Functions and Graphs I

SUMMARY KEYWORDS
function, inverse, inverse function, equal, ordered pairs, domain, squared, topics, definitions, economics, streetlight, number, green light, mapping, y value, students, negative, moving, evaluate, question

SPEAKER
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Hello and welcome to ALEKS walkthrough video number five. My name is Robert McKeown and today we're going to be talking about functions and graphs, students. In my experience at York University, and I'm not been here very long, students find this topic extremely challenging. I don't think it's particularly hard. But what you have to do is you have to memorize a number of definitions, and then apply those definitions to different situations. And that can be a little bit difficult because you actually have to learn the definitions, you actually have to do the work doesn't matter how smart you are smart is not going to help you if you don't have the knowledge necessary to answer the question. And so this is actually the longest series of videos of all a topics because there's just so much that I want to explain to you. And then you just need to know. So you can think of maybe the first four topics as being rather introductory, didn't require a lot of background information or background knowledge, background knowledge, now we're building off the knowledge you learned in topics 123, and four, and we're going forward into more advanced mathematics and into economic applications. So again, before we get started, make sure you have a pencil, a pen, paper, or if you're really fancy, get yourself a tablet with a writing pen. That's a really great tool to have. I was so impressed with the my students who had one that I went out and bought one. And I use it in class. If you've ever had a lecture with me, you'll see it for sure. So without any further ado, let's get down to the problems. To answer the topic five questions correctly, you have to understand a few definitions. And our first definition is what is a function. So you can read a formal definition on the slides. One thing I'd like you to think about is I'd like you to think about a streetlight. So functions don't have to be mathematical in nature, we could have an X variable, which is the streetlight can be a red light, can be red. Or it can be a green light. And there's a function that transforms that observing that light and to an outcome, which is stop your car forever the red light, and if it's a green light, it's go. And so we've got two ordered pairs here, we've got an ordered pair that's red light, and associated with stop and another ordered pair, that's a green light. And it leads to go. So that's an example of a function in everyday life. We're gonna focus on numerical functions here. But in economics, sometimes we don't want to use numbers, we want to use concepts like this, and we do so here is another definition, maybe a simpler definition of what a function is. So we've got a function something like this. And we might have y is equal to some function of x. So let's consider a function x squared. So if x is equal to zero, y is going to also be equal to zero. And if x is equal to one, y will be equal to one. If x is equal to negative one, y is also equal to one. And so I'll tell you that x squared is a function for every x there is one Y value. Now I haven't proved it. But if you want, you can go home, you can put in any number you want into that. And you're only going to get one y value, whatever number you put into that function, you're going to get only one y value. In fact, that's what makes it a function. Let me show you an example of something that is not a function. So this is a function. What if I have? I can't use that terminology, what if I have y squared plus x squared is equal to 25? Well, I've got y is equal to a function of x. So why don't I rearrange this thing over here and isolate y on one side, so that it looks like the function I showed you on the left hand side of the screen. So I can take, I know that y squared is going to be equal to 25 minus x squared, I can write y, I'll take the square root of both sides of the expression. So I've got 25 minus x squared. But notice that this is going to be plus or minus, whenever I take the square root of something that's been squared, it could be positive or negative. And so with this example, for each x value, there are two possible I'm not going to say each, let's just say for some, there are two possible y values. So it's not a function, the language we like to use in mathematics and economics is that the x values are in something called the domain. And the y values are in something called the range. So if I've got a set of axes here, and maybe I've got one, or 01, and two, here, this is the domain. And then there's some function that takes us to, I'm gonna just change this one to a negative one, like our previous example. And negative one also maps to Y equals one. So the function is mapping from the domain into the range. And if you want to read more about it, I found a neat website down here that you can read more about domain and range and what a function is what a function is not. There is a special kind of function, and it's called a one to one function. Now, when we had this previous example, we had a function that was equal to x squared. But you can see here that, and I'll get rid of that because we don't care about it. But you can see that there's more than one x value factor or two that lead to one y value. So this is not a one to one function. Now let me see. I could write a one to one function, which is just f of x is equal to x And notice that each number in the domain leads to a unique number in the range. So f of x equal to x is a one to one function. So there's, this is a function over here, but it's not a one to one function. It's a function but not a one to one function over here. This is a function and it is a one to one function. One to One functions are important because every one to one function has an inverse function has an inverse function. So let's consider another one to one function, I'll have 01 and two in my domain. And this time, I'll just say let the function be equal to two x. And there's my domain. And I'm going to have Y values here, if x is zero, y is zero. If x is one, y is two, and if x is two, if x is two, y is four. Now let's consider the inverse. So if we have the inverse of this function, this functions unique inverse, then it's going to reverse, essentially just going to reverse the mapping. And the inverse of this function is equal to one half times x. So inverse functions are not very, very complicated, in their fundamental or simplest form, they're just reversing. Where we're moving from, are we moving from X to get y or are we moving? Are we taking y to get X inverse functions have a very interesting property. So this little operator here creates a composite. So this is a to operator to create a composite function. And more commonly, it's written like this, which is basically to say that we have the inverse function and we plug it into the original function. We're going to end up with just x whatever that x value is. So let me give you an example from the previous one, we had f of x is equal to two the inverse of that function or excuse me, that was two x, the inverse was equal to one half x. and say I wanted to evaluate we always start with the inner right whatever is in the brackets is the first order of operations. So F inverse three We have one half times three, which gives us three over two. Now let's take that three over two and plug it into the original function. And we get two times three over two, which is equal to three. And that is exactly what this special property is all about. And it says that if we take the inverse of x, and then we take that inverse of x and take the function of the whole thing, we get x. We've gone through an introduction together, let's apply what we've learned to answer the next three questions. So we're given two functions. The first function g doesn't have any calculations, it's just a set of ordered pairs. So given us the ordered pairs directly, we don't have to do any calculation to find them. The second function age is a function as you're used to seeing it, where we're going to have to perform some calculations. Now the first question is asking us to find the inverse of G. So remember that if we have x, mapping into y, according to G, now we've got the inverse of G where we're mapping from y to x. So g inverse is going to take the y value here, move it to the first position where the domain is. And if y is equal to seven, we're going to map to x equal to negative five. And we're going to do that for each of the ordered pairs. And the squirrely back, it represents a set beginning and the ending of the set. And it's a set of ordered pairs. And we're done. So finding the inverse function, when you're given a set of ordered pairs, is very straightforward, you just switch the positions of the domain and the range. So we're asked to find the inverse of h of x. So I'm going to start it off by writing y is equal to three x plus 13. And now I'm going to solve for x. So I'm going to isolate x on the left hand side of the equation. So I'm going to have three x is equal to y minus 13. And then x is equal to y minus 13 divided by three, then I'm going to replace x with y, and y with x. So I'm going to have y is equal to x minus 13 over three, so the inverse is equal to x minus 13 over three, so that's the inverse of the function. So it started switching the x's and y's around. But that's just because we want to show the inverse function should be a function of x. Our next question is asking for the composite function h composite with its inverse. And it wants to wants us to evaluate that composite function when x is equal to four. So why don't I do this the hard way, and then we can I can show you and remind you how to do it the easy way. So another way of writing In this is to say that we have H inverse of four, and then that that inverse is actually within the original function. I've taken the inverse function we came up with on the previous slide, and brought it with me. And so why don't we do that one first. So H, one of four is going to be equal to four minus 13 divided by three, which is equal to negative nine over three, which is equal to negative three. Now I'm going to use, I'm going to plug that negative three into the original function through three times negative three, plus 13 is equal to negative nine plus 13, which is equal to four. Right? And that's exactly what we should get because we know that the composite of the H function and its inverse I'll delete that little x when it's evaluated at four whether it's x is going to be equal to four. Right? Just like if we didn't have a number to evaluated at, we would get our x value