

Chapter 12

Philolaos and “Those Called Pythagoreans”

1. Philolaos’ life and book(s)

In the generation after Empedokles and Anaxagoras there was a great flowering of the Pythagorean school; and thanks to its most notable representative, Philolaos of Croton, we have some fairly definite information about what the Pythagorean study-groups were doing, and what doctrine was taught. We do not know how much of this doctrine goes right back to the Master himself. But Philolaos wrote a book (44 A 1), which Plato managed to obtain; and from that we can gather that by 450 BCE the Pythagoreans had far outstripped the Ionians in astronomy.ⁱ

In fact, Philolaos may have written more than one book; but some scholars of an older generation, would not allow that anything worth having had reached us from any of them. Fortunately, more moderate views have prevailed, and we do not now have to contend with the absurd argument that the “fragments” are too “Platonic” to be genuine.ⁱⁱ

Philolaos was born in Croton, somewhere round 475 BCE. He is said to have been taught by Lysis (who was supposed to have been a pupil of Pythagoras himself).ⁱⁱⁱ But the Lysis with whom he was certainly associated would not have been old enough to have studied with the Master. They both had to leave home (when Philolaos was still young) because the “Pythagorean party” was finally expelled from Croton in about 454 BCE. The Lysis who was a friend of Philolaos eventually

went to Thebes, where he taught Epaminondas, and wrote a book, or books, ascribed (in the proper orthodox way) to Pythagoras. Perhaps Philolaos went with him at that time. He certainly lived in various Pythagorean communities in Italy — for instance at Metapontum; and at some stage of his life he was living in Thebes (A 1a — *Phaedo* 61e).^{iv} He thought and taught in the intellectual turmoil created by the Sophists; and this — together with his exile — may have turned his mind towards writing out the truth about God, Nature and human existence. We do not know when he died; but he probably spent his last years in Tarentum, and he may have been dead before the execution of Sokrates in 399 BCE.^v

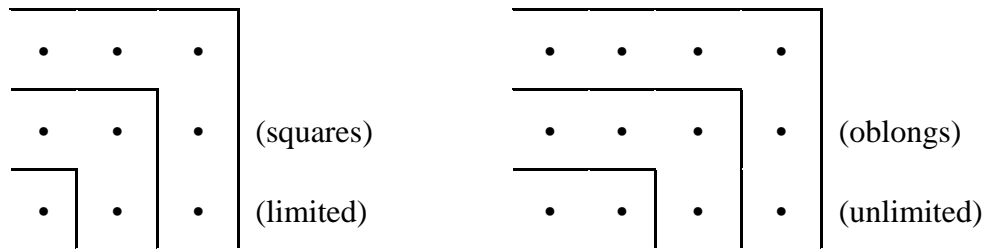
Aristotle's account of "those called Pythagoreans" harmonizes well in many places with the fragments of Philolaos. This is what we should expect, since the book of Philolaos was certainly one of Aristotle's main sources (probably he had the writings of Lysis also; and certainly those of Archytas). It is interesting, therefore, that Aristotle ascribes almost nothing to Philolaos himself (or to either of the others).^{vi} He thought of the doctrine in Philolaos' book as belonging to the Pythagoreans generally; and he could not be sure (any more than we can) how much of it went back to the Master himself. In my summary account I shall ascribe everything in Aristotle's reports that appears to be consistent with the general testimony about Philolaos, to Philolaos personally. He should be thought of as the most important voice of the Pythagorean school — and certainly the best mind in it — between 445 and 410 BCE.

2. Limit and the Unlimited

The book that Plato bought began with the assertion: "Nature in the *kosmos* was harmonized from unlimited <things> and limiting <things>, both the *kosmos* as a whole and everything in it" (B 1).^{vii} The use of the plural is already interesting. Pythagoras may have spoken of Limit (meaning the One) and the Unlimited (meaning the Boundless Air of Anaximenes, which was probably already synonymous in his mouth with the "Void"). In the mind of Philolaos there is a universal *kosmos*, but there are also a great many subordinate forms of order. There is a universal *harmonia*,

but also everything can be harmoniously numbered in its own distinctive way; and — as the Master himself probably understood and taught — all of the numbers are “limits.” Mathematical reasoning is the criterion of truth, both in general, and in particular; and according to Philolaos “geometry is the principle and mother-city of the other kinds of learning” (A 29, A 7a). Thus, when we study the numbers geometrically, we can see why (although the numbers are all limits) it is also the case that the odd numbers are “limiters” while the even numbers are “unlimited.”^{viii}

The Pythagorean number-theorists of Philolaos’ time (and probably earlier) were classifying the series of odd numbers as “square numbers” and the series of even numbers as “oblong.” Thus $1 + 3$ is the square of 2, $1 + 3 + 5$ is the square of 3, and so on. For the “oblong” numbers we need a diagram which shows how the Pythagoreans used their right-angled *gnomons*:



In the “oblongs” the geometrical ratio changes continually. It is never 1:1 as it is in the “squares.” Thus the even numbers are “Unlimiteds.” (One thing that this development in number theory clearly shows is that by this time the table of ten pairs of Opposites has reached its canonical shape — Square and Oblong have arrived on the list.)^{ix}

Aristotle tells us that the Pythagoreans “were the first to bring mathematics forward; and being brought up in mathematical studies, they thought that the principles of mathematics were the principles of all things” (58 B 4 — *Metaphysics* 985 b 23-25). What they *said* (and perhaps the Master said it first) was that “things *are* numbers” (*Metaphysics* 1090 a 22); but what this meant varied in different minds. It is quite evident that the senses do not present us directly with numbers;

but we have to remember that (by the time of Philolaos) Parmenides has demonstrated that there is a great gulf between “what appears” and “what is.” In the Parmenidean perspective, the orthodox Pythagoreans were pluralists. But Philolaos believes (as did others) that “what is” is a great mathematical structure (a cosmic *harmonia* behind appearances); and he believes further that the bridge between Being and Seeming can be constituted by the discovery of lesser harmonies everywhere.^x

“What is” consists of numbers that are harmonizing *powers* (on one side); and a “boundless” material (on the other). “Everything that is known has a number; for nothing could be thought or known without this” (B 4; cf. B 6, B 11).^{xi} It seems natural to respond that the “unlimited or boundless” *can* be thought. But Philalos means that it cannot be *grasped*; and the proposition “nothing without a number can be thought” has to be understood as referring to *determinate* things.) (Huffman^{xii} believes that the doctrine “things are numbers” was an Aristotelian *interpretation*, and not an actual Pythagorean dictum.)

There are three kinds of number. The even numbers begin with 2, the odd numbers with 3; and the number 1 (there being no zero in Greek mathematics) is in a special category by itself. It is both odd and even because it transforms the odd and even categories alternately (B 5).^{xiii} The Dekad is the “perfect” number; and as *Tetraktys* (1 2 3 4) it contains the nature of point, line, plane and solid (A 13).^{xiv}

Only God knows (or can know) the causal numbers that actively create the *harmonia* of the *kosmos*. The eternal being of the objects in our world is beyond our ken (B 6).^{xv} We know only the mathematical patterns that these “limiters” and the unlimited stuff create in the world that appears to us; but we do know that the number-limits and the boundless stuff are necessary for the existence of our world of appearance, and that the number-limits *harmonize* the boundless stuff.^{xvi} The stuff itself is a togetherness of opposites; for if it were not it would not need *harmonizing*. It cannot subsist (as it eternally must) without the operation of the *harmonia* that makes it into a *kosmos* (B 16).^{xvii}

The mathematical conception of *harmonia* had already been exemplified by earlier Pythagoreans in their account of the musical scale. For Philolaos this became the demonstrative connection between Being and Seeming. The same fragment that tells us how things must be, in order for us to have the certain mathematical knowledge that we do have, goes on directly to define the musical intervals that constitute a full octave.

The creative power that produced the harmony of the *kosmos* is quite beyond us; we do not know (either in general, or in most of the particular cases) how the harmony is brought about. But geometry is the key (and the ideal paradigm) of what we must seek. The regular solids can be generated first by the motion of a point, and then by that of the surface generated. In the *Timaeus* Plato assigned the shapes of the regular solids to the atoms of Fire, Air, Water and Earth. That hypothesis probably came from Philolaos.^{xviii} The curious research program of Philolaos' pupil Eurytos (which will be described below) rested on the same belief that "geometry is the mother-city of the other sciences." Ultimate reality may be as unknowable as Parmenides thought it was — and as Melissos may by now have confirmed. But every kind of material difference that can be reduced to a mathematical expression becomes "knowable" in spite of that.^{xix} It seems obvious to Aristotle that there cannot be knowledge without some *material* assumption. But he did not have to take the Eleatic challenge as seriously as Philolaos and his friends.^{xx}

3. The *kosmos*

In spite of his commitment to Eleatic formalism in theory, Philolaos continued to work in the school-tradition. He was quite willing to give an empirically descriptive account of the cosmic harmony. "The first <thing> harmonized, the One, in the middle of the Sphere, is called *Hestia* [the Hearth]" (B 7),^{xxi} he says. Notice that in speaking of the world as we know it, Philolaos uses the passive voice. The One in our world is not the all-creative cause, but "the first thing harmonized." This "Hearth" at the center, is a fire around which everything revolves. We cannot see it, although

the Earth is its planet, because we live on the outside face of the Earth (cf. A 17). We look outwards toward the Moon and Sun, which are the next planets in the cosmic sequence. Our Earth is actually the second planet from the Hearth, the closest being the “Counter-Earth.” This (like the Hearth itself) is behind us, and so not visible to us. Thus, counting the heaven of the fixed stars, there are ten spheres that circle round the Hearth. This is the cosmological theory ascribed to “those called Pythagoreans” by Aristotle (59 B 1, *Metaphysics* 986 a 3).^{xxii} But Aetios also tells us that Philolaos called the Heaven of the Fixed Stars “Olympos,” and distinguished it from the *kosmos* that was below (i.e. within) its comprehensive sphere (A 16).^{xxiii} That is part of an allegory of divine and human knowledge.^{xxiv} The Hearth with nine moving “planets” (including the Earth) constitute the Dekad. But one of the planets moves invisibly in perfect unison with the Earth. So the eight singular (and visible) moving spheres produce the musical octave (58 B 37);^{xxv} while the unmoving center, with the fast-moving (and indefinitely numerous) periphery represent the One in its original self-sufficiency, and its creative fruitfulness. Beyond the Dekad of units we see the infinite array of the numbers that it can generate. Thus the Heaven that *contains* the dekadic *kosmos* is the best image that we can have of our Father and King; and it is only right for us philosophers to recognize it as “Olympos” — the throne of Zeus himself.^{xxvi}

Our Sun is like a burning glass. It receives heat and light from the Hearth, and focusses it for us. The Moon, on the other hand, not being fiery (but shining by the Sun’s reflected light) is another Earth — with bigger and more beautiful living beings (A 19, A 20).^{xxvii} There are thus *three* “Earths” — the Counter-Earth, which (presumably) looks on the Hearth itself, and the Earth and Moon that depend on light and heat reflected from the Sun — the fourth moving body. The importance of the first four numbers (the *Tetraktys*) is clearly figured. Then there are five smaller lights, before we reach “Olympos” which is the infinite encompassing power of the creative One.

There was a pre-Parmenidean religious allegory about the *kosmos* — or perhaps several alternative allegories — in which the created (or “harmonized”) One was not distinguished from the divine creative power. Aristotle reports that some Pythagoreans called the motionless Central Fire the “Throne of Zeus,” the “Tower of Zeus” and the “Guardroom of Zeus” (58 B 37).^{xxviii} Philolaos

moved the Homeric “throne of Zeus” (i.e., Olympos), so that Zeus reigns visibly over the whole *kosmos* in his absolutely stable Heaven. Hence Philolaos had to find a new name for the Hearth. He called his “harmonized One” the “Mother of the Gods.” This expressed its relation to the eight moving numbers of the cosmic harmony. It was also the “binding measure of Nature,” because it is the “odd-even” ratio (A 16).^{xxxix}

Secondly, we know that the hypothesis of the Central Fire (at least) is older than Philolaos, because we find the theory of “two Suns” (ours being like a burning glass) in Empedokles.^{xxx} It is very likely that the name (and hypothesis) of the “Tower (or Guardroom) of Zeus” goes back to the Master himself, since he believed in an unmoving fiery center which devoured the Air (or “Void”) in order to generate the *kosmos*. In that case, the movement of the Earth may have been a primitive given in Pythagorean astronomy; and since the Pythagoreans were bound to be looking for the *Tetraktys* at the base of the cosmic *harmonia*, the hypothesis of the Counter-Earth would have arisen as soon as they got the Moon, Sun and planets in order.^{xxxi}

The relation of the moving Earth to the Sun (with the necessary rotation of the Earth as it goes round the Hearth) accounts for day and night. Philolaos reckoned the length of our Solar Year to be $364\frac{1}{2}$ rotations (A 22).^{xxxii} There is no absolute “up” and “down” in the spherical order of his *kosmos*; so there are people and animals at the antipodes of the Earth, for whom our world is upside down, just as theirs is for us (B 17).^{xxxiii}

Philolaos calculated the length of the Great Year in which the Solar and the Lunar calendar would be harmonized. According to his reckoning Sun and Moon came back to their original places in fifty-nine years with twenty-one intercalated lunar months — A 22.)^{xxxiv} There *were* Pythagoreans in this generation who thought that after the greater cosmic time period in which all of the heavenly bodies returned to their initial positions, history repeated itself perfectly down to the tiniest detail. Eudemos illustrated the doctrine in a lecture by saying: “I shall be talking to you holding my pointer, as you sit here, and everything else will be as it is now” (58 B 24).^{xxxv} This outrageously excessive interpretation of the astronomical cycle (a cycle which may have been

grasped in essentials by Pythagoras himself)^{xxxvi} was forced upon the later school by the desire to meet the standard of Parmenidean Being. It is not consistent either with the ethics of Philolaos, or with his generally tentative attitude toward the explanatory “harmonies” of the sensible world. But we cannot be quite certain that he was not guilty of this fantasy (among his others).

It may seem doubtful that he had any such belief, because he held that there is a twofold “destruction” on Earth by fire from heaven, and by water from the Moon. This was apparently a cyclic process in which the *kosmos* is *nourished* by “exhalations” of the fire and water (A 18).^{xxxvii} The report is rather indefinite, but it is at least plausible to assume that Philolaos was accounting for the weather cycle of the seasons; and in that case he could still have believed in a perfect “eternal return” in the *kosmos* as a whole.

It was probably Philolaos who invented the famous hypothesis of the “harmony of the Spheres.” The eight (observable) moving spheres in his *kosmos* produce a *harmonia* of the whole octave.^{xxxviii} Certainly he was deeply interested in musical theory — and in the detectable intervals of musical sound. The ratios of the pentatonic scale were by this time known; how much Philolaos contributed we cannot say. The reports of his work on semitones in different scales, make one think of the rude comments of Plato in the *Republic* about the empirical research of Pythagorean music-theorists (B 6; A 25, A 26).^{xxxix} But Plato would have been referring to a later generation (and not to Philolaos himself).

Proklos tells us that Philolaos expounded his mathematical (and especially his geometric) mystical theology in a work called the *Bacchae* (B 19).^{xl} This was probably *his* title (or at least the school-title) for his book — or for one part of it. It is easy to fit everything we know into this hypothesis, if we assume a basic allegory involving *three* levels of cognition: sensory experience, philosophical science, and divine knowledge. All of us, in our ordinary lives, are “Bacchae.” We live in “this over-roofed Cave” — as Empedokles called it (31 B 120).^{xli} Our sensory heaven — the blue sky while daylight lasts — is actually a “roof” that hides the real *kosmos* of God from us; so we live in the Underworld, and our proper Gods are Demeter, Persephone and Dionysos (Bacchos).^{xlii}

The philosophers who study the *kosmos* revealed at night, are the true worshippers of the Olympian Gods. But “Zeus” alone on the *scientific* “Olympos” — the “heaven” of the Fixed Stars — knows the absolute truth, i.e. the truly creative numbers of the divine *Tetraktys*.

The mathematical constructions of the philosophical *kosmos* were — according to the hypothesis adopted here, which is at least highly plausible — those found partly in Plato’s *Timaeus*, and taken over by Speusippos in his book on “Pythagorean Numbers” (A 13).^{xliii} The theological application (for the “Bacchants”) can be seen in the report of Plutarch, who says that the right angle belonged to Rhea, Aphrodite, Demeter, Hestia and Hera; the angles of the <equilateral> triangle to Hades, Dionysos and Ares. The dodekahedron belonged to Zeus (obviously because it was his “House,” or the geometric shape of the *kosmos*); and the fifty-six sided figure to Typhon — the inclusion of one of the Titans, traditionally confined in Tartaros, was matter of remark for Plutarch, but no explanation is given (A 14).^{xliv}

4. Ethics and Psychology

Without a doubt, the views of Philolaos about human life were those of the Pythagorean schools in his time; and very largely those of the Master himself. The doctrine that “the soul is yoked to the body, and buried in it as in a tomb” (B 14),^{xlv} surely reflects a genuine *ipse dixit* of Pythagoras: *sōma, sema* (“the body, a tomb”). The rational soul is a spark of the divine fire; it comes from the Hearth, and it should return to “Olympos.”

But the soul must not try to leave the intermediate *kosmos* of time and becoming, until God permits it to do so. Our world is a fallen place of sadness,^{xlvi} but a philosopher who understands the unity of the divine life, must not think that (s)he can go to God by her own motion. Suicide is a wrong committed against God, whose creatures and property we are. The Gods need us as their servant-worshippers; and it is part of our duty to marry and have children, in order that this service to them may be maintained (B 15 — *Phaedo* 61de).^{xlvii}

When we die, our soul goes on to existence in another body. (We do not find this in any report about Philolaos, but if he had doubted or denied it, he would have been the greatest of all Pythagorean heretics, and we should certainly find some mention of *that*). The “tomb” metaphor strongly suggests that if we live well (and especially if we live *philosophically*) we can legitimately hope that our soul will become one of the Fixed Stars, and so return to union with God; but the soul of a suicide might perhaps be embodied in a tree (as in the myths upon which *Dante’s* vision depended).^{xlviii}

The Pythagorean “way of life” of which Plato speaks — the ethics of daily existence in our sensory world — was probably not completely described in Philolaos’ book.^{xlix} Plato himself observed it (and talked to Archytas about it). The school tradition on political ethics is securely enshrined in the four speeches that the Master was supposed to have given to different audiences after his arrival in Croton.¹ He told the young people to honor and obey their elders (especially their parents); to be *friendly* with their fellows generally; to be *temperate* (self-controlled, *sōphrōn*) in their general behavior; and to develop their intellect in every way for the benefit of the whole community. Being then invited to address the City Council, he advised them to set up a temple for the Muses (who were a divine model of *concord* [*homonoia*] and harmony [*harmonia*]); to regard their governing responsibility as a divine trust; to speak truthfully always, so as not to need oaths; to deal *lovingly* with their wives and children; to have sexual relations only with their wives; to be diligent, and do every good deed at the opportune moment (*kairos*); to seek (and strictly *deserve*) a good reputation.

The Council asked him to talk to the boys and to the women. So he urged the boys not to be abusive — even to those who spoke abusively; to remember how the Gods loved them, and how close they were to the Gods; and to be obedient, in preparation for the time when they would give the orders. He spoke to the womenfolk first about prayer and sacrifice. They must give things that they made themselves to the Gods (and not *bloody* sacrifices); they must love and be obedient to their husbands (above all others); sexual intercourse with their husbands was not a religious

pollution; they should be generous in lending to, and sharing with, one another. Finally he named four Goddesses for the four periods of their life (girlhood, marriage, motherhood, grandmotherhood).^{li}

The emphasis on the *concord* of human society as a *hierarchy* of authority is very plain. In this context it is noteworthy that in the first speech Zeus is presented as *both father and mother* (to Athena) and Hera as *both mother and father* (to Hephaestus). The sexes are not socially equal; that is part of the natural order. But *for the Gods* (or in the philosophical life) sex is quite indifferent. Even in social life the ideal of concord is to be *friends*; and friends are ideally equals. When Aristotle laid it down (at the climax of his own discussion) that “a friend is another self” he was probably citing a Pythagorean maxim.^{lii} It is clear that Aristotle’s two books on Friendship (*Ethics* VIII-IX) leaned heavily on Pythagorean sources. The conservative attitude of the Pythagorean community towards the wider society is indicated in the four speeches. But their own “way of life” was a perfect communism of rational friendship. “Female” was subordinate to “Male” in Nature; but this difference disappeared when the “desires of the body” were forgotten in philosophy.

Returning now to psychological theory, some of “those called Pythagoreans” compared the soul to the dancing motes of dust in a sunbeam. We can be confident that Philolaos was one of those who distinguished between the visible mote, and the soul that caused it to move (58 B 40).^{liii} He apparently regarded the human soul as an aetheric power that *harmonizes* the body. Simmias (in the *Phaedo*) shows himself to be a very poor student of Philolaos when he makes the alarming discovery that if the soul is a “harmony” it must perish with the body (*Phaedo* 86c).^{liv} Plato himself only means us to see that *harmonia* means something different to an executive musician, from what it means to a listener.^{lv} It is true, of course, that the soul as an active *harmonia* will only be “immortal” as long as it passes from embodiment to embodiment — and the “tomb” metaphor encourages us to hope for a higher kind of life. But then perhaps it is only *while in service in this kosmos* that the soul is a *harmonia of the body*. “Olympos” may be a *community* of immortal souls *in harmony* under Zeus, their king. (Plato suggested in the *Timaeus* (41d) that the Creator assigns the created souls to the stars.)^{lvi}

What our sources say about the *physical* harmony of the human body (according to Philolaos) is not easy to understand. That we are sustained by a fire-soul is orthodox Pythagorean doctrine; but that we have no “cold” component until we are born, and begin to breathe the Air, sounds like a fantastic misreading (or over-reading) of whatever Philolaos said (A 27).^{lvii} Even if he was no physician — and it seems clear that he was not — the Pythagorean doctors upon whom he depends, would not have missed the significance of the birth-waters.

Disease in the living body is due to bile, blood and phlegm. Phlegm is a fiery element, and is the cause of fever. What Philolaos called “bile” is apparently lymph (the water element), not a secretion of the liver. It is “non-existent,” or [in other words?] “useless” (A 27, A 28).^{lviii} (The curious locution “non-existent” may spring from restriction of “existence proper” to “harmonies” of the active kind. It certainly seems to be influenced by Parmenides.)

We hear nothing of any political views in Philolaos. He may (quite probably) have had views about the relation of popular and philosophical theology, but they are lost to us.^{lix} (He was not able — or willing? — to come to terms with those who drove out their Pythagorean governors. But we cannot draw any safe conclusions from that. He died contentedly enough — as far as we know — in the democratically constituted city of Tarentum.)

5. Eurytos

Eurytos of Croton (or perhaps of Tarentum) is mentioned several times as a student of Philolaos, and as his associate. Except that he was younger than Philolaos, and that he lived — at least for a time — in Tarentum, we know nothing about his dates and career.^{lx}

According to what we may call the “geometric theory of numbers” that was developed by Philolaos (and probably by other mathematically-minded Pythagoreans of his generation, and the one before it) *one* is the number of the point, *two* of the line, and *three* of the triangle. With *four* the

Tetraktys is completed, and number becomes plurally generative. Four points will give us not only the square in plane geometry (and justice in ethics), but the pyramid in solid geometry. When we reach *eight* we have the number, not only of the cube, but of the musical octave (and these two “harmonies” should be thought of as generated by quite different creative powers).^{lxi}

Eurytos set out to apply this method of geometric construction to the world of organic becoming. He thought that it should be possible to determine the “real number” of Man and Horse. For this purpose, he would draw a shaded picture, and place pebbles on all the salient points. In this way one could determine (perhaps) just how small the creative number that expressed itself as “horse” or “man” was (45 A 2, A 3).^{lxii}

This was certainly one of the oddest excesses of Pythagorean speculation. We are naturally moved to wonder what *use* this sort of inquiry would be (for anything except a children’s game, or a beginning class in graphic art).^{lxiii} But that reaction only shows that we do not appreciate the *religious* context of Pythagorean inquiry. To discover a geometric *harmonia* was to draw near to God. (However, unless we can discover something about the way in which the geometric number *particularizes* itself, the supposed *harmonia* will be, even more vividly, an index of how far we are from God. Instead of smiling at the folly of Eurytos, we should reflect on how his program illuminates the practical futility of *Plato’s* “unwritten doctrines.” Plato was writing busily to the day of his death. So he surely knew just what he was doing when he left his theological speculations “unwritten.”)

6. Archytas

Archytas of Tarentum was an older contemporary of Plato. Cicero says he was a pupil of Philolaos. He was probably the best mathematician that the Pythagorean school produced; and he was also the most noteworthy political leader after the Master himself. Diogenes Laertios says that he was the elected “general” (military commander) of Tarentum seven times, and that he was “never

defeated”; but that once he resigned because popular opinion was against him. Tarentum was a “democracy”; and Archytas seems to have understood political compromise better than the Pythagorean leaders of an earlier generation; he may well have partly inspired (or confirmed) Plato’s belief in the possibility of a Philosopher King (47 A 1, A 5).^{lxiv}

His most notable achievement as a mathematician was an elegant geometrical construction for the duplication of the cube. But he also did important work in music theory, by determining the basic intervals in three types of scale: diatonic, chromatic, enharmonic (A 14, A 16).

Archytas also studied mathematical mechanics (B 7; cf. A 1, A 15); and he was a good engineer. Among other things, he invented toys for children. One of our sources attests to his love of children (A 10, A 2; cf. A 8).

He defended the Pythagorean concept of the Unlimited (and clarified its significance as “Void,” or empty *space*) by an appeal to mathematical intuition that became famous. Empedokles (and no doubt some other readers of Parmenides) took reality to be a perfectly full *Sphere*. But, said Archytas, let us suppose that I am at the circumference of the Sphere, and I thrust out my stick. Where does it go? (A 24).^{lxv}

Archytas declared that all of the mathematical sciences were “sisters.” He praises (Pythagorean) mathematicians because “having passed an excellent judgment on the nature of the wholes they were bound to see well how things are in the parts. They have handed on to us a clear judgment about the speed of the stars, and their rising and setting, as well as about [plane] geometry and arithmetic, and solid geometry, and not least about music” (B 1; cf. B 4).^{lxvi} These are the sciences that Plato arrays as the philosophical curriculum of the *Republic*.

Archytas discovered the relation between the pitch of a sound, and the motion of the causal medium. There could be no sound, he said, without the striking of two objects against each other; and he argued that there were sounds too great for us to hear, as well as too small (or too far away)

(B 1). In this way he dealt with two problems: Zeno's paradox of the Millet Seed, and the reason why we cannot hear "the harmony of the Spheres." The Pythagorean musical theorists had developed (before his time, and possibly before that of Philolaos) the doctrine that the motion of the heavenly spheres produces an actual musical *harmonia*; but it is "too great" for us to hear.^{lxvii}

It would be appropriate to end the chapter with this, the most famous of all the Pythagorean speculations. But we must add some final remarks about the ethical views of Archytas. First, this truly great mathematician understood clearly the practical, social importance of logical neutrality:

Calculation [*logismos*], when completed, puts an end to civil strife, and makes concord grow; for when it has come to be, there is no taking more than one's share [*pleonexia*] but equality. By Calculation we come to terms about business agreements, and through it the poor receive from those with means. The rich give to the needy, both trusting through it to get what is fair. Being the standard and deterrent of those who do wrong, it deters those who know how to calculate before they do wrong, convincing them that they will not escape detection, when they come against it; and it deters [others] from wrongdoing by showing those who do not know how to calculate that in this way they do wrong (B 3).

Secondly, Archytas maintained the ethical tradition of the School. In his little dialogue on old age, Cicero lets Cato the Elder quote a speech of Archytas in which "the pleasure of the body" was condemned as the most pernicious of human passions (A 9). One must try to detach oneself from bodily desires, as far as that is possible. Then, in Cicero's other little dialogue on Friendship, Laelius cites another speech in which Archytas said that we need a friend to share our joys. If the Master did not say that "a friend is another self" then it was Archytas who coined the proverb.^{lxviii}

Finally, this man of great practical wisdom also wrote a treatise *On the Dekad* (B 5) (i.e. the *Tetraktys*).^{lxix} Kathleen Freeman says (quite correctly) that “A mass of mystical lore on numbers was transmitted by Philolaos.”^{lxx} But we must remember, when we read the superstitious rubbish with which later Pythagorean literature is filled, that some of it was handed down by a scientist whose remains show few signs of any inclination to Philolaic fantasy. The division between the Akousmatics (the religious Pythagoreans) and the Mathematicians came to pass only after the exile of Philolaos; and in the circle of a mathematician who still exercised political power, it did not yet take place at all. Archytas was as seriously devoted in his religious belief and practice, as he was in his mathematical studies.

Notes

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- i. Probably Anaxagoras was the first to have the book in Athens. The story that Plato “copied out” the *Timaeus*, only shows that the “Pythagorean” inspiration of that dialogue was generally recognized. It may have been invented by writers who did not *have* the book of Philolaos itself for comparison; but J.E. Raven (KR 308) blames Aristoxenos for it — and Aristoxenos did know what he was talking about. What is more important, is that the book of Philolaos was not easy to get hold of. The Pythagoreans only released it to known sympathizers. Both Speusippos and Aristotle used it later — for Speusippos see 44 A 13. But by their time, there may already have been forgeries in circulation.
 - ii. We do still have to worry about the distinction between the Pythagoreanism of Philolaos and that of Plato and the Academy. Aristotle, who disliked the Pythagorean influence, consciously distinguished between Plato and “those called Pythagoreans.” But most of the other early witnesses did not feel that this distinction was important.
 - iii. The Lysis who taught Epaminondas (46 A 1) cannot have been much older than Philolaos (and may have been a few years younger). He was not therefore a pupil of Pythagoras himself. Either there were two Pythagoreans called Lysis, or (more probably) two political reactions against the Pythagoreans were confused later in the school tradition. The Thebes connection strongly suggests that the Lysis who went into exile with Philolaos was not *his* teacher, but a younger man who *did* teach Epaminondas.
 - iv. For the date of his exile see Guthrie, 1962, I, 329. The supposition that Plato got the book directly from Philolaos (A 1, 85; cf. A 5) is absurd on all counts. We should even ignore the

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- dramatic date of the *Phaedo* in estimating when Philolaos lived in Thebes. It is more relevant, probably, in estimating the date of his death.
- v. Both he and Eurytos are sometimes identified as Metapontines or Tarentines; and Cicero says that he was the teacher of Archytas (who himself recorded memories of Eurytos). See 44 A 3 and 45 A 2.
 - vi. For a good short survey of the points of agreement see W. Burkert, 1972, 234 n. 83. Aristotle mentions Philolaos only once. In the *Eudemian Ethics* (1225a 30) he quotes a “saying” of his that “there are some *logoi* that are stronger than we are” (B 16). The *logos* of truth’s anonymity — which Aristotle was clearly quite willing to accept — has proved stronger than the personal authorship of Philolaos himself. In justification of Aristotle, it should be said that much of what Philolaos wrote was the older traditional doctrine of the School — and that was what Aristotle was interested in. We shall see that even with respect to cosmology, Philolaos probably only made some creatively imaginative improvements upon the astronomical theory of his immediate predecessors and contemporaries.
 - vii. This is given by Diogenes Laertios (VIII, 85) from Demetrios of Magnesia.
 - viii. What this means is illustrated a little further on. M. Schofield is right in maintaining the interpretation suggested here (KRS 326). He is only wrong in thinking that the odd/even distinction was the only important meaning of the opposition between “limiters” and “unlimited” in the mind of Philolaos. The doctrine has many different applications; but 58 B 28 (*Physics* 203a 10) shows why “odd and even” was one of them. Compare also 44 B 5.
 - ix. See 58 B 28 (Aristotle, *Physics*, 203a 1).

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- x. Thus the cube is a “geometric *harmonia*.” It has 12 edges, 8 angles, 6 faces — and 8 is the “harmonic mean” between 6 and 12 (A 24).
- xi. B 4; cf. B 6 and B 11. As far as Philolaos is concerned, we need not distinguish in Aristotle’s reports between those who said that things are “made of” numbers (as bodies are of flesh and bone, cf. 58 B 4, *Metaphysics* 985b 23ff; B 8, *Metaphysics* 987a 9ff; B 9, *Metaphysics* 1080b 16ff; B 10, *Metaphysics* 1083b 8ff; B 13, *Metaphysics* 987b 22ff; B 22, *Metaphysics* 989b 29ff; B 25, *Metaphysics* 1036b 8) and those who said that things depend on, or are determined by, their numbers (cf. 58 B 4, *Metaphysics* 985b 23ff; B 8, *Metaphysics* 987a 9ff; B 22, *Metaphysics* 989b 29ff; B 27, *Metaphysics* 1092b 26ff). Philolaos could have expressed himself in either way. He could have referred either to the actual “Olympian” harmonizing power, or to the number that is passively harmonized at the level of cosmic becoming (in the “infinite” or “boundless” — i.e., quite featureless matter).
- xii. See Huffman, 1993, 57-64. The point is not vital, but surely the Master may have said it. What matters, in any case, is what it *meant* to Philolaos — and one thing that was certainly important to him was the Parmenidean status of numbers.
- xiii. Compare Aristotle, *Metaphysics*, 986a 15 (58 B 5).
- xiv. Philolaos noted that the Dekad contained as many primes (1, 2, 3, 5, 7) as composites (4, 6, 8, 9, 10); and the *Tetraktys* is the *arche* of health.
- xv. Later on Ekphantos of Syracuse — influenced perhaps by Leukippos and Demokritos — said

that the creative numbers were “indivisible *bodies*.” Theophrastos thought that he was the first to say this (51 A 2 — Guthrie, I, 324-325). Philolaos was far removed from this sort of materialism. All that we can say for certain about his creative numbers is that — since he had certainly read Zeno — he does not regard the numbers that only God can know as subject to Zeno’s attack. The dates of Ekphantos are quite uncertain.

- xvi. The view of J. Barnes (1979, II, 85-87) that the “Unlimiteds” of Philolaos are “stuffs” (copper, tin, oil, etc.) is correct as far as it goes, but simplistic, and not general enough. The *difference* between copper and tin is caused by their “limiters”; and the Void (or Time) provide us with examples of “unlimiteds” that are not “stuffs.”

- xvii. The language of the fragment is vague. Probably Philolaos has not seen the book of Anaxagoras. Anaxagoras, on the other hand, may have Philolaos’ book; and he has certainly talked to some philosophically inclined Pythagorean exiles.

- xviii. *Timaeus*, 55d Compare 44 A 13, A 14, A 15, and B 12. (B 12 is suspect; and almost all of the other evidence is Academic. So unless one concedes that this part of the *Timaeus* was “copied out” of Philolaos, our report cannot establish that the doctrine was pre-Platonic. But the research program of Eurytos helps to convince me that it was.) Another pointer is the association of the dodekahedron with Zeus — 44 A 14. This was the structural shape of his “House” — compare note 29 below.

- xix. It seems to me probable that Zeno’s arguments were directed more against this mathematical faith of the Pythagoreans, than at anyone else (compare the argument of M.C. Nussbaum, 1979).

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- xx. See especially 58 B 8 (*Metaphysics*, A 5 987a 9 ff). The report of Aristotle, *Metaphysics*, 109a 13 (58 B 26) and elsewhere, that the Pythagoreans attributed generation to eternal things, springs (if my hypothesis is correct) from a revision of the more primitive view of Pythagoras by Philolaos, in response to Parmenides. Pythagoras would have said simply that the One (or Limit) “breathed in the surrounding Unlimited Air (or Void).” Philolaos distinguished between the realms of Being and Becoming, and said that God (the “Zeus” of eternal Olympos) planted the One of the cosmic cycle, which expresses itself in the great rounds of time and space; from this created (or generated) One, all of the *perpetual* numerical order of the *kosmos* was born.
- xxi. Compare A 16, A 17 (Aetios). It is vitally important that what is *harmonized* — the *kosmos* — is at a different level than the logical elements — limiters, unlimiteds, and *harmonia*. These elements refer us to the divine activity that is beyond our knowledge; but our knowledge itself is concerned with the *kosmos*. Aristotle’s complaint that the Pythagoreans “generated” eternal things is unjust; his complaint that they did not explain the generation of the *sensible kosmos* is quite correct (*Metaphysics* 1090a 32-35). But Philolaos said clearly that this could not be done.
- xxii. Compare further, Aristotle, Fragment 203 (from Alexander).
- xxiii. It is at least probable that Philolaos distinguished the *aether* as the fifth element that belongs to the Heavenly sphere (and to the Hearth) — see B 12. (Thus “the first thing harmonized” was the vital Fire of the *kosmos*. This was at the center because that is the “best” position; and it was Fire because Fire is the “best” element — compare *On the Heaven*, 293a 18n. But the Whole as the unity of the Dekad is what is truly divine.)

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- xxiv. As W. Burkert says, the astronomy of Philolaos was a “mélange of myth and *physiologia*” (1972, 350). It seems clear that the report that the *Ouranos* was between Earth and Moon arose from a literal reading of the allegory. The sky of our daytime experience is between us and the Moon; and Philolaos used this daytime limitation to represent the unscientific sensory consciousness of the uneducated masses. The infinity of the Fixed Stars in the night sky represented *divine* consciousness. The *kosmos* itself — *between* our sky-roof and the Fixed Stars — represented human philosophical consciousness. (It is definitely wiser to admit the confused mingling of an allegory of cognition with the scientific cosmology in our report, than to suppose that the report is based partly on the genuine book, and partly on a later forgery — as Burkert and Huffman both do. We shall find that the allegory helps us with other problems of interpretation.)
- xxv. This is Aristotle, *On the Heavens*, 293a-b; compare 44 A 16, A 17, A 21 (Aetios on Philolaos).
- xxvi. In 50 A 1 Cicero reports that Theophrastos said Hiketas of Syracuse was the first to say that the vault of heaven is motionless, and the earth revolves round the Central Fire. But Diogenes Laertios — 44 A 1, 85 — mentions Hiketas and Philolaos together in connection with the Earth’s movement. If they were contemporaries, either of them may have suggested the stillness of the Heaven to the other. (The name “Olympos” for the Heaven *guarantees* stillness, if it refers to the physical world directly.)
- xxvii. The odd claim that Moon-animals “do not excrete” probably reflects the influence of Empedokles’ theory of “whole natures”; and probably the Moonies were fifteen times as strong as we are, because the Moon’s “day” [in the light of the Central Fire?] is fifteen times as long as our day and night. (The belief that the Moon is another Earth like ours is found in “Orphic” sources. But this is probably because of Pythagorean influence upon “Orphism”

— cf. M.L. West, 1983, 92-93.)

- xxviii. 58 B 37 — *On the Heaven*, 293b 1ff. One (or more) of these names may have been invented by the Master himself. Philolaos could have called the *center*, the “tower” or the “Guardroom,” if he called the whole Heaven-structure the “house” — see note 29; however, *these* names are not ascribed to him.
- xxix. He cannot have called the center *alone* “the *House of Zeus*.” But he may have used this name for the aetherial unity of center and circumference (within which the *kosmos* dwells).
- xxx. The account in our sources is confused (31 A 56; cf. 31 A 30); but it suffices to show that the hypothesis of the Central Fire — as the “Tower” or “Guardroom” of Zeus — was part of Pythagorean astronomy before Philolaos, and that Empedokles almost certainly got his theory of the Sun from them. Similarly the discovery that the Moon shines by reflected light was probably made by Parmenides or some other Pythagorean, before Parmenides wrote his poem. (To believe that Anaxagoras was first here, because Plato’s Sokrates speaks of it as a new discovery of his, is almost certainly a mistake. Anaxagoras “discovered” it in Parmenides or Empedokles; and it promptly became a *new scandal* attached to his name in Athens.)
- xxxi. This development was certainly gradual. We must think of Pythagorean astronomy as a *communal* achievement. There was at least *one* (and given the variety of names for the Central Fire, probably more than one) important Pythagorean astronomer between Pythagoras and Philolaos. (Guthrie — I, 289 — says that Aristotle, *On the Heaven* suffices to show that before Philolaos no one thought that the Earth moves. But all it shows is that until Plato and Aristotle got hold of Philolaos’ book, no outsider *knew* what the general view

of the school was. Parmenides and Empedokles revealed enough for us to be sure that astronomical observation and speculation was going on.

xxxii. The length of our year is thus 729 days and nights. The *a priori* mathematical aspect of this calculation is brought out by Plato in the ninth book of the *Republic*.

xxxiii. There were Pythagoreans who applied the concept of “Right/Left” to the *kosmos* (compare 58 B 30 — *On the Heaven* 284b 6ff — with the comments of Simplicios). Thus our side of the Earth must be the “left” side, because we do not face the Central Fire. For Philolaos, who identified the periphery of the world-system as “Olympos,” this became an untenable view. There cannot be a “right and left” in the *kosmos*, just as there is no “up and down.” This need not have affected his attitude towards the Table of Opposites as a guide of human life; and his book may still have been the source of Aristotle’s knowledge of the Table. But there was uncertainty about the general Pythagorean acceptance of the Table in Aristotle’s mind, and almost certainly this sprang from his sources — see 58 B 5 (*Metaphysics* 986a 15ff). So quite probably the Table was not in Philolaos’ book; Aristotle found it in some other source (Lysis or Archytas?).

xxxiv. The total number of months is once more 729 (the cube of 9).

xxxv. Eudemos, fragment 88 — see Guthrie, I, 281.

xxxvi. Compare 14 A 8a (and Chapter 5, p. 000 above).

xxxvii. The “measures” of Herakleitos may have influenced Philolaos here; and the view of

Xenophanes that the Sun is “nourished” by “exhalations” of a “wet” kind. But any interpretation is necessarily very tentative.

- xxxviii. See above (at note 18) and below (at note 58).
- xxxix. Compare *Republic*, . Plato was probably well acquainted with the work of Archytas who completed — or refined — the work of Philolaos in this area.
- xl. Compare B 17, B 18. B 17 is a report that the *kosmos* is a sphere organized from the center, so that the difference of “above” and “below” is not significant. B 18 is a heading that has lost its quotation (which ought at this point in Stobaeos to be about the Sun — but the heading may have become detached from somewhere else). For the sort of mystical doctrine that B 19 refers to see also A 14.
- xli. I take the doctrine that the daylight sky is a “roof” to be part of the original teaching of the Master; but Empedokles is our earliest evidence for it. The allegory was made originally by connecting the story of Orpheus in the Underworld, with the story of his death, through the Pythagorean hypothesis that Orpheus passed on from being a devotee of Bacchos (in the Underworld) to become a devotee of Apollo (in the Heaven). The devotees of Bacchos tore Orpheus to pieces in revenge (but this, in turn, is an allegorical expression of what the life of the senses does to us if we don’t worship Apollo — see note 49).
- xlii. The Master himself probably used Persephone (“the Bride”) as an image of the ambiguous natural/spiritual destiny of womanhood (see note 51 below). The whole metaphor of our double life in the realms of “Day and Night” may have passed down from his study of

Hesiod to Parmenides and Philolaos.

- xliii. This should be read together with the passages from Nikomachos cited by Guthrie I, 260-2. See also *Timaeus* . The mathematical foundations for all this were laid by Hippasos (and probably published in his “Sacred Logos” (to which Proklos refers in B 19). We can concede to W. Burkert that our hypothesis is probably not quite literally true. Both Plato and Speusippos may have made improvements (further refinements) upon the work of Philolaos (and *his* predecessors).
- xliv. Rhea (“Mother of the Gods”) should probably be identified with Hestia (“the Hearth”). Then the Square has four Goddesses. The Triangle has three Gods (though in the report of Proklos himself Kronos, Rhea’s husband, is added). Damaskios says that the half-circle (of Day and Night?) was assigned to the Dioskouri. But he also tells us that the Triangle was given to Athena, and the Square to Hermes. We must be cautious about interpretation because Speusippos has worked over what Philolaos left, and others have come after him. But it is certainly significant that Typhon is an *entombed* soul. When we consider his role in the *Theogony*, we are bound to think of him as the anti-Pythagorean horde that is the last enemy of Zeus on “Olympos.” (For the “astrological” interpretation of all this material see W. Burkert, 1972, 349-350.)
- xlv. This fragment has been regarded with quite unjustifiable suspicion. The reference to “ancient theologians and prophets” speaks for it, rather than against it. Philolaos was simply repeating what the Master himself had taught (and the Master saw himself as one of a long succession of sages) — compare Chapter 5, p. 000 above.
- xlvi. Empedokles clearly took over this same pessimistic view from the Pythagoreans. (The

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- Master got it from the writings or oral teachings of the *Orphic* communities. It did not originate with him.)
- xlvi. See also 58 C 4 (Iamblichos, *Life of Pythagoras*, 86). Apparently the doctrine about suicide was not in Philolaos' book. At least "Sokrates" has to explain it to Simmias and Kebes — and this (paradoxically) is probably because Sokrates (or Plato) learned it from them (rather than from the book itself). In the dialogue, of course, they are not very clever about *harmonia* generally. (I am simply *assuming* that what Iamblichos tells us about our religious duty to have children goes right back to the Master himself — cf. Plato, *Laws*, 773e and Chapter 5, p. 000 [81, at note 35].)
- xlvi. See *Inferno*, Canto XIII. (There is no evidence that Philolaos actually held any such view, but he had myths like the story of Daphne to interpret — and she was fleeing from *Apollo*.)
- xlix. If the book (or an important part of it) was called *Bacchae* then it surely contained some account of our ordinary life "in the Underworld." But only the Orphic allegory can be reconstructed with confidence. Dionysos is the god who releases the musical inspiration of Orpheus. With his lyre Orpheus can charm Hades and Persephone, so that they allow Eurydice to return to the sunlight of Apollo with him. The *orgiastic* worship of Dionysos is the *death* of Orpheus. The drunkard loses his "singing head" of philosophical reason; and even the "ethical life" of his body is torn into pieces (cf. Iamblichos, *Life of Pythagoras* 240: "One must not pull apart the god within oneself").
1. Iamblichos, *Life of Pythagoras*, 37-57. C.J. de Vogel (1966, chap. VI) has shown fairly convincingly that these speeches go back to a Pythagorean source before 400 BCE.

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- li. At this point, the editor calls Pythagoras “the inventor of names.” One of his “goddesses” (the Bride) is Persephone (who belongs both to our world and to the Underworld); but the supposed *Doric* tradition that Maia is “Grandmother” is not familiar to me (Hermes is her son; so Aethalides-Pythagoras is her actual grandson).

 - lii. *Ethics* IX, 9 (1169 b 6). Pythagoras certainly had a hierarchical concept of friendship (between the Gods, humans and animals). But in our sources it has been Platonized — thus Iamblichos, *Vita* 229, speaks of “the rational part of the soul” and “the irrational parts.” The paradigm stories of friendship between Pythagoreans — especially Damon and Phintias (Aristoxenos in Porphyry, *Vita* 60-61) — assume equality. (For “another self” see Archytas below.) [just before note 68]

 - liii. Aristotle, *On the Soul* 404a 17.

 - liv. Compare Aristotle, *De Anima*, 407b and 58 B 41 — *Politics* 1340b 18.

 - lv. We have noted already how significant the use of the passive voice is in B 7. (Much of the discussion of the “soul is a harmony” doctrine is misguided, because of the obstinate adherence of students to the erroneous interpretation that “Sokrates” obviously meant to correct. We have to make the same distinction between the speaker of a *logos* and the listener, in order to understand Herakleitos and Empedokles.)

 - lvi. The assignment of each soul to its own star, gives it its place in “Olympos.” The subsequent “sowing” of the souls in the “instruments of time” (42d) represents the “choice of life” and placement in the temporal *kosmos*. Of course, we cannot prove that Plato got this idea from Philolaos’ book. But it is consistent with the doctrine of salvation that Pythagoras taught

(and it solves the problem of the “Orphic” salvation of the soul from its “imprisonment” (Chapter 5, p. 000 [83, at note 39] above). My own guess is that the Master taught that the human soul comes from and should finally return to its own star; and that Philolaos (or someone between the Master and himself) finally justified the distinction between the Fixed Stars and the planets — or between “Olympos” and the *kosmos* — by postulating the motion of the Earth. “Olympos” is the completely stable “House of Zeus” with “the Hearth” at its center. (We cannot give any weight to the report of Claudianus Mamertus that “When separated from the body by death, the soul leads a disembodied life in the world” — 44 B 22. A report of that kind definitely shows that there *were* forgeries ascribed specifically to Philolaos.)

- lvii. See Chapter 14 [at note 21] for a plausible interpretation of this paradoxical report.

- lviii. The statement that “bile *is not* in us” probably reflects the influence of the Eleatics. That it is “useless” provides the *explanation* for this strange usage. But the doctrine of how blood “thickens” and “rarefies” can safely be ascribed to Pythagoras himself, because it is so evidently inspired by Anaximenes. It is quite probable that all of Philolaos’ medical theories came directly from the teaching of the Master.

- lix. Compare note 24 above.

- lx. Both of them probably spent some years in Tarentum when Archytas was young. Hence Archytas became one of our recorded sources for Eurytos (45 A 2).

- lxi. See for example Aristotle, *Metaphysics*, 989b 30 - 990a (58 B 22), 1002a 4, 1028b 16 (58 B 23), 1080b 16 (58 B 9), 1083b 8, 1090a 20, 1090b 5 (58 B 24), 1091a 12; *Physics* 201 A 10

(58 B 28); *Ethics* 1132 B 21 (58 B 4); *On the Heaven* 300a 17 (58 B 38).

- lxii. A 3 begins with Aristotle, *Metaphysics* N 5, 1092b; the commentary of Alexander is surely mistaken in supposing that Eurytos *began* with a definite number of pebbles.
- lxiii. The relation between the square and the cube shows why a profile drawing was sufficient for Eurytos' purpose. Being "even" is the *infinite* aspect of things — cf. 58 B 28 and 58 B 23 in note 61. As soon as division is rationally possible at all, it can be repeated *ad infinitum*. (Plato was expressing the meaning of the Pythagorean view that "the Even is Unlimited" when he called the Unlimited "the Indefinite Dyad." But Aristotle says categorically that that development was *his* — 58 B 13, *Metaphysics* 987b 22ff.)
- lxiv. A 1 is from Diogenes Laertios; A 5 is from Plato's Seventh Letter. See also 44 A 3 (from Cicero).
- lxv. Melissos had already grasped this point — and probably Parmenides understood it before him. But it was Archytas who saw how to make it evident to ordinary consciousness.
- lxvi. We should take note of the feminine-passive ("harmonized") aspect of the image. The "sister" relation makes the necessary distinctness of the "number-causes" in different fields explicit; and this testimony of Archytas is the best evidence that we have for crediting much of the mathematical work that we hear of in *Academic* "Pythagorean" sources to the generation of Philolaos. Our general prejudices ought to be in that direction, rather than in favor of Plato and Speusippos.)

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- lxvii. The earliest clear statement of the theory is in the Myth of Er (*Republic*, X). But Plato's account seems to be evidently founded on the way Philolaos reconciled the octave of the moving *kosmos* with the stability of the One as *Dekad*. That the doctrine was Pythagorean is testified by Aristotle (*On the Heaven*, 290b 12 — 58 B 35); that it was older than Archytas is fairly surely guaranteed by the alternative explanation Aristotle cites for our failure to hear the cosmic music. (The sound is always there, so we are acclimatized from birth — this may have been the Philolaic theory.)
- lxviii. Cicero, *Cato Maior* 39-41; *Laelius*, 88. Cicero's story of how Cato "heard the speech" is probably a fiction. But there is no reason to doubt that Cicero himself had read the ethical texts of Archytas in Athens. The list of works dismissed as spurious in 47 B 9 probably reflects (with some inevitable duplications) what was preserved in the Academy. (This much is true, even if the purported fragments of those works are mostly forged. Forgers must often have begun from a list of genuine titles.)
- lxix. B 5 is from Theo of Smyrna. The report is late, but there is no valid reason to suppose that the treatise was simply a forgery. Aristotle had some sources other than Philolaos for the — often conflicting — numerological beliefs that he reports; and Plato would certainly have brought the writings of Archytas back to Athens.
- lxx. 1959, 224. (See, for example, 58 B 27 — Aristotle, *Metaphysics*, 1093a 23.)