Functions & Graphs

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Pre-Calculus Mathematics for Business and Economics

Sub-Topics

1. Functions: One-to-One and Inverse Functions.
2. Transforming Graphs
3. Graph Parabola
4. Finding real zeroes of a function
5. Writing an Equation for a Horizontal and Vertical Transformations
6. Variable expressions as inputs of functions
7. Graph Cubic Functions
8. Set Theory: Union and Intersection

Functions

A function is a **relation** for which each value from the set the first components of the ordered pairs is associated with exactly one value from the set of second components of the ordered pair.

Functions

A simple definition of a function is an equation for which any x that can be plugged into the equation will yield exactly one y out of the equation.

Domain and Range

The **domain** of an equation is the set of all x's that we can plug into the equation and get back a real number for y. The **range** of an equation is the set of all y's that we can ever get out of the equation.

`https://tutorial.math.lamar.edu/classes/alg/functiondefn.aspx`

Inverse and One-to-One Functions

An one-to-one function has every element of the range of the function corresponds to exactly one element of the domain.

Inverse and One-to-One Functions

An one-to-one function, $f(x)$, has an inverse function, $f^{-1}(x)$. An inverse function reverses the mapping!

Inverse and One-to-One Functions

An inverse function, $f^{-1}(x)$ has a special property:

$(f \ o \ f^{-1})(x) = f(f^{-1}(x)) = x$

Functions: Consider the following one-to-one functions:
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$h(x) = 3x + 13$ Find: g^{-1}

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Find: $\left(h \; o \; h^{-1}\right)(4)$

Transforming Graphs: $y = f(x)$

$y = (2)f(x)$:
"you receive twice the y for the same $f(x)$ "

$y = f(2x)$:
"you only need half the x to have the same y"

See video for a practice question on ALEKS

Graph Parabolas:

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x	$f(x)$		y
-3	$3(-3)^2 + 6(-3) - 2$	$=$	7
-2	$3(-2)^2 + 6(-2) - 2$	$=$	-2
-1	$3(-1)^2 + 6(-1) - 2$	$=$	-5
0	$3(0)^2 + 6(0) - 2$	$=$	-2
1	$3(1)^2 + 6(1) - 2$	$=$	7

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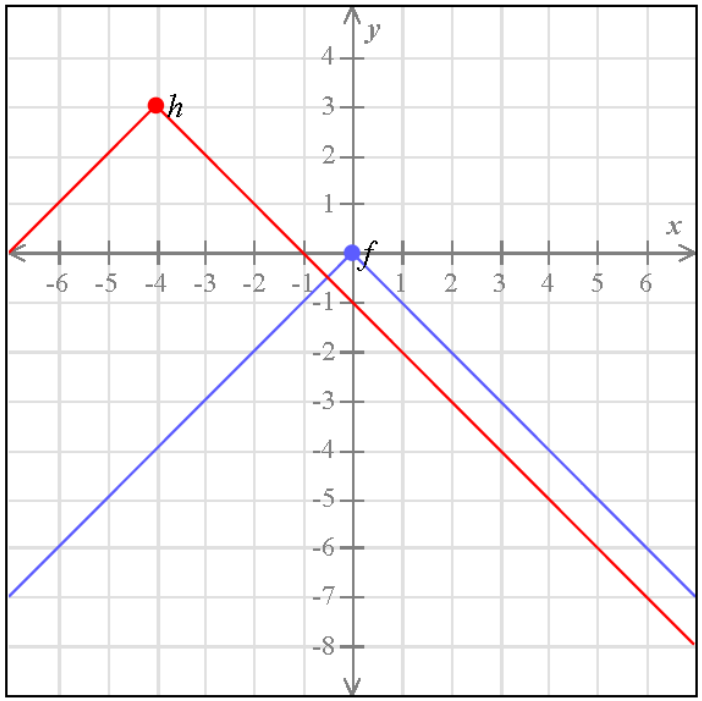
Graph the points: (-3,7), (-2,-2), (-1,-5), (0,-2), (1,7) on ALEKS

Find all real zeros of a polynomial function:

$$g(x) = -2x\left(x^2 + 16\right)\left(x^2 - 1\right)$$

The function f is defined by $f(x) = -|x|$

Write down the expression for $h(x)$.



Variable expressions as inputs of functions: Find $g(5z)$

$g(x) = 2x^2 - 1$

Graph Cubic Functions: Plot five points on the graph of the function: one point with $x = 0$, two points with negative -values, and two points with positive -values. Then click on the graph-a-function button. $y = \left(\frac{7}{4}\right) x^3$

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x	$f(x)$			y	Coordinates
-2	$\left(\frac{7}{4}\right)(-2)^3$	$=$		-14	$(-2, -14)$
-1	$\left(\frac{7}{4}\right)(-1)^3$	$=$		-1.75	$(-1, -1.75)$
0	$\left(\frac{7}{4}\right)0^3$	$=$		0	$(0, 0)$
1	$\left(\frac{7}{4}\right)1^3$	$=$		1.75	$(1, 1.75)$
2	$\left(\frac{7}{4}\right)2x^3$	$=$		14	$(2, 14)$

Set Theory: Union and Intersection: Find the Union of dark haired people and tall people. Then find the intersection of dark-hair and tall people.

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Tall $= \{Frank, Xu, Coco\}$

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Union: People who are either tall or have dark hair.

Intersection: People who are both tall and have dark hair.

Set Theory: Union and Intersection: Find the Union of G and M. Find the intersection of G and M.

$G = \{d, h, j\}$
$M = \{e, f, g\}$