Finite-time thermodynamics: Engine performance improved by optimized piston motion

(Otto cycle/optimized heat engines/optimal control)

MICHAEL MOZURKEWICH AND R. S. BERRY

Department of Chemistry and the James Franck Institute, The University of Chicago, Chicago, Illinois 60637

Contributed by R. Stephen Berry, December 29, 1980

ABSTRACT **The methods of finite-time thermodynamics are used to find the optimal time path of an Otto cycle with friction and heat leakage. Optimality is defined by maximization of the work per cycle; the system is constrained to operate at a fixed frequency, so the maximum power is obtained. The result is an improvement of about 10% in the effectiveness (second-law efficiency) of a conventional near-sinusoidal engine.**

Finite-time thermodynamics is an extension of conventional thermodynamics relevant in principle across the entire span of the subject, from the most abstract level to the most applied. The approach is based on the construction of generalized thermodynamic potentials (1) for processes containing time or rate conditions among the constraints on the system (2) and on the determination of optimal paths that yield the extrema corresponding to those generalized potentials.

Heretofore, work on finite-time thermodynamics has concentrated on rather idealized models (2–7) and on existence theorems (2), all on the abstract side of the subject. This work is intended as a step connecting the abstract thermodynamic concepts that have emerged in finite-time thermodynamics with the practical, engineering side of the subject, the design principles of a real machine.

In this report, we treat a model of the internal combustion engine closely related to the ideal Otto cycle but with rate constraints in the form of the two major losses found in real engines. We optimize the engine by "controlling" the time dependence of the volume—that is, the piston motion. As a result, without undertaking a detailed engineering study, we are able to understand how the losses are affected by the time path of the piston and to estimate the improvement in efficiency obtainable by optimizing the piston motion.

THE MODEL

Our model is based on the standard four-stroke Otto cycle. This consists of an intake stroke, a compression stroke, a power stroke, and an exhaust stroke. Here we briefly describe the basic features of this model and the method used to find the optimal piston motion. A detailed presentation will be given elsewhere.

We assume that the compression ratio, fuel-to-air ratio, fuel consumption, and period of the cycle all are fixed. These constraints serve two purposes. First, they reduce the optimization problem to finding the piston motion. Also, they guarantee that the performance criteria not considered in this analysis are comparable to those for a real engine. Relaxing any of these constraints can only improve the performance further.

We take the losses to be heat leakage and friction. Both of these are rate dependent and thus affect the time response of the system. The heat leak is assumed to be proportional to the instantaneous surface of the cylinder and to the temperature difference between the working fluid and the walls (i.e., Newtonian heat loss). Because this temperature difference is large only on the power stroke, heat loss is included only on this stroke. The friction force is taken to be proportional to the piston velocity, corresponding to well-lubricated metal-on-metal sliding; thus, the frictional losses are directly related to the square of the velocity. These losses are not the same for all strokes. The high pressures in the power stroke make its friction coefficient higher than in the other strokes. The intake stroke has a contribution due to viscous flow through the valve.

The function we have optimized is the maximum work per cycle. Because both fuel consumption and cycle time are fixed, this also is equivalent to maximizing both efficiency and the average power.

In finding the optimal piston motion, we first separated the power and nonpower strokes. An unspecified but fixed time t was allotted to the power stroke, with the remainder of the cycle time given to the nonpower strokes. Both portions of the cycle were optimized with this time constraint and were then combined to find the total work per cycle. The duration t of the power stroke was then varied and the process was repeated until the net work was a maximum.

The optimal piston motion for the nonpower strokes takes a simple form. Because of the quadratic velocity dependence of the friction losses, the optimum motion holds the velocity constant during most of each stroke. At the ends of the stroke, the piston accelerates and decelerates at the maximum allowed rate. Because the friction losses are higher on the intake stroke, the optimal solution allots more time to this stroke than to the other two. The piston velocity as a function of time is shown in Fig. 1.

The power stroke was more difficult to optimize because of

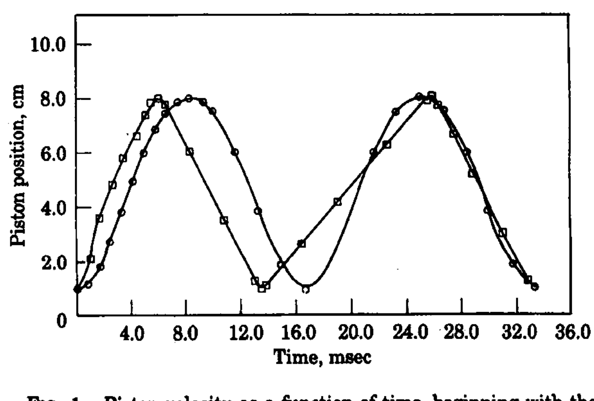

FIG. 1. Piston velocity as a function of time, beginning with the power stroke. The maximum allowed acceleration is 2×10^4 m/sec^2.

Table 1. Engine parameters*

Mechanical parameters:		
Compression ratio = 8		
Piston position at minimum volume = 1 cm; displacement = 7 cm		
Cylinder bore (b) = 7.98 cm		
Cylinder volume (V) = 400 cm^3		
Cycle time (τ) = 33.3 msec at 3600 rpm		
Thermodynamic parameters:	Compression stroke	Power stroke
Initial temperature	333 K	2795 K
Mol of gas	0.0144	0.0157
Constant-volume heat capacity	2.5 R	3.35 R
Cylinder wall temperature (T_w) = 600 K		
Reversible work per cycle (W_R) = 435.7 J		
Reversible power (W_R/τ) = 13.1 kW		
Loss terms:		
Friction coefficient (α) = 12.9 kg/sec		
Heat leak coefficient (κ) = 1305 kg/deg per sec^3		
Work lost, per cycle, to time loss and bearing friction (W_B) = 50 J		

* The parameters are based on data from ref. 10.

the presence of the heat leak. The problem was solved by using the variational technique of optimal control theory (8). The formalism yields the equation of motion of the piston as a fourth-order set of nonlinear differential equations. These were solved numerically. The resulting motion is shown in Fig. 1 for the entire cycle.

The asymmetric shape of the piston motion on the power stroke arises from the trade-off between friction and heat leak losses. At the beginning of the stroke the gases are hot, capable of yielding high efficiency, and the rate of heat loss is high. It is therefore advantageous to make the velocity high on this part of the stroke. As work is extracted, the gases cool and the rate of heat leakage diminishes relative to frictional losses. Consequently the optimal path moves to lower velocities as the power stroke proceeds.

The solutions were obtained first with unlimited acceleration and then with limits on acceleration and deceleration. The latter situation yields a result familiar in other contexts under the name of "turnpike" solution (9). The system tries to operate as long as possible at its optimal forward and backward velocities, by accelerating and decelerating between these velocities at the maximum rates. In this way, the system spends as much time as possible moving along its best or turnpike path.

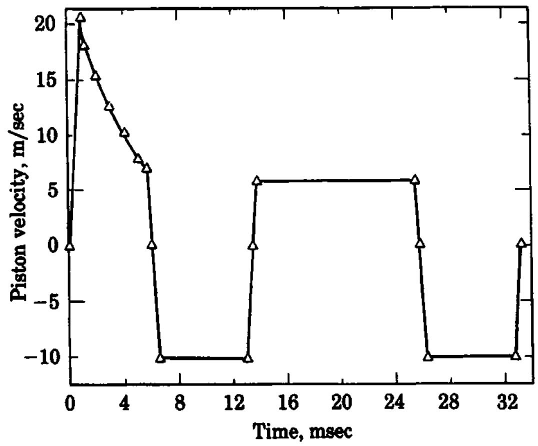

FIG. 2. Comparison of the optimal (□) and conventional (○) piston motions over the power, exhaust, intake, and compression strokes. The optimal path is constrained to have a maximum acceleration of 2×10^4 m/sec^2.

RESULTS

Parameters for the computations were taken from ref. 10 or, in the case of the friction coefficient, adjusted to give frictional losses of the magnitude cited in ref. 10. Those parameters are given in Table 1. The results of the calculations of some typical cases are given in Table 2, where they are compared with the conventional Otto cycle engine having the same compression ratio but a standard, near-sinusoidal motion. The effectiveness ε (the ratio of the work done to the reversible work, also called the second-law efficiency) is slightly higher for the optimized engine whose piston acceleration is limited to 5×10^3 m/sec^2, the maximum of the conventional engine of the first row. If the piston is allowed to have 4 times the acceleration of the conventional engine, the effectiveness increases 9%; if the acceleration is unconstrained, the improvement in effectiveness goes up to 11%.

These values are typical, not the most favorable. If the total losses of the conventional engine are held approximately constant but shifted to correspond to about 80% larger heat loss and about 60% smaller friction loss, the gain in effectiveness goes up, reaching more than 17% above the effectiveness of the corresponding conventional engine.

The principal source of the improvement in use of energy in this analysis is in the reduction of heat losses when the working fluid is near its maximum temperature. This is why the improvement is greater for engines with large heat leaks and low friction than for engines with relatively better insulation but higher friction.

Table 2. Results (all energies in joules)

Max. acc., m/sec^2	v_{max}, m/sec	t', msec	W_p	W_T	W_F	W_Q	Q	T_f, K	ε	% increase in ε	% decrease in W_Q	% decrease in W
Conventional	13.6	8.33	503	276	67	43	224	1095	0.633	—	—	—
5×10^3	17.1	7.62	500	279	63	43	210	1130	0.640	1.1	−0.14	5
1×10^4	18.6	6.63	508	293	58	34	186	1160	0.672	6.2	21	12
2×10^4	20.5	6.07	513	300	58	28	172	1180	0.689	8.7	34	14
5×10^4	22.4	5.90	516	304	57	24	167	1185	0.698	10.1	44	14
Unconstrained	25.4	5.48	518	307	58	21	156	1200	0.705	11.2	53	13

t', Time spent on power stroke; W_p, work done on power stroke; W_T, net work for one cycle; W_f, friction losses; W_Q, work lost due to heat leak; Q, heat leak; T_f, temperature at end of power stroke; ε, effectiveness.

Finally, it is instructive to examine the path of the piston in time, for the optimized engine and for its conventional counterpart. The position of the piston as a function of time is shown for these two cases in Fig. 2.

In closing, let us emphasize the unconventional approach to optimizing a thermodynamic system illustrated by this work. Instead of controlling heat rates, heat capacities, conductances, friction coefficients, reservoir temperatures, or other usual parameters of thermodynamic engines, we have controlled the time path of the engine volume.

We thank Dr. Morton Rubin for helpful comments and suggestions. This work was supported in part by a grant from the Exxon Education Foundation.

1. Hermann, R. (1973) *Geometry, Physics and Systems* (Dekker, New York).
2. Salamon, P., Andresen, B. & Berry, R. S. (1977) *Phys. Rev. A* **14**, 2094–2102.
3. Curzon, F. L. & Ahlborn, B. (1975) *Am. J. Phys.* **43**, 22–24.
4. Andresen, B., Berry, R. S., Nitzan, A. & Salamon, P. (1977) *Phys. Rev. A* **15**, 2086–2093.
5. Rubin, M. (1979) *Phys. Rev. A* **19**, 1272–1276, 1277–1289.
6. Salamon, P., Nitzan, A., Andresen, B. & Berry, R. S. (1980) *Phys. Rev. A* **21**, 2115–2129.
7. Gutkowicz-Krusin, D., Procaccia, I. & Ross, J. (1978) *J. Chem. Phys.* **69**, 3898–3906.
8. Hadley, C.F.G. & Kemp, M. C. (1971) *Variational Methods in Economics* (North-Holland, Amsterdam).
9. Sen, A., ed. (1970) *Growth Economics* (Penguin, Baltimore, MD).
10. Taylor, C. F. (1966) *The Internal Combustion Engine in Theory and Practice* (MIT Press, Cambridge, MA), Vol. 1, pp. 158–164; Vol. 2, pp. 19–20.

Correction

Correction. In the article "Finite-time thermodynamics: Engine performance improved by optimized piston motion" by Michael Mozurkewich and R. S. Berry, which appeared in the April 1981 issue of *Proc. Natl. Acad. Sci. USA* (78, 1986–1988), an undetected printer's error resulted in incorrect placement of the illustrations. The picture shown as Fig. 1 actually is Fig. 2 and that shown as Fig. 2 actually is Fig. 1.