ESSAYS ON PREFERENTIAL TRADE AGREEMENTS UNDER UNCERTAINTY

Shafkat Ali

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ABSTRACT

This dissertation studies the formation of preferential trade agreements using a coalition formation approach in both certain and uncertain frameworks. It is at the intersection of international trade and cooperative game theory.

In chapter 2 we consider a three-country model of oligopoly and trade under demand uncertainty. We endogenize the coalition structure that forms in a three stage game. We find that for small volatilities countries prefer global free trade. The more positively correlated two countries are the more likely they are to form a customs union. We also find that countries may wish to stand alone under certain variance-covariance configurations.

In chapter 3 we add exogenous trade costs under both certainty and uncertainty. We find that trade costs critically affect choice of output by firms and choice of tariffs and coalitions by governments. With symmetric trade costs as trade costs vary we find different coalitions forming in equilibrium. The introduction of demand uncertainty affects coalition choices by changing the cutoff trade costs at which a country may be indifferent between two different coalitions. Further, coalitions that may form under certainty or low uncertainty may not form with high uncertainty. On the other hand under different configuration of trade costs coalitions that may not be feasible under certainty may be shown to be possible under uncertainty. In both cases, as long as trade costs are not prohibitive, as volatility in every market increases without bound, we get global free trade with probability one. As a special case we show that under

certain conditions two geographically distant countries may choose to form a coalition excluding a nearby country if the market volatility and correlation between partner countries is high enough.

To my parents.

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Chapter 1
Introduction

The aim of this research is to study the process of formation of customs unions using a coalition formation approach. We hope to bring together two areas of microeconomics, the theory of customs unions in international trade and the endogenous formation of coalitions from cooperative game theory. With the recent proliferation of regional trade agreements the customs union issue has become topical. Bhagwati has been at the forefront of bringing the issue of regionalism in the limelight. He has raised the question whether regional trading blocs are building blocks or stumbling blocks towards global free trade. He expressed the opinion that this second wave of regionalism, unlike the first in the sixties and seventies was temporary. It has not been rigorously established in the literature whether Preferential Trading Agreements (PTA's) are stepping stones towards free trade.

It was Jacob Viner who initiated the theory of regional trade agreements as a separate field in economic theory in his classic work 'The Customs Union Issue'. Viner noted that the formation of customs unions had two effects: trade creation and trade diversion. The former refers to moving production from high-cost non-member country to low-cost member country while the latter refers to moving production from low-cost non-member country to high-cost member country. The former he thought of as welfare enhancing and the latter as welfare reducing although latter analysis showed that trade diversion was not necessarily welfare reducing. Authors such as Viner, Lipsey and Meade were concerned with the static welfare effects of customs unions. Bhagwati [1993] initiated what he called the dynamic time path question relating to the continued reductions in trade barriers asking whether PTA's were building blocks or stumbling blocks to worldwide freeing of trade.

The theory of PTAs is in some sense a special case of the theory of coalition formation. In the game theoretic context, the problem of coalition formation was first explicitly considered by von Neumann and Morgenstern in their seminal work on game theory. During the initial stage research was conducted using the characteristic function approach where the worth of each coalition depends on the members of the coalition only. One of the first papers to consider the endogenous formation of coalitions is Hart and Kurz[1983]. Prior to their work it was assumed that the coalition structure was given exogenously and attention was focused on splitting the surplus. In their paper the authors obtain the coalition structure as an endogenous outcome of their model. The existence of coalitions implies that interactions between coalitions will be conducted at two levels: among coalitions and within each coalition. Hart and Kurz establish a valuation criterion for each individual player in each given coalition structure and proceed to study various stability concepts based on this criterion.

A strand of literature relevant to this paper is the non-cooperative theory of coalition formation initiated by Bloch, Ray and Vohra and Yi. In these papers the common underlying theme is the analysis of equilibrium coalition structures. Bloch [1996]. examines an infinite-horizon Coalition Unanimity Game in which a coalition forms if and only if all potential members agree to form the coalition. Ray and Vohra [1997] study the Equilibrium Binding Agreements rule under which coalitions are allowed to break up into smaller subcoalitions only. Yi [1996] investigates the Open Membership game in which non-members can join an existing coalition without the permission of the existing members. The first explicit link between customs unions and coalitions seems to have been Riezman[1985]. He modelled customs union formation as a two-stage game. In the first stage countries make coalitional choices according to core theory. In the second stage optimal tariffs are determined. The model is that of a pure exchange economy where which coalition form depends crucially upon the initial endowments, through examples he goes on to show that a customs union can be a equilibrium even when both countries do better at free trade. A more substantial model is that of Yi[1996]. He conducts his analysis in a framework of monopolistic competition with ex ante similar countries. Based on certain simplifying assumptions he obtains the payoff to each country as a function of the coalition structure alone. It should be mentioned that the aforementioned model is a special case of coalition structures with externalities, i.e. where members outside the coalition are either positively or negatively affected by the formation of the coalition. Since the formation of customs unions is likely to impact the countries outside the union, coalitional games with spillovers seem the appropriate model to use. Here the payoffs to players is given by a partition function rather than a characteristic function.

Although these models give valuable insight into the customs unions issue they still leave some questions unanswered. For example Yi [1996] obtains the per member partition function in his model of customs unions but exactly which unions will form does not seem clear if the rule of formation is not open membership. The theory of coalition formation under uncertainty seems to be a somewhat neglected topic, at least as far as international trade is concerned. The only paper that seeks to explain the existence of customs unions as consequence of uncertainty is Fries[1984]. The source of uncertainty there is in commodity prices and domestic output in a framework of incomplete markets. Uncertainty in the form of correlated markets is a well-studied phenomena in industrial organization. Chiang and Brown [2002]show that if firm's demand is subject to additive random disturbances and these disturbances are correlated then this will have an effect on the coalitions firms form. The idea in this thesis is to incorporate correlated markets across countries and see what effects this will have on the coalitions countries form.

In Chapter 2 we will take a three-country model to examine the welfare effects of the formation of customs unions. The idea is to embed the problem into a class of more general problems: the theory of coalition formation. This poses several conceptual difficulties. One has to specify the rules of formation, whether the process is simultaneous or sequential, the solution concepts, etc. That is, all the technical problems that plague cooperative game theory with the additional complexity that the structure of the world trading system adds. Even though global free trade maximizes aggregate welfare, we see the existence of trading blocs. The first question that comes to mind is: why do they exist ? Various arguments such as political economy considerations, geographical proximity, etc. have been proposed. After establishing the baseline model, we introduce uncertainty in the form of correlated markets in different countries and attempt to show that market volatility and correlation can impact the formation of customs unions.

The central force driving the results is option value. It was Appelbaum and Melatos [2012] (henceforth AM) who first put forward the idea that trade agreements have option value. This paper builds on their idea and it would be an understatement to say we owe them a huge intellectual debt. A trade agreements option value reflects the value of being able to make some decisions after uncertainty is resolved. AM argue that the introduction of uncertainty fundamentally alters the cost-benefit analysis associated with trade agreement formation that goes beyond insurance considerations already considered in the literature. According to them, while all trade agreements have option value, these option values differ across agreements because trade agreements are characterized by different rules and imply different behaviour. AM consider three types of trade agreements: 1) Stand Alone, where each can choose its external tariffs as it pleases, 2) a free trade area (FTA),which permits members to choose their own external tariff rates, they must agree to: (i) free trade with their partner and (ii) a schedule of “rules-of-origin” that determine the duty-free status of goods originating in non-member nations but traded within the FTA and 3) a customs union (CU), requires members to commit to: (i) intra-union free trade, (ii) jointly determine a common external tariff rate to levy on non-members and (iii) share the resulting CET revenue according to an agreed formula. AM do not consider global free trade which Pareto dominates all other arrangements under certainty.

A key difference in this chapter between their paper is that we allow for the possibility of global free trade, but ignore FTAs. Also in their model the third country is "passive" in the sense it does not sign trade agreements.[1] In our model we allow for the possibility of union with the third country. There is therefore a strong degree of symmetry in the model. We are thus attempting to fully endogenize the coalition structure that occurs in equilibrium.

A further contribution of the paper is that by specifying the exact source of uncertainty we are able to analyze the dynamics of the model in detail and obtain insight into how the choice of trade agreements is affected by the variation in the variances in different markets as well as the correlation between the markets.

Our findings are quite straightforward. We find that for small volatilities countries prefer global free trade. The more positively correlated two countries are the more likely they are to form a customs union. We also find that countries may wish to stand alone under certain variance-covariance configurations.

Chapter 3 advances the literature on the formation of PTAs in international trade by incorporating trade costs and uncertainty in our model of oligopoly and

[1] Indeed this is the reason they are able to dispense with the case of global free trade.

trade. As before we adopt a three-country oligopolistic framework and show that trade costs (in the form of transportation costs) influence the decisions of firms (in their output choices) and governments (in their tariff and coalition choices). When we introduce uncertainty in the form of demand shocks, we find that these too affect the choices made by firms and governments. Indeed we find that in certain extreme situations, trading arrangements that we would not observe under certainty may obtain as equilibrium outcome under certain configuration of trade costs, demand shocks and their covariances.

So far the literature has been rather silent on the relation of trade costs to PTAs, even under certainty, so the first question that needs to be addressed is "why consider trade costs ?" There are compelling reasons to incorporate trade costs in trade models. We take the view adopted by Anderson and van Wincoop [2004] that "distance is not dead." In their survey they find that trade costs matter- in their implications for welfare, economic policy and economic geography. Obstfeldt and Rogoff [2001] argue that all major puzzles in international macroeconomics hang on trade costs.

In the first part of the paper we consider the implications that trade costs have on the formation of PTAs, where the only PTAs we consider here will be customs unions (CUs), in which member countries drop tariff barriers among themselves and impose a common tariff against non-member countries subject to MFN rules. Starting with Viner an extensive literature has developed on the formation of CUs not only in the trade literature but also in the game-theoretic literature. Early literature focused on perfect competition but the persistence of economic profits motivate us to consider an oligopolistic framework in this paper. The first paper to extensively analyze oligopolistic firms and customs unions seem to have been Reizman in an unpublished manuscript. Before him we can only mention Gasiros [1987] and Gehrig [1990] who made much more stringent assumptions and whose results have a much different flavour to the ones Reizman (and we) consider. None of these papers consider either trade costs or uncertainty.

The logic behind trade in this oligopolistic framework is essentially increased competition. Helpman and Krugman [1989] have noted that, " the idea that international trade increases competition goes back to Adam Smith and it has long been one of the reasons that economists give for believing that the gains from trade and the costs from protection are larger than their own models seem to suggest."

In terms of assumptions regarding market structure, this paper is closest in spirit to Brander and Krugman's "reciprocal markets" model, first laid out by Brander [1981] which seeks to explain intra-industry trade. Brander [1981] develops a model in which rivalry of oligopolistic firms serve as an independent cause of trade and leads to a two-way trade in identical goods. Brander and Krugman [1983] build on this paper and show that oligoplistic rivalry of firms naturally gives rise to "reciprocal dumping": each firm dumps into the other firm's home market and this phenomenon is shown to be robust to fairly general specification of firm's behavior and market structure. In this paper we will maintain the Cournot assumption but we do not allow for free entry. Under certainty we find as do Brander and Krugman that welfare is quadratic (U-shaped) in trade costs. However, the addition of a third country in our model fundamentally alters the number and type of coalitions countries can form. We constrain countries to form customs unions (cu), stand alone (sa)- where they set independent discriminatory tariffs against each other, global free trade (GFT), where there are no tariff barriers between any country, or autarky (Aut)- when there is no trade at all. Indeed we find that under different trade costs all these different coalition structures can occur in equilibrium. (Brander and Krugman in their two-country model consider only multilateral trade liberalization and autarky). A further difference is that in our paper tariffs are endogenous and obtained by maximizing a well-defined welfare function.

The addition of uncertainty in the form of (potentially correlated) demand shocks in each market fundamentally alters the results obtained under certainty. Appelbaum and Melatos [2012] have shown that trade agreements have option value i.e. the value emanating from being able to make at least *some* decisions after the resolution of uncertainty. In our model uncertainty is resolved "early" before tariffs are set abut after coalition decisions are made. We consider this to be the timing that gives rise to interesting results in our model. As Appelbaum and Melatos have pointed out, the cases where uncertainty is resolved before all decisions are made and the case where uncertainty is resolved after all decisions are made are of little significance in their differences from the certainty case. Different choices of coalitions give rise to different option values and countries must take into account the new costs and benefits associated with option values when making their coalition choices. Indeed equilibrium coalition structures are altered significantly when the option value effect is taken into account. Increasing volatility ceteris paribus when trade costs are homogenous has the effect of increasing the likelihood of global free trade, decreasing the likelihood of stand alone and an ambiguous effect on the likelihood of customs union. Beyond a certain cutoff level of volatility no customs union forms for any range of values of trade costs.

The incorporation of so many diverse elements in the model : coalition formation, endogenous tariffs, trade costs and uncertainty necessarily entails making a number of simplifying assumptions to be able to obtain tractable results and say something meaningful about these results. First of all we assume that this is a one-shot simultaneous move game without renegotiation. We assume there is one firm in each country that is a monopolist in a non-numeraire good. There is a numeraire good that is traded costlessly to balance trade.[2]We assume utility is additively separable in the numeraire and non-numeraire good, linear in the numeraire and quadratic in the non-numeraire. Linearity of the demand function for the non-numeraire good follows immediately from this assumption. We further assume that marginal cost of production for the non-numeraire good is constant and the same for every country.

[2] This is another distinction from Brander and Krugman who consider a partial equilibrium setup. This is a general equilibrium framwork albeit a very special case of general equilibrium where we have no income effects and intersectoral substitution effects on the demand side and no cost changes on the factor market side. (Leahy and Neary actually label as partial equilibrium for the case in which the industry is not large enough to give rise to the aforementioned effects).

Chapter 2
The Pure Uncertainty Problem

This chapter considers a three country model of oligopoly and trade where customs union between any two countries is possible. We consider the effect demand uncertainty has on the output firms choose, the tariffs set by the governments and the coalitions countries form by considering a three-stage game. Our goal is to endogenize the coalition structures that form in equilibrium.

In a customs union (such as the European Union) member countries abolish tariff barriers among themselves and set a common external tariff against non-members subject to the most favored nation (MFN) clause.

It seems that Appelbaum and Melatos ([2012],[2014]) were the first to deeply analyze the role of uncertainty in the choice of trade agreements. This chapter is heavily indebted to their paper in the method used to study the formation of customs unions. It is in a sense a simplification of their model in that the source of uncertainty is less general and we do not consider cost uncertainty. Further unlike them we do not allow for the possibility of free trade areas so this is a customs union game. The crucial difference is that we allow all countries to be active and therefore we can (and need to) consider the possibility of global free trade. Indeed we find that the possibility of global free trade has a significant bearing on the predictions of the model. Also, by being particular about the nature of uncertainty we are able to analyze in some detail how market volatility and market correlation impact the formation of coalitions. For different values of the correlation coefficient we are able to give a fairly complete characterization of the coalitions that form in equilibrium in terms of the parameters of the model

As a benchmark case we consider the model with no uncertainty and find that global free trade dominates all other arrangements. Even with symmetric uncertainty we find that global free trade is the equilibrium coalition structure. With asymmetric variances of the demand shocks we find for different variance-covariance configurations different coalitions can form.

In the following sections we present the ingredients of the model: the coalition formation game, the underlying trade model and the source and resolution of uncertainty. we then study the various coalition structures under different variance-covariance configurations when the third country is non-stochastic. For various values of the correlation coefficient we completely characterize the equilibrium coalition structure.

2.1 The General Framework

We consider a three-country customs union game. Players (countries indexed by i, j and k) are engaged in a three-stage game: First they form coalitions, second they set tariffs optimally and third firms choose output. The only forms of coalitions we will be considering are stand alone, global free trade and customs unions. When a country chooses to stand alone it sets discriminatory tariffs against the other two countries. Global free trade entails abolishing all tariffs between all three countries. If i and j form a customs union they abolish tariffs among themselves and set a *common* tariff against the third country. Setting the common tariff entails maximizing the *joint* welfare by each country by members of the union. In the following three subsections we describe the three ingredients of the model in detail: the coalition formation game, the trade model and the nature and timing of resolution of uncertainty.

2.1.1 The Coalition Formation Game

Consider N countries, $N = \{i, j, k\}$.

There are five possible coalition structures : $\{\{i\}, \{j\}, \{k\}\}$ corresponding to stand alone (sa), $\{\{i, j, k\}\}$ corresponding to the grand coalition or global free trade (GFT), $\{\{i, j\}, \{k\}\}$ corresponding to a customs union between i and j, $\{\{i, k\}, \{j\}\}$ corresponding to a customs union between i and k and $\{\{j, k\}, \{i\}\}$ corresponding to a customs union between j and k.

For each country i, for each coalition structure there is associated a payoff or welfare $W^i : \Sigma(S) \rightarrow \mathbb{R}$, where S is a coalition $(\varnothing \neq S \subseteq N)$ and $\Sigma(S)$ is a nonempty set (the set of strategies of S).[3] $W^i = \pi_i + CS^i + T^i$, where π_i is the producer surplus, CS^i is the consumer surplus and T^i is tariff revenue. Each country

[3] We abuse notation slightly and use i for a generic country here.

has four strategies : sa where it forms no coalitions; GFT where it agrees to the grand coalition; and cu for customs union with either of the two other countries.

As payoffs are given for *individual* players rather than each coalition we think this game should be modelled as one of *nontransferable utility* (NTU).[4]

We can view this game as a *cooperative game in strategic form*; i.e., a triple

$$\left(N, (\Sigma(S))_{\varnothing \neq S \subseteq N}, \left(W^i\right)_{i \in N}\right)$$

The solution concept however is essentially non-cooperative: Countries compare welfares under various coalition structures and choose the one that gives them the highest welfare.

2.1.2 Underlying Trade Model

There are three large countries indexed by i, j and k. Countries are ex ante similar except for their demand conditions.

We first describe the market structure in each country. It is essentially based on the model of oligopoly considered by Dixit [1984] who cites economies of scale and scope, entry barriers and product differentiation as being compelling reasons for considering such a market structure. Also as pointed out by Leahy and Neary in the Palgrave handbook of international trade, a key contribution of oligopoly theory to

4 Allowing for transfers does not fundamentally alter any of our results., as is explained in proposition 2 in [5].

trade theory is the focus on a real world feature: the persistence of pure profits and the strategies by firms to raise them.

The model is a very special case of a general equilibrium model where we ignore income effects and assume exogeneity of factor prices. Since factor prices are exogenous to firms so is its cost function, precluding the endogenous determination of international differences in costs. Thus we do not consider the pattern of inter-industry trade. Trade in this model will be of intra-industry type.

Each country has one firm which produces a numeraire good which is costlessly traded to balance trade. It also produces a homogenous non-numeraire good. Demand is assumed to be linear with a constant and unit slope and an additive disturbance term.

Demand in each market is subject to exogenous shocks that are potentially correlated. Casual observation suggests that market correlation is an empirically plausible phenomenon.

The inverse demand function in country j is given by

$$P_j = A - Q_j + e_j \tag{2.1}$$

Where P_j is the price of the non-numeraire good in country j, Q_j the total output to the domestic market, e_j is the random disturbance term.

The non-numeraire good is produced at constant marginal cost c in each country. We set $A - c = a$.

Conjectures are Cournot and firms choose output to maximize profit.

The model can be viewed as a simplification of that considered by Yi [1996] where we have assumed goods are perfect substitutes.

We consider (specific) tariffs as the only policy instruments used by countries. Tariffs are endogenous in this model: governments set tariffs to maximize a social welfare function that comprises of producer surplus, consumer surplus and tariff revenues. Tariff revenues are rebated back to consumers in the form of lump sum transfers.

2.1.3 The Source and Resolution of Uncertainty

We assume that the demand shocks (e_i, e_j, e_k) are the only source of uncertainty

$$E(e_j) = 0, E(e_j^2) = \sigma_j^2 \text{ and } E(e_j e_i) = \sigma_{ji} \tag{2.2}$$

$$E(e_i) = 0, E(e_i^2) = \sigma_i^2 \text{ and } E(e_k) = 0, E(e_k^2) = \sigma_k^2 \tag{2.3}$$

where σ_{ji} measures the correlation between markets j and i and $\sigma_i^2, \sigma_j^2, \sigma_k^2$ are the variances of e_i, e_j, e_k respectively.

Our benchmark case will be when uncertainty is resolved before stage 1, i.e. before *all* decisions are made, which is essentially the certainty case. Uncertainty may be resolved late after all decisions are made, a case not considered here. This case is not important as then the welfare functions are linear is the random variables, and these have mean zero by assumption. We will be concerned with the "early" resolution of uncertainty: before tariff decisions are made but after the trade regime is chosen, which we consider to be the most interesting case. We assume the realized values of the random variables are common knowledge to all players.

2.2 The Model

In this section we explicitly solve the model staring with stage 3. The trade regime is chosen before the uncertainty is resolved and tariffs and output is chosen after the state of the world is known.

2.2.1 Stage 3: Output Choice

All three firms choose their output simultaneously in Cournot fashion.

Given the demand function (1), and the tariffs chosen by the three countries the problem of firm i in j is

$$\underset{\{q_{ij}\}}{Max}\ \pi_{ij}(a, Q_j, e_j, t_{ji}, q_{ij}) \tag{2.4}$$

$$\text{where } \pi_{ij} = (a - Q_j + e_j - t_{ji})q_{ij}, \text{ is the profit of } i \text{ in } j \tag{2.5}$$

$$\text{and } Q_j = q_{ij} + q_{jj} + q_{kj} \text{ is the total output to } j, \tag{2.6}$$

$$\text{we let } a = A - c, \tag{2.7}$$

Let $\pi^*_{ij}(a, Q_j, e_j, t_{ji}, q_{ij})$ denote the maximized Nash level of profits of i in j. From all the first order conditions from firm i's problems, we obtain the Nash level of output.[5]

$$q^*_{ij} = q^*_{ij}(a, , e, t) \tag{2.8}$$

$$\text{where, } e = (e_i, e_j, e_k) \text{ is the vector of random variables,} \tag{2.9}$$

$$\text{and } t = (t_i, t_j, t_k) \text{ is the tariff vector} \tag{2.10}$$

2.2.2 Stage 2: Tariff Choice

In stage 2 countries choose their tariffs given that the trade regime has been chosen in stage 1, by maximizing a social welfare function.

$$W^i = CS^i + \pi_i + TR^i \tag{2.11}$$

[5] The explicit functional forms are given in the appendix. We note the optimal quantities are a function of the first moments of the random variables only.

Which may be written:

$$W^i = \frac{1}{2}Q_i^{*2} + (\pi_{ij}^* + \pi_{ii}^* + \pi_{ik}^*) + (q_{ji}^* t_{ij} + q_{ki}^* t_{ik}) \qquad (2.12)$$

We consider the tariff structure for all three types of trade regimes.

Denote by EW_j^{sa} the expected welfare of j under stand alone, EW_j^{GFT} the expected welfare of j under global free trade and $EW_j^{\{\{i,j\},\{k\}\}}$ the expected welfare of j when it forms a customs union with i leaving out k.

Tariff Choice for Stand Alone

When countries choose to stand alone they set discriminatory tariffs against each other subject to the MFN rules. The MFN rules require $t_{ij} = t_{ik}$, $t_{ji} = t_{jk}$, $t_{ki} = t_{kj}$, so each country effectively chooses one tariff. We denote by $t_i^{*sa}(a, e)$ the Nash equilibrium level of tariffs in country i under stand alone. The explicit expressions for these tariffs are:

$$\begin{aligned} t_i^{*sa}(a,e) &= \frac{3}{10}a + \frac{3}{10}e_i \\ t_j^{*sa}(a,e) &= \frac{3}{10}a + \frac{3}{10}e_j \\ t_k^{*sa}(a,e) &= \frac{3}{10}a + \frac{3}{10}e_k \end{aligned}$$

Substituting the Nash equilibrium tariffs into the welfare functions yields the Nash equilibrium level of welfare:

$$
\begin{aligned}
EW_i^{*sa}(a,e) &= \frac{21}{50}a^2 + \frac{1}{100}\sigma_j^2 + \frac{2}{5}\sigma_i^2 + \frac{1}{100}\sigma_k^2 \\
EW_j^{*sa}(a,e) &= \frac{21}{50}a^2 + \frac{1}{100}\sigma_i^2 + \frac{2}{5}\sigma_j^2 + \frac{1}{100}\sigma_k^2 \\
EW_k^{*sa}(a,e) &= \frac{21}{50}a^2 + \frac{1}{100}\sigma_j^2 + \frac{2}{5}\sigma_k^2 + \frac{1}{100}\sigma_i^2
\end{aligned}
$$

We note the welfare functions are convex in all the random variables. As pointed out by Appelbaum and Melatos it is this convexity that is the source of a trade agreement's option value.

Tariff Choice for Customs Union

Now if i and j form a Customs Union they eliminate tariffs between themselves $(t_{ij} = t_{ji} = 0)$ and impose a common tariff t against country k. Let the Nash equilibrium common external tariff be given by $t^*(a,e)$.To find the optimal common tariff, country j maximizes the joint welfare of i and j with respect to the common tariff to obtain :

$$t^*(a,e) = \frac{5}{38}(2a + e_i + e_j) \tag{2.13}$$

We find country k's (the outsider's) tariff against i and j be:

$$t_{ki}^{*cu}(a,e) = t_{kj}^{*cu}(a,e) = \frac{3}{10}a + \frac{3}{10}e_k$$

Given these we can find the Nash level of welfares for i and j to be:

$$EW_i^{*\{\{i,j\},\{k\}\}} = \frac{167}{2432}\sigma_j^2 + \frac{25}{1216}\sigma_{ij} + \frac{871}{2432}\sigma_i^2 + \frac{1}{100}\sigma_k^2 + \frac{869}{1900}a^2 \quad (2.14)$$

$$EW_j^{*\{\{i,j\},\{k\}\}} = \frac{167}{2432}\sigma_i^2 + \frac{25}{1216}\sigma_{ij} + \frac{871}{2432}\sigma_j^2 + \frac{1}{100}\sigma_k^2 + \frac{869}{1900}a^2 \quad (2.15)$$

$$EW_k^{*\{\{i,j\},\{k\}\}} = \frac{377}{11\,552}\sigma_i^2 - \frac{345}{5776}\sigma_{ij} + \frac{377}{11\,552}\sigma_j^2 + \frac{2}{5}\sigma_k^2 + \frac{732}{1805}a^2 \quad (2.16)$$

The first 3 stochastic expressions in the social welfare functions represent the gain in welfare from the option value accruing to j and k respectively from being able to wait for the resolution of uncertainty. We note that for j social welfare is increasing in the market volatilities of all markets since $\partial EW_j/\partial\sigma_i^2 > 0$, $\partial EW_j/\partial\sigma_j^2 > 0$, $\partial EW_j/\partial\sigma_k^2 > 0$, $\partial EW_j/\partial\sigma_{ij} > 0$. The case of country i is similar. Country k's welfare is also increasing in the market volatilities but decreasing in the market correlation between i and j, since $\partial EW_k/\partial\sigma_{ij} < 0$. Also the welfare of all three countries do not depend on the correlation of i and j with k.

Global free Trade

We now calculate the welfare of each country under the grand coalition.

The following are calculated in a manner analogous to the previous section except that all tariffs are set to 0. Firms maximize profit in each market. There is no stage 2 since there are no tariff decisions. The resulting Nash welfares are

$$EW_j^{*GFT} = \frac{1}{16}\sigma_i^2 + \frac{11}{32}\sigma_j^2 + \frac{1}{16}\sigma_k^2 + \frac{15}{32}a^2 \quad (2.17)$$

$$EW_i^{*GFT} = \frac{1}{16}\sigma_j^2 + \frac{11}{32}\sigma_i^2 + \frac{1}{16}\sigma_k^2 + \frac{15}{32}a^2 \quad (2.18)$$

$$EW_k^{*GFT} = \frac{1}{16}\sigma_i^2 + \frac{11}{32}\sigma_k^2 + \frac{1}{16}\sigma_j^2 + \frac{15}{32}a^2 \quad (2.19)$$

2.2.3 Stage 1: Endogenous Formation of Coalitions

We are interested in the equilibrium coalition structure that obtains under various configuration of variances and covariances of the demand shocks. Specifically, we want to know under what conditions each coalition can be expected to form.

Given that countries are risk neutral we assume countries consider the same welfare functions as previously used, make pairwise comparisons between the welfares under each coalition structure and choose the coalition structure that gives them the highest welfare

Now each coalition structure is given by a system of nine inequalities, three for each country.

Global Free Trade $(\{\{i, j, k\}\})$ **is the Equilibrium Coalition Structure.**

GFT would be the equilibrium if and only if the following inequalities hold:

For country i :

$$EW_i^{GFT} > EW_i^{sa}$$

$$EW_i^{GFT} > EW_i^{\{\{i,j\},\{k\}\}}$$

$$EW_i^{GFT} > EW_i^{\{\{i,k\},\{j\}\}}$$

We have similar inequalities for the other two countries

In terms of the parameters of the model:

For country i :

$$EW_i^{GFT} - EW_i^{sa} = \frac{1}{16}\sigma_j^2 - \frac{1}{32}\sigma_i^2 + \frac{1}{16}\sigma_k^2 + \frac{3}{32}a^2 > 0 \tag{2.20}$$

$$EW_i^{GFT} - EW_i^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200}a^2 - \frac{15}{2432}\sigma_j^2 - \frac{35}{2432}\sigma_i^2 + \frac{21}{400}\sigma_k^2 - \frac{25}{1216}\sigma_{ij} > 0 \tag{2.21}$$

$$EW_i^{GFT} - EW_i^{\{\{i,k\},\{j\}\}} = \frac{173}{15\,200}a^2 + \frac{21}{400}\sigma_j^2 - \frac{35}{2432}\sigma_i^2 - \frac{15}{2432}\sigma_k^2 - \frac{25}{1216}\sigma_{ik} > 0 \tag{2.22}$$

For country j :

$$EW_j^{GFT} - EW_j^{sa} = \frac{1}{16}\sigma_i^2 - \frac{1}{32}\sigma_j^2 + \frac{1}{16}\sigma_k^2 + \frac{3}{32}a^2 > 0 \tag{2.23}$$

$$EW_j^{GFT} - EW_j^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200}a^2 - \frac{15}{2432}\sigma_i^2 - \frac{35}{2432}\sigma_j^2 + \frac{21}{400}\sigma_k^2 - \frac{25}{1216}\sigma_{ij} > 0 \tag{2.24}$$

$$EW_j^{GFT} - EW_j^{\{\{j,k\},\{i\}\}} = \frac{173}{15\,200}a^2 + \frac{21}{400}\sigma_i^2 - \frac{35}{2432}\sigma_j^2 - \frac{15}{2432}\sigma_k^2 - \frac{25}{1216}\sigma_{jk} > 0 \tag{2.25}$$

For country k :

$$EW_k^{GFT} - EW_k^{sa} = \frac{1}{16}\sigma_i^2 - \frac{1}{32}\sigma_k^2 + \frac{1}{16}\sigma_j^2 + \frac{3}{32}a^2 > 0 \tag{2.26}$$

$$EW_k^{GFT} - EW_k^{\{\{i,k\},\{j\}\}} = \frac{173}{15\,200}a^2 - \frac{15}{2432}\sigma_i^2 - \frac{35}{2432}\sigma_k^2 + \frac{21}{400}\sigma_j^2 - \frac{25}{1216}\sigma_{ik} > 0 \tag{2.27}$$

$$EW_k^{GFT} - EW_k^{\{\{j,k\},\{i\}\}} = \frac{173}{15\,200}a^2 + \frac{21}{400}\sigma_i^2 - \frac{35}{2432}\sigma_k^2 - \frac{15}{2432}\sigma_j^2 - \frac{25}{1216}\sigma_{jk} > 0 \tag{2.28}$$

So a country i prefers GFT to CU with k if $\sigma_j^2$is high, σ_i^2 is low, σ_k^2 is low and correlation with j, σ_{ij} is low.

Therefore a country prefers GFT to CU if volatility of the two markets is low and the volatility of the country that is potentially excluded is high.

A Customs Union between i and j, $(\{\{i, j\}, \{k\}\})$ is the Equilibrium Coalition Structure.

This will be the case if the following inequalities hold:

$$EW_i^{\{\{i,j\},\{k\}\}} > EW_i^{sa}$$

$$EW_i^{\{\{i,j\},\{k\}\}} > EW_i^{GFT}$$

$$EW_i^{\{\{i,j\},\{k\}\}} > EW_i^{\{\{i,k\},\{j\}\}}$$

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_j^{sa}$$

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_j^{GFT}$$

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_i^{\{\{j,k\},\{i\}\}}$$

These relate to conditions for i and j. Note that if i and j do indeed form an union, country k cannot do anything about it so we do not need conditions for k.

These conditions are equivalent to:

$$EW_i^{\{\{i,j\},\{k\}\}} - EW_i^{sa} = \frac{71}{1900}a^2 + \frac{3567}{60\,800}\sigma_j^2 - \frac{509}{12\,160}\sigma_i^2 + \frac{25}{1216}\sigma_{ij} > 0 \quad (2.29)$$

$$EW_i^{\{\{i,j\},\{k\}\}} - EW_i^{GFT} = -\frac{173}{15\,200}a^2 + \frac{15}{2432}\sigma_j^2 + \frac{35}{2432}\sigma_i^2 - \frac{21}{400}\sigma_k^2 + \frac{25}{1216}\sigma_{ij} > 0 \quad (2.30)$$

$$EW_i^{\{\{i,j\},\{k\}\}} - EW_i^{\{\{i,k\},\{j\}\}} = \frac{3567}{60\,800}\sigma_j^2 - \frac{3567}{60\,800}\sigma_k^2 + \frac{25}{1216}\sigma_{ij} - \frac{25}{1216}\sigma_{ik} > 0 \tag{2.31}$$

$$EW_j^{\{\{i,j\},\{k\}\}} - EW_j^{sa} = \frac{71}{1900}a^2 + \frac{3567}{60\,800}\sigma_i^2 - \frac{509}{12\,160}\sigma_j^2 + \frac{25}{1216}\sigma_{ij} > 0 \tag{2.32}$$

$$EW_j^{\{\{i,j\},\{k\}\}} - EW_j^{GFT} = -\frac{173}{15\,200}a^2 + \frac{15}{2432}\sigma_i^2 + \frac{35}{2432}\sigma_j^2 - \frac{21}{400}\sigma_k^2 + \frac{25}{1216}\sigma_{ij} > 0 \tag{2.33}$$

$$EW_j^{\{\{i,j\},\{k\}\}} - EW_j^{\{\{j,k\},\{i\}\}} = \frac{3567}{60\,800}\sigma_i^2 - \frac{3567}{60\,800}\sigma_k^2 + \frac{25}{1216}\sigma_{ij} - \frac{25}{1216}\sigma_{jk} > 0 \tag{2.34}$$

Therefore, from i's perspective, it would want to form a CU with j if it's own volatility is low, partner's volatility is high and correlation with it's partner is high. We have a symmetric condition from j's perspective. For a customs union between any other two countries to be an equilibrium we can derive similar conditions

Stand Alone $(\{\{i\},\{j\},\{k\}\})$ **is the Equilibrium Coalition Structure.**

This will be the case if the following hold for i:

$$EW_i^{sa} > EW_i^{GFT}$$

$$EW_i^{sa} > EW_i^{\{\{i,j\},\{k\}\}}$$

$$EW_i^{sa} > EW_i^{\{\{i,k\},\{j\}\}}$$

We have similar conditions for j and k.

Or,

$$EW_i^{sa} - EW_i^{GFT} = \frac{9}{160}\sigma_i^2 - \frac{21}{400}\sigma_k^2 - \frac{21}{400}\sigma_j^2 - \frac{39}{800}a^2 > 0 \quad (2.35)$$

$$EW_i^{sa} - EW_i^{\{\{i,j\},\{k\}\}} = \frac{63}{1520}\sigma_i^2 - \frac{449}{7600}\sigma_j^2 - \frac{3}{152}\sigma_{ji} - \frac{71}{1900}a^2 > 0 \quad (2.36)$$

$$EW_i^{sa} - EW_i^{\{\{i,k\},\{j\}\}} = \frac{63}{1520}\sigma_i^2 - \frac{449}{7600}\sigma_k^2 - \frac{3}{152}\sigma_{ik} - \frac{71}{1900}a^2 > 0 \quad (2.37)$$

$$EW_j^{sa} - EW_j^{GFT} = \frac{9}{160}\sigma_j^2 - \frac{21}{400}\sigma_k^2 - \frac{21}{400}\sigma_i^2 - \frac{39}{800}a^2 > 0 \quad (2.38)$$

$$EW_j^{sa} - EW_j^{\{\{i,j\},\{k\}\}} = \frac{63}{1520}\sigma_j^2 - \frac{449}{7600}\sigma_i^2 - \frac{3}{152}\sigma_{ji} - \frac{71}{1900}a^2 > 0 \quad (2.39)$$

$$EW_j^{sa} - EW_j^{\{\{j,k\},\{i\}\}} = \frac{63}{1520}\sigma_j^2 - \frac{449}{7600}\sigma_k^2 - \frac{3}{152}\sigma_{jk} - \frac{71}{1900}a^2 > 0 \quad (2.40)$$

$$EW_k^{sa} - EW_k^{GFT} = \frac{9}{160}\sigma_k^2 - \frac{21}{400}\sigma_j^2 - \frac{21}{400}\sigma_i^2 - \frac{39}{800}a^2 > 0 \quad (2.41)$$

$$EW_k^{sa} - EW_k^{\{\{i,k\},\{j\}\}} = \frac{63}{1520}\sigma_k^2 - \frac{449}{7600}\sigma_i^2 - \frac{3}{152}\sigma_{ki} - \frac{71}{1900}a^2 > 0 \quad (2.42)$$

$$EW_k^{sa} - EW_k^{\{\{j,k\},\{i\}\}} = \frac{63}{1520}\sigma_k^2 - \frac{449}{7600}\sigma_j^2 - \frac{3}{152}\sigma_{jk} - \frac{71}{1900}a^2 > 0 \quad (2.43)$$

Thus a country is likely to stand alone if its own variance is high and the variances of the other two countries are low.

We can formally state from observation of the previous inequalities:

Proposition 1 *Under linear demand and constant marginal cost of production a country prefers GFT to CU if volatility of the two markets is low and the volatility of the country that is potentially excluded is high. If countries i and j form a customs union social welfare of all three countries is increasing in the market volatilities. The welfare of i and j is increasing in the market correlation between i and j while the welfare of the excluded country k is decreasing in the market correlation between i and j. Moreover the welfare of all three countries is independent of the market* *correlation of i and j with k. A country is likely to stand alone if its own variance is high and the variances of the other two countries are low.*

We provide some intuition behind this result. Increased variance essentially increases the option value associated with any particular coalition structure other things being equal. This is well explained in Appelbaum and Melatos and we can do no better than paraphrase from their paper [2012]. Under uncertainty, trade among members gives rise to an extra cost: an additional constraint that reduces member's trade policy freedom in response to a shock in the trading environment, so the addition of uncertainty results in another source of welfare in the form of option value. Without renegotiation the impact of uncertainty varies according to the depth of integration, i.e. by coalition type which must be taken into account at the time of coalition formation. A particular type of trade agreement imposes particular trade policy restrictions on their members. Given this insight from AM we can explain Proposition 1 as follows: any two countries would prefer global free trade over a customs union with each other if their own variance is low and that of the potentially excluded country is high because by including the third country they can enjoy the option value associated with the third country. (Of course the third country is likely to stand alone). If i and j form a union , given that their welfare is convex in the random variables increasing variance increases the option value from being together and positive correlation amplifies this effect. Also the option value to the third country increases with increasing volatility if it stands alone since its welfare is convex in the random variables. Countries prefer to stand alone if their own variance is high and that of the other two are low since by being alone they can enjoy their own high option value and there is not much option value to be gained from the other two countries anyway.

2.3 Benchmark Case : No Uncertainty and Symmetric Uncertainty

This case is quite trivial from the coalition formation perspective as global free trade is preferred by all countries. it is a standard result in trade that global free trade maximizes aggregate welfare and in this model it actually translates to all countries agreeing to global free trade under certainty. Even when all variances of the demands are the same we get global free trade dominating all other arrangements. We formalize this as follows :

Proposition 2 *With linear demand and constant marginal cost of production, in the absence of uncertainty, GFT dominates all other coalition structures. Moreover if all variances of the demand shocks are the same then also GFT dominates all other arrangements.*

Proof. *This is easy to see as*

$$EW_l^{GFT} - EW_l^{sa} = \frac{3}{32}a^2 > 0 \textit{ for } l = i, j, k \tag{2.44}$$

$$EW_l^{GFT} - EW_l^{\{\{l,m\},\{n\}\}} = \frac{173}{15\,200}a^2 > 0 \text{ for } l,m,n \in \{i,j,k\} \tag{2.45}$$

For the second claim we set $\sigma_i^2 = \sigma_j^2 = \sigma_k^2 = \sigma^2$ *and note that*

$$EW_l^{GFT} - EW_l^{sa} = \frac{3}{32}a^2 + \frac{5}{32}\sigma^2 > 0 \text{ for } l = i,j,k \tag{2.46}$$

and

$$EW_l^{GFT} - EW_l^{\{\{l,m\},\{n\}\}} = \frac{173}{15\,200}a^2 + \frac{971}{30\,400}\sigma^2 - \frac{25}{1216}\rho\sigma^2 \tag{2.47}$$

Now the maximum value ρ *can take is +1 so the minimum value of* $\frac{971}{30\,400}\sigma^2 - \frac{25}{1216}\rho\sigma^2$ *is* $\frac{173}{15\,200}\sigma^2$ *which is greater than zero, so*

$$EW_l^{GFT} - EW_l^{\{\{l,m\},\{n\}\}} = \frac{173}{15\,200}a^2 + \frac{971}{30\,400}\sigma^2 - \frac{25}{1216}\rho\sigma^2 > 0 \text{ for } l,m,n \in \{i,j,k\} \tag{2.48}$$

■

With symmetric countries the effect of increased competition dominates all other effects under certainty, making GFT Pareto-dominate all other arrangements.

With symmetric variance and no correlation, countries are still essentially symmetric. By being together countries can actually pool their option value so increasing volatility makes Global free trade more likely other things being equal

2.4 Equilibrium Coalition Structures under Asymmetric Shocks

Thus we focus on the case when there is some degree of asymmetry in the uncertainty across countries

To make the exposition simple and renderable graphically we will assume that k's market is non-stochastic and has no correlation with the other two markets. (i.e.,$\sigma_k^2 = 0$, $\sigma_{ki} = \sigma_{jk} = 0$)

We assume $\sigma_{ij} = \rho\sigma_i\sigma_j$, where ρ is the correlation coefficient between i and j, $-1 \leq \rho \leq 1$.

We break down the analysis into three cases: $\rho = 0$ (no correlation), $\rho = 1$ (perfect positive correlation) and $\rho = -1$(perfect negative correlation).

This is enough to highlight the how the equilibrium coalition structure is affected as the market correlation changes. Given the continuity of the welfare functions the indifference curves vary smoothly so we ignore the intermediate cases.

We partition the $\sigma_i - \sigma_j$ plane into region and identify the coalition structures in each region. The various dividing lines are the indifferences between pairs of coalition structures.

We treat Case 1 where there is no correlation as the benchmark case. This is because option value is the driving force behind the results and it works through the variances; the covariances affect only at the margin.

Case 1 ($\rho = 0$ (no correlation))

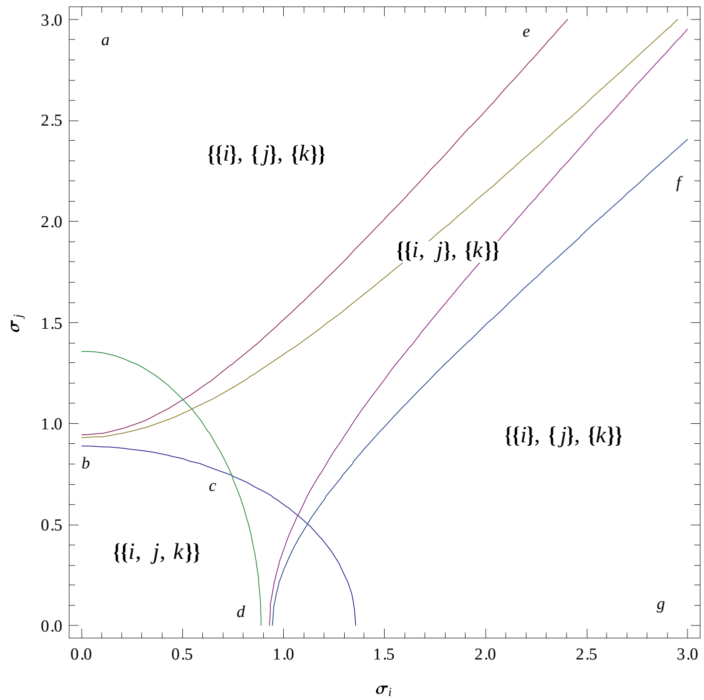

Figure 1

We notice that for small variances GFT is the equilibrium (region bcd). The intuition behind this is that when variances are small there is very little option value associated with not committing to form any sort of union. Thus the benefits from free trade dominate. On the other hand if the variance of j is high and that of i is moderately high sa results. The intuition behind this result is as follows: if your own variance is high you would like to stand alone and enjoy high option value from standing alone. i however would prefer a union but if j does not cooperate this is not possible so sa results. We have a symmetric result for high σ_i and moderate σ_j. Now if variances of both i and j are moderate to high and they are similar, they will prefer to form a customs union. The intuition behind this is that they appropriate some gains from trade while retaining some option value .The trade-off from the gains from trade and the option value that results from high variance in their markets induce i and j to form a union.

Case 2 ($\rho = 1$ (perfect positive correlation))

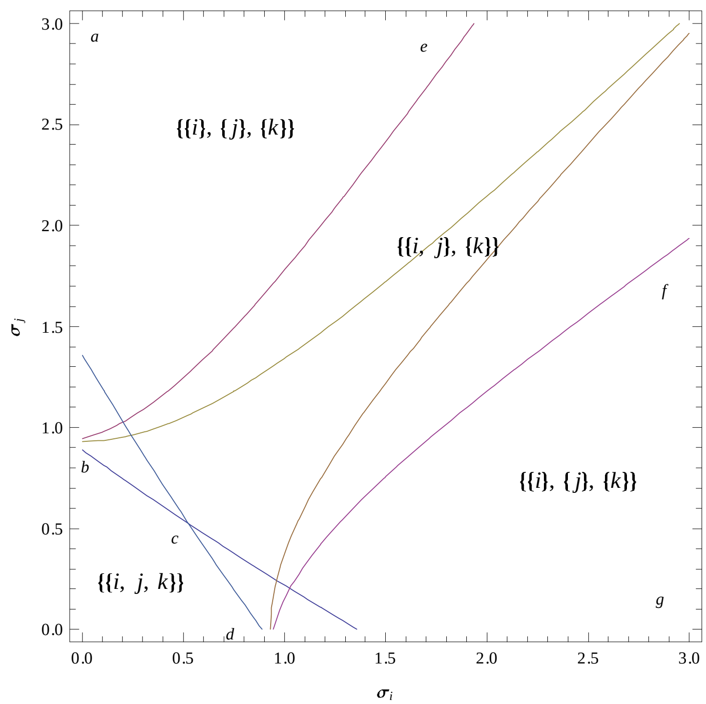

Figure 2

Case 3 ($\rho = -1$ (perfect negative correlation))

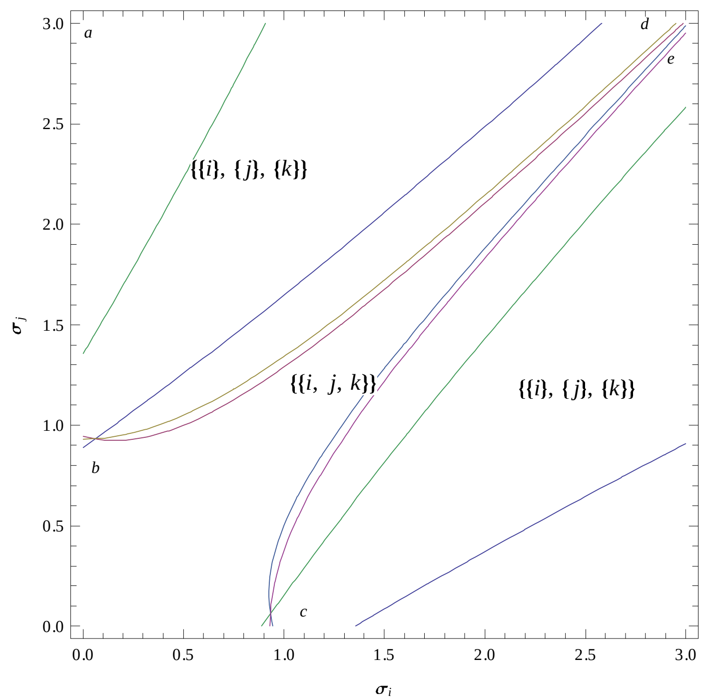

Figure 3

We notice from the previous graphs that as the correlation coefficient decreases from +1 to -1 the area bcd increases and that of Customs union between i and j diminishes until in the limit $\rho = -1$ we actually do not observe any customs union at all.

The trend when ρ goes from +1 to -1 may be explained as follows : decreasing ρ dampens the effect of variance on option value and makes GFT more attractive relative to CU. We observe that the likelihood of CU diminishes as ρ decreases for similar reasons. Given the continuity of the welfare functions in the variances and covariances these results are quite robust. We collect the aforementioned observations into the following propositions.

Proposition 3 *Under linear demand and constant marginal cost and with the third country non-stochastic, the likelihood of global free trade is increasing in* σ_i^2 *and* σ_j^2.

Proposition 4 *Under linear demand and constant marginal cost and with the third country non-stochastic, as* ρ *goes from +1 to -1, the likelihood of GFT increases while that of customs union between* i *and* j *diminishes.*

2.5 Summary

In this chapter we considered a three-country model of oligopoly and trade under demand uncertainty. We endogenized the coalition structure that forms in a three stage game. We find that for small volatilities countries prefer global free trade. The more positively correlated two countries are the more likely they are to form a customs union. We also find that countries may wish to stand alone under certain variance-covariance configurations.

Chapter 3
The Model with Trade Costs

This chapter can be viewed as a logical continuation of our earlier chapter on the formation of coalitions by countries under uncertainty without explicit trade costs.

The rest of the chapter is structured as follows: In section 3.1 we describe our model. Section 3.2 considers the case where trade costs faced by each country is the same. Section 3.3 we consider a case where one country is situated between the other two. Both these cases are considered under certainty and uncertainty. We characterize the equilibrium coalition structures that arise in a three stage game in several propositions and provide some explanation and intuition behind these results. The derivation of the optimal quantities, optimal tariffs and welfares under different coalition structures is given in an appendix.

3.1 The Model

The model is essentially the same as that of chapter 1 except that we add trade costs in the form of per unit transportation costs

The per unit transportation cost for a firm in i exporting to a firm in j is d_{ij} and the cost for a firm in j exporting to a firm in i is d_{ji}.In what follows we will assume $d_{ij} = d_{ji}$.

There are three stages to the model: in stage one countries choose their trade regime, in stage two tariffs are set optimally, in stage three firms choose their output in each market. The model is solved backwards starting with stage three to obtain the equilibrium coalition structure. A coalition structure is defined to be an equilibrium if from each countries perspective it is preferred to all other coalition structures. Thus each equilibrium coalition structure is given by a set of nine inequalities, three for each country.

The model is solved for general trade costs and different levels of uncertainty and unspecified covariances in the Appendix . This is a one-shot simultaneous-move game. The welfares resulting from each coalition structure to each country is derived as functions of the demand scale parameter, trade costs and the variances and covariances of the demand shocks. The welfares in the special two structures considered in section 2 are obtained by specialization on these welfare functions.

We need to put some structure to the demand uncertainty and trade costs to be able to draw tractable results and explain how trade costs and uncertainty affect the choice of output, tariffs and coalitions.

3.2 Symmetric Trade Costs

In this section we assume i, j and k are ex ante similar in all respects and the firm in each country faces an exogenous per unit transportation cost of d. We name this scenario the "triangular trade cost problem", because it can be interpreted as i, j and k being equidistant from each other at the vertices of an equilateral triangle. The strong symmetry of the model allows us to focus on the relation between trade costs and coalition choice. Overall transportation costs are proportional to exports. We solve the model backwards to obtain optimal tariffs and welfare under various coalition structures as functions of trade costs alone under certainty in subsection 3.1 and as function of trade costs and the variances of the demand shocks and their covariances under uncertainty in section 3.2.

3.2.1 The Model under Certainty

We collect here the welfare expressions for various coalition structures and then decompose these welfares into their components from country j's perspective.

For stand alone:

$$EW_j^{sa} = \frac{42}{100}a^2 - \frac{36}{100}ad + \frac{72}{100}d^2 \tag{3.1}$$

$$CS_j^{sa} = \frac{9}{50}a^2 - \frac{6}{25}ad + \frac{2}{25}d^2 \tag{3.2}$$

$$\pi_j^{sa} = \frac{9}{50}a^2 - \frac{4}{25a}ad + \frac{12}{25}d^2 \tag{3.3}$$

$$TR_j^{sa} = \frac{3}{50}a^2 - \frac{7}{25}ad + \frac{4}{25}d^2 \tag{3.4}$$

For global free trade:

$$EW_j^{GFT} = \frac{15}{32}a^2 - \frac{20}{32}ad + \frac{28}{32}d^2 \tag{3.5}$$

$$CS_j^{GFT} = \frac{1}{32}(3a-2d)^2 \tag{3.6}$$

$$\pi_j^{GFT} = \frac{1}{16}(a-2d)^2 \tag{3.7}$$

For customs union between i and j :

$$EW_j^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 - \frac{269}{475}ad + \frac{401}{475}d^2 \tag{3.8}$$

$$CS_j^{\{\{i,j\},\{k\}\}} = \frac{169}{722}(a - \frac{8}{13}d)^2 \tag{3.9}$$

$$\pi_j^{\{\{i,j\},\{k\}\}} = \frac{7561}{36100}a^2 - \frac{1622}{9025}ad + \frac{6069}{9025}d^2 \tag{3.10}$$

$$TR_j^{\{\{i,j\},\{k\}\}} = \frac{1}{361}(5a-6d)(a-5d) \tag{3.11}$$

Autarkic Trade Costs

It is useful to note that there are two cases of stand alone- one where countries trade with discriminatory tariffs in place and autarky- where they do not trade at all.

The possible distinct coalition structures are still three- GFT, cu and sa.

Here we find the level of trade costs for which exporting becomes unprofitable, i.e. the autarkic level of trade cost which we will denote by $d_{p.}$

To this end we look at market j and find the value of d for which profits of i in j becomes zero. The symmetry of the model will ensure that this value of d is the same for all markets. We do so for all possible coalition structures

1 *Stand Alone*

Now profit of i in j is given by

$$\pi_{ij} = q_{ij}^2 = \left(\frac{1}{10}a - \frac{2}{5}d\right)^2 \tag{3.12}$$

This gives

$$d_p^{sa} = \frac{1}{4}a \tag{3.13}$$

2 *GFT*

$$\pi_{ij} = q_{ij}^2 = \left(\frac{1}{4}(a - 2d)\right)^2 \tag{3.14}$$

$$d_p^{GFT} = \frac{1}{2}a \tag{3.15}$$

3 *Customs Union between i and j*

$$\pi_{ij} = q_{ij}^2 = \left(\frac{1}{4}(\frac{2}{5}a - \frac{8}{5}d)\right)^2 \tag{3.16}$$

$$d_p^{CU} = \frac{1}{4}a \tag{3.17}$$

Equilibrium Coalition Structure

To find the equilibrium coalition structures under various trade regimes we need to evaluate the differences in welfare under various trade regimes and assume that countries choose the coalition that yields highest welfare. In what follows we set $a = 1$.

So a coalition structure will be an equilibrium if each country prefers it to all other coalition structures. Thus in general each equilibrium coalition structure is given by a set of nine inequalities, three for each country.

GFT would be the equilibrium if and only if the following inequalities hold:

For country i :

$$EW_i^{GFT} > EW_i^{sa}$$

$$EW_i^{GFT} > EW_i^{\{\{i,j\},\{k\}\}}$$

$$EW_i^{GFT} > EW_i^{\{\{i,k\},\{j\}\}}$$

With similar conditions for the other two countries.

$\{\{i,j\},\{k\}\}$, a customs union between i and j would be an equilibrium iff

$$EW_i^{\{\{i,j\},\{k\}\}} > EW_i^{sa}$$

$$EW_i^{\{\{i,j\},\{k\}\}} > EW_i^{GFT}$$

$$EW_i^{\{\{i,j\},\{k\}\}} > EW_i^{\{\{i,k\},\{j\}\}}$$

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_j^{sa}$$

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_j^{GFT}$$

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_i^{\{\{j,k\},\{i\}\}}$$

These relate to conditions for i and j. Note that if i and j do indeed form an union, country k cannot do anything so we do not write down the conditions for k.

Stand Alone $(\{\{i\},\{j\},\{k\}\})$ is the Equilibrium Coalition Structure iff

$$EW_i^{sa} > EW_i^{GFT}$$

$$EW_i^{sa} > EW_i^{\{\{i,j\},\{k\}\}}$$

$$EW_i^{sa} > EW_i^{\{\{i,k\},\{j\}\}}$$

With similar conditions for the other countries.

In what follows we solve for values of d for which a country will be indifferent between a pair of coalitions. By calculating these cutoff values of d we are able to partition the real line into intervals and specify the equilibrium coalition structures in these intervals.

From country j's perspective:

$$\text{Solving } EW_j^{GFT} - EW_j^{sa} = \frac{39}{800} - \frac{53}{200}d + \frac{31}{200}d^2 = 0 \text{ for } d,$$

$$d = 1.5 \text{ or } d = 0.20968 \tag{3.18}$$

$$\text{Solving } EW_j^{GFT} - EW_j^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200} - \frac{271}{3800}d + \frac{117}{3800}d^2 = 0 \text{ for } d, \tag{3.19}$$

$$d = 2.1438 \text{ or } d = 0.17243 \tag{3.20}$$

$$\text{Solving } EW_j^{\{\{i,j\},\{k\}\}} - EW_j^{sa} = \frac{71}{1900} - \frac{92}{475}d + \frac{59}{475}d^2 = 0 \text{ for } d, \tag{3.21}$$

$$d = 1.3338 \text{ or } d = 0.22556 \tag{3.22}$$

$$EW_j^{GFT} > EW_j^{\{\{i,j\},\{k\}\}} \text{ when } d \in [0, 0.17243)$$

[6]

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_j^{GFT} \text{ when } d > 0.17243$$

---

[6] We use the tie breaking rule that larger coalitions are slightly more expensive to form. So for example in a tie between GFT and $\{\{i,j\},\{k\}\}$, $\{\{i,j\},\{k\}\}$ would dominate.

$$EW_j^{\{\{i,j\},\{k\}\}} > EW_j^{sa} \text{ when } d < 0.225\,56 \text{ or } d > 1.\,333\,8$$

$$EW_j^{sa} > EW_j^{\{\{i,j\},\{k\}\}} \text{ when } d > 0.225\,56$$

Thus, we have the following proposition characterizing equilibrium coalition structures :

Proposition 5 *Under linear demand and constant marginal cost of production, when each country faces the same trade costs we have the following equilibrium coalition structures as trade costs vary:*

$$GFT \text{ if } d \in [0, 0.172\,43) \tag{3.23}$$

$$\{\{i,j\},\{k\}\} \text{ if } d \in [0.172\,43, 0.225\,56) \tag{3.24}$$

$$sa \text{ if } d \in [0.225\,56, \infty) \tag{3.25}$$

We note we have symmetric conditions for the other two countries, hence GFT, sa are unambiguously equilibrium coalition structure in the relevant range but for $d \in [0.172\,43, 0.225\,56)$, we can have any of $\{\{i,j\},\{k\}\}$, $\{\{i,k\},\{j\}\}$ or $\{\{j,k\},\{i\}\}$.

Before giving the intuition behind this result It is worthwhile to take a moment at this point to compare our model to Brander and Krugman's.

We note that as in Brander and Krugman welfare as well as each component of welfare is quadratic in trade costs for all possible coalition structures although it is not smooth at the points where there is a change in coalition structure.[7]. We recall in Brander and Krugman consumer surplus is rises monotonically as trade costs fall and this holds true in our model also since a reduction in trade costs makes all goods cheaper to consumers. In Brander and Krugman in the region of free trade profits are decreasing in trade costs but increasing in them in the neighborhood of autarky. With linear demands it follows that profits are U-shaped in trade costs reaching their maximum at autarky and their minimum at free trade. The reasoning behind this is as follows: starting from free trade exports are harmed more by an increase in the firm's own costs than home sales are helped than home sales are helped by an increase in rival's costs. Hence total sales and profits fall for a small increase in d at free trade. Next starting from autarky exports are initially zero so a small fall in trade costs has a negligible effect on profits in the export market. But home sales are at the monopoly level so a small fall in foreign firm's trade costs has a first-order effect on home market profits. Hence overall profits fall for a small fall in d at autarky. Adding the U-shaped profit function to the falling consumer surplus schedule yields a U-shaped welfare function. To consider the entire welfare schedule, welfare is clearly falling in the neighborhood of free trade so we focus on the region of autarky: starting from autarky a small fall in d induces a rise in consumer surplus because the price falls

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[7] Thus the entire welfare schedule for $d \in [0, \infty]$ is continuous but non-smooth.

but profits in the home market fall because of fall in price and sales are reduced. The price effect cancels so the fall in profit outweighs the rise in consumer surplus. Thus the overall welfare is U-shaped reaching its maximum at free trade but it minimum before prohibitive level of d.

In proposition 4 we notice starting from autarky, if trade costs fall a little countries would find it profitable to trade, albeit with discriminatory tariffs in place. Each country gains in consumer surplus, since the home firm's monopoly power is diminished. It also gains in tariff revenues relative to autarky. Profits fall by the same logic as in Brander and Krugman. The *overall* effect however is different. The welfare of each country increases starting from autarky for a small fall in trade costs. The reason is three-fold: our model incorporates tariff revenues which increase when moving from autarky to stand alone; the presence of three rather than two countries gives a bigger boost to consumer surplus; even though overall profits fall the domestic firm has access to an extra market compared to Brander and Krugman. So starting from autarky as trade costs fall countries trade but government curtails trade through tariffs In fact as trade increases governments increase tariffs so they can extract more rent. To a certain extent tariffs and trade costs offset each other but tariff increases as a fraction of d.[8][9]

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[8] This is noted from the expression for optimal tariffs as a function of trade costs obtained in the second stage of the three-stage game.

[9] It is a little puzzling at first sight why stand alone should ever occur in equilibrium. Why not simply agree to global free trade ? We should remember however this is a model of intra-industry trade. As in Brander and Krugman trade at high transportation costs may be socially inefficient but firms still trade. The government curtails trade through tariff and stand alone emerges in equilibrium. This seems to be a manifestation of the prisoner's dilemma.

When trade costs fall further i and j have the incentive to form a customs union. Under CU countries internalize the benefits from tariffs. Rent-extraction is a dominant factor here. By forming a CU member countries gain in consumer surplus since consumers have access to the same good at a lower price. The domestic firm loses some monopoly power at home but gains tariff-free access to the other member's market. Tariff revenue falls. The pro-competitive effect dominates and as a result overall welfare under the customs union structure rises monotonically with falling trade costs for both i and j.

With a further reduction in trade costs the pro-competitive effect is even stronger and global free trade is the equilibrium. The gain in consumer surplus dominates the loss in tariff revenue. The domestic firm loses further monopoly power at home but now gains tariff-free access to *two* markets. The overall effect is that welfare under global free trade also increases monotonically as trade costs fall.

3.2.2 Triangular Trade Costs with Symmetric Shocks

In this section we set $Var(e_j) = Var(e_i) = Var(e_k) = \sigma^2$ and for the moment ignore correlation between markets.[10]

The symmetric structure of the model allows us to focus purely on how the degree of uncertainty affects coalition choices in the presence of trade costs.

[10] We do not lose much from this by way of insight. Adding correlations would not qualitatively affect the results since they affect only at the margins.

Welfare

We collect here the welfare expressions for various coalition structures from country j's perspective in terms of trade costs and variances of the shocks.

For stand alone:

$$EW_j^{sa} = \frac{42}{100}a^2 - \frac{36}{100}ad + \frac{72}{100}d^2 + \frac{42}{100}\sigma^2 \tag{3.26}$$

For global free trade:

$$EW_j^{GFT} = \frac{15}{32}a^2 - \frac{20}{32}ad + \frac{28}{32}d^2 + \frac{15}{32}\sigma^2 \tag{3.27}$$

For customs union between i and j :

$$EW_j^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 - \frac{269}{475}ad + \frac{401}{475}d^2 + \frac{13279}{30400}\sigma^2 \tag{3.28}$$

Equilibrium Coalition Structure

To find the equilibrium coalition structures under various trade regimes we need to evaluate the differences in welfare under various trade regimes and assume that countries choose the coalition that yields highest welfare. In what follows we set $a = 1$. The method is similar to the method used under certainty, except that the cutoffs will now be a function of σ^2.We note however that the prohibitive level of trade costs is independent of σ^2 (and the same as under certainty). This is because the decision not to trade if trade costs are too high is a decision by firms, made after uncertainty has been resolved. Firms will stop exporting when profits are zero: the resulting trade cost is linear in the shock so on taking expectations it disappears.

We also note beyond a certain cutoff variance we do not observe customs unions which is expressed in the following lemma:

Lemma 6 *For $\sigma^2 > 0.126$ we do not observe customs union.*

Proof. To see this we note that if the cutoff d for which a country is indifferent between GFT and cu and the cutoff d for which a country is indifferent between cu and sa coincide then the probability of customs union forming becomes zero. This critical level of uncertainty is found by solving for σ^2 for which $\frac{1}{62}\left(53-\sqrt{1600-1209\sigma^2}\right) = \frac{1}{472}\left(386-\sqrt{68400-30149\sigma^2}\right)$ which gives a value $\sigma^2 = 0.126$. So for $\sigma^2 > 0.126$ cu does not occur for any value of d. ■

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We note that essentially two parameters determine the equilibrium coalition structure: d and σ^2. Hence, given the welfares we can partition the $d-\sigma^2$ plane into disjoint regions and identify the equilibrium coalition structures in each region. As in the certainty case, each equilibrium coalition structure is given by a set of nine inequalities. We characterize the equilibrium coalition structures in the following proposition:

Proposition 7 *Under linear demand and constant marginal cost of production when countries face a per unit transportation cost of d and each market is subject to an exogenous shock with variance σ^2 as trade costs vary we observe the following coalition structures in equilibrium:*

GFT *is the equilibrium coalition structure if ordered pair (d, σ^2) is such that:*

$$EW_l^{GFT} - EW_l^{sa} > 0 \text{ for } l = i, j, k$$
$$EW_l^{GFT} - EW_l^{\{\{l,m\},\{n\}\}} > 0, \ \ l, m, n \in (i, j, k), \ l \neq m \neq n$$

This is equivalent to

GFT is the equilibrium coalition structure if

$$d \in [0, 0.17), \ \sigma^2 \in [0, \infty) \text{ and } \frac{173}{15\,200} - \frac{271}{3800}d + \frac{117}{3800}d^2 + \frac{971}{30400}\sigma^2 > 0$$
$$d \in [0.241, 0.5), \ \sigma^2 \in [0.126, \infty) \text{ and } \frac{39}{800} - \frac{53}{200}d + \frac{31}{200}d^2 + \frac{39}{800}\sigma^2 > 0$$

cu is the equilibrium coalition structure if ordered pair (d, σ^2) is such that :

$$EW_l^{\{\{l,m\},\{n\}\}} - EW_l^{sa} > 0 \text{ , } l, m, n \in (i, j, k), \ l \neq m \neq n$$
$$EW_l^{\{\{l,m\},\{n\}\}} - EW_l^{GFT} > 0 \text{ , } l, m, n \in (i, j, k), \ l \neq m \neq n$$

This is equivalent to

cu is the equilibrium coalition structure if [11][12]

[11] cu can be any of $\{\{i, j\}, \{k\}\}, \{\{i, k\}, \{j\}\}, \{\{j, k\}, \{i\}\}$.

[12] We do not need the $EW_l^{\{\{l,m\},\{n\}\}} - EW_l^{GFT} > 0$, $l, m, n \in (i, j, k)$, $l \neq m \neq n$ condition as

$$d \in [0.17, 0.241),\ \sigma^2 \in [0, 0.126) \text{ and } \frac{71}{1900} - \frac{92}{475}d + \frac{59}{475}d^2 + \frac{511}{30400}\sigma^2 > 0 \tag{3.29}$$

sa is the equilibrium coalition structure if the ordered pair (d, σ^2) is such that:

$$\begin{aligned} EW_l^{sa} - EW_l^{GFT} &> 0 \text{ for } l = i, j, k \\ EW_l^{sa} - EW_l^{\{\{l,m\},\{n\}\}} &> 0,\ \ l, m, n \in (i, j, k),\ l \neq m \neq n \end{aligned}$$

This is equivalent to

sa is the equilibrium coalition structure if

$$\begin{aligned} d &\in [0.225, 0.241),\ \sigma^2 \in [0, 0.126) \text{ and } -\frac{71}{1900} + \frac{92}{475}d - \frac{59}{475}d^2 - \frac{511}{30400}\sigma^2 > 0 \\ d &\in [0.241, 2.14),\ \sigma^2 \in [0.126, 0.936) \text{ and } -\frac{173}{15\,200} + \frac{271}{3800}d - \frac{117}{3800}d^2 - \frac{971}{30400}\sigma^2 > 0 \end{aligned}$$

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the $GFT - cu$ indifference locus lies entirely above the $cu - sa$ indifference locus.

The above characterization is somewhat involved and a graph in the $d - \sigma^2$ plane will make things clearer

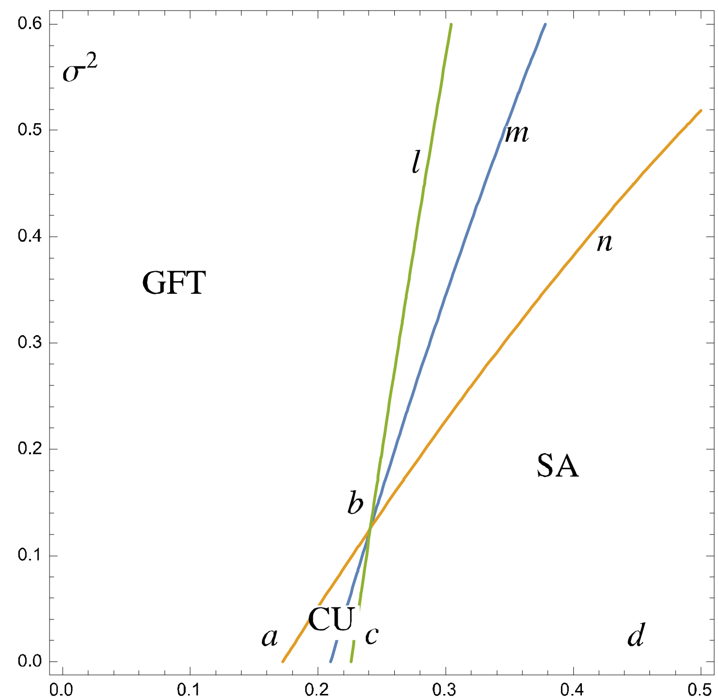

Figure 4 (Triangular Trade costs)

We explain how the graph partitions the plane into equilibrium coalition structures. Line l refers to when a country is indifferent between sa and forming cu. m is the indifference between GFT and sa and n is the indifference between GFT and cu

When variances are high all countries want to be together to pool their option values as well as enjoy the benefits of free trade.

For a certain range of intermediate values of trade costs and low variance in each market any two countries would like to form a customs union. The reasoning for this is similar to the certainty case. However, as can be seen from the graph the range of values for d diminishes as σ^2 increases because GFT becomes more attractive due to the option value effect.

For a greater value of trade costs and low variance stand alone is the equilibrium by reasoning similar to the certainty case.

It is interesting to note that as σ^2 increases without bound with $d \in [0, \infty)$ GFT dominates all other arrangements. In fact the region where GFT occurs is potentially infinite while the regions of cu and sa are finite. This leads to the following proposition:

Proposition 8 *Under linear demand and constant marginal cost of production when countries face a per unit transportation cost of d and each market is subject to an exogenous shock with variance σ^2 ,when $d \in [0, \infty)$,as $\sigma^2 \to \infty$. GFT occurs with probability one, while cu and sa occur with probability zero*

We may compare our results to our earlier paper, Ali [2014] where we were concerned with the pure uncertainty problem without trade costs. There we found that with symmetric variances global free trade *always* dominated all other arrangements when countries were symmetric. Here we get the possibility of all coalition structures as trade costs vary. It is thus a non-trivial generalization and shows how the incorporation of trade costs into trade models can affect results.

3.3 Asymmetric Trade Costs

In this section we consider a different configuration of trade costs. We assume two countries are geographically distant and another country is situated between these two. This motivates us to call the scenario the "linear trade cost problem" as countries can be seen to be situated in a straight line with one in between the other. As before they are involved in a coalition formation game.

3.3.1 The Model under Certainty

In this section we assume the structure of trade costs are

$$d_{ij} = d_{jk} = d,\ d_{ik} = 2d \tag{3.30}$$

The interpretation is that j is exactly in the "middle" of k and i.

Autarkic Trade Costs

First we find the level of trade costs beyond which we have autarky.

From j's perspective: j will stop exporting to i and k if profits are zero in those markets.

Profits as a function of trade costs is given by:

$$\pi_{ji} = q_{ji}^2 = \frac{1}{4}\left(\frac{4}{10}a - \frac{9}{20}d\right)^2 \tag{3.31}$$

$$\text{so} \qquad d_{p,j}^{sa} = \frac{8}{9}a \tag{3.32}$$

It will be apparent later that this is the prohibitive level that will be relevant when we take i and k's coalition preferences into account.

From i's perspective: i will stop exporting to j if profits are zero in that market.

Profits as a function of trade costs is given by:

$$\pi_{ij} \;=\; q_{ij}^2 = \frac{1}{4}\left(\frac{4}{10}a - \frac{36}{20}d\right)^2 \tag{3.33}$$

$$\text{so} \qquad d_{p,i}^{sa} = \frac{2}{9}a \tag{3.34}$$

Welfare

We collect here the welfare expressions for various coalition structures in terms of trade costs , for i and j. (k's is symmetric to i's given the cost structure)

For stand alone:

$$EW_j^{sa} = \frac{42}{100}a^2 - \frac{34}{100}ad + \frac{129}{200}d^2 \tag{3.35}$$

For global free trade:

$$EW_j^{GFT} = \frac{15}{32}a^2 - \frac{20}{32}ad + \frac{1}{2}d^2 \tag{3.36}$$

For customs union between i and j :

$$EW_j^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 - \frac{1223}{1900}ad + \frac{10691}{7600}d^2 \tag{3.37}$$

For stand alone:

$$EW_i^{sa} = \frac{42}{100}a^2 - \frac{11}{20}ad + \frac{763}{400}d^2 \tag{3.38}$$

For global free trade:

$$EW_i^{GFT} = \frac{15}{32}a^2 - \frac{17}{16}ad + \frac{85}{32}d^2 \tag{3.39}$$

For customs union between i and j :

$$EW_i^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 - \frac{1033}{1900}ad + \frac{6131}{7600}d^2 \tag{3.40}$$

Equilibrium Coalition Structure

To find the equilibrium coalition structures under various trade regimes we need to evaluate the differences in welfare under various trade regimes but this is not enough. Given the asymmetry the conditions for i and k will be similar but for j will be different. Thus coalition preferences will not necessarily be aligned for all countries. To resolve this we first rank the coalition choices in certain defined intervals on the real line for each country. As we did in the triangular trade costs problem we find the cutoff d's for which j and i are indifferent between any two coalitions. We find the differences in the welfares under the two coalition structures :

$$EW_j^{GFT} - EW_j^{sa} = \frac{39}{800} - \frac{7}{200}d - \frac{29}{200}d^2$$

$$EW_j^{GFT} - EW_j^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200} + \frac{1021}{3800}d - \frac{6891}{7600}d^2 \quad (3.41)$$

$$EW_j^{\{\{i,j\},\{k\}\}} - EW_j^{sa} = \frac{71}{1900} - \frac{577}{1900}d + \frac{5789}{7600}d^2 \quad (3.42)$$

$$EW_i^{GFT} - EW_i^{sa} = \frac{39}{800} - \frac{41}{80}d + \frac{599}{800}d^2$$

$$EW_i^{GFT} - EW_i^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200} - \frac{3943}{7600}d + \frac{28113}{15200}d^2 \quad (3.43)$$

$$EW_i^{\{\{i,j\},\{k\}\}} - EW_i^{sa} = \frac{71}{1900} + \frac{3}{475}d - \frac{4183}{3800}d^2 \quad (3.44)$$

and equate to zero this expression and solve for d.

From j's perspective we have the following coalition preferences:

$$for\ d \in [0, 0.33),\ GFT \succ CU \succ sa \quad (3.45)$$

$$for\ d \in [0.33, 0.47),\ CU \succ GFT \succ sa \quad (3.46)$$

$$for\ d \in [0.47, \infty),\ CU \succ sa \succ GFT \quad (3.47)$$

From i's (as well as k's) perspective we have the following coalition preferences:

$$for\ d \in [0, 0.024),\ GFT \succ CU \succ sa \tag{3.48}$$

$$for\ d \in [0.024, 0.11),\ CU \succ GFT \succ sa \tag{3.49}$$

$$for\ d \in [0.11, 0.18),\ CU \succ sa \succ GFT \tag{3.50}$$

$$for\ d \in [0.18, \infty),\ sa \succ CU \succ GFT \tag{3.51}$$

We make the following claims about the equilibrium coalition structures given the above coalition preferences:

1 *For* $\in [0, 0.024),\ GFT$ *is the equilibrium coalition structure.*

This is straightforward, as in this interval, all countries prefer global free trade.

2 *A customs union between* i *and* j *is an equilibrium coalition structure for* $d \in .[0.024, 0.18)$

We justify this by reasoning as follows: For values of d in the interval $[0, 0.18)$, i prefers $\{\{i, j\}, \{k\}\}$ while j prefers GFT. GFT however requires the consent of all parties including i's. However $\{\{i, j\}, \{k\}\}$ happens to be the second preferred choice for j, so given j cannot achieve GFT it will agree to a coalition with i since it the best it can do.

From k's perspective it would prefer a union with j but if i does not want a union with k and i and j have already formed a CU, k would not want to join that union even if were accepted since it would then effectively be GFT which is k's *last* choice.[13]

3 *For* $d \in [0.18, 0.22)$*,* sa *is the equilibrium coalition structure.*

The reasoning behind this is simple: in this interval k and i prefers sa, and there is nothing j can do but stand alone.

3.3.2 The Model under Uncertainty

We add uncertainty in the as before and solve the three-stage model.

Welfare

In this section we set $Var(e_j) = Var(e_i) = Var(e_k) = \sigma^2$, $\sigma_{ij} = \rho\sigma^2$.

We collect here the welfare expressions for various coalition structures in terms of trade costs and variances of the shocks and their covariances. , for i and j. (k's is symmetric to i's given the cost structure)

For stand alone:

$$EW_j^{sa} = \frac{42}{100}a^2 - \frac{34}{100}ad + \frac{129}{200}d^2 + \frac{42}{100}\sigma^2 \tag{3.52}$$

For global free trade:

$$EW_j^{GFT} = \frac{15}{32}a^2 - \frac{20}{32}ad + \frac{1}{2}d^2 + \frac{15}{32}\sigma^2 \tag{3.53}$$

[13] We note by this reasoning $\{\{j, k\}, \{i\}\}$ could be an equilibrium too for $d \in [0.024, 0.18)$.

For customs union between i and j :

$$EW_j^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 - \frac{1223}{1900}ad + \frac{10691}{7600}d^2 + \frac{13279}{30400}\sigma^2 + \frac{25}{1216}\sigma_{ij} \quad (3.54)$$

For stand alone:

$$EW_i^{sa} = \frac{42}{100}a^2 - \frac{11}{20}ad + \frac{763}{400}d^2 + \frac{42}{100}\sigma^2 \quad (3.55)$$

For global free trade:

$$EW_i^{GFT} = \frac{15}{32}a^2 - \frac{17}{16}ad + \frac{85}{32}d^2 + \frac{15}{32}\sigma^2 \quad (3.56)$$

For customs union between i and j :

$$EW_i^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 - \frac{1033}{1900}ad + \frac{6131}{7600}d^2 + \frac{13279}{30400}\sigma^2 + \frac{25}{1216}\sigma_{ij} \quad (3.57)$$

Equilibrium Coalition Structure

To find the equilibrium coalition structures under various trade regimes we evaluate the differences in welfare under various trade regimes but we are faced with a situation similar to the certainty case of the linear trade costs problem. We resolve it in a similar manner, the only difference being that cutoffs now will be a function of the variances and covariances of the demand shocks. The prohibitive trade costs are the same as in the certainty case by the same reasoning for the triangular trade cost problem.

$$EW_j^{GFT} - EW_j^{sa} = \frac{39}{800} - \frac{7}{200}d + \frac{31}{200}d^2 - \frac{29}{200}\sigma^2$$

$$EW_j^{GFT} - EW_j^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200} - \frac{1021}{3800}d - \frac{6891}{7600}d^2 + \frac{971}{30400}\sigma^2 - \frac{25}{1216}\sigma^2\rho \tag{3.58}$$

$$EW_j^{\{\{i,j\},\{k\}\}} - EW_j^{sa} = \frac{511}{30\,400}\sigma^2 + \frac{25}{1216}\sigma^2\rho - \frac{577}{1900}d + \frac{71}{1900} + \frac{5789}{7600}d^2 \tag{3.59}$$

$$EW_i^{GFT} - EW_i^{sa} = \frac{39}{800} - \frac{41}{80}d + \frac{500}{800}d^2 + \frac{39}{800}\sigma^2$$

$$EW_i^{GFT} - EW_i^{\{\{i,j\},\{k\}\}} = \frac{173}{15\,200} - \frac{3943}{7600}d - \frac{28113}{15200}d^2 + \frac{971}{30400}\sigma^2 - \frac{25}{1216}\sigma^2\rho \tag{3.60}$$

$$EW_i^{\{\{i,j\},\{k\}\}} - EW_i^{sa} = \frac{511}{30\,400}\sigma^2 + \frac{25}{1216}\sigma^2\rho + \frac{3}{475}d + \frac{71}{1900} - \frac{4183}{3800}d^2 \tag{3.61}$$

First we set $\rho = 0$ and characterize the equilibrium coalition structures in the following proposition:

Proposition 9 *Under linear demand and constant marginal cost of production under the linear cost structure with each market subject to an exogenous shock with variance σ^2 as trade costs vary we observe the following coalition structures in equilibrium:*

GFT *is the equilibrium coalition structure if each element in the ordered pair* (d, σ^2) *satisfies:*

$$\begin{aligned} EW_l^{GFT} - EW_l^{sa} &> 0 \textit{ for } l = i, j, k \\ EW_l^{GFT} - EW_l^{\{\{l,m\},\{n\}\}} &> 0, \;\; l, m, n \in (i, j, k), \; l \neq m \neq n \end{aligned}$$

This is equivalent to

GFT is the equilibrium coalition structure if

$$\begin{aligned} d &\in [0, 0.21), \; \sigma^2 \in [0, \infty) \text{ and } \frac{173}{15\,200} - \frac{3943}{7600}d - \frac{28113}{15200}d^2 + \frac{971}{30400}\sigma^2 > 0 \\ d &\in [0.21, 0.22), \; \sigma^2 \in [0.0.52, \infty) \text{ and } \frac{39}{800} - \frac{41}{80}d + \frac{500}{800}d^2 + \frac{39}{800}\sigma^2 > 0 \end{aligned}$$

cu is the equilibrium coalition structure if ordered pair (d, σ^2) is such that :

$$\begin{aligned} EW_l^{\{\{l,m\},\{n\}\}} - EW_l^{sa} &> 0 \text{ , } l, m, n \in (i, j, k), \; l \neq m \neq n \\ EW_l^{\{\{l,m\},\{n\}\}} - EW_l^{GFT} &> 0 \text{ , } l, m, n \in (i, j, k), \; l \neq m \neq n \end{aligned}$$

This is equivalent to

cu is the equilibrium coalition structure if

$$d \in [0.021, 0.22),\ \sigma^2 \in [0, 0.78) \text{ and } -\frac{173}{15\,200} + \frac{3943}{7600}d + \frac{28113}{15200}d^2 - \frac{971}{30400}\sigma^2 > 0$$

[14][15]

sa is the equilibrium coalition structure if the ordered pair (d, σ^2) is such that:

$$EW_l^{sa} - EW_l^{GFT} > 0 \text{ for } l = i, j, k$$
$$EW_l^{sa} - EW_l^{\{\{l,m\},\{n\}\}} > 0,\ \ l, m, n \in (i, j, k),\ l \neq m \neq n$$

This is equivalent to

sa is the equilibrium coalition structure if

$$d \in [0.18, 0.21),\ \sigma^2 \in [0, 0.52) \text{ and } -\frac{511}{30\,400}\sigma^2 - \frac{3}{475}d - \frac{71}{1900} + \frac{4183}{3800}d^2 > 0$$
$$d \in [0.21, 0.89),\ \sigma^2 \in [0.52, 0.79) \text{ and } -\frac{173}{15\,200} + \frac{271}{3800}d - \frac{117}{3800}d^2 - \frac{971}{30400}\sigma^2 > 0$$

---

[14] cu can be any of $\{\{i, j\}, \{k\}\}, \{\{i, k\}, \{j\}\}, \{\{j, k\}, \{i\}\}$.

[15] The first condition is redundant as is clear from Fig 2

The graph below will help clarify the proposition.

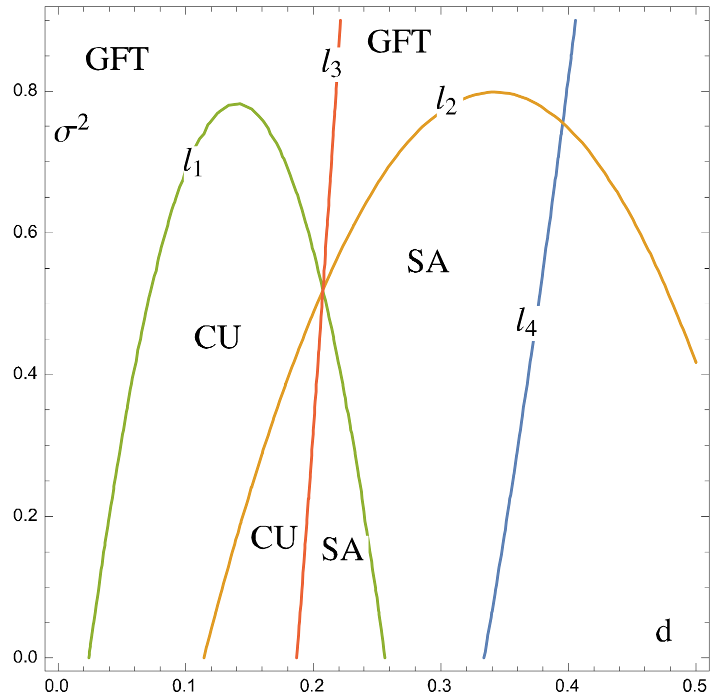

Figure 5 (Linear Trade Costs)

l_1 is the $GFT - CU$ indifference locus, l_2 is the $GFT - SA$ indifference locus, l_1 is the $CU - SA$ indifference locus, all from country i and k's perspective.

l_4 is the $GFT - CU$ from j's perspective.

First of all let us note that in the non-autarkic range of trade costs for i and k from j's perspective, GFT is preferred over all other arrangements.(as shown by line l_4).

The reason for this is that j being in the center has more incentive to trade with the other two countries since it's costs from doing so is low relative to the other two countries. i and k only agree to GFT for a smaller range of values as their costs from complete integration is higher. However increasing volatility makes GFT more and more likely since then countries can enjoy the option value associated with being together. Indeed as in the triangular case a cursory look at the graph above shows that within the non-autarkic region as volatility increases without bound the area of GFT becomes infinity so we have an analogue of Proposition 7 for the linear case as well.

Beyond a certain level of trade costs i wants to form a union with j and exclude, since k is far away and trade with k is costly. Beyond a certain value of d ,i and k want to stand alone since by a reasoning similar to the certainty case, even though j by virtue of its position would still prefer GFT. However it is forced to stand alone as i and k want to stand alone. Beyond a certain value of d, i and k do not export to j. This is not autarky, however since j may find it profitable to export to i and $k,$ so we actually have stand alone effectively

A Special Case

In this section we investigate conditions under which the somewhat pathological case where i and k (the countries far apart) form a customs union leaving out the middle country j.

We may motivate this section by considering the case of the US and Japan. These are two quite geographically distant countries but their bilateral trade volume is very high under any measure. There are many countries much nearer to US with which it has little trade and the same applies to Japan. We think this example will in some small way shed light on this phenomenon. Of course there are many reasons for the high level of trade between these two countries and we are merely suggesting market correlation can be one of them.

We first note that the welfare of i when it forms a union with k is (under the linear cost structure) :

$$EW_i^{\{\{i,k\},\{j\}\}} = \frac{869}{1900}a^2 - \frac{488}{475}ad + \frac{10837}{3800}d^2 + \frac{1}{100}\sigma_j^2 + \frac{871}{2432}\sigma_k^2 + \frac{167}{2432}\sigma_i^2 + \frac{25}{1216}\sigma_{ik} \tag{3.62}$$

We can have $\{\{i,k\},\{j\}\}$ as an equilibrium outcome iff

$$\begin{aligned} EW_i^{\{\{i,k\},\{j\}\}} &> EW_i^{\{\{i,j\},\{k\}\}} \\ EW_i^{\{\{i,k\},\{j\}\}} &> EW_i^{GFT} \\ EW_i^{\{\{i,k\},\{j\}\}} &> EW_i^{sa} \end{aligned}$$

and

$$\begin{aligned} EW_k^{\{\{i,k\},\{j\}\}} &> EW_k^{\{\{i,j\},\{k\}\}} \\ EW_k^{\{\{i,k\},\{j\}\}} &> EW_k^{GFT} \\ EW_k^{\{\{i,k\},\{j\}\}} &> EW_k^{sa} \end{aligned}$$

In what follows we set $a = 1$,

We focus on i since k's case is symmetric.

The first three conditions are equivalent to

$$\begin{aligned}
\frac{919}{1900}d + \frac{15543}{7600}d^2 - \frac{21167}{60800}\sigma_j^2 + \frac{21167}{60800}\sigma_k^2 + \frac{25}{1216}\sigma_{ik} - \frac{25}{1216}\sigma_{ij} &> 0 \\
-\frac{173}{15200} + \frac{267}{7600}d + \frac{2973}{15200}d^2 - \frac{669}{2432}\sigma_i^2 - \frac{21}{400}\sigma_j^2 + \frac{719}{2432}\sigma_k^2 + \frac{25}{1216}\sigma_{ik} &> 0 \\
\frac{71}{1900} - \frac{907}{1900}d + \frac{7177}{7600}d^2 - \frac{4029}{12160}\sigma_i^2 + \frac{21167}{60800}\sigma_k^2 + \frac{25}{1216}\sigma_{ik} &> 0
\end{aligned}$$

Under certainty,

$$\frac{919}{1900}d + \frac{15543}{7600}d^2 > 0 \tag{3.63}$$

$$-\frac{173}{15200} + \frac{267}{7600}d + \frac{2973}{15200}d^2 > 0 \tag{3.64}$$

$$\frac{71}{1900} - \frac{907}{1900}d + \frac{7177}{7600}d^2 > 0 \tag{3.65}$$

These conditions are equivalent to the following holding simultaneously:

$$d > 0.237 \tag{3.66}$$

$$d > 0.17 \tag{3.67}$$

$$d < 0.1 \text{ or } d > 0.41 \tag{3.68}$$

However under certainty in the region of $d > 0.41$ we have autarky so we have the intuitive result:

Proposition 10 *With linear trade costs, under certainty, countries that are farthest apart will not form a union leaving out a country in the centre.*

We now make j non stochastic ($\sigma_j^2 = 0$), set $\sigma_i^2 = \sigma_k^2 = \sigma^2$ and $\sigma_{ik} = \sigma_{ij} = \sigma_{jk} = 0$.

Now for $\{\{i,k\},\{j\}\}$ to be an equilibrium outcome,

$$\frac{919}{1900}d + \frac{15543}{7600}d^2 - \frac{21167}{60800}\sigma^2 > 0 \tag{3.69}$$

$$-\frac{173}{15200} + \frac{267}{7600}d + \frac{2973}{15200}d^2 + \frac{50}{2432}\sigma^2 > 0 \tag{3.70}$$

$$\frac{71}{1900} - \frac{907}{1900}d + \frac{7177}{7600}d^2 + \frac{511}{30400}\sigma^2 > 0 \tag{3.71}$$

If we fix d and increase σ^2 the above three inequalities will ultimately hold, so we have the somewhat counterintuitive result:

Proposition 11 *Under linear costs, if the middle country is non-stochastic and the volatilities in the two extreme countries high enough, the extreme countries may wish to form a union excluding the middle country.*

The intuition behind the result is that even though the two countries are far apart their volatility may be so high that there is considerable option value from a union while none from j's market.

3.4 Summary

In this chapter we considered a three-country model of trade and oligopoly with endogenous tariffs and exogenous trade costs under both certainty and uncertainty. We have found that trade costs critically affect choice of output by firms and choice of tariffs and coalitions by governments With symmetric trade costs as trade costs vary we find different coalitions forming in equilibrium. With uncertainty affects coalition choices change by changing the cutoff trade costs at which a country may be indifferent between two different coalitions. Further, coalitions that may form under certainty or low uncertainty may not form with high uncertainty in the case where trade costs are the same for all countries. On the other hand under different configuration of trade costs coalitions that may not be feasible under certainty may be shown to be possible under uncertainty. In both cases, as long as trade costs are not prohibitive, as volatility in every market increases without bound, we get global free trade with probability one. As a special case we show that under certain conditions two geographically distant countries may choose to form a coalition excluding a nearby country if the market volatility and correlation between partner countries is high enough. Thus we see that trade costs matter when negotiating a trade agreement. We think we have demonstrated crucial link between the choice of trade regime and trade costs, a connection that is not explicitly explored in the literature.

Conclusion

This thesis was concerned with the effect uncertainty and trade costs have on the coalition choices by countries in a model of oligopoly and trade by considering a three-stage simultaneous move game. We endogenized the coalition structure that forms in the three stage game. We found that for small volatilities countries prefer global free trade. The more positively correlated two countries are the more likely they are to form a customs union. We also found that countries may wish to stand alone under certain variance-covariance configurations.

We then added exogenous trade costs under both certainty and uncertainty. We found that trade costs critically affect choice of output by firms and choice of tariffs and coalitions by governments With symmetric trade costs as trade costs vary we found different coalitions forming in equilibrium. The introduction of demand uncertainty affects coalition choices by changing the cutoff trade costs at which a country may be indifferent between two different coalitions. Further, coalitions that may form under certainty or low uncertainty may not form with high uncertainty. On the other hand under different configuration of trade costs coalitions that may not be feasible under certainty may be shown to be possible under uncertainty. In both cases, as long as trade costs are not prohibitive, as volatility in every market increases without bound, we get global free trade with probability one. As a special case we showed that under certain conditions two geographically distant countries may choose to form a coalition excluding a nearby country if the market volatility and correlation between partner countries is high enough

We admit there are many limitations to this model. It does not allow for renegotiation . We have not considered Free Trade Areas in this paper although they seem to be the preferred choice of PTAs these days. Another extension would be to allow for substitutability between goods. Accommodating a "love for variety" parameter would enrich the model.

Although we do not expect to see any significant qualitative changes to our results from these extensions, we nevertheless expect changes at least at the margins.

One major extension we hope to see is the incorporation of foreign direct investment in our model. We expect this to significantly change the qualitative predictions in our framework.

The most challenging extension would be to account for more than three countries, something we did not do for reasons of tractability. Nevertheless, for this sort of model to have any empirical relevance, we need to be able to go beyond three countries. Nevertheless the point of the research was to show that trade costs and uncertainty matter in trade agreements. We think we have demonstrated crucial link between the choice of trade regime, uncertainty and trade costs. We hope this will motivate others to pursue both theoretical and empirical research on this issue.

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Appendix

In this section we derive the welfare functions for various coalition structures in chapter 1

Optimal tariffs and welfare under $\{\{i, j\}, \{k\}\}$

In stage 3 each firm chooses its output in each market to maximize its profits.

Profit of firm i in j's market is

$$\pi_{ij}(q_{ij}, q_{jj}, q_{kj}, e_j, t_{ji}) = (a - q_{jj} - q_{ij} - q_{kj} + e_j)q_{ij} - t_{ji}q_{ij} \tag{1.72}$$

Where q_{ij} is the quantity sold by i to j's market and t_{ji} is the tariff imposed by j on i's exports. k sets tariffs t_{ki}, t_{kj} on i and j respectively.

The first-order condition for profit maximization in country j is

$$-2q_{ij} + a - q_{jj} - q_{kj} + e_j - t_{ji} = 0 \tag{1.73}$$

This may be written as

$$a - q_{ij} - Q_j + e_j - t_{ji} = 0 \tag{1.74}$$

Now if i and j form a Customs Union they eliminate tariffs between themselves $(t_{ij} = t_{ji})$ and impose a common tariff t against country k.

The first-order conditions gives $q_{ij}, q_{jj}, q_{kj}, q_{ji}, q_{ii}, q_{ik}, q_{ki}, q_{kk}, q_{jk}, Q_i, Q_j, Q_k$ as functions of tariffs and the random variables as follows

$$q_{ij} = \frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_j \tag{1.75}$$

$$q_{jj} = \frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_j \tag{1.76}$$

$$q_{kj} = \frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_j \tag{1.77}$$

$$q_{ji} = \frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_i \tag{1.78}$$

$$q_{ii} = \frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_i \tag{1.79}$$

$$q_{ki} = \frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_i \tag{1.80}$$

$$q_{ik} = \frac{1}{4}a - \frac{3}{4}t_{ki} + \frac{1}{4}t_{kj} + \frac{1}{4}e_k \tag{1.81}$$

$$q_{kk} = \frac{1}{4}a + \frac{1}{4}t_{kj} + \frac{1}{4}t_{ki} = \frac{1}{4}e_k \tag{1.82}$$

$$q_{jk} = \frac{1}{4}a + \frac{1}{4}t_{ki} - \frac{3}{4}t_{kj} + \frac{1}{4}e_k \tag{1.83}$$

In what follows, we concentrate on country j.

Total output to country j is given by

$$Q_j = \frac{3a - t_{ji} - t_{jk} + 3e_j}{4} \tag{1.84}$$

Profit of j is

$$\begin{aligned}\pi_j(t, t_{ki}, t_{kj}, e_i, e_j, e_k) &= q_{ji}^2 + q_{jj}^2 + q_{jk}^2 = \left(\frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_j\right)^2 + \left(\frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_i\right)^2 \\ &+ \left(\frac{1}{4}a + \frac{1}{4}t_{ki} - \frac{3}{4}t_{kj} + \frac{1}{4}e_k\right)^2 \end{aligned} \tag{1.85}$$

Consumer surplus in j is

$$CS_j = \frac{1}{32}(3a - t + 3e_j)^2 \tag{1.86}$$

Tariff revenue for j is

$$T_j = t\left(\frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_j\right) \tag{1.87}$$

The expressions for the profit, consumer surplus and tariff revenues for i are similar with subscript i interchanged for j.

Expressions for the profit, consumer surplus and tariff revenues for country k can be found by substitution similarly as follows

$$\pi_k(t, t_{ki}, t_{kj}, e_i, e_j, e_k) = q_{ki}^2 + q_{kk}^2 + q_{kj}^2 = \left(\frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_i\right)^2 + (\frac{1}{4}a + \frac{1}{4}t_{kj} + \frac{1}{4}t_{ki} + \frac{1}{4}e_k)^2 + \left(\frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_j\right)^2 \quad (1.88)$$

$$CS_k = \frac{1}{32}(3a - t_{kj} + t_{ki} + 3e_k)^2 \quad (1.89)$$

$$T_k = t_{kj}\left(\frac{1}{4}a + \frac{1}{4}t_{ki} - \frac{3}{4}t_{kj} + \frac{1}{4}e_k\right) + t_{kj}\left(\frac{1}{4}a - \frac{3}{4}t_{ki} + \frac{1}{4}t_{kj} + \frac{1}{4}e_k\right) \quad (1.90)$$

In stage 2 governments set tariffs by maximizing a social welfare function.

The expected welfare of a country is taken to be the sum of producer surplus, consumer surplus and tariff revenues.

$$EW_j(t, t_{ki}, t_{kj}, e_i, e_j, e_k) = E(\pi_j + CS_j + T_j) \quad (1.91)$$

We found expressions for these in the previous section. By substitution one obtains

$$EW_j(t, t_{ki}, t_{kj}, e_i, e_j, e_k) = E(\left(\frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_j\right)^2 + \left(\frac{1}{4}a + \frac{1}{4}t + \frac{1}{4}e_i\right)^2 + \left(\frac{1}{4}a + \frac{1}{4}t_{ki} - \frac{3}{4}t_{kj} + \frac{1}{4}e_k\right)^2 + \frac{1}{32}(3a - t + 3e_j)^2 + t\left(\frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_j\right)) \quad (1.92)$$

To find the optimal common tariffs, country j maximizes the joint welfare of i and j with respect to the common tariff to obtain :

$$t = \frac{5}{38}\left(2a + e_i + e_j\right) \tag{1.93}$$

Similarly the expected welfare of k is found to be

$$\begin{aligned} EW_k(t, t_{ki}, t_{kj}, e_i, e_j, e_k) &= E\left[\left(\frac{1}{4}a - \frac{3}{4}t + \frac{1}{4}e_i\right)^2 + (\frac{1}{4}a + \frac{1}{4}t_{kj} + \frac{1}{4}t_{ki} + \frac{1}{4}e_k)^2\right. \\ &\quad + \tfrac{1}{32}(3a - t_{kj} - t_{ki} + 3e_k)^2 + t_{kj}\left(\tfrac{1}{4}a + \tfrac{1}{4}t_{ki} - \tfrac{3}{4}t_{kj} + \tfrac{1}{4}e_k\right) \\ &\quad \left. + t_{kj}\left(\tfrac{1}{4}a - \tfrac{3}{4}t_{ki} + \tfrac{1}{4}t_{kj} + \tfrac{1}{4}e_k\right)\right] \end{aligned} \tag{1.94}$$

Again to find optimal tariffs we take the first-order conditions with respect to t_{ki}, t_{kj} to obtain

$$t_{ki} = t_{kj} = \frac{3}{10}a + \frac{3}{10}e_k \tag{1.95}$$

Substituting these values into the social welfare functions one obtains

$$EW_j^{\{\{i,j\},\{k\}\}} = \frac{167}{2432}\sigma_i^2 + \frac{25}{1216}\sigma_{ij} + \frac{871}{2432}\sigma_j^2 + \frac{1}{100}\sigma_k^2 + \frac{869}{1900}a^2 \tag{1.96}$$

$$EW_k^{\{\{i,j\},\{k\}\}} = \frac{377}{11\,552}\sigma_i^2 - \frac{345}{5776}\sigma_{ij} + \frac{377}{11\,552}\sigma_j^2 + \frac{2}{5}\sigma_k^2 + \frac{732}{1805}a^2 \tag{1.97}$$

Welfare under Global Free Trade $\{\{i, j, k\}\}$

We now calculate the welfare of each country under the grand coalition.

We focus on country j.

The following are calculated in a manner analogous to the previous section except that all tariffs are set to 0. Firms maximize profit in each market. There is no stage 2 since there are no tariff decisions

$$q_{ij} = \frac{1}{4}a + \frac{1}{4}e_j \tag{1.98}$$

$$Q_j = \frac{3}{4}a + \frac{3}{4}e_j \tag{1.99}$$

$$\pi_{ij}(e_i, e_j, e_k) = (\frac{1}{4}a + \frac{1}{4}e_j)^2 + (\frac{1}{4}a + \frac{1}{4}e_i)^2 + (\frac{1}{4}a + \frac{1}{4}e_k)^2 \tag{1.100}$$

$$CS_j = \frac{9}{32}(a + e_j)^2 \tag{1.101}$$

$$EW_j(e_i, e_j, e_k) = E(\pi_j + CS_j) \tag{1.102}$$

$$EW_j^{FT} = \frac{1}{16}\sigma_i^2 + \frac{11}{32}\sigma_j^2 + \frac{1}{16}\sigma_k^2 + \frac{15}{32}a^2 \tag{1.103}$$

Stand Alone $\{\{i\},\{j\},\{k\}\}$

Stand alone implies that all three countries have discriminatory tariffs in place.

Focusing on country j.

The profit of firm i in j is

$$\pi_{ij}(q_{ij}, q_{jj}, q_{kj}, e_j, t_{ji}) = (a - q_{jj} - q_{ij} - q_{kj} + e_j)q_{ij} - t_{ji}q_{ij} \tag{1.104}$$

Where q_{ij} is the quantity sold by i to j's market and t_{ji} is the tariff imposed by j on i's exports.

Similar profit functions are obtained for each firm in each country. From the profit maximizing conditions we derive optimal quantities as functions of the disturbances and tariffs

As before expected welfare of a country is taken to be the sum of producer surplus, consumer surplus and tariff revenues.

Expected welfare of j is:

$$EW_j^{sa} = \pi_j + CS_j + TR^j \tag{1.105}$$

Plugging the values of the quantities obtained in stage 3 into the welfare functions and taking the first order conditions with respect to tariffs give the optimal tariffs:

$$t_{ij} = t_{ik} = \frac{3}{10}a + \frac{3}{10}e_i \tag{1.106}$$

$$t_{ji} = t_{jk} = \frac{3}{10}a + \frac{3}{10}e_j \tag{1.107}$$

$$t_{ki} = t_{kj} = \frac{3}{10}a + \frac{3}{10}e_k \tag{1.108}$$

Finally we can obtain the expected welfare of j as:

$$EW_j^{sa} = 0.42a^2 + 0.01\sigma_i^2 + 0.4\sigma_j^2 + 0.01\sigma_k^2 \tag{1.109}$$

Similarly welfare of i and k is given by:

$$EW_i^{sa} = 0.42a^2 + 0.01\sigma_j^2 + 0.4\sigma_i^2 + 0.01\sigma_k^2 \tag{1.110}$$

$$EW_k^{sa} = 0.42a^2 + 0.01\sigma_j^2 + 0.4\sigma_k^2 + 0.01\sigma_i^2 \tag{1.111}$$

In this section we derive the welfare functions in chapter 2 as functions of the demand scale parameter, the trade costs and the variances and covariances of the demand functions. The welfare expressions in the paper are obtained by specialization on these formulas.

Stand Alone$\{\{i\},\{j\},\{k\}\}$

Stand alone implies that all three countries have discriminatory tariffs in place.

Focusing on country j.

The profit of firm i in j is

$$\pi_{ij}(q_{ij}, q_{jj}, q_{kj}, e_j, t_{ji}, d_{ij}) = (a - q_{jj} - q_{ij} - q_{kj} + e_j)q_{ij} - t_{ji}q_{ij} - d_{ij}q_{ij} \quad (1.112)$$

Where q_{ij} is the quantity sold by i to j's market and t_{ji} is the tariff imposed by j on i's exports and d_{ij} is the unit transportation cost for exporting to j.

The first-order condition for profit maximization in country j is

$$-2q_{ij} + a - q_{jj} - q_{kj} + e_j - t_{ji} - d_{ij} = 0 \quad (1.113)$$

This may be written as

$$a - q_{ij} - Q_j + e_j - t_{ji} - d_{ij} = 0 \quad (1.114)$$

Expected welfare of j is:

$$EW_j^{sa} = \pi_j + CS_j + TR^j \tag{1.115}$$

Plugging the values of the quantities obtained in stage 3 into the welfare functions and taking the first order conditions with respect to tariffs give the optimal tariffs:

From all the first-order conditions in market i, j and k gives $q_{ij}, q_{jj}, q_{kj}, q_{ji}, q_{ii}, q_{ik},$ $q_{ki}, q_{kk}, q_{jk}, Q_i, Q_j, Q_k$ as functions of tariffs, unit transportation costs and the random variables as follows

$$q_{ij} = \frac{1}{4}(a - 3d_{ij} + d_{jk} + t_{ji} - 3t_{jk} + e_j)$$

$$q_{jj} = \frac{1}{4}(a + d_{ij} + d_{jk} + t_{ji} + t_{jk} + e_j)$$

$$q_{kj} = \frac{1}{4}(a - 3d_{jk} + d_{ij} + t_{ji} - 3t_{jk} + e_j)$$

$$q_{ji} = \frac{1}{4}(a - 3d_{ij} + d_{ik} + t_{ij} - 3t_{ik} + e_i)$$

$$q_{ii} = \frac{1}{4}(a + d_{ij} + d_{ik} + t_{ij} + t_{ik} + e_i)$$

$$q_{ki} = \frac{1}{4}(a - 3d_{ik} + d_{ik} + t_{ik} - 3t_{ij} + e_i)$$

$$q_{ik} = \frac{1}{4}(a - 3d_{ik} + d_{jk} + t_{ki} - 3t_{kj} + e_k)$$

$$q_{kk} = \frac{1}{4}(a + d_{ik} + d_{jk} + t_{ki} + t_{kj} + e_k)$$

$$q_{jk} = \frac{1}{4}(a - 3d_{jk} + d_{ik} + t_{ki} - 3t_{kj} + e_k)$$

The the optimal tariffs are obtained by plugging in the optimal quantities into the welfare function, maximizing with respect to the tariffs and solving the resulting set of equations.

In what follows, we concentrate on country j.

Total output to country j is given by

$$Q_j = \frac{1}{4}(3a - d_{ij} - d_{jk} - t_{ji} - t_{jk} + 3e_j)$$

$$t_{ij} = \frac{3}{10}a - \frac{7}{20}d_{ij} + \frac{3}{20}d_{ik} + \frac{3}{10}e_i$$

$$t_{ik} = \frac{3}{10}a - \frac{7}{20}d_{ik} + \frac{3}{20}d_{ij} + \frac{3}{10}e_i$$

$$t_{ji} = \frac{3}{10}a - \frac{7}{20}d_{ij} + \frac{3}{20}d_{jk} + \frac{3}{10}e_j$$

$$t_{jk} = \frac{3}{10}a - \frac{7}{20}d_{jk} + \frac{3}{20}d_{ij} + \frac{3}{10}e_j$$

$$t_{ki} = \frac{3}{10}a - \frac{7}{20}d_{ik} + \frac{3}{20}d_{jk} + \frac{3}{10}e_k$$

$$t_{kj} = \frac{3}{10}a - \frac{7}{20}d_{jk} + \frac{3}{20}d_{ik} + \frac{3}{10}e_k$$

Finally we can obtain the expected welfare of j as:

$$\begin{aligned} EW_j^{sa} &= \frac{21}{50}a^2 + \frac{171}{400}d_{ij}^2 + \frac{171}{400}d_{jk}^2 + \frac{1}{200}d_{ik}^2 + d_{ij}\left(-\frac{19}{100}a - \frac{9}{200}d_{ik} - \frac{1}{20}d_{jk}\right) \\ &+ d_{ik}\left(\frac{1}{50}a - \frac{9}{200}d_{jk}\right) - \frac{19}{100}ad_{jk} + \frac{1}{100}\sigma_k^2 + \frac{2}{5}\sigma_j^2 + \frac{1}{100}\sigma_i^2 \end{aligned} \quad (1.116)$$

Similarly welfare of i and k is given by:

$$\begin{aligned} EW_i^{sa} &= \frac{21}{50}a^2 + \frac{171}{400}d_{ij}^2 + \frac{171}{400}d_{ik}^2 + \frac{1}{200}d_{jk}^2 + \\ & d_{ij}\left(-\frac{19}{100}a - \frac{9}{200}d_{jk} - \frac{1}{20}d_{ik}\right) + d_{jk}\left(\frac{1}{50}a - \frac{9}{200}d_{ik}\right) \\ & -\frac{19}{100}ad_{ik} + \frac{1}{100}\sigma_k^2 + \frac{2}{5}\sigma_i^2 + \frac{1}{100}\sigma_j^2 \end{aligned} \quad (1.117)$$

$$\begin{aligned} EW_k^{sa} &= \frac{21}{50}a^2 + \frac{171}{400}d_{ik}^2 + \frac{171}{400}d_{jk}^2 + \frac{1}{200}d_{ij}^2 + \\ & d_{ik}\left(-\frac{19}{100}a - \frac{9}{200}d_{ij} - \frac{1}{20}d_{jk}\right) + d_{ij}\left(\frac{1}{50}a - \frac{9}{200}d_{jk}\right) \\ & -\frac{19}{100}ad_{jk} + \frac{1}{100}\sigma_j^2 + \frac{2}{5}\sigma_k^2 + \frac{1}{100}\sigma_i^2 \end{aligned} \quad (1.118)$$

Global Free Trade $\{\{i, j, k\}\}$

In the case of global free trade, all countries set their tariffs to zero, so there is no stage 2.

The resulting welfare functions are:

$$\begin{aligned} EW_j^{GFT} &= \frac{15}{32}a^2 - \frac{7}{16}ad_{ij} + \frac{21}{32}d_{ij}^2 + \frac{1}{4}ad_{ik} - \frac{3}{8}d_{ij}d_{ik} + \frac{1}{8}d_{ik}^2 - \frac{7}{16}ad_{jk} + \frac{3}{16}d_{ij}d_{jk} \\ &\quad -\frac{3}{8}d_{ik}d_{jk} + \frac{21}{32}d_{jk}^2 + \frac{1}{16}\sigma_k^2 + \frac{11}{32}\sigma_j^2 + \frac{1}{16}\sigma_i^2 \end{aligned} \tag{1.119}$$

$$\begin{aligned} EW_i^{GFT} &= \frac{15}{32}a^2 - \frac{7}{16}ad_{ij} + \frac{21}{32}d_{ij}^2 + \frac{1}{4}ad_{jk} - \frac{3}{8}d_{ij}d_{jk} + \frac{1}{8}d_{jk}^2 - \frac{7}{16}ad_{ik} + \frac{3}{16}d_{ij}d_{ik} \\ &\quad -\frac{3}{8}d_{ik}d_{jk} + \frac{21}{32}d_{ik}^2 + \frac{1}{16}\sigma_k^2 + \frac{11}{32}\sigma_i^2 + \frac{1}{16}\sigma_j^2 \end{aligned} \tag{1.120}$$

$$\begin{aligned} EW_k^{GFT} &= \frac{15}{32}a^2 - \frac{7}{16}ad_{ik} + \frac{21}{32}d_{ij}^2 + \frac{1}{4}ad_{ij} - \frac{3}{8}d_{ij}d_{ik} + \frac{1}{8}d_{ij}^2 - \frac{7}{16}ad_{jk} + \frac{3}{16}d_{ik}d_{jk} \\ &\quad -\frac{3}{8}d_{ij}d_{jk} + \frac{21}{32}d_{jk}^2 + \frac{1}{16}\sigma_j^2 + \frac{11}{32}\sigma_k^2 + \frac{1}{16}\sigma_i^2 \end{aligned} \tag{1.121}$$

Customs Union $\{\{i, j\}, \{k\}\}$

The expected welfares of i and j if they form a customs union are given below. They eliminate tariffs between themselves and impose a joint common tariff on k

$$EW_j^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 + \frac{9}{19}ad_{ij} + \frac{13}{19}d_{ij}^2 + \frac{81}{400}d_{jk}^2 + d_{jk}\left(-\frac{9}{100}a - \frac{9}{200}d_{ik}\right) + \frac{1}{100}ad_{ik} + \frac{1}{400}d_{ik}^2 + \frac{1}{100}\sigma_k^2 + \frac{167}{2432}\sigma_i^2 + \frac{871}{2432}\sigma_j^2 + \frac{25}{1216}\sigma_{ij} \quad (1.122)$$

$$EW_i^{\{\{i,j\},\{k\}\}} = \frac{869}{1900}a^2 + \frac{9}{19}ad_{ij} + \frac{13}{19}d_{ij}^2 + \frac{81}{400}d_{ik}^2 + d_{ik}\left(-\frac{9}{100}a - \frac{9}{200}d_{jk}\right) + \frac{1}{100}ad_{jk} + \frac{1}{400}d_{jk}^2 + \frac{1}{100}\sigma_k^2 + \frac{167}{2432}\sigma_j^2 + \frac{871}{2432}\sigma_i^2 + \frac{25}{1216}\sigma_{ij} \quad (1.123)$$