Essays in Incomplete Information and Trade Policy

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Abstract

Strategic trade policy has been one of the most intensively researched areas in theory of industrial organization and international trade over the last three decades. The fundamental motivation is that governments adopt trade polices to confer strategic advantage to their respective domestic firms when firms are imperfectly competing with each other. However, most of existing literature focuses on markets with certainty and complete information among firms. This dissertation introduces incomplete information at industrial level into uncertainty markets in various trade models, and it also integrates the concept of option value from financial economics into equilibrium analysis.

In Chapter 1, incomplete information at industrial level is introduced into an importing country model in which the domestic market demand is uncertain, and the policy is chosen before the uncertainty is resolved. Unlike the classical findings on the issue of equivalence of tariffs and quotas under certainty and complete information, it is shown that a tariff is superior to a quota regardless of the degree of uncertainty. Moreover, a prohibitive quota that results in autarky is always preferred to a quota at the free-trade level as long as quota is concerned.

Chapter 2 studies the design of trade policies in an uncertain third market with incomplete information. It is shown that the country with firm having information disadvantage tends to choose the direct quantity control, while the country with well-informed firm would use export subsidy (export quota) when the degree of uncertainty is sufficiently high (low).

Finally, Chapter 3 extends the conventional literature on strategic trade policy in reciprocal dumping model to the context that involves market demand uncertainty and incomplete information. Incomplete information at industrial level redistributes the option value associated with better information to the country with well-informed firm. As a result, both governments tend to choose tariffs over export subsidies in the Nash equilibrium of the simultaneous strategic trade policy games under complete and incomplete information. This yields a second best outcome. Moreover, Nash equilibrium outcome is shown to be inferior to free-trade outcome.

Dedicated to my beloved grandfather Yifeng Ning

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Chapter 1

On the Equivalence of Tariffs and Quotas with Incomplete Information

1.1 Introduction

The equivalence/non-equivalence of tariffs and quotas has attracted considerable attention ever since the seminal work by Bhagwati (1965, 1968). Bhagwati (1965) sets up a two-sector general equilibrium model to demonstrate that the equivalence of tariffs and quotas holds if domestic market is perfectly competitive but not if it is monopolized. Subsequently, Shibata (1968) shows that the equivalence can still hold as long as the domestic market is perfectly competitive but imports come from a foreign monopolist.

Although Bhagwati (1965, 1968) and Shibata (1968) recognize the importance of market structure in studying the equivalence between tariffs and quotas, the effect of market structure on the issue of equivalence had not been rigorously analyzed until 1980s, notably by Itoh and Ono (1984). They construct a Bertrand duopoly model with heterogeneous products and show that the domestic price is lower under a tariff than under an equivalent quota. Using a conjectural variation approach under duopolistic quantity competition, Hwang and Mai (1988) show that the equivalence holds only under Cournot equilibrium. The domestic price is higher (lower) under a tariff than the equivalent quota if the market becomes less (more) competitive than Cournot. Similarly, Fung (1989) compares the effects of tariffs and quotas under Cournot-Nash and Stackelberg, and argues that tariffs and quotas can only be equivalent if firms are Cournot-Nash producers with heterogeneous goods. The domestic prices are lower under a tariff than a quota if the domestic firm is a Stackelberg leader, but the two prices are equal if the domestic firm behaves like a Cournot-Nash producer. While most studies primarily focus on the price equivalence of tariffs and quotas, little attention has been given to the equivalence in terms of social welfare. In our opinion, the social welfare equivalence is more important than the price equivalence since the governments consider not only producers but also consumers as a whole in prescribing their policies.

It is clear that the market structure plays an important role in determining the equivalence of tariffs and quotas. Other than the market structure, demand uncertainty can be another important factor that leads to the result of non-equivalence between tariffs and quotas. Weitzman (1974) considers the choice between price and quantity regimes under uncertainty. He shows that the flexibility provided by a price control regime is more desirable with sufficiently high market volatility, but such a flexibility ceases to exist when price control is replaced by quantity control. Fishelson and Flatters (1975) compare tariffs and quotas for a country facing a less than perfectly elastic foreign supply curve. They point out that, even in a perfectly competitive market, the equivalence of tariffs and quotas breaks down when the domestic and the foreign supply and demand are stochastic. They argue that the stochastic behavior of market uncertainty can arise from the random disturbances of supply and demand. Dasgupta and Stiglitz (1977) show a tariff is unambiguously superior to a quota in a general equilibrium model with uncertain demand.

Along the same line, Cooper and Riezman (1989) construct a third-country model based on this insight and show that the choice between export subsidies and export quotas depend on the degree of market uncertainty. Specifically, they demonstrate that export subsidies are potentially more desirable from the point of view of the exporting country with high uncertainty when firms have more information than governments. By contrast, Chen and Hwang (2006) examine a similar issue in the context of an importing country and demonstrate that when firms have more information than the government, flexibility becomes less desirable as degree of market uncertainty rises. In short, a tariff would be preferred to a quota by the importing country when market volatility is sufficiently low and the reverse is true otherwise.

Clearly, both market structure and uncertainty are known to be the two major factors affecting the equivalence between tariffs and quotas.[1] No attention, however, has been given to the role of incomplete or asymmetric information[2] on the issue of equivalence. To highlight this, we assume that the domestic firm has more information about the demand conditions than its foreign rival.

[1] In Chapter 2, I extend Cooper and Riezman (1989) by incorporating incomplete information at industrial level. I show that flexibility is no longer desirable when one firm has more information about the third market than the other firm, and thus export quota becomes a strictly dominant strategy for the country with less informed firm. In Chapter 3, I integrate incomplete information at industrial level into a reciprocal dumping model. On the other hand, Matschke (2003) constructs a screening model with Cournot competition and shows that asymmetric information can influence the equivalence of tariffs and quotas. However, she assumes that domestic government captures the entire quota rent to ensure an interior solution for an optimal quota level. This assumption is quite arbitrary as admitted by the author, and will not be made in this chapter.

[2] In this chapter, we use incomplete information and asymmetric information interchangeably.

The information disadvantage facing the foreign firm could arise because of consumers' preferences that are less transparent or idiosyncratic shocks that are far less predictable to the foreign-based company. The purpose of this chapter is to reexamine the issue of tariff-quota equivalence under incomplete information in a stochastic duopoly model and then compare the social welfare rankings with one fully-informed domestic firm competing with one uninformed foreign firm in the domestic market à la Cournot.

Unlike the findings in Chen and Hwang (2006), we show that the domestic country turns out to have the same amount of option values under tariffs and quotas. Moreover, we show that a tariff is always superior to a quota regardless of market uncertainty. Under incomplete information, a quota at the free trade level can never emerge as an equilibrium outcome. By decomposing the expected social welfare under different policy instruments, we find that flexibility offered by tariffs increases consumer's surplus and tax revenue that more than offsets decrease in producer's surplus when the domestic government moves from a quota to a tariff policy.

This chapter is organized as follows. Section 1.2 outlines our theoretical framework. Section 1.3 derives the expected level of social welfare for both tariff and quota regimes using a game theoretical approach. Section 1.4 ranks the expected social welfare under different policy instruments. Finally, section 1.5 presents the concluding remarks.

1.2 The Model

Following Hwang and Mai (1988), we assume that there are two firms, one domestic firm (denoted as firm 1) and one foreign firm (denoted as firm 2), both of which produce a homogeneous product for the domestic market.[3] These firms are identical except for their country of operation if there is no government intervention. Assume each firm produces final goods at a constant marginal cost of $c > 0$. Profits from the production of q_i units of output for firm i is therefore $\pi_i : q_i \to \mathbb{R}^+ \cup \{0\}$:

$$\pi_i = (p - c)\, q_i,$$

for $i = 1, 2$, and p is the market price of the final goods firm i is selling.

All consumers in question reside in country 1 (i.e., the domestic country). For simplicity, we assume that the domestic government is the sole decision maker in choosing the instruments and levels of trade intervention against the foreign firm, whereas the foreign government is passive. The demand in the domestic country is uncertain, assumed to take the following linear form

$$p = a - bQ + \varepsilon,$$

where $Q = q_1 + q_2$ is the total output produced by the domestic firm (q_1) and the foreign firm (q_2). The parameters a ($> c$) and b are both positive and ε is a random variable which reflects domestic market uncertainty. For illustration purpose, we assume ε could take only two possible values

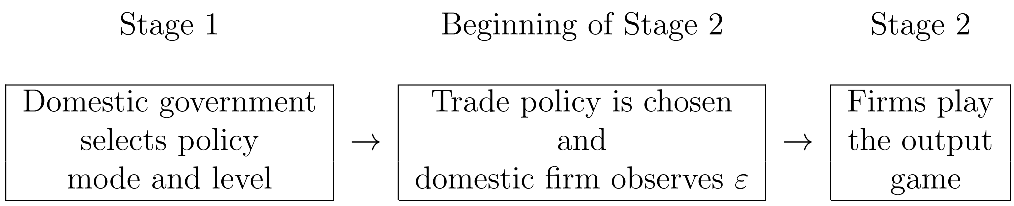
$$\varepsilon \in \Omega \equiv \{-V, V\},$$

where $V \in \mathbb{R}$ and the subjective common prior probability measure over Ω is assumed to be $(\theta, 1 - \theta)$. That is, we assume the bad state $-V$ and the good state V occur with probability θ and $1 - \theta$, respectively.

[3] Our results are readily to be extended with N_1 numbers of domestic firms and N_2 numbers of foreign firms.

The game consists of two stages. In the first stage, the domestic government chooses the optimal policy mode and level in terms of tariff or quota to maximize its expected social welfare level before the realization of ε. At the beginning of stage two, the random variable ε is realized, but the information is only available to the domestic firm. Both firms then select their output to maximize profits given the optimal policy level imposed by the domestic government. Figure 1.1 shows the timing and the partial resolution of uncertainty. In this setting, only the domestic firm has the complete information about the true state of the world, neither the domestic government nor the foreign firm possess this piece of information.

Figure 1.1: Timing of Two-Stage Trade Game with Incomplete Information



Without loss of generality, we assume $\theta = \frac{1}{2}$ in the subsequent analysis. Given this, the expected value of the random variable is $E(\varepsilon) = 0$, and the corresponding variance is $Var(\varepsilon) = V^2 = \sigma^2$. In what follows, a backward induction approach is used to solve the two-stage trade game with incomplete information. Since stage two Cournot game is a strategic game with incomplete information, we shall adopt Bayesian Nash equilibrium concept in solving equilibrium outputs for both firms. By using this solution concept, we can guarantee the equilibrium is a perfect Bayesian equilibrium.

1.3 Subgame Equilibrium

In this section, we derive equilibrium output levels for both firms in stage two using Bayesian Nash equilibrium concept under tariffs and quotas, respectively. The expected social welfare level of the domestic government is then calculated by reverting back to stage one given that the domestic government anticipates Bayesian Nash equilibrium output levels in the subsequent stage.

Given that the domestic firm has full information about the domestic market demand in stage two, we shall not exclude the possibility that the domestic firm may choose to opt out of the market when the realized ε is too low. However, for presentation purpose, we assume that random variable V falls in a range such that the domestic firm's output is positive under tariffs even when realized domestic market demand is low. The welfare ranking under different policy instruments still holds if we relax this assumption. A more detailed analysis of welfare ranking under tariffs and quotas when this assumption of stochastic term V is relaxed can be found in Appendix A.

1.3.1 Import Tariffs

Without loss of generality, assume that the demand is sufficiently high so that both firms remain active under tariffs even if the realized ε is $-V$. As shown below, this happens when $V < \frac{8}{9}(a - c)$.[4]

Suppose the domestic government imposes a tariff on imported goods at a rate of t. Since the domestic firm is able to observe the true state of domestic market demand at the beginning of stage two, the problems for the domestic

[4] The equilibrium analysis under tariff policy for $V \geq \frac{8}{9}(a - c)$ can be found in Appendix A.1.

firm are

$$\max_{q_{1L}^T} \pi_{1L}^T = \left(a - b\left(q_{1L}^T + q_2^T\right) - V\right) q_{1L}^T - cq_{1L}^T, \qquad (1.1)$$
$$\max_{q_{1H}^T} \pi_{1H}^T = \left(a - b\left(q_{1H}^T + q_2^T\right) + V\right) q_{1H}^T - cq_{1H}^T, \qquad (1.2)$$

where q_{1L}^T, q_{1H}^T, π_{1L}^T and π_{1H}^T are output decision and profits of the domestic firm under tariff policy regime (denoted by superscripts T) if the true domestic market demand is low or high, respectively. The foreign firm cannot observe the true market demand in the domestic country, hence its objective function under tariff regime given the common prior is

$$\max_{q_2^T} E\left(\pi_2^T\right) = \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^T + q_2^T\right) + \varepsilon\right) - cq_2^T - tq_2^T, \qquad (1.3)$$

where $q_1^T \in \{q_{1L}^T, q_{1H}^T\}$[5] depend on the realization of ε. The best response functions for the above maximization problems are

$$BR_{1L}(q_2^T) = q_{1L}^T \in \arg\max \pi_{1L}^T,$$
$$BR_{1H}\left(q_2^T\right) = q_{1H}^T \in \arg\max \pi_{1H}^T,$$
$$BR_2\left(q_{1L}^T, q_{1H}^T\right) = q_2^T \in \arg\max E\left(\pi_2^T\right).$$

Given these best response functions, we can obtain the Bayesian Nash equilibrium points for any given tariff t as follows:

$$q_{1L}^T = \frac{1}{3b}(a - c + t) - \frac{V}{2b},$$
$$q_{1H}^T = \frac{1}{3b}(a - c + t) + \frac{V}{2b},$$
$$q_2^T = \frac{1}{3b}(a - c - 2t).$$

[5]Henceforth, we use subscript L and H to denote the output level and profits for the domestic firm in low and high domestic market demand, respectively.

The solutions characterize the Bayesian Nash equilibrium in the domestic market given (t, V). Note that in order to make the domestic firm active when the market demand is low, we must have $V < \frac{2}{3}(a - c + t)$ for any given level of tariff set by the domestic government in the previous stage.[6] Also notice that a higher value of t increases output of the domestic firm, and a higher value of t decreases output of the foreign firm. Furthermore, It is worth noting that the bigger the value of V, the higher (lower) the output of domestic firm in high (low) demand states.

Going back to the first stage, the domestic government maximizes the expected social welfare with respect to tariff, given that the domestic government can fully anticipate output of both firms at Bayesian Nash equilibrium level. The social welfare function of the domestic country under tariff regime is specified as the sum of producer's surplus (profits of the domestic firm), consumer's surplus, and tariff revenue. The domestic government therefore sets the tariff rate so as to maximize the expected social welfare function:

$$\begin{aligned}
\max_t E\left(SW^T\right) &= \frac{1}{2}\left(PS_L^T + CS_L^T\right) + \frac{1}{2}\left(PS_H^T + CS_H^T\right) + TR \\
&= \frac{1}{2}\left(\pi_{1L}^T + \frac{b}{2}\left(q_{1L}^T + q_2^T\right)^2\right) + \frac{1}{2}\left(\pi_{1H}^T + \frac{b}{2}\left(q_{1H}^T + q_2^T\right)^2\right) \\
&+ \; tq_2^T, \qquad (1.4)
\end{aligned}$$

where

$$\begin{aligned}
\pi_{1L}^T &= \frac{1}{36b}\left(2\left(a - c + t\right) - 3V\right)^2, \\
\pi_{1H}^T &= \frac{1}{36b}\left(2\left(a - c + t\right) + 3V\right)^2.
\end{aligned}$$

[6] It is straightforward to show that the domestic firm will always produce in high domestic market demand by our assumption of V and relevant parameters.

Solving yields

$$t^* = \frac{1}{3}(a-c). \tag{1.5}$$

Note that the introduction of uncertainty has no effect on the optimal tariff level. This is not surprising since the domestic government does not know the true state when making the decision by assumption, and the subjective common prior over Ω makes the expected value of random variable zero.[7] Substituting (1.5) into best response function of the domestic firm in low demand market, we can update the critical point of V such that the domestic firm shuts down in low demand state. This gives us $V < \frac{8}{9}(a-c)$ or $\sigma^2 < \frac{64}{81}(a-c)^2$ in terms of variance.

Substituting the optimal tariff into Bayesian Nash equilibrium output levels yields the equilibrium levels of output for both firms under the tariff regime:

$$q_{1L}^T = \frac{4}{9b}(a-c) - \frac{V}{2b}, \tag{1.6}$$

$$q_{1H}^T = \frac{4}{9b}(a-c) + \frac{V}{2b}, \tag{1.7}$$

$$q_2^T = \frac{1}{9b}(a-c). \tag{1.8}$$

For comparison, we now examine the case where both firms are able to observe ε. In this case, the second stage Cournot game becomes strategic game with complete information when there is full information at the industrial level. Both firms now have flexibility by adjusting their output levels based on true market demand under tariff regime. Given this, one can easily verify that the optimal tariff rate is identical to the one given in equation (1.5) and the corresponding optimal output levels (denoted as $'$ for complete information case)

[7] In a more general subjective common prior setting, i.e., a probability measure $(\theta, 1-\theta)$ over uncertainty space, the optimal tariff rate is $t^* = \frac{1}{3}(a - c + E(\varepsilon))$. We can see that the magnitude of optimal tariff is affected by the (subjective) expected value of random variable.

for each firm are

$$\begin{aligned}
q'_{1L} &= \frac{4}{9b}(a-c) - \frac{V}{3b}, \\
q'_{1H} &= \frac{4}{9b}(a-c) + \frac{V}{3b}, \\
q'_{2L} &= \frac{1}{9b}(a-c) - \frac{V}{3b}, \\
q'_{2H} &= \frac{1}{9b}(a-c) + \frac{V}{3b}.
\end{aligned}$$

A comparison with the outputs under incomplete information is now in order. Clearly, the domestic firm tends to be more conservative (aggressive) when the demand is low (high) under incomplete information than under full information. This can be understood as follows. The uninformed foreign firm produces an output that maximizes its expected profits. Such an output is state independent, which is seen in equation (1.8) as a weighted average of q'_{2L} and q'_{2H}. In response to this, the domestic firm is able to take advantage of full information about market conditions by producing relatively less (more) with an intention to capture the "monopoly" rents in bad (good) market. The relative aggressiveness by the domestic firm can therefore be viewed as a strategic response to information disadvantage facing the foreign firm,

It can be easily verified that the expected social welfare for the domestic country under tariff regime with complete information is

$$E(SW') = \frac{7}{18b}(a-c)^2 + \frac{1}{3b}\sigma^2,$$

whereas the corresponding expected social welfare for the domestic country under tariff regime with incomplete information is

$$E\left(SW^T\right) = \frac{7}{18b}(a-c)^2 + \frac{3}{8b}\sigma^2. \tag{1.9}$$

It is straightforward to show that $E\left(SW^{T}\right) - E\left(SW'\right) > 0$ given any level of σ^2. To summarize, we have the following proposition:

Proposition 1.1. *Under tariffs with incomplete information, the informed domestic firm produces less (more) relative to the full information case when the realized demand is low (high), whereas the uninformed foreign firm produces a moderate level of output. Moreover, the domestic firm's information advantage against the foreign firm results in higher expected social welfare for the domestic country.*

It is worth noting that the second term in the expected social welfare expression of the domestic country $E\left(SW^{T}\right)$ is the option value associated with information advantage. The intuition behind this is simple. Being well-informed, the domestic firm is able to fully capture the benefit that results from the flexibility it enjoys. This consequently enhances firm's profits and hence the social welfare. The idea is similar to the option value in financial economics where the option value is positively related to market volatility. To see this, calculate $\frac{\partial E\left(SW^{T}\right)}{\partial \sigma^2} = \frac{3}{8b} > 0$, yielding

Proposition 1.2. *Under tariff regime, higher degree of uncertainty implies higher expected social welfare for the domestic country due to the option value effects.*

Next, we examine the expected social welfare for the domestic country under quotas.

1.3.2 Import Quotas

Instead of imposing a tariff on the imported goods, the domestic government can alternatively choose to limit the quantity of imports through quotas. Our task is to evaluate the effect of such quantity restrictions on firm's behavior and the social welfare.

With no information about true domestic demand, the foreign firm is assumed to produce an output that is set by the domestic government. That is, the quota facing the foreign firm is assumed to be binding. Given this, the domestic firm would, therefore, choose the level of output to maximize its profit after the demand becomes known in stage two. Let $\overline{q}_2^Q$ be the maximum quantity that the foreign firm is permitted to sell in the domestic market, where the superscript Q is used to represent the variable choice under quota. The objective functions for the domestic firm in different states of nature are:

$$\max_{q_{1L}^Q} \pi_{1L}^Q = \left(a - b\left(q_{1L}^Q + \overline{q}_2^Q\right) - V\right) q_{1L}^Q - c q_{1L}^Q, \quad (1.10)$$

$$\max_{q_{1H}^Q} \pi_{1H}^Q = \left(a - b\left(q_{1H}^Q + \overline{q}_2^Q\right) + V\right) q_{1H}^Q - c q_{1H}^Q. \quad (1.11)$$

Solving these optimization problems yields optimal outputs across states under quotas:

$$\begin{aligned} q_{1L}^Q &= \frac{1}{2b}(a - c - V) - \frac{1}{2}\overline{q}_2^Q, \\ q_{1H}^Q &= \frac{1}{2b}(a - c + V) - \frac{1}{2}\overline{q}_2^Q. \end{aligned}$$

The social welfare under the quota regime is defined as the sum of producer's surplus and consumer's surplus. Specifically, we can write the problem for the domestic government as

$$
\begin{aligned}
\max_{\overline{q}_2^Q} E\left(SW^Q\right) &= \frac{1}{2}\left(PS_L^Q + CS_L^Q\right) + \frac{1}{2}\left(PS_H^Q + CS_H^Q\right) \\
&= \frac{1}{2}\left(\pi_{1L}^Q + \frac{b}{2}\left(q_{1L}^Q + \overline{q}_2^Q\right)^2\right) \\
&+ \frac{1}{2}\left(\pi_{1H}^Q + \frac{b}{2}\left(q_{1H}^Q + \overline{q}_2^Q\right)^2\right), \qquad (1.12)
\end{aligned}
$$

where

$$
\begin{aligned}
\pi_{1L}^Q &= \frac{1}{4b}\left(a - c - V - b\overline{q}_2^Q\right)^2, \\
\pi_{1H}^Q &= \frac{1}{4b}\left(a - c + V - b\overline{q}_2^Q\right)^2.
\end{aligned}
$$

The associated first- and second-order conditions for welfare maximization with respect to quotas are

$$
\begin{aligned}
\frac{\partial E\left(SW^Q\right)}{\partial \overline{q}_2^Q} &= \frac{3}{4}b\overline{q}_2^Q - \frac{1}{4}(a-c) = 0, \\
\frac{\partial^2 E\left(SW^Q\right)}{\partial\left(\overline{q}_2^Q\right)^2} &= \frac{3}{4}b > 0.
\end{aligned}
$$

Clearly, the second-order condition for social welfare maximization is violated. Hence, an interior solution does not exist.[8] Given this, the optimal quota level will be either zero or at the free-trade level. To determine which one is better for the domestic country, it requires us to compare the social welfare levels between these two choices.

[8] Eldor and Levin (1990) and Chen and Hwang (2006) also derive similar outcome.

Zero Quota

In this case, $\overline{q}_2^Q = 0$. Given our assumption that $V < \frac{8}{9}(a-c)$, the domestic firm will always be active under zero quota regardless of what the realized demand is. By substitution, q_{1L}^Q and q_{1H}^Q are reduced to (relabelling them by superscript ZQ):

$$q_{1L}^{ZQ} = \frac{1}{2b}\left(a-c-V\right), \tag{1.13}$$
$$q_{1H}^{ZQ} = \frac{1}{2b}\left(a-c+V\right). \tag{1.14}$$

These are the outputs the domestic firm produces under zero quota in bad state and good state, respectively. By substitution, the corresponding social welfare under zero quota is therefore given by

$$\begin{aligned} E\left(SW^{ZQ}\right) &= \frac{1}{2}\left(PS_L^{ZQ} + CS_L^{ZQ}\right) + \frac{1}{2}\left(PS_H^{ZQ} + CS_H^{ZQ}\right) \\ &= \frac{1}{2}\left(\pi_{1L}^{ZQ} + \frac{b}{2}\left(q_{1L}^{ZQ}\right)^2\right) + \frac{1}{2}\left(\pi_{1H}^{ZQ} + \frac{b}{2}\left(q_{1H}^{ZQ}\right)^2\right) \\ &= \frac{3}{8b}\left((a-c)^2 + \sigma^2\right), \end{aligned} \tag{1.15}$$

where

$$\pi_{1L}^{ZQ} = \frac{1}{4b}\left(a-c-V\right)^2,$$
$$\pi_{1H}^{ZQ} = \frac{1}{4b}\left(a-c+V\right)^2.$$

Needless to say, the domestic country is self sufficient under zero quota.[9]

[9] Appendix A.2 discusses the quota regime (zero quota and free-trade level quota) when the assumption of $V < \frac{8}{9}(a-c)$ is relaxed.

Free-Trade Quota

Alternatively, the quota can be set at the expected free-trade level, i.e., the level of output the foreign firm wishes to produce for the domestic market when there is no government intervention of any kind. Obviously, the free-trade level quota, if imposed, is always binding since it is the level the foreign firm wants to export. Given that the foreign import is fixed by quota (which is state-independent), it is likely that the fully informed domestic firm may find it unprofitable to participate in production after the market condition becomes known. This occurs if the bad state of nature $-V$ is sufficiently negative. Our analysis below also takes this special scenario into consideration in addition to the usual case where two firms remain active in the market. As usual, we use superscript FT to indicate the variables under expected free-trade quota regime.

Our task here is to determine the free-trade level quota when it is set. Starting from stage two, the objective functions for the domestic firm in good and bad states are

$$\pi_{1L}^{FT} = \left(a - b\left(q_{1L}^{FT} + q_2^{FT}\right) - V\right) q_{1L}^{FT} - cq_{1L}^{FT}, \tag{1.16}$$

$$\pi_{1H}^{FT} = \left(a - b\left(q_{1H}^{FT} + q_2^{FT}\right) + V\right) q_{1H}^{FT} - cq_{1H}^{FT}. \tag{1.17}$$

Similarly, the problem for the foreign firm given common prior is

$$\max_{q_2^{FT}} E\left(\pi_2^{FT}\right) = \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^{FT} + q_2^{FT}\right) + \varepsilon\right) - cq_2^{FT}, \tag{1.18}$$

where $q_1^{FT} \in \{q_{1L}^{FT}, q_{1H}^{FT}\}$. The best response functions for the above maximization problems are

$$\begin{aligned} BR_{1L}(q_2^{FT}) &= q_{1L}^{FT} \in \arg\max \pi_{1L}^{FT}, \\ BR_{1H}\left(q_2^{FT}\right) &= q_{1H}^{FT} \in \arg\max \pi_{1H}^{FT}, \\ BR_2\left(q_{1L}^{FT}, q_{1H}^{FT}\right) &= q_2^{FT} \in \arg\max E\left(\pi_2^{FT}\right). \end{aligned}$$

Given these best response functions, the Bayesian Nash equilibrium output in absence of any government intervention are

$$q_{1L}^{FT} = \frac{1}{3b}(a-c) - \frac{1}{2b}V, \tag{1.19}$$

$$q_{1H}^{FT} = \frac{1}{3b}(a-c) + \frac{1}{2b}V, \tag{1.20}$$

$$q_2^{FT} = \frac{1}{3b}(a-c). \tag{1.21}$$

The corresponding profits for the domestic firm in good and bad states are

$$\begin{aligned} \pi_{1L}^{FT} &= \frac{1}{36b}\left(2(a-c) - 3V\right)^2, \\ \pi_{1H}^{FT} &= \frac{1}{36b}\left(2(a-c) + 3V\right)^2, \end{aligned}$$

respectively. Clearly, the domestic firm is active in low demand if and only if $V < \frac{2}{3}(a-c)$ or $\sigma^2 < \frac{4}{9}(a-c)^2$ and becomes inactive otherwise. Two cases are considered:

1. $V < \frac{2}{3}(a-c)$

   In this case, both firms remain active. One can obtain the domestic country's social welfare as follows:

$$\begin{aligned}
E\left(SW_A^{FT}\right) &= \frac{1}{2}\left(PS_L^{FT}+CS_L^{FT}\right)+\frac{1}{2}\left(PS_H^{FT}+CS_H^{FT}\right) \\
&= \frac{1}{2}\left(\pi_{1L}^{FT}+\frac{b}{2}\left(q_{1L}^{FT}+q_2^{FT}\right)^2\right) \\
&+ \frac{1}{2}\left(\pi_{1H}^{FT}+\frac{b}{2}\left(q_{1H}^{FT}+q_2^{FT}\right)^2\right) \\
&= \frac{1}{3b}(a-c)^2+\frac{3}{8b}\sigma^2, \qquad (1.22)
\end{aligned}$$

where the subscript A indicates the case which the domestic firm is active even in low market demand.

2. $V \geq \frac{2}{3}(a-c)$

   In this case, the domestic firm is not active when the bad state occurs. That is, the domestic firm sets $q_{1L}^{FT}=0$. The best response functions for firms' problems can be rewritten as

$$\begin{aligned}
BR_{1H,NA}\left(q_{2,NA}^{FT}\right) &= q_{1H,NA}^{FT}\in\arg\max \pi_{1H}^{FT}, \\
BR_{2,NA}\left(q_{1L}^{FT}=0, q_{1H,NA}^{FT}\right) &= q_{2,NA}^{FT}\in\arg\max E\left(\pi_2^{FT}\right),
\end{aligned}$$

   where subscript NA denotes the case when the domestic firm is not active when $-V$ realizes as true state. This yields

$$\begin{aligned}
q_{1H,NA}^{FT} &= \frac{2}{7b}(a-c)+\frac{4}{7b}V, \qquad (1.23) \\
q_{2,NA}^{FT} &= \frac{3}{7b}(a-c)-\frac{1}{7b}V. \qquad (1.24)
\end{aligned}$$

Accordingly, the expected social welfare for the domestic country is given by

$$\begin{aligned} E\left(SW_{NA}^{FT}\right) &= \frac{1}{2}\left(CS_{L,NA}^{FT}\right) + \frac{1}{2}\left(PS_{H,NA}^{FT} + CS_{H,NA}^{FT}\right) \\ &= \frac{1}{2}\left(\frac{b}{2}\left(q_{2,NA}^{FT}\right)^2\right) + \frac{1}{2}\left(\pi_{1H,NA}^{FT} + \frac{b}{2}\left(q_{1H,NA}^{FT} + q_{2,NA}^{FT}\right)^2\right) \\ &= \frac{3}{14b}\left(a-c\right)^2 + \frac{2}{7b}\left(a-c\right)\sigma + \frac{3}{14b}\sigma^2, \qquad (1.25) \end{aligned}$$

where

$$\pi_{1H,NA}^{FT} = \frac{4}{49b}\left(\left(a-c\right) + 2V\right)^2.$$

Zero Quota vs. Free-Trade Quota

Given Cases 1 and 2 above, a straightforward comparison yields the following proposition.

Proposition 1.3. *Under quota with incomplete information against the foreign firm, zero quota is always preferred by the domestic country.*

Proof. We prove the proposition in two parts, depending on whether the domestic firm is active in low demand or not. First, we compare the expected social welfare level of the domestic country between zero quota and free-trade level quota if the domestic firm remains active in low demand. Denote

$$\Delta_1 = E\left(SW^{ZQ}\right) - E\left(SW_A^{FT}\right) = \frac{1}{24b}\left(a-c\right)^2 > 0, \qquad (1.26)$$

implying that zero quota dominates free-trade quota.

Next, compare the expected social welfare for the domestic country between zero quota policy and the free-trade quota policy if the domestic firm is not active in low demand. Calculate

$$\begin{aligned}\Delta_2 &= E\left(SW^{ZQ}\right) - E\left(SW^{FT}_{NA}\right) \\ &= \frac{9}{56b}(a-c)^2 - \frac{2}{7b}(a-c)\sigma + \frac{9}{56b}\sigma^2. \end{aligned} \tag{1.27}$$

Δ_2 appears to be strictly convex since $\frac{\partial \Delta_2}{\partial \sigma} = -\frac{2}{7b}(a-c) + \frac{9}{28b}\sigma \gtreqless 0$ and $\frac{\partial^2 \Delta_2}{\partial \sigma^2} = \frac{9}{28b} > 0$. The minimum of Δ_2 occurs at $\sigma_{\min} = \frac{8}{9}(a-c)$. At $\sigma = \sigma_{\min}$, $\Delta_2(\sigma_{\min}) = \frac{17}{504b}(a-c)^2 > 0$. One can therefore conclude that $\Delta_2 > 0$ for all σ^2. In short, both $\Delta_1 > 0$ and $\Delta_2 > 0$, thus making zero quota a superior choice. The proposition is proven. □

This proposition shows that if the domestic government uses a quota as a policy instrument, it will limit the volume of foreign import to zero, thereby making its own firm a monopolist. This result seems counter intuitive and can be understood by referring to Table 1.1 and Table 1.2. These two tables decompose the expected social welfare of the domestic country into two separate components (consumer's surplus and producer's surplus) under autarky (i.e., zero quota) and a quota at the free-trade level with active and inactive domestic firm in low market demand, respectively.

As seen in Table 1.1, a change from a quota at the free-trade level to autarky increases profits for the domestic firm (or producer's surplus) at the expense of consumers. This is because autarky makes the domestic firm a monopolist, allowing it to earn the monopoly rent. As a result, consumers suffer due to an increase in price, which is captured by the reduction in consumer's surplus. Nevertheless, the increase in producer's surplus more than offsets the decrease in consumer's surplus, resulting in a net increase in social welfare. The same pattern also holds for the case where the domestic firm becomes inactive when the realized demand is too low (see Table 1.2). In other words, prohibitive

Table 1.1: The Decomposition of SW^{ZQ} and SW_A^{FT}: $V < \frac{2}{3}(a-c)$

	Zero Quota	Free-Trade Quota	Δ_1
Producer's Surplus	$\frac{1}{4b}\left((a-c)^2+\sigma^2\right)$	$\frac{1}{9b}(a-c)^2+\frac{1}{4b}\sigma^2$	$\frac{5}{36b}(a-c)^2$
Consumer's Surplus	$\frac{1}{8b}\left((a-c)^2+\sigma^2\right)$	$\frac{2}{9b}(a-c)^2+\frac{1}{8b}\sigma^2$	$-\frac{7}{72b}(a-c)^2$
Social Welfare	$\frac{3}{8b}\left((a-c)^2+\sigma^2\right)$	$\frac{1}{3b}(a-c)^2+\frac{3}{8b}\sigma^2$	$\frac{1}{24b}(a-c)^2$

quota turns out to be the social-welfare-maximizing policy. In our model with asymmetric information, autarky is more desirable than quota at the free-trade level, since option value accrues exclusively to the domestic firm in the zero quota case. This result holds for all σ^2. It is confirmed that in the present case, the loss in consumer's surplus is more than offset by the gain in producer's surplus. This is seen in the last column of Table 1.2 where the welfare of the domestic country is shown to be positively correlated with σ^2.

Note that the second term associated with variance in equation (1.15) is the option value accruing to the domestic country, since the domestic firm, being fully informed, is able to make the output decision after the resolution of uncertainty. It is straightforward to show that $\frac{\partial E\left(SW^{ZQ}\right)}{\partial \sigma^2} > 0$. Hence, an increase in market volatility increases the expected social welfare for the domestic country. This is summarized in the following proposition.

Proposition 1.4. *Under the quota regime, the expected social welfare of the domestic country increases with market volatility.*

Table 1.2: The Decomposition of SW^{ZQ} and SW^{FT}_{NA}: $V \geq \frac{2}{3}(a-c)$

	Zero Quota	Free-Trade Quota	Δ_2
Producer's Surplus	$\frac{1}{4b}\left((a-c)^2+\sigma^2\right)$	$\frac{2}{49b}(a-c+2\sigma)^2$	$\frac{41}{196b}(a-c)^2$ $-\frac{8}{49b}(a-c)\sigma$ $+\frac{17}{196b}\sigma^2$
Consumer's Surplus	$\frac{1}{8b}\left((a-c)^2+\sigma^2\right)$	$\frac{17}{98b}(a-c)^2$ $+\frac{6}{49b}(a-c)\sigma$ $+\frac{5}{98b}\sigma^2$	$-\frac{19}{392b}(a-c)^2$ $-\frac{6}{49b}(a-c)\sigma$ $+\frac{29}{392b}\sigma^2$
Social Welfare	$\frac{3}{8b}\left((a-c)^2+\sigma^2\right)$	$\frac{3}{14b}(a-c)^2$ $+\frac{2}{7b}(a-c)\sigma$ $+\frac{3}{14b}\sigma^2$	$\frac{9}{56b}(a-c)^2$ $-\frac{2}{7b}(a-c)\sigma$ $+\frac{9}{56b}\sigma^2$

1.4 Choice of Policy Regimes

In this section, we examine stage one problem regarding the choice of regime by the domestic government. As discussed in the previous section, autrky (or zero quota) is preferred to a quota at the free-trade level, regardless of the degree of uncertainty. Therefore, we need only compare tariff regime with the autarky.

Subtracting equation (1.15) from equation (1.9) yields

$$\begin{aligned}\Delta_3 &= E\left(SW^T\right) - E\left(SW^{ZQ}\right)\\ &= \left(\frac{7}{18b}(a-c)^2 + \frac{3}{8b}\sigma^2\right) - \left(\frac{3}{8b}\left((a-c)^2+\sigma^2\right)\right)\\ &= \frac{1}{72b}(a-c)^2. \qquad (1.28)\end{aligned}$$

Note that Δ_3 does not contain any option value term. The option value terms in equation (1.15) and equation (1.9) cancels out since tariff and zero quota regimes provide the domestic country with the same portion of the option value. Therefore, we can establish the following proposition.[10]

Proposition 1.5. *In the market with incomplete information under demand uncertainty, the social welfare is unambiguously higher under a tariff than under a quota.*

This result is in sharp contrast to Chen and Hwang (2006) in that the optimal policy is independent of the level of uncertainty. In their model, optimum policy is autarky (i.e., zero quota), quota at the free-trade level and tariff for high, intermediate and low levels of uncertainty respectively. The underlying reason behind their result is that a quota allows the domestic country to limit foreign firm's access to option values, particularly when the degree of uncertainty is high. This is because both foreign and domestic firms are fully informed and a tariff regime results in equal access to option values. However, in the present model with incomplete information, the foreign firm is uninformed about the true state of nature, and thus has no choice but to produce a state-independent output, which excludes it from capturing option value, associated with the ability to make decisions after the resolution of uncertainty. Therefore, the lack of information on the part of the foreign firm itself forces the option value to be redistributed in favor of the domestic firm. In this sense, keeping option values for the domestic firm through quotas can be similarly achieved when the domestic firm has information advantage over the foreign firm. This consequently marginalizes the need for a quota. But if a quota is used, it would be set at zero (see Proposition 1.3).

[10] Appendix A.3 presents the detailed welfare ranking when the assumption of $V < \frac{8}{9}(a-c)$ is relaxed.

When it comes to the choice between autarky and a tariff, it is useful to focus on the relative magnitudes of consumer's surplus, producer's surplus and tax revenues as the policy regime changes from autarky to a tariff. From Table 1.3, it is clear that there is a significant loss in producer's surplus when the market structure changes from a monopoly to a duopoly as the country moves away from zero quota. At the same time, such a move clearly benefits consumers because of a lower output price due to competition. This is seen in Δ_3, the last column of Table 1.3. One can easily verify that the loss in producer's surplus is greater than an increases in consumer's surplus. Nevertheless, as we take into account tarrif revenues $\frac{1}{27b}(a-c)^2$ (see Row 3 of Table 1.3), the country ends up having higher social welfare.

The conventional result that views tariffs better than quotas is based on the argument that tariffs generate revenues for the country, but quotas do not. This holds in the absence of uncertainty. In the present model with a stochastic demand, our result is consistent with this strand of the literature. While the superiority of tariffs over quotas is explained by tariff revenues in the conventional literature, in our case it is driven by the role of information and its effect on the choice of policy regimes.

1.5 Conclusion

With regard to welfare equivalence of tariffs and quotas, it is well-known in the literature that tariffs are superior to quotas in models with no uncertainty. When demand is uncertain, the result becomes ambiguous. Specifically, the optimal policy can be autarky, a quota at the free-trade level, or even a tariff depending on whether the degree of uncertainty is high, medium, or low. The present chapter considers a duopoly model with two firms (domestic and

Table 1.3: Comparison and Decomposition of SW^T and SW^{ZQ}

	Tariff	Zero Quota	Δ_3
Producer's Surplus	$\frac{16}{81b}(a-c)^2 + \frac{1}{4b}\sigma^2$	$\frac{1}{4b}\left((a-c)^2+\sigma^2\right)$	$-\frac{17}{324b}(a-c)^2$
Consumer's Surplus	$\frac{25}{162b}(a-c)^2 + \frac{1}{8b}\sigma^2$	$\frac{1}{8b}\left((a-c)^2+\sigma^2\right)$	$\frac{19}{648b}(a-c)^2$
Tax Revenue	$\frac{1}{27b}(a-c)^2$	0	$\frac{1}{27b}(a-c)^2$
Social Welfare	$\frac{7}{18b}(a-c)^2 + \frac{3}{8b}\sigma^2$	$\frac{3}{8b}\left((a-c)^2+\sigma^2\right)$	$\frac{1}{72b}(a-c)^2$

foreign) competing against each other in the domestic market and the output game involves information asymmetry. The incomplete information adds a new dimension to the issue of welfare equivalence between tariffs and quotas. Our analysis of tariffs and quotas under incomplete information has generated the following novel results:

1. Under tariffs with incomplete information, the well-informed domestic firm produces less (more) relative to the full information case when the realized demand is low (high), whereas the ill-informed foreign firm tends to produce a moderate level of output.

2. Higher degree of uncertainty implies higher expected social welfare for the domestic country due to option value effects, regardless of whether it is a tariff or a quota.

3. With incomplete information, a tariff is superior to a quota for all σ^2.

Our analysis is based on a set of simplified assumptions: two firms (domestic and foreign) with identical marginal cost, the linear demand with additive uncertainty, and a binary common prior over the uncertainty space that each state occurs with equal probability. With these admittedly restrictive assumptions, we are able to derive some interesting results that are new to the literature in our view. Nevertheless, our results can readily be extended to take into account more general cases such as the cost function being non-linear, an oligopoly with more than two firms, and different priors placed over uncertainty space in a dynamic setting. Our conjecture is that tariffs may still be superior as long as incomplete information persists at the industrial level, but we leave it for future research.

Chapter 2

Choice of Trade Policy with Incomplete Information

2.1 Introduction

Strategic trade policy, which combines theories of international trade and industrial organization, has been brought to our attention due to the seminal work by Brander and Spencer (1985). Through these policies, governments influence the behavior of their domestic firms in their subsequent strategic interaction with foreign firms. Strategic trade policy is designed to shift profits towards domestic firms when market in the destination country is imperfectly competitive. Brander and Spencer (1985) show that an export subsidy can shift rents from the foreign to the domestic firm, hence provide a new explanation for the use of export subsidies. Moreover, Krishna (1989) shows that direct quantity constraints on exports can perform a similar role in oligopolistic markets under certain conditions.

A weakness of these results is that these theoretical models constrain the policy instrument selected by governments. For instance, Brander and Spencer

(1985) restrict their attention to export subsidy or tax. Another shortcoming of conventional strategic trade policy models is that they assume that governments have complete information about export markets. However, real policymakers are unlikely to meet the information requirements assumed by theorists. Cooper and Riezman (1989) is the first paper to introduce uncertainty and incomplete information in the context of strategic trade policy.[1] In Cooper-Riezman three-country trade model à la Brander and Spencer (1985), policymakers can use either price incentives (export subsidies) or direct quantity controls (export quotas), but they are not fully informed about demand in the export market. Cooper and Riezman (1989) argue that governments will subsidize domestic firms when the degree of export market uncertainty is high and export quotas when uncertainty is low. In addition, they show that the number of firms plays a critical role in equilibrium. Countries with a large number of firms will tax exports, while countries with fewer firms will subsidize exports in a bilateral subsidy game.

The role of uncertainty in strategic trade policy models have been examined in other contexts as well. Shivakumar (1993) analyzes the importance of demand uncertainty for the optimal choice of trade policy instrument as in Cooper and Riezman (1989). However, Shivakumar (1993)'s focus is the timing of policy implementation, the choice being whether to do it before or after the resolution of uncertainty. Grant and Quiggin (1997) show that whether the equilibrium modes of trade intervention in the presence of uncertainty is a specific, ad valorem or quadratic trade tax, emerges endogenously from the pa-

[1] In Chapter 1, I introduce incomplete information at industrial level in examining social welfare equivalence issue of tariffs and quotas from the importing country's perspective. I show that a tariff is always superior to a quota as long as incomplete information persists at industrial level. Moreover, I also introduces incomplete information into reciprocal dumping model in Chapter 3, and I show that the equilibrium outcome from choosing between tariff and subsidy results a prisoner dilemma outcome.

rameters of the model. They use an equilibrium concept using supply functions proposed by Klemperer and Meyer (1989). Caglayan (2000) extends Cooper and Riezman (1989) framework to the context where firms from both countries have imperfect information about uncertain demand in the third country but receive a signal about the stochastic term. Although, in this case, firms have imperfect information about demand in the destination country, the signal firms received equips them with better information than policymakers.

In addition to the choice of trade policy instruments under uncertainty, which is the main focus of the previous papers, specific policy instrument and information asymmetry have also been considered in the context of strategic trade policy. Qui (1994) compares the impact of complete information and incomplete information on export subsidies in a model where only one government is active, and the cost of a firm is unknown to both government and the rival firm. For Cournot competition, he finds the government prefers to be informed and thus offers complete-information subsidies. Maggi (1999) examines equilibrium trade policies when firms have better information than governments about the profitability of the industry. The main result is that, governments offer non-linear export subsidies inducing firms to reveal information. The firms respond by expanding outputs, thereby adding to the Brander–Spencer prisoner's dilemma problem. Similarly, Creane and Miyagiwa (2008) study an information acquisition model in which firms have incentive to disclose information to the governments in a three-country trade model. They prove that firms disclose demand and cost information to governments in a Cournot competition, and governments are caught in an informational prisoner's dilemma. Anam and Chiang (2000) further extend Brander and Spencer (1985) single export market framework with demand uncertainty to the context of multiple correlated export markets. They demonstrate that when firms engage in quantity competition in two stochastic and positively correlated markets, it may be optimal to tax exports to the more volatile market while subsidizing it in the relatively stable market when firms are risk-averse.

The current chapter investigates the effect of incomplete information at industrial level on choice of trade policies. We model a three-country international trade model à la Brander and Spencer (1985) with demand uncertainty in the third country, which is the destination of the other two countries' exports. Similar to Cooper and Riezman (1989), we focus on the choice of trade policy, assuming governments from exporting countries have no information about demand condition in the destination country. Unlike Cooper and Riezman (1989), we assume that one of the exporting firms has an information advantage over the other. Specifically, we let firm from one of the exporting country have complete information about demand in the third country, while firm from the other country is assumed to be incompletely informed.[2] Our equilibrium results are significantly different from Cooper and Riezman (1989) and Caglayan (2000). We show that direct quantity control becomes the dominant strategy for country with the incompletely-informed firm. By contrast, the equilibrium choice between export subsidy and export quota for the country with the fully informed firm depends on the degree of uncertainty in the third market. We demonstrate that what drives our results is the option value associated with the ability to make decisions armed with more information about the uncertainty parameters.

This chapter is organized as follows. Section 2.2 outlines the basic third-market model and information partition. Section 2.3 derives sub game equilib-

[2]This can be thought as the firm with complete information is a mature firm in exporting its goods to the third country, while the firm with incomplete information is new to export its goods to the third market.

rium for various pairs of strategies. Section 2.4 characterizes and analyzes the optimal choice of policy regimes. Finally, section 2.5 concludes the chapter.

2.2 The Model

Following Brander and Spencer (1985), we assume that there are two firms, one domestic (denote as firm 1) and one foreign (denote as firm 2), producing a homogeneous product exclusively for exports.[3] The export competition takes place in a neutral third country where the demand is subject to some random disturbances. For simplicity, assume that the inverse demand function in this export market is

$$p = a - b\left(q_1 + q_2\right) + \varepsilon,$$

where q_1 and q_2 represent the export of the domestic and foreign firms respectively. The parameters a and b are both positive and ε is a random variable, defined over a finite set Ω, which reflects stochastic demand conditions. For tractability, it is assumed that the disturbance term ε is binary, which takes only two possible values: $\varepsilon \in \Omega = \{\varepsilon_l, \varepsilon_h\}$, where $\varepsilon_h > \varepsilon_l$ and $\varepsilon_l, \varepsilon_h \in \mathbb{R}$. The subjective common prior for ε_l and ε_h to occur are θ and $1-\theta$, respectively. Hence the expected value of random variable is $E(\varepsilon) = \theta\varepsilon_l + (1-\theta)\varepsilon_h$, and the variance of random variable is $var\left(\varepsilon\right) = E\left(\varepsilon^2\right) - \left(E\left(\varepsilon\right)\right)^2 = \sigma^2$.

The cost of production for firm i $(i = 1, 2)$ is assumed to be linear in output, i.e.,

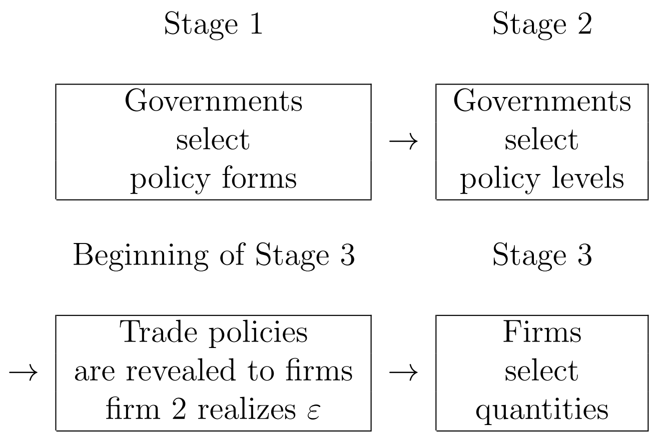
$$C_i = cq_i.$$

Following Cooper and Riezman (1989), our model consists of three stages. In stage one, each government commits to a policy instrument before the real-

[3] Our model is readily to be extended to have N_1 number of firms in the domestic country and N_2 number of firms in the foreign country as in Cooper and Riezman (1989).

ization of random variable ε. The levels of the policy instruments are then set in the second stage, again before the realization of ε. The values of ε becomes known at the beginning of stage three. Unlike the Cooper-Riezman's model, we assume that ε is fully observable to the foreign firm, but such information is not available to the home firm. However, the home firm does know the distribution of ε.[4] That is, the home firm faces information disadvantage in competing with the foreign firm in export competition. This permits us to gain some insights into the effect of incomplete information on policy choices by each country. Both firms play a Cournot-Nash game and set outputs to maximize profits under incomplete information in stage three, given optimal policies chosen by the governments in previous stages. Since the foreign firm sets output after observing realized market demand, it stands to capture the option value associated with being able to wait for the resolution of uncertainty. Figure 2.1 shows the timing of moves and the (partial) resolution of uncertainty.

Figure 2.1: Three-stage Trade Game with Incomplete Information



Before getting into the heart of present model, it may be useful to consider the case where there is no government interventions and firms can fully observe the market condition when ε is realized. Under this limiting case, firm i's profit

[4]This is reflected in our common prior assumption.

is therefore

$$\pi_i = pq_i - cq_i,$$

for $i = 1, 2$. One can easily verify that the Cournot-Nash equilibrium of output is

$$q_i^* = \frac{a - c + \varepsilon}{3b},$$

for $i = 1, 2$. For the output to be non-negative, it requires $a - c + \varepsilon \geq 0$. This benchmark case serves as a basis for comparison when the governments seek to shift profits in favor of their native firms by providing an export subsidy (see Brander and Spencer (1985)).

In what follows, we construct a three-stage perfect Bayesian equilibria for various policy regimes using backward induction. Subsidy and quota games are examined in sequence and their welfare implications are compared. By using this approach, we can guarantee that the equilibrium is a perfect Bayesian equilibrium.

2.3 Equilibrium of Subgames

In this section, we derive equilibrium levels of output produced by firms and levels of government intervention using Bayesian Nash solution concept in each subgame (stage two and stage three). In other words, we derived the equilibrium points when two governments choose the following four combinations of forms of intervention:

i. (S, S): both governments grant export subsidy;

ii. (Q, Q): both governments impose export quota;

iii. (Q, S): government 1 imposes export quota while government 2 provides export subsidies;

iv. (S, Q): government 1 grants export subsidy while government 2 imposes export quotas.

Denote $q_i\,(a_1, a_2)$ as firm i's decision on output level as a reaction to a pair of strategy in forms of intervention chosen by both governments, where $a_1 \in A_1 = \{S, Q\}$ and $a_2 \in A_2 = \{S, Q\}$.[5] For each of the following subsections, expected social welfare for both countries are also derived.

2.3.1 Bilateral Subsidy Game: (S,S)

We first consider the game, where subsidy is committed to by governments as the instrument of protection, in stage one. The solution through backward induction starts in stage three when ε becomes known to firm 2.

The game involves two firms and has two states of nature (ε_l and ε_h) that are asymmetrically revealed to the firms involved. The possible actions of each player are the amount of outputs (or exports) which is defined over $[0, \infty)$. Technically, it is a strategic game with incomplete information (or Bayesian game). In what follows, we characterize the Bayesian Nash equilibrium. Specifically, each firm maximize its (expected) profit by setting quantities given conjectures on the quantities chosen by its rival. In a Bayesian Nash equilibrium, these conjectures will be confirmed.

Let s_1 and s_2 be the amount of subsidy given by governments 1 and 2, respectively, to their own firm. Being unable to observe the true state of nature,

[5] A_i is the set of actions for government i in the stage one game. We use S stands for export subsidy and Q stands for export quota.

firm 1 thus chooses $q_1(S,S)$ to maximize the expected profits, given by

$$\begin{aligned} E(\pi_1(S,S)) &= \theta \left(\left(a - b\left(q_1(S,S) + q_2^l(S,S)\right) + \varepsilon_l\right) q_1(S,S)\right) \\ &+ (1-\theta) \left(\left(a - b\left(q_1(S,S) + q_2^h(S,S)\right) + \varepsilon_h\right) q_1(S,S)\right) \\ &- cq_1(S,S) + s_1 q_1(S,S), \end{aligned} \tag{2.1}$$

where θ and $1-\theta$ are the subjective probabilities associated with ε_l and ε_h.

On the other hand, firm 2 is able to observe the true state of nature once it is realized. Hence, it can make its production decision according to the demand condition. If $\varepsilon = \varepsilon_l$, firm 2 chooses $q_2^l(S,S)$ to maximize its profit:

$$\begin{aligned} \pi_2^l(S,S) &= \left(a - b\left(q_1(S,S) + q_2^l(S,S)\right) + \varepsilon_l\right) q_2^l(S,S) \\ &- cq_2^l(S,S) + s_2 q_2^l(S,S). \end{aligned} \tag{2.2}$$

Conversely, if $\varepsilon = \varepsilon_h$, firm 2 sets $q_2^h(S,S)$ so as to maximize

$$\begin{aligned} \pi_2^h(S,S) &= \left(a - b\left(q_1(S,S) + q_2^h(S,S)\right) + \varepsilon_l\right) q_2^h(S,S) \\ &- cq_2^h(S,S) + s_2 q_2^h(S,S). \end{aligned} \tag{2.3}$$

The best response functions for the above maximization problems are

$$\begin{aligned} BR_1(q_2^l, q_2^h) &= q_1 \in \arg\max E(\pi_1(S,S)), \\ BR_{2l}(q_1) &= q_2^l \in \arg\max \pi_2^l(S,S), \\ BR_{2h}(q_1) &= q_2^h \in \arg\max \pi_2^h(S,S). \end{aligned}$$

Given these, we obtain the Bayesian Nash equilibrium points as follows:

$$
\begin{aligned}
q_1(S,S) &= \frac{a-c+2s_1-s_2}{3b} + \frac{E(\varepsilon)}{3b}, \\
q_2^l(S,S) &= \frac{a-c-s_1+2s_2}{3b} + \frac{\varepsilon_l}{2b} - \frac{E(\varepsilon)}{6b}, \\
q_2^h(S,S) &= \frac{a-c-s_1+2s_2}{3b} + \frac{\varepsilon_h}{2b} - \frac{E(\varepsilon)}{6b}.
\end{aligned}
$$

These equations characterize the Bayesian Nash equilibrium points in market 3 for given values of $(s_1, s_2, \varepsilon_l, \varepsilon_h, \theta)$. Notice that higher values of s_1 leads increasing output by country 1's firm, while higher values of s_2 from its rival country causes firm 1's output to fall. This is due to the fact that governments recognize increasing export subsidy levels lead to output expansions by their firms and output reductions by rival firms.[6]

In stage two, both governments maximize their expected social welfare by choosing export subsidy levels, and they take into account the responses from firms (completely and incompletely informed) in stage three. That is, a government's objective is to choose a value for the ex post subsidy that maximizes the expected value of its firm's profit net of subsidies since we can ignore consumer's surplus due to absence of domestic consumers. This implies that income distribution is not an important determinant of social welfare for each country.

Information partition for both countries indicates both countries have no information about true demand in country 3, and country 1 only anticipates its firm has one reaction function of output level in the following stage (due to lack of information). We can write expected social welfare for country 1 as

$$E(SW_1(S,S)) = E(\pi_1(S,S)) - s_1 q_1(S,S), \tag{2.4}$$

[6] E.g., traditional view on profit shifting motivation as in Brander and Spencer (1985).

where

$$E(\pi_1(S,S)) = \frac{(a - c + 2s_1 - s_2 + E(\varepsilon))^2}{9b}.$$

On the other hand, country 2 knows its firm will have complete information about true demand in market 3, then country 2's expected social welfare given the common prior $(\theta, 1-\theta)$ on Ω can be written as

$$\begin{aligned} E(SW_2(S,S)) &= \theta \left(\pi_2^l(S,S) - s_2 q_2^l(S,S)\right) \\ &+ (1-\theta)\left(\pi_2^h(S,S) - s_2 q_2^h(S,S)\right), \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} \pi_2^l(S,S) &= \frac{(2a - 2c - 2s_1 + 4s_2 + 3\varepsilon_l - E(\varepsilon))^2}{36b}, \\ \pi_2^h(S,S) &= \frac{(2a - 2c - 2s_1 + 4s_2 + 3\varepsilon_h - E(\varepsilon))^2}{36b}. \end{aligned}$$

Similar to solving Bayesian Nash equilibrium points in the third stage, each country maximize its expected social welfare by setting export subsidy levels given conjectures on the levels chosen by the other government in the second stage.

The best response functions for stage two game are

$$\begin{aligned} BR_1(s_2) &= s_1 \in \arg\max E(SW_1(S,S)), \\ BR_2(s_1) &= s_2 \in \arg\max E(SW_2(S,S)). \end{aligned}$$

Solving yields the Bayesian Nash equilibrium level of subsidy rate as

$$s_1^* = s_2^* = \frac{a-c}{5} + \frac{E(\varepsilon)}{5}. \tag{2.6}$$

Note that $s_i > 0$ for $i = 1, 2$. That is, both governments choose to subsidize their own firms. It is worth noting that the equilibrium export subsidy levels are symmetric for both governments even with incomplete information at industrial level. This is because governments 1 and 2 hold the same prior beliefs about random variable ε and $q_1(S,S) = \theta q_2^l(S,S) + (1-\theta)(q_2^h(S,S))$. Also note that when $E(\varepsilon) = 0$, the equilibrium export subsidy is reduced the export subsidy obtained by Cooper and Riezman (1989) even though information symmetry is assumed in their paper.

Substituting equation (2.6) into the expected social welfare functions for both governments, we get

$$E(SW_1(S,S)) = \frac{2((a-c)+E(\varepsilon))^2}{25b}, \quad (2.7)$$

$$E(SW_2(S,S)) = \frac{2((a-c)+E(\varepsilon))^2}{25b} + \frac{\sigma^2}{4b}. \quad (2.8)$$

Notice that $E(SW_2(S,S))$ is the sum of firm's profits net of export subsidy and the variance term (which is absent in $E(SW_1(S,S))$). This indicates that higher variance of ε benefits the country with better information (i.e., country 2 in our model). This can be seen by calculating $\frac{\partial E(SW_2(S,S))}{\partial \sigma^2} = \frac{1}{4b} > 0$ and $E(SW_2(S,S)) - E(SW_1(S,S)) = \frac{1}{4b}(\sigma^2) > 0$. Obviously, if there is no uncertainty in demand (hence no information problem), we obtain the classical result of Brander and Spencer (1985). This gives us

Proposition 2.1. *Under a bilateral subsidy game, the difference in expected social welfare between two countries is the option value effect enjoyed by the country with well-informed firm. Moreover, the expected social welfare for country with better informed firm increases with market volatility.*

Having more information at industrial level does not always benefit the foreign firm. This occurs especially when $\varepsilon = \varepsilon_l$. Specifically, firm 2's expected profit is lower (higher) than firm 1's expected profit if the true market demand in country 3 is low (high). To see this, calculate

$$\begin{aligned} E(\pi_1(S,S)) &= \frac{4(a-c+E(\varepsilon))^2}{25b}, \\ E(\pi_2^l(S,S)) &= \frac{(4a-4c+5\varepsilon_l-E(\varepsilon))^2}{100b}, \\ E(\pi_2^h(S,S)) &= \frac{(4a-4c+5\varepsilon_h-E(\varepsilon))^2}{100b}. \end{aligned}$$

It can be easily verified that $E\left(\pi_2^l\left(S,S\right)\right) < E\left(\pi_1\left(S,S\right)\right) < E\left(\pi_2^h\left(S,S\right)\right)$. This is because firm 1 with incomplete information always produces the moderate level of output (i.e., a weighted average output level given $(\theta, 1-\theta)$ over Ω). Nevertheless, when the true demand is low, firm 2 will be forced to produce less than expected, given that firm 1 is incapable of reacting to it for lack of information. Conversely, when the true demand is high, firm 2 will respond by producing much more to take advantage of good market conditions, given that it anticipates no action from firm 1.

It is also worth noting the differences among $E(\pi_2^l(S,S))$, $E(\pi_1(S,S))$ and $E(\pi_2^h(S,S))$ depend on the common prior $(\theta, 1-\theta)$ over Ω. For any fixed parameters, if firm 1 is more pessimistic (i.e., higher θ), it will lose more profit relative to firm 2 in high demand. But if firm 1 is more optimistic (i.e., lower θ), the opposite is true.

2.3.2 Bilateral Quota Game: (Q,Q)

The equilibrium analysis under bilateral export quota subgame is easy to analyze. In this case, governments impose export level restrictions for their own firms in the second stage. For simplicity, assume that export quota levels are binding for both firms in both countries. That is, both firms will produce positive outputs in stage three, given export restrictions set by their home governments in stage two.

Each government sets an export ceiling to the third market for their respective firms in stage two. Since the actions taken by both governments occur before the resolution of random variable, each government chooses $\overline{q}_i\,(Q,Q)$ for $i=1,2$ so as to maximize their expected social welfare given by

$$\begin{aligned} E(\pi_1(Q,Q)) &= \theta\left(\left(a-b\left(\overline{q}_1(Q,Q)+\overline{q}_2(Q,Q)\right)+\varepsilon_l\right)\overline{q}_1(Q,Q)\right) \\ &+ (1-\theta)\left(\left(a-b\left(\overline{q}_1(Q,Q)+\overline{q}_2(Q,Q)\right)+\varepsilon_h\right)\overline{q}_1(Q,Q)\right) \\ &- c\overline{q}_1(Q,Q), \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} E\left(\pi_2\left(Q,Q\right)\right) &= \theta\left(\left(a-b\left(\overline{q}_2\left(Q,Q\right)+\overline{q}_1\left(Q,Q\right)\right)+\varepsilon_l\right)\overline{q}_2\left(Q,Q\right)\right) \\ &+ (1-\theta)\left(\left(a-b\left(\overline{q}_2\left(Q,Q\right)+\overline{q}_1\left(Q,Q\right)\right)+\varepsilon_h\right)\overline{q}_2\left(Q,Q\right)\right) \\ &- c\overline{q}_2\left(Q,Q\right), \end{aligned} \tag{2.10}$$

respectively. The best response functions for above optimization problems are

$$\begin{aligned} BR_1(\overline{q}_2\left(Q,Q\right)) &= \overline{q}_1\left(Q,Q\right) \in \arg\max E(\pi_1(Q,Q)), \\ BR_2(\overline{q}_1\left(Q,Q\right)) &= \overline{q}_2\left(Q,Q\right) \in \arg\max E(\pi_2(Q,Q)). \end{aligned}$$

Solving yields the Bayesian Nash equilibrium level of quota:

$$\overline{q}_1^*\left(Q,Q\right) = \overline{q}_2^*\left(Q,Q\right) = \frac{a-c}{3b} + \frac{E\left(\varepsilon\right)}{3b}. \tag{2.11}$$

Substituting equation (2.11) into each firm's expected profit functions, we obtain the expected social welfare for both countries as follows:

$$E\left(SW_1\left(Q,Q\right)\right)=E\left(SW_2\left(Q,Q\right)\right)=\frac{\left(\left(a-c\right)+E\left(\varepsilon\right)\right)^2}{9b}. \tag{2.12}$$

It is worth noting that both government end up with same level of social welfare under export quota game. In this case, there is no option value to be had for firm 2 even though it has more information about the market condition than firm 1 does. With output being state-independent under quota, the well-informed firm loses the option of setting output according to the market conditions and therefore, there is no option value effect associated with information advantage for firm 2.

Next, we turn to examine equilibrium for mixed games.

2.3.3 Export Quota vs. Export Subsidy: (Q,S)

So far we have derived subgame equilibrium when both governments choose symmetric strategies. We now consider the scenarios that governments 1 and 2 choose to intervene with asymmetric policy instruments. Suppose that government 1 chooses direct quantity control and government 2 decides to subsidize its own firm. Under this circumstance, government 1 imposes an output constraint on firm 1, while government 2 selects the amount of subsidy for firm 2 in stage one. Given our assumption that a quota is binding, this limits what firm 1 can sell to the third market. Letting $\overline{q}_1\left(Q,S\right)$ be the quota imposed on firm 1 by government 1, firm 2's profits for $\varepsilon=\varepsilon_h$ and $\varepsilon=\varepsilon_l$ are

$$\begin{aligned}\pi_2^l\left(Q,S\right) &= \left(a-b\left(\overline{q}_1\left(Q,S\right)+q_2^l\left(Q,S\right)\right)+\varepsilon_l\right)q_2^l\left(Q,S\right)\\ &- cq_2^l\left(Q,S\right)+s_2q_2^l\left(Q,S\right),\end{aligned} \tag{2.13}$$

$$\pi_2^h(Q,S) = \left(a - b\left(\overline{q}_1(Q,S) + q_2^h(Q,S)\right) + \varepsilon_l\right) q_2^h(Q,S) - cq_2^h(Q,S) + s_2 q_2^h(Q,S). \quad (2.14)$$

Therefore, in stage three, firm 2 chooses $q_2^l(Q,S)$ to maximize $\pi_2^l(Q,S)$ and $q_2^h(Q,S)$ to maximize $\pi_2^h(Q,S)$.

The best response functions for firm 2's problems are

$$BR_{2l}(\overline{q}_1(Q,S)) = q_2^l(Q,S) \in \arg\max \pi_2^l(Q,S),$$
$$BR_{2h}(\overline{q}_1(Q,S)) = q_2^h(Q,S) \in \arg\max \pi_2^h(Q,S).$$

Solving yields

$$q_2^l(Q,S) = \frac{a - c + s_2 - b\overline{q}_1(Q,S) + \varepsilon_l}{2b},$$
$$q_2^h(Q,S) = \frac{a - c + s_2 - b\overline{q}_1(Q,S) + \varepsilon_h}{2b},$$

given any level of s_2.

Given these, government 1 then chooses the export quota level while government 2 selects optimal subsidy level in stage two. For our third market model, the expected social welfare for country 1 can therefore be written as

$$E(SW_1(Q,S)) = \theta\left(\left(a - b\left(\overline{q}_1(Q,S) + q_2^l(Q,S)\right) + \varepsilon_l\right)\overline{q}_1(Q,S)\right) + (1-\theta)\left(\left(a - b\left(\overline{q}_1(Q,S) + q_2^h(Q,S)\right) + \varepsilon_h\right)\overline{q}_1(Q,S)\right) - c\overline{q}_1(Q,S). \quad (2.15)$$

Similarly, the expected social welfare for country 2 is specified as producer's surplus net of subsidy, given by

$$\begin{aligned} E\left(SW_2\left(Q,S\right)\right) &= \theta\left(\pi_2^l\left(Q,S\right) - s_2 q_2^l\left(Q,S\right)\right) \\ &+ \left(1-\theta\right)\left(\pi_2^h\left(Q,S\right) - s_2 q_2^h\left(Q,S\right)\right), \end{aligned} \tag{2.16}$$

where

$$\begin{aligned} \pi_2^l\left(Q,S\right) &= \frac{\left(a-c+s_2-b\overline{q}_1\left(Q,S\right)+\varepsilon_l\right)^2}{4b}, \\ \pi_2^h\left(Q,S\right) &= \frac{\left(a-c+s_2-b\overline{q}_1\left(Q,S\right)+\varepsilon_h\right)^2}{4b}. \end{aligned}$$

Government 1 chooses $\overline{q}_1\left(Q,S\right)$ to maximize its expected social welfare, while government 2 chooses s_2 to maximize its expected social welfare simultaneously. The best response functions are

$$\begin{aligned} BR_1(s_2) &= \overline{q}_1\left(Q,S\right) \in \arg\max E\left(SW_1\left(Q,S\right)\right), \\ BR_2(\overline{q}_1\left(Q,S\right)) &= s_2 \in \arg\max E\left(SW_2\left(Q,S\right)\right). \end{aligned}$$

Solving yields Bayesian Nash equilibrium policy level:

$$\overline{q}_1^*\left(Q,S\right) = \frac{a-c}{2b} + \frac{E\left(\varepsilon\right)}{2b}, \tag{2.17}$$

$$s_2^* = 0. \tag{2.18}$$

Hence, the optimal policy for government 2 is not to intervene at all. The reason behind this result is that firm 2 knows what firm 1 will produce given the quota is binding and it can also fully observe the true market demand and act accordingly. Profit maximization by firm 2 ensures social welfare maximization for the entire country. Therefore, there is no role for government 2 to play. This explains $s_2^* = 0$.

Substituting $s_2^* = 0$ and $\overline{q}_1^*(Q, S)$ into firm 2's best response function, we get the expected output for firm 2 in equilibrium:

$$E(q_2^l(Q,S)) = \frac{a-c}{4b} + \frac{\varepsilon_l}{2b} - \frac{E(\varepsilon)}{4b}, \tag{2.19}$$

$$E(q_2^h(Q,S)) = \frac{a-c}{4b} + \frac{\varepsilon_h}{2b} - \frac{E(\varepsilon)}{4b}. \tag{2.20}$$

The corresponding social welfare for each country under (Q, S) are

$$E(SW_1(Q,S)) = \frac{((a-c)+E(\varepsilon))^2}{8b}, \tag{2.21}$$

$$E(SW_2(Q,S)) = \frac{((a-c)+E(\varepsilon))^2}{16b} + \frac{\sigma^2}{4b}. \tag{2.22}$$

Note that the second term in country 2's social welfare is the option value associated with better information. It is easy to verify that the expected social welfare for country 2 increases as market volatility increases. Moreover, it is straightforward to obtain

$$E(SW_2(Q,S)) - E(SW_1(Q,S)) > (<)\ 0,$$

if and only if $\sigma^2 > (<) \frac{((a-c)+E(\varepsilon))^2}{4}$. This implies that country 2 is better off (worse off) relative to country 1 when the degree of uncertainty is sufficiently high (low).

2.3.4 Export Subsidy vs. Export Quota: (S,Q)

Here, we consider the last pair of strategies (S, Q) that are selected by two governments in the first stage: with government 1 chooses subsidy and government

2 imposes direct quantity control. As usual, our analysis starts from the last stage. Firm 1 does not know the true market demand in the third market, but it can however observe the export quota level $\overline{q}_2(S,Q)$ set by government 2. Firm 1's expected profit function is

$$\begin{aligned} E(\pi_1(S,Q)) &= \theta((a-b(q_1(S,Q)+\overline{q}_2(S,Q))+\varepsilon_l)\,q_1(S,Q)) \\ &+ (1-\theta)((a-b(q_1(S,Q)+\overline{q}_2(S,Q))+\varepsilon_h)\,q_1(S,Q)) \\ &- cq_1(S,Q)+s_1q_1(S,Q). \end{aligned} \tag{2.23}$$

Being unable to observe the realized demand, firm 1 produces a fixed state-independent output for the third market, given $\overline{q}_2(S,Q)$. Thus, it chooses $q_1(S,Q)$ to maximize its profit and the best response function is

$$BR_1(\overline{q}_2(S,Q)) = q_1(S,Q) \in \arg\max E(\pi_1(S,Q)).$$

Solving yields

$$q_1(S,Q) = \frac{a-c+s_1-b\overline{q}_2(S,Q)}{2b} + \frac{E(\varepsilon)}{2b},$$

given any level of s_1.

In stage two, government 1 sets the subsidy level s_1 to maximize its expected social welfare (specified as producer's surplus net of subsidy) given by

$$E(SW_1(S,Q)) = E(\pi_1(S,Q)) - s_1q_1(S,Q), \tag{2.24}$$

where

$$E(\pi_1(S,Q)) = \frac{(a-c+s_1+E(\varepsilon)-b\overline{q}_2(S,Q))^2}{4b}.$$

At the same time, government 2 selects the export quota $\overline{q}_2(S,Q)$ to maximize its social welfare (specified as producer's surplus):

$$\begin{aligned} E(SW_2(S,Q)) &= \theta\left(\left(a - b\left(q_1(S,Q) + \overline{q}_2(S,Q)\right) + \varepsilon_l\right)\overline{q}_2(S,Q)\right) \\ &+ (1-\theta)\left(\left(a - b\left(q_1(S,Q) + \overline{q}_2(S,Q)\right) + \varepsilon_h\right)\overline{q}_2(S,Q)\right) \\ &- c\overline{q}_2(S,Q). \end{aligned} \tag{2.25}$$

The best response functions for the above optimization problems are

$$\begin{aligned} BR_1(\overline{q}_2(S,Q)) &= s_1 \in \arg\max E(SW_1(S,Q)), \\ BR_2(s_1) &= \overline{q}_2(S,Q) \in \arg\max E(SW_2(S,Q)). \end{aligned}$$

Solving yields Bayesian Nash equilibrium policy rate:

$$s_1^* = 0, \tag{2.26}$$

$$\overline{q}_2^*(S,Q) = \frac{(a-c)}{2b} + \frac{E(\varepsilon)}{2b}. \tag{2.27}$$

Surprisingly, the optimal policy regime is symmetric as in (Q,S). Here, government 1 now grants no subsidy to firm 1 if government 2 imposes an export quota on firm 2. This can be explained as follows. Government 1 makes its choice based on its conjecture on government 2's best response. If there is no deviation from government 2's actual choice, then the Bayesian Nash equilibrium holds in stage two. Since government 1 and firm 1 have same information partition over Ω and firm 1 is well informed about the export quota imposed by government 2 at the beginning of stage three, firm 1 actually knows what firm 2 will produce. In this case, firm 2 loses the option of responding to market conditions since its output is fixed by the quota and is therefore no longer state contingent. Therefore, government 1 would leave full decision to its own firm (i.e., firm 1) since profit maximization ensures social welfare maximization in the third market setting. In short, government 1 would choose no subsidy.

Substituting stage two equilibrium points into firm 1's best response function, we get the expected output by firm 1 in stage three:

$$E(q_1 (S, Q)) = \frac{a-c}{4b} + \frac{E(\varepsilon)}{4b}. \tag{2.28}$$

The corresponding expected social welfare for both countries under (S, Q) are

$$E(SW_1 (S, Q)) = \frac{((a-c) + E(\varepsilon))^2}{16b}, \tag{2.29}$$

$$E(SW_2 (S, Q)) = \frac{((a-c) + E(\varepsilon))^2}{8b}. \tag{2.30}$$

This together with the finding under (Q, Q) imply that the export quota essentially eliminates firm's ability to react to market conditions after the resolution of uncertainty. Hence no option value available for the country with the better-informed firm. This can be seen in $E(SW_2(Q, Q))$ and $E(SW_2(S, Q))$ that have no σ^2 term. This gives us

Proposition 2.2. *The export quota eliminates option values for the country with the better-informed firm.*

We are now ready to endogenize the choice of policy regimes in stage one. To this end, a normal form game is constructed next.

2.4 Choice of Trade Policy

The equilibrium analysis for each sub-game in the previous section indicates that different forms of intervention do not lead to the same welfare outcome. We can now pose the obvious question: what will the governments choose in stage one when the foreign firm has better information about the market condition over the domestic firm? We answer this question with the help of normal form game outlined below.

Let a n-tuple $G \equiv < N, (A_i), (SW_i), f, (\succeq_i) >$ be a strategic game of choice of trade policy. The specification of the game G is as follows:

i. the finite set of players N consist of two players: government 1 and 2;

ii. for each government $i \in N$, the set of actions $A_i \equiv \{Subsidy, Quota\}$;

iii. the set SW_i represents the set of expected social welfare for government i;

iv. a function $f : A_i \rightarrow SW_i$ associates consequences with action profiles;

v. for each government $i \in N$, the preference relation of government i is $\succeq_i$ over SW_i.[7]

We introduce a set of expected social welfare SW here because each government cares about its social welfare, but not about the profiles of export subsidy or export quota level that generate that social welfare level.

Normal form representation of game G is illustrated in Table 2.1. Government 1 is the row player and government 2 is the column player. The first expression in each cell is the expected social welfare level for country 1 and the second expression in each cell is the expected social welfare level for country 2.

[7]For example, an action $a_i \succeq a_i'$ if and only if $f(a_i, a_{-i}) \succeq f(a_i', a_{-i})$ $\forall a_i, a_i' \in A_i$.

Table 2.1: Normal form representation of game G

Government 1	Government 2: Subsidy	Quota
Subsidy	$\frac{2((a-c)+E(\varepsilon))^2}{25b}, \frac{2((a-c)+E(\varepsilon))^2}{25b}+\frac{\sigma^2}{4b}$	$\frac{((a-c)+E(\varepsilon))^2}{16b}, \frac{((a-c)+E(\varepsilon))^2}{8b}$
Quota	$\frac{((a-c)+E(\varepsilon))^2}{8b}, \frac{((a-c)+E(\varepsilon))^2}{16b}+\frac{\sigma^2}{4b}$	$\frac{((a-c)+E(\varepsilon))^2}{9b}, \frac{((a-c)+E(\varepsilon))^2}{9b}$

In order to illustrate Nash equilibrium of game G, we adopt the rationalizability concept by Pearce (1984) for presentation purpose. It follows that the game G is dominant solvable. We begin by proving the following lemma.

lemma 2.1. *Subsidy is a strictly dominated strategy for government 1.*

Proof. See Appendix B.1 □

Therefore, the *subsidy* will never be chosen by government 1. In other words, government 1 will always prefer to choose the *quota* irrespective of the policy choice of government 2. In view of *subsidy* being a strictly dominated strategy for government 1, government 2 will respond by choosing a quota (subsidy) if $\sigma^2 \leq (\geq) \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$. This is proven in the following lemma.

lemma 2.2. *The optimal strategy for government 2 depends on the variance over* Ω*. Specifically, a quota (subsidy) is chosen if* $\sigma^2 \leq (\geq) \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$.

Proof. See Appendix B.2 □

As we can see, for a small variance, setting *Quota* becomes the optimal strategy for government 2. Since neither governments has information advantage over each other, there is no option value to country 2 and both countries end up having the same expected level of social welfare. On the other hand, for sufficiently large variance, choosing *Subsidy* becomes the optimal strategy for government 2.[8] The option value accounts for a significant part of social welfare for country 2. By lemmas 2.1 and 2.2, we have the following proposition.

Proposition 2.3. *Nash equilibrium of game G is*

i. (Q, Q) *if* $\sigma^2 \leq \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$;

ii. (Q, S) *if* $\sigma^2 \geq \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$.

Our result is in sharp contrast to Cooper and Riezman (1989) and Caglayan (2000). In their studies, both firms have complete information about demand condition in the third market, and any of four pairs of strategies can become Nash equilibrium trade policy choice, depending on the market volatility. Here, we show that *Quota* becomes the dominant strategy for country with the less informed firm. Hence the possibility of getting (S, S) and (S, Q) are eliminated. As the flexibility provided by setting subsidy can only be beneficial to the country with a well-informed firm, incomplete information itself redistributes the option value from the less-informed to the well-informed country. But the option value is accrued to the country with a better informed firm only if the variance is sufficiently large.

2.5 Conclusion

The literature on the choice of strategic trade policy in oligopolistic industry under uncertainty points to a variety of combinations of modes of intervention that could emerge as an equilibrium outcome in a three-country setting. When we

[8]When $\sigma^2 = \frac{7}{36}\left((a-c)+E(\varepsilon)\right)^2$, government 2 is indifferent between choosing quota and subsidy.

consider policymakers choosing between export subsidy/tax and export quota, the optimal choice of trade policy depends on the degree of uncertainty in the third market, and all four combinations of subsidy and quota could be equilibrium under demand uncertainty. The novelty of this chapter is that it analyzes the optimal choice of strategic trade policy in a third market model when information is incomplete across the duopoly exporters to the third market.

Our principle results can be summarized as follows. In a choice of trade policy game, export subsidy is a dominated strategy for country with incompletely informed firms. In other words, imposing direct quantity control is always optimal for country with ill-informed firm irrespective of what form of interventions is chosen by the other country. This holds for any degree of market uncertainty. For country with completely informed firm, the optimal choice of trade policy depends on the volatility of market demand in the third market. If the market in the third country is relatively stable, imposing export quota is optimal. Conversely, if market demand in the third country is relatively volatile, subsidy turns out to be the optimal choice. This is driven by the option value effect associated with better information.

Our analysis rests heavily on a number of simplifying assumptions. Firstly, the specification of the demand structure is linear with an additive shock. A more general setting with respect to the demand function could be introduced. Secondly, the source of uncertainty is unique in our theoretical framework. We only consider the single source of uncertainty in market demand. In addition to country specific shocks, one may introduce a richer environment of uncertainty by taking firm specific shocks into consideration. Thirdly, we impose a common prior assumption on beliefs for all players involved. Further research could consider a more general setting by allowing different prior beliefs about the states of nature. Lastly, we have assumed that consumers only reside in the third country. This assumption is only appropriate if the good is produced solely for export or if the domestic market can be isolated from trade policies by the use of consumption subsidies/taxes. This assumption simplifies our analysis since consumer's surplus is excluded from social welfare calculation. A more realistic setting may include the domestic consumption of goods produced by the domestic firm.

Chapter 3

Strategic Trade Policy in Reciprocal Dumping Model with Incomplete Information

3.1 Introduction

Strategic trade policy has been one of the most intensively researched areas of international trade over the last three decades following the path-breaking seminal work by Brander and Spencer (1985). In their work, both governments adopt trade polices to confer strategic advantage to their respective domestic firm when firms are imperfectly competing with each other. The essence of this literature explains how these strategic trade policies are beneficial at the national level. Further research along this line examines a wide variety of alternative scenarios which involves (but not limited to): the nature of oligopolistic competition in terms of conjectural variation and conduct parameters, available trade policy instruments, information structure, timing and leadership structure, etc.[1]

Most early papers along this line of research in strategic trade policy adopt the third-market setting: the entire output of the rival oligopolistic firms are exported to a market other than the one where firms themselves are located. These papers exclusively focus on how governments can shift oligopoly rents by using export subsidy in favor of their respective domestic firms. (e.g., see Cooper and Riezman (1989), Shivakumar (1993) and Caglayan (2000)). On the other hand, in the so-called reciprocal market model, first analyzed by Brander and Krugman (1983), domestic and foreign firms are assumed to compete in each other's market. The typical case, where firms engage in Cournot competition in homogeneous product gives rise to intra-industry trade of reciprocal dumping variety. An implication of this setting for strategic trade policy is that consumer's surplus becomes significantly important in determining social welfare for governments. Brander and Spencer (1985) and Dixit (1984) are the pioneering papers that analyze equilibrium in strategic trade policy in the deterministic reciprocal market. In the reciprocal dumping model, import restriction becomes an additional viable policy regime. Nash equilibrium levels of subsidy game and tariff game usually involves positive subsidy and tariff rate. However, the welfare ranking of the policy regimes is generally ambiguous.

Although the so-called third market model is a useful simplification for isolating rent-shifting motive for strategic trade policy, reciprocal dumping model is apparently the more realistic scenario. Important policy issues such as export subsidies and countervailing duties can only be analyzed in the reciprocal dumping model. However, a limitation of the literature to date is that policy making

[1] For a complete survey of these early literature incorporating extensions in strategic trade policy, see Brander (1995).

in reciprocal market is analyzed with no uncertainty. Most recently, Anam and Chiang (2018) extend Cooper and Riezman (1989) to reciprocal markets that are interdependent, and they replace the constant marginal cost assumption with quadratic cost functions. They show that quadratic cost functional form has significant impact on the sign of export subsidy, and the market correlation plays an important role in determining Nash equilibrium in choosing optimal policy regime.

Unlike Anam and Chiang (2018), this chapter retains basic assumption of constant marginal cost and segmented markets, but the added value of this chapter is to introduce incomplete information at industrial level.[2] In order to highlight the impact of uncertainty, the simplest framework for the reciprocal market is assumed. There is one firm in the home and foreign countries producing a homogeneous good at constant marginal cost. The firms compete in both markets as Cournot duopolists, similar to Brander (1981). The two markets are segmented, and in one market demand is deterministic while in the other demand is stochastic.[3] In stage one, both governments are assumed to simultaneously commit to either a tariff or an export subsidy policy before the resolution of uncertainty. In stage two, the level of each policy instrument is set again before random variable is revealed. In order to highlight the role of incomplete information at industrial level, we discuss two possible information partitions: (i) both firms are able to observe the realized demand, and (ii) only

[2] I introduce incomplete information at industrial level in examining social welfare equivalence issue of tariff and quota from the importing country's perspective in Chapter 1. I demonstrate that a tariff is always superior to a quota as long as incomplete information persists at industrial level. In Chapter 2, I extend Cooper and Riezman (1989) by incorporating incomplete information at industrial level in a third market model. I show that flexibility is no longer desirable when one firm has more information about the third market than the other firm, and thus export quota becomes a strictly dominant strategy for the country with less informed firm.

[3] The asymmetric demand might due to political conditions in each country. For example, the stochastic demand could reflects the prevailing political instability in that country.

the domestic firm can observe true state of nature. The true market demand is revealed according to different information partition at the beginning of stage three. Finally, in stage three, the firms set their profit-maximizing output in both markets taking the present strategic trade policy level as given. As in the conventional analysis, both governments arrive at the Nash strategic trade policy equilibrium by backward induction. An important feature of this setup (in both information partition cases) is that governments and the firms stand to capture option value associated with the ability to make strategic decisions after the resolution of uncertainty.

We construct a game in which each government chooses the policy regime between tariffs and export subsidies in stage one. This gives rise to four possible scenarios: subsidy v.s. subsidy (S, S), tariff v.s. tariff (T, T), tariff v.s. subsidy (T, S) and subsidy v.s. tariff (S, T). The Bayesian Nash equilibrium levels of each policy is determined in stage two. The firms (one domestic and the other foreign) then compete against each other. The multi-stage game is then solved by backward induction. Two different information partitions (complete information and incomplete information at industrial level) are considered for each pair of strategies. National welfare associated with each policy combination is then used to characterize the choice of policy regimes.

Several interesting results emerge from our analysis. We show that Nash equilibrium export subsidy dominates the corresponding tariff equilibrium for both information partitions. Our analysis demonstrates that incomplete information at industrial level redistributes the option value associated with better information. Incomplete information is shown to shift option value to the country with the more informed firm. As in the optimal strategic choice of trade policy games, we show that (T, T) is the unique Nash equilibrium policy combination for both trade games with complete and incomplete information.

Moreover, the trade games with complete and incomplete information have a prisoner dilemma property since (S, S) is a Pareto outcome. Furthermore, we also show that the Bayesian Nash equilibrium outcome is inferior to the outcome without any government interventions.

This chapter is organized as follows. Section 3.2 presents the basic reciprocal dumping model. Section 3.3 derives the sub-game equilibrium for the trade game with incomplete information. Section 3.4 characterizes and analyzes the optimal choice of policy regimes. Section 3.5 relates our results to free-trade outcome. Finally, section 3.6 concludes this chapter.

3.2 The Model

Consider a two-country (home and foreign) model in which each country has only one firm producing a homogeneous good. These firms are identical except for their country of operation if there is no government intervention. For computational as well as expositional ease, we assume that the demand function in each country is linear with constant slope b. In order to highlight the effect of incomplete information, we assume the demand in country 1 is uncertain, but the demand in country 2 is deterministic. Thus, the inverse demand functions for countries 1 and 2 are given by

$$p_1 = a - b\,(q_1 + x_2) + \varepsilon,$$

and

$$p_2 = a - b\,(q_2 + x_1)\,,$$

where p_i is the commodity price in country i, q_i is the delivery of the local firm to the local market, x_j is the export of firm j to market i, and ε is a random disturbance term in the domestic market demand for all $i, j = 1, 2$ and $i \neq j$. Following the convention, country 1 is the home country and country 2 is the foreign country.

For simplicity, we assume the random variable ε is binary. Specifically,

$$\varepsilon \in \Omega \equiv \{-V, V\},$$

where $V \in \mathbb{R}$. The subjective common prior probability measure over Ω is assumed to be $(\theta, 1-\theta)$. In other words, we assume that the bad state $-V$ occurs with probability θ and the good state V occurs with probability $1-\theta$, respectively.

Assume each firm produces final goods at a constant marginal cost of $c > 0$. The cost function for firm i is therefore assumed to be

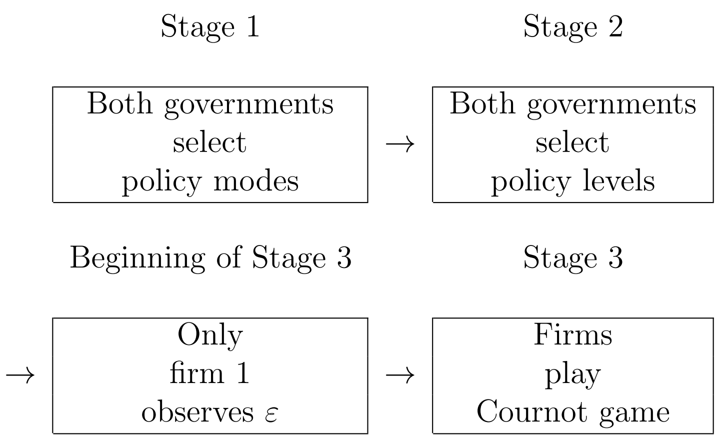
$$C_i = c\,(q_i + x_i),$$

for $i = 1, 2$. Homogeneous product with identical cost structure implies that the trade is entirely of the intra-industry type.

Following Anam and Chiang (2018), our trade game consists of three stages. In stage one, each government commits to a policy instrument to be either export subsidy or tariff. The corresponding levels of the policy regimes are then set in stage two once the specific trade policy is prescribed in the previous stage. At the beginning of stage three, the random variable ε is either revealed to both firms (trade game of complete information) or only to the domestic firm (trade game of incomplete information). Both firms then play a Cournot game by setting outputs (both for local and foreign delivery) to maximize their profits, given the optimal policies chosen by the governments in previous stages. To ease presentation, the equilibrium analysis for stage sub-game under complete information is relegated to the Appendix C. In the main text, we exclusively focus our analysis on the stage game under incomplete information.

Figure 3.1 shows the timing of uncertainty resolution for the trade game under incomplete information. Note that the random variable ε is realized at the beginning of stage three. The random variable, once realized, is only observed by firm 1. Given that the better informed firm would stand to capture option values at the expense of the less informed firm, the implication that comes out of the game under incomplete information is expected to be substantially different from that under perfect information. Since the random variable becomes known at the beginning of stage three, both governments make their policy choices without having any knowledge of true demand in the home market.

Figure 3.1: Timing of Three-Stage Trade Game with Incomplete Information



Our model differs from Anam and Chiang (2018) in three ways. Firstly, in their model, the cost function for each firm is assumed to be quadratic. The increasing marginal cost in their model has significant impact on the levels of trade policies.[4] In our model, however, the market equilibrium is separable given segmented markets and constant marginal costs in order to highlight

[4]Specifically, quadratic cost function can make optimal export subsidy level negative.

the role of information. Secondly, they assume identical market demand in both countries while we assume asymmetric market demand. Lastly, they only consider the complete information trade game while we also conduct equilibrium analysis for incomplete information at industrial level.

Without loss of generality, we assume $\theta = \frac{1}{2}$ in the subsequent analysis. Given this, the expected value of random variable is $E(\varepsilon) = 0$, and its variance is $Var(\varepsilon) = \sigma^2 = V^2$. This simplified assumption allows us to see clearly the effect of market volatility on the choice of policy regime.

In what follows, we derive the perfect Bayesian equilibrium for various policy regimes by backward induction, beginning with stage three.

3.3 Subgame Equilibrium

In this section, we derive equilibrium output levels for both firms in stage three for trade game with incomplete information under various trade policies. The expected social welfare level for both governments is then calculated by reverting back to previous stages given that both governments anticipate equilibrium output levels in the last stage. We use Bayesian Nash solution concept in solving equilibrium output levels for trade game with incomplete information. The complete derivation of subgame equilibrium for trade game with complete information at industrial level can be found in the Appendix C. We use superscript CI to denote variable choices for trade game with complete information, and we use superscript II to denote variable choices for trade game with incomplete information. We also use (a_1, a_2) to denote pair of strategies each government chooses in the first stage, where $a_i \in A_i \equiv \{S, T\}$ for $i = 1, 2$.[5]

[5] For example, (S, T) represents that government 1 chooses export subsidy and government 2 imposes tariff.

Next, we examine the Bayesian-Nash equilibrium for our three-stage game for four policy combinations: (S,S), (T,T), (T,S), and (S,T).

3.3.1 Subsidy Game (S,S)

We start with the subsidy game in which both governments choose to subsidize their firms' exports in the first stage. The domestic firm, being able to observe the true demand in market 1, solves the following maximization problem:

$$\begin{aligned} \max_{q_{1L}^{II}(S,S),x_1^{II}(S,S)} \pi_{1L}^{II}(S,S) &= \left(a - b\left(q_{1L}^{II}(S,S) + x_2^{II}(S,S)\right) - V\right) q_{1L}^{II}(S,S) \\ &+ \left(a - b\left(q_2^{II}(S,S) + x_1^{II}(S,S)\right)\right) x_1^{II}(S,S) \\ &- c\left(q_{1L}^{II}(S,S) + x_1^{II}(S,S)\right) + s_1 x_1^{II}(S,S), \end{aligned} \tag{3.1}$$

if $\varepsilon = -V$ and

$$\begin{aligned} \max_{q_{1H}^{II}(S,S),x_1^{II}(S,S)} \pi_{1H}^{II}(S,S) &= \left(a - b\left(q_{1H}^{II}(S,S) + x_2^{II}(S,S)\right) + V\right) q_{1H}^{II}(S,S) \\ &+ \left(a - b\left(q_2^{II}(S,S) + x_1^{II}(S,S)\right)\right) x_1^{II}(S,S) \\ &- c\left(q_{1H}^{II}(S,S) + x_1^{II}(S,S)\right) + s_1 x_1^{II}(S,S), \end{aligned} \tag{3.2}$$

if $\varepsilon = V$, where s_1 is the subsidy granted by government 1, and the subscripts L and H stand for the variable choice for each firm in the respective states.[6] Nevertheless, the foreign firm, being unable to observe the true state of nature

[6]Henceforth, we use subscript L to represent variable choice when the random variable is $-V$, and we use subscript H to represent variable choice when the random variable is V.

in market 1, solves the following maximization problem:

$$\begin{aligned}
\max_{q_2^{II}(S,S),x_2^{II}(S,S)} E\left(\pi_2^{II}(S,S)\right) &= \left(a-b\left(q_2^{II}(S,S)+x_1^{II}(S,S)\right)\right)q_2^{II}(S,S) \\
&+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a-b\left(q_1^{II}(S,S)+x_2^{II}(S,S)\right)+\varepsilon\right) \\
&\times x_2^{II}(S,S) \\
&- c\left(q_2^{II}(S,S)+x_2^{II}(S,S)\right) \\
&+ s_2 x_2^{II}(S,S), \qquad (3.3)
\end{aligned}$$

where $q_1^{II}(S,S) \in \left\{q_{1L}^{II}(S,S), q_{1H}^{II}(S,S)\right\}$ depends on the realization of ε and s_2 is the subsidy given by the foreign government.

The best response functions for above optimization problems are

$$\begin{aligned}
BR_{1L} &= q_{1L}^{II}(S,S) \in \arg\max \pi_{1L}^{II}(S,S), \\
BR_{1H} &= q_{1H}^{II}(S,S) \in \arg\max \pi_{1H}^{II}(S,S), \\
BR_{1x} &= x_1^{II}(S,S) \in \arg\max \pi_{1L}^{II}(S,S), \\
BR_2 &= q_2^{II}(S,S) \in \arg\max \pi_2^{II}(S,S), \\
BR_{2x} &= x_2^{II}(S,S) \in \arg\max \pi_2^{II}(S,S).
\end{aligned}$$

Solving these equations simultaneously yields Bayesian Nash equilibrium points for stage three game given any (s_1, s_2, V):

$$\begin{aligned}
q_{1L}^{II}(S,S) &= \frac{1}{6b}\left(2(a-c) - 3V - 2s_2\right), \\
q_{1H}^{II}(S,S) &= \frac{1}{6b}\left(2(a-c) + 3V - 2s_2\right), \\
x_1^{II}(S,S) &= \frac{1}{3b}\left(a - c + 2s_1\right), \\
q_2^{II}(S,S) &= \frac{1}{3b}\left(a - c - s_1\right), \\
x_2^{II}(S,S) &= \frac{1}{3b}\left(a - c + 2s_2\right).
\end{aligned}$$

It is evident that a higher export subsidy provided by the domestic government increases the export of the domestic firm to the foreign market, but a higher export subsidy given by the foreign government to its own firm lowers the local delivery of the domestic firm to the domestic market. This is due to profit shifting as noted in Brander and Spencer (1985).

Folding back to the second stage, both governments then set subsidy rates to maximize their respective social welfare functions, given that they anticipate the Bayesian Nash equilibrium output level in the subsequent stage. The social welfare functions for both governments under the subsidy game are the sum of producer's surplus and consumer's surplus net of total subsidy, given by

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{II}(S,S)\right) &= PS_1^{II}(S,S) + CS_1^{II}(S,S) - S_1^{II}(S,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{II}(S,S) + \pi_{1H}^{II}(S,S)\right) \\
&+ \frac{1}{2}\Big(\frac{b}{2}\left(q_{1L}^{II}(S,S) + x_2^{II}(S,S)\right)^2 \\
&+ \frac{b}{2}\left(q_{1H}^{II}(S,S) + x_2^{II}(S,S)\right)^2\Big) \\
&- s_1 x_1^{II}(S,S), \qquad (3.4)
\end{aligned}$$

$$
\begin{aligned}
\max_{s_2} E\left(SW_2^{II}(S,S)\right) &= PS_2^{II}(S,S) + CS_2^{II}(S,S) - S_2^{II}(S,S) \\
&= \pi_2^{II}(S,S) + \frac{b}{2}(q_2^{II}(S,S) + x_1^{II}(S,S))^2 \\
&- s_2 x_2^{II}(S,S). \qquad (3.5)
\end{aligned}
$$

The best response functions of stage two game are

$$
\begin{aligned}
BR_1(s_2) &= s_1 \in \arg\max E\left(SW_1^{II}(S,S)\right), \\
BR_2(s_1) &= s_2 \in \arg\max E\left(SW_2^{II}(S,S)\right).
\end{aligned}
$$

Solving yields Bayesian Nash equilibrium level of export subsidies

$$
s_1^* = s_2^* = \frac{1}{4}(a-c). \qquad (3.6)
$$

Clearly, the expected level of Bayesian Nash equilibrium subsidy is the same for both countries mainly because both governments make their policy choices on the basis of the distribution of ε. With no surprise, these optimal subsidy rates are identical to the solutions under perfect information (see equation (C.7) in Appendix C.1).

By substitution, we can obtain the expected output levels as follows:

$$
\begin{aligned}
E\left(q_{1L}^{II}(S,S)\right) &= \frac{1}{4b}(a-c) - \frac{1}{2b}V, \\
E\left(q_{1H}^{II}(S,S)\right) &= \frac{1}{4b}(a-c) + \frac{1}{2b}V, \\
E\left(x_1^{II}(S,S)\right) &= \frac{1}{2b}(a-c), \\
E\left(q_2^{II}(S,S)\right) &= \frac{1}{4b}(a-c), \\
E\left(x_2^{II}(S,S)\right) &= \frac{1}{2b}(a-c).
\end{aligned}
$$

Now we are able to compare these output levels for the subsidy game under incomplete information with the output levels under complete information. Note that $E\left(x_2^{II}\left(S,S\right)\right)$ is the weighted average of $E\left(x_{2L}^{CI}\left(S,S\right)\right)$ and $E\left(x_{2H}^{CI}\left(S,S\right)\right)$[7], given our assumption that each state occurs with equal probability. Without being able to observe the true market demand in the domestic country, the foreign firm now can only supply a state-independent export to the domestic market. As a consequence, the domestic firm tends to deliver lower (higher) local output in low (high) market demand compare to the complete information case under subsidy game. This can be seen as $E\left(q_{1L}^{II}\left(S,S\right)\right) < E\left(q_{1L}^{CI}\left(S,S\right)\right)$ and $E\left(q_{1H}^{II}\left(S,S\right)\right) > E\left(q_{1H}^{CI}\left(S,S\right)\right)$. Since the foreign market involves no uncertainty, there is no change in the foreign firm's local delivery and the domestic firm's export to the foreign market.

Given optimal subsidy levels with incomplete information, the expected social welfare for both countries under subsidy game with incomplete information are

$$E\left(SW_1^{II}\left(S,S\right)\right) = \frac{15}{32b}\left(a-c\right)^2 + \frac{3}{8b}\sigma^2, \quad (3.7)$$

$$E\left(SW_2^{II}\left(S,S\right)\right) = \frac{15}{32b}\left(a-c\right)^2. \quad (3.8)$$

It is worth noting that now the expected social welfare for the foreign country contains no term associated with option value. The incomplete information at industrial level shifts the entire option value to the domestic country. As the domestic firm is able to make output decisions after it observes the random variable, incomplete information for the foreign firm enables the domestic firm to fully capture the option value. As a result, the domestic country ends up with higher expected social welfare relative to the foreign country due to option

[7]See Appendix C.1 for expected output levels under subsidy game with complete information.

value effects. This can be seen from $E\left(SW_1^{II}\left(S,S\right)\right)-E\left(SW_2^{II}\left(S,S\right)\right)=\frac{3}{8b}\sigma^2$. Moreover, it can be easily verified that $\frac{\partial E\left(SW_1^{II}(S,S)\right)}{\partial\sigma^2}>0$.[8] This gives us the following proposition.

Proposition 3.1. *For export subsidy game with incomplete information at industrial level,*

i. the entire option value shifts to the domestic country due to incomplete information against the foreign firm;

ii. social welfare for the domestic country increases as variance increases;

iii. the domestic country ends up with higher social welfare due to option value effects.

Next, we turn to examine sub game equilibrium with incomplete information under tariffs.

3.3.2 Tariff Game (T,T)

Under tariffs, the domestic firm solves the following maximization problem:

$$\begin{aligned}\max_{q_{1L}^{II}(T,T),x_1^{II}(T,T)}\pi_{1L}^{II}\left(T,T\right) &= \left(a-b\left(q_{1L}^{II}\left(T,T\right)+x_2^{II}\left(T,T\right)\right)-V\right)q_{1L}^{II}\left(T,T\right)\\ &+ \left(a-b\left(q_2^{II}\left(T,T\right)+x_1^{II}\left(T,T\right)\right)\right)x_1^{II}\left(T,T\right)\\ &- c\left(q_{1L}^{II}\left(T,T\right)+x_1^{II}\left(T,T\right)\right)-t_2x_1^{II}\left(T,T\right), \quad (3.9)\end{aligned}$$

[8] A similar result obtained by Cooper and Riezman (1989), Chen and Hwang (2006) and Anam and Chiang (2018).

if $\varepsilon = -V$ and

$$\begin{aligned}\max_{q_{1H}^{II}(T,T),x_1^{II}(T,T)} \pi_{1H}^{II}(T,T) &= \left(a - b\left(q_{1H}^{II}(T,T) + x_2^{II}(T,T)\right) + V\right) q_{1H}^{II}(T,T) \\ &+ \left(a - b\left(q_2^{II}(T,T) + x_1^{II}(T,T)\right)\right) x_1^{II}(T,T) \\ &- c\left(q_{1H}^{II}(T,T) + x_1^{II}(T,T)\right) - t_2 x_1^{II}(T,T), \quad (3.10)\end{aligned}$$

if $\varepsilon = V$, where t_2 is the tariff imposed by the foreign government on exports from the domestic firm.

On the contrary, the foreign firm maximizes the expected profit function given a common prior:

$$\begin{aligned}\max_{q_2^{II}(T,T),x_2^{II}(T,T)} E\left(\pi_2^{II}(T,T)\right) &= \left(a - b\left(q_2^{II}(T,T) + x_1^{II}(T,T)\right)\right) q_2^{II}(T,T) \\ &+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^{II}(T,T) + x_2^{II}(T,T)\right) + \varepsilon\right) \\ &\times x_2^{II}(T,T) \\ &- c\left(q_2^{II}(T,T) + x_2^{II}(T,T)\right) \\ &- t_1 x_2^{II}(T,T), \quad (3.11)\end{aligned}$$

where $q_1^{II}(T,T) \in \left\{q_{1L}^{II}(T,T), q_{1H}^{II}(T,T)\right\}$ depend on the realization of ε, and t_1 is the tariff imposed by the domestic government on exports from the foreign firm.

The corresponding best response functions for above maximization problems are

$$\begin{aligned} BR_{1L} &= q_{1L}^{II}(T,T) \in \arg\max \pi_{1L}^{II}(T,T), \\ BR_{1H} &= q_{1H}^{II}(T,T) \in \arg\max \pi_{1H}^{II}(T,T), \\ BR_{1x} &= x_{1}^{II}(T,T) \in \arg\max \pi_{1L}^{II}(T,T), \\ BR_{2} &= q_{2}^{II}(T,T) \in \arg\max \pi_{2}^{II}(T,T), \\ BR_{2x} &= x_{2}^{II}(T,T) \in \arg\max \pi_{2}^{II}(T,T). \end{aligned}$$

Solving yields Bayesian Nash equilibrium output levels given any (t_1, t_2, V):

$$\begin{aligned} q_{1L}^{II}(T,T) &= \frac{1}{6b}\left(2(a-c) - 3V + 2t_1\right), \\ q_{1H}^{II}(T,T) &= \frac{1}{6b}\left(2(a-c) + 3V + 2t_1\right), \\ x_{1}^{II}(T,T) &= \frac{1}{3b}\left(a - c - 2t_2\right), \\ q_{2}^{II}(T,T) &= \frac{1}{3b}\left(a - c + t_2\right), \\ x_{2}^{II}(T,T) &= \frac{1}{3b}\left(a - c - 2t_1\right). \end{aligned}$$

Going backward to the second stage, each government chooses a tariff rate t_i to maximize its expected social welfare given that each government anticipates firms' strategic behavior in the last stage. The expected social welfare for each government is specified as the sum of producer's surplus, consumer's surplus, and tariff revenue. Given the common prior assumption, we can write each government's problem as

$$\begin{aligned}\max_{t_1} E\left(SW_1^{II}(T,T)\right) &= PS_1^{II}(T,T)+CS_1^{II}(T,T)+TR_1^{II}(T,T)\\ &= \frac{1}{2}\left(\pi_{1L}^{II}(T,T)+\pi_{1H}^{II}(T,T)\right)\\ &+ \frac{1}{2}(\frac{b}{2}\left(q_{1L}^{II}(T,T)+x_2^{II}(T,T)\right)^2\\ &+ \frac{b}{2}\left(q_{1H}^{II}(T,T)+x_2^{II}(T,T)\right)^2)\\ &+ t_1x_2^{II}(T,T), \end{aligned} \tag{3.12}$$

$$\begin{aligned}\max_{t_2} E\left(SW_2^{II}(T,T)\right) &= PS_2^{II}(T,T)+CS_2^{II}(T,T)+TR_2^{II}(T,T)\\ &= \pi_2^{II}(T,T)+\frac{b}{2}(q_2^{II}(T,T)+x_1^{II}(T,T))^2\\ &+ t_2x_1^{II}(T,T). \end{aligned} \tag{3.13}$$

The best response functions for stage two game are

$$\begin{aligned}BR_1(t_2) &= t_1 \in \arg\max E\left(SW_1^{II}(T,T)\right),\\ BR_2(t_1) &= t_2 \in \arg\max E\left(SW_2^{II}(T,T)\right).\end{aligned}$$

Hence the Bayesian Nash equilibrium policy rates are

$$t_1^* = t_2^* = \frac{1}{3}(a-c). \tag{3.14}$$

The optimal tariff rates set by the governments are the same under incomplete information. They are again identical to that under complete information (see Appendix C.2) due to no change in information partition at the national level. Given these optimal tariff rates, we can get the expected output levels in stage three as follows:

$$\begin{aligned}
E\left(q_{1L}^{II}(T,T)\right) &= \frac{4}{9b}(a-c)-\frac{1}{2b}V,\\
E\left(q_{1H}^{II}(T,T)\right) &= \frac{4}{9b}(a-c)+\frac{1}{2b}V,\\
E\left(x_{1}^{II}(T,T)\right) &= \frac{1}{9b}(a-c),\\
E\left(q_{2}^{II}(T,T)\right) &= \frac{4}{9b}(a-c),\\
E\left(x_{2}^{II}(T,T)\right) &= \frac{1}{9b}(a-c).
\end{aligned}$$

Similar to the argument we made for subsidy game under incomplete information, the well-informed firm is more aggressive (conservative) in the good (bad) market 1, and the ill-informed firm tends to deliver a state-independent export to the market 1. This is due to the effect of incomplete information at the industrial level. Given the optimal tariff rates, we can calculate the expected social welfare for each country as

$$E\left(SW_{1}^{II}(T,T)\right) = \frac{65}{162b}(a-c)^{2}+\frac{3}{8b}\sigma^{2}, \tag{3.15}$$

$$E\left(SW_{2}^{II}(T,T)\right) = \frac{65}{162b}(a-c)^{2}. \tag{3.16}$$

As we can see, the option value associated with better information now completely shifts to the domestic country under tariffs when the domestic firm is a information monopolist. One can also easily verify that $\frac{\partial E\left(SW_1^{II}(T,T)\right)}{\partial\sigma^2} > 0$. The above can be summarized in the following proposition.

Proposition 3.2. *For tariff game under incomplete information,*

i. the entire option value shifts to the domestic country due to the information deficiency of the foreign firm;

ii. social welfare for the domestic country increases as variance increases;

iii. the domestic country ends up with higher social welfare due to option value effects.

Next, we examine equilibrium points under mixed games with incomplete information.

3.3.3 Mixed Game (T,S)

For the mixed game, we first analyze the sub-game equilibrium when the domestic government imposes tariff on the goods imported to market 1 and the foreign government subsidizes firm 2's export.

In stage three, the maximization problems for the domestic firm are

$$\begin{aligned} \max_{q_{1L}^{II}(T,S),x_1^{II}(T,S)} \pi_{1L}^{II}(T,S) &= \left(a - b\left(q_{1L}^{II}(T,S) + x_2^{II}(T,S)\right) - V\right) q_{1L}^{II}(T,S) \\ &+ \left(a - b\left(q_2^{II}(T,S) + x_1^{II}(T,S)\right)\right) x_1^{II}(T,S) \\ &- c\left(q_{1L}^{II}(T,S) + x_1^{II}(T,S)\right), \end{aligned} \tag{3.17}$$

if $\varepsilon = -V$ and

$$\begin{aligned} \max_{q_{1H}^{II}(T,S),x_1^{II}(T,S)} \pi_{1H}^{II}(T,S) &= \left(a - b\left(q_{1H}^{II}(T,S) + x_2^{II}(T,S)\right) + V\right) q_{1H}^{II}(T,S) \\ &+ \left(a - b\left(q_2^{II}(T,S) + x_1^{II}(T,S)\right)\right) x_1^{II}(T,S) \\ &- c\left(q_{1H}^{II}(T,S) + x_1^{II}(T,S)\right), \end{aligned} \tag{3.18}$$

if $\varepsilon = V$. The ill-informed foreign firm solves the following maximization problem, given a common prior:

$$
\begin{aligned}
\max_{q_2^{II}(T,S),x_2^{II}(T,S)} E\left(\pi_2^{II}(T,S)\right) &= \left(a - b\left(q_2^{II}(T,S) + x_1^{II}(T,S)\right)\right) q_2^{II}(T,S) \\
&+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a - b\left(q_1^{II}(T,S) + x_2^{II}(T,S)\right) + \varepsilon\right) \\
&\times x_2^{II}(T,S) \\
&- c\left(q_2^{II}(T,S) + x_2^{II}(T,S)\right) \\
&- t_1 x_2^{II}(T,S) + s_2 x_2^{II}(T,S), \qquad (3.19)
\end{aligned}
$$

where $q_1^{II}(T,S) \in \left\{q_{1L}^{II}(T,S), q_{1H}^{II}(T,S)\right\}$ depend on the realization of ε, t_1 is the tariff imposed by government 1 and s_2 is the export subsidy provided by government 2.

The corresponding best response functions for the stage three game are

$$
\begin{aligned}
BR_{1L} &= q_{1L}^{II}(T,S) \in \arg\max \pi_{1L}^{II}(T,S), \\
BR_{1H} &= q_{1H}^{II}(T,S) \in \arg\max \pi_{1H}^{II}(T,S), \\
BR_{1x} &= x_1^{II}(T,S) \in \arg\max \pi_{1L}^{II}(T,S), \\
BR_{2} &= q_2^{II}(T,S) \in \arg\max \pi_2^{II}(T,S), \\
BR_{2x} &= x_2^{II}(T,S) \in \arg\max \pi_2^{II}(T,S).
\end{aligned}
$$

Solving yields Bayesian Nash equilibrium points for stage three game given any (t_1, s_2, V):

$$\begin{aligned}
q_{1L}^{II}(T,S) &= \frac{1}{6b}\left(2(a-c) - 3V - 2s_2 + 2t_1\right), \\
q_{1H}^{II}(T,S) &= \frac{1}{6b}\left(2(a-c) + 3V - 2s_2 + 2t_1\right), \\
x_1^{II}(T,S) &= \frac{1}{3b}(a-c), \\
q_2^{II}(T,S) &= \frac{1}{3b}(a-c), \\
x_2^{II}(T,S) &= \frac{1}{3b}(a-c+2s_2-2t_1).
\end{aligned}$$

In stage two, the domestic government chooses tariff rate and the foreign government chooses subsidy rate in order to maximize their expected social welfare given that both governments anticipate that firms behave strategically. The expected social welfare for the domestic country is the sum of producer's surplus, consumer's surplus, and tariff revenue, while the expected social welfare for the foreign country is the sum of producer's surplus and consumer's surplus net of total subsidy. Formally, we have

$$\begin{aligned}
\max_{t_1} E\left(SW_1^{II}(T,S)\right) &= PS_1^{II}(T,S) + CS_1^{II}(T,S) + TR_1^{II}(T,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{II}(T,S) + \pi_{1H}^{II}(T,S)\right) \\
&+ \frac{1}{2}(\frac{b}{2}\left(q_{1L}^{II}(T,S) + x_2^{II}(T,S)\right)^2 \\
&+ \frac{b}{2}\left(q_{1H}^{II}(T,S) + x_2^{II}(T,S)\right)^2) \\
&+ t_1 x_2^{II}(T,S), \qquad (3.20)
\end{aligned}$$

$$\begin{aligned}\max_{s_2} E\left(SW_2^{II}(T,S)\right) &= PS_2^{II}(T,S) + CS_2^{II}(T,S) - S_2^{II}(T,S) \\ &= \pi_2^{II}(T,S) + \frac{b}{2}(q_2^{II}(T,S) + x_1^{II}(T,S))^2 \\ &- s_2 x_2^{II}(T,S). \end{aligned} \tag{3.21}$$

The corresponding best response functions are

$$\begin{aligned} BR_1(s_2) &= t_1 \in \arg\max E\left(SW_1^{II}(T,S)\right), \\ BR_2(t_1) &= s_2 \in \arg\max E\left(SW_2^{II}(T,S)\right). \end{aligned}$$

Solving stage two game gives us the Bayesian-Nash equilibrium policy rates:

$$t_1^* = \frac{5}{14}(a-c), \tag{3.22}$$

$$s_2^* = \frac{1}{14}(a-c). \tag{3.23}$$

Note that these optimal policy rates are identical to the mixed game with complete information.[9] Given these, the expected output in stage three can be summarized as follows:

$$\begin{aligned} E\left(q_{1L}^{II}(T,S)\right) &= \frac{3}{7b}(a-c) - \frac{1}{2b}V, \\ E\left(q_{1H}^{II}(T,S)\right) &= \frac{3}{7b}(a-c) + \frac{1}{2b}V, \\ E\left(x_1^{II}(T,S)\right) &= \frac{1}{3b}(a-c), \\ E\left(q_2^{II}(T,S)\right) &= \frac{1}{3b}(a-c), \\ E\left(x_2^{II}(T,S)\right) &= \frac{1}{7b}(a-c). \end{aligned}$$

[9]Optimal rates for complete information can be found in Appendix C.3. The same reasoning in previous sections is also applied here.

The corresponding expected social welfare functions are

$$E\left(SW_1^{II}(T,S)\right) = \frac{449}{882b}(a-c)^2 + \frac{3}{8b}\sigma^2, \tag{3.24}$$

$$E\left(SW_2^{II}(T,S)\right) = \frac{101}{294b}(a-c)^2. \tag{3.25}$$

Obviously, the expected social welfare for the domestic country increases with market volatility.

3.3.4 Mixed Game (S,T)

In this case, the domestic government subsidizes firm 1's exports to the foreign market, while the foreign government imposes tariff on the imports from firm 1.

As above, the problems for the domestic firm is

$$\begin{aligned}\max_{q_{1L}^{II}(S,T),x_1^{II}(S,T)} \pi_{1L}^{II}(S,T) &= \left(a - b\left(q_{1L}^{II}(S,T) + x_2^{II}(S,T)\right) - V\right) q_{1L}^{II}(S,T) \\ &+ \left(a - b\left(q_2^{II}(S,T) + x_1^{II}(S,T)\right)\right) x_1^{II}(S,T) \\ &- c\left(q_{1L}^{II}(S,T) + x_1^{II}(S,T)\right) \\ &- t_2 x_1^{II}(S,T) + s_1 x_1^{II}(S,T),\end{aligned} \tag{3.26}$$

if $\varepsilon = -V$ and

$$\begin{aligned}\max_{q_{1H}^{II}(S,T),x_1^{II}(S,T)} \pi_{1H}^{II}(S,T) &= \left(a - b\left(q_{1H}^{II}(S,T) + x_2^{II}(S,T)\right) + V\right) q_{1H}^{II}(S,T) \\ &+ \left(a - b\left(q_2^{II}(S,T) + x_1^{II}(S,T)\right)\right) x_1^{II}(S,T) \\ &- c\left(q_{1H}^{II}(S,T) + x_1^{II}(S,T)\right) \\ &- t_2 x_1^{II}(S,T) + s_1 x_1^{II}(S,T),\end{aligned}$$

if $\varepsilon = V$, where t_2 is the tariff imposed by the foreign government and s_1 is the export subsidy given by the domestic government.

The foreign firm solves the following maximization problem, given a common prior

$$\begin{aligned}\max_{q_2^{II}(S,T),x_2^{II}(S,T)} E\left(\pi_2^{II}(S,T)\right) &= \left(a-b\left(q_2^{II}(S,T)+x_1^{II}(S,T)\right)\right)q_2^{II}(S,T) \\ &+ \frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a-b\left(q_1^{II}(S,T)+x_2^{II}(S,T)\right)+\varepsilon\right) \\ &\times x_2^{II}(S,T) \\ &- c\left(q_2^{II}(S,T)+x_2^{II}(S,T)\right), \end{aligned} \tag{3.27}$$

where $q_1^{II}(S,T) \in \left\{q_{1L}^{II}(S,T), q_{1H}^{II}(S,T)\right\}$ depend on the realization of ε.

The best response functions for stage three game are

$$\begin{aligned} BR_{1L} &= q_{1L}^{II}(S,T) \in \arg\max \pi_{1L}^{II}(S,T), \\ BR_{1H} &= q_{1H}^{II}(S,T) \in \arg\max \pi_{1H}^{II}(S,T), \\ BR_{1x} &= x_1^{II}(S,T) \in \arg\max \pi_{1L}^{II}(S,T), \\ BR_2 &= q_2^{II}(S,T) \in \arg\max \pi_2^{II}(S,T), \\ BR_{2x} &= x_2^{II}(S,T) \in \arg\max \pi_2^{II}(S,T). \end{aligned}$$

Solving yields the Bayesian-Nash equilibrium points for stage three game given any (s_1, t_2, V):

$$\begin{aligned}
q_{1L}^{II}(S,T) &= \frac{1}{6b}\left(2(a-c)-3V\right), \\
q_{1H}^{II}(S,T) &= \frac{1}{6b}\left(2(a-c)+3V\right), \\
x_1^{II}(S,T) &= \frac{1}{3b}\left(a-c+2s_1-2t_2\right), \\
q_2^{II}(S,T) &= \frac{1}{3b}\left(a-c-s_1+t_2\right), \\
x_2^{II}(S,T) &= \frac{1}{3b}\left(a-c\right).
\end{aligned}$$

In stage two, the domestic government chooses subsidy rate and the foreign government chooses tariff rate to maximize their expected social welfare:

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{II}(S,T)\right) &= PS_1^{II}(S,T) + CS_1^{II}(S,T) - S_1^{II}(S,T) \\
&= \frac{1}{2}\left(\pi_{1L}^{II}(S,T) + \pi_{1H}^{II}(S,T)\right) \\
&+ \frac{1}{2}(\frac{b}{2}\left(q_{1L}^{II}(S,T) + x_2^{II}(S,T)\right)^2 \\
&+ \frac{b}{2}\left(q_{1H}^{II}(S,T) + x_2^{II}(S,T)\right)^2) \\
&- s_1 x_1^{II}(S,T), \qquad (3.28)
\end{aligned}$$

$$\begin{aligned}
\max_{t_2} E\left(SW_2^{II}(S,T)\right) &= PS_2^{II}(S,T) + CS_2^{II}(S,T) + TR_2^{II}(S,T) \\
&= \pi_2^{II}(S,T) + \frac{b}{2}(q_2^{II}(S,T) + x_1^{II}(S,T))^2 \\
&+ t_2 x_1^{II}(S,T). \qquad (3.29)
\end{aligned}$$

The corresponding best response functions for stage two game are

$$\begin{aligned} BR_1(t_2) &= s_1 \in \arg\max E\left(SW_1^{II}(S,T)\right), \\ BR_2(s_1) &= t_2 \in \arg\max E\left(SW_2^{II}(S,T)\right). \end{aligned}$$

Solving yields Bayesian Nash equilibrium level of policy rates:

$$s_1^* = \frac{1}{14}(a-c), \tag{3.30}$$
$$t_2^* = \frac{5}{14}(a-c). \tag{3.31}$$

Given these optimal policy rates, we can then calculate the expected output decisions by firms in the last stage:

$$\begin{aligned} E\left(q_{1L}^{II}(S,T)\right) &= \frac{1}{3b}(a-c) - \frac{1}{2b}V, \\ E\left(q_{1H}^{II}(S,T)\right) &= \frac{1}{3b}(a-c) + \frac{1}{2b}V, \\ E\left(x_1^{II}(S,T)\right) &= \frac{1}{7b}(a-c), \\ E\left(q_2^{II}(S,T)\right) &= \frac{3}{7b}(a-c), \\ E\left(x_2^{II}(S,T)\right) &= \frac{1}{3b}(a-c). \end{aligned}$$

The corresponding expected social welfare functions are

$$E\left(SW_1^{II}(S,T)\right) = \frac{101}{294b}(a-c)^2 + \frac{3}{8b}\sigma^2, \tag{3.32}$$
$$E\left(SW_2^{II}(S,T)\right) = \frac{449}{882b}(a-c)^2. \tag{3.33}$$

One can easily verify that the expected social welfare for the domestic country increases with market volatility. This together with the findings for (T,S) yield the following proposition:

Proposition 3.3. *For mixed game (either (T,S) or (S,T)) with incomplete information,*

i. the entire option value shifts to the domestic country due to incomplete information;

ii. social welfare for the domestic country increases as variance increases.

3.4 Choice of Regimes

Given that we have derived subgame equilibrium points for trade game with incomplete information and trade game with complete information (see Appendices C.1 to C.4), we are now ready to investigate what governments would do in order to maximize their social welfare in stage one. This can be modeled as two strategic games.

To this end, we construct two strategic games that involve choosing policy regimes by two governments simultaneously in the first stage. We denote G_1 and G_2 as the choice of policy games with expected payoffs derived under incomplete information and under complete information (see Appendix C), respectively. For both games:

i. the set of players are government 1 and 2;

ii. the set of actions are $\{S, T\}$;

iii. the payoffs are in terms of expected social welfare;

iv. the preference relations of governments are over expected social welfare.

Table 3.1 and 3.2 illustrate the normal form representation of game G_1 and G_2, respectively. Government 1 is the row player and government 2 is the column player in both games.

Table 3.1: Normal form representation of game G_1

Government 1	Government 2: Subsidy	Government 2: Tariff
Subsidy	$E\left(SW_1^{II}(S,S)\right)$, $E\left(SW_2^{II}(S,S)\right)$	$E\left(SW_1^{II}(S,T)\right)$, $E\left(SW_2^{II}(S,T)\right)$
Tariff	$E\left(SW_1^{II}(T,S)\right)$, $E\left(SW_2^{II}(T,S)\right)$	$E\left(SW_1^{II}(T,T)\right)$, $E\left(SW_2^{II}(T,T)\right)$

Table 3.2: Normal form representation of game G_2

Government 1	Government 2: Subsidy	Government 2: Tariff
Subsidy	$E\left(SW_1^{CI}(S,S)\right)$, $E\left(SW_2^{CI}(S,S)\right)$	$E\left(SW_1^{CI}(S,T)\right)$, $E\left(SW_2^{CI}(S,T)\right)$
Tariff	$E\left(SW_1^{CI}(T,S)\right)$, $E\left(SW_2^{CI}(T,S)\right)$	$E\left(SW_1^{CI}(T,T)\right)$, $E\left(SW_2^{CI}(T,T)\right)$

We are now ready to conduct equilibrium analysis of these two games using conventional Nash solution concept. Our presentation here follows rationalizability concept proposed by Pearce (1984). Let us first consider the game G_1. Assuming that government 2 chooses subsidy to begin with, the best response for government 1 is to choose tariff since $E\left(SW_1^{II}(T,S)\right) - E\left(SW_1^{II}(S,S)\right) > 0$. On the other hand, given that government 2 chooses tariff, the best response for government 1 is to choose tariff as well since $E\left(SW_1^{II}(T,T)\right) - E\left(SW_1^{II}(S,T)\right) > 0$. Both best responses from government 1 is independent of market volatility, and government 1 strictly prefers to impose tariff on imports from firm 2 no matter which trade policy government 2 prescribed. Therefore, subsidy is strictly dominated by tariff from government 1's perspective. After strategy subsidy is eliminated by government 1, tariff is also a best response from government 2's point of view. As a result, (T, T) is the only pair of strategy that survives the iterated elimination of strictly dominant strategy. Hence (T, T) is the unique Nash equilibrium of G_1. This gives us

Proposition 3.4. *The unique Nash equilibrium of game G_1 is (T, T), regardless of market volatility.*

The similar argument and reasoning from rationalizability can also be applied to G_2, yielding

Proposition 3.5. *The unique Nash equilibrium of game G_2 is (T, T), regardless of market volatility.*

This is not surprising since game G_1 and G_2 are actually variants of classical *prisoner's dilemma* game. Although the pair of strategy (S, S) gives better payoffs to both governments than the payoffs from (T, T), (S, S) is never achievable as long as we do not allow the governments to cooperate. Our result is consistent with the conventional wisdom that a Cournot-Nash game played by governments and firms will lead them to choose a tariff in equilibrium with certainty.

It is worth noting that the unique Nash equilibrium is independent of variance. This is because the option value associated with better information is the same under all pairs of strategies chosen by governments. For example, the option value to the domestic country is $\frac{3}{8b}\sigma^2$ for all pair of strategies with incomplete information. Therefore, the market volatility plays no role in determining optimal strategy in choosing between subsidy and tariff. This results is in sharp contrast to Anam and Chiang (2018) in that the choice of trade policy game is shown to depend on the correlation between two markets. This is not the case in the current chapter because we assume that two markets are stochastically independent. Moreover, our results are also different from Cooper and Riezman (1989) in that the choice is shown to depend on the degree of uncertainty. While their model assumes that the information about market demand is symmetrically revealed to both firms, we here focus on the effect of information asymmetry on the choice of trade policies.

Notice that although incomplete information for the foreign firm does not affect the optimal strategic choice between tariff and subsidy, it, nevertheless, redistributes the option value associated with better information. As shown in trade game with incomplete information, the well-informed domestic firm is able to capture entire option value, since it is able to make output decision after the resolution of uncertainty.

Because Nash equilibrium (T, T) implies a prisoner's dilemma outcome, we shall consider a special case which both governments agree on free-trade agreement. Then we compare these social welfare levels from free-trade to our Nash equilibrium output to determine which trade policy (tariff or free-trade agreement) is optimal.

3.5 Free-Trade with Incomplete Information

In this section, we show that no intervention is superior to the Bayesian Nash equilibrium (T, T). To see this, first consider the domestic firm's maximization problem:

$$\begin{aligned}\max_{q_{1L}^{II},x_1^{II}} \pi_{1L}^{II} &= \left(a-b\left(q_{1L}^{II}+x_2^{II}\right)-V\right)q_{1L}^{II}+\left(a-b\left(q_2^{II}+x_1^{II}\right)\right)x_1^{II}\\ &- c\left(q_{1L}^{II}+x_1^{II}\right), \qquad (3.34)\end{aligned}$$

if $\varepsilon=-V$ and

$$\begin{aligned}\max_{q_{1H}^{II},x_1^{II}} \pi_{1H}^{II} &= \left(a-b\left(q_{1H}^{II}+x_2^{II}\right)+V\right)q_{1H}^{II}+\left(a-b\left(q_2^{II}+x_1^{II}\right)\right)x_1^{II}\\ &- c\left(q_{1H}^{II}+x_1^{II}\right), \qquad (3.35)\end{aligned}$$

if $\varepsilon=V$. The foreign firm, being ill-informed, solves the following maximization problem given common prior:

$$\begin{aligned}\max_{q_2^{II},x_2^{II}} E\left(\pi_2^{II}\right) &= \left(a-b\left(q_2^{II}+x_1^{II}\right)\right)q_2^{II}+\frac{1}{2}\sum_{\varepsilon\in\Omega}\left(a-b\left(q_1^{II}+x_2^{II}\right)+\varepsilon\right)x_2^{II}\\ &- c\left(q_2^{II}+x_2^{II}\right), \qquad (3.36)\end{aligned}$$

where $q_1^{II}\in\left\{q_{1L}^{II},q_{1H}^{II}\right\}$ depend on the realization of ε.

The best response functions for above problems are

$$\begin{aligned}BR_{1L} &= q_{1L}^{II}\in\arg\max\pi_{1L}^{II},\\ BR_{1H} &= q_{1H}^{II}\in\arg\max\pi_{1H}^{II},\\ BR_{1x} &= x_1^{II}\in\arg\max\pi_{1L}^{II},\\ BR_2 &= q_2^{II}\in\arg\max\pi_2^{II},\\ BR_{2x} &= x_2^{II}\in\arg\max\pi_2^{II}.\end{aligned}$$

Solving yields

$$\begin{aligned}
q_{1L}^{II} &= \frac{1}{3b}(a-c) - \frac{1}{2b}V, \\
q_{1H}^{II} &= \frac{1}{3b}(a-c) + \frac{1}{2b}V, \\
x_{1}^{II} &= \frac{1}{3b}(a-c), \\
q_{2}^{II} &= \frac{1}{3b}(a-c), \\
x_{2}^{II} &= \frac{1}{3b}(a-c).
\end{aligned}$$

By substitution, one obtains the expected social welfare functions for home and foreign countries as

$$\begin{aligned}
E\left(SW_1^{II}\right) &= PS_1^{II} + CS_1^{II} \\
&= \frac{1}{2}\left(\pi_{1L}^{II} + \pi_{1H}^{II}\right) + \frac{1}{2}\left(\frac{b}{2}\left(q_{1L}^{II} + x_2^{II}\right)^2 + \frac{b}{2}\left(q_{1H}^{CI} + x_2^{II}\right)^2\right) \\
&= \frac{4}{9b}(a-c)^2 + \frac{3}{8b}\sigma^2, \qquad (3.37) \\
E\left(SW_2^{II}\right) &= PS_2^{II} + CS_2^{II} \\
&= \pi_2^{II} + \frac{b}{2}(q_2^{II} + x_1^{II})^2 \\
&= \frac{4}{9b}(a-c)^2, \qquad (3.38)
\end{aligned}$$

respectively.

It follows that

$$\begin{aligned}
E\left(SW_1^{II}\right) - E\left(SW_1^{II}(T,T)\right) &> 0, \\
E\left(SW_2^{II}\right) - E\left(SW_2^{II}(T,T)\right) &> 0.
\end{aligned}$$

This result is summarized in

Proposition 3.6. *With incomplete information at industrial level, Nash equilibrium outcome for game G_1, (T,T), is inferior to free-trade outcome for both countries.*

3.6 Conclusion

The main focus of this chapter is to reexamine the strategic trade policy in the context of reciprocal markets under incomplete information, an issue that has been ignored in the literature. Several departures from conventional strategic trade policy wisdom are observed. Firstly, an export subsidy is superior to a tariff if one country is active and can unilaterally choose a trade policy against its rival. But interestingly, tariffs turn out to be a non-cooperative equilibrium outcome when both governments are active and set trade policies against each other. The result holds regardless of market volatility and information partition. Incomplete information for the foreign firm redistributes the option value, associated with ability to make decision after the resolution of uncertainty, to the country with more information. Secondly, we show that the Nash equilibrium outcome under incomplete information (i.e., (T, T)) is inferior to the equilibrium outcome absent of any form of government intervention (i.e., *Laissez-faire*).

This chapter adopts some simplified assumptions to highlight our assertions. These include a trade game being simultaneous-move, policy modes being restricted to tariffs and export subsidies, marginal costs being constant, and the random disturbance being additive in form. A natural extension of current model would be to expand policy set to include quota as well as a sequential game structure. Therefore, the countervailing policy model can be revisited to take into account the possibility that one country proactively intervenes by adopting a strategic trade policy while the other responds optimally by retaliating with appropriate instruments in response to the leader's choice. The equilibrium choice of the leader government would then be determined by backward induction as usual. It is possible that equilibrium response to a subsidy may be something other than a countervailing duty. But we leave it for future research.

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Appendix A

Appendix to Chapter 1

A.1 Import Tariffs

Here, we consider the case in which the domestic firm finds it unprofitable to remain in the market when the realized demand is too low, i.e., $V \geq \frac{8}{9}(a-c)$. Our task is to find the optimal tariff rate, Bayesian Nash equilibrium level of output and expected social welfare for the domestic country when the domestic firm shuts down in a depressed market. We use subscript NA to denote variables in this case.

Suppose now that the domestic firm set $q^T_{1L} = 0$ if it observes the domestic market demand is low, the new best response functions for the domestic firm and the foreign firm are

$$\begin{aligned} BR_{1H,NA}\left(q^T_{2,NA}\right) &= q^T_{1H,NA} \in \arg\max \pi^T_{1H}, \\ BR_{2,NA}\left(q^T_{1L} = 0, q^T_{1H,NA}\right) &= q^T_{2,NA} \in \arg\max E\left(\pi^T_2\right). \end{aligned}$$

Given these new best response functions, we can solve for the new Bayesian Nash equilibrium points for any given tariff t_0 as

$$\begin{aligned} q^T_{1H,NA} &= \frac{2}{7b}(a-c+t_0)+\frac{4}{7b}V, \\ q^T_{2,NA} &= \frac{3}{7b}(a-c)-\frac{4}{7b}t_0-\frac{1}{7b}V. \end{aligned}$$

Folding back to stage one, the domestic government chooses tariff t_0 to maximizes the expected social welfare, given that the domestic government anticipates the strategic behavior of both firms in the second stage. Note that there is a change in the expected social welfare function for the domestic government in low market demand since the domestic firm now provides zero output in low market demand. Formally, the objective function for the domestic government is

$$\begin{aligned} \max_{t_0} E\left(SW^T_{NA}\right) &= \frac{1}{2}\left(CS^T_{L,NA}\right)+\frac{1}{2}\left(PS^T_{H,NA}+CS^T_{H,NA}\right)+TR_{NA} \\ &= \frac{1}{2}\left(\frac{b}{2}\left(q^T_{2,NA}\right)^2\right)+\frac{1}{2}\left(\pi^T_{1H,NA}+\frac{b}{2}\left(q^T_{1H,NA}+q^T_{2,NA}\right)^2\right) \\ &+ t_0 q^T_{2,NA}, \end{aligned} \tag{A.1}$$

where

$$\pi^T_{1H,NA}=\frac{4}{49b}(a-c+t_0+2V)^2.$$

Solving the optimization problem for the domestic government yields

$$t_0^*=\frac{1}{3}(a-c). \tag{A.2}$$

Note that $t_0^*=t^*$. This implies the choice of optimal tariff rate is not sensitive to whether the domestic firm is active or not in the bad state. Substituting t_0 into the Bayesian Nash equilibrium levels of output for the two firms, one obtains

$$q^T_{1H,NA} = \frac{8}{21b}(a-c) + \frac{4}{7b}V, \quad \text{(A.3)}$$
$$q^T_{2,NA} = \frac{5}{21b}(a-c) - \frac{1}{7b}V. \quad \text{(A.4)}$$

Substituting these equilibrium values into first-order condition of equation (1.1) yields $V_0 = \frac{8}{9}(a-c)$ – the critical point of V so that the domestic firm stops producing in low market demand. Hence we can conclude that for any value of $V \in [\frac{8}{9}(a-c), \infty)$, the domestic firm will not be active in the low market demand.

Given these equilibrium values, we can now calculate the expected social welfare for the domestic country if the domestic firm is not active in low market demand under tariff regime as

$$E\left(SW^T_{NA}\right) = \frac{11}{42b}(a-c)^2 + \frac{2}{7b}(a-c)\sigma + \frac{3}{14b}\sigma^2. \quad \text{(A.5)}$$

A.2 Import Quotas

There are three cases that need to be considered under quota policy depending on whether the domestic firm is active or not when the market realizes low demand:

1. the domestic firm is active in low market demand under both zero quota policy and free-trade quota policy;
2. the domestic firm is active in low market demand under zero quota policy but inactive in low market demand under free-trade quota policy;

3. the domestic firm is not active in low market demand under either zero quota policy or free-trade quota policy.

Note that for the domestic firm to be active in low demand under zero quota policy we need $V < (a-c)$, and the domestic firm is active in low demand under free-trade quota if $V < \frac{2}{3}(a-c)$. Therefore, it is not possible that the domestic firm is active in low demand under free-trade quota but inactive in low demand under zero quota policy.

In section 1.3.2, we have conducted the analysis for the Case 1 (see equation (1.26)) and Case 2 (see equation (1.27)). We now conduct the equilibrium analysis for Case 3. Note that from equation (1.13), we can see that the domestic firm will not produce in low market demand under aurtaky if $V \geq (a-c)$. If this is the case, the expected social welfare for the domestic country (denoted as subscript NA as usual) is

$$\begin{aligned} E\left(SW_{NA}^{ZQ}\right) &= \frac{1}{2}\left(PS_H^{ZQ} + CS_H^{ZQ}\right) \\ &= \frac{1}{2}\left(\pi_{1H}^{ZQ} + \frac{b}{2}\left(q_{1H}^{ZQ}\right)^2\right) \\ &= \frac{3}{16b}\left(a-c+V\right)^2. \end{aligned} \tag{A.6}$$

We are now ready to compare the expected social welfare under zero quota and free-trade quota if the domestic firm is not active when low market demand realizes under either policies. Subtracting equation (1.25) from equation (A.6) and denoting difference as Δ_4, we have

$$\begin{aligned} \Delta_4 &= E\left(SW_{NA}^{ZQ}\right) - E\left(SW_{NA}^{FT}\right) \\ &= \left(\frac{3}{16b}\left(a-c+V\right)^2\right) - \left(\frac{3}{14b}\left(a-c\right)^2 + \frac{2}{7b}\left(a-c\right)V + \frac{3}{14b}V^2\right) \\ &= -\frac{1}{112b}\left(3\left(a-c\right)-V\right)\left(a-c-3V\right). \end{aligned} \tag{A.7}$$

Δ_4 is strictly concave since $\frac{\partial^2 \Delta_4}{\partial V^2} = -\frac{3}{56b} < 0$. The maximum of Δ_4 occurs at $V_{\max} = \frac{5}{3}(a-c)$. At $V = V_{\max}$, $\Delta_4(V_{\max}) = \frac{1}{21b}(a-c)^2 > 0$. Moreover, $\Delta_4 = 0$ happens at $V = \frac{1}{3}(a-c)$ and $V = 3(a-c)$. Given that $V \geq (a-c)$ in our Case 3, we then can conclude following results regarding to optimal quota policy:

- When $(a-c) \leq V < 3(a-c)$, $\Delta_4 > 0$, zero quota is preferred to free-trade quota;
- When $V = 3(a-c)$, $\Delta_4 = 0$, the domestic government is indifference between zero quota and free-trade quota;
- When $V > 3(a-c)$, $\Delta_4 < 0$, free-trade quota is preferred to zero quota.

Note that if we have a sufficient large $V(> 3(a-c))$, free-trade quota seems to be optimal for the domestic country. This result is slight different from what we have analyzed in section 1.3.2. However, this result will not affect welfare ranking when choosing between tariffs and quotas as show in Appendix A.3.

A.3 Welfare Comparison

When we relax our assumption that random variable $V < \frac{8}{9}(a-c)$ in our main text, we shall consider four different possible cases regarding to whether the domestic firm will active or not in low market demand under different policy instruments similar to Appendix A.2 for analysis of quotas. Hence we have

1. The domestic firm is active in low market demand under both tariff and quota;
2. The domestic firm is active in low market demand under tariff, but inactive in low market demand under quota;

3. The domestic firm is not active in low market demand under tariff, but active in low market demand under quota;

4. The domestic firm is not active in low market demand under either tariff or quota.

We have covered Case 1 and Case 2 in section 1.4 as we assumed $V < \frac{8}{9}(a-c)$ throughout our equilibrium analysis. For easy comparisons, we first restate our results from the main text (Case 1 and Case 2, respectively) and then conduct our welfare analysis for Cases 3 and 4 where the domestic firm becomes inactive. By analyzing all four cases, we can show that a tariff is optimal for all $V \in \mathbb{R}$.

Case 1

In this case, the random variable V satisfies $V < \frac{2}{3}(a-c)$. Given that the optimal quota policy is zero quota, Δ_3 (see equation (1.28)) implies a tariff is preferred to a quota at zero leval.

Case 2

The random variable V falls in the range $\left[\frac{2}{3}(a-c), \frac{8}{9}(a-c)\right)$ in this case. Similar to Case 1, given that the optimal quota policy is autarky, Δ_3 (see equation (1.28)) implies that a tariff is preferred.

Case 3

In this case, the stochastic term V shall falls in the range $\left[\frac{8}{9}(a-c), (a-c)\right)$. Note that the optimal quota policy is also zero quota if quota is considered.

Define

$$
\begin{aligned}
\Delta_5 &= E\left(SW_{NA}^{T}\right) - E\left(SW^{ZQ}\right) \\
&= \left(\frac{11}{42b}(a-c)^2 + \frac{2}{7b}(a-c)\sigma + \frac{3}{14b}\sigma^2\right) - \left(\frac{3}{8b}\left((a-c)^2+\sigma^2\right)\right) \\
&= \frac{19}{168b}(a-c)^2 + \frac{2}{7b}(a-c)\sigma + \frac{9}{56b}\sigma^2. \qquad \text{(A.8)}
\end{aligned}
$$

Δ_5 is strictly concave since $\frac{\partial \Delta_5}{\partial \sigma} = \frac{2}{7b}(a-c) - \frac{9}{28b}\sigma \gtreqless 0$ and $\frac{\partial^2 \Delta_5}{\partial \sigma^2} = -\frac{9}{28b} < 0$. The maximum of Δ_5 occurs at $\sigma_{\max} = \frac{8}{9}(a-c)$. At $\sigma = \sigma_{\max}$, $\Delta_5(\sigma_{\max}) = \frac{1}{72b}(a-c)^2 > 0$. Moreover, $\Delta_5(\sigma = (a-c)) = \frac{1}{84b}(a-c)^2 > 0$. We can conclude that $\Delta_5 > 0$ given $V \in \left[\frac{8}{9}(a-c), (a-c)\right)$. Hence, a tariff is preferred to a quota under Case 3.

Case 4

The last case is $V \geq (a-c)$. According to Appendix A.2, the optimal quota policy is either zero quota or a quota at the free-trade level depending on the cut-off point $V = 3(a-c)$.

For $V \leq 3(a-c)$, we subtract equation (A.6) from equation (A.5) and define

$$
\begin{aligned}
\Delta_6 &= E\left(SW_{NA}^{T}\right) - E\left(SW_{NA}^{ZQ}\right) \\
&= \left(\frac{11}{42b}(a-c)^2 + \frac{2}{7b}(a-c)V + \frac{3}{14b}V^2\right) - \left(\frac{3}{16b}(a-c+V)^2\right) \\
&= \frac{1}{336b}\left(5(a-c) - 3V\right)^2, \qquad \text{(A.9)}
\end{aligned}
$$

which is always positive.

On the other hand, for $V > 3(a-c)$, we shall instead subtract equation (1.25) from equation (A.5) and define

$$\begin{aligned}\Delta_7 &= E\left(SW_{NA}^{T}\right) - E\left(SW_{NA}^{FT}\right) \\ &= \left(\frac{11}{42b}(a-c)^2 + \frac{2}{7b}(a-c)\sigma + \frac{3}{14b}\sigma^2\right) \\ &- \left(\frac{3}{14b}(a-c)^2 + \frac{2}{7b}(a-c)\sigma + \frac{3}{14b}\sigma^2\right) \\ &= \frac{1}{21b}(a-c)^2, \end{aligned} \tag{A.10}$$

which is positive.

Given that $\Delta_6 > 0$ and $\Delta_7 > 0$, the optimal policy is again a tariff under Case 4.

Cases 1-4 allow us to conclude that a tariff is always superior to a quota for all $V \in \mathbb{R}$.

Appendix B

Appendix to Chapter 2

B.1 Proof of Lemma 2.1

Let $B_1(a_2)$ be the set of government 1's best actions

$$B_1(a_2) \equiv \{a_1 \in A_1 | (a_1, a_2) \succeq_1 (a_1', a_2) \forall a_1' \in A_1\}.$$

Given government 2 chooses *Subsidy*, we have

$$B_1(Subsidy) = \{Quota\},$$

since $f(Quota, Subsidy) \succ_1 f(Subsidy, Subsidy)$ or

$$\frac{((a-c)+E(\varepsilon))^2}{8b} > \frac{2((a-c)+E(\varepsilon))^2}{25b}.$$

Similarly, given government 2 chooses *Quota*, we have

$$B_1(Quota) = \{Quota\},$$

since $f\left(Quota, Quota\right) \succ_1 f\left(Subsidy, Quota\right)$ or

$$\frac{\left((a-c)+E\left(\varepsilon\right)\right)^2}{9b} > \frac{\left((a-c)+E\left(\varepsilon\right)\right)^2}{16b}.$$

Hence we have

$$B_1\left(a_2\right) = \{Quota\} \quad \forall a_2 \in A_2.$$

B.2 Proof of Lemma 2.2

Let $B_2\left(Quota\right)$ be the set of government 2's best responses to government 1's strategy $Quota$

$$B_2\left(Quota\right) \equiv \left\{a_2 \in A_2 |\left(Quota, a_2\right) \succeq_2 \left(Quota, a_2'\right)\right\}.$$

If $f\left(Quota, Quota\right) \succ_2 f\left(Subsidy, Quota\right)$, we have

$$\frac{\left((a-c)+E\left(\varepsilon\right)\right)^2}{9b} \geq \frac{\left((a-c)+E\left(\varepsilon\right)\right)^2}{16b} + \frac{\sigma^2}{4b},$$

that is

$$B_2\left(Quota\right) = \{Quota\}, \qquad if \quad \sigma^2 \leq \frac{7}{36}\left((a-c)+E\left(\varepsilon\right)\right)^2.$$

On the other hand, if $f\left(Subsidy, Quota\right) \succ_2 f\left(Quota, Quota\right)$, we have

$$\frac{\left((a-c)+E\left(\varepsilon\right)\right)^2}{16b} + \frac{\sigma^2}{4b} \geq \frac{\left((a-c)+E\left(\varepsilon\right)\right)^2}{9b},$$

that is

$$B_2\left(Quota\right) = \{Subsidy\}, \qquad if \quad \sigma^2 \geq \frac{7}{36}\left((a-c)+E\left(\varepsilon\right)\right)^2.$$

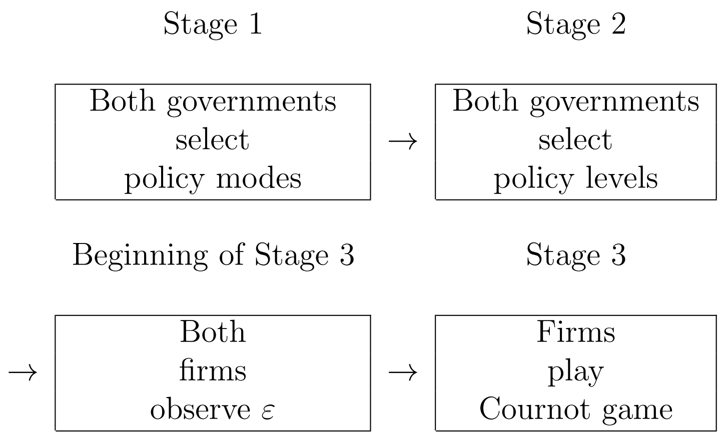
Thus the best response to government 1's strategy *Quota* for government 2 depends on σ^2.

Appendix C

Appendix to Chapter 3

In this Appendix, we derive the equilibrium outputs, subsidies/tariffs, and policy regimes in stage three for trade game with complete information under various trade policies. As in the main text, the actions taken by various decision makers are summarized in the following figure:

Figure C.1: Timing of Three-Stage Trade Game with Complete Information



Four policy combinations, (S, S), (T, T), (T, S) and (S, T), under complete information are examined in sequence.

C.1 Subsidy Game (S,S)

We first consider the game, where export subsidy is committed to by both governments, as the policy instrument in stage one. Under complete information assumption, the solution through backward induction starts in stage three when the random variable ε is known to both domestic and foreign firms.

Starting from the last stage, if the true market condition is $-V$ in country 1, each firm's problem is

$$\begin{aligned}\max_{q_{1L}^{CI}(S,S),x_1^{CI}(S,S)} \pi_{1L}^{CI}(S,S) &= \left(a-b\left(q_{1L}^{CI}(S,S)+x_{2L}^{CI}(S,S)\right)-V\right)q_{1L}^{CI}(S,S)\\ &+ \left(a-b\left(q_1^{CI}(S,S)+x_1^{CI}(S,S)\right)\right)x_1^{CI}(S,S)\\ &- c\left(q_{1L}^{CI}(S,S)+x_1^{CI}(S,S)\right)+s_1x_1^{CI}(S,S)\,, \quad \text{(C.1)}\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(S,S),x_{2L}^{CI}(S,S)} \pi_{2L}^{CI}(S,S) &= \left(a-b\left(q_2^{CI}(S,S)+x_1^{CI}(S,S)\right)\right)q_2^{CI}(S,S)\\ &+ \left(a-b\left(q_{1L}^{CI}(S,S)+x_{2L}^{CI}(S,S)\right)-V\right)x_{2L}^{CI}(S,S)\\ &- c\left(q_2^{CI}(S,S)+x_{2L}^{CI}(S,S)\right)+s_2x_{2L}^{CI}(S,S)\,. \quad \text{(C.2)}\end{aligned}$$

Similarly, if the true market demand is high in country 1, each firm's problem can be written as

$$\begin{aligned}\max_{q_{1H}^{CI}(S,S),x_1^{CI}(S,S)} \pi_{1H}^{CI}(S,S) &= \left(a-b\left(q_{1H}^{CI}(S,S)+x_{2H}^{CI}(S,S)\right)+V\right)q_{1H}^{CI}(S,S)\\ &+ \left(a-b\left(q_2^{CI}(S,S)+x_1^{CI}(S,S)\right)\right)x_1^{CI}(S,S)\\ &- c\left(q_{1H}^{CI}(S,S)+x_1^{CI}(S,S)\right)+s_1x_1^{CI}(S,S)\,, \quad \text{(C.3)}\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(S,S),x_{2H}^{CI}(S,S)} \pi_{2H}^{CI}(S,S) &= \left(a-b\left(q_2^{CI}(S,S)+x_1^{CI}(S,S)\right)\right)q_2^{CI}(S,S)\\ &+ \left(a-b\left(q_{1H}^{CI}(S,S)+x_{2H}^{CI}(S,S)\right)+V\right)x_{2H}^{CI}(S,S)\\ &- c\left(q_2^{CI}(S,S)+x_{2H}^{CI}(S,S)\right)+s_2x_{2H}^{CI}(S,S), \quad \text{(C.4)}\end{aligned}$$

where s_i is the subsidy per unit of export for $i=1,2$.

The best response functions for the above maximization problems are

$$\begin{aligned}BR_{1L} &= q_{1L}^{CI}(S,S)\in\arg\max\pi_{1L}^{CI}(S,S),\\ BR_{1H} &= q_{1H}^{CI}(S,S)\in\arg\max\pi_{1H}^{CI}(S,S),\\ BR_{1x} &= x_1^{CI}(S,S)\in\arg\max\pi_{1L}^{CI}(S,S),\\ BR_2 &= q_2^{CI}(S,S)\in\arg\max\pi_{2L}^{CI}(S,S),\\ BR_{2xL} &= x_{2L}^{CI}(S,S)\in\arg\max\pi_{2L}^{CI}(S,S),\\ BR_{2xH} &= x_{2H}^{CI}(S,S)\in\arg\max\pi_{2H}^{CI}(S,S).\end{aligned}$$

Given these best response functions, we can obtain the Nash equilibrium points in each state for any given level of s_1 and s_2 as follows:

$$\begin{aligned}q_{1L}^{CI}(S,S) &= \frac{1}{3b}(a-c-V-s_2),\\ q_{1H}^{CI}(S,S) &= \frac{1}{3b}(a-c+V-s_2),\\ x_1^{CI}(S,S) &= \frac{1}{3b}(a-c+2s_1),\\ q_2^{CI}(S,S) &= \frac{1}{3b}(a-c-s_1),\\ x_{2L}^{CI}(S,S) &= \frac{1}{3b}(a-c-V+2s_2),\\ x_{2H}^{CI}(S,S) &= \frac{1}{3b}(a-c+V+2s_2).\end{aligned}$$

The solutions characterize the Nash equilibrium in each market given (s_1,s_2,V).

Folding back to the second stage, both governments set subsidy rates to maximize respective social welfare given that they anticipate the Nash equilibrium output levels in stage three across different states of nature. The social welfare functions for both governments under export subsidy are specified as the sum of producer's surplus and consumer's surplus net of total subsidy. Hence, the problem for each government can be written as

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{CI}(S,S)\right) &= PS_1^{CI}(S,S) + CS_1^{CI}(S,S) - S_1^{CI}(S,S) \\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(S,S) + \pi_{1H}^{CI}(S,S)\right) \\
&+ \frac{1}{2}\Big(\frac{b}{2}\left(q_{1L}^{CI}(S,S) + x_{2L}^{CI}(S,S)\right)^2 \\
&+ \frac{b}{2}\left(q_{1H}^{CI}(S,S) + x_{2H}^{CI}(S,S)\right)^2\Big) \\
&- s_1 x_1^{CI}(S,S), \qquad \text{(C.5)}
\end{aligned}$$

$$\begin{aligned}
\max_{s_2} E\left(SW_2^{CI}(S,S)\right) &= PS_2^{CI}(S,S) + CS_2^{CI}(S,S) - S_2^{CI}(S,S) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(S,S) + \pi_{2H}^{CI}(S,S)\right) \\
&+ \frac{b}{2}(q_2^{CI}(S,S) + x_1^{CI}(S,S))^2 \\
&- \frac{1}{2}\left(x_{2L}^{CI}(S,S) + x_{2H}^{CI}(S,S)\right) s_2. \qquad \text{(C.6)}
\end{aligned}$$

The best response functions of stage two game are

$$\begin{aligned}
BR_1(s_2) &= s_1 \in \arg\max E\left(SW_1^{CI}(S,S)\right), \\
BR_2(s_1) &= s_2 \in \arg\max E\left(SW_2^{CI}(S,S)\right).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium level of export subsidies

$$s_1^* = s_2^* = \frac{1}{4}(a-c). \tag{C.7}$$

By Substitution, one can obtain the expected output levels for each firm in stage three as follows:

$$\begin{aligned}
E\left(q_{1L}^{CI}(S,S)\right) &= \frac{1}{4b}(a-c) - \frac{1}{3b}V, \\
E\left(q_{1H}^{CI}(S,S)\right) &= \frac{1}{4b}(a-c) + \frac{1}{3b}V, \\
E\left(x_{1}^{CI}(S,S)\right) &= \frac{1}{2b}(a-c), \\
E\left(q_{2}^{CI}(S,S)\right) &= \frac{1}{4b}(a-c), \\
E\left(x_{2L}^{CI}(S,S)\right) &= \frac{1}{2b}(a-c) - \frac{1}{3b}V, \\
E\left(x_{2H}^{CI}(S,S)\right) &= \frac{1}{2b}(a-c) + \frac{1}{3b}V.
\end{aligned}$$

Given equation (C.7), the expected social welfare for both countries are

$$E\left(SW_1^{CI}(S,S)\right) = \frac{15}{32b}(a-c)^2 + \frac{1}{3b}\sigma^2, \tag{C.8}$$

$$E\left(SW_2^{CI}(S,S)\right) = \frac{15}{32b}(a-c)^2 + \frac{1}{9b}\sigma^2. \tag{C.9}$$

It is straightforward to show that $\frac{\partial E\left(SW_i^{CI}(S,S)\right)}{\partial \sigma^2} > 0$ for $i = 1, 2$. That is, the expected social welfare increases with market volatility. Notice that the second term in expected social welfare functions is the option value associated with better information. Apparently, $E\left(SW_1^{CI}(S,S)\right) - E\left(SW_2^{CI}(S,S)\right) > 0$, indicating that the domestic country enjoys higher option values than the foreign country.

Next, we turn to examine the sub game equilibrium under tariff game.

C.2 Tariff Game (T,T)

We now turn to the tariff game between the two countries. Similar to the equilibrium analysis in the previous section, we start from the last stage in which both firms observe the true market condition in country 1.

If the true market demand in country 1 is low, we can write firms' problems as

$$\begin{aligned}
\max_{q_{1L}^{CI}(T,T),x_1^{CI}(T,T)} \pi_{1L}^{CI}(T,T) &= \left(a - b\left(q_{1L}^{CI}(T,T) + x_{2L}^{CI}(T,T)\right) - V\right) q_{1L}^{CI}(T,T) \\
&+ \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) x_1^{CI}(T,T) \\
&- c\left(q_{1L}^{CI}(T,T) + x_1^{CI}(T,T)\right) \\
&- t_2 x_1^{CI}(T,T), \qquad \text{(C.10)}
\end{aligned}$$

$$\begin{aligned}
\max_{q_2^{CI}(T,T),x_{2L}^{CI}(T,T)} \pi_{2L}^{CI}(T,T) &= \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) q_2^{CI}(T,T) \\
&+ \left(a - b\left(q_{1L}^{CI}(T,T) + x_{2L}^{CI}(T,T)\right) - V\right) x_{2L}^{CI}(T,T) \\
&- c\left(q_2^{CI}(T,T) + x_{2L}^{CI}(T,T)\right) \\
&- t_1 x_{2L}^{CI}(T,T). \qquad \text{(C.11)}
\end{aligned}$$

On the other hand, if the true market demand in country 1 is high, the profit functions for both firms are

$$\begin{aligned}
\max_{q_{1H}^{CI}(T,T),x_1^{CI}(T,T)} \pi_{1H}^{CI}(T,T) &= \left(a - b\left(q_{1H}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right) + V\right) q_{1H}^{CI}(T,T) \\
&+ \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) x_1^{CI}(T,T) \\
&- c\left(q_{1H}^{CI}(T,T) + x_1^{CI}(T,T)\right) \\
&- t_2 x_1^{CI}(T,T), \qquad \text{(C.12)}
\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(T,T),x_{2H}^{CI}(T,T)} \pi_{2H}^{CI}(T,T) &= \left(a - b\left(q_2^{CI}(T,T) + x_1^{CI}(T,T)\right)\right) q_2^{CI}(T,T) \\ &+ \left(a - b\left(q_{1H}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right) + V\right) x_{2H}^{CI}(T,T) \\ &- c\left(q_2^{CI}(T,T) + x_{2H}^{CI}(T,T)\right) \\ &- t_1 x_{2H}^{CI}(T,T), \qquad \text{(C.13)}\end{aligned}$$

where t_i is the tariff imposed by the home government on foreign imports for $i = 1, 2$.

The best response functions for above problems are

$$\begin{aligned} BR_{1L} &= q_{1L}^{CI}(T,T) \in \arg\max \pi_{1L}^{CI}(T,T), \\ BR_{1H} &= q_{1H}^{CI}(T,T) \in \arg\max \pi_{1H}^{CI}(T,T), \\ BR_{1x} &= x_1^{CI}(T,T) \in \arg\max \pi_{1L}^{CI}(T,T), \\ BR_2 &= q_2^{CI}(T,T) \in \arg\max \pi_{2L}^{CI}(T,T), \\ BR_{2xL} &= x_{2L}^{CI}(T,T) \in \arg\max \pi_{2L}^{CI}(T,T), \\ BR_{2xH} &= x_{2H}^{CI}(T,T) \in \arg\max \pi_{2H}^{CI}(T,T). \end{aligned}$$

Given these best response functions, the Nash equilibrium output levels given any (t_1, t_2, V) for stage three game are

$$\begin{aligned} q_{1L}^{CI}(T,T) &= \frac{1}{3b}(a - c - V + t_1), \\ q_{1H}^{CI}(T,T) &= \frac{1}{3b}(a - c + V + t_1), \\ x_1^{CI}(T,T) &= \frac{1}{3b}(a - c - 2t_2), \\ q_2^{CI}(T,T) &= \frac{1}{3b}(a - c + t_2), \\ x_{2L}^{CI}(T,T) &= \frac{1}{3b}(a - c - V - 2t_1), \\ x_{2H}^{CI}(T,T) &= \frac{1}{3b}(a - c + V - 2t_1). \end{aligned}$$

Moving backward to the second stage, each government selects the tariff rate to maximize its social welfare given that each government anticipates Nash equilibrium outputs from both firms in stage three. The maximization problem facing each government under tariffs is the sum of producer's surplus, consumer's surplus and tariff revenue, given by

$$\begin{aligned}
\max_{t_1} E\left(SW_1^{CI}(T,T)\right) &= PS_1^{CI}(T,T) + CS_1^{CI}(T,T) + TR_1^{CI}(T,T) \\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(T,T) + \pi_{1H}^{CI}(T,T)\right) \\
&+ \frac{1}{2}\big(\frac{b}{2}\left(q_{1L}^{CI}(T,T) + x_{2L}^{CI}(T,T)\right)^2 \\
&+ \frac{b}{2}\left(q_{1H}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right)^2\big) \\
&+ \frac{1}{2}t_1\left(x_{2L}^{CI}(T,T) + x_{2H}^{CI}(T,T)\right), \qquad \text{(C.14)}
\end{aligned}$$

$$\begin{aligned}
\max_{t_2} E\left(SW_2^{CI}(T,T)\right) &= PS_2^{CI}(T,T) + CS_2^{CI}(T,T) - TR_2^{CI}(T,T) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(T,T) + \pi_{2H}^{CI}(T,T)\right) \\
&+ \frac{b}{2}(q_2^{CI}(T,T) + x_1^{CI}(T,T))^2 \\
&+ t_2 x_1^{CI}(T,T). \qquad \text{(C.15)}
\end{aligned}$$

The best response functions of stage two game are

$$\begin{aligned}
BR_1(t_2) &= t_1 \in \arg\max E\left(SW_1^{CI}(T,T)\right), \\
BR_2(t_1) &= t_2 \in \arg\max E\left(SW_2^{CI}(T,T)\right).
\end{aligned}$$

Solving yields Bayesian Nash equilibrium level of tariff

$$t_1^* = t_2^* = \frac{1}{3}(a-c). \qquad \text{(C.16)}$$

It is worth noting that both governments impose same amount of tariff rate since there is no information advantage of one country over another country at national level. Substituting the optimal tariff rate in equation (C.16) into best response functions of both firms, we can get the expected output in stage three as

$$\begin{aligned}
E\left(q_{1L}^{CI}(T,T)\right) &= \frac{4}{9b}(a-c) - \frac{1}{3b}V, \\
E\left(q_{1H}^{CI}(T,T)\right) &= \frac{4}{9b}(a-c) + \frac{1}{3b}V, \\
E\left(x_{1}^{CI}(T,T)\right) &= \frac{1}{9b}(a-c), \\
E\left(q_{2}^{CI}(T,T)\right) &= \frac{4}{9b}(a-c), \\
E\left(x_{2L}^{CI}(T,T)\right) &= \frac{1}{9b}(a-c) - \frac{1}{3b}V, \\
E\left(x_{2H}^{CI}(T,T)\right) &= \frac{1}{9b}(a-c) + \frac{1}{3b}V.
\end{aligned}$$

Given these, the expected social welfare for both countries can then be written as

$$E\left(SW_{1}^{CI}(T,T)\right) = \frac{65}{162b}(a-c)^2 + \frac{1}{3b}\sigma^2, \quad \text{(C.17)}$$

$$E\left(SW_{2}^{CI}(T,T)\right) = \frac{65}{162b}(a-c)^2 + \frac{1}{9b}\sigma^2. \quad \text{(C.18)}$$

Hence, $\frac{\partial E\left(SW_i^{CI}(T,T)\right)}{\partial \sigma^2} > 0$ for $i = 1, 2$. This implies that higher market volatility, both at home and abroad, enhances the expected social welfare. The stochastic term represents the gain in social welfare associated with the option value accruing to firms from being able to wait for the resolution of uncertainty. It is also worth noting that the domestic country ends up with higher expected social welfare due to higher option value effects.

Next, we turn to examine the mixed game, either (T,S) or (S,T), under complete information in the next two sub-sections.

C.3 Mixed Game (T,S)

We now turn to a scenario where country 1 imposes a tariff on firm 2's export, while country 2 subsidizes its exports. As above, the three-stage game is solved by the backward induction, beginning with the last stage.

In stage three, both firms observe the random variable ε before making their output decisions. If $\varepsilon = -V$, their problems can be written as

$$\begin{aligned}\max_{q_{1L}^{CI}(T,S),x_1^{CI}(T,S)} \pi_{1L}^{CI}(T,S) &= \left(a - b\left(q_{1L}^{CI}(T,S) + x_{2L}^{CI}(T,S)\right) - V\right) q_{1L}^{CI}(T,S) \\ &+ \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) x_1^{CI}(T,S) \\ &- c\left(q_{1L}^{CI}(T,S) + x_1^{CI}(T,S)\right), \qquad \text{(C.19)}\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(T,S),x_{2L}^{CI}(T,S)} \pi_{2L}^{CI}(T,S) &= \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) q_2^{CI}(T,S) \\ &+ \left(a - b\left(q_{1L}^{CI}(T,S) + x_{2L}^{CI}(T,S)\right) - V\right) x_{2L}^{CI}(T,S) \\ &- c\left(q_2^{CI}(T,S) + x_{2L}^{CI}(T,S)\right) \\ &+ s_2 x_{2L}^{CI}(T,S) - t_1 x_{2L}^{CI}(T,S). \qquad \text{(C.20)}\end{aligned}$$

Conversely, if $\varepsilon = V$, firms make

$$\begin{aligned}\max_{q_{1H}^{CI}(T,S),x_1^{CI}(T,S)} \pi_{1H}^{CI}(T,S) &= \left(a - b\left(q_{1H}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right) + V\right) q_{1H}^{CI}(T,S) \\ &+ \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) x_1^{CI}(T,S) \\ &- c\left(q_{1H}^{CI}(T,S) + x_1^{CI}(T,S)\right), \qquad \text{(C.21)}\end{aligned}$$

$$
\begin{aligned}
\max_{q_2^{CI}(T,S),x_{2H}^{CI}(T,S)} \pi_{2H}^{CI}(T,S) &= \left(a - b\left(q_2^{CI}(T,S) + x_1^{CI}(T,S)\right)\right) q_2^{CI}(T,S) \\
&+ \left(a - b\left(q_{1H}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right) + V\right) x_{2H}^{CI}(T,S) \\
&- c\left(q_2^{CI}(T,S) + x_{2H}^{CI}(T,S)\right) \\
&+ s_2 x_{2H}^{CI}(T,S) - t_1 x_{2H}^{CI}(T,S), \qquad \text{(C.22)}
\end{aligned}
$$

where t_1 is the tariff imposed by the domestic government on the imports from the foreign firm, and s_2 is the export subsidy granted by the foreign government to its own firm's export to market 1.

The corresponding best response functions for the above maximization problems are

$$
\begin{aligned}
BR_{1L} &= q_{1L}^{CI}(T,S) \in \arg\max \pi_{1L}^{CI}(T,S), \\
BR_{1H} &= q_{1H}^{CI}(T,S) \in \arg\max \pi_{1H}^{CI}(T,S), \\
BR_{1x} &= x_1^{CI}(T,S) \in \arg\max \pi_{1L}^{CI}(T,S), \\
BR_{2} &= q_2^{CI}(T,S) \in \arg\max \pi_{2L}^{CI}(T,S), \\
BR_{2xL} &= x_{2L}^{CI}(T,S) \in \arg\max \pi_{2L}^{CI}(T,S), \\
BR_{2xH} &= x_{2H}^{CI}(T,S) \in \arg\max \pi_{2H}^{CI}(T,S).
\end{aligned}
$$

Given these best response functions, the Nash equilibrium outputs given (t_1, s_2, V) for stage three game are

$$\begin{aligned}
q_{1L}^{CI}(T,S) &= \frac{1}{3b}(a-c-V-s_2+t_1),\\
q_{1H}^{CI}(T,S) &= \frac{1}{3b}(a-c+V-s_2+t_1),\\
x_1^{CI}(T,S) &= \frac{1}{3b}(a-c),\\
q_2^{CI}(T,S) &= \frac{1}{3b}(a-c),\\
x_{2L}^{CI}(T,S) &= \frac{1}{3b}(a-c-V+2s_2-2t_1),\\
x_{2H}^{CI}(T,S) &= \frac{1}{3b}(a-c+V+2s_2-2t_1).
\end{aligned}$$

In stage two, the expected social welfare for the domestic country is the sum of producer's surplus, consumer's surplus and tariff revenue, while the expected social welfare for the foreign country is the sum of producer's surplus and consumer's surplus net of total subsidy. Government 1 chooses t_1 and government 2 chooses s_2 to maximize their expected social welfare:

$$\begin{aligned}
\max_{t_1} E\left(SW_2^{CI}(T,S)\right) &= PS_1^{CI}(T,S)+CS_1^{CI}(T,S)+TR_1^{CI}(T,S)\\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(T,S)+\pi_{1H}^{CI}(T,S)\right)\\
&+ \frac{1}{2}(\frac{b}{2}\left(q_{1L}^{CI}(T,S)+x_{2L}^{CI}(T,S)\right)^2\\
&+ \frac{b}{2}\left(q_{1H}^{CI}(T,S)+x_{2H}^{CI}(T,S)\right)^2)\\
&+ \frac{1}{2}t_1\left(x_{2L}^{CI}(T,S)+x_{2H}^{CI}(T,S)\right), \qquad \text{(C.23)}
\end{aligned}$$

$$
\begin{aligned}
\max_{s_2} E\left(SW_2^{CI}(T,S)\right) &= PS_2^{CI}(T,S) + CS_2^{CI}(T,S) - S_2^{CI}(T,S) \\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(T,S) + \pi_{2H}^{CI}(T,S)\right) \\
&+ \frac{b}{2}(q_2^{CI}(T,S) + x_2^{CI}(T,S))^2 \\
&- \frac{1}{2}\left(x_{2L}^{CI}(T,S) + x_{2H}^{CI}(T,S)\right) s_2. \qquad \text{(C.24)}
\end{aligned}
$$

The best response functions of stage two game are

$$
\begin{aligned}
BR_1(s_2) &= t_1 \in \arg\max E\left(SW_1^{CI}(T,S)\right), \\
BR_2(t_1) &= s_2 \in \arg\max E\left(SW_1^{CI}(T,S)\right).
\end{aligned}
$$

Solving yields

$$
t_1^* = \frac{5}{14}(a-c), \qquad \text{(C.25)}
$$

$$
s_2^* = \frac{1}{14}(a-c). \qquad \text{(C.26)}
$$

This implies that the optimal tariff imposed by government 1 and the optimal export subsidy set by government 2 are positive. Moreover, the tariff imposed by government 1 is higher than the export subsidy granted by government 2. With these optimal policy levels, we can obtain the expected Nash equilibrium outputs for both firms:

$$
\begin{aligned}
E\left(q_{1L}^{CI}(T,S)\right) &= \frac{3}{7b}(a-c) - \frac{1}{3b}V, \\
E\left(q_{1H}^{CI}(T,S)\right) &= \frac{3}{7b}(a-c) + \frac{1}{3b}V, \\
E\left(x_1^{CI}(T,S)\right) &= \frac{1}{3b}(a-c), \\
E\left(q_2^{CI}(T,S)\right) &= \frac{1}{3b}(a-c), \\
E\left(x_{2L}^{CI}(T,S)\right) &= \frac{1}{7b}(a-c) - \frac{1}{3b}V, \\
E\left(x_{2H}^{CI}(T,S)\right) &= \frac{1}{7b}(a-c) + \frac{1}{3b}V.
\end{aligned}
$$

Moreover, the expected social welfare for home and foreign countries are given by

$$E\left(SW_1^{CI}(T,S)\right) = \frac{449}{882b}(a-c)^2 + \frac{1}{3b}\sigma^2, \quad \text{(C.27)}$$
$$E\left(SW_2^{CI}(T,S)\right) = \frac{101}{294b}(a-c)^2 + \frac{1}{9b}\sigma^2. \quad \text{(C.28)}$$

It is straightforward to verify that $\frac{\partial E\left(SW_i^{CI}(T,S)\right)}{\partial \sigma^2} > 0$ for $i = 1,2$. That is, social welfare under mixed policies (T,S) increases with market volatility.

C.4 Mixed Game (S,T)

Finally, we examine the last possible pair of strategies chosen by the governments in stage one under complete information. That is, country 1 subsidizes its exports to market 2, while country 2 imposes a tariff on imported goods from firm 1.

If the true state in market 1 is $-V$, the profit functions for each firm are

$$\begin{aligned}\max_{q_{1L}^{CI}(S,T),x_1^{CI}(S,T)} \pi_{1L}^{CI}(S,T) &= \left(a - b\left(q_{1L}^{CI}(S,T) + x_{2L}^{CI}(S,T)\right) - V\right) q_{1L}^{CI}(S,T) \\ &+ \left(a - b\left(q_2^{CI}(S,T) + x_1^{CI}(S,T)\right)\right) x_1^{CI}(S,T) \\ &- c\left(q_{1L}^{CI}(S,T) + x_1^{CI}(S,T)\right) \\ &+ s_1 x_1^{CI}(S,T) - t_2 x_1^{CI}(S,T), \quad \text{(C.29)}\end{aligned}$$

$$\begin{aligned}\max_{q_2^{CI}(S,T),x_{2L}^{CI}(S,T)} \pi_{2L}^{CI}(S,T) &= \left(a - b\left(q_2^{CI}(S,T) + x_1^{CI}(S,T)\right)\right) q_2^{CI}(S,T) \\ &+ \left(a - b\left(q_{1L}^{CI}(S,T) + x_{2L}^{CI}(S,T)\right) - V\right) x_{2L}^{CI}(S,T) \\ &- c\left(q_2^{CI}(S,T) + x_{2L}^{CI}(S,T)\right). \quad \text{(C.30)}\end{aligned}$$

On the other hand, if the true state in market 1 is V, the profit functions are

$$\begin{aligned}
\max_{q_{1H}^{CI}(S,T),x_1^{CI}(S,T)} \pi_{1H}^{CI}(S,T) &= \left(a - b\left(q_{1H}^{CI}(S,T) + x_{2H}^{CI}(S,T)\right) + V\right) q_{1H}^{CI}(S,T) \\
&+ \left(a - b\left(q_2^{CI}(S,T) + x_1^{CI}(S,T)\right)\right) x_1^{CI}(S,T) \\
&- c\left(q_{1H}^{CI}(S,T) + x_1^{CI}(S,T)\right) \\
&+ s_1 x_1^{CI}(S,T) - t_2 x_1^{CI}(S,T), \qquad \text{(C.31)}
\end{aligned}$$

$$\begin{aligned}
\max_{q_2^{CI}(S,T),x_{2H}^{CI}(S,T)} \pi_{2H}^{CI}(S,T) &= \left(a - b\left(q_2^{CI}(S,T) + x_1^{CI}(S,T)\right)\right) q_2^{CI}(S,T) \\
&+ \left(a - b\left(q_{1H}^{CI}(S,T) + x_{2H}^{CI}(S,T)\right) + V\right) x_{2H}^{CI}(S,T) \\
&- c\left(q_2^{CI}(S,T) + x_{2H}^{CI}(S,T)\right), \qquad \text{(C.32)}
\end{aligned}$$

where s_1 is the export subsidy granted by government 1 on firm 1's export to market 2, and t_2 is the tariff rate imposed by government 2 on firm 1's export to market 2.

The best response functions for above optimization problems are

$$\begin{aligned}
BR_{1L} &= q_{1L}^{CI}(S,T) \in \arg\max \pi_{1L}^{CI}(S,T), \\
BR_{1H} &= q_{1H}^{CI}(S,T) \in \arg\max \pi_{1H}^{CI}(S,T), \\
BR_{1x} &= x_1^{CI}(S,T) \in \arg\max \pi_{1L}^{CI}(S,T), \\
BR_2 &= q_2^{CI}(S,T) \in \arg\max \pi_{2L}^{CI}(S,T), \\
BR_{2xL} &= x_{2L}^{CI}(S,T) \in \arg\max \pi_{2L}^{CI}(S,T), \\
BR_{2xH} &= x_{2H}^{CI}(S,T) \in \arg\max \pi_{2H}^{CI}(S,T).
\end{aligned}$$

This gives

$$\begin{aligned}
q_{1L}^{CI}(S,T) &= \frac{1}{3b}(a-c-V),\\
q_{1H}^{CI}(S,T) &= \frac{1}{3b}(a-c+V),\\
x_{1}^{CI}(S,T) &= \frac{1}{3b}(a-c+2s_1-2t_2),\\
q_{2}^{CI}(S,T) &= \frac{1}{3b}(a-c-s_1+t_2),\\
x_{2L}^{CI}(S,T) &= \frac{1}{3b}(a-c-V),\\
x_{2H}^{CI}(S,T) &= \frac{1}{3b}(a-c+V).
\end{aligned}$$

These are Nash equilibrium outputs across states.

In stage two, government 1 chooses s_1 and government 2 chooses t_2 to maximize their expected social welfare given by

$$\begin{aligned}
\max_{s_1} E\left(SW_1^{CI}(S,T)\right) &= PS_1^{CI}(S,T)+CS_1^{CI}(S,T)-S_1^{CI}(S,T)\\
&= \frac{1}{2}\left(\pi_{1L}^{CI}(S,T)+\pi_{1H}^{CI}(S,T)\right)\\
&+ \frac{1}{2}(\frac{b}{2}\left(q_{1L}^{CI}(S,T)+x_{2L}^{CI}(S,T)\right)^2\\
&+ \frac{b}{2}\left(q_{1H}^{CI}(S,T)+x_{2H}^{CI}(S,T)\right)^2)\\
&- s_1 x_1^{CI}(S,T), \qquad \text{(C.33)}
\end{aligned}$$

$$\begin{aligned}
\max_{t_2} E\left(SW_2^{CI}(S,T)\right) &= PS_2^{CI}(S,T)+CS_2^{CI}(S,T)+TR_2^{CI}(S,T)\\
&= \frac{1}{2}\left(\pi_{2L}^{CI}(S,T)+\pi_{2H}^{CI}(S,T)\right)\\
&+ \frac{b}{2}(q_2^{CI}(S,T)+x_1^{CI}(S,T))^2\\
&+ t_2 x_1^{CI}(S,T). \qquad \text{(C.34)}
\end{aligned}$$

The best response functions for stage two game are

$$\begin{aligned} BR_1(t_2) &= s_1 \in \arg\max E\left(SW_1^{CI}(S,T)\right), \\ BR_2(s_1) &= t_2 \in \arg\max E\left(SW_2^{CI}(S,T)\right). \end{aligned}$$

Solving gives us Bayesian Nash equilibrium level of policy rate

$$s_1^* = \frac{1}{14}(a-c), \tag{C.35}$$

$$t_2^* = \frac{5}{14}(a-c). \tag{C.36}$$

With no surprise, the optimal policy rates for export subsidy and tariff is identical that under (T, S). Given s_1^* and t_2^*, the expected outputs for both firms can be obtained as follows:

$$\begin{aligned} E\left(q_{1L}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c-V), \\ E\left(q_{1H}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c+V), \\ E\left(x_1^{CI}(S,T)\right) &= \frac{1}{7b}(a-c), \\ E\left(q_2^{CI}(S,T)\right) &= \frac{3}{7b}(a-c), \\ E\left(x_{2L}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c-V), \\ E\left(x_{2H}^{CI}(S,T)\right) &= \frac{1}{3b}(a-c+V). \end{aligned}$$

The corresponding social welfare for both countries are

$$E\left(SW_1^{CI}(S,T)\right) = \frac{101}{294b}(a-c)^2 + \frac{1}{3b}\sigma^2, \tag{C.37}$$

$$E\left(SW_2^{CI}(S,T)\right) = \frac{449}{882b}(a-c)^2 + \frac{1}{9b}\sigma^2. \tag{C.38}$$

Again, we have $\frac{\partial E\left(SW_i^{CI}(S,T)\right)}{\partial \sigma^2} > 0$ for $i = 1, 2$. In other words, social welfare under mixed policies (S, T) is also enhanced with higher variance. This together with the sub-game equilibrium analysis under (T, S), we can conclude that the social welfare increases with market volatility for mixed games under complete information.

These equilibrium social welfare levels in different sub-games under complete information are used to construct choice of policy game in section 3.4.