Functions and Graphs III

SUMMARY KEYWORDS
equal, function, negative, squared, coordinates, intersection, represented, union, question, set, graph, tall, absolute value, diagram, hair, dark, dark haired, zoo, values, unknown variable

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Any number multiplied by zero is just equal to zero. So when we're given a question like this, it says find all the real zero zeros of a polynomial function. So it's saying what find the values of x, such that g of x is equal to zero? Well, there's a few things. So all I have to do is show that one of these factors is equal to zero. And I've shown that g of x itself is equal to zero. So here with this one, if x is equal to zero, then g of x will be equal to zero, no matter what the the other values are. So that was one What about the second factor? Well, I can see that if x is equal to four, then x squared is equal to 16. But wait a second, it's plus 16. Is there any way to get to square a number and have a negative value? know there's no, there's no possible real number that I can plug into x square plus 16 and come up with a zero value? So there's no way no way to make x squared plus 16 equal to zero? No way. So there's no solution for that factor anyway. What about three? This one's a little more promising because we have a negative sign there. If x is equal to one, then g of x, or I should say g of one is equal to one squared minus one, which is just equal to zero. So if x is equal to one, g of x will be equal to zero. Is there any other value? Well, what if x was equal to negative one? Negative one square minus one is equal to zero as well. So there are three answers to the question if x is equal to zero, if x is equal to negative one, and if x is equal to one, then g of x is going to be equal to zero. Now let's go to ALEKS. And see if we did it correctly, I've gone ahead and added the values into ALEKS. So we've got negative one, zero and one, and I'm going to click on the check button. And we got the correct answer. We've been given a function f of x, which we know is the negative of the absolute value of x. And that represents the graph in a blue color with the origin coordinates 00. Another function has been drawn an H function whose highest point is at coordinates negative four, three. And the question is asking us to write down the expression for the function h of x. So let me write out F of X, which is equal to negative the absolute value of x. And there's an implication here that it's plus zero. Why do I know that because if x is equal to zero, we get the coordinates 00. Which takes us right to here and we know just looking at the diagram, the graph we know that that's the maximum. Now we want an equation for h of x, and it's gonna look something like this. Well, I'll read it very generally. Or maybe Alright as negative the absolute value of x plus a some unknown variable A plus B. And so we want to essentially solve for A and B. Remember that a is going to move the diagram left and right, B is going to move, or I shouldn't say the diagram, it's going to move the function left and right, and B is going to move the function up and down. Now I know that the H function has a max at the coordinates negative four, three, to move the max up or make it to move the max up three units. set B equal to well, its original value zero, plus three. So set B equal to three means I'm gonna have h of x is equal to negative x plus a plus three. Let's call this one. And this

is kind of our step one here. And step two. Notice h of x is maximized when x is equal to negative four, so let's set x equal to negative four. There's a couple different ways I could show you how to do this, let's recognize that this function is equal to our Y axis value. If I do that, I can rewrite this whole thing as y is equal to what it's equal to. Three, which is also equal to negative the absolute value of negative four plus a plus three. And if I subtract three off both sides of the equation, I'm going to get zero is equal to negative negative four plus a. What value of a makes this true? Well, a has to be equal to four. And now we can add everything together. And we see that h of x is going to be equal to negative f of x plus four, plus three, or h of x is equal to negative x plus four, plus three. Looking at ALEKS, I'm going to press negative and then I'm going to touch this little button over here, which is going to create an absolute value sign for me. I've got x minus four. Excuse me, that's x plus four, plus three. And let's see if we've got the right expression for h of x. I'll click the check button. And we got the right answer. And this is a great question, because it shows you a more general way of inputting values into a function. You can take a function and input it into another function, as this example demonstrates. This comes up a lot in calculus. This comes up a lot in economic applications. A lot of students are not familiar with it. It's not hard. And so but students since they're not familiar with it, they often panic and don't understand what's happening. So let's look at this question. The question says, here's a function g of x, find g of five z. So everywhere There's an X, I'm going to plug in five z. So we've got two times five z squared minus one. Now I'm going to simplify or expand, I guess I'm going to expand this expression. And I've got five squared. C squares, notice that the exponent comes into both the factors, both the five and the Z are going to be squared, and the minus one stays outside of everything. When I work through oops, getting ahead of myself. 50 c square forgetting the minus one, I'm going a little too fast. But there is the answer. So if we want to evaluate g of x, at five z, we can do that. Plug it in everywhere there's an X, you're going to plug in a five z. Now I've entered our answer on ALEKS. And I'm going to click the check button. And we got the correct answer. We're going to take a look at cubic functions. It's important to be familiar with cubic functions and the shape that they take on. This question is similar to one we did a little bit earlier where we had to evaluate a number of points using the function and then graphing it. And so I'm not going to take too much time redoing that again. But I want to make sure that you understand and you know how to use the ALEKS tool and how to graph a function like this. Now, I've gone ahead and solve for it. Here are the solutions. Notice that we've got fractions for in this case is represented as a decimal, but it's a fraction. Or it could be represented as a fraction. And the problem with this is, it's going to be awfully hard for you to find the point negative 1.75 on the ALEKS diagram. So let me show you again, the tool that we're going to use to solve a question like this on ALEKS. Here's the question on ALEKS. And I'm going to use the plot anywhere function. So I'm going to click on this highlighted little image of a diagram. And it's asking me just to show you know, it says enter the x&y coordinates of the point. And now I've got the coordinates over here, to the right of the screen. So that's going to make it easy for me to input them in. So we've got negative two, negative 14. And then I click plot point, negative one, negative 1.75. Oops, there's a mistake, make sure I move over 1.75. And I'm going to plot the point zero and zero. That's an easy one, plot the point, one, and 1.75. plot the point, and two, and 14. And I'm going to plot the point. So it looks like I did it properly. Hopefully, I didn't make an input error. I'm going to close this box here. And then I'm going to click on these little axes with the curvy line through them. And ALEKS should know that how I want the cubic function to be drawn. Now I'll move back up so you can see the question. And I'm going to click on the check box. And happily, I did it successfully. Let's take a look at set theory. Set Theory theory is really, really quite fun. And it's really, really important for understanding how to read mathematics, how to communicate

mathematics. I'll show you this example. It's really, really easy. But the key is understanding the language of mathematics. If you understand the language of mathematics, you'll be able to read your textbook. You'll be able to read academic papers and really understand what those authors are trying to tell you. And you'll be able to follow those instructions. And you're going to be, that's a very employable skill. That's a really great skill to have. So let's take a look at a simple example that I put together. The question says, find the union of dark haired people and tall people, then find the intersection of dark hair and tall people. And we're given two sets. We've got our sets. We've got our dark haired set and our tall set. Now, when it says find the union, what that means in mathematical language is find the people who are either tall, or have dark hair. So in this example, well, Ahmed has dark hair, so he's in that union. zoo, is in that union. They have dark hair. Mo has dark hair. Frank is tall. So even though he doesn't have dark hair, that's okay. He's in the union of dark haired and tall people. And zoo is also tall. But we've already included them in the Union, and we don't need to record them twice. There's nothing wrong with recording them twice. But generally, it's better if we just record them once, because we don't want to count them twice. If we were adding up all the numbers of people in this set. And Coco, Coco is tall. She doesn't have dark hair. But that's okay. She's in the Union. She's still in that union of dark hair and tall people. Now, what about the intersection? Well, the intersection of dark haired and tall people are those who are both tall and have dark hair. And I guess I could say about the union is that we'd say that this thing up here is equal to dark you tall at the union of dark hair, and tall people is equal to this set with five names. Now find the intersection of people who are both tall and have dark hair. So if I want dark, an upside down you that's a little bit bigger than a you this, this symbol. The dark, the intersection of dark and tall people is going to be equal to well, Ahmed is only has dark hair. And he's not tall, so he's not tall. Xu has both dark hair and is tall. Moe has dark hair, but he's not tall. Frank is tall, but he does not have dark hair. And Coco is tall, but she does not have dark hair. So the intersection of these spaces is just zoo. Now here's the question that we see on ALEKS. So we're given two sets. The sets contain unknown variables represented by the letters. And we're asked to find the union of G and M. and then find the intersection of G and M. So applying what we learned in the previous example, the union of G and m is going to be equal to well, any of these variables that occur in either the set G or the set M. So we're gonna have D, E, F, G, H, and J. Now, I tried to keep it in alphabetical order, because that's the proper way to represent it. It's probably not necessary, but it's the it's the appropriate it's the best way to represent it. And so the union of G and M has six different variables in it. When we look at the intersection well, neither of these variables to pay And both the set G and the set m. so we can say that the intersection of these two sets is empty, or it is the null set. It's the empty set. So it's empty. Or it sounds a little bit cooler in English to say it's the null set. But you can certainly call it the empty set that's completely legitimate. Here's our question on ALEKS. So the union of these two sets, I'm going to press this little button here to get the swirly brackets represented as a set. And so the union it has six variables in it. The intersection is the empty set. And let's see if I did it correctly. And I didn't that's okay. I just made a mistake with the language of ALEKS. And that language is I shouldn't have squirrely brackets around by empty set. So I've got the empty set here without squirrely brackets and now it tells me that I got the correct answer. So remember that when you're answering questions on ALEKS, it wants the null set with no squirrely brackets around it.