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A Finite Element Model to Study the Effect of Porosity Location on the Elastic Modulus of a Cantilever Beam

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Abstract— The effect of the location of porosity concentration on elastic modulus of a cantilever beam is investigated. First, two-dimensional investigation with beam theory, Euler-Bernoulli and Timoshenko, was performed to estimate the modulus based on load-deflection curve. Second, three-dimensional finite element model (FEM) in Abaqus was developed to identify the effect of porosity concentration. The use of macro-models such as beam theory and three-dimensional FEM enabled enhanced understanding of the effect of porosity on modulus.

Keywords-porosity, elastic modulus, FEA, micro-mechanical, cantilever

I. INTRODUCTION

A. Background

It is known that porosity affects the mechanical properties of metals. In many materials, increases in macroscale pore sizes have shown to decrease ultimate strength, yield stress, and fatigue life [1]. However, due to advancements in manufacturing, pores in metals tend to be on the microscale instead of macroscale. This presents new concerns since the effect on a material's mechanical properties due to this microporosity is unknown [2]. This is critical because without understanding how microporosity affects a material, the ability to predict behaviour due to loading throughout its life cycle is difficult. Throughout the life cycle of a structure, exposure to various environmental conditions, sometimes harsh, is possible. Due to these environmental conditions, it is possible that porosity can be increased in the material. As such, structures may be affected in various locations, and in differing amounts, depending on the exposure to the environment. It is critical to understand if the location of porosity has an effect on a structure.

B. Purpose of Study

It is known that porosity has an effect on elastic modulus. The work of Morrissey and Nakhla [3] presented a literature review on existing models available in literature. These models, mostly empirical, describe the effect of porosity on elastic modulus. Morrissey and Nakhla developed a two-dimensional finite element model (FEM) that successfully captured the effect of porosity on elastic modulus in tension.

In the current work, a three-dimensional FEM is developed to investigate the effect of porosity on modulus. The effects of uniform distribution or concentrated zones of porosity was investigated. All FEM results were compared to test data reported in literature.

II. PROCEDURES

A. Understanding Experimental Setup and Data

The first step in this study was to examine experimental load versus deflection data for micro-cantilevers. For this study, the work by Gong [4] was first analyzed to understand the correct beam theory to apply for determining elastic modulus, as well as to develop the three-dimensional FEM. It is reported in [4], for beam 5, an experimental elastic modulus of 147 GPa. As well, their three-dimensional FE model captured the trend of porosity reduction with an average error in prediction of 38% compared to experimental.

It is reported in [4] that samples were heat treated such that an average grain size of 8-10 µm was obtained. From these samples, the micro-cantilevers were produced at the University of California, Berkeley (UCB) using a focused ion beam (FIB). The FIB was used to cut three trenches using a 7-15 nA beam current – forming a U-shaped trench that had a width of 20-30 µm and a depth of 10 µm. Then using a 1-3 nA beam current, the outline of the beam was refined. Lastly, the sample was rotated 45° both clockwise and counter-clockwise around the length of the beam to allow for cutting of the triangular bottom of the beam. Using a MicroMaterials nanoindenter with a square tip, UCB was able to obtain the load and deflection data for the micro-cantilevers. The depth of indentation into bulk material was removed from the experimental deflection to ensure only the displacement due to bending is accounted. Lastly, the load was applied with a displacement rate of 10 nm/s to the tip of the micro-cantilever until fracture.

The micro-cantilever is approximately 7.5 µm tall, 4 µm wide, and 28 µm long – load is applied at approximately 27 µm from the root, this will be taken to be the length since deflection is also measured at this location. The cross-section is pentagonal shaped, proposed in [5]. The beam is considered to be short and stubby with a length-to-height ratio of less than four. The beam is not undergoing uniform bending (concentrated load introduces shear forces) and the beam is not rigidly connected to the support. Due to these factors, shear effects may be highly pronounced at the root. Therefore, both Euler-Bernoulli and Timoshenko beam theories were used in the current study.

Using the load-deflection curve reported in [4], the data was extracted and the slope for the linear section was determined to be approximately 1932 N/m. Next, the modulus was calculated using Euler-Bernoulli and Timoshenko beam theory, resulting in 148 GPa and 154 GPa, respectively. This is compared to the reported modulus in [4] in Table I below. Due to the beam geometry, boundary conditions, and the manner at which the load is applied, it is most likely that Timoshenko beam theory is more accurate because the effect due to shear is included. As such, Timoshenko beam theory will be the method of calculating the elastic modulus within this study.

TABLE I. COMPARISON OF CALCULATED MODULUS

Beam Theory	This Study	Reported in [4]
Euler-Bernoulli	148 GPa	147 GPa
Timoshenko	154 GPa	-

For this study, several assumptions are made. Uranium Dioxide is highly anisotropic [6-7]; however, it is assumed that the material acts as an isotropic material since the micro-cantilevers are ideally contained within a single crystal-grain. It is reported in [4] that not all micro-cantilevers are within a single grain; however, without additional information on number of grains and grain orientation, the assumption will remain. The FEM assumes the beam is solid, homogeneous and has a constant cross-section. Lastly, it is assumed that the effect on Poisson's ratio for porosities less than 5% is negligible [8-9]. The FEM assumes uniform porosity distribution across the cross-section.

B. Finite Element Model – Three-Dimensional Beam

To build the FEM, a three-dimensional, deformable solid part was created in Abaqus. The substrate was sketched and extruded to create a cube. From the front face, the geometry was sketched and extruded to create the beam. The actual beam from [4] and the currently developed FEM is shown in Fig.1. The beam and substrate were then partitioned to allow for separate modification of material properties and mesh development. The beam was further partitioned into three segments of equal length, shown in Fig. 2. This allows for different material properties to be applied to each segment. The next step was to develop the mesh. The mesh was refined differently within the beam than the substrate. The beam had 11,088 3D Stress Hex Quadratic (Reduced Integration) elements, while the substrate had 4,464 3D Stress Hex Quadratic (Reduced Integration) elements.

The next step was to apply boundary conditions to the FEM. The side, rear, and bottom faces of the substrate have fixed boundary condition, while the top and front faces, as well as the beam, are free surfaces, as shown in Fig. 3 (lighter colours indicate free while dark indicate fixed). Next, a tip load was applied to the beam. The deflection is measured from the bottom side, directly under the location of the applied load, to ensure indentation into the top surface of the beam due to the load was not accounted, thus skewing the deflection data.

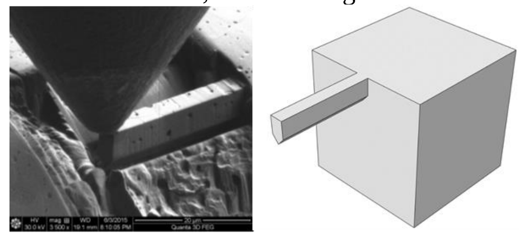

Figure 1. SEM Picture of Experimental Setup [4] (left), Abaqus FEM this study (right)

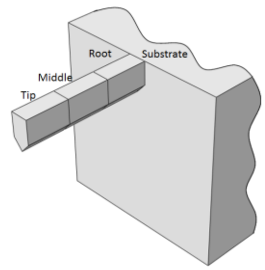

Figure 2. Beam Sections (From left to right: Tip, Middle, Root, Substrate).

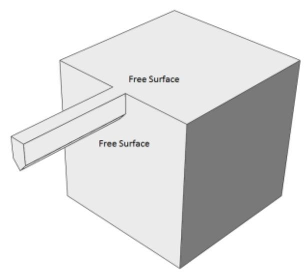

Figure 3. Boundary Conditions (Top, Front, and Beam are free surfaces; all other sides fixed).

Once the FEM was developed, the first test was to determine the calculated modulus of a perfect beam with nominal modulus and zero porosity. This was to provide a baseline for the predicted modulus due to the boundary conditions alone. However, it was unsure if the size of the substrate would have an effect on the results and as such, the FEM was first tested with various substrate sizes.

The substrate was constrained to be a solid cube in each case. Four substrate sizes were considered and the dimensions were scaled due to the beam's largest dimension – height. Sizes were labelled A through D, with A being the smallest and D the largest. Substrate size A was approximately the same size as the height of the beam. Substrate sizes B, C, and D were approximately one-and-a-half times, three-times, and six-times larger than the beam height, respectively. A comparison of the moduli obtained for the four cases is shown below in Table II. As can be seen, the size of the substrate does affect the response of the beam; however, as long as the substrate is at least three-times the height of the beam, the effect is insignificant. This is expected since when the substrate is small, the fixed boundary condition has a greater effect on the rigidity of the root of the beam, thus, stiffening the beam and over-predicting the modulus. As the substrate size is increased, this effect is reduced to a point such that the modulus is unchanged since the fixed boundary condition is sufficiently far enough away from the root of the beam. For this study, Substrate size D was chosen for the FEM to ensure the fixed boundary condition did not influence the results.

TABLE II. COMPARISON OF SUBSTRATE SIZE

Substrate Size	Modulus Calculated (GPa)
A	186.7
B	182
C	180
D	180

With the size of the substrate determined, the modulus obtained due to the boundary conditions alone was 18% less than the nominal value. This will be considered the base case value and the effect on modulus due to porosity amount and location will be compared.

C. Finite Element Model – Reduction in Modulus due to Porosity under Bending

Before an analysis can be completed, a FEM must first be developed to determine the percent reduction in elastic modulus versus the position of porosity concentration along the beam, similar to [3]. A two-dimensional FEM was developed to determine this reduction. The FEM was a long, slender beam, with uniform cross-section, and rigidly supported at the root.

The FEM is assumed to have a nominal elastic modulus with zero pores to establish a base case. Next, the elastic modulus is calculated from load and deflection data at the tip. Using this calculated modulus, it was normalized with the nominal value to provide a percent reduction in modulus. This was repeated with various pore locations and pore sizes to simulate various porosities and porosity concentration locations. The results of this FEM are shown below in Fig. 4.

As the porosity concentration moves further away from the root, the percent reduction in elastic modulus decreases. These percent reduction values were applied to the nominal elastic modulus for Uranium Dioxide and new moduli were calculated for when porosity is concentrated within different sections of the beam. A similar process was completed for when the porosity is uniformly distributed over the entire length of the beam. In this case, the percent reduction was approximately equal to the reduction experienced when porosity was concentrated at the tip for both porosities.

Next, these reduced moduli were imported into the three-dimensional FEM for the various setups. These moduli were applied to specific sections of the beam where the porosity was to be concentrated while the remaining sections of the FEM, including the substrate, were considered to be equal to the nominal modulus of 219 GPa for Uranium Dioxide [10].

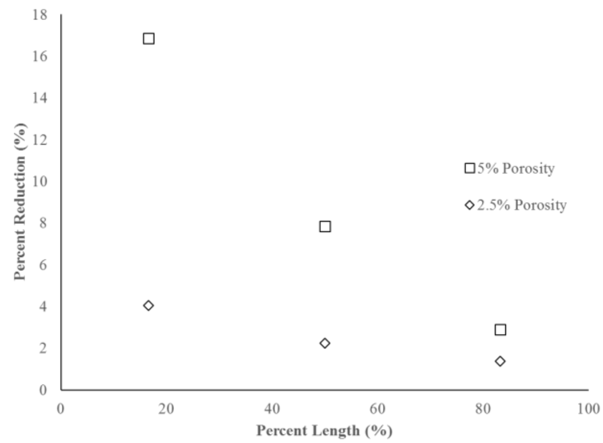

Figure 4. Percent reductions in modulus due to porosity concentration along percent length of the beam.

III. RESULTS AND DISCUSSION

The results for moving the porosity concentration from the root to the tip is shown in Table III. When the porosity is concentrated at the root, the reduction in modulus is the greatest, while moving towards the tip this reduction decreases. This trend is further supported by the Reduction in Modulus due to Porosity under bending FEM. Furthermore, Gong has successfully demonstrated in [4] that the location of pore concentration does affect the modulus, with the largest effect at the root and decreasing away from the root.

When comparing the percent reductions determined from the two-dimensional and three-dimensional FE models in this study, the values differ. There are several reasons for these discrepancies: 1) different applied boundary conditions, 2) different cross-section, thus different second moments of area – three-dimensional FEM is pentagonal whereas two-dimensional represents rectangular, and 3) general errors converting from two-dimensional to three-dimensional FEM.

When comparing to Gong's results, there is a large difference in the values. There are several reasons for these discrepancies: 1) pore location relative to neutral plane in Single Pore FEM [4] is unknown – pores further away will have a larger effect on reducing modulus, 2) pore distribution is not uniform across cross-section in Cluster Pore FEM [4] – pores constrained within rectangular-portion of the beam (no pores located in the triangular section), and 3) uncertain boundary conditions – Gong initially completed a substrate-size sensitivity-analysis, but for the FEM of the reconstructed beam, the substrate appears to be approximately the same size as the beam which would greatly influence the results due to the boundary conditions stiffening the beam.

In Fig. 5, the Abaqus FE models completed for this study are compared with experimental load versus deflection data reported in [4]. From this, it is evident that the FE models capture the trend of the experimental results from [4] with average error of 14.8%. However, there is some error which can be contributed to the assumptions made in this study. The FE models completed in this study assumed a uniform, constant cross-section, free of imperfections, which is not the case when observing SEM images of the beam in [4]. As well, the FE models were isotropic due to the single-grain assumption, but as reported in [4] this was not true. Lastly, the porosity concentration in the FE models completed in this study do not include the effect of pores away from the neutral plane – it is assumed the porosity is concentrated uniformly across the cross-section with no bias away from the neutral plane.

In Fig. 6, all FE models by this study and by Gong are compared with the experimental data. It is evident that Gong's Single Pore Root FEM has the largest effect, comparable to this study's FEM for 5% porosity concentrated at the root. The placement of this single pore from the neutral plane is unknown and as such it is difficult to compare to the FE models completed in this study.

In Fig. 7, all 2.5% porosity models are compared with the experimental data. Again, Gong's Single Pore Root FEM has the largest effect on the load-deflection response. However, when comparing the Cluster Pore FE models from [4] to the FE models completed in this study, it is evident that the Cluster Pore FE models are over-predicting the elastic modulus. This may be due to the orientation chosen by Gong to model the cluster of pores. This orientation differs from the assumed uniform porosity concentration for the FE models completed in this study, hence the lower reductions.

In Fig. 8, a comparison is shown between Gong's experimental data and FE models. Gong's FE models show a much better agreement to the experimental data than is reported in [4] with an average error of 21.8% with experimental.

The FE models completed in this study, which assume uniform porosity distribution across the cross-section, show with certainty that porosities of 5% have a large effect on the behaviour of the beam when concentrated close to the root. However, if these large porosities are uniformly distributed over the length, or concentrated in a location away from the root – at or beyond half the beam length – the effect decreases drastically. Whereas, for porosities of 2.5% and lower, it can be concluded that there is minimal effect on the beam's modulus regardless of distribution and concentration throughout the length.

IV. CONCLUSIONS

This study has proven that the amount of porosity and the location of said porosity has an effect on the elastic modulus. Several cases were analyzed and it was determined that porosities concentrated at the root have the largest effect on the elastic modulus of a cantilever, while porosities uniformly distributed over the length, or concentrated away from the root, have minimal effect on elastic modulus.

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TABLE III. REDUCTION IN MODULUS DUE TO POROSITY

Finite Element Model	Porosity	Reduction in Modulus (2D FEM)	Reduction in Modulus (3D FEM)	Reductions in Modulus Reported in [4][a]	
				Single Large Pore[b]	***Cluster of Pores***[c]
Perfect Beam	0%	0%	0%	0%	
Root	2.5%	-3.0%	-1.38%	-20%	-4.5%
	5.0%	-15.8%	-7.82%	-	-
Middle	2.5%	-1.2%	-0.23%	-8.6%	-2.7%
	5.0%	-6.8%	-1.25%	-	-
Tip	2.5%	-0.3%	-0.01%	-8.6%	-2.7%
	5.0%	-1.8%	-0.06%	-	-
Entire	2.5%	-0.3%	-0.26%	-	-
	5.0%	-1.1%	-0.79%	-	-

a.Values reported in [4] differ from reported load-deflection values in [4], evident when plotted in Figures

b. Location of Pore relative to neutral axis unknown.

c. Pore Clusters occupy only rectangular cross-section, no pores in triangular portion.

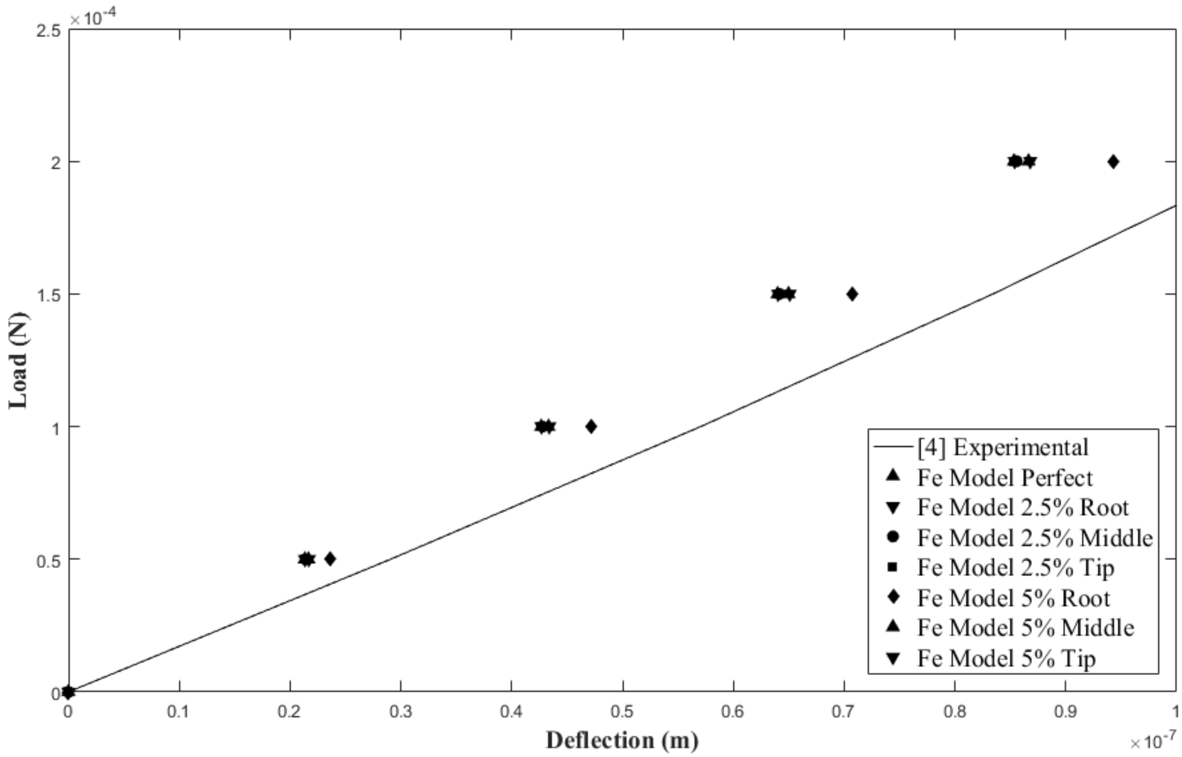

Figure 5. Load-Deflection Comparison of This Study with [4] Experimental.

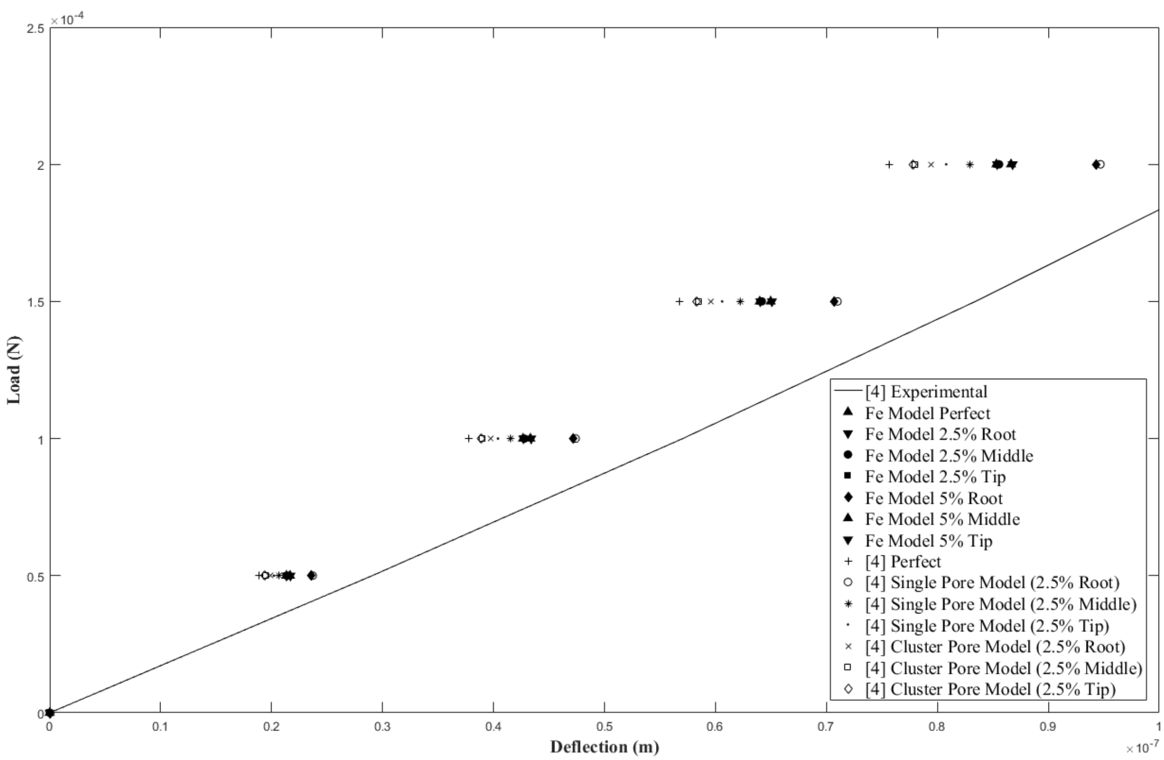

Figure 6. Load-Deflection Comparison of All Data.

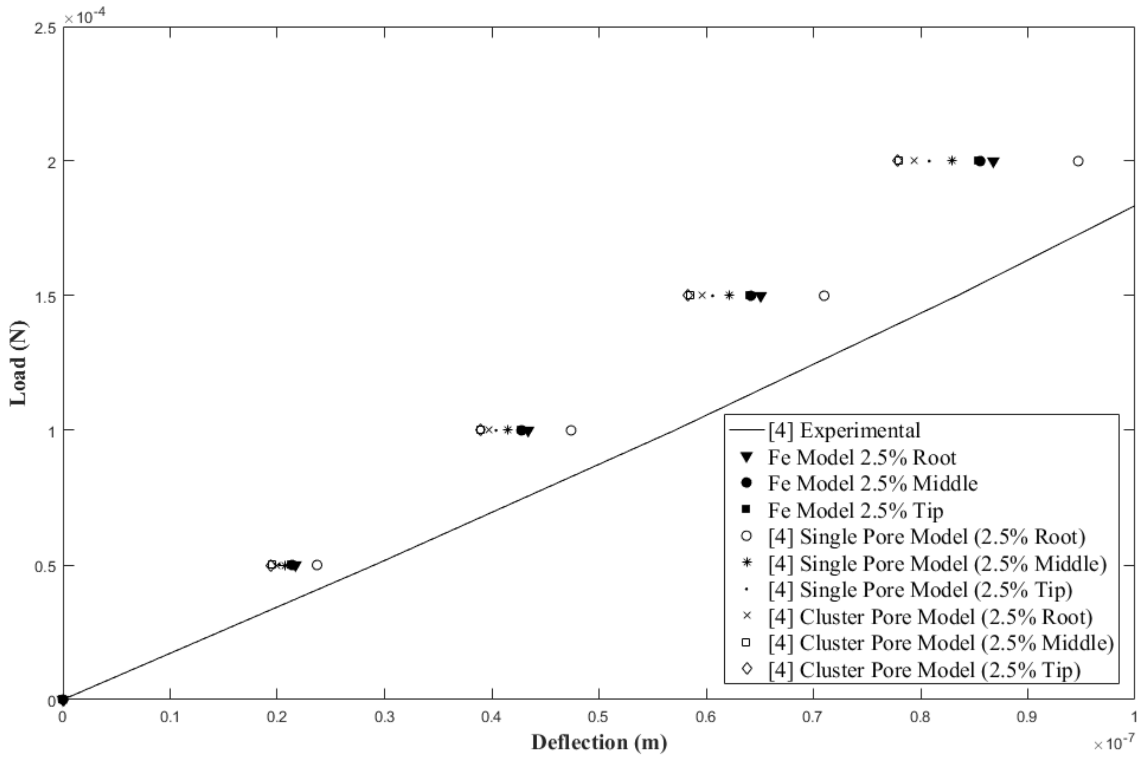

Figure 7. Load-Deflection Comparison of All 2.5% Porosity FE Models with Experimental [4].

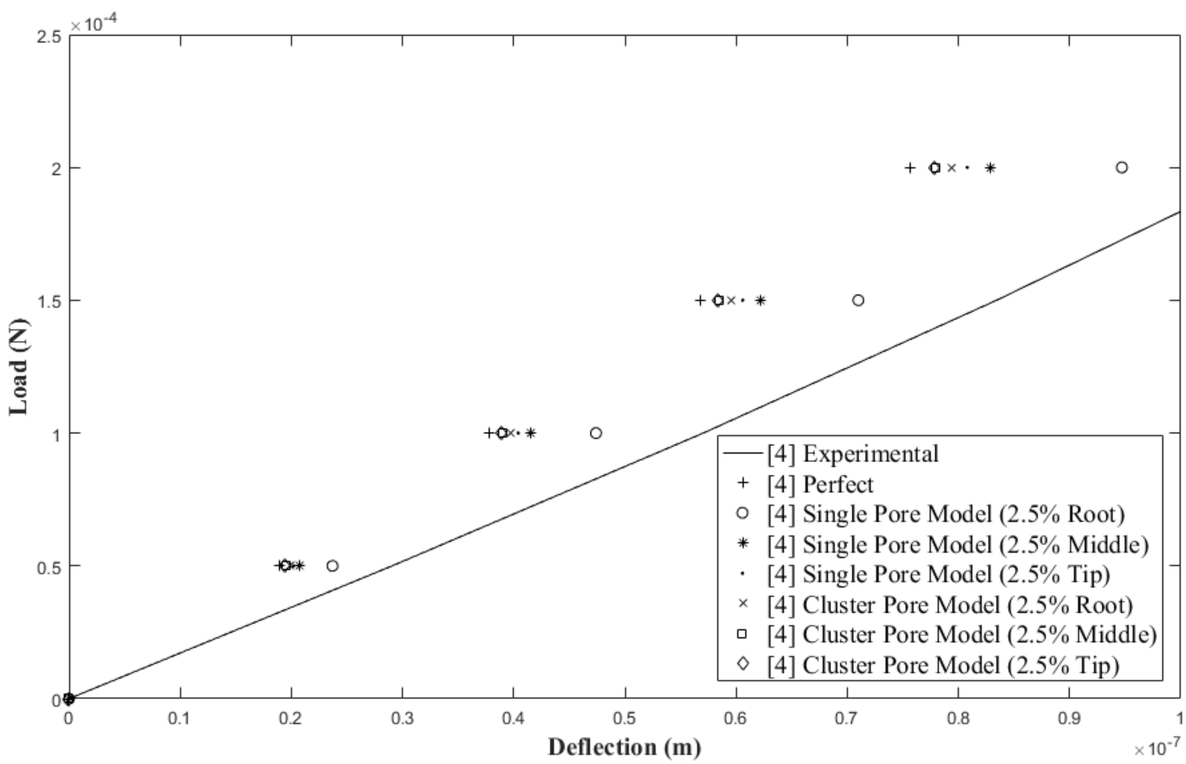

Figure 8. Load-Deflection Comparison of [4] FE Models and Experimental Data.