Exponents and Polynomials I

SUMMARY KEYWORDS
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SPEAKER
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Hello and welcome to ALEKS walkthrough video number three. My name is Robert McKeown, and I'm very happy that you decided to check out my video. We're moving along now, we're still talking about some very fundamental algebra skills. But we're going to introduce two new concepts to you. The first is the idea of a quadratic equation, where we possibly have more than one solution. So that's going to come up today if it didn't come up before. And we're going to look at exponents and the rules for exponents. Now, students often tell me or when they get this part in the course, they say, Oh, you know, this is very easy. I kind of know this, or they see the rules like, Oh, yeah, yeah, I get it. Well, you have to know it inside and out. Because you're going to be given problems in economics, you could be given math problems in your math courses. And you're expected to be absolute masters of these topics. And you're going to have to do a whole bunch of different operations, and exponents are just going to be one of them. So it's really, really important to nail the rules now, how exponents can be manipulated, so that when you get more complicated problems, you don't make a silly mistake, and then end up getting the wrong answer. So per usual, I want you to get your pencil, I want you to get your paper, your booklet, you can print out the slides, you can. If you have an extra device, if you have a tablet, you can write your answers on the slides electronically and save them saving them as optional. But you should work through these problems with me work through them together. That's how you're going to master algebra. That's how you're going to master mathematics by practicing and doing yourself. Now, let's get to it. Let's take a look at our first question. The question is asking us to graph the solution to the following inequality. So what is it mean? Well, let's take a look at the slides. And let's go into some more detail. So there's the question written again. What is the solution to this inequality? Well, what it's really telling us and I'll show it to you, I'll type it out is find the x values, or the values for x? And which the equality is true? So remember, that if we're multiplying two positive numbers, the answer is also going to be positive. If we're multiplying a negative and a positive, the answer is negative. And two negatives, multiplied by each other gives us a positive Not to be confused with two wrongs making a right. So we need to find the values of x for which this thing is strictly greater than zero. Looking at the expression, I want you to notice two things. Notice that if x is equal to negative six, or maybe I should say if x is a number smaller than negative six, then x plus six will be less than zero. And if x is equal to five, or again, I'm going to change it from equal to less than five. x minus five will also be less than zero. Now in general, how do you solve this type of problem? Where we've got two factors and, and they're multiplied by each other? What are we going to do to solve this? We're going to create a sine diagram. And I had my students on the econ 1530 final exam. Last year, my students have to draw sign diagrams to answer questions that are a little more complicated than this, but are the same idea. So I'm going to show you how to create a sign diagram. And if you're in my class, you're going to want to be able to do it, because I'm going to give you marks if you can do it, too. So how do we create a sine diagram? Well, I'm going to start off by writing the two factors over here. And then I'm going to draw, well, maybe I'll draw a line like that. And I'll have negative infinity over here. I'll draw a line over here. I guess I could put a matching line there if I wanted to. And I'm going to mark off these important numbers, the negative six and the five, they're going to be important. And zero might be important as well, or might not so but I'll put in the zero as well. Now, if x takes on a value, less than negative six, x plus six will be negative. And if x takes on a value less than five, x minus five, will be negative. Now, if x is greater than negative six, x plus six is going to be positive. And if x is greater than five, x minus five will be positive. And I'll just since I've added in those vertical lines, we'll add that negative symbol there now. two negatives. A negative times negative is a positive. A positive times a negative is a negative. And two positives, a positive times a positive gives us a positive. So we can see here that if x is less than negative six, this inequality will hold. And if x is greater than five, this inequality will also hold. And notice that if x is equal to five, it does not hold because the answer will be 00 times any real number is equal to zero. And similarly, if x is equal to negative six, this expression will be equal to zero, and therefore it is not strictly greater than zero. Now let's put our answer into ALEKS. I'm going to use the circle that sort of hollow. And I'm going to choose negative six, and five. And I want my lines to be where x is greater than five, and x is greater than negative six. Now let's see if we've got the right answer. And we did it successfully, we use the sign diagram to make sure that we got the right answer. So these questions are a little more, a little more time consuming, a little more challenging than you might think if you try and eyeball it, you might get the answer, but you also might make a mistake. So in this question, we have squared x, and it's equal to 50. It tells us that x is a real number. But what it wants us to do is to simplify our answer as much as possible. And I'll explain what the question means when it says that. In fact, you can just see at the top there, there's something about this square root property, so I'm going to show you the square root property. The other thing to be aware of is that it's possible that there is no solution to This question. And it's also possible that there's more than one solution. Looking at the slides, we have the same question. I've just added in that little hint. ALEKS really wants us to use the square root property here. As a first step, I want to isolate X on one side of the equation equation. Now I've got x squared on one side of the equation, but what I really want is x. So I'm going to take, well, I could just write it like that, I'm going to take the square root of both sides of the equation. But as I'm doing that, notice that if I had x squared is equal to 16. And I'll just use, you know, an easy number here, x could be equal to four, or x could be equal to negative four. So I'm taking the square root of both sides of the equation. But I want to recognize that x could be positive or negative. So I'm going to have x over here. And I'm going to say that's equal to plus minus the square root of 50. Now you probably notice and saying, Well, look, I've got x squared is equal to 50, I could just take the square root of 50. And I've got the numeric solution. That's not what ALEKS wants us to do once to test our algebra skills. There are reasons in the real world that we might not want to just solve away for this, when you get into 1540. And you're looking at multivariate situations, there's not going to be a just a simple solution, like the square root of 50 is equal to a number a little bigger than seven. So what can I do to further simplify this? Well, 50 I can do something with that 50. So what if I wrote 50 as 25 times two. And I know that x is equal to plus minus, what's the square root of 25, it's five. So I could bring the five out and leave the square root of two, which of course is going to be an irrational number. Now let's see how I did we're going to go back to ALEKS and we're going to plug in this solution, notice there's two solutions. So x could be equal to five times the square root of two or x could be equal to negative five times the square root of two. So here we are an ALEKS x is going to be equal to five. And I click on the square root symbol after I put in the five times the square root of two, and then I want another O. So I'm going to start off by clicking this little button here. And then I'm going to make sure that my cursor stays on the left hand side there. And then I want five times the square root of two. And then moving into the second box, I want negative five times the square root of two. And let's see if I did it right. And I did. So we have the right answer here. And there were two solutions. This little question that I've added to demonstrate the introduction to factoring. And so here the question, we have an expression, and we're being asked to expand it. So how are you going to expand it? Well, it's good to remember that the expansion of this thing is going to be one times three, plus one times four, plus two, times three, plus two, times four. So you do the outer first and then the inner. Expanding, we get x times x plus x times four, minus one times x minus one times four x squared plus four x minus x minus four. And we can collect the like terms. And we're left with x squared plus three x minus four. And this is known as a quadratic expression. It's an expression because it's not equal to anything. It's not a, it's not a function, right. But we'll see. We'll see what a function is later. Now let's look at a more complicated problem. So I haven't showed it to you and ALEKS will show you we'll put the answer into ALEKS later. But the questions asking us to factor the following completely. This is a tricky, factoring, question. So to start off, how are we going to find out what y is equal to that's another way. Another thing that we're trying to do here, when we are asked to factor it into its simplest multiplicative multiplicative bits? Well, as a first step, why don't I factor out why four, and so we're left with nine y squared minus 24, y plus 16. And you can see that this inner thing is a quadratic expression. Now, how can we solve and factor this quadratic? Well, one method would be to use the quadratic formula, you can look up the quadratic formula online. That would be one way that you could go about factoring this. But if I look at a little more closely, I see that I could rewrite this just a little bit. Oops. And this is kind of, as we'll see, these are equivalent. As we'll see in the next question, I can factor this out. Like so where you can see that if I multiplied this thing out, we'd have minus 24, y in the middle 12 y minus 12 y, and the two ends are going to get us back to this thing up here. The trick is to see that both three and four are factors of 24. And when three times four, we get 1212 minus 12 gives us 24. That's sort of the secret. So let's see if it's the right answer on ALEKS. Here's the problem on ALEKS. So we had y and I'm going to press sift while my keyboard shift six and I can automatically get an exponent for I hit the right move key and I start open up a bracket and I'll do it I just realized I made a little mistake over here. I in my head. I knew I was doing it correctly, but I should have had a negative sign over there. Now going back to ALEKS. Here we are. I'm going to click the check button and we got the right answer.