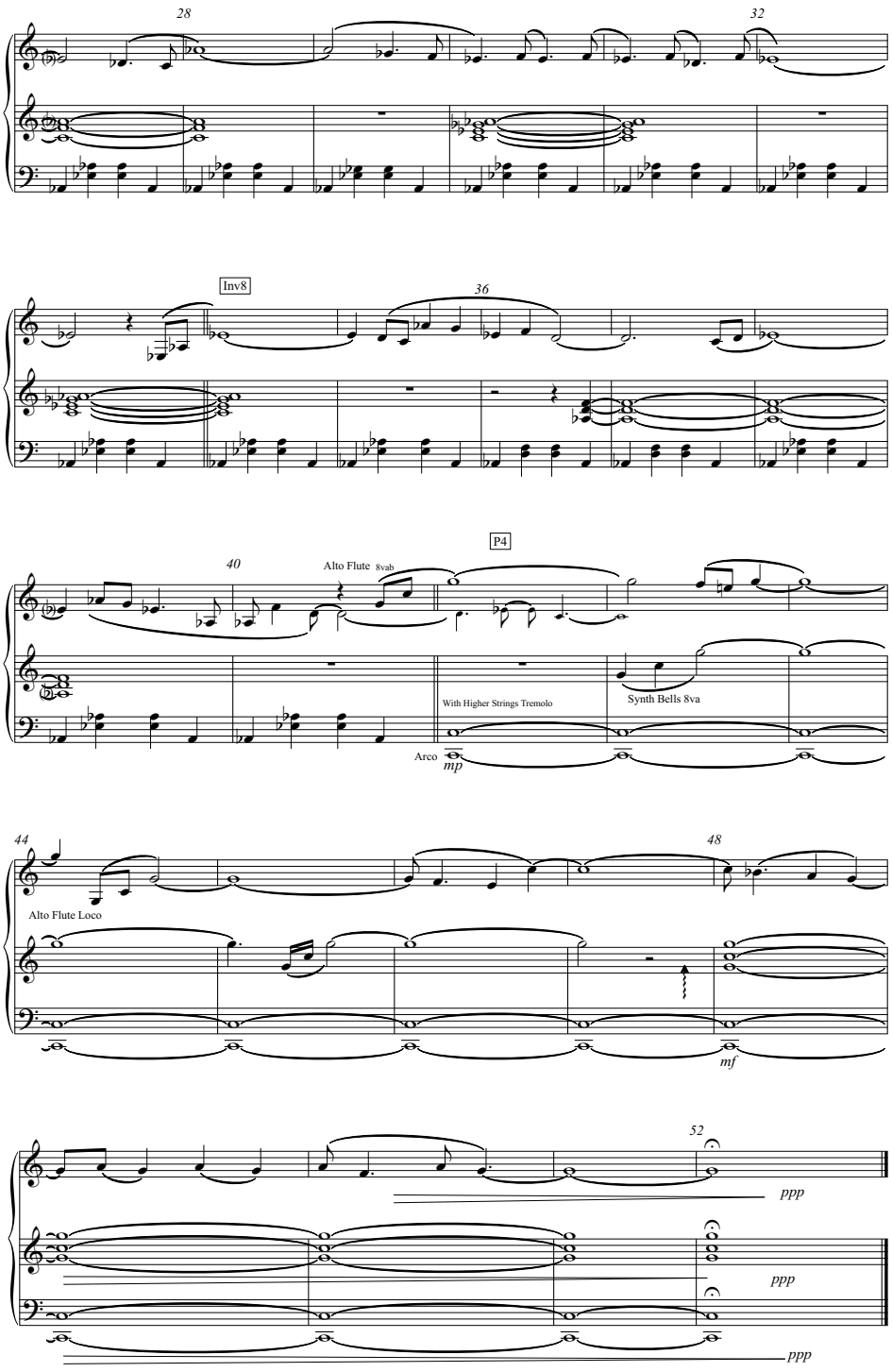
6-25 [013568] Locrian Mode

From a series of compositions scoring the spoken word, *Anima* represents a journey from darkness to light. To produce this transition, the 6-25 hexachord is revealed in two different ways. The "dark" section uses scale-like, eighth-note motion in the lower voices, bars 1-12 – **Inv8** and **P8**, and slightly dissonant twists in bars 13-21 – **P7, P3,** and **Inv0**. A shift occurs moving into bar 22 as the clarinet introduces the "light" section with wider intervals followed by a less active melody **– P0** over a low ostinato. At bar 34, echoing the material introduced by the clarinet, another lift is created with the entry of the alto flute – **P4**. The ostinato figure has

ceased. Shimmering bells and tremolo strings support this final statement.

Figure 44: *Anima* – Score Reduction




28
32
Inv8
36
P4
40
Alto Flute 8vab
With Higher Strings Tremolo
Synth Bells 8va
Arco
mp
44
48
Alto Flute Loco
mf
52
ppp
ppp
ppp

Untitled 2

6-48 [012579]

Interval vector [232341]

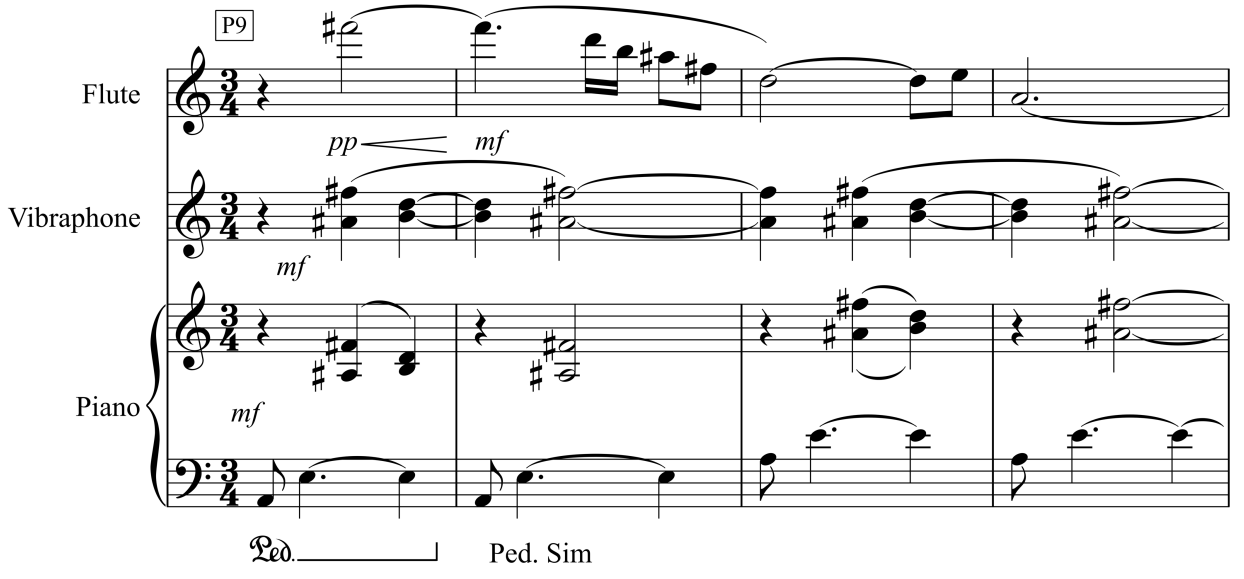
Invariance Vector <11000000>

Z-mate 6–z26

Technique: Use of Z-mate, completion of the aggregate

As outlined earlier, not all hexachord pairs that form the aggregate are related by transposition. In this case the 6-z26 hexachord complements the 6-48 hexachord. *Untitled 2* explores a simple combination of these hexachords in both melody and accompaniment. This short piece begins with a simple **P9** expression of the 6-48 hexachord – the flute and clarinet, with piano and vibraphone accompaniment.

Figure 45: *Untitled 2* –Bars 1- 6



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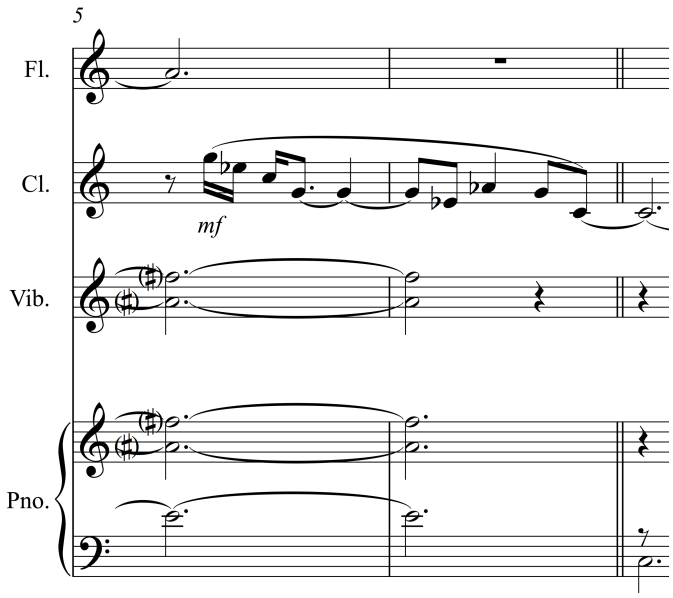
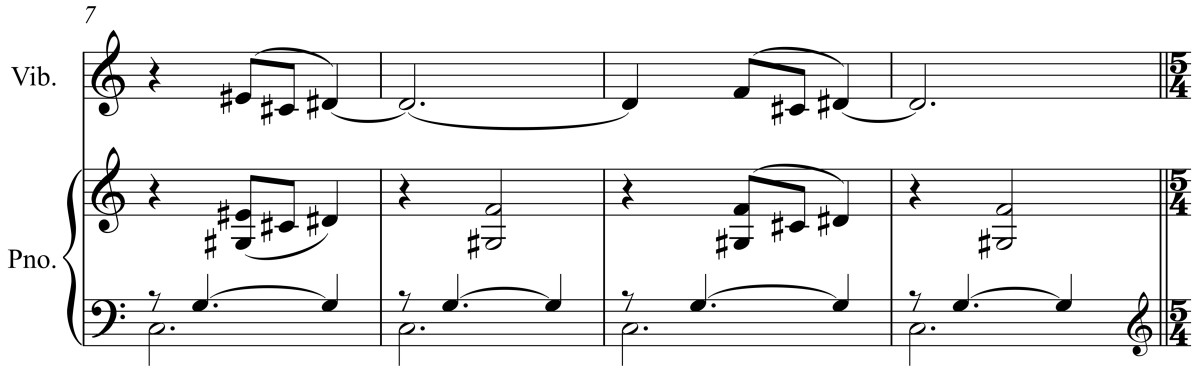


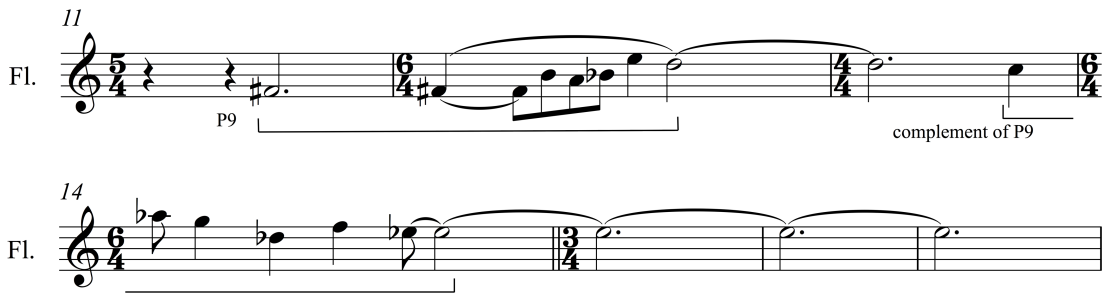
A brief transition follows (bars 7-10) in which both piano and vibraphone play the complement of **P9 –** material derived from 6-z26.

Figure 46: *Untitled 2* **–** Piano and Vibraphone Bars 7-10



The accompaniment shifts back to **P9** as the flute plays all twelve pitch-classes – a melody expressing both **P9** and the complement of **P9** (6-48 and 6-z26) (figure 47).

Figure 47: *Untitled 2* – Flute Bars 11-17



A brief piano and vibraphone transition follows in bars 15-22 (complement of **P9**) followed by a return to **P9** with a clarinet statement (bars 23-26).

During bars 27-34 the piano and vibraphone shift to the complement of **P1** (figure 48) while the flute and clarinet play a melody created from the complement of **P1** and **P1** (6-48 and 6-z26) (figure 49) .

Figure 48: *Untitled 2* – Piano and Vibraphone Bars 27-28

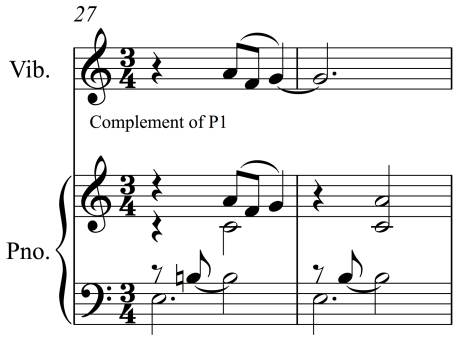
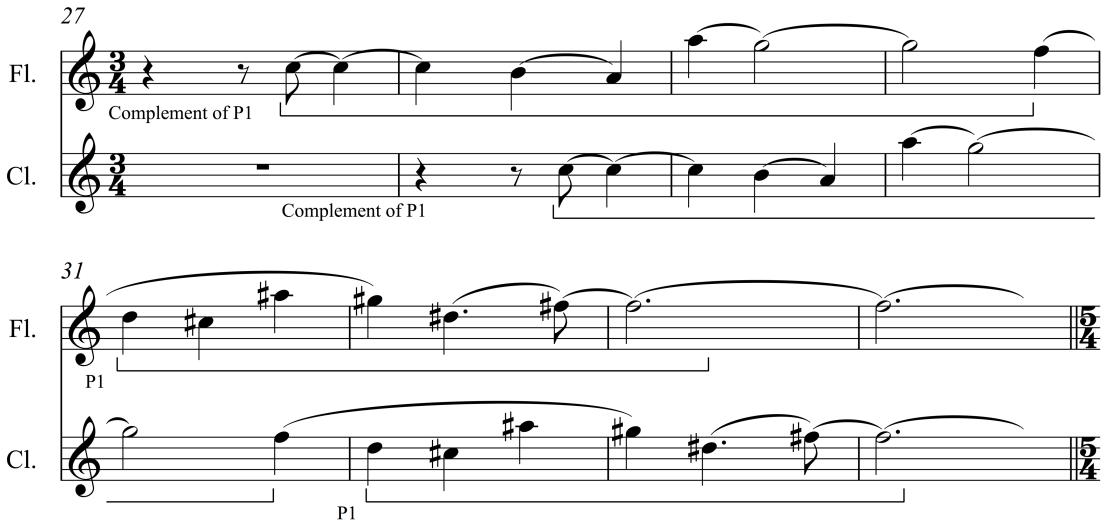
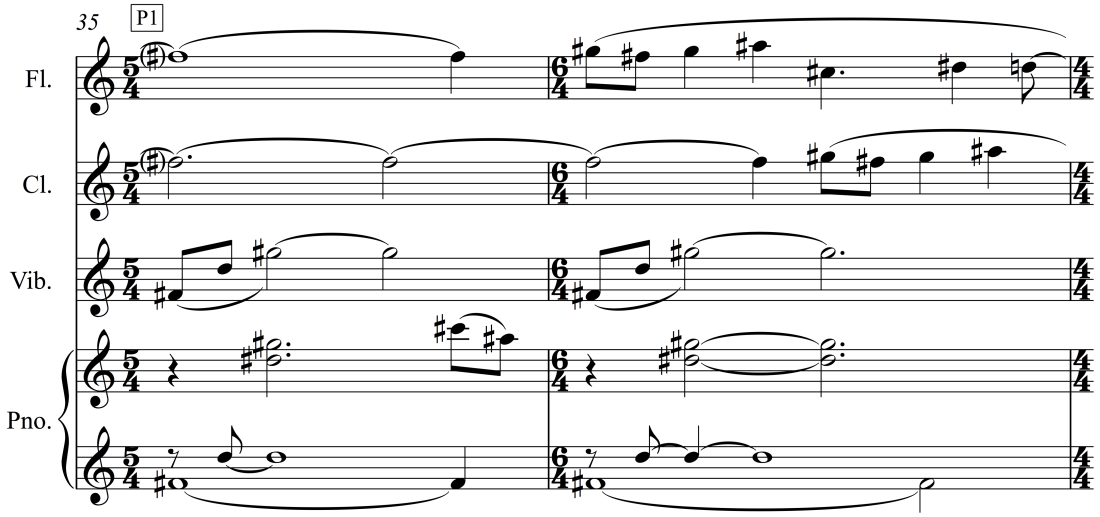


Figure 49: *Untitled 2* – Flute and Clarinet Bars 27-3



For the final five bars there is a shift back to **P1** in all parts

Figure 50: *Untitled 2* – Bars 35-39



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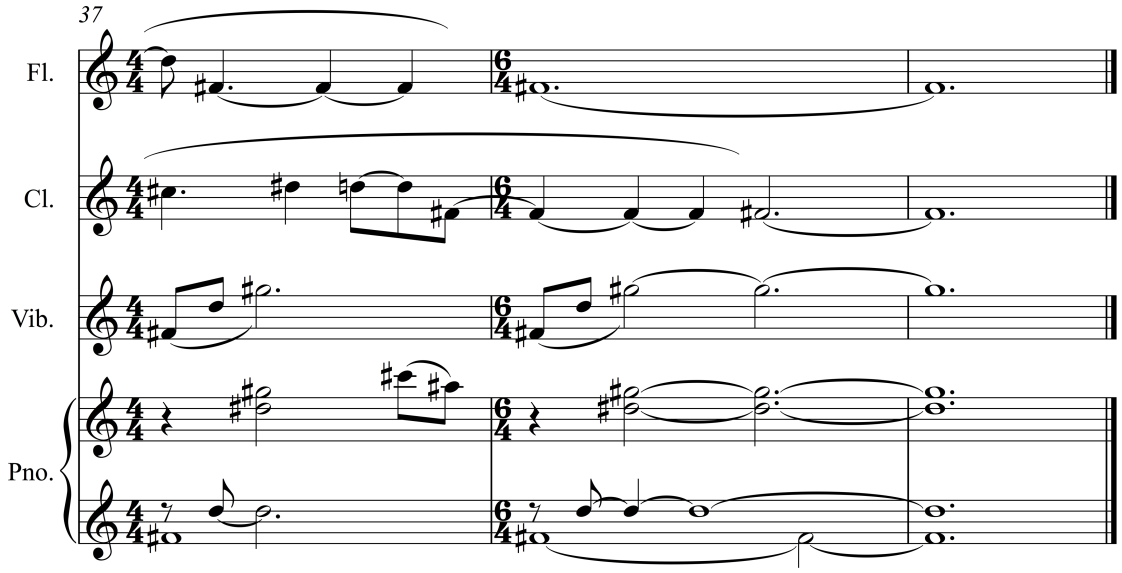


A Potion in Motion

6-32 [024579]

Interval Vector [142250]

Invariance Vector <11001100>

Technique: Combining hexachord transpositions to create new "tonality"

Hexachords may be manipulated to reproduce familiar scalar structures.

It should be noted that the 6-32 hexachord [024579] is very similar to the heptachord 7-35 [013568t] which can, in normal form, be [e024579] – a collection from which all of the classical modes may be extracted. The missing pitch-class eleven (e) in the 6-32 hexachord collection presents an opportunity that may be used to the composer's advantage. Let us consider the 6-32 collection in prime form (pitch-class 0 = C). When translated to notes and [024579] becomes C, D, E, F, G, A. If this collection is combined with a transposition of the same collection from **P0** to **P7** (G, A, B, C, D, E), all of the notes of the 7-35 heptachord are available (with duplications),

and thus any of the classical modes may be reproduced. Combining **P0** with **P5** (F, G, A, Bb, C, D) has a similar result, replicating a transposition of the same 7-35 heptachord.

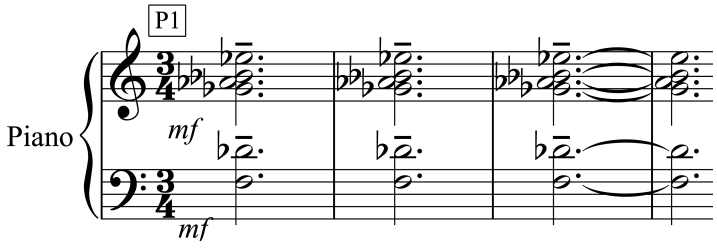
Going to the trouble of realizing these combinations begs the question – why not just use the classical modes as a starting point; why even bother with hexachords? Would the end result not be the same? I believe that the end result would *not* be the same. Manipulating the collections as hexachords, keeping the hexachords in different voices, and creating melodic lines and harmonic structures derived from the transformations of the six-note collection is quite different than creating a work with the Lydian or Mixolydian mode.

A Potion in Motion explores the simultaneous use of the **P0** and **P7**, and then the **P0** and **P5** versions of the 6-32 hexachord. The different versions are never combined in the same voice, but rather are set against each other.

This piece is a theme and variations in which each repeat of the theme, first presented in bars 5-21, offers a different treatment of the original statement. This is done, not through elaboration or rhythmic variation, but by recombining the original two-voice/ two-hexachord statement in different ways – by re-harmonizing it, by transposition and/or by canonic entries of the thematic material. Meanwhile each repeat of a second theme, introduced in bars 22-37, remains (almost) the same.

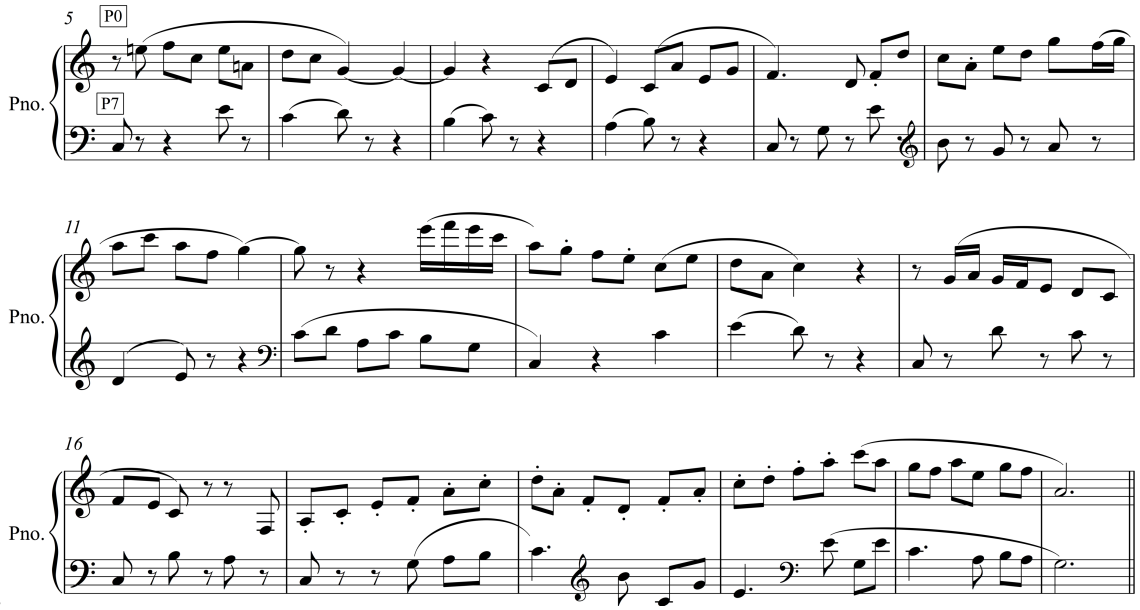
We begin with three repeats of a chord containing all notes of the **P1** transposition. Note that, because the 6-32 Hexachord is invariant through inversion, any **Px** = **Invx-3**, so for example, **P1** = **Inv10**.

Figure 51: *A Potion in Motion* – Introduction



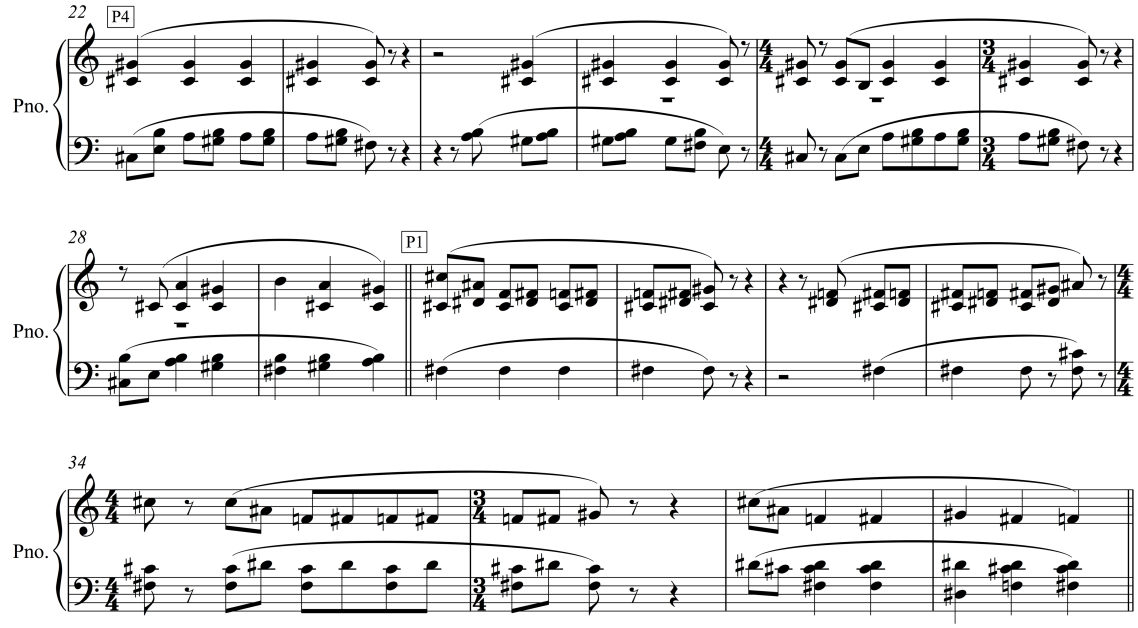
In the first exposition of the thematic material, the upper voice is limited to **P0** and the lower voice limited to **P7** the combination of the two hexachords offers all pitch-classes of a 7-35 heptachord. However, because the two transpositions of the hexachord are kept separate, a distinct sound is achieved.

Figure 52: *A Potion in Motion* – First Theme Bars 5-21



The second theme (bars 22-37) is less linear, with staggered rhythms, vertical structures, and a contained melody concealed in the inner voices. The two-voice counterpoint of the first theme has ended and one hexachord provides for a polyphonic treatment, with **P4** and then **P1**.

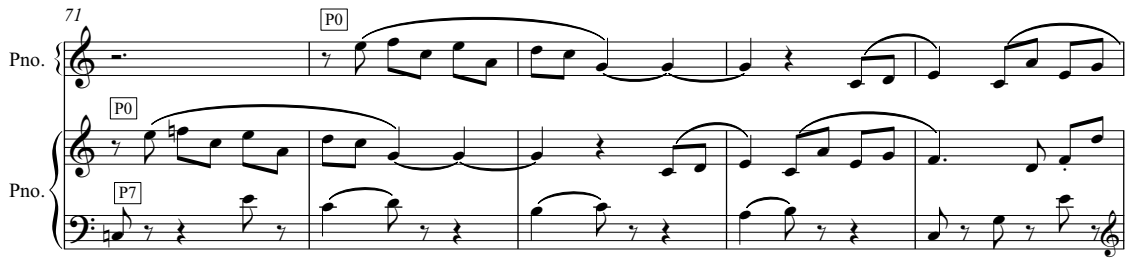
Figure 53: *A Potion in Motion* – Second Theme Bars 22-37



The first variation (bars 38-54) follows; here, the upper voice is transposed from **P0** to **P7**, and the lower voice is transposed from **P7** to **P0**. The upper voice is harmonized by the addition of another higher voice playing the **P0** transposition in bars 49-54. When the second theme returns, it is combined with another, more linear, voice derived from **P4.**

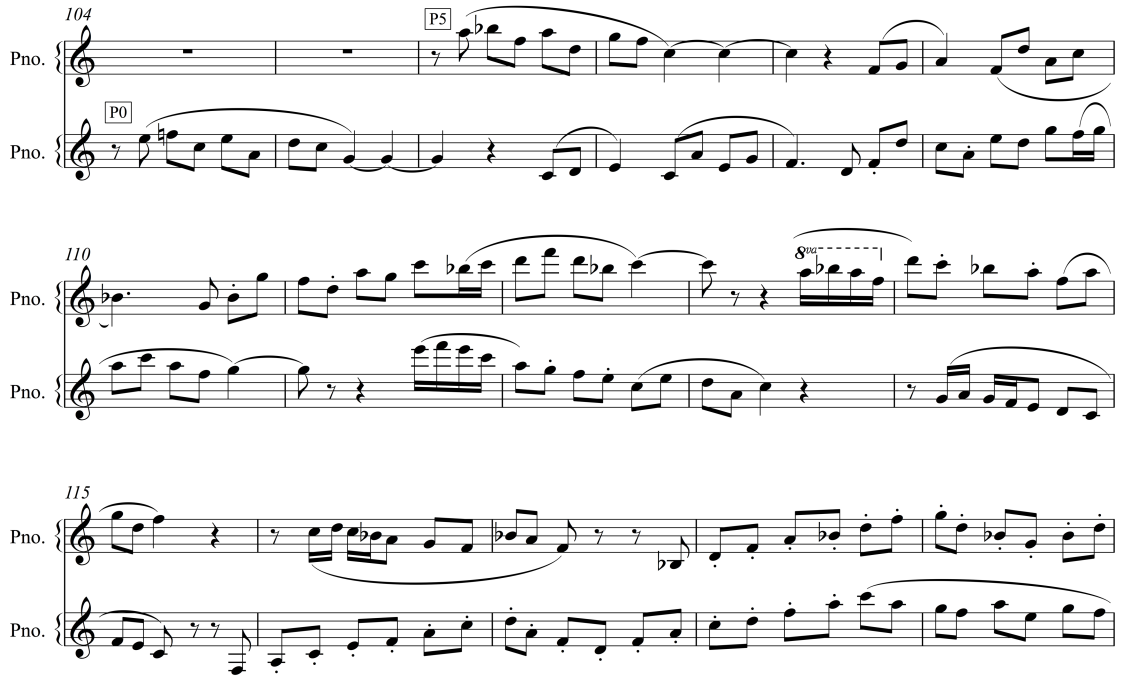
The second variation of the first theme (bars 71-87) sees a return to the original hexachord pairing, but this time the upper voice is joined by another voice echoing the theme a bar later – both upper voices are **P0** and the lower voice **P7**.

Figure 54: *A Potion in Motion* – Second Variation Bars 71-75



On the second repeat of the second theme (bars 88-103) the additional linear line is extended. The third and final repeat of the first theme follows, beginning at bar 104. All previous renderings of the first theme employed some combination of the **P0** and **P7** transpositions of the 6-32 hexachord. This time, with the combination of the **P0** and **P5** transpositions, a slight harmonic shift occurs. Further, both voices are now playing, what was previously the upper voice, in different transpositions – with the entry of the **P5** transposition delayed two bars. The piece concludes with a truncated, transposed version of the second theme, followed by an extended repeat of the sustained introductory chords.

Figure 55: *A Potion in Motion* – Last Repeat of Both Themes and Conclusion



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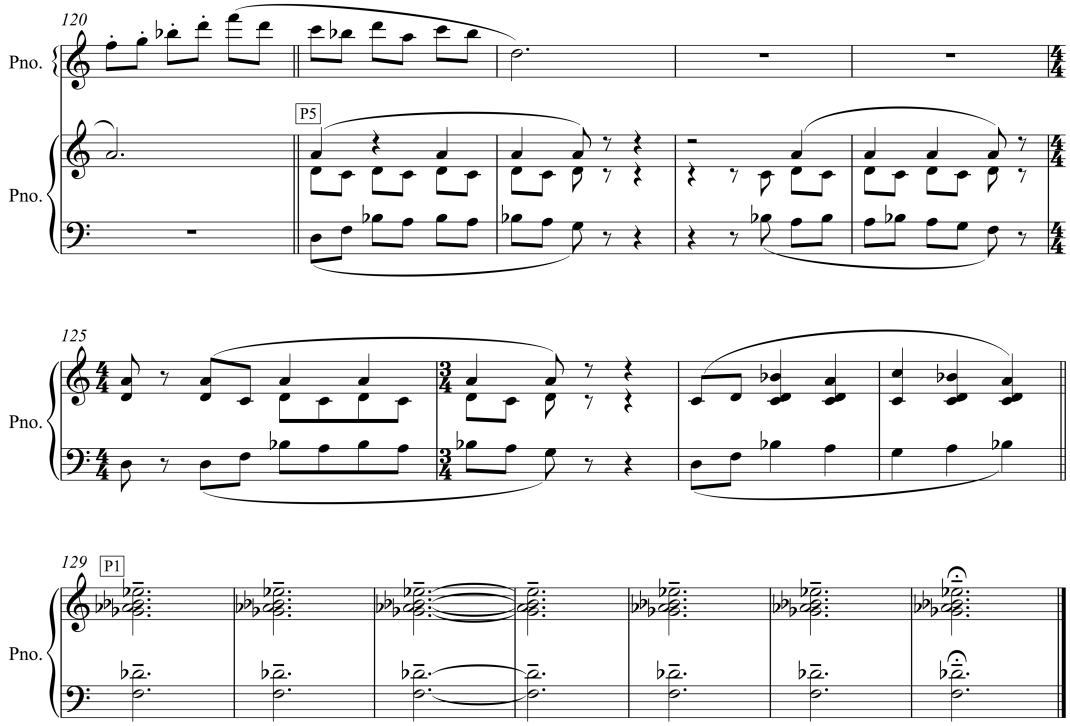


Study for Flute Viola and Harp

6-17 [012478]

Interval Vector [322332

Invariance Vector <1001000>

Z-mate 6-z43

Characteristic: All trichord hexachord

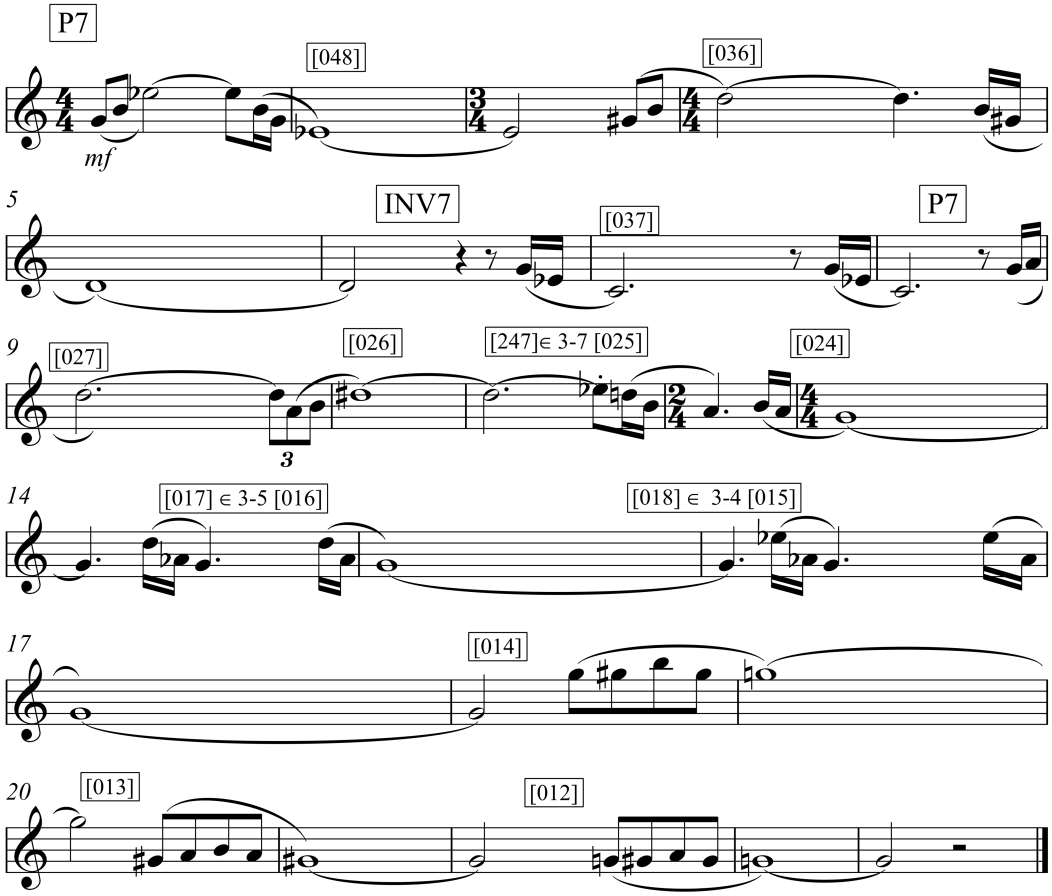
The hexachord 6-17 has the unique characteristic of containing all possible three-note sets (trichords). This piece explores each of the trichords available within the **P7** transposition of the hexachord (G, G#, A, B, D, Eb).

Study for Flute Viola and Harp begins with the trichord that covers the largest intervallic distance [048]. The size of the trichords is gradually reduced until the final melodic statement

[012]. Note that the largest four trichords are triadic formations familiar to more functional harmony –augmented [048] and diminished [036], suspended [057] ([027] in prime form) and major [047] (minor [037] in prime form).

All material is generated from one transposition of the hexachord with the exception of the descending figures in bars 6 and 7 which are derived from the inversion of the same hexachord. The viola and harp parts (not shown) provide support to the melodic and harmonic progression.

Figure 56: *Study for Flute Viola and Harp* – Flute



The following list itemizes each trichord used in order of appearance in the composition, followed by each trichord's set class number, the trichord in prime form, and, if required, a brief

explanation of how the trichord used was derived from prime form. (The symbol ∈ means "is a member of.")

[048] ∈ 3-12 [048] Augmented Trichord

[147] ∈ 3-10 [036] Diminished Trichord – [147] becomes [036] when transposed by -1

[047] ∈ 3-11 [037] Minor or Major Trichord – [047] becomes [037] when inverted and placed in prime form

[027] ∈ 3-9 [027] Suspended Trichord – [027] becomes [057] when inverted and transposed by -5

[248] ∈ 3-8 [026] – [248] becomes [026] when transposed by -2

[247] ∈ 3-7 [025] – [247] becomes [025] when transposed by -2

[024] ∈ 3-6 [024]

[017] ∈ 3-5 [016] – [017] becomes [016] when inverted and placed in Prime form

[018] ∈ 3-4 [015] – [018] becomes [015] when inverted and placed in Prime form

[014] ∈ 3-3 [014]

[124] ∈ 3-2 [013] – 124] becomes [013] when transposed by -1

[012] ∈ 3-1 [012]

IMPROVISATION

When I started composing with hexachords in 2015, I planned to explore their applications in various musical genres. I began recording some of these compositions with other musicians in 2017. Many of these pieces were composed with a jazz aesthetic in mind – the instrumentation, the form, and the rhythm all implying that genre. The harmonic language, of course, has been replaced. These hexachordal compositions have no key signatures, no familiar triadic harmony or harmonic motion, and no chord symbols. Improvisation in these pieces involves the free expression of the unordered pitch-classes of the chosen hexachord and its various transformations.

When asked to improvise, the soloist is provided with the appropriate transposition or inversion of the hexachord. This serves as both a harmonic and melodic resource. All improvised pitch choices are limited to and guided by, the transformations of the hexachord. Playing over the "changes" takes on a new meaning. The improvisor must abandon most familiar patterns and sequences.

I emphasize that this restriction is my creative decision. I could have, for instance, permitted the improvisor to add approach tones and enclosures. Perhaps I could have constructed a more familiar "solo section" based on tonal chord changes similar to the harmonic movement of the hexachord, and return the player, at least for the improvisation, to a familiar environment. However, at this point in my work with hexachords, a strict limiting of note choices seems best suited to my exploration. These limitations notwithstanding, a prepared improviser can produce satisfying and sometimes exciting results, as demonstrated in recordings that accompany this dissertation. All of the improvisors participating in my recordings are eclectic musicians but would probably be identified as jazz musicians. It is striking that in all of these pieces, despite

the severe limitation of note choices, the gestures, rhythm, feel, and aesthetic of that tradition are maintained.

By comparison, let us consider the score for Gunther Schuller's *Abstraction* (1959) – an example of a "third stream" jazz/twelve-tone fusion. This composition is scored for alto sax, electric guitar, percussion, two violins, viola, cello, and two acoustic basses. The piece is five minutes long; most of the non-improvised music is derived from a twelve-tone row. The string parts for *Abstraction* are complex, angular and dense. The improvisation is provided by the alto sax player, who is directed to "ad-lib" for most of the piece. The implication is that the sax player will interpret what the rest of the ensemble is playing – a fragmented twelve-tone row – and freely improvise. Schuller's composition is created with an ordered twelve-tone row. The composer determined that giving the improvisor complete freedom would produce results best aligned with the intent of the piece. Of course, performance depends on the availability of musicians capable of convincingly interpreting such directions. As mentioned earlier, when this piece was recorded for John Lewis's *Jazz Abstractions* (1960), the music was performed by some of the greatest players of the era – including extended improvisations by Ornette Coleman on saxophone. While improvising with ordered sets is extremely limiting and challenging, I have no doubt that if asked to restrict his note choices to the ordered twelve-tone row Coleman would have produced an equally convincing performance.

Each of the following five compositions contains a section of improvisation.

The Sleuth

6-6 [012567]

Interval Vector [421242]

Invariance Vector <11000011>

Z-mate 6-z38

Techniques: Creating movement within hexachord, variation created by inversion, improvisation

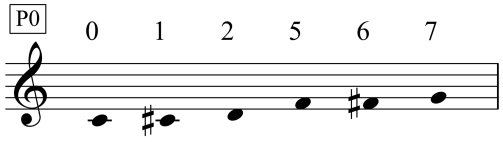
My intent with this composition was to suggest a cue from a film noir score. *The Sleuth* is an example of a jazz/blues aesthetic adapted to the hexachord paradigm. In other words, rhythmic figures, phrasing, and instrumentation associated with jazz and blues are imposed upon the prescribed six-note collection.

The 6-6 hexachord contains sufficient intervallic material to approximate some of the idiomatic sonic structures associated with film noir while offering the composer unique melodic and harmonic opportunities.

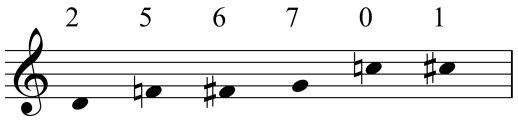
In pre-composition, I examined the hexachord and extracted harmonic material that I thought might be useful. This was done, as shown below, by reordering the pitch-classes of the prime form of the hexachord and creating verticals.

Figure 57: Reordering and Verticals

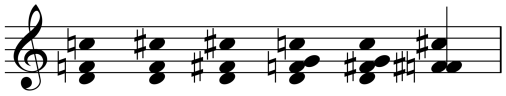
Hexachord in prime form:



Hexachord reordered:

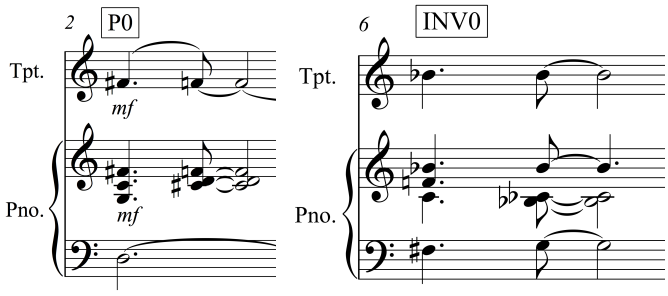


Some of the possible verticals:



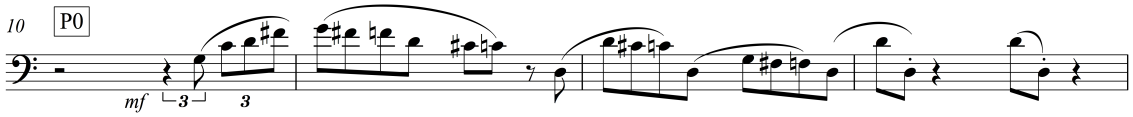
In this piece, as in most of my hexachord compositions, harmonic motion is created by movement within hexachords and by movement through different transformations of the hexachord. The introductory figure played by the piano and trumpet demonstrates movement within the hexachord.

Figure 58: *The Sleuth* – Movement Within the Hexachord



The rhythm of the melody performed by the bass and tenor saxophone, suggests be-bop jazz lines. The note choices are limited to the **P0** hexachord. When the supporting harmony changes to **Inv0**, the melody continues as **P0.**

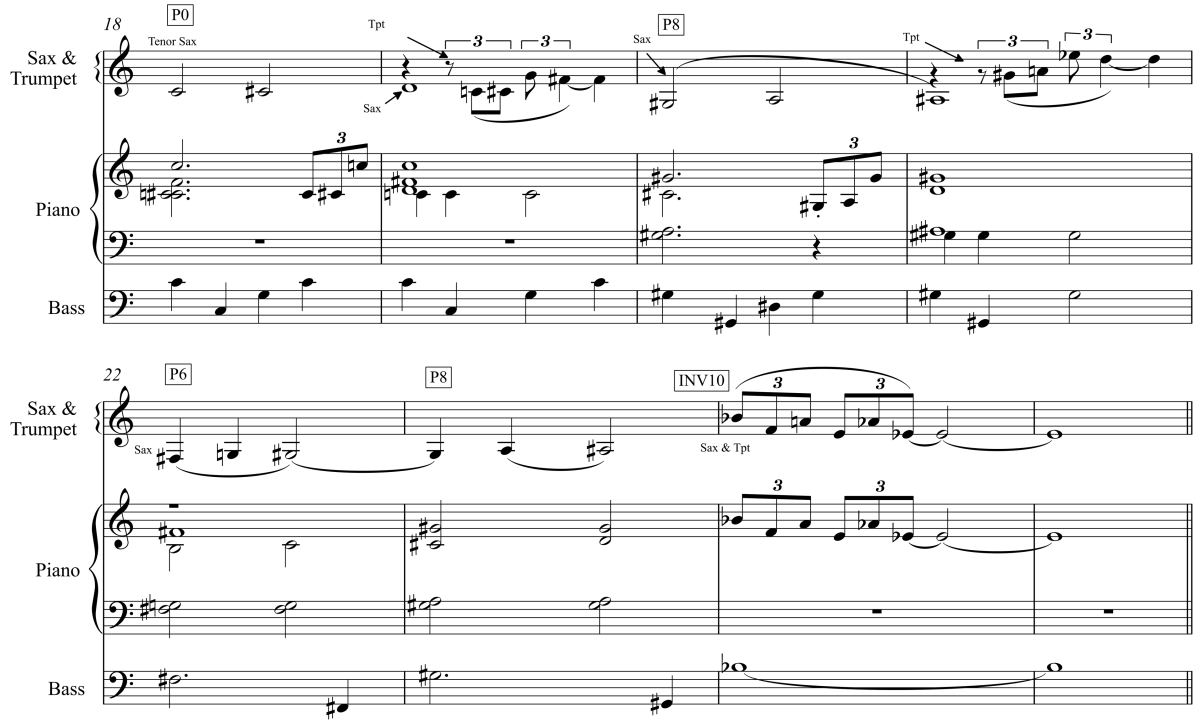
Figure 59: *The Sleuth* – Melody



The contrasting release of the **B** section at bar 18 shows movement through different versions of the hexachord. This eight-bar sequence, built from **P0, P8, P6, P8,** and **Inv10,** is transposed when repeated – up a minor third to **P3, P11, P9, P11,** and **Inv 1.**

Figure 60: *The Sleuth* – Transformations

Harmonic Motion Through Transformations of Hexachord

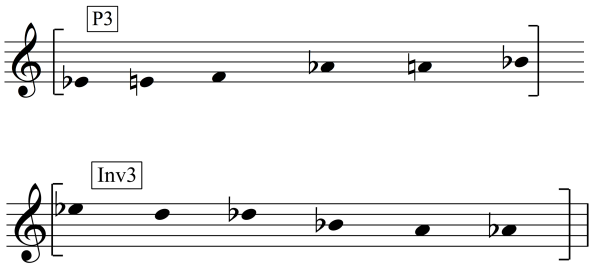


This composition includes an improvised piano solo. As outlined previously, the note choices for improvisations in these hexachord pieces are guided not by chord changes, but by the transformations of the hexachord. Restrictions of note choices notwithstanding, the listener

unfamiliar with the genesis of the composition would probably not notice anything unusual about the solo beyond a slight twist in the musical syntax.

The sixteen bars of the solo are divided into four sections – four bars each of **P3, Inv3, P3,** and **Inv3**. The improviser is provided the note choices shown in figure 61 for each transformation.

Figure 61: *The Sleuth* **–** Piano Solo Note Selection



Late One Night

6-39 [023458]

Interval Vector [333321]

Invariance Vector <10000000>

Z-mate- 6-z10

Technique: Dividing hexachord sequence into two trichord sequences, improvisation

This piece is from the first collection of hexachordal compositions that I recorded with improvising musicians. Although I am reluctant to call these pieces "jazz," they were inspired by the jazz innovators of the past – those who attempted to challenge existing conventions.

In 1959 Miles Davis released *Kind of Blue* one of the most innovative and successful of all

jazz albums. Davis introduced a re-thinking/readjustment of the jazz paradigm– abandoning traditional chord progressions and the well-established urgency of be-bop. A new sound was created using different constraints and a new, leaner, more relaxed aesthetic.

In my hexachordal compositions, familiar reference points, like chord symbols and key signatures, are not present. The improvisor is compelled to reconsider his or her technical and aesthetic engagement with the music. Most well practiced patterns and "licks" are meaningless. Nonetheless, I believe that the creative voices of the players are still evident; the language has shifted but they are able to speak with eloquence.

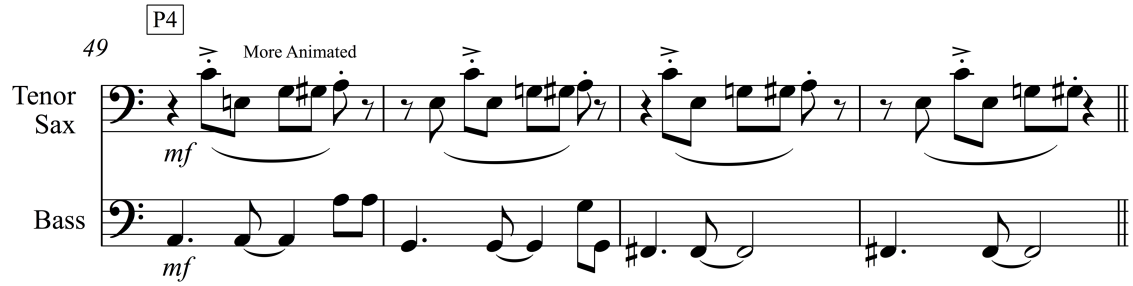
For *Late One Night* I have extracted intervallic structures from the hexachord that suggest familiar tonal gestures. **P0** (C, D, Eb, E, F, Ab) contains an F minor triad and the pitches E, Eb, and D. I created a two-bar pattern for use as a recurring motif – an adaptation of the common descending line (Fm - Fm/Eb - Fm/D). Although the piece is constructed around this simple movement, variation and development are created through transposition, inversion and, as explained later, trichord extraction.

The piece begins with the bass and piano establishing the descending figure described above and continues at **A** with a statement of a simple contained melody expressed in **P0** then **P7** by the tenor saxophone.

Figure 62: *Late One Night* – Melody at A

At **B**, bar 49, a transition begins. This connecting passage will occur twice more in the piece always remaining, despite transposition of the **A** material, at **P4.**

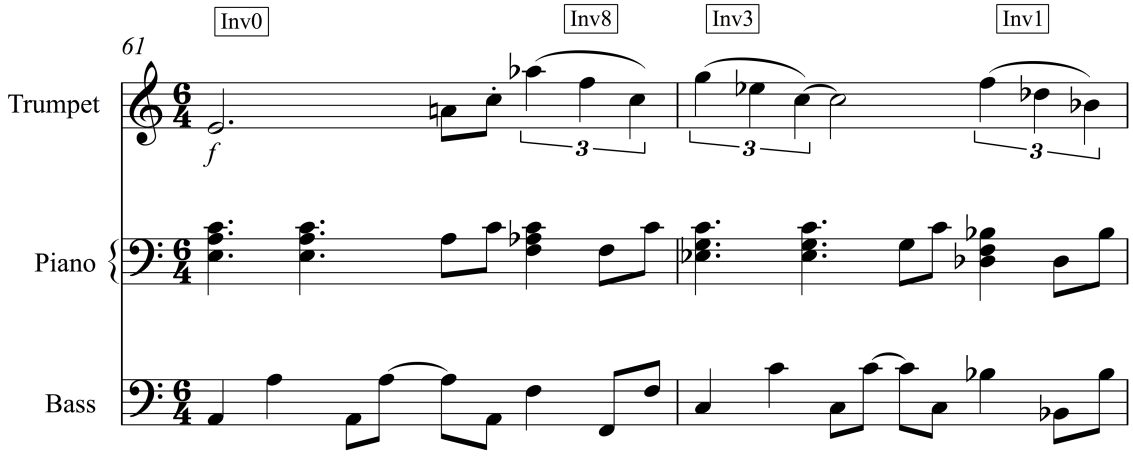
Figure 63: *Late One Night* – Riff



A melody, played by the trumpet, rises and falls over an octave and a half in the **C** section. Here I am fragmenting the hexachord. At bar 61 a passage begins in which a series of trichords are extracted from a sequence of four hexachords – **Inv0**, **Inv8**, **Inv3**, and **Inv1**. Thus, only three pitch-classes of each hexachord are used as each transformation of the hexachord passes. From bar 61- 64 the trichords are minor triads. As can be seen in figure 64, the trumpet actively outlines all notes of each trichord in the first two bars and only one note of each trichord as the sequence repeats.

Figure 64: *Late One Night* – Fragmentation 1

Trichords as Minor Triads



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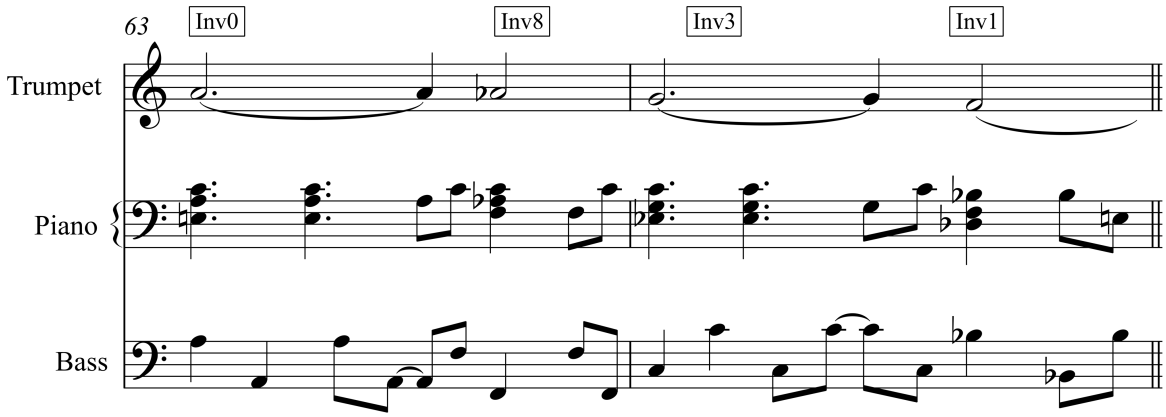
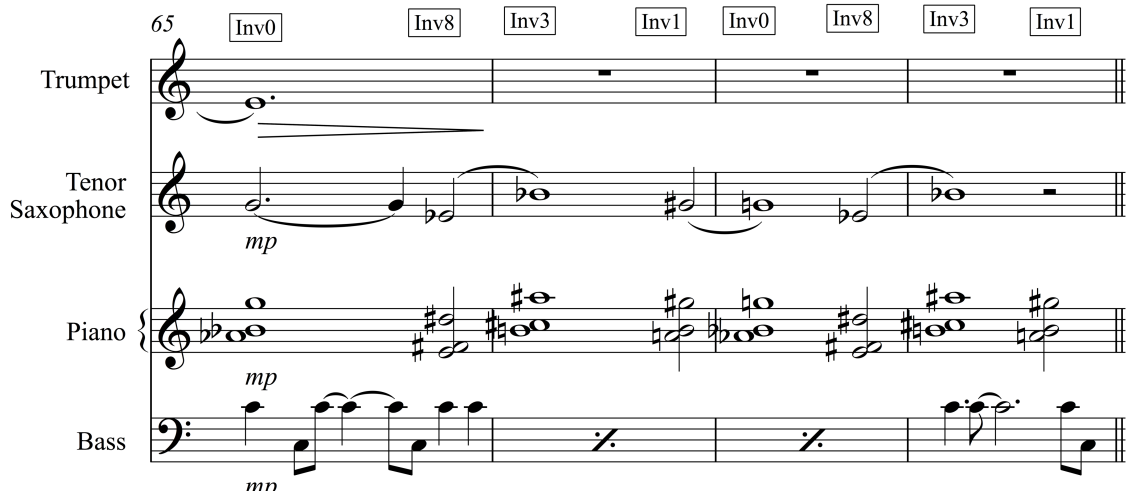


The three remaining notes from each hexachord in the sequence are used in the following four bar sequence (beginning at bar 65). The bass pedals a C beneath these three-note chords. The C is common to each of the inversions **Inv8**, **Inv0**, I**nv3**, but is *not* a member of **Inv1** – my strict application of hexachord exclusivity was set aside here for musical reasons.

Figure 65: *Late One Night* **–** Fragmentation 2

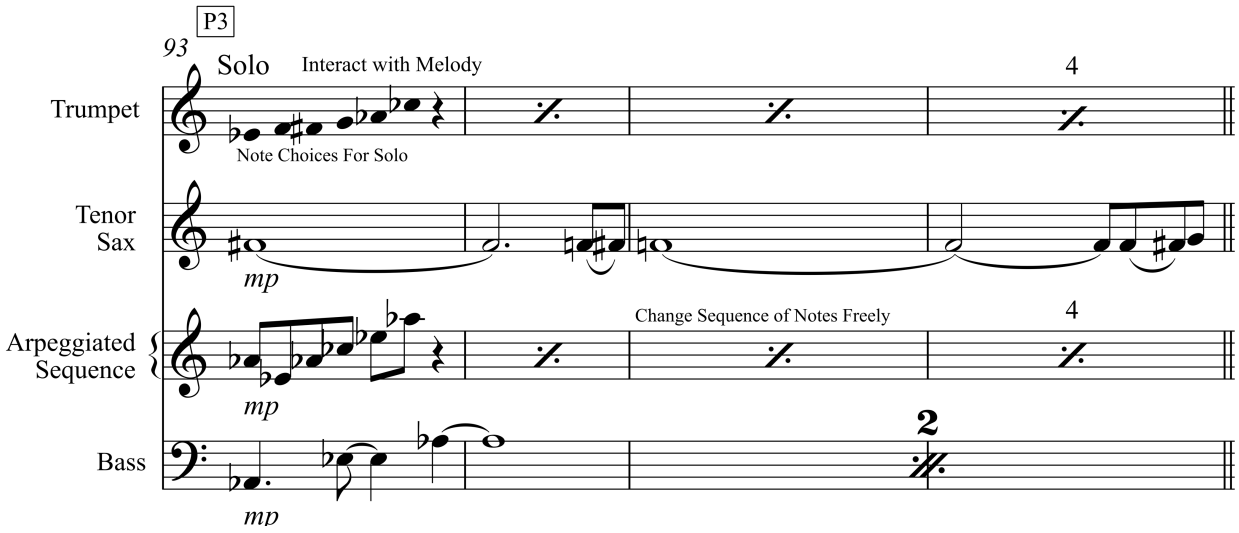
Remaining three notes of each hexachord



The connecting/transition riff returns – followed by a repeat of the introduction material transposed from **P0** to **P3.** In this piece I abandon the familiar solo section so often present in

jazz compositions. Instead, the improvisor engages with a restatement of the melody. A trumpet solo begins with the transposed **A** melody serving as a background to the solo. The soloist was directed to interact with the melody and to use only the note choices offered by the current transformations of the hexachord – **P3** and **P10**. Figure 66 displays the note choices for improvisation in the first four bars of the trumpet solo.

Figure 66: *Late One Night* – Trumpet Solo



Late One Night concludes with a return to the connecting/transition riff – **P4**, the trumpet continuing with short adlib exchanges.

Untitled 3

6-29 [013689] (Forte Version)

Interval Vector [224232]

Invariance Vector <11000000>

Z-mate 6-z50

Techniques: Emulation of functional harmony, improvisation

As outlined in an earlier footnote (6), Allen Forte and John Rahn categorized hexachord collections in a slightly different manner. Although the pitch-class set tables they created are almost identical, there are two hexachords that differ in prime form – 6-29 and 6-31. I have composed pieces using both the Rahn and Forte versions of these collections. While the differences between the two are slight (all of the pitch-classes in Rahn's 6-29 [023679] can be obtained by transposing Forte's version), using the Forte version [013689] instead of Rahn's [023679] subtly redirects the choices made by the composer.

While this piece exhibits a number of characteristics that establish a sense of functional harmony, the hexachordal design offers a curious twist. Although each of the **A**, **B**, and **Solo** sections remains on one transformation of the hexachord for eight bars or longer, the motion within the hexachord creates a sense of harmonic movement. The length of each section and the transformations used are shown below.

Introduction bars 1-17– **Inv0 P0**

A bars 17-33 – **P0**

Transition bars 33-35 – **Inv0**

A^2 bars 35-43 – **P8**

B bars 43-59– **P2 P10**

Solo bars 58-77 – **P0**

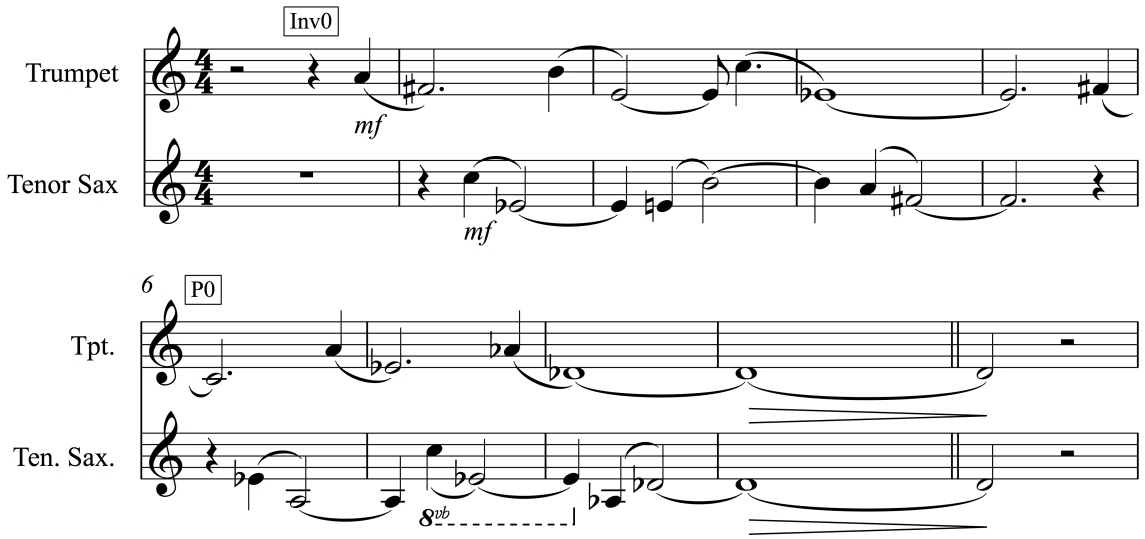
A² bars 77-85 – **P8**

B bars 85-101 – **P2 P10**

Coda bars 101-113 – **P0 Inv0**

Initially, the trumpet and tenor saxophone outline an ambiguous tonality alternating between the inverted and prime form of the hexachord, a progression echoed in bars 9 -16 by the bass clarinet and vibraphone.

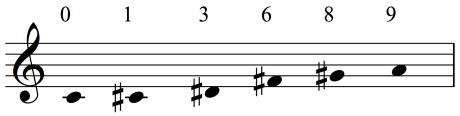
Figure 67: *Untitled 3* – Introduction Bars 1-10



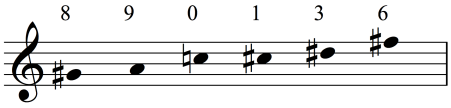
After the introduction a repeating two-bar figure in the piano is introduced suggesting a dominant seventh chord with a flattened second degree. During pre-composition, the sequence of pitch-classes of the hexachord were re-arranged and focused to emphasize this.

Figure 68: *Untitled 3* – Pre-composition Extraction

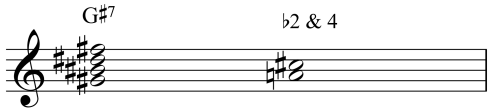
Prime Form **P0**



Re-ordered **P0**

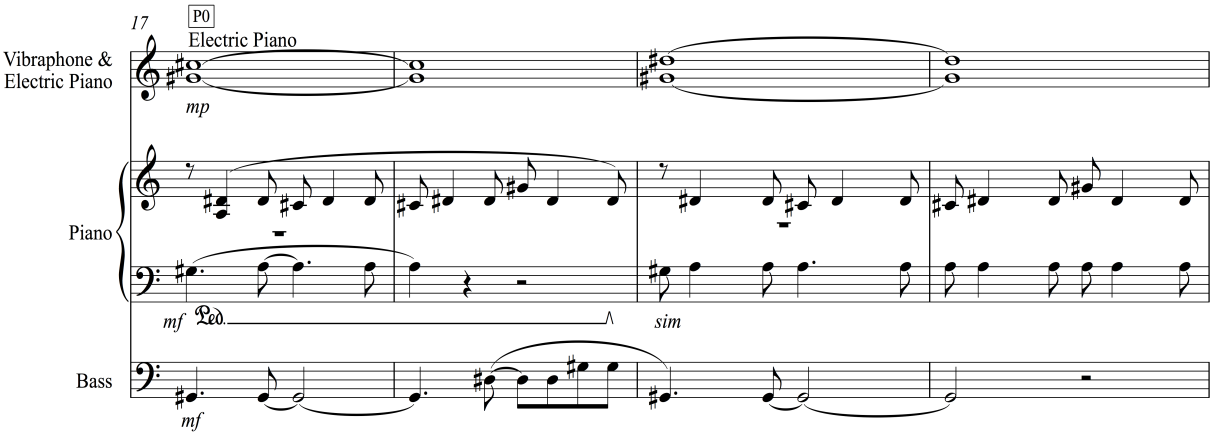


Extracted Verticals (with enharmonic re-spelling)



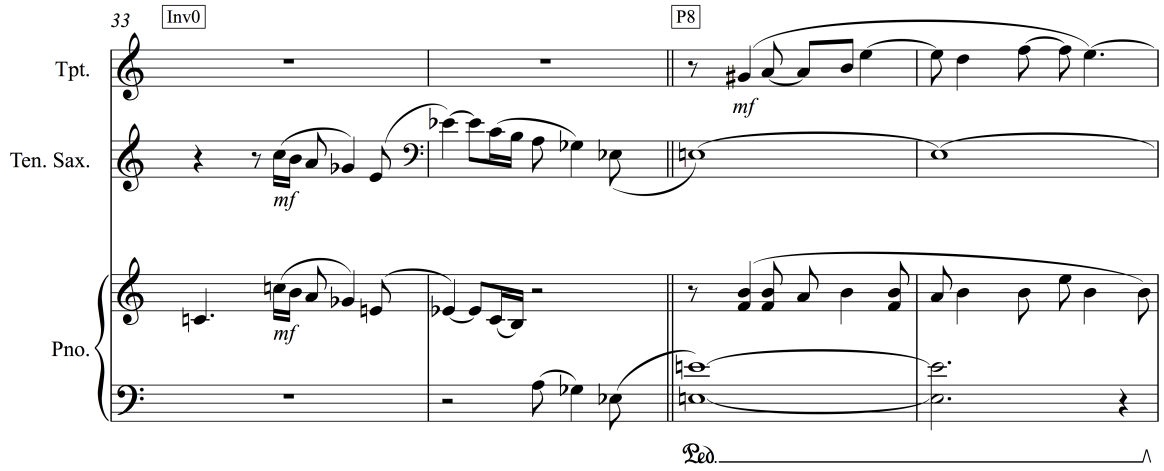
When the rhythm section enters at bar 17 the bass and piano establish a tonal center around G#, reinforced later by a simple sax melody.

Figure 69: *Untitled 3* – Bar 17



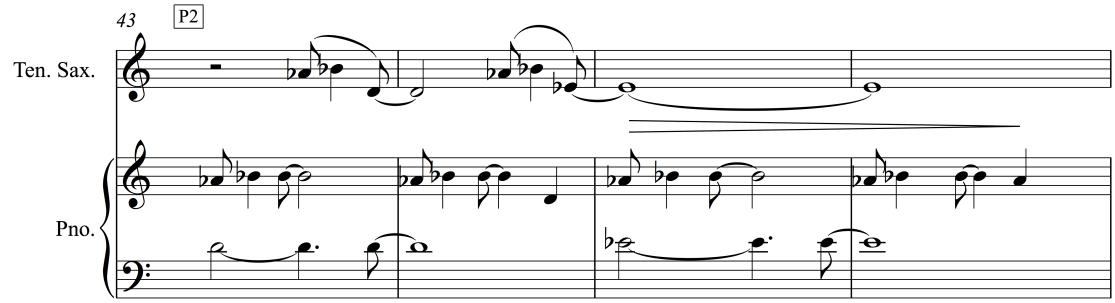
The piano and tenor sax play a transition line derived from the inversion of **P0**. New melodic material is introduced over a transposition to **P8.**

Figure 70: *Untitled 3* – Inv 0 Transition to P8



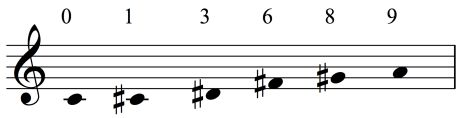
The release beginning at bar 43 approximates a dominant to tonic resolution; sounds from the world of functional harmony are borrowed, but here the significance of these sounds has changed. This harmonic sequence is related to what comes before and after, and because it is derived from the same hexachord, the melodic and intervallic choices are restricted. The progression is imitating functional harmony, while performing as a hexachordal cell, even though only four pitch-classes of the hexachord are employed. The phrase introduced at **P2** in bar 43 is repeated, then transposed to **P10** at bar 53.

Figure 71: *Untitled 3* – Bars 43 - 46



The note choices for the improvised solo beginning in bar 58 are restricted to **P0.** Here, as in the rest of the composition, the listener may not be aware of the underlying constraints.

Figure 72: *Untitled 3* – Note Choices for Solo



After the solo, the transition figure returns us to the two-bar piano ostinato **P8**, followed once again by the release with slight rhythmic alterations. The piece then concludes with a brief coda, comprised of the ostinato, a reinterpretation of the melody **P0**, and the transition **Inv0**.

Out of Sight

6-44 [012569]

Interval Vector [313431]

Invariance Vector <10000010>

Z-mate 6-z19

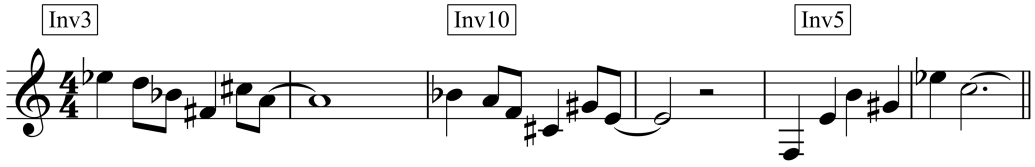
Technique/Characteristic: Simultaneous use of two contrasting melodic lines, improvisation

Initially composed for tenor saxophone and trumpet, accompanied by vibraphone, bass, and drums, this piece was inspired first by the Eric Dolphy recording *Out to Lunch* (1964). After a few rewrites and a few instrument changes, the piece is now perhaps more reminiscent of the Gary Burton recording of Carla Bley compositions – *Dreams So Real* (1975). Nonetheless, the

intent of this composition is to emulate, using the hexachordal paradigm, the adventurous harmonic and melodic styles of some of the mid twentieth-century jazz innovators. Although originally conceived as simple **AABA** form, repetition of the form was abandoned in favour of a simple sixteen-bar solo section introduced by an eight-bar transition.

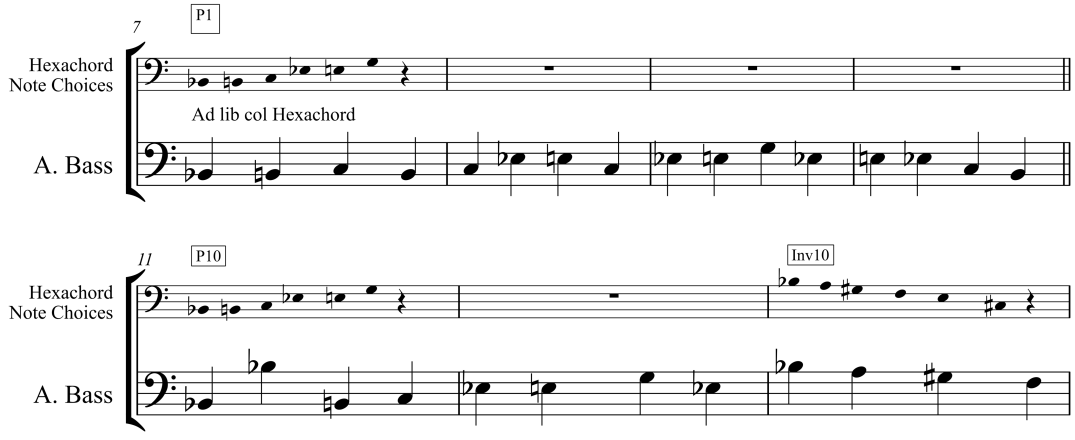
The piece begins and ends with a six-bar phrase.

Figure 73: *Out of Sight* – Intro and Ending



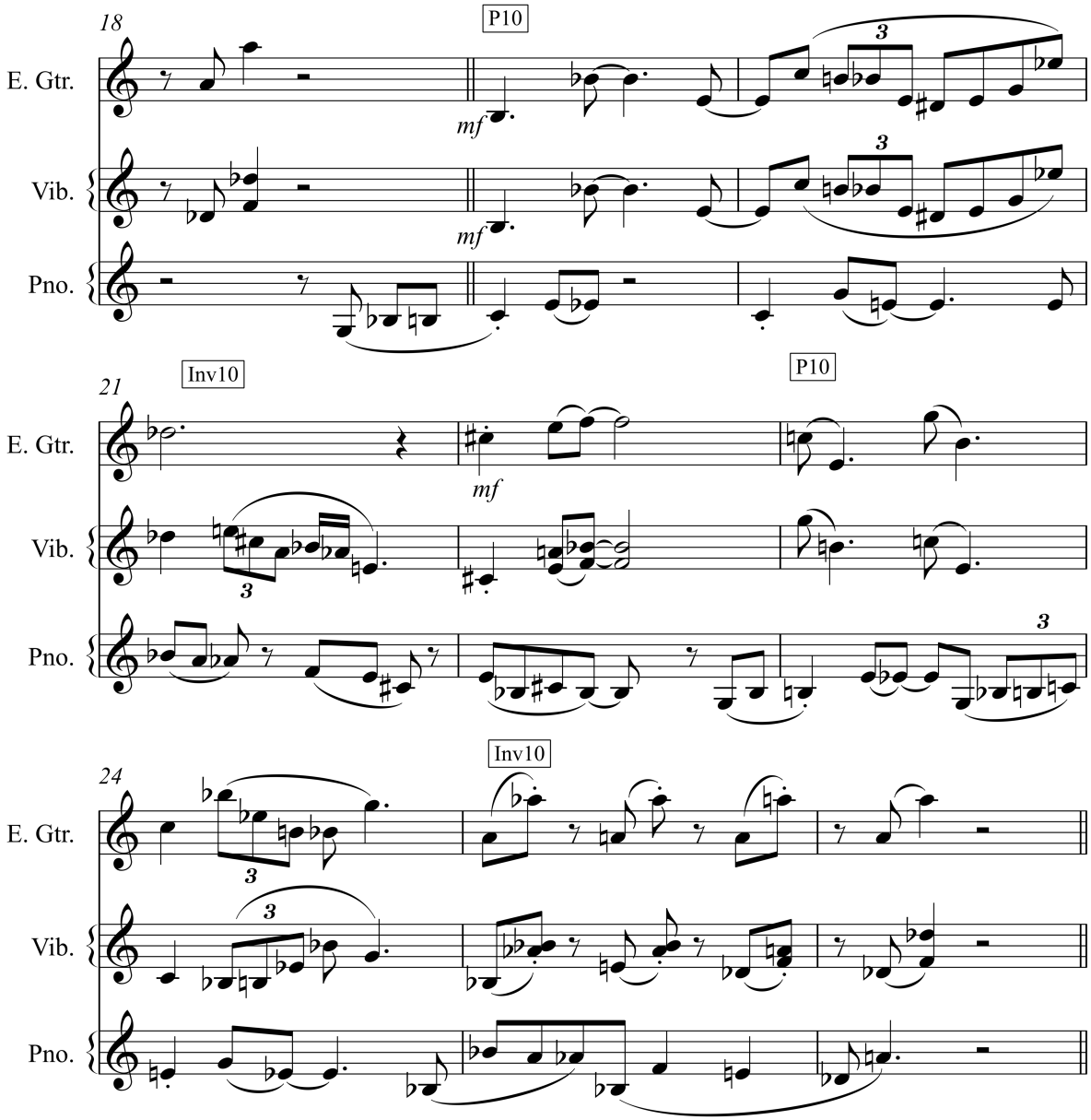
It is common practice for the members of a jazz rhythm section to create their own part, based on the chord changes provided, and guided further by the melody, form, and context of the composition. The "walking bass" is most often an improvised linear line that serves to both propel the rhythm and guide the harmonic motion. In this piece a suggested line was sketched in for the bass player, and the current hexachord note choices were also displayed. The player was instructed to create a part based on these directions.

Figure 74: *Out of Sight* – Bass



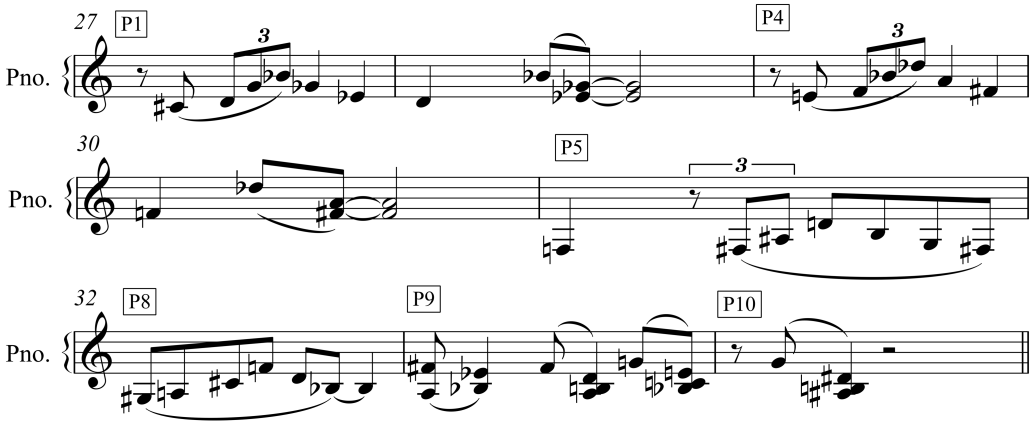
For the **A** sections, two contrasting themes were created. The first theme, played by vibraphone and guitar, is introduced the first time through **A**. The second, which is introduced by the piano on the first repeat of **A**, serves to offset the rhythm and contrast the angular contour of the first. Both themes were created from a repeating four bar expression of **P10** and **Inv10**.

Figure 75: *Out of Sight* – Themes



For the **B** section, a rotation was mapped out using the six transpositions of the hexachord that contain Bb. The acoustic bass pedals this common tone. In figure 75, the piano melody is shown along with the transformations of the hexachord. Note that as the hexachords change, each melodic phrase contains a Bb (or A#).

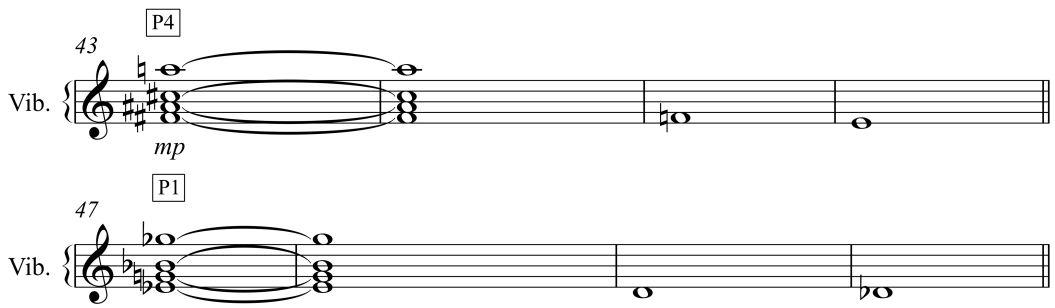
Figure 76: *Out of Sight* – B Rotation



When the **A** section returns, the two contrasting melodies are emphasized in three ways: the acoustic bass doubles the piano melody, the drums are silent, and all of the harmonic and melodic material is shifted up a tritone.[30] The transition that follows guides us back, over eight bars, to the **P10** transformation and the solo section. In this transition, a triad with a #9 above the root, followed by a descent, suggests the blues.

[30] While such a transformation would certainly be called a *modulation* in tonal music, the term implies a home key, movement from that key, and key signatures – elements that are perhaps suggested in this music but are not present.

Figure 77: *Out of Sight* – Transition



It is not surprising that musicians performing these pieces, especially those asked to improvise, interpret this music with a tonal bias. Gary Schwartz, the guitarist on this piece, commented that he regarded the hexachord used in this composition as a dominant seventh chord with an additional flatted third and major seventh – easy to visualize and convenient for a jazz/blues tinged solo. Although the prescribed solo section of this piece is not a twelve-bar blues, the sixteen-bar form, combined with the simple harmonic movement, certainly guide the improvisor in that direction.

The following is an overview of the transformations of the hexachord used in this piece:

Introduction

Inv3/Inv3/ Inv10/ Inv10/Inv5/Inv5

P10/P10/P10/P10

A

P10/ P10/ Inv 10/ Inv 10/

P10/ P10/ Inv 10/ Inv 10/

A Repeat with counter melody on Piano

B Rotation

P1/ P1/ P4/ P4/

P5/ P8/ P9/ P10

A Transposed

P4/P4/ Inv4/ Inv/4

P4/P4/ Inv4/ Inv/4

Transition

P4/ P4/ P4/ P4/

P10/ P10/ P10/ P10/

Solo Section

P10 x8

P3 x4

P10 x4

Coda – same as introduction

A Sigh for a Cipher

6-45 [023469]

Interval Vector [234222]

Invariance Vector <11110000>

Z-mate 6-z23

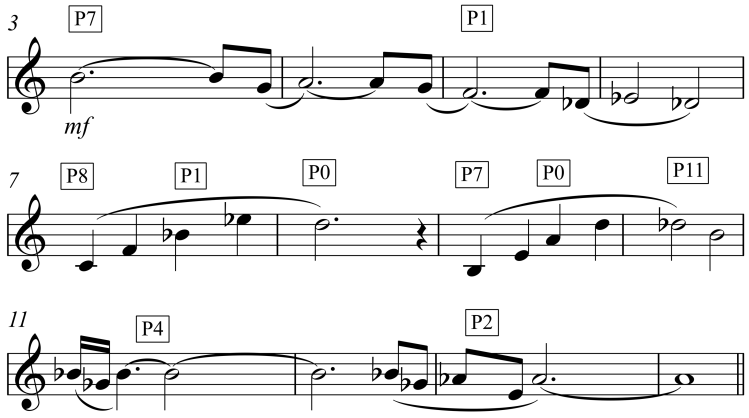
Technique/Characteristic: Complex solo section

In *A Sigh for a Cipher*, a complex hexachord progression underlies the initial languid melodic twelve-bar statement. When the form returns to this progression, after the ethereal floating mood created in the second and third sections, the saxophone is asked to improvise. Harmonic/hexachordal movement within "solo sections," as demonstrated in previous

compositions, has thus far been kept to a minimum. However, this twelve-bar section includes ten transformations of the hexachord. A transcription of the solo in figure 83 is accompanied by the note choices available to the player. The result is gratifying for me, for it demonstrates that fluid, soulful improvisation is possible in this environment. And why would it not be? The jazz improvisor is accustomed to deciphering chord symbols, choosing which scales to apply, and creating appropriate lines, riffs, and figures. In this case, the "scales" are provided, and with some preparation, the improvisor may create convincing lines over complex harmonic movement. In addition to the improvisation, the sax player, Colleen Allen, added enhancements to most of the written melodic lines – all of these embellishments are derived from the shifting transformations of the hexachord.

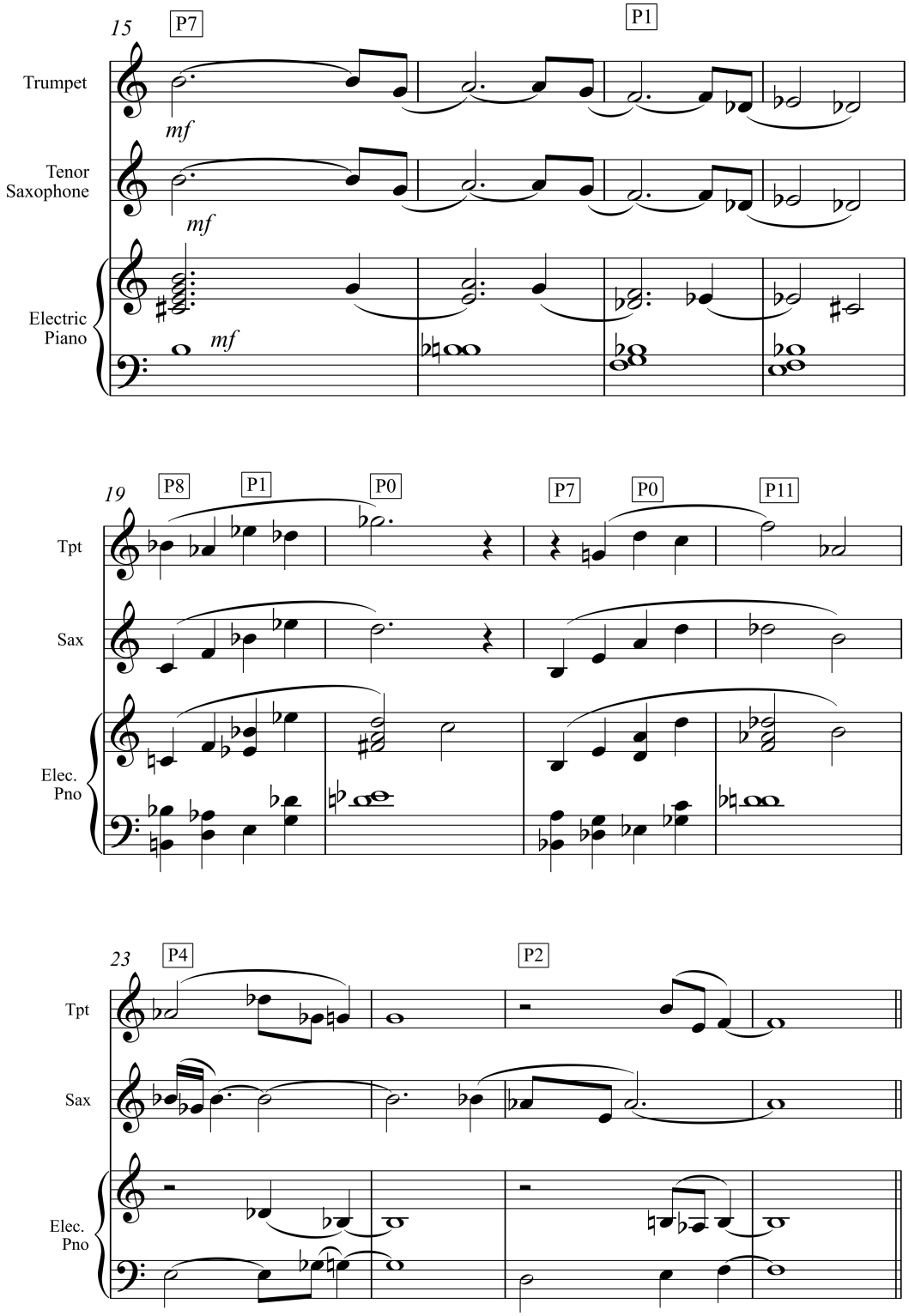
Figure 78 shows the first iteration of the sax melody with the ten transformations of the hexachord. Note that in the recorded version, the sax player has enhanced the written melody with octave shifts and slight embellishments.

Figure 78: *A Sigh for a Cipher* – Sax Melody



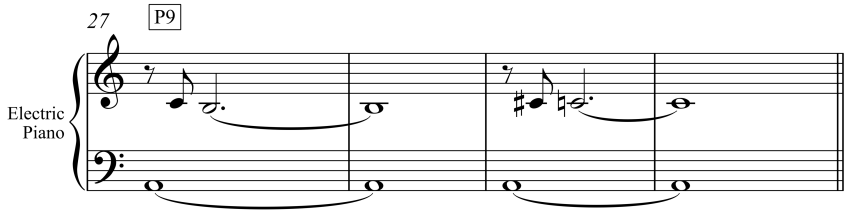
This section is repeated with a counter-melody added by the trumpet on bar19.

Figure 79: *A Sigh for a Cipher* – Melody and Counter-Melody



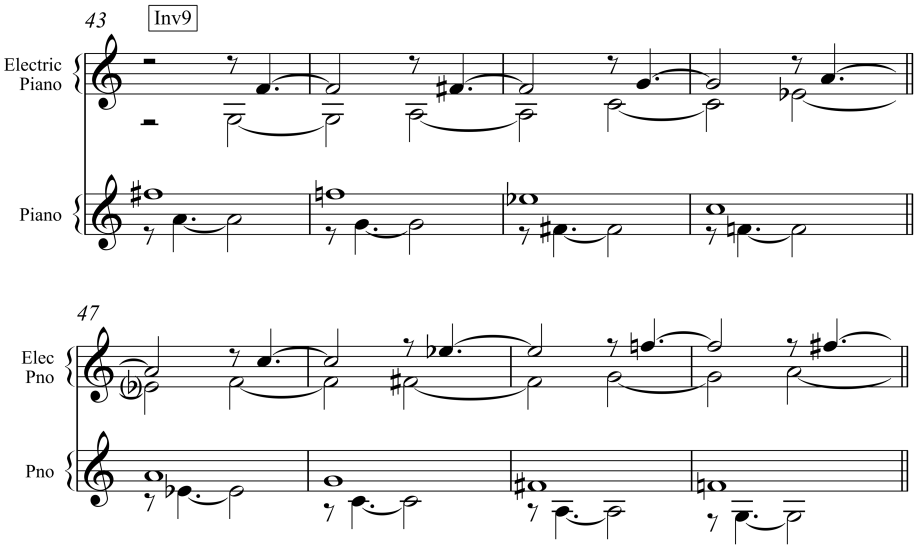
A contrasting section follows, differing in both the amount of movement and the orchestration. Using **P9,** lower strings, trumpet and violins exchange short phrases over a repeating four bar phrase. The electric piano outlines four notes of the hexachord with shifting minor-seconds over a pedal tone.

Figure 80: *A Sigh for a Cipher* – Shifting Minor-Seconds



More motion is introduced at bar 43 as the piano and electric piano introduce an exchange of rising and falling intervals derived from **Inv9.** Notice the crossing of parts that occurs between the fourth and fifth measure of this eight-bar phrase.

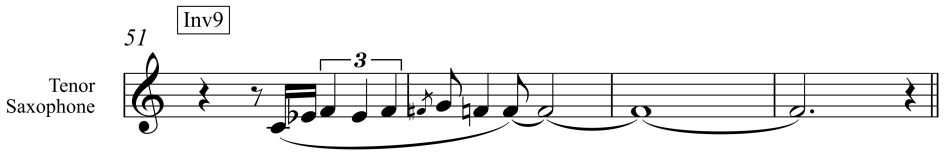
Figure 81: *A Sigh for a Cipher* – Crossing Parts



As this sequence repeats, the tenor sax reappears with a simple repeating figure.

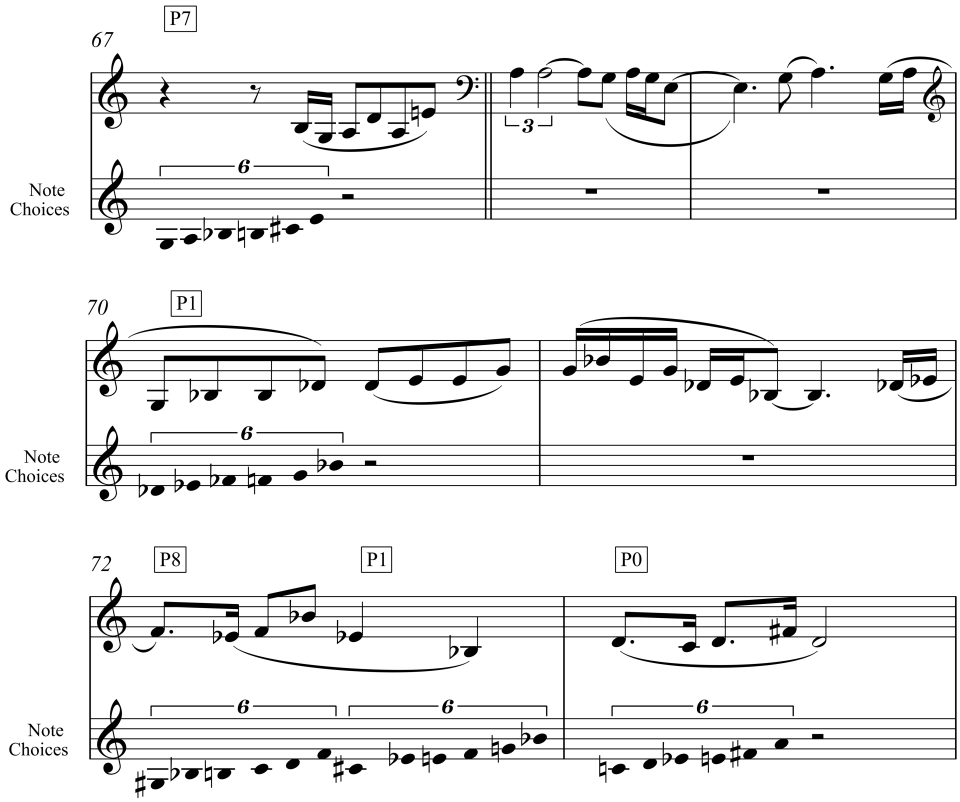
Once again, note the enhancements added by the performer on the recorded version.

Figure 82: *A Sigh for a Cipher* – Sax Figure

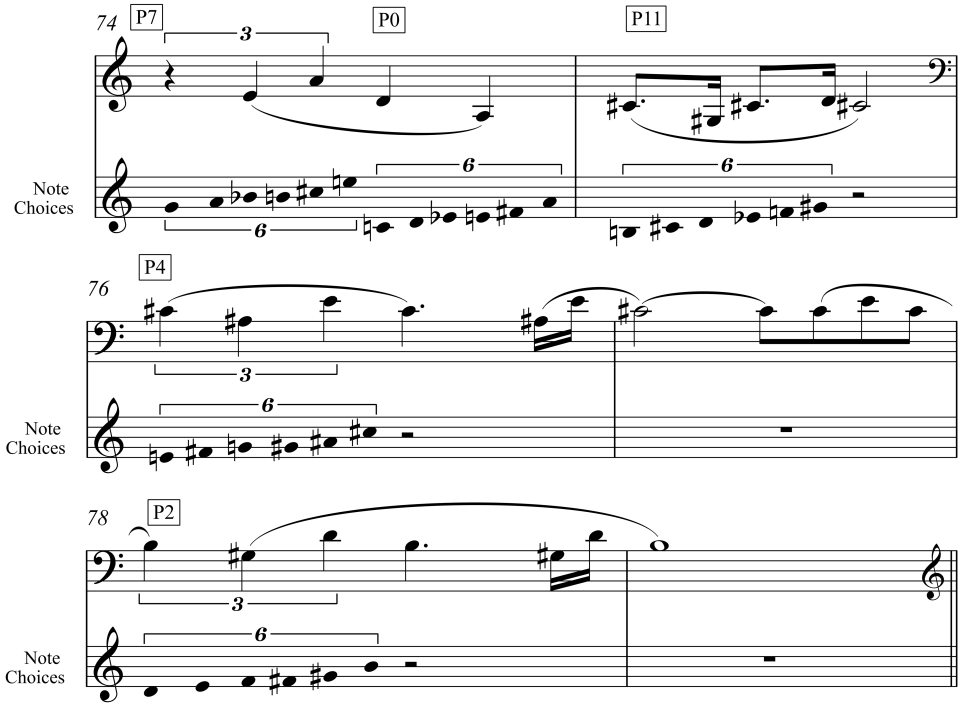


On returning to the original twelve-bar progression, the sax improvises over the "changes." The transcription of the solo below also displays the note choices.

Figure 83: *A Sigh for a Cipher* –Solo Transcription with Note Choices



Continued on next page



The solo is followed by a return to the melody with counter-melody, and the piece concludes with a brief restatement of the contrasting minor-second motif from bar 27.

CONCLUSION

> Of all the innovative techniques of the first three quarters of the twentieth century, perhaps the most significant is the twelve-tone technique.... The method of composition based on tone rows is pervasive and has had astonishingly swift dissemination. Few modern composers have escaped the influence of this method. (David Baker 1990, 92)

I now add myself to the list of composers influenced by twelve-tone technique. A decade ago, this harmonic world seemed unattractive and remote. Like many music students, I had listened to and analyzed a few pieces of twelve-tone music, understood the technique in its simplest form, and was not attracted to, what seemed to me at the time, the harshness and unsatisfying nature of the music. While my exploration of hexachords has immersed me in the *techniques* of twelve-tone music, it has also introduced me to a great many complex, beautiful, and sometimes challenging works that I find myself listening to again and again. Even more surprising to me are the many ways that twelve-tone technique has been, and is being, adapted. I have come to appreciate the significant differences in the twelve-tone works of Schoenberg, Hauer, Webern, Berg, Gerhard, Wolpe, Stravinsky, Babbitt, Wuorinen, Perle, Finney, Boulez and others.

My experience with hexachords is a personal one, in that I have created my own rules of engagement. While other composers may find my methods useful, it is my hope that further exploration and investigation of hexachords may yield ever more interesting, creative results. It may be helpful in retrospect to consider the pros and cons, the opportunities and obstacles, connected to and presented by hexachordal technique.

Let us first consider the challenges. Just as it is possible to compose tonal music without a knowledge of music theory, it is quite possible to attain satisfying results with six-note collections without a knowledge of atonal theory. However, unlike tonal music, which most listeners in the western world have been constantly exposed to, atonal music and hexachordal

technique is, for most listeners and composers, something apart from the familiar. Thus, it is helpful for the composer to be familiar with atonal theory, pitch-class set theory, and integer notation. Some composers may be frustrated by the imposed constraints and apparent lack of freedom – key signatures, movement through the cycle of fifths, chords built in thirds, and most of the formal gestures of tonal music must be abandoned. It takes a while to get used to and to find ways of engaging with collections which can at first seem murky, somber, and lackluster. There can be a sense of sameness that permeates compositions created with this technique – perhaps the greatest challenge for the composer is to distill or extract something special from each collection.

On the other hand, let us consider the attractions. The properties of hexachords are well documented. Despite the focus on twelve-tone music, there is a rich academic catalog of articles discussing history and techniques. Most of the great composers of the past century, in a variety of ways, have engaged with hexachords. This is an alternate harmonic world that is great in expressive power, yet largely ignored, apart from twelve-tone music. Although the harmonic landscape may seem unfamiliar, motivic development, inversion, and transposition remain as essential compositional tools. The identical intervallic structure in the remaining six notes that complete the aggregate, and in the transformations of the hexachord, provides an underlying, intrinsic architecture to each composition.

I suggested at the beginning of this paper that twelve-tone technique, while usually concealed beneath the surface of contemporary musical experience, occasionally rises as something startling, innovative and unique. A recent example is *Real Enemies,* a 2016 recording by Darcy James Argue's Secret Society. Paying homage to film composers Michael Small and David Shire (both producers of twelve-tone scores in the 1970s), the work is a rare instance of

serial composition transcending the need for additional narrative, even though narrative *is* provided through theme and occasional voice-over. Sixty years ago, it was difficult to find players that could read complex notation, make intricate rhythmic settings "groove," and improvise at a very high level – one of the reasons that Schuller's third stream fusion of serialism and jazz failed to endure. The musicians contributing to Argue's project are functioning at an exceedingly high level on all fronts. His work shows us that twelve-tone music can be meaningful, electrifying, contemporary, dark, and beautiful.

While the third stream movement of the mid twentieth century may have failed to convince listeners and critics of the value of a twelve-tone technique/ jazz fusion, more recent explorations by John O'Gallagher, Argue, and others have been more successful. Although certainly not twelve-tone music, my work with hexachords is also a descendant of this lineage. Musicians engaged for performance on my hexachordal "jazz" recordings have been receptive to this alternate harmonic world. It is crucial, however, to prepare the players before any performance or recording session. As outlined earlier, this is unfamiliar territory, especially for improvisors – the charts have no key signatures or chord changes, and improvisation is guided by different transpositions or inversions of a hexachord rather than chord progressions. Despite these alterations to the harmonic palette of jazz, it is still possible to allow *all* players the freedom to interpret the "changes." Musicians accompanying a composed melody or solo may improvise their parts based on the note choices of the changing hexachords, much as a bass or piano player would do when interpreting chord changes.

A lifetime of responding to and, for this composer, employing the emotional cues of tonal music provides an inevitable backdrop to most of my creative choices. Additionally, scoring picture, dance, theatre, or the spoken word compounds the challenges of using a limited or

controlled choice of musical material. What is striking, however, is the range of emotion, the variety of colour, and the programmatic suggestiveness that may be drawn from each hexachord collection. Just as the familiar major scale may produce varied emotional responses, each hexachord may, in its own way, yield variety. Over the last few years, as my interest and knowledge have expanded, I have been incorporating hexachords more and more into my compositional language.

There are, of course certain aesthetic and practical challenges. It is all but impossible to work as a "composer for hire" if you are unwilling to make changes. The requests of a client preclude strict adherence to any personally imposed harmonic directive. However, on a few recent projects I have been both composer and co-producer, allowing me significantly more creative freedom. A recent project involved providing background accompaniment for a collection of twenty-one poems. I made a decision to score each poem with a hexachord selected sequentially from Rahn's set-class table, rather than attempt to match the "sound" of a hexachord to the temper of each poem. By so doing, I was encouraged to distill specific elements from each collection that enhanced the intent and framed the emotion of the poem.

Much attention has been paid to hexachords as a component of twelve-tone composition. This abundance of research notwithstanding, theorists, and composers largely ignore their use as an exclusive compositional resource. My research and the eighteen compositions presented in this dissertation may spark a realignment in the consideration and practice of hexachordal composition. Hexachordal technique is a practical creative method related to, but separate from, twelve-tone composition. It affords a unique ability to create original, nuanced and focused compositional sound-worlds for creators of western art music and media composers. Further, my hexachordal works for jazz ensemble present a new approach to composition and improvisation

within that genre. I believe the compositions included in this research demonstrate the success of this reinterpretation of jazz – a paradigm shift that yields a vast range of distinctive and colourful harmonic and melodic environments.

For many composers, including myself, creativity thrives when limitations are imposed. These limitations can take many forms: the length of the piece, the orchestration, deadlines, synchronization with other media, and in this case, the choice of musical material and the techniques applied.

It is often impossible, even for the trained ear, to detect the intricate transformations and permutations that occur in serial and hexachordal techniques. The devices are present but hidden beneath the surface of the music. Nonetheless, they serve the composer – generating material, providing cohesion, and setting limits. For me, the results of the constraints I have chosen are unique, grounded, and satisfying.

After working with hexachords for a few years, I am aware of a curious duality that is especially present in the jazz compositions. Because these pieces are created with a tonal bias, it is, in most cases, impossible to eliminate a suggestion of tonal gravity. When this suggested tonality is combined with other distinguishing characteristics of jazz, such as instrumentation, rhythm and form, the listener will probably not question the genesis of the piece and enjoy or dismiss the piece accordingly. However, the music also exists as a carefully rendered hexachordal composition with perhaps more significance and meaning in its non-tonal incarnation. This is multi-dimensional music – it can mean two completely different things at the same time.

As I conclude, it is important to remember once again that most of the techniques used in my work originated either in the works of the serial composers of the twentieth century

(Schoenberg, Hauer, Stravinsky, *et al*.) or were borrowed from the theorists who sought to better understand atonal and twelve-tone music.

In twelve-tone music every piece has a unique setting determined by the ordering of the row. That setting may be rendered in a variety of radically different ways: by observing strict ordering of the row and its transformations; by ignoring the ordering but maintaining hexachord integrity; by creating permutations through rotation, through verticalization, and manipulation of arrays; by re-interpretation of the technique (as in the work of George Perle); or by expansion of the technique into *integral serialism* or *multi- serialism.*

Schoenberg and other composers of twelve-tone music ask us to listen to this music in a new way. The musical idea should be considered perhaps as an object suspended in three-dimensional space, something we can regard from multifarious angles, each perspective revealing something new. Unfortunately, it is all but impossible for a listener steeped in western art music and/or popular music to abandon the powerful reference points of tonality.

On its own, twelve-tone music usually lacks forward motion. The musical idea, although vigorously transformed and refashioned in a composition, often remains static. However, forward motion and extended form can sometimes be created by combining the music with another medium and using the narrative of that medium to propel the music. In addition, many composers animate serial technique by referencing or incorporating tonality. Looking ahead to the future from 1923, Schoenberg mused about a possible union of twelve-tone technique and other styles of composition (Schoenberg 1975, 207). To a great extent, it is these unions with tonal composition and with other media that have helped to sustain an interest in serial composition. As we have seen, much of my own work with hexachords purposely seeks to reference tonality.

In 1952, Josef Rufer, a student of and one-time assistant to Schoenberg, wrote of the universal appeal of twelve-tone technique in *Die Komposition mit zwölf Tönen*: "...it could be used in an entirely personal manner, varied only by the quality and degree of the creative imagination" (1969, 1). Certainly, the techniques introduced almost a century ago by Schoenberg and, to a certain extent, Hauer have gone through many transformations and adaptations. Every composer approaching serialism has interpreted the concepts in a different way, leaving a rich but scattered and confusing tradition. Indeed, for many critics, the serial discourse is more concerned with the intricate machinations of a composition's genesis than the music itself. Nonetheless, a tradition has endured, perpetuated in part by persistent interest in the twelve-note equal tempered scale. Witness, for example, the resolute conviction displayed by Charles Wuorinen. The introduction to his 1979 book *Simple Composition* begins with a striking statement, dismissive of anything tonal: "...while the tonal system, in an atrophied or vestigial form, is still used in popular and commercial music, and even in the works of backward-looking serious composers of the mainstream, it has been replaced or succeeded by the twelve- tone system" (3).

Wuorinen's comments notwithstanding, Covach (2002, 625) observes a trend in the 1990s away from the familiar prescriptive aspects of twelve-tone music and toward an interest in the expanded or alternative techniques of composers like Krenek and Hauer. Rather than declining, serial technique is transforming, although I would argue that has been transforming from the very beginning. As Lansky and Perle point out, it may well be the *techniques* of serialism that endure, rather than the distinct idea of twelve- tone composition:

> Perhaps the most important influence of Schoenberg's method is not the 12-note idea in itself, but with it the individual concepts of permutation, inversional symmetry and complementation, closed systems, properties of adjacency as compositional determinants, transformations of musical surfaces through predefined operations, and so on. ... In this sense the development of

> the serial idea may be viewed not as a radical break with the past, but as a particularly brilliant coordination of musical ideas which had developed in the course of recent history. (Headlam *et al* 2013, 25)

Indeed, it is these "individual concepts of permutation" that I am incorporating into my own practice – creating alternative musical landscapes with hexachords.

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