

# Functions and Graphs II

## SUMMARY KEYWORDS

parabola, vertex, negative, function, equal, squared, symmetry, minimum, point, guess, twice as much, manipulations, diagram, give, draw, form, plot, straight line, correct, intuition

## SPEAKER

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The slides have some tips on how to transform graphs. Essentially, you want to think about having a function in its general form like that. And then there's two manipulations that might be made. One is to double the function itself. In English, what does that mean? Well, if you have the same amount of  $f$  of  $x$ , right, if  $x$  is the same value, you're going to get twice as much  $y$ . Now, what happens if we put that now notice that the two here is outside the function? Looking at the next example, we're inside, it's inside the function. Well, what does that mean? Well, that means that you can have the same  $f$  of  $x$  the same  $y$  with half as much  $x$  half as much  $x$ . So we're trying to do that to give you a little bit of intuition. So here we are on ALEKS, we've got a question. And it says that we've got one half of  $X$  the function of one half  $x$ , this means that we're going to need twice as much  $X$  to get the same amount of  $y$ . So if I want to get  $y$  equal to negative four, and before, that would require two  $X$ , now it's going to take four  $x$ , so I'm going to draw a little blue cross make a little point there. Now, if we can see by the original function here, that if  $x$  is equal to zero,  $y$  will be equal to zero. So multiplying  $x$ , or sorry, zero by one half is not going to change that. So the origin stays the same. And similarly, on the other side of the diagram, if negative two  $x$  got us negative four  $y$ , now it's going to take negative four  $X$  to get negative four  $y$  is going to take twice as much  $X$  to get us there. I'm going to just draw the lines, now I got to draw two lines. And so we stretch the function out to the left and the right, because we need more  $X$  to get the same  $y$ . So let's see if that was correct. And it was correct. So we needed twice as much  $X$  to get the same amount of  $y$ . So a parabola looks something like this. Or maybe like this. And it just keeps going off in that direction. This form here would be something like  $y$  is equal to negative two  $x$  squared plus four  $x$  plus four, something like that. And when this is negative, we're going to get out max up here. The one on the right, could be something like two  $x$  squared minus four  $x$ . And it looks like maybe negative four. Something like that. And where if this term is positive, this is gonna be a min and so this one over here is decreasing. As the absolute value of  $x$  increases, and over here,  $y$  is increasing as the absolute value of  $x$  increases. Here, the max it, this is the vertex. The vertex is a max here. The men is also the vertex, the vertex is the min. So the question is asking us to find the vertex. There are a few ways to find the vertex. I'm just going to show you how to do it by trial and error. Essentially. There are other ways but we don't use them. They don't come up very much in economics, so I won't spend too much time on that unless you really want me to. You're more than welcome to ask me questions. I guess via email if we're doing distance or after class if we're in person. So for trial and error, I usually just, I will just guess that  $x$  is equal to zero. If  $x$  is equal to zero,  $y$  in this case is going to be equal to negative two. parabolas have certain properties that are going to help me with my gas, I know that this parabola, the square term is positive. So the vertex is going to occur at the minimum, so I can plug and play with some

numbers and see if I can find that minimum. I know that a parabola has a certain amount of symmetry, if I draw a straight line coming up from the vertex, then the distance from that straight line to each of its wings is going to be the same. Looking back at my guess, when  $x$  was equal to zero,  $y$  was equal to negative two. Well, why don't I guess? Okay, well, what happens if  $x$  is equal to one, well, then I get three times one squared, plus six minus two, and that's going to give me seven. So it's quite a bit higher than negative two. Now, if I can show that  $x$  is at negative one is greater than  $y$  is equal to negative two, then I'll know that at  $x$  zero,  $y$  equal negative two, we've got the vertex. So I'm going to guess  $x$  is equal to negative one, I get three times negative one squared, plus six times negative one, minus two, and I end up with minus five. Now I know that  $x$  zero,  $y$  negative two is not, well, maybe I should make it a little more formal here. I know that the pair's zero, negative two is not the vertex. But all hope is not lost. Because the coordinate negative one, negative five, might be the vertex. So I'm gonna guess.  $x$  is equal to negative two. And I've got  $y$  is going to be equal to negative two squared times three, plus six times negative two, minus two. And I've got  $12 - 12 - 2$ , which equals to negative two. So therefore, negative one, negative five is the vertex. I did that by guessing. And I know that Cindy has a problem with the square term, this term right here being positive. I know that once I find the minimum point, that's going to be the vertex. And because of symmetry, any other value of  $x$  is going to give a higher value of  $y$ . We can now go over to ALEKS. And I'll show you how to graph this parabola and ALEKS, anywhere feature and ALEKS. And that's that this little diagram here with the X on it, so I'm going to click on that, and it says, plot a point anywhere. And I'm going to go down to here. And I'm just going to put in all our coordinates that we calculated before. And notice that every time I do that and I click the plot a point button, a little Little x appears. Okay, so I've plotted the points of the parabola. And then I'm good. All I had to do is press this button right here. And ALEKS has sort of knows what to do with the plot points. Now let's see if it's correct. And it was correct. So we got the right answer.