ENRICHING MATHEMATICS EXPECTATIONS IN GRADE ONE WITH A REGGIO-INSPIRED EMERGENT CURRICULM

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Abstract

This thesis researches the mathematics learning of young children. In a school seriously invested in the Reggio Emilia experience for ten years, grade-one learning is project based. I pose the question is it possible to enrich numeracy expectations in the Ontario mathematics curriculum for grade one, while engaging in a Reggio-inspired emergent curriculum? As a teacher/researcher I observed and listened carefully to the students' interactions. I facilitated discussions, problem solving and directions for the project, while remaining open to the children's mathematical theories. I paid close attention to their hypotheses about proportion as well as their exploration into perimeters, and I supported their math manipulatives "factory." Pedagogical documentation in the form of graphic novels demonstrates how emergent curriculum weaves play-based inquiry with sophisticated mathematical thinking. I argue that emergent curriculum and provincial expectations can co-exist in a grade one classroom to infuse complex mathematical thinking with joy and beauty.

Dedication

To my friend John Townley Dean

Acknowledgements

I would like to thank my “Reggio-inspired” mentor Professor Carol Anne Wien for her support and encouragement for the past ten years, and for giving her time so generously to our school professional development and growth. I would also like to acknowledge Pat Margerm my “ mathematics” mentor who first suggested that I research my two areas of interest. I appreciate the input of Professor Esther Fine for her invaluable help and ideas when editing. My colleagues at school, Justine Montgomery and Kerry Embrey have been an inspiration and invaluable to my professional growth through our meetings, discussions and inquiries. I would also like to acknowledge the work of Francesca Giorgiani , from Reggio Emelia who worked along side me and helped me develop a deeper understanding of the Reggio Emelia experience. Finally I would like to thank my family, Henry, Catherine and Victoria for their patience as I was focused on my work.

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Chapter One: Tomorrow's World

Ten years ago, in consultation with my principal and colleagues, a Reggio-inspired experience was initiated in the kindergarten and grade one classes of our school. The school where we work has a long tradition of educational innovation. The school was founded in 1887, as a place of learning for the daughters of clergy, at a time when few women were formally educated. The school is for junior kindergarten to grade 12 girls, ethnically diverse with boarding students from more than sixteen different countries. The students are predominantly from high socio-economic backgrounds with several girls on scholarship from Nepal and the Greater Toronto Area. The school is an independent school with high tuition that affords state-of-the-art facilities and a junior school building recently erected with Reggio principles in mind. The classrooms feature light coloured wood and a pale, calm palette of colours. The indoor walls are glass to afford transparency of learning and visibility into the hallways and between rooms.

The educational implications of a Reggio–inspired experience with its rich philosophy and social–constructivist approach, appealed to the faculty but how would our parents react to a radical change in the school's direction? The school already held a reputation for excellence, with one hundred percent of grade 12 students going on to university. Would the parents agree to a change in pedagogy, as the Reggio experience is very different from the education parents experienced? The school felt a change in pedagogy was necessary to prepare the students for the needs of a changing world and the demands of a changing workforce. Many western, urban societies have recognized that memorizing and regurgitating information is no longer sufficient or practical but skill in locating and utilizing information to solve problems is paramount. Many factors, including the revolution in technology, increase in information, globalization, and global warming, are requiring educators to prepare citizens for a constantly changing world. I was developing a Reggio-inspired experience in a grade one classroom when most of the literature and work in Reggio was centred on pre-school-age children.

Most of the parents at the school are high achievers and many of the mothers had attended the school themselves. In the past the parents had been very vocal about changes, as they needed to know that "rigour" would be sustained. Rigour as interpreted by the administration, parents and teachers would benefit from having a common meaning. The definition of rigour in the classroom can be contentious, but in this case I define classroom rigour as consisting of multi-faceted, challenging experiences. I wondered if rigour could not only be maintained in a Reggio-inspired approach, but also be enriched and enhanced. Prior to the introduction of a Reggio-inspired approach, I had relied on the Ontario Curriculum to teach my grade one class (Ontario Ministry of Education, 2005).

I had worked with increasing misgivings as to whether the Ontario Curriculum provided the best vehicle for preparing children for the future. I had experienced many occasions of frustration where I felt that the curriculum fragmented the children's learning by being subject-specific and content-focused. The layout of the curriculum with lists of expectations did not seem to honour the child as a "meaning-maker" (Fraser, & Gestwicki, 2000, p. 249) or address the need for the children to grasp the relevance of what they were learning. One of the pioneers and writers about the Reggio philosophy, Carla Rinaldi, when asked about "curriculum" responded,

> The term curriculum is unsuitable for representing the complex and multiple strategies that are necessary for sustaining children's knowledge building processes. (2006, p. 132)

I will outline principles of Reggio-inspired education and The Ontario Curriculum (Ontario Ministry of Education, 2005), before presenting data that acted as a provocation to my research.

Principles of the Reggio Emilia experience

The Reggio Emilia pedagogy relevant to this study includes the image of the child, reciprocity and relationships, the role of the teacher, the hundred languages of learning, documentation, progettazione, and emergent curriculum. The following section describes these principles.

The image of the child. Children are viewed as intelligent, capable and full of potential (Malaguzzi, 1994, p. 52). Each child is seen as intellectually capable of constructing his/her own learning with the teacher serving as a facilitator. Children are to be honoured and taken seriously and have rights to interact with each other, their parents and teachers. Children are seen as full members of society as citizens, not as aspiring–to–be citizens. Pre-conceived limits are not placed on children's capabilities but rather the children are challenged to collaborate, theorize and refine hypotheses within a social constructivist approach. Malaguzzi and the educators in Reggio Emilia developed this pedagogy, which strongly respects the rights and potential of all children. Ideas of identity, culture and context resonate in Rinaldi's words:

> The young child is the first great researcher. Children are born searching for and, therefore, researching the meaning of life, the meaning of the self in relation to others and to the world. Children are born searching for the meaning of their existence...the meaning of the conventions, customs and habits we have, and the rules and answers we provide. (Rinaldi, 2006, p. 63)

Reciprocity and relationships. Reciprocity is "mutual exchange" (Rinaldi, 1998, p.121) among children, teachers and parents as well as within the community. Education is seen as socially and culturally situated with everyone working collaboratively and co-operatively. The well being of children, parents and community depends on the well being of all the protagonists. There is a recursive cycle of reciprocity, exchange and dialogue. Conflict of ideas, discussion and negotiation are common components of this reciprocity. Children rely on one another's competencies and children view their schooling as a collaborative experience where the common good is sought along with individual progress (Rinaldi, 1998, p.119).

The role of the teacher. The teacher acts as an observer, listener, learner, nurturer, partner, and researcher (Edwards, Gandini & Forman, 1998, p.118). The image of the child shapes the role of the Reggio teacher. Teachers develop a pedagogy of listening to enable a pedagogical dialogue to occur. The teachers listen attentively to the childrens' conversations and observe their interests closely. Teachers share in the interest, excitement and joy of learning, scaffolding experiences so that children reach unaccustomed heights of thinking and feeling. The teacher listens to the children's ideas and helps them represent their ideas through the hundred languages. The hundred languages of learning is a phrase used from a poem written by Malaguzzi called, "No way. The hundred is there" about the multiplicity of childrens' ideas, thoughts and ways of representing their world in different media (Malaguzzi, 1998, p. 3). A teacher is a facilitator of reciprocity and relationships. A teacher encourages the children to be active participants in their own learning during in-depth investigation and projects. There is no rigid timetable and time is given generously for ideas to emerge. Teachers are seen as researchers of the children's learning and they work alongside the children as partners in the learning process.

The hundred languages. The hundred languages of children is a metaphor for different forms of symbolization invented by children–verbal, graphic, plastic, musical, gestural, iconic, expressive, emotional. The metaphor refers to symbolic representations and understandings expressed in multiple ways. These different "languages" are explored in an atelier, a place where rich materials, tools and professional atelieristas or artists assist the children and teachers. Experimentation with materials and techniques, aesthetics, creativity and discovery are all elements of the metaphor of the hundred languages. The expression "The Hundred Languages of learning" is a metaphor for the powerful learning of early childhood. The book *The Hundred languages of Children: The Reggio Emilia Approach- Advanced Reflections* presents an overview of the educational philosophy and practices of the Reggio Emilia experience (Edwards, Gandini & Forman, 2012).

Documentation. The value that the educators in Reggio Emilia place on children's work and on communicating with others about the children's experiences is shown through documentation. Documentation panels have photographs of children working, samples of children's products, and text describing the process of what the children are doing. The panels are aesthetically pleasing and often include childrens' artwork. The panels are intended to be windows into childrens' thinking and their learning processes. The panels may show the development of a project or the changing ideas of the children as they investigate and think deeply and critically. The panels have multiple audiences: the children, teachers, parents and the community. Also the documentation panels have multiple purposes, one being to propel further learning. These purposes also include recording a significant moment of learning, displaying ideas graphically, and informing parents and the community about what and how the children learn. Documentation enables reflection, interpretation and an opportunity for the teachers to learn about the children's learning. Teachers rely on documentation of the learning process to reflect upon and direct the process of learning (Giudici, Rinaldi, & Krechevsky, 2001, p. 18).

Progettazione. Rinaldi (1998), states that flexible planning occurs with teachers, parents or administrators and involves the life of the school and community. Preparation and organization of materials, space, thoughts, provocations and occasions for learning are all carefully considered in the development of a project.(p. 114) General educational objectives are considered within a flexible plan. The study of documentation helps teachers decide on objectives and direction and curriculum develops from this study of children's curiosities and interests. Reggio educators prefer to talk about "*progettazione,*" and maintain it cannot be translated into English adequately (Wien, 1997). "*Progettazione*" is not curriculum, as the term implies, but rather a forward-looking strategy to investigate, or a "reconnaissance" that can be altered or enriched as it develops. Malaguzzi said of social constructivism:

> This theory holds that knowledge is gradually constructed by people becoming each other's student, by taking a reflective stance towards each other's constructs, and by honouring the power of each other's initial perspective for negotiating a better understanding of subject matters. (1994, p. xix)

Emergent curriculum

I wondered if a more emergent curriculum would address the misgivings I had about the conventional Ontario curriculum and if there was room for the two to co-exist in my classroom. Emergent curriculum is not decided in advance, although teachers provide provocations, have goals and intentions, and form hypotheses about the direction that projects might take. Projects can range from a few days' experience to several months work. The teachers and children collaborate, discuss, predict and accommodate the direction or emergence of a project with the help of a pedagogista or curriculum or pedagogy expert (Rinaldi, 2003, p. 5-6). Emergent curriculum means teaching and learning in a complex, sophisticated way that encompasses intention yet is reciprocally planned or an emergent process (Wien, 2008, p. 5).

> The course of this curriculum is not known at the outset. It is emergent – that is, its a trajectory that develops as participants bring their own genuine responses to

> the topic and collaboratively create the course to follow out of these multiple connections. (Wien, 2008, p. 5)

There is flexibility in the way a project develops with collaboration, reflection and discussion along the way. Unlike a prescribed pre-mapped curriculum written by experts, emergent curriculum is created by and with the teachers, children and parents in response to their interests and community. It is a social constructivist approach, as indicated by Elizabeth Jones:

> The goal of emergent curriculum is to respond to every child's interests. Its practice is open-ended and self-directed. It depends on teacher initiative and intrinsic motivation, and it lends itself to a play-based environment. Emergent curriculum emerges from the children, but not only from the children. (2012, p.67)

Knowledgeable others assist the teachers in the emergent curriculum process, such as an atelierista or art specialist in an atelier, used by all the children and teachers, as they explore ideas through "one hundred languages" or expressions in a range of media. A pedagogista travels among schools to participate in discussions about projects, documentation and teacher research into the learning. Learning is not limited by fixed curricula and evaluation methods. Numbers or letters as a symbol or metaphor to evaluate a complex learning process are inappropriate for young children, as Wien writes,

> Grades A, B, C, and D are unfitting metaphors for young children's learning. I argue, in fact, that teachers of early childhood years (K–Grade 2) *not* be required

> to grade children in this manner, for such grading is presumptuous, forced, contrived, and unreasonable. Adequate time and space in which to learn are necessary as a condition prior to judgement of learning. A linear, segmented curriculum in particular shows no knowledge of the rhythms of living of young children. (2004, p. 150)

An emergent curriculum does not see knowledge as "transmittable" in the sense of filling an empty vessel through direct instruction. Rather emergent curriculum is constructivist, a complex series of assessing interconnections among prior knowledge, collaboration, exchange, reciprocity and inquiry. Emergent curriculum is a comprehensive approach that includes educational experiences, reflection and adjustment of practice as the learning takes place. In North America, emergent curriculum, while inspired by Reggio Emilia, is interpreted in various ways according to a community's politics, context and culture. Forman and Fyfe describe emergent curriculum as,

> an intentional planning process and the need for negotiation between teachers and learners in determining content and teaching strategies. The terms negotiated curriculum and *progettazione* (Rinaldi, 1998, p. x1), have been proposed as concepts that more effectively capture this active role of the teacher in curriculum development. (1998, p.239)

In emergent curriculum specific learning is not pre-determined although there are predictions about the course the learning will take: the learning is dynamic and adaptable to the interests of the children. Teacher observation, documentation and participation, scaffolds prior knowledge and current input and results in shifts in directions of learning.

Reggio philosophy and mathematics

A project in Reggio that included mathematical content was *Shoe and Metre* (Castagnetti, & Vecchi, 1997), where the children needed another art table in their classroom, a real–life situation that led to a measurement project. I have also written about emergent work in mathematics (Hislop & Armstrong, 2008), in Canada, a piece also about a measurement project but involving flower boxes. Little research has been done on the emergence of numeracy in integrated projects or daily classroom life. An exception to this is a major research paper by Monica Rohel titled, "Documenting Children's Understanding of Number Concepts: Interpreting the Reggio Emilia Approach" (2010). Clearly there is room for more research into emergent curriculum and emergent numeracy. Many educators believe in an integrated approach, for as Bateson claims,

> Break the pattern that connects the items of learning and you necessarily destroy all quality. (Bateson, 1979, p. 7)

Malaguzzi's metaphor of the "Tangle of spaghetti" (cited in Tarr, 2006, p. 7) challenges the idea of a tree of knowledge expanding in an ever-increasing, hierarchical fashion. The plate of spaghetti is intertwined and tangled and the strands of pasta cannot be looked at individually. As Rinaldi says, "The timing and styles of learning are individual and cannot be standardized with those of others" (2006, p. 125). Malaguzzi writes,

> We as educators are already prisoners of a model that ends up as a funnel. I think, moreover, that the funnel is a detestable object, and it is not much appreciated by the children either. Its purpose is to narrow down what is big into what is small. This choking device is against nature. (1994, p.52)

Project work in Reggio is always an integrated series of small narratives, rather than knowledge organized into disciplines or subject areas and this image of learning has greatly affected my own teaching.

The Principles of the Ontario Curriculum, Numeracy Strand

There are principles of The Ontario Curriculum that support a Reggio-inspired emergent experience and some principles that support direct instruction of a fixed curriculum.

Principles that support an emergent approach to mathematics. The process expectations in the Ontario curriculum appear congruent with an emergent curriculum. They mandate that students shall

> apply developing problem-solving strategies as they pose and solve problem and conduct investigations, to help deepen their mathematical understanding
>
> apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others)
>
> demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem e.g., by explaining to others why they think their solution is correct)
>
> select and use a variety of concrete, visual, and electronic learning tools and

appropriate computational strategies to investigate mathematical ideas and to solve problems

make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts

create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols)

make connections among them, and apply them to solve problems

communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations. (Ministry of Education, 2005, p. 36)

I argue that these processes appear congruent with an integrated, Reggio-inspired emergent curriculum. These process skills promote an image of a capable child creating and posing problems, and allow for "the hundred languages of learning" and documentation to be incorporated in communicating and representing their learning. Problem solving and deepening thinking are also aspects of progetazzione. The teacher's role is to be a facilitator of these investigations and relationships are honoured as mathematical thinking is communicated.

Principles that support direct instruction of a fixed curriculum. The Ontario curriculum expectations are presented to teachers as a body of mathematical content that must be learned by each child. The principles behind this aspect are learning, accountability, evaluation and assessment. While acknowledging the "diversity" of grade one students, the curriculum proceeds to list academic expectations to be attained by the end of grade one by every student. The content in the curriculum is listed as expectations, presented in a linear, fragmented way as,

> Number Sense and Numeration:
>
> representing and ordering whole numbers to 50;
>
> establishing the conservation of number;
>
> representing money amounts to 20¢;
>
> decomposing and composing numbers to 20;
>
> establishing a one-to-one correspondence when counting the elements in a set;
>
> counting by 1's, 2's, 5's, and 10's;
>
> adding and subtracting numbers to 20. (Ministry of Education, 2005, p. 32)

The question for the teacher is how to approach this content. These linearly –stated expectations support a principle of direct instruction and fragmentation of mathematics. The listing of expectations can be seen as a focus on student deficits if expectations are not reached. Other learning, which may occur, might not be mentioned on the list. How will a child be evaluated if the learning they construct is rich and sophisticated but not a specified expectation? The expectations are very specific and do not seem to suggest an emergent approach initiated by children's interest.

In The Ontario Curriculum, (Ministry of Education, 2005) the principle behind the expectations seems to be one of breadth of learning as opposed to depth. The nature of the expectations, although allowing teacher choice in pedagogy, leads the educator towards a cursory glance at ideas with everyone looking at the same ideas. This approach can defeat the purpose of small group and interactive learning. These expectations promote individual learning and uniformity of learning. Prior knowledge of the individual is ignored if everyone must reach the same expectation from the curriculum. If the children are already proficient at the listed expectations what then should the educator do?

The mathematics curriculum is further divided into strands such as patterning and algebra, numeration, measurement, data management and probability, geometry and spatial-sense, yet all of these inter-relate. This list of expectations undermines the inter-related nature of mathematical concepts across the strands and other subject areas; it may in fact be limiting student achievement in mathematics. The teaching approach chosen is vital in order to make connections and integrate strands and disciplines.

Not only does the presentation of linear curriculum expectations suggest limiting the teaching in a multi-disciplinary way, it fails to emphasize emotional and motivational aspects of children's meaning making. Important layers of enjoyment, aesthetic sensibility, spirituality or emotion and focus on celebrations and exhibits of their learning are missing. Constance Kamii expresses the importance of interest and emotion in mathematics:

> Every normal student is capable of good mathematical reasoning if attention is directed to activities of his [her] interest, and if by this method the emotional inhibitions that too often give him [her] a feeling of inferiority in lessons in this area are removed. (1999, p. 98-99)

Ontario teachers are required to assess whether the children have reached the mandated expectation, the specified understanding and then assign a numerical grade from 1- 4 to represent the learning. Such grading and testing has also been associated with low confidence, anxiety, boredom, distaste and low performance in mathematics (Wigfield & Meece, 1988). Regarding evaluation the curriculum document states:

> Level 1 identifies achievement that falls much below the Provincial Standard while still reflecting a passing grade.
>
> Level 2 identifies achievement that approaches the standard.
>
> Level 3 identifies achievement that reaches the standard.
>
> Level 4 identifies achievement that surpasses the standard.

> It should be noted that achievement at level 4 does not mean that the student has achieved expectations beyond those specified for a particular grade. It indicates that the student has achieved all or almost all of the expectations for that grade, and that he or she demonstrates the ability to use the knowledge and skills specified for that grade in more sophisticated ways than a student achieving at level 3 (Ministry of Education, 2005, p. 23).

Emergent mathematics in grade one

In the following two anecdotes I will outline emergent numeracy experiences that I felt enriched the expectations and indicated rigour and sophistication in mathematical thinking.

At the beginning of the year boxes of educational supplies (pushpins, tape, binders etc.), arrived for me to check off against a purchasing request. The children began to get excited and asked if the parcels were for them. When I explained they were full of materials for us to use, the children asked to open them. What followed were two hours of irresistible learning. Ignoring the timetable, I explained we would have to check the items against a purchasing request to check that the order was correct or if anything was missing. We would then have to inform the purchasing office if anything needed re-ordering and if the amounts charged were correct.

The children had clipboards to record amounts of each purchase and were amazed we would use 1400 plastic pockets for portfolios and 200 pushpins. They added together two boxes of items such as 100 plus 100 paper fasteners to get a total of 200. They counted in 3s to thirty as the masking tape was packaged in groups of 3. They organized themselves into collaborative groups, helping each other count and record. They were indignant when they discovered that mistakes had been made with the order. They even noticed the pricing on the form, and with the help of a calculator checked to see if we were charged correctly. For example, only 4 erasers arrived when 4 *boxes* were ordered and only 12 glue sticks arrived instead of 12 packages. They learned adults make mistakes too! We found omissions in the order and we were able to visit and inform the purchasing office of our findings. What struck me from this experience was the emotional reaction of excitement, their engagement in trying to discover new concepts like how to count in 3s, their manipulation of large numbers and how much further than the curriculum they went, representing numbers to 50 and counting in twos, fives and tens. Money up to one dollar comes under the numeracy strand and demanded children double quantities and calculate much larger amounts than the curriculum specified.

I was gathering some anecdotal evidence that children went beyond expectations through emergent enjoyable tasks. I was wondering if multi-disciplinary experiences enabled the learning to be deeper and connect more areas of learning.

Another interesting example was when the grade six teacher, who supervises the grade ones at recess, was telling them about her son's pet snake, Jethro. The grade ones decided to write to the snake asking about what he ate, what kind of a snake he was, etc. Over a three-month period, Jethro was kind enough to contribute his shedded skins. This skin shedding fascinated the children, and they measured each skin during consecutive months. The shedding seemed to happen once a month and they began to notice patterns. Comparing the skins they realized that the snake grew the equivalent of two unifix bricks in length each month, and they became concerned that he would outgrow his tank. The girls used unifix bricks as non-standard units of measurement to firmly establish the idea of a "unit" before using traditional measurement units such as centimetres. They asked for the dimensions of Jethro's tank and came up with a t-chart and growing pattern of how long he would be by the following summer if he continued growing at the same rate.

These two stories highlight some of the rich possibilities for mathematical learning when the experiences are emergent.

These experiences led me to research further into emergent curriculum with a focus on mathematics. How could I maintain the playful, aesthetically pleasing math inquiries that arose during project work and yet be accountable for meeting the curriculum expectations? I suspected that in fact rich inquiries would exceed expectations.

Research questions

What are some of the ways it might be possible to enrich the numeracy expectations for grade one with an emergent curriculum so that student work is more extensive, complex and sophisticated?

What are some of the elements that contribute to an emergent curriculum that enriches thinking in Numeracy?

These questions raise issues I continually think about. How do I define enrich? Is this more complex learning than that outlined in the Ontario expectations? By complex I mean learning that consists of many intricate, interconnected parts. Is enriching to make richer by the addition or increase of some desirable quality, attribute, and/or ingredients including beauty? Could the achievements of students in this study outside the listed expectations be called enriching? (i.e. if they can expand upon the curricula) or are these experiences interruptions in the learning process? The curriculum suggests giving the highest grade to students that demonstrate the ability to use knowledge and skills in sophisticated ways. Emotions and aesthetics also play a role in learning and should be considered when exploring enrichment. These are some of the issues raised as I began thinking about this research.

Chapter Two: Research Design

This chapter describes the research perspective, site, participants, time frame, type of data generation, collection and analysis of data. There is a gap in the research literature on emergent mathematics in a grade one classroom in relation to The Ontario Curriculum (Ministry of Education, 2005) and numeracy. I researched how Reggio principles and practice, in my interpretation, affect an emergent numerical understanding in young girls. I researched the children's learning in my grade one class with reference to the Ontario expectations and processes with particular attention to numeracy. This qualitative research was naturalistic, descriptive data concerned with processes of learning. The research included the description of complex classroom situations. This study was designed to explore the following questions:

> What are some of the ways it might be possible to enrich the numeracy expectations for grade one with an emergent curriculum so that student work is more extensive, complex and sophisticated?
>
> What are some of the elements that contribute to an emergent curriculum that enriches thinking in Numeracy?

The view of the researcher

My framework for teaching in my view is a critical, democratic, feminist, socialist perspective. My educational stance is that of social constructivism. I have been an elementary teacher for 29 years, and taught in Egypt, Cyprus, the United Kingdom, Malaysia and, most recently, in Toronto, Canada. I have been investigating the Reggio Emilia philosophy for the past 10 years in a grade one classroom and have presented at national conferences and written about my work (Hislop & Armstrong, 2008). I visited Reggio in 2006 on a study tour and worked with the Reggio pedagogista, Francesca Giorgiani, examining Ontario curriculum and emergent curriculum during the school year 2008-2009.

Context

Working in a Reggio-inspired grade one, I generated data in the classroom context throughout the course of a project on identity. I have described the school setting in chapter one (p. 1). The data I collected was from daily emergent experiences in a Reggio-inspired grade one setting and were not specifically orchestrated for the research. Research is a regular part of the teaching expectation at the school and part of my personal philosophy of teaching.

Participants and ethical procedures

This ethnographic study involved my Reggio Emilia-inspired grade one classroom, which includes twenty grade one girls, one full time teacher and a part-time support teacher and the part-time pedagogista or curriculum expert. I provided participants from the grade one class with an informed consent form for their parents to sign (see appendix 1), and a simplified consent form suitable for grade one students. I invited the children to participate in this interesting numeracy research project. I informed parents that I would like to investigate whether the Reggio Emilia philosophy with its emergent curriculum is a catalyst for the development of children's understanding about numeracy. Would we exceed the expectations? I explained that conversations would be transcribed, so we could recall what was said, sample work would be kept, and these materials would be developed into documentation panels to be shared with the children, parents and others who came into our classroom. These panels might also be shared with other educators, at conferences, workshops or in educational journals. I asked permission to use the child's words, representations, writings, and image (still or video) in my thesis, and later in presentations to other educators, and possibly in articles submitted for publication to education journals. I informed the parents that the primary purpose for the project was to further my own understanding of learning processes and improve children's mathematical understanding.

I sought and was granted ethical consent from The York Faculty of Graduate Studies Ethics Review Committee and by The School Ethical Review Committee. I invited the children to participate or not, as they wished. I assured the parents that confidentiality would be provided to the fullest extent possible by law. Since the data (words, images, samples of work) is held in the public place of the classroom and within the educational community, the documentation panels would include children's photos and actual names. I asked permission to use children's first name only, to recognize and honour their work. If parents preferred their child to remain anonymous in the thesis, they could choose a pseudonym. All research participation is voluntary, and parents might withdraw permission at any time for their daughter's materials to be included in the thesis. A participant's decision not to participate would not influence the individual's relationship with the researcher or York University, now or in the future. All twenty permission slips were returned by the parents and children and all agreed to their child's first name to be used alongside images. There were no restrictions indicated on images or sharing the children's work. The parents provided permission to the school and permission for the work to be shared in the thesis and beyond.

Data generation procedures

I conducted the data generation over one semester, from September to December 2011 in a grade one class of 6-7 year olds. I analyzed the data from January 2012 to October 2012. I researched specific processes of emergent curriculum derived from my interpretation of the Reggio Emilia experience. As the methods employed by the research must be consistent with the theoretical paradigm of the study (Glesne, & Peshkin, 1992, p. 82), for this interpretive study I utilized an ethnographic design, qualitative data collection methods (i.e. transcripts of questions, theories and problem solving, participant observations, conferences, children's work, documentation, photographs and field-notes), and interpretive data analysis methods. The pedagogical documentation consists of text, photographs, children's work, panels, illustrations, graphic novels and analysis of the processes involved as described in the following sections.

Morning meetings

I conducted morning meetings and conferences with the children to learn more about the children's implicit and explicit understandings within the expectations of numeracy. These morning meetings would begin in an informal way with the children gathering on the carpet, sitting in a circle and discussing items of interest. Usually the morning meeting is an invitation to discuss project work and possible directions it may take. As my research was focused on emergent mathematics, I transcribed in particular children's mathematical understanding and ideas. Morning meeting was a venue to review each day's activities, discoveries and problems with the whole class.

At the beginning of the semester the discussion concerned their project focus on identity. In chapter three I brought a provocation and questions to the morning meeting to elicit theories. In chapter four a problem was brought to morning meeting by the cleaning staff and revisited at further meetings. Thus problems were brought to the whole class, worked on by individuals or small groups and then discussed again with the whole class in a recursive, daily cycle. Morning meetings were transcribed and re-read daily. Transcriptions of the morning meeting were essential for deciding upon the trajectory of the childrens' investigations.

Records and transcriptions

All the data I generated was in the context of a project that emerged about identity. The classroom pet budgies had hatched eggs over the summer and were now parents. We began to explore not only the budgies' identity, but also the childrens' identity, what it included and how it could change. The children were considering physical and social characteristics. I researched the emergent mathematics in September that occurred when the children were invited to comment about themselves as mathematicians in grade one. I wondered about their understanding of mathematics. In the course of a conversation some of the children did not know explicitly what mathematics was, despite having many mathematical experiences in kindergarten. I felt that they had a lot of prior experiences in kindergarten with mathematics and were struggling to articulate explicitly what math was. As a provocation to explore their mathematical understanding and make their implicit understandings more explicit, the support teacher and I decided that illustrations and theory building might reveal the extent of their understanding. Below I outline the recurring process for each chapter of findings.

The children illustrated their theories individually with blank paper and sharpies (black markers). Each child dictated a personal theory about mathematics in answer to questions, and I typed each theory into the computer, then printed and glued the theory onto their illustration page. Their theories were transcribed onto the computer word for word. This theory dictation and illustration took two days to complete. All the theories were presented at morning meeting to the whole group to find similarities, differences and what conceptions the children had. The children came up to the board and posted their theory after explaining it. Those with similar theories would post theirs near each other. Different theories would each have their own column until categories emerged. I took many photographs of the morning meeting. The pedagogista and I reviewed the theories and illustrations and researched the mathematical implications of their ideas. This theory building, illustrating, sorting and categorization took a week to complete and was chosen from other classroom experiences to be highlighted in chapter three, as it pertained to mathematics, their developing understanding of numeracy and might include expectations from the curriculum (Ministry of Education, 2005).

In chapter four, the class brainstormed a way to solve a problem brought to our attention by the school housekeeping staff. I recorded on my computer the initial conversation between the children and housekeeping staff and their brainstorming ideas and initial strategy to solve the problem. A small group of children decided to investigate the problem. I observed and transcribed a narrative onto the computer of how their initial strategy was problematic and encouraged them to seek advice of their peers at another meeting. I recorded their conversations with each other and their self-talk as they experimented. Complex mathematical questions arose as the investigation revealed a counterintuitive disparity between area and perimeter. I recorded their questions and wonderings. The whole group brainstormed an alternative way to proceed in measuring. I transcribed proceedings onto the computer and took photographs as the whole class then experimented with an alternative strategy. I helped facilitate the trajectory of the emergent math from the whole group to small group then back to the whole group over a period of four days until the children remedied the problem. Transcripts and photographs were recorded of all meetings and conversations pertaining to the experiences.

In chapter five, as we were gathering for morning meeting, the whole group of children were discussing the numerical addresses of their homes and the support teacher transcribed their conversation and movements on the computer. This interest reminded me of a child-made numberline of houses, made by a former years class that I had in the cupboard. I brought this sectioned numberline out as a provocation to see if the class would be interested in re-assembling it and ordering the numbers to 100. I transcribed their conversations as they worked and talked among themselves and took many photographs of the process. This activity led to the children ordering, reassembling and creating a number line and formulating their own questions. The narrative was recorded on paper and later typed up on the computer. The photographs were analyzed and juxtaposed with the text. They problem-solved their own questions in pairs, with a variety of different strategies, which they shared with the whole group. The names and words and written representations of the children's work were shared as a whole class and recorded.

In chapter six, at morning meeting, the children ventured to hypothesize about the relationship between two different variables. After this initial morning meeting the children voiced other questions and interests which I transcribed onto the computer recording each child's name and words. The idea of ratio arose from one of the children and I encouraged the class to pursue these interests. The whole class worked with partners and spent a day investigating. I took photographs with the aid of the support teacher and we both typed up their conversations and comments as they worked. These transcripts were then shared the next day with the whole class. This led to the children making a further hypothesis to investigate. Another day was spent investigating and again the support teacher and I transcribed methods, strategies and conversations onto the computer. At morning meeting, the findings were discussed, transcribed and generalizations made by the children with the team teacher and myself facilitating. At home the children also conducted research and brought data back to morning meeting. This project lasted three weeks as they went on to explore many aspects of ratio.

Artwork and graphic representations

In each of the findings, chapters, three, four, five and six, the pedagogista and I studied between 2- 3 examples of drawings from each child. Each child did individual drawings and the pedagogista and I sat down and studied the eighty illustrations to give our different perspectives and thoughts on them. Even if a child was working with a group or partner they recorded their own thinking.We studied different representations of math theories, area measurement, acrylic paintings of budgies, problem solving illustrations, painted life-size portraits and claywork. The pedagogista and I observed representations of student thinking.

Childrens' written work and oral narratives

I interpreted their written and oral work for depth of understanding and sophistication, for example in their dictated math theories in chapter three. I examined their use of mathematical vocabulary and ability to communicate mathematical ideas through text and oral narratives for clarity, conceptions and integration with literacy. In chapter four I recorded their experimentation and conversations about area and perimeter. I also recorded their use of mathematical terminology. In chapter five I recorded their conversations by note taking of their original words as they made their numberline and presented their problem solving strategies. In chapter six I kept written problem-solving sheets for analysis as they investigated body to head ratio as well as their oral narratives of the body measuring process and the resulting mathematical generalizations they formulated.

Photographs

I took photographs of the learning processes and used them in documentation and/or for data analysis. Photographs or series of photographs gave additional information to the pedagogical documentation by illustrating expressions, emotions, context, sequences, body language, reciprocity and other facets of learning not always clear in observation notes. Every day I took between 30 -50 photographs on math from which to select. I took photographs of the different stages of the project on a daily basis to complement the oral narratives and student work samples. I studied them to select the most revealing shots for mathematical documentation.

Pedagogical Documentation and Documentation Panels

I created documentation and panels about numeracy that traced the learning processes of children's investigations, as well as my colleagues interpretations of the learning. I created the documentation with the team teacher and pedagogista in order to gain multiple perspectives of the learning. Six panels were created for children's theories about math and problem solving, and a further six panels about solving the problem of the chaff. I selected the children's conversations about the learning, a series of photos and original artwork by the children. We interpreted the children's activities, thinking, relationships and collaboration. I recorded the context of the learning as well as the ways childrens' ideas, theories and understandings propelled the project or investigation .

Analysis

I used the data to study the children's individual and collaborative numerical learning expectations and processes. The data was organized into folders of transcripts, photographs in contact sheet form and graphic representations for each experience. With a copy of the math curriculum to refer to, I searched the transcripts with two colleagues to identify examples of learning that appeared to be more enriched, advanced, integrated, sophisticated or complex than those listed in the expectations and we discussed daily experiences and next steps in the project. One such conversation began,

Susan: The girls claimed they didn't know what math is. I know they need to make their implicit understanding explicit but I predict they do have a rich mathematical understanding.

Kerri: Maybe they could represent their understandings more explicitly through drawing what math is.

Justine: After drawing they could talk about their illustration and this might help them become more explicit. I think you need questions to provoke the drawings.

I analyzed the work for Reggio principles such as emotional and aesthetic contribution to learning as well as the illustration of mathematical expectations and processes. Various levels of abstraction and symbolization appeared evident to me in the recordings. The artwork illustrated mathematical strategies, theories or thinking used by individuals or groups. I saw that the artwork represented numeracy in many ways, through, painting, clay work, and drawing. I identified spatial relationships, patterning, originality, creativity and the elements of design as contributing to the children's mathematical understandings, and I gathered and analyzed many samples.

As a team of educators we would sit with the transcripts, math books about developmentally appropriate practice, and Ontario curriculum expectations, and select examples that best addressed the research questions. For the children's theories I consulted the recent research on mathematics and its history. For the sixth chapter I consulted the mathematical research, as I was unsure that the investigation of ratio was age appropriate. I met with the pedagogista twice a week for half an hour and the support teacher daily for half an hour for collaborative discussions. For assembling panels longer periods of time after school were necessary. For example we all agreed that sorting and categorization of ideas, such as theories of mathematics, was an enriched experience compared to the usual sorting of classroom objects often experienced in grade one classes. For all the chapters I coded transcriptions of small and large group conversations, self-talk, actions and gestures for major themes of numeracy, process skills and Reggio principles. I created graphic novels to show these collaborative interpretations.

Graphic Novels

I produced four graphic novels of the children's experiences with mathematics for the children to share with their families. I used the computer program 'Comic Life' as a template for the graphic novels. With the team teacher I selected photographs that displayed the mathematical inquiry and illustrated accomplishment of the curriculum expectations. I selected specific children's words from the transcriptions, which best portrayed the mathematical narrative, to insert into speech bubbles. The photographs and accompanying speech bubbles provided an interesting format for the parents to share their daughter's learning and participation in mathematics. I included some of the children's paintings and artwork, problem solving strategies and photographs of the process, actions, gestures and expressions. I asked my colleagues' permission to use their images in the graphic novels.

Analysis of numeracy and process expectations

I coded themes placing a code letter beside the transcripts according to which numeracy expectation was met or enriched. I also coded the transcripts for the presence of the mathematical processes, for example applying developing problem-solving strategies as they posed and solved problems and conducted investigations, to help deepen their mathematical understanding. In chapter five they assembled a numberline to 100 when the curriculum expects them to work with numbers to 50. They problem solved how many houses each child should make, a division problem not on the curriculum until grade three. After making 55 houses for their own numberline they worked out how many more they still needed, solving a two-digit subtraction problem in creative ways when the curriculum stipulates one-digit subtraction. Themes also included the children's ability to apply reasoning skills, reflect, use manipulatives, make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts.

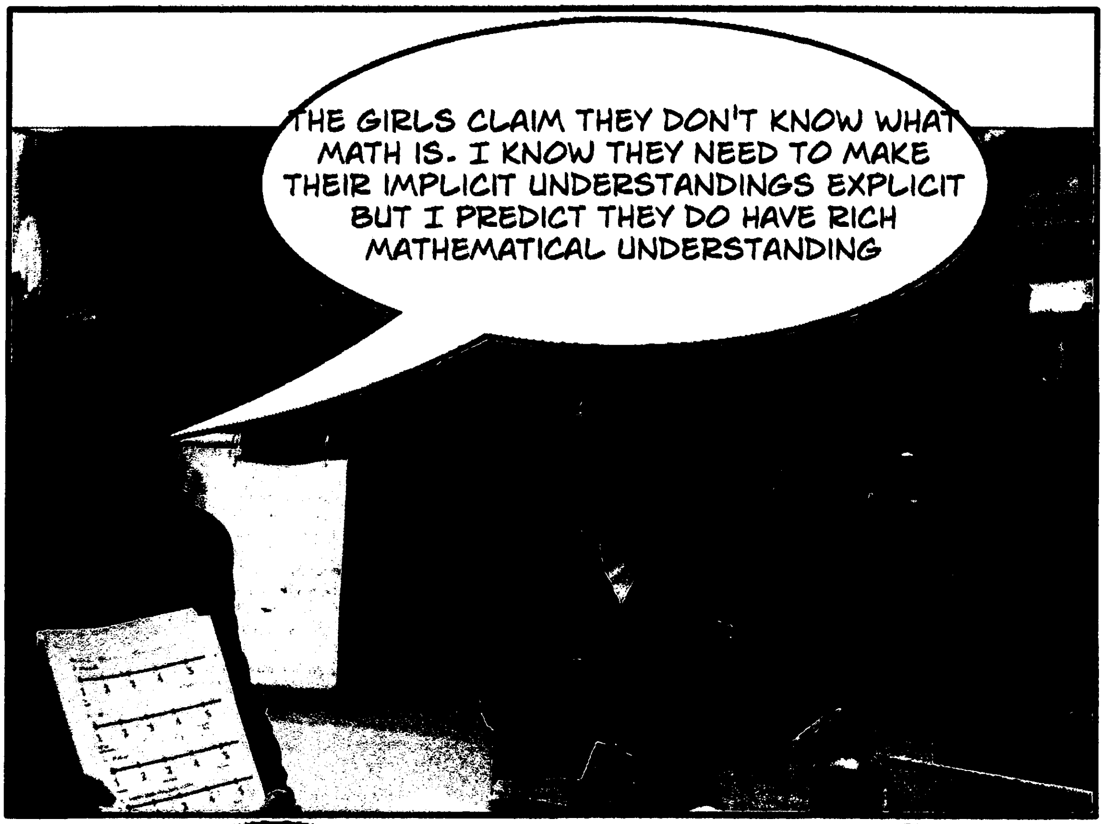
Analysis of emergent curriculum

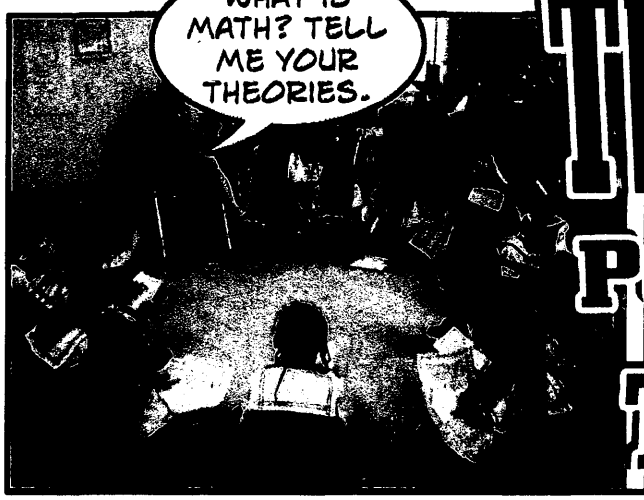
I also coded the transcripts for the presence of the Reggio principles mentioned in chapter one and emergent curriculum such as connections and integration with other areas of the curriculum. For instance in the last chapter the girls hypothesized about variables and through a series of explorations and conversations made generalizations about body ratios at different ages and fractional generalizations about where facial features appear on a human head. Just as in the Reggio experience, children were encouraged to formulate their own hypotheses, make predictions, explore, prove or disprove collaboratively to make informed generalizations.

Themes that contribute to enrichment

Once I had a data set or a set of experiences that I felt showed enrichment, I looked for emerging themes that might contribute to sophistication, complexity, integration or advancement beyond the expectations noted in the Ontario Curriculum, or sophisticated ways in which the expectations were used. These emerging themes, if identified, might help me understand the nature of enrichment and elements necessary for enrichment to occur. I hoped these themes, patterns or categories might shed light on aspects of collaboration, confidence, interest, and enjoyment in mathematical learning. For example all the chapters incorporate all the process skills and Reggio principles of emergent curriculum. Initial analysis indicated multi-strand experiences occurred in each chapter. Also, in chapter five, I extended the use of child-made math manipulatives to see if the children still maintained a more vested interest in self-made tools than store-bought resources. The children went on to make number counting books, numeral in nature photographic posters, dictionaries, and clay tablets of numeral orientation.

Before the next four chapters of findings, readers will find first an interlude in the form of a graphic novel. This graphic novel documentation was shared with the parents, children and other teachers in school. Following the graphic novel is a presentation and discussion of data related to the research questions.



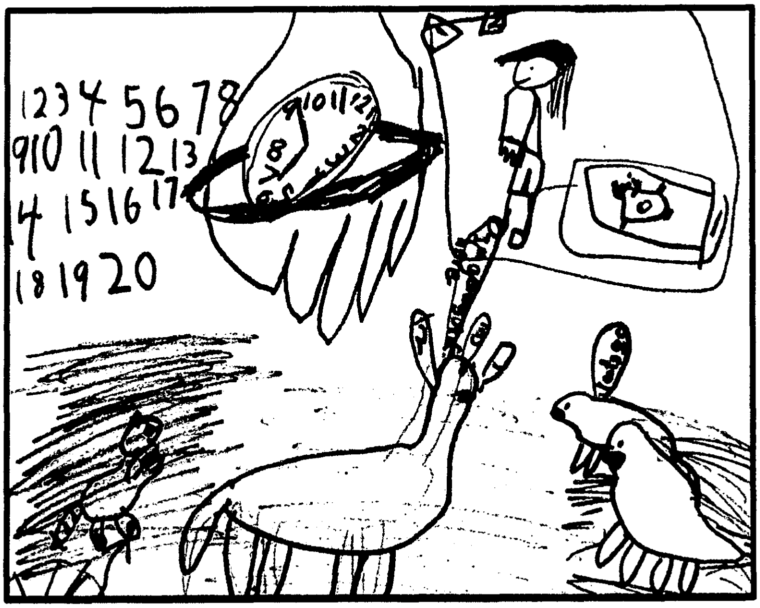


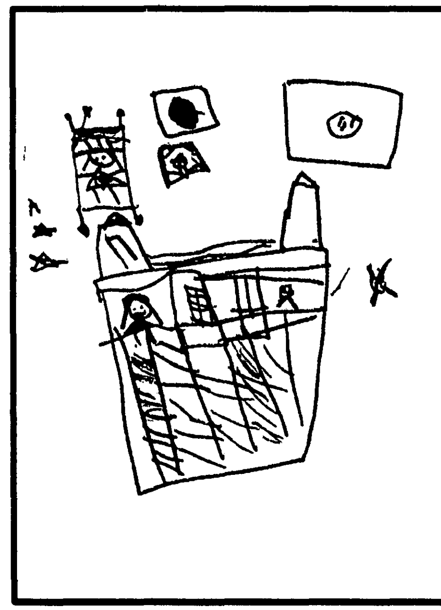
Theories Pertaining To Math

Susan Hislop

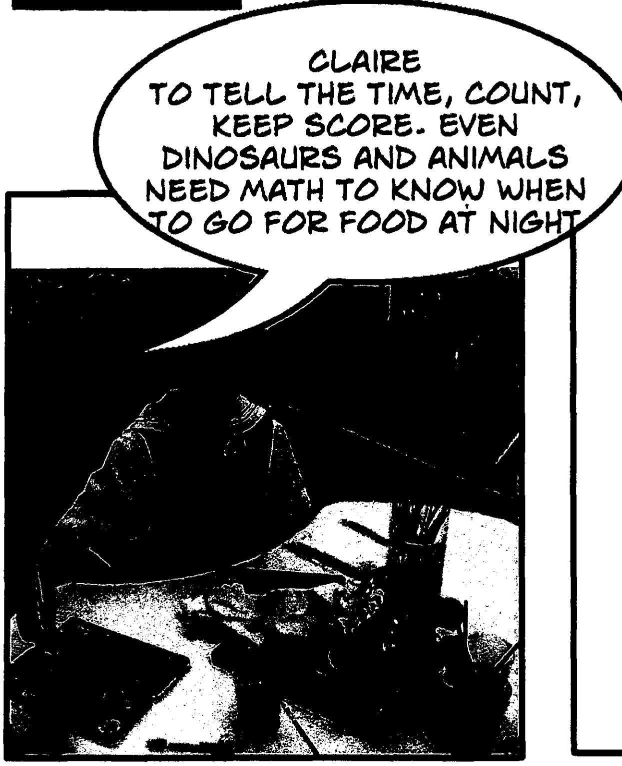
& Grade One

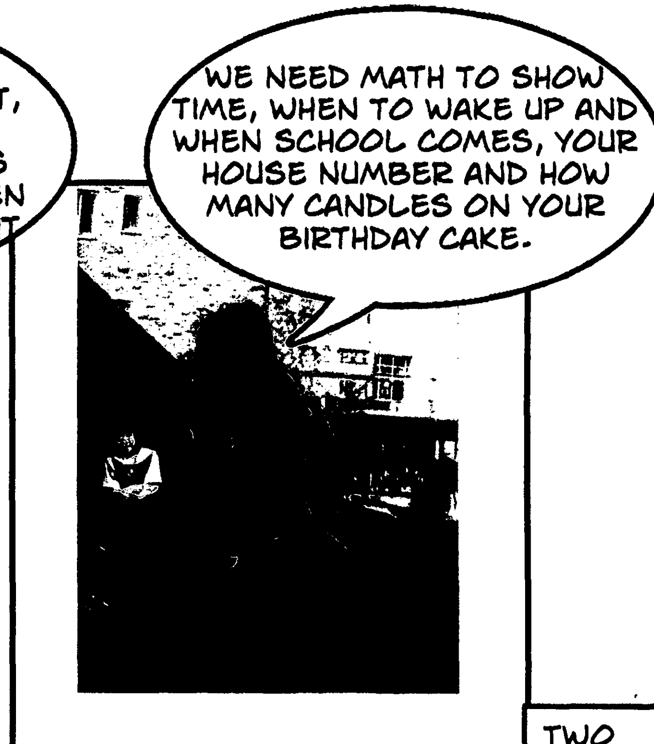
What is Math for ?





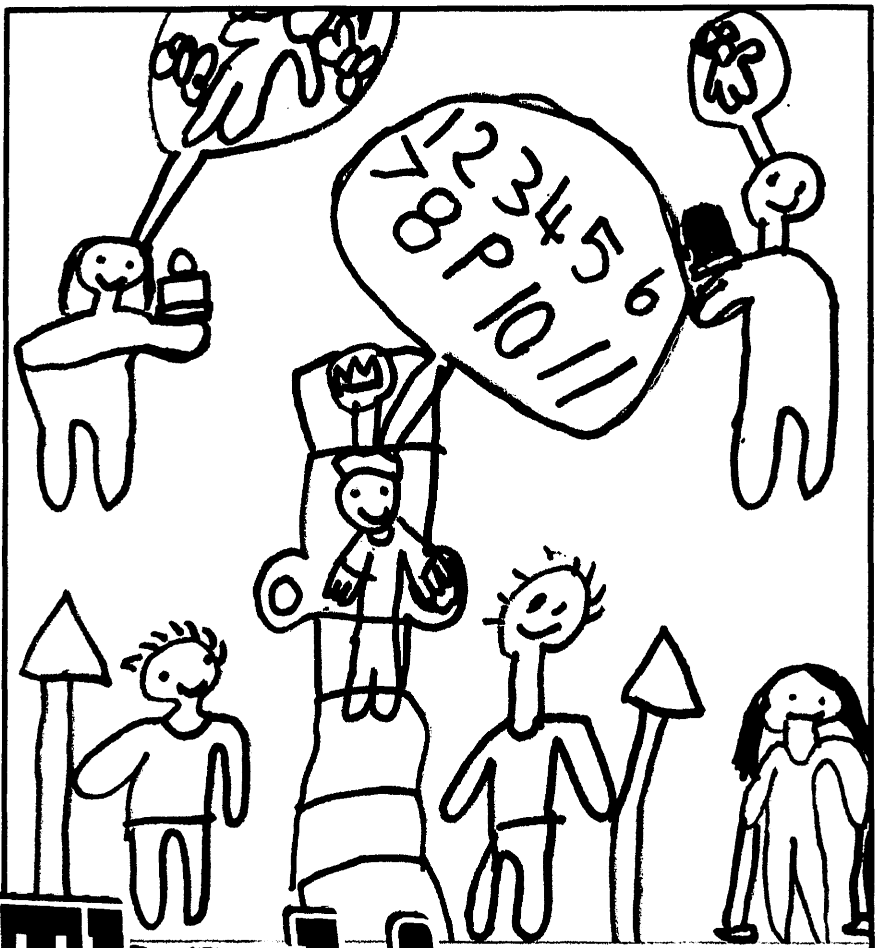
ANIMALS DEMONSTRATE NUMBER SENSE: LIONS DEFEND THEIR TERRITORIES AGAINST INTRUDERS, BUT THEY WILL ATTACK ONLY IF THEY OUT-NUMBER THE OTHER PRIDE. BRIAN BUTTERWORTH





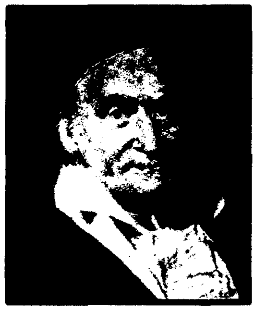
Where did math come from ?

The Egyptians
PAIGE THE EGYPTIANS INVENTED MATH TO COUNT THEIR PYRAMIDS
LAURELLE MATH WAS INVENTED BY ONE PERSON FOR THE WHOLE WORLD
LEXI LONG AGO SOMEONE INVENTED MATH
A person



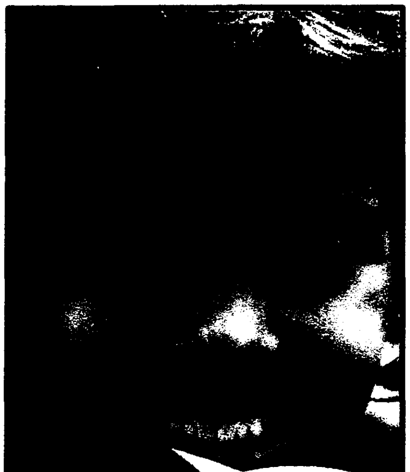
The king Cavemen

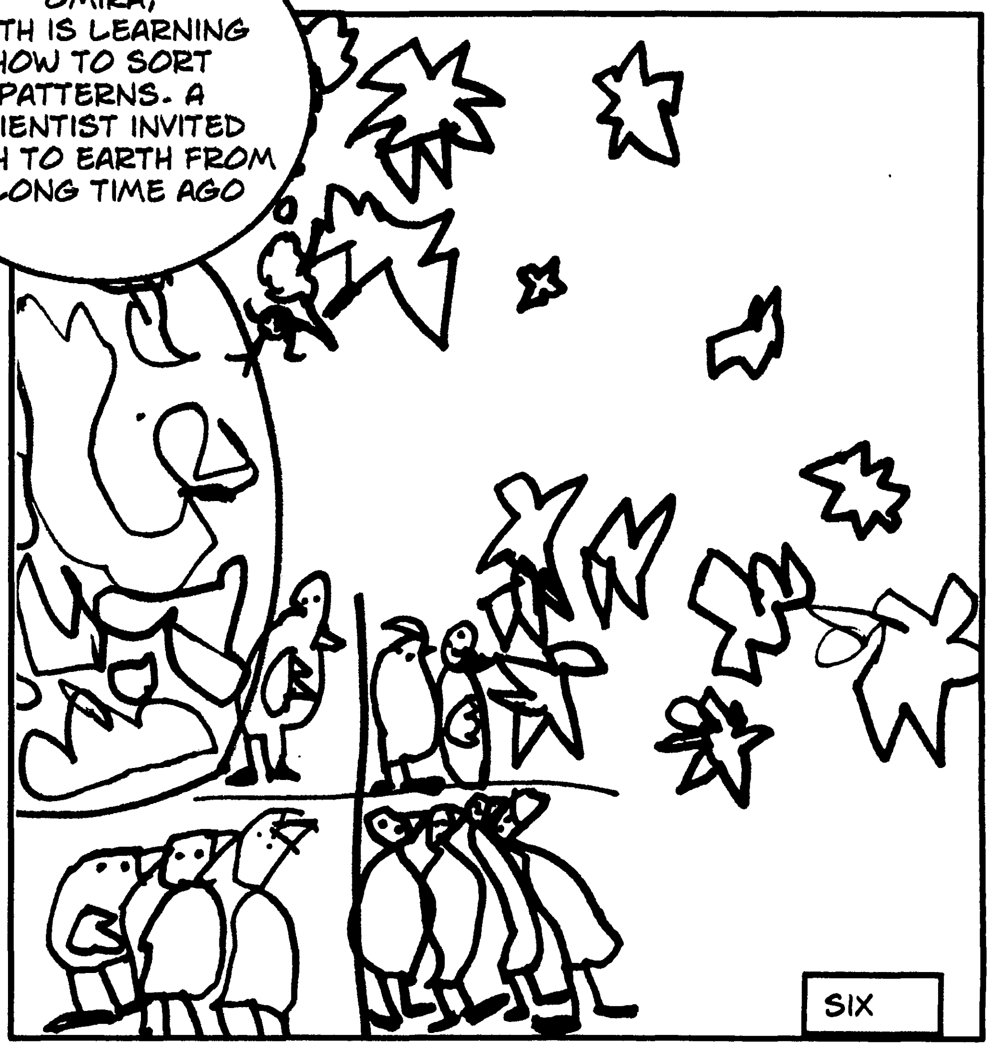
PREHISTORIC PEOPLE RECOGNIZED ABSTRACT (TIME AND SEASONS) AND CONCRETE (ANIMALS, FOOD) QUANTITIES AS MENTIONED IN THE GIRLS THEORIES. NUMERACY PRE-DATES WRITING AND PROBABLY CAME ABOUT DUE TO TRADE.



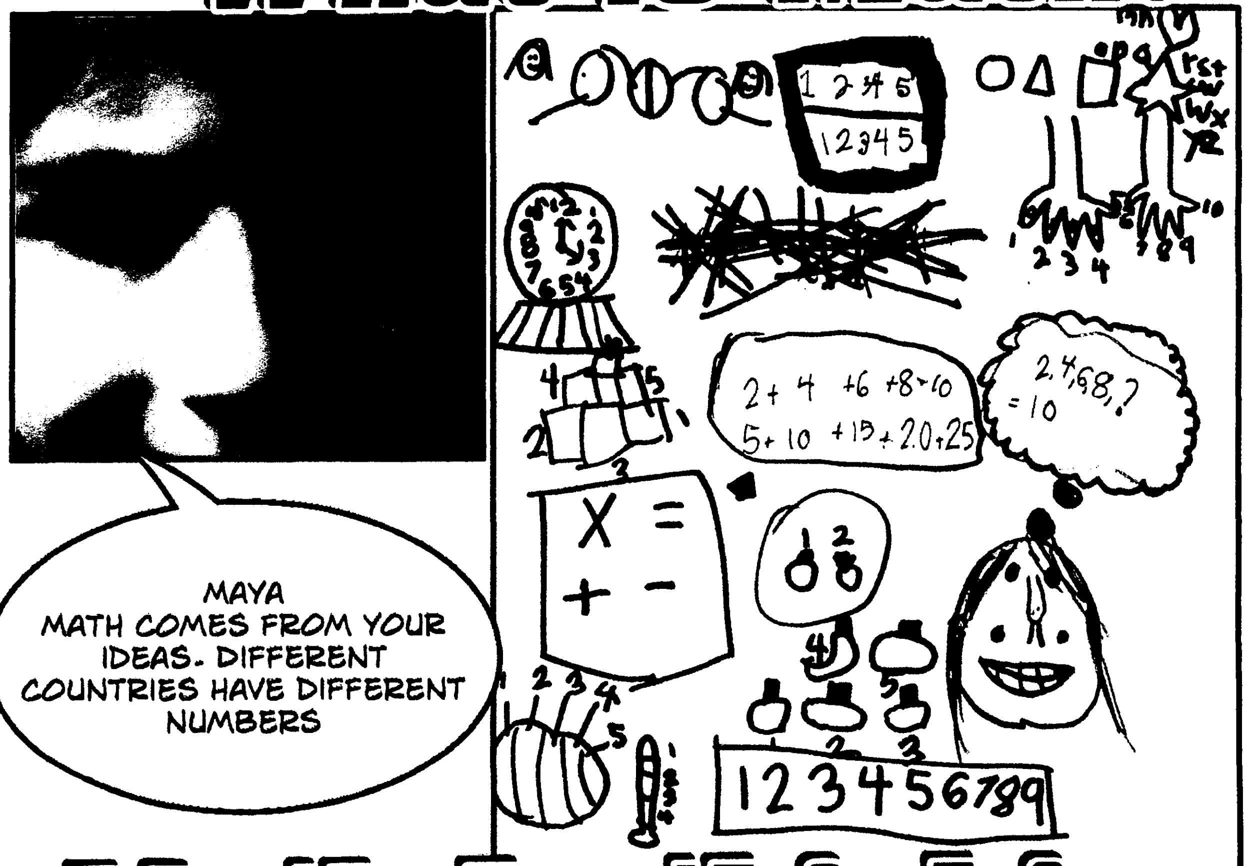
CARL FRIEDRICH GAUSS A GREAT GERMAN MATHEMATICIAN AND PHYSICAL SCIENTIST TALKED ABOUT MATH AS THE QUEEN OF SCIENCES

Scientist

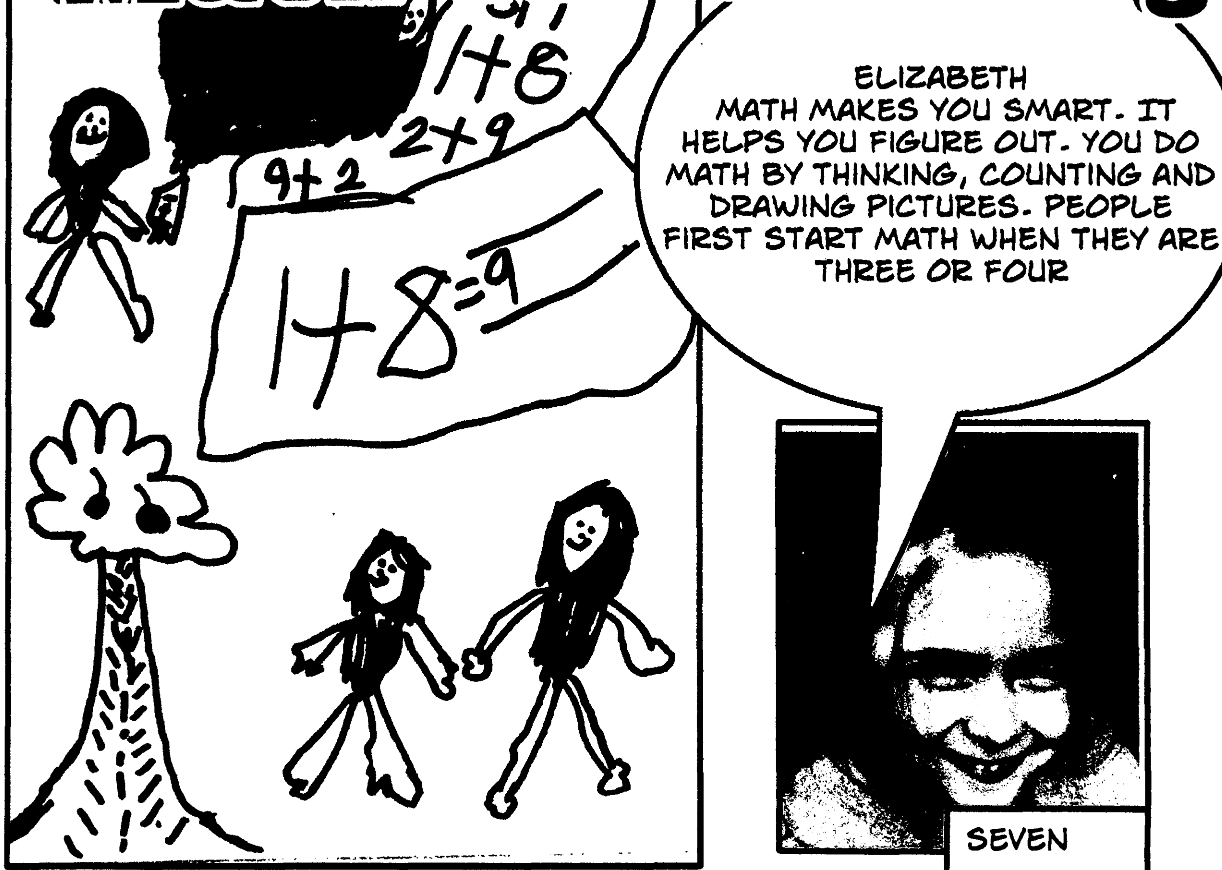


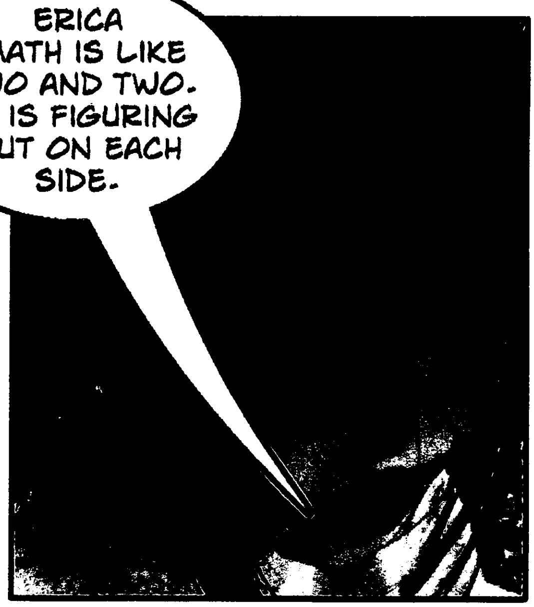


What is Math?



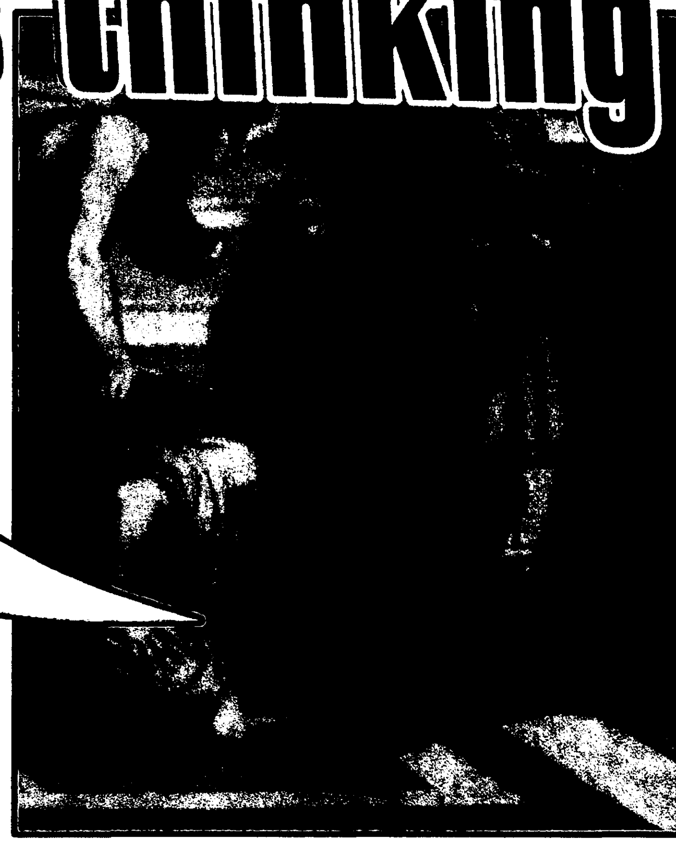
Math Is thinking



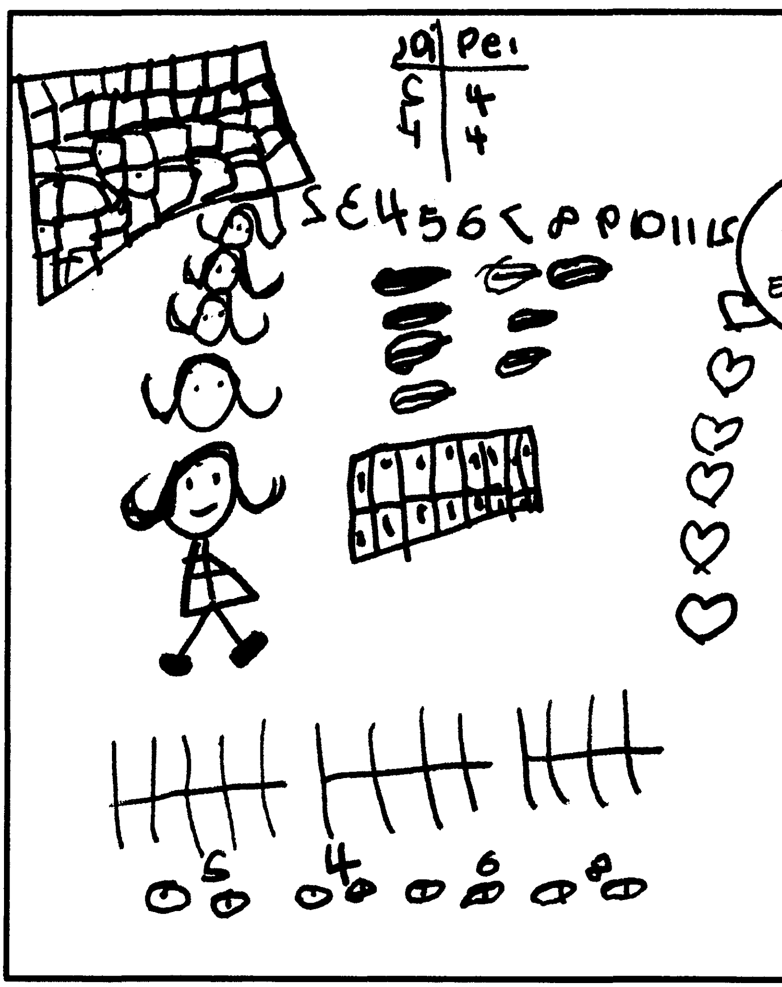


Math is Counting

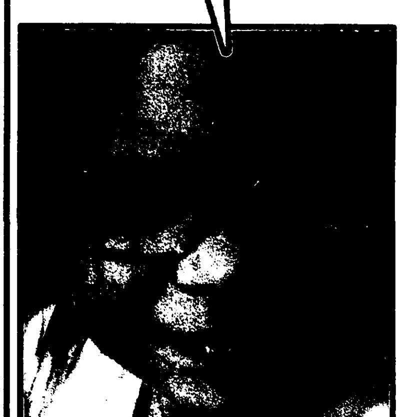
Math Is thinking



MANY GIRLS REFER TO MATH AS LEARNING AND SOLVING PROBLEMS AND THAT IT IS ACCOMPLISHED BY THINKING. MATHEMATICS COMES TO US FROM THE GREEK MEANING KNOWLEDGE , STUDY AND LEARNING.



ELLE
MATH HELPS YOU LEARN HOW TO COUNT. EVERYBODY CAN DO MATH WHO THINKS

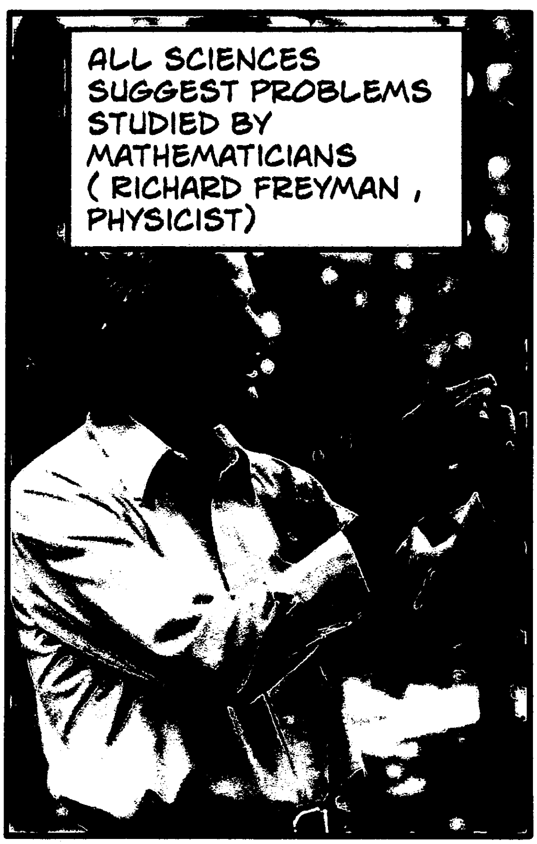


Children's ideas mirror Those of early scientists And mathematicians

MATH IS SEEN AS PATTERNS AND THEIR PRIOR UNDERSTANDINGS ACKNOWLEDGE GEOGRAPHICAL AND HISTORICAL DIFFERENCE IN NUMBER SYSTEMS. THEY PORTRAY MATH AS AN " INVENTION" ATTRIBUTED TO GOD, CAVEMEN, AND SCIENTISTS

THE GIRLS ASSOCIATED MATH AND SCIENCE AND EVEN ASTROLOGY MENTIONING SCIENTISTS " INVITING MATH TO EARTH" ASTROLOGY WAS AN EARLIER MEANING OF MATHEMATICS.

THE CHILDREN'S ABILITY TO BE EXPLICIT ABOUT THEIR UNDERSTANDINGS REQUIRED TEACHER SCAFFOLDING AND QUESTIONING.



Leonhard Euler, who created and popularized much of the mathematical notation used today

Chapter Three: Theories Pertaining to Mathematics

At the beginning of the school year the grade ones were intrigued by the new classroom environment, and especially by the class pets, budgies that had hatched four eggs. As the budgies had become parents we had been discussing identity and how it can change. They researched budgie identity as including their wild habitat, food sources, life cycle and physical characteristics. This was extended to an investigation into children's identity and their own changed status within the school. The transition from kindergarten to grade one was seen as a huge milestone by the children and they approached the new school year with some trepidation. Parents and siblings had told them they were now in grade school. Thus we began a project on identity, looking at the budgie's changed identity (as parents to four chicks), and at the girls' identity, and the changes from kindergarten.

I asked about their feelings towards mathematics. Could a project on identity address apprehension, as the research suggested is present among Ontario students towards mathematics. Would an emergent curriculum increase confidence? The literature has shown math anxiety and confidence play a significant role in a child's success in math. "The myth of ability" often makes children believe that capacity in math is some kind of special gift or talent not accessible to all (Meighton, 2007).

In the principles of the Reggio experience, children's theories have been described as understandings or conceptions in their search for meaning, not to be judged as right or wrong or seen as misunderstandings or naiive theories (Rinaldi, 2006, p. 112). Young children seek to interpret their world creatively through theory building, a process of intentional questioning and searching for answers. Interpretive theories are a right of competent children, in Reggio-inspired education. The meanings that children produce, the explanatory theories they develop in an attempt to give answers, are of the utmost importance. They strongly reveal the ways in which children perceive, question and interpret reality and their relationship with it.

After talking about the exciting experiences we would be having in mathematics in grade one, some of the children replied that they did not know what math was. This lack of recognition of math terminology was surprising, as I knew that they had explored math centres in kindergarten and conducted many mathematical inquiries within projects. The research indicates children's numeracy will be sophisticated and enable them to solve problems when it is interesting, meaningful, and enjoyable and educational practices enhance this interest, meaning and enjoyment (Resnick & Ford, 1981).

After discussion with colleagues I decided to ask the children to illustrate their understanding of math and then dictate a theory to me answering the questions, "What is Math?" "What is Math for" and "Where does Math come from?" reminding them about math centres they had explored in kindergarten. The cognitive development literature claims that much of children's knowledge is implicit rather than explicit (Karmiloff-Smith, 1992; Nelson, 1995) and "intuitive rather than formal"(Kuhn, 1989). I suspected they had an implicit understanding about mathematics, which they found difficult to make explicit and articulate through structured language.

Theory generation, sorting and categorizing

The children each dictated and illustrated a personal theory in response to the questions about mathematics. The theories and illustrations were presented, by the children, at morning meeting to find similarities, differences and prior knowledge. The children came up to the board, one by one, and posted their theory after explaining it; anyone with a similar theory would post theirs beside that one. Each set of different theories would have its own column so that categories emerged. For example four children who believed math came from God created a category, six other childrens' ideas that a person had invented math created another category. In essence the children were sorting and categorizing their theories and then coming to a consensus and a title for each group. The groups were, math as: an invention, fun, tools and symbols, geographically and historically represented, patterns, and math as practical and theoretical knowledge.

Math as an invention. To the question, "Where did math come from?" responses ranged from God to the Egyptians. Figures credited by the children with the invention of mathematics were the king, cavemen, inventors and scientists. Except for Paige, their central agreement was that an authority figure designed math and that it was a male. I find it interesting that the general agreement was that math was invented and this invention happened long ago and across the whole world, as in these examples:

God made it up and told inventors what to do.

God made numbers so people could learn.

God made math so we can have fun.

God invented math so we could have fun learning.

Egyptians invented math to count their pyramids.

Math was invented by one person, for the whole world.

Someone invented math long ago.

Math as fun. The children saw math as fun or entertainment as well as learning and only Paige alluded to a practical use for math in counting the pyramids. The girls mention math as "fun for learning" or God invented "math for us to have fun." The children's previous experiences in a Reggio-inspired kindergarten seems to have fostered a sense that math is fun and enjoyable, not a subject for anxiety. Mathematicians find there is often a definite aesthetic and enjoyable aspect to much of mathematics such as Euclid's proofs that there are infinitely many prime numbers (Bellos, 2010, p. 16).

Math tools and symbols. The girls illustrated their ideas of math with many math tools and symbols. They drew numberlines to 10 and some to 20, thus meeting the Ontario numeracy expectations representing and ordering numbers. The children were familiar with equations for addition. Some girls drew teeth as representations of mathematical ideas — perhaps how many teeth the child had lost, complete with the number 9 represented in different ways on dot cards, a symbol and numberline. Rulers, shapes, patterns, clocks, Roman Numerals, ten frames, t- charts for growing patterns and abaci were drawn displaying understanding of typical math representation of manipulatives, symbols and thinking.

Math geographically and historically. These ideas went beyond the expectations in their knowledge of other number systems such as Roman numerals. The children acknowledged geographical and historical differences in number systems in their prior knowledge and theory building.

> The King of China invented math for everyone
>
> Math is from cavemen. They needed to share food so everyone got the same (amount), they invented numbers.
>
> Math is learning how to sort patterns. A scientist invited math to earth from long ago.

Prehistoric people did in fact recognize abstract (time, seasons) and concrete (animals) quantities as mentioned in the girls' theories (Bellos 2010, p.11). The research states that numeracy predates writing and that trade, or " sharing" as stated by the girls, stimulated invention of numeracy (Bellos 2010, p.11).

Math as patterns. Mathematics as patterns is common in both the girls' and mathematician's ideas. Lexi and Erica in their illustrations use simple shape and symbol repeating patterns. Omira talks about math as learning how to sort patterns and draws a growing pattern in a t-chart about budgies, as does Elle with numbers. Maya in her illustrations writes out a counting by 2s pattern. Three different types of pattern are represented, a simple repeating shapes pattern, a counting by multiples pattern (2s) and a growing pattern in a t-chart. Elementary students are introduced to the concept of functions by investigating growing patterns, visual patterns formed with manipulatives that allow them to concretely build an input-output table or t-chart to organize data about the number used for each stage of the pattern. The table helps quantify the pattern so that students see both the growing pictures and the growing numbers in the table. They can note the change from stage to stage and predict a general rule that will work for any stage of the pattern without having to build it or know how many blocks were used in the stage before it. This is an important abstraction of the pattern and the rule must make sense to students and be in their own words or in their own mathematical notation that reflects the level of their current understanding.

Math as practical and theoretical knowledge. The girls said math is, "To tell the time, count, keep score. Even dinosaurs and animals need math to know when to go for food at night." Indeed it is inferred that animals do demonstrate number sense; lions defend their territories against intruders, but they will only attack if they out-number the other pride (Butterworth, 2005). Such attacks could also be due to size, mass or estimation but these are all parts of number sense.

The girls had practical knowledge of math in the following examples.

We need math to show when to wake up and when school comes, your house number and how many candles on your birthday cakes.

Math is to count, for houses and teams.

Erica: Math is like two and two. It is figuring out each side (of an equation).

The children illustrated real world applications of math, e.g. for telling the time, thermometers, measuring, shapes and patterns and keeping score in games. The third example is a more theoretical or abstract interpretation of what math entails such as addition and equations or algorithms and problem solving. When asked, many girls referred to math as "learning and solving problems" accomplished "by thinking". These can be seen in the accompanying graphic novel.

Maya: Math comes from ideas. Different countries have different numbers.

Elizabeth: Math makes you smart. It helps you figure out. You do math by thinking, counting and drawing pictures. People first start math when they are 3 or 4 years old.

Maya and Elizabeth also see math as ideas or abstract or theoretical knowledge. However they both make real life connections about its geographical occurrence, differences and how and when math is explored in school.

Elle: Math helps you learn how to count. Everybody can do math who thinks.

Elle sees math as a tool or help in thinking and talks of its availability to everyone. Clearly she does not have the image of math as a special gift or ability but as a resource accessible for everyone.

Analysis

The children's theories and their sorting and categorization of them require a heavier cognitive load than that laid out in the numeracy and process expectations. Their initial comments of " not knowing what math is" proved unfounded. Further probing raised their consciousness of what they did in fact know. With young children, implicit understanding is often difficult for them to articulate without careful questioning and probing. Their theories displayed their ability to be explicit about what they know but required scaffolding by the teachers. Butterworth (2005, p.10), when exploring children's number sense agreed with Brownell advocating "meaningful learning" rather than drill (Brownell, 1935).

Enrichment of numeracy expectations

Breadth of numeracy knowledge. The children were aware of a wide range of components of mathematics: invention of math, emotions and math, math tools and symbols, geographical and historical implications, patterns, theoretical and practical implications. Many of their theories were developing conceptions but this sophisticated overview of math was impressive for such young children.

Representation of numbers. Many of the illustrations showed representation of the numerals and ordering beyond 10 at the beginning of the year. Claire and Lexi wrote out the numerals to 20, Laurelle to 23.

One to one correspondence. Maya showed one to one correspondence by drawing her hands and assigning a numeral to each digit from 1 to 10.

Quantity and money. Erica represented ideas of more and less by drawing a person with hands outstretched and differing amounts of coins in each hand. She labeled "more" and drew three coins and "less" and drew two coins. She further represented her ideas of quantity and magnitude by drawing two leaves and writing less and five dots and writing more. In the first example she is comparing sets of the same object, coins, but in the second example she is focusing on two different sets of objects, leaves and dots and revealing an understanding of magnitude regardless of set objects.

Skip counting and multiples. Maya wrote out 2,4,6,8,10 and then composed her own question, 2,4,6,8, ? Distinguishing between questions and answers and formulating questions are enriched understandings for young children. She also wrote out

5,10,15, 20, 25. Moving away from unitizing towards counting in multiples, an important mathematical step.

Addition and Commutative property. Elizabeth wrote out addition equations, 1 + 8, 2 + 9, 9 + 2. This string of equations reflects learning of related algorithms. Teachers often bring attention to relationships between equations and ask carefully formulated "strings" of equations to show one more than 8 is 9 and build a relationship with 9. Her juxtapositioning of 2+9 and 9+2 illustrated she understood the commutative property in addition. (The order in which the two numbers are added does not change the total.)

Fractions. Maya drew a circle and partitioned it into five approximately equal parts to represent fifths. She also drew two people sharing a cake cut in two equal sized pieces or halves shedding light on her understanding of equal amounts and the concept of a half and division by sharing.

Enrichment of the process expectations

The process expectations were similarly enriched by the trajectory of illustrating, theorizing, sorting and categorization.

Reasoning and Proving. Comparing their theories and drawings, there was evidence of pattern recognition and classification. The children recognized similarities between theories such as " invention" as a theory for where math came from. They also differentiated between theories seeing the theoretical and practical examples of math. Category formulation and naming was a whole class activity that widened everyone's perspective and offered opportunity for idea revision. The teachers brought the research literature to the children so they could consider the "proof" of their theories.

Reflecting. During morning meetings the children considered each theory and demonstrated their ability to reflect on and monitor their thinking. The process involved meta-cognitive thinking, which requires higher-order thinking skills and sophisticated use of the process skills. Using the hundred languages of learning to represent ideas and discussing different perspectives requires all the process skills. The opportunity to debate, negotiate, reflect, evaluate and seek meaning enabled them to reach a higher level of thinking collaboratively rather than individually.

Selecting and using tools. Knowledge about the variety of tools for learning is another process skill reflected in the children's illustrations, symbols and words. There were t-charts, clocks, thermometers, dot plates, ten frames, numberlines, equations, tallies, patterns, abacuses, geometric figures and numerals.

Making connections. In this theory generation and illustration many mathematical ideas and the curriculum strands were interconnected. Numeracy was represented but also patterning and algebra in the repeating and growing patterns. Geometry was intertwined with the patterning, using different geometric figures to create repeating patterns. Measurement was represented in the form of clocks, money, thermometers and rulers. Data management and probability were addressed as the categories of different theories were organized in a graph-like formation displaying prevalence of ideas. The probability of each theory was discussed and proofs we could find to back their ideas e.g. Butterworth's theories on animals and math.

Representing. Mathematical ideas were represented using pictures, words numbers, diagrams and invented symbols. The girls were given no guidance with their illustrations but created original, individual illustrations of their theories.

Communicating. Theory generation in young children is sophisticated because they are often not limited by current "correct" information and are willing to take risks. As David Elkind writes:

> Children have their own curriculum priorities and construct their own math, science and technology concepts. These concepts while age-appropriate, may appear wrong from an adult perspective...young children's thinking has to be understood on its own terms and in its own context, not from the perspective of adult thought. (1998, p. 3)

Eliciting theories about mathematics helped the children to articulate their thinking and ideas about mathematics. Theory generation was a creative process and appealed to their emotions, aesthetic sensibility and intuitive conceptions about math as a discipline. It is a sophisticated and complex process to create a theory and articulate it through words and represent it through illustrations.

The children met many numeracy and process expectations at the beginning of the term in September in an enriched way with this initial experience. Generating theories about math highlighted that following the content list of numeracy expectations would be redundant for many in the class. It was crucial to honour prior knowledge and theory building and differentiate future experiences accordingly.

Comparison with early mathematicians

Their theories display much understanding about mathematics and mirror understandings written by early mathematicians and scientists. Researchers of cognitive development have argued that young childrens' cognitive behaviours approximate those of scientists and mathematicians (Ness, and Farenga, 2007). As in the Reggio experience, children were seen as capable and sophisticated in their ability to generate theories. The children's intuitive ideas about the longevity of math are interesting. Alex Bellos, in his book, *Alex's Adventures in Numberland,* claims,

> No-one knows for certain, but numbers are probably no more than about 10,000 years old. By this I mean a working system of words and symbols. One theory is that such a practice emerged together with agriculture and trade. (Bellos, 2010, p15)

In "The Wonder Of Learning", a Reggio Emilia exhibit that I visited in in Indianapolis in 2009, confirmed that"Children's theories have an amazing proximity to early scientists and mathematicians' theories down through the centuries" (Reggio Children, 2009). Similarly, Kamii's research in mathematics used two groups of children; a group learning from text books and a group learning through constructivist practices and found constructivist learners achieved superior results in solving word problems. She concluded that when first graders are encouraged to do their own thinking, they are much more likely to develop potential for logical reasoning (1999, p.122).

Nominal, cardinal and ordinal representation.

Sophistication was present in the children's ability to articulate nominal, cardinal and ordinal representations of number. The children's illustrations displayed ordering numbers to 20 so they had a sense of the ordinality of numbers. Ordinal numbers tell the order of things in a set—first, second, third, etc. Ordinal numbers do not show quantity. They show rank or position. Houses with numbers displayed and team players wearing numbers displayed a nominal use of number. A nominal number names something—a telephone number, a player on a team. Items were drawn in piles and labeled more and less displaying their understanding of relative quantities and the cardinality of numbers. Thus cardinality or how many in a set, a quantity, as well as ordinality and nominality were represented. It was sophisticated thinking when the children illustrated all three meanings of number — cardinal, ordinal, and nominal.

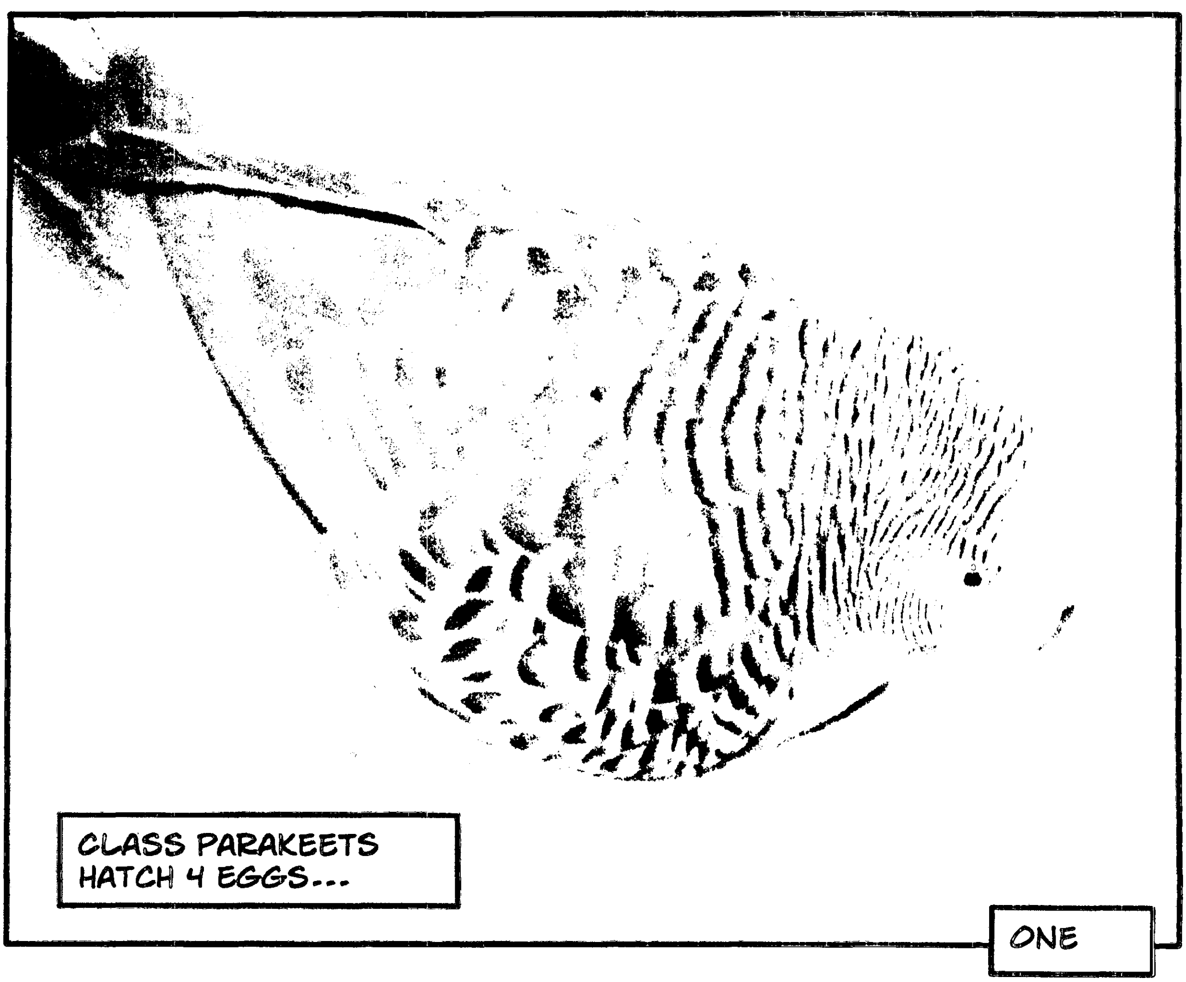
Sorting and categorization

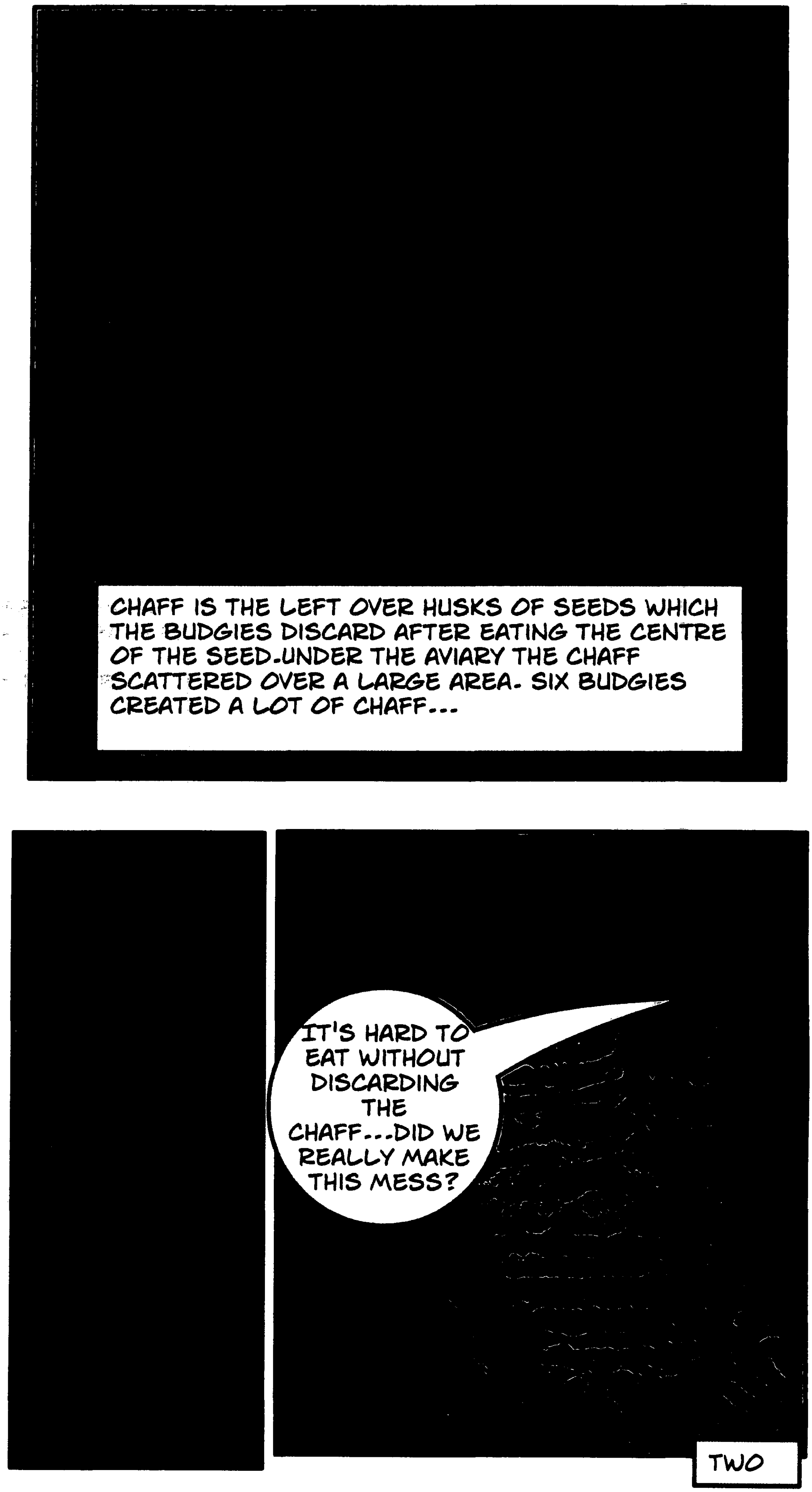
The complex sorting and categorizing of theories by the children exceeded mathematical expectations in data management and included practice in comparing quantities as they enumerated the columns of theories. This was a sophisticated and abstract sorting of ideas. In a traditional classroom the children would be asked to sort classroom items or toys, not ideas. This activity leads to graphing and falls under the data management and probability strand of the curriculum. However, numeracy is also present as they assess prevalence of certain theories and count the number of similar theories in each category.

Initial experiences emerging within the identity project revealed the need to enrich the expectations in numeracy due to the skills and knowledge already achieved. Many of the numeracy expectations were met as the children entered grade one. I needed to provide opportunities for mathematical processes to be cultivated further. In particular developing the mathematical process of communication was encouraged, as the children needed teacher scaffolding to articulate their understandings.

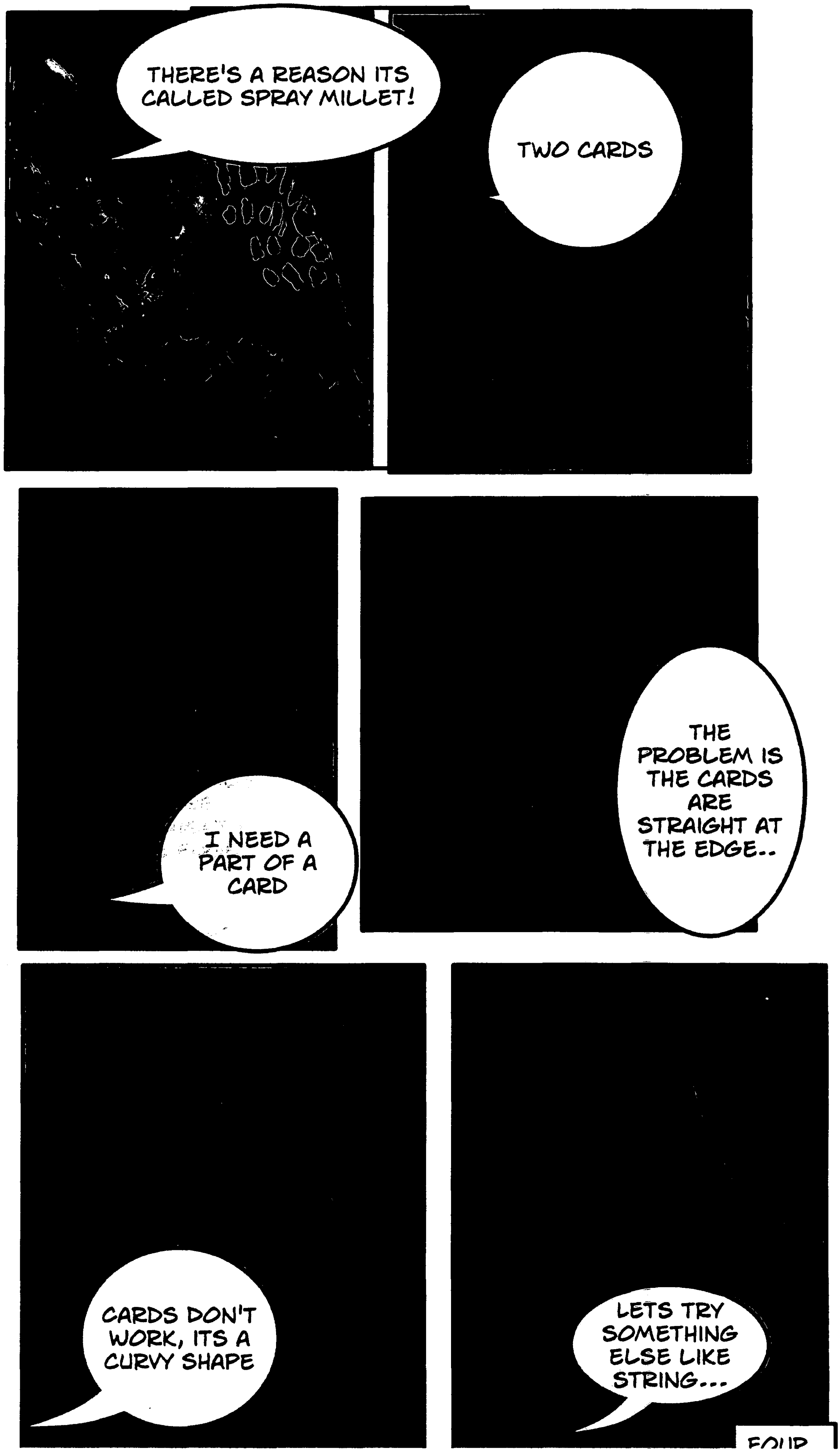
Skirting the chaff with Math

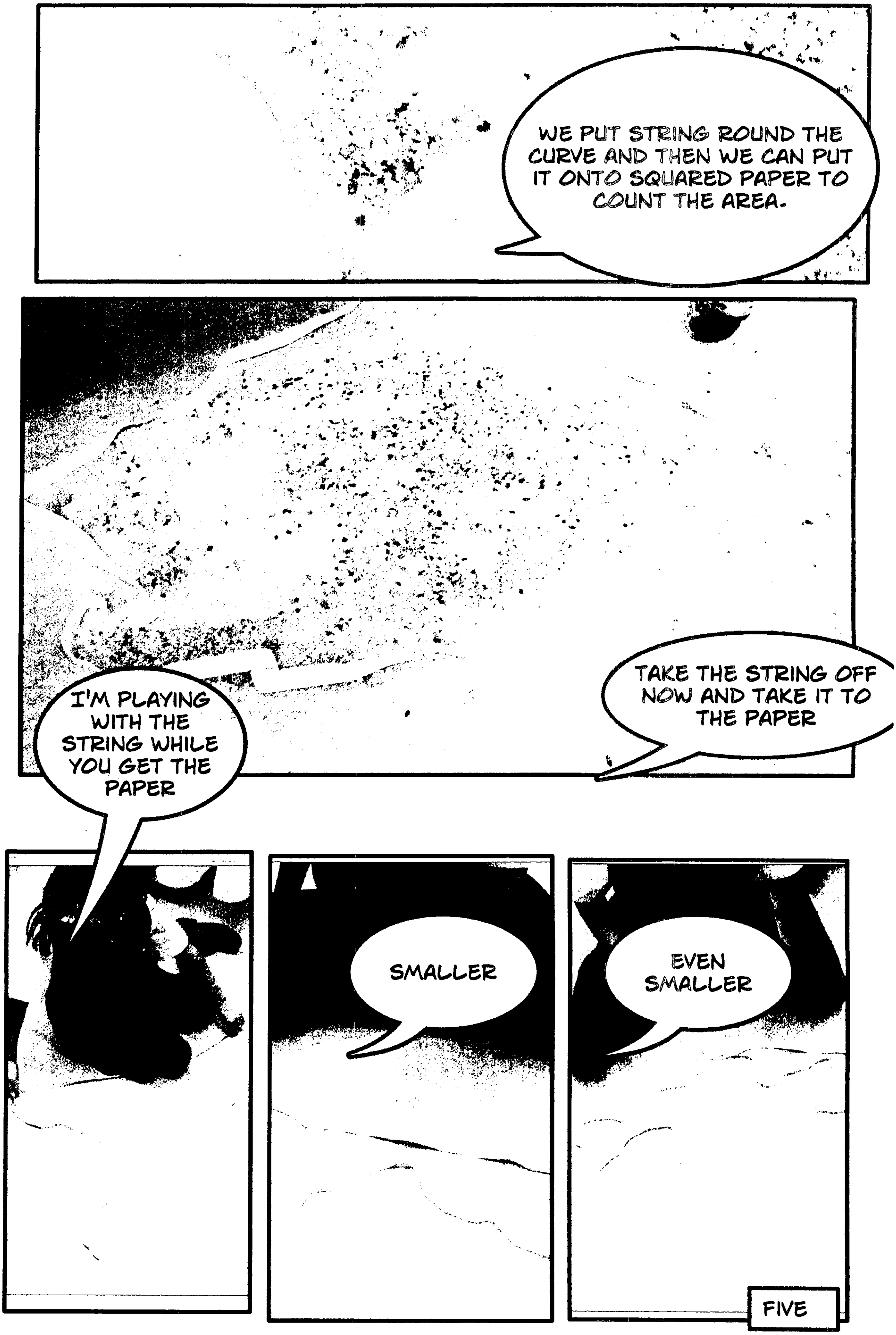
by Susan Hislop & Grade One

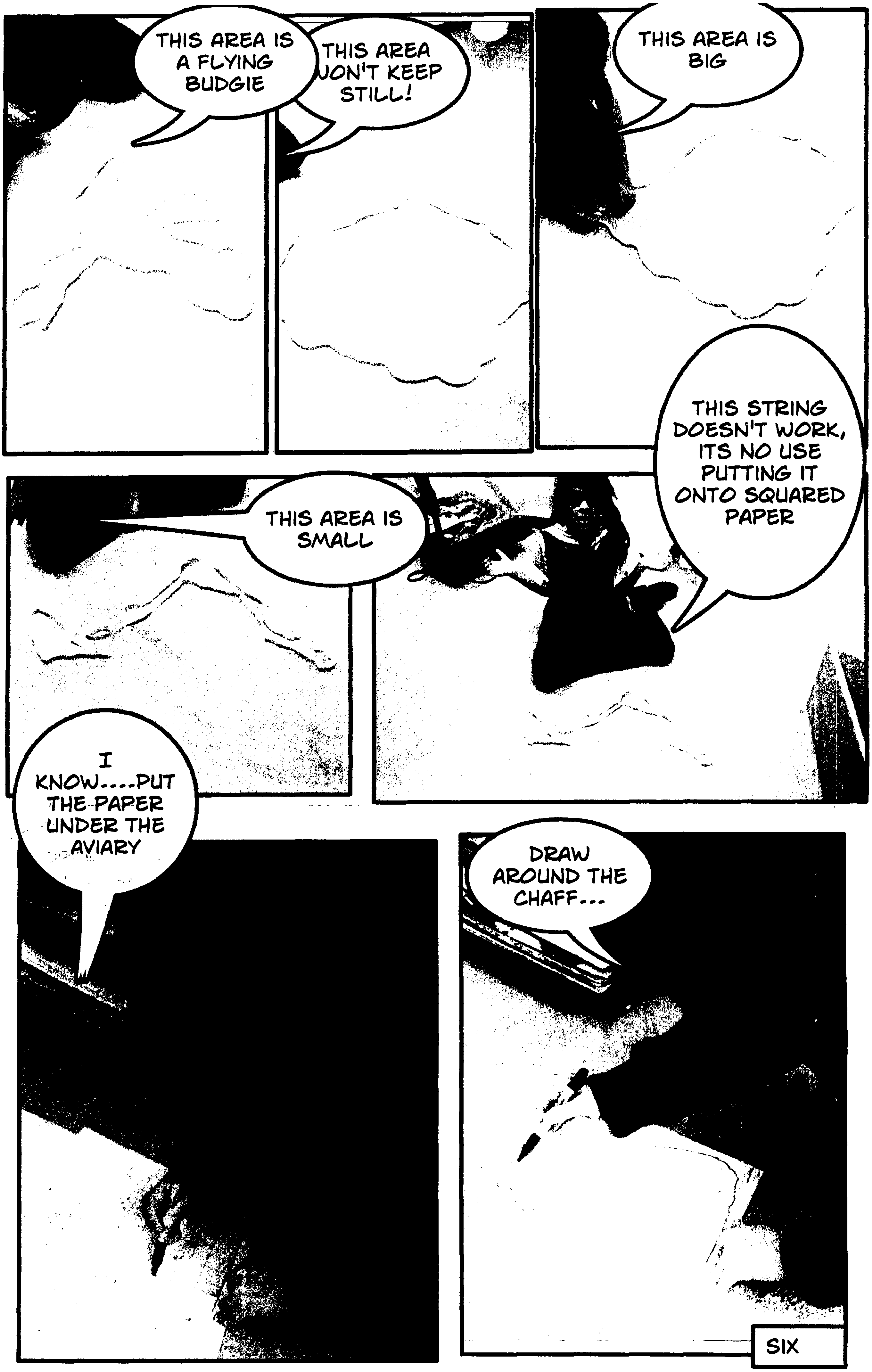



CHAFF IS THE LEFT OVER HUSKS OF SEEDS WHICH THE BUDGIES DISCARD AFTER EATING THE CENTRE OF THE SEED. UNDER THE AVIARY THE CHAFF SCATTERED OVER A LARGE AREA. SIX BUDGIES CREATED A LOT OF CHAFF...
IT'S HARD TO EAT WITHOUT DISCARDING THE CHAFF...DID WE REALLY MAKE THIS MESS?
TWO


THESE BUDGIES MAKE A LOT OF MESS
YOU MAKE A MESS
IT A GINORMOUS MESS
HOW DO WE SOLVE THE PROBLEM OF THE CHAFF?
LETS MEASURE THE MESS!
OH DEAR I HAVE TO EAT!
LET'S MEASURE BY COVERING THE MESS WITH CARDS
THREE

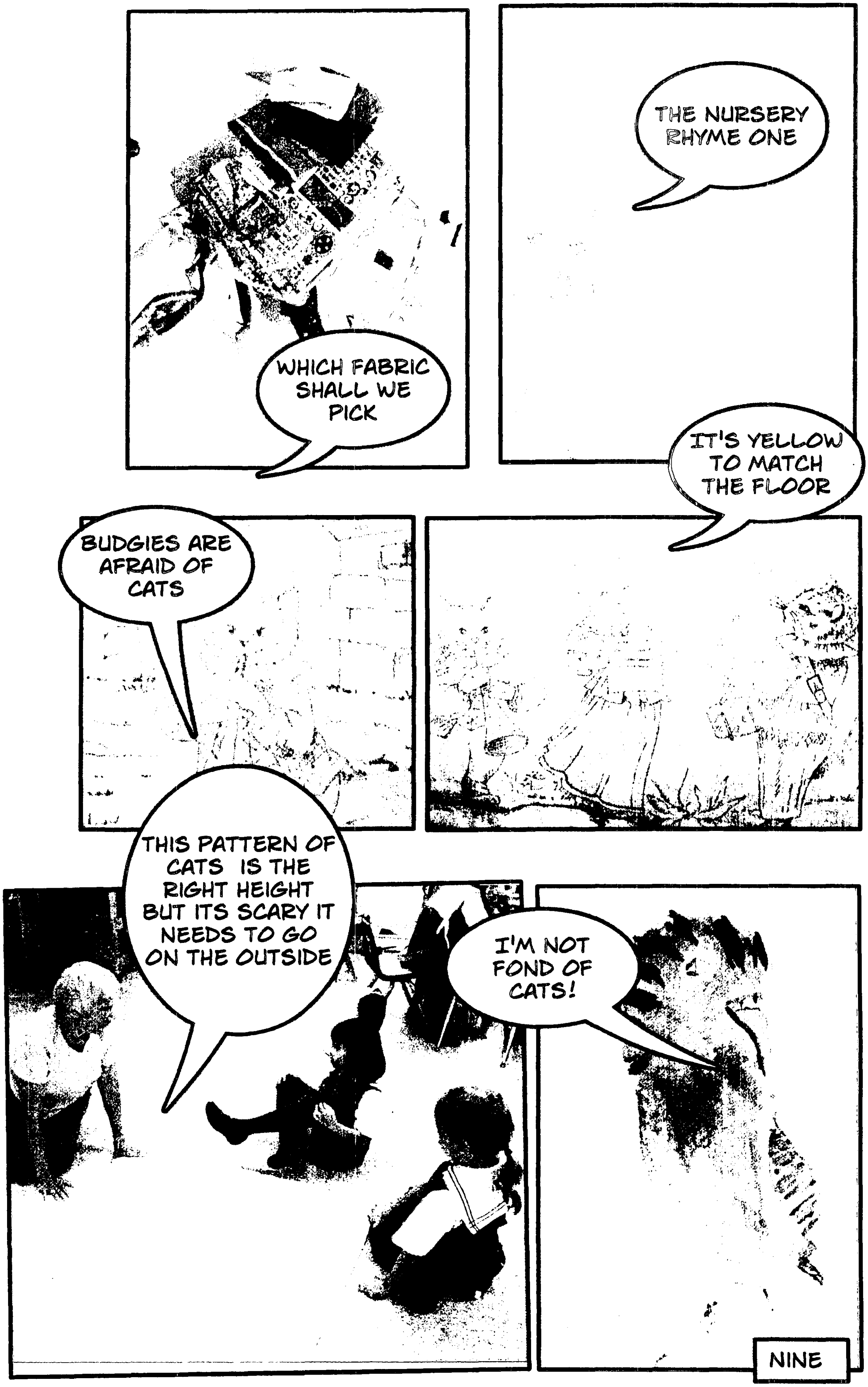

THERE'S A REASON ITS CALLED SPRAY MILLET!
TWO CARDS
I NEED A PART OF A CARD
THE PROBLEM IS THE CARDS ARE STRAIGHT AT THE EDGE..
CARDS DON'T WORK, ITS A CURVY SHAPE
LETS TRY SOMETHING ELSE LIKE STRING...


WE PUT STRING ROUND THE CURVE AND THEN WE CAN PUT IT ONTO SQUARED PAPER TO COUNT THE AREA.
TAKE THE STRING OFF NOW AND TAKE IT TO THE PAPER
I'M PLAYING WITH THE STRING WHILE YOU GET THE PAPER
SMALLER
EVEN SMALLER


THIS AREA IS A FLYING BUDGIE
THIS AREA WON'T KEEP STILL!
THIS AREA IS BIG
THIS AREA IS SMALL
THIS STRING DOESN'T WORK, ITS NO USE PUTTING IT ONTO SQUARED PAPER
I KNOW....PUT THE PAPER UNDER THE AVIARY
DRAW AROUND THE CHAFF...


THEN I CAN COUNT THE SQUARES
NOW WE KNOW HOW MUCH MESS , CAN WE MAKE A SKIRT FOR THE BUDGIE CAGE? IT HAS TO STOP THE CHAFF FALLING OUT. CAN YOU HELP US ? WE CAN MEASURE FIRST...
LONGWAYS
THAT'S CALLED LENGTH
SEVEN


LETS USE STRING
WIDTH
SIDEWAYS
THIS IS TH WIDTH OF CAGE
WHAT SHALL WE DO ABOUT UPWARDS?
HEIGHT
IS THIS FOR US? HOW KIND
THE FABRIC IS NOT LONG ENOUGH...CAN WE SEW PIECES TOGETHER?


WHICH FABRIC SHALL WE PICK
THE NURSERY RHYME ONE
IT'S YELLOW TO MATCH THE FLOOR
BUDGIES ARE AFRAID OF CATS
THIS PATTERN OF CATS IS THE RIGHT HEIGHT BUT ITS SCARY IT NEEDS TO GO ON THE OUTSIDE
I'M NOT FOND OF CATS!


WE NEED TO CONSIDER FEMINIST PERPECTIVES...WHAT LITERATURE WILL REASSURE PARENT CONCERNS ABOUT SEWING?
SHE SAYS" The relationship between feminism and domesticity has recently come in for renewed interest in popular culture".
GIILLIS AND HOLLOWS SPRINGS TO MIND
SHE TALKS ABOUT " how these domesticities might be understood from a feminist perspective"
WHO KNEW WE WOULD CAUSE ALL THIS PROBLEM SOLVING?
TEN


I CAN PUSH THE PEDAL
THE ELASTIC NEEDS TO BE SHORTER THAN THE FABRIC SO IT WILL GATHER IN.
WE NEED TO ALLOW MORE FOR THE SEAM
THE ELASTIC NEEDS TO SNAP
THE SEEDS ARE STAYING IN THE CAGE MORE...ONLY A FEW ARE ESCAPING NOW... THE SKIRT IS WORKING. MS. JUSTINA WILL BE HAPPY.
LOOK AT THE LOVELY SKIRT

Chapter Four: Skirting the Chaff with Math

I will outline an unexpected event with the classroom pets, the budgies/parakeets. The grade one's class budgies hatched four babies and I purchased a lovely, big aviary to house the six parakeets. When the budgies eat seeds, they spit out the chaff – the external casing of the seed – and it falls onto the floor. Understandably, the cleaning staff was unhappy with the area of scatter of chaff around the aviary, from the six birds. Vera Justina, our head of housekeeping, brought our attention to the problem during morning meeting. The girls responded:

Victoria: Look at all the mess!

Maya: It's big.

Paige: It's a ginormous mess. Let's measure it!

A purposeful math experience

The children felt a desire to quantify the area of scatter before the problem could be resolved. The emerging math experiences had a purpose, to solve the problem of the chaff scatter, and so the girls were motivated and engaged. The whole class had the opportunity to invent a strategy for measuring the chaff. In actual fact, measuring the extent of the scatter of chaff did not seem essential to me but it was important for me to honour their strategies and allow them to take ownership and work out, through trial and error, what information they needed to solve the problem. I have found often that following a circuitous route leads to more incidental learning. The problem of the chaff illustrates the utility of purposeful math, for when an authentic situation such as this presents itself the math takes on an urgency and relevance, enhancing student engagement. The whole class, in small groups, at different times throughout the day, selected units of measurement such as cards, sheets of plain paper and squared paper, but was I leading them to think in terms of area and not linear measurement? Was this underestimating their ability to make their own distinctions about area and length and units of measurement and selection of appropriate tools to measure?

Relationship between area and perimeter. Following their own ideas of measuring the scatter led to an investigation of the relationship between the perimeter and area of the scatter of chaff. This was a complex task that involved measuring an irregular or organic shape. The girls believed they needed to know the extent of the problem before it could be solved. First they tried measuring with cards but they lamented that the straight edges did not cover the edge of scatter adequately. The girls suggested that if they measured the scatter with string and then placed the string on squared paper they could measure the area of scatter by counting the squares. However, this process proved problematic. The string could suggest different areas depending on how it was placed on the paper.

Elle: This is no good, the area won't keep still.

Elle Sophie: If I move the string into this shape the area gets smaller. If I move the string like that it gets bigger. String is useless to measure… we need something else.

They saw that they needed a more stable measure.

Marion Small says,

> Measuring perimeter indirectly with string helps students see that perimeter is still a type of length measurement — it is length around an object or space. Students are often surprized to find shapes with the same area can have different perimeters and that shapes with the same perimeter can have different areas. (2009, p. 402)

A period of revising initial ideas took place and much discussion and experimentation ensued until Laurelle said, "Let's put the squared paper under the aviary and draw straight onto the paper around the chaff."

Counting beyond a hundred. The class investigated the area of scatter and found that the area covered was quite extensive, over a hundred squares. Numeracy in the form of counting thus became entwined with geometry and measurement. Not only ordinality, but cardinality and magnitude became part of the mathematical learning as each square or group of squares was accorded a count. The girls counted in 5s, 10s and 1s, assisting each other all the way to over 100 and comparing efficiencies of counting methods. For example one child said, " It's much quicker to count by 5s than 2s, or 10s than 5s." This is an important step away from unitizing or counting by ones.

Fractional squares and rounding up. Another problem that arose was the covering of part of a square. Some children chose to ignore partial covering but others argued it would not truly reveal the extent of the problem unless partial squares were included. Ideas of approximation and rounding up from half a square and rounding down from a partial square (under a half) were considered. Establishing the visual idea of half, more than and less than a half is important to their developing understanding of fractions. Results were compared and the girls were satisfied with a rounding up and down of squares to get as accurately as possible the area of scatter. Recognition that measurement is not truly accurate but an approximation was also an important discovery. The girls commented, "We are counting the parts of squares and adding them together to make wholes but its kind of like estimating, like when we measured ourselves with straws and said we were five and a bit straws or almost six straws long". The girls recognized the need to be more accurate than using "a bit" or "almost" and fractions gave them a more accurate measure for partial squares.

Finding the root of the problem. The girls wondered about the cause of the scatter and compared the old bird cage with the new aviary: "The old cage was less tall and not as wide. I think it's the height that makes the chaff fall so far out". Experiments dropping food from different heights involved much hilarity and joy and led to a new determination.The girls determined that the height of the aviary was to blame for the area of scatter and we discussed possible solutions, such as positioning the foodtrays or perches lower. Vera Justina, our head of housekeeping, suggested a fabric skirt to go around the bottom of the cage, telling the girls that she had a skirt on her parrot's cage. The girls were keen to pursue this possible solution and proceeded to design a skirt to fit around the aviary.

Solving the problem. The children suggested we invite the principal's mother, Mrs. McDonald, to come in as an expert seamstress. She had worked previously with the class, on sewing, as a member of our extended community. The two girls involved in this project were new to the school and they found working with a "grandmother" figure comforting. They relished the extra attention and patience Mrs. MacDonald was able to give. Karen, in particular, instantly relaxed when working with Mrs. MacDonald, sharing stories of her own grandparents in China.

Grandmother: What attributes shall we measure?

Elizabeth: Longways, sideways, and upwards

Grandmother: Do you mean length, width, and height?

The next problem was to figure out which attributes of the cage to measure. I left out non-standard units of measure, such as straws, bricks, and pipecleaners but the girls had ideas of their own. They found the string again. The girls decided to measure the aviary width and length and cut their string accordingly. Karen and Elizabeth identified the attributes, length and width, and performed transitivity skillfully (using an object, the string, to measure and compare with the fabric). They realized they needed two lengths and two widths of material. The fabric was not long enough, so they debated whether they could cut two pieces and sew them together. The aesthetic choice of fabric was important to the children and they decided upon a nineteenth century reproduction of a nursery rhyme that included personification of animals. This choice was based on the fact that they tell the budgies nursery rhymes to achieve a positive response/interaction with them; the budgies join in chirping. The whole group deliberated the colour and design of the fabric. The class eventually picked a pastel yellow to blend with the colour and tones of the classroom and because, as Georgia said, "It is a happy colour."

The next decision to be made was the height of the skirt, "What height should the skirt be?" (Already abandoning her prior approximate language of "upwards" to mean height.)

There are cats on the fabric. We should make the skirt as tall as the cats estimating the height the skirt should be by eyeballing).

The cats may scare the budgies.

Not if the pattern goes on the outside.

The girls displayed further problem solving and development of math vocabulary as extra fabric was allowed for seams. The girls decided upon elastic as suitable for holding the skirt in place. Elastic was chosen over a variety of different fasteners, including velcro, buttons and zippers. A long conversation occurred as to how to measure elastic and how it should be shorter than the fabric length to allow for gathering and tension. Important ideas of the properties of materials were developing.

Analysis

The girls went beyond the curriculum expectations in many ways in this problem solving. Some of the elements contributing to an enriched curriculum will now be discussed.

Enrichment of numeracy expectations

Counting in multiples. One of the numeracy expectations for grade one is to understand whole numbers to fifty but in this experience the girls counted to over a hundred (289). The girls discussed the efficiency of the different modes of counting and acknowledged that the larger the number, the higher should be the group to count by: "10s is best for a number over one hundred". The girls who counted by 2s lamented the length of time it had taken them when the other groups shared their more efficient strategy.

Two- and three- digit adding. The girls in the adding of partial squares also went beyond addition of single digits to twenty and involved two-digit addition above 100. In all there were 289 squares covered with chaff so the girls were counting to 219 and then adding on the number of partially–covered squares.

Developing fractional ideas . Not only did they consider whole numbers but also fractional numbers when counting partial squares and rounding up and down accordingly. The number they counted was in the context of a measurement situation and denoted area. A partially covered square was counted if it was more than half covered and if it was less than half it was not counted. Much rich language was evident, such as "This square is only about a third covered" and "This just has a quarter (covered)". The expectations recommend describing only halves, fourths and quarters.

Establishing one-to-one correspondence. One-to-one correspondence was displayed as the girls pointed to a square or group of squares and counted one for each square, or groups of squares as they counted the area of chaff. Counting the elements in a set included moving beyond counting in 1s to counting in groups, an important step in moving from ordinality to cardinality or moving from looking at number order to quantifying and ascribing magnitude. The girls were not merely counting using one-to-one correspondence but also recognizing an area value. The squared paper contained a hundred squares and the girls were able to articulate, "100, 200 squares," displaying counting by 100s.

Using 10s as benchmarks. Conservation of number enabled the girls to note the number of whole squares and record it and then add on the partial squares. The girls, in the adding of partial squares, also went beyond addition of single digits to twenty and demonstrated two and three digit addition above 100, as in "I have 100, 200 squares and nineteen more, that's 219 full squares" and "If I add 219 and then the part squares are 70 that's almost 220 and 70 which would be 290. Then I would take away one to get 289." This shows the sophisticated use of 10s as benchmarks in adding 220 instead of 219 and then adjusting the total by subtracting one.

Measuring area. Following the evaluation, discussion and revision of strategies everyone in the class tested the relationship between perimeter and area and this led to the change in strategy to catch the chaff directly onto the squared paper. The children had assumed that string of the same length would always give the same area no matter how it was arranged. Initially they had measured around the chaff on the floor with string and carried the string to the squared paper and discovered they could create vastly different areas with the same length of string.

Enrichment of the process expectations

Much rich content was learned in a deep and meaningful way when the girls used all the mathematical processes described in the ministry expectations: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing and communicating (Ministry of Education, 2005, p.32) as the following section will show.

Problem Solving. The girls problem solved and investigated the relationship between the perimeter of the scatter and the area of the scatter. This deepening of their understanding was evident in solving the problem of how to measure the area of an organic shape, how to find an alternative strategy, which manipulative to use to measure and how to solve the problem of the chaff. They also solved problems involving addition of three digit numbers, using a variety of strategies, of how to attach the skirt to the cage, and how to add extra fabric for seams. Problem solving through an emergent approach led to acceptable solutions to problems such as using two pieces of fabric to reach the required length for the skirt, and using a shorter piece of elastic than the fabric length.

Reasoning and proving. The development of reasoning skills was evident as the girls decided partial squares could not be ignored. Through discussion with others, that partial squares count, they developed a rounding up and down method. They made and investigated conjectures such as that the same perimeter can have different areas. "Looking for patterns and drawing logical conclusions should be integral aspects of elementary math instruction"(National Council of Teachers of Mathematics, 1989). Conclusions can stem from intuitive, deductive or inductive reasoning according to Baroody (1998, p. 2-24). In their quest to measure the area of chaff all three types of reasoning were apparent.

Reflecting. The girls reflected on their strategies that did not work to their satisfaction and concluded that they needed to let the chaff fall upon the squared paper directly. They refined and monitored their thinking by explaining to others why they thought their solution or strategy was correct. As they reflected and refined their use of measuring tools they said, " Cards don't work, It's a curvy shape" and " Let's try string." As they monitored the relationship between area and perimeter they concluded, " The area won't keep still!" They helped clarify a conception, grasping that the same perimeter did not always give the same area and explained their finding to each other, the teachers and the grandmother. Reflecting on their initial strategies led to the final solution of positioning squared paper directly under the cage to catch the chaff.

Selecting appropriate tools. The girls selected and used a variety of concrete and visual learning tools. Initially they tried using cards to measure the scatter of chaff but found the straight edges unsuitable for measuring an organic shape. They selected string for its capacity to curve around the area of scatter. The squared paper was useful for counting the squares and calculating area but had to be positioned under the cage and string. The string could not be carried to the paper or the exact perimeter would be compromised. Calculators were used to help count by 5s, 10s and 100s and add on partial squares, to record the area of scatter of chaff. The girls also made connections among simple mathematical concepts and procedures, and related mathematical ideas to situations drawn from everyday contexts, their class pet problem.

Connecting. In direct math instruction the strands are taught separately in distinct units, breaking down the natural connections in real-life math. In contrast the problem of the chaff was an authentic problem grounded in the environment and the experience of the children helped reveal, not hide, the mathematics in daily living, as some artificial contexts do. Some mathematical problems created by teachers lack authenticity and emergent mathematical problems are more tangible. An interconnected, multi-strand inquiry (measurement, numeracy, geometry) enabled depth of learning as opposed to looking at each strand individually.

Representing. The girls created representations of mathematical ideas, such as, drawings of areas. Then they used concrete materials such as squared paper and string and made connections between the perimeter and area, and applied these connections to solve problems. Throughout this process they communicated mathematical thinking orally and visually to the teacher, using everyday language, a developing mathematical vocabulary, and a variety of representations. This happened when Kiran took the string from around the chaff over to the squared paper on the table and said, "We put string around the curve and then we can put it onto squared paper. To count the area." When this strategy failed Karen revised the strategy and said, "Put the paper under the aviary, draw around the chaff then count the squares."

Enriching mathematics process expectations

Student work reached beyond the listed process expectations in many ways. Student work was a complex series of intricate, interconnected, multi-strand, multi-disciplinary problems. The work went beyond mathematics to include choice, exploration of properties of different materials such as various fasteners, and to include caring for parakeets and understanding their characteristics. These multi-strand problems included concepts from the measurement, and geometry strands (perimeter, area) and numeracy strand (estimation and fractions) outlined in the curriculum (Ministry of Education, 2005, p.31). The following are several examples of going beyond the process expectations.

Discovering further problems

Attempting to measure area led to discoveries about perimeter, rounding up and down, and fractions. There was opportunity to revise and refine ideas, which enabled a greater depth of learning.

Experiencing satisfaction and aesthetics

Satisfaction and aesthetics are important elements in motivating and engaging the interest of the children. Aesthetics in my view means the nature of art and beauty and connects with the creation and appreciation of beauty. Aesthetics include sensory-emotional values and can include critical reflection on art and beauty. Aesthetics played an important role in the choice of yellow cloth, "yellow, as a happy colour" that blended into the school environment, tastefully. There was an aesthetic appreciation of the shapes the string could create, as in "This area is a flying budgie" and also the enhancement of the aviary, as in "Look at the lovely skirt". Satisfaction was found by making meaning and finding a solution to a problem. Satisfaction also implies motivation to perform in the future, and finding meaning and satisfaction in the group learning. The children felt a need to measure the extent of the mess and there was satisfaction in exploring their ideas and following a plan, as: "Now we know how much mess, we can make a skirt for the budgie cage." There was great satisfaction and wonder in their discovery that a certain perimeter can make different areas, as in: "This area won't keep still." The children were satisfied with the efficiency of the skirt and concluded, "The seeds are staying in the cage more...only a few are escaping now... the skirt is working." There was also reciprocity between aesthetics and satisfaction that encouraged the children to persevere with difficult tasks.

Multiple strategies

The children could solve a problem using a strategy of their choice and comfort level with mathematics. Problems with multiple strategies to solve contributed to an enriched curriculum by extending each child's thinking in an appropriate way. All the children tried to work out the area of the chaff in small groups of three, throughout the day and then shared their findings with the whole group at the end of the day.The children who were able to count the number of squares by 2s or 5s or 10s, or even 1s, did so as they felt comfortable. A calculator was chosen by some children to add on the partial squares. The problem of calculating area was accessible for all the children to solve in their own way and they benefited by sharing each other's strategies and ideas.

A higher cognitive demand and load attested to the role of the children as protagonists in their own learning. The girls decided to measure area and perform as accurately as possible. There were important conceptual layers in the task to develop reasoning, communication, thinking and metacognition.

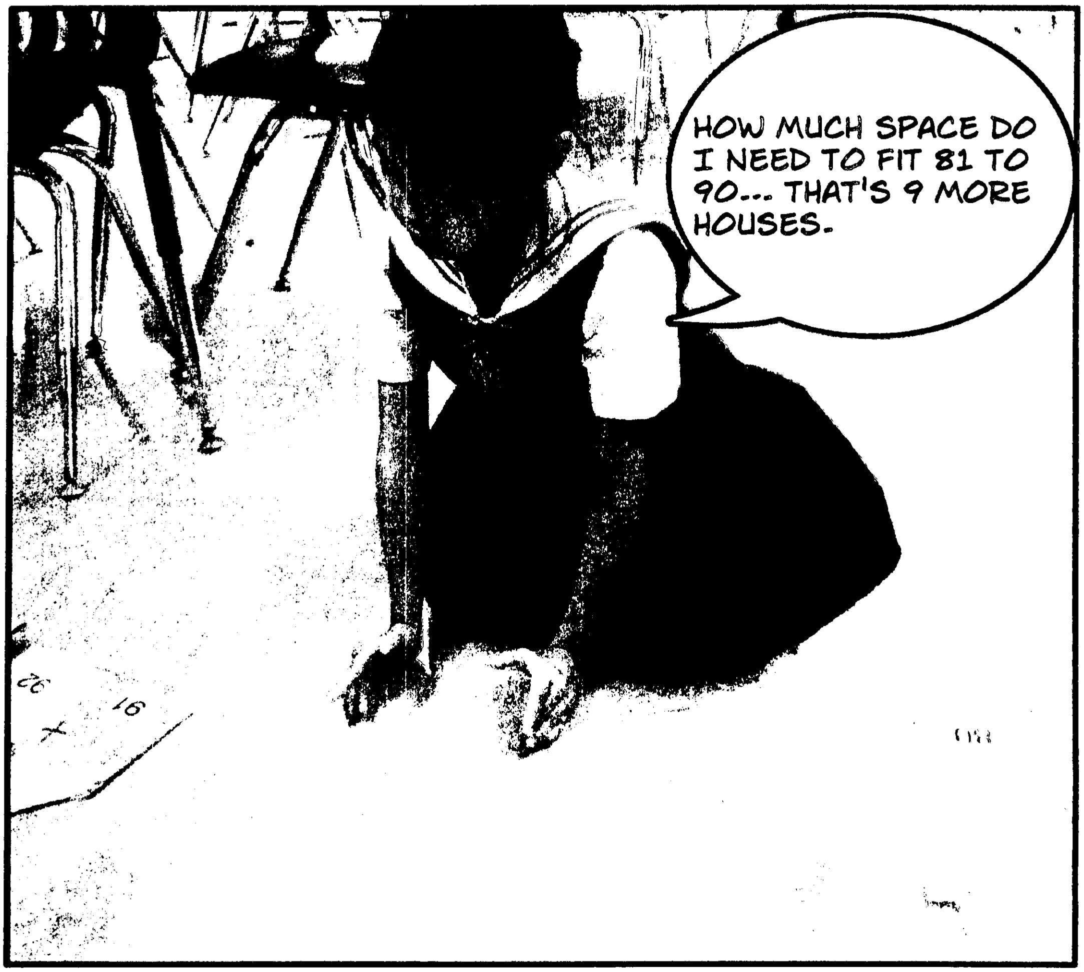
The mathematical content that emerged in this problem of the chaff was extensive, rich and sophisticated. Measuring, relationship between area and perimeter, fractions, multi-digit addition, counting in multiples, and using benchmarks of 10 all displayed this richness. As a teacher I could not have designed or anticipated such a problem and the enriching opportunities it afforded. The children made important mathematical discoveries themselves about perimeters and fractions. I think the depth of learning came not only from the mathematical content and use of all the process expectations. In fact in the following chapters these same aspects of the process skills are present but I will not repeat the analysis. The depth of learning also came from the children's emotions; their empathy with the housekeeping staff, surprise about the relationship between area and perimeter, joy in experimenting into the root of the problem, satisfaction in the exchanges with others and aesthetics in creating and noticing beauty. The skirt was " lovely" or beautiful to them, the shapes you could make with string were charming. I notice that often in emergent curriculum, facilitating convoluted pathways in problem solving leads to deep learning, engagement and discoveries and the sensory-emotional responses of the children act as motivators.

Making a Number neighbourhood

I HAVE FOUND THAT WHEN CHILDREN MAKE THEIR OWN MANIPULATIVES THEY ARE MORE MEANINGFUL. I SHOWED THE GRADE ONE CLASS A NUMBER NEIGHBOURHOOD NUMBERLINE FROM 0 – 100 MADE BY A PREVIOUS CLASS.

THEY IMMEDIATELY WANTED TO REASSEMBLE THE SECTIONS IN ORDER AND MAKE THEIR OWN NUMBERLINE AS A CLASS.

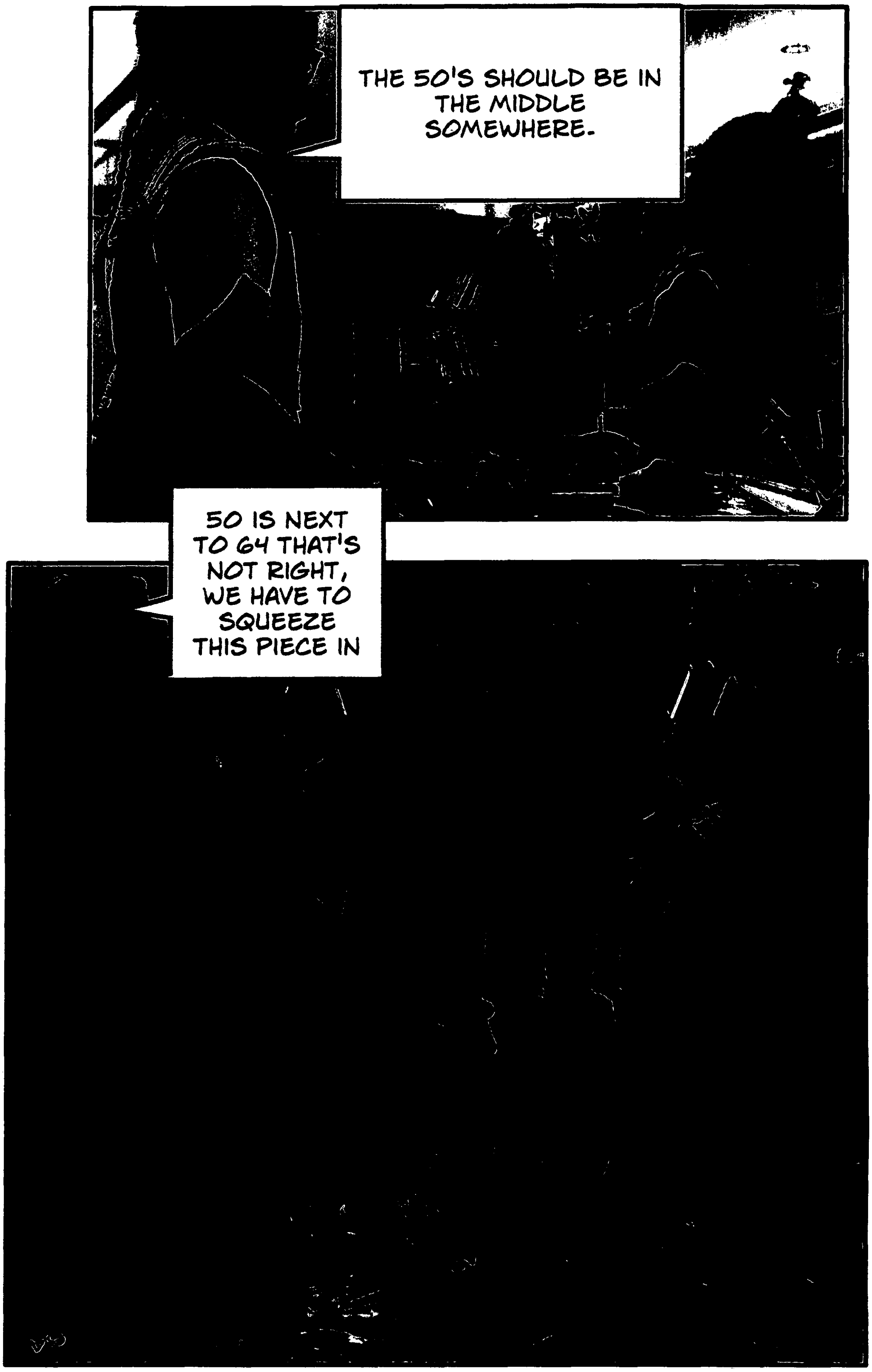
THEY ESTIMATED THE NUMBERLINE WOULD FIT ALONG THE FRONT OF THE CLASS AND BEGAN TO ORDER AND SORT THE NUMBERS.

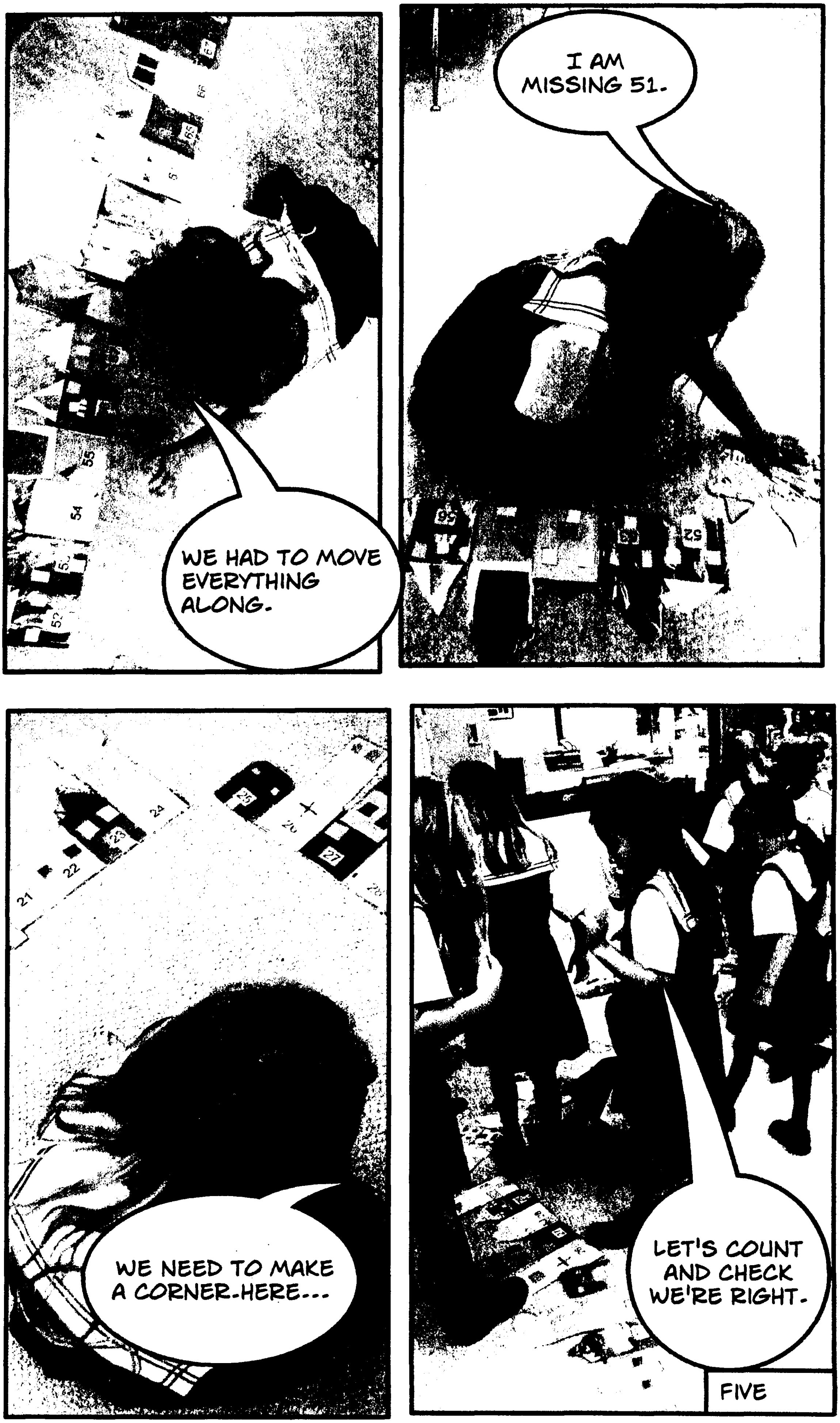

HOW MUCH SPACE DO I NEED TO FIT 81 TO 90... THAT'S 9 MORE HOUSES.


THE 90'S GO NEAR THE END SOMEWHERE.
THIS IS 20. ITS AT THE BEGINNING OF THE NUMBERLINE.
TWO


76 COMES AFTER 75
IT'S NEAR THE END
OF THE NUMBER LINE
CLOSE TO 100.


WE HAVE TO
SQUEEZE 44 IN
BEFORE 45.
THREE


THE 50'S SHOULD BE IN THE MIDDLE SOMEWHERE.
50 IS NEXT TO 64 THAT'S NOT RIGHT, WE HAVE TO SQUEEZE THIS PIECE IN


WE HAD TO MOVE EVERYTHING ALONG.
I AM MISSING 51.
WE NEED TO MAKE A CORNER-HERE...
LET'S COUNT AND CHECK WE'RE RIGHT.

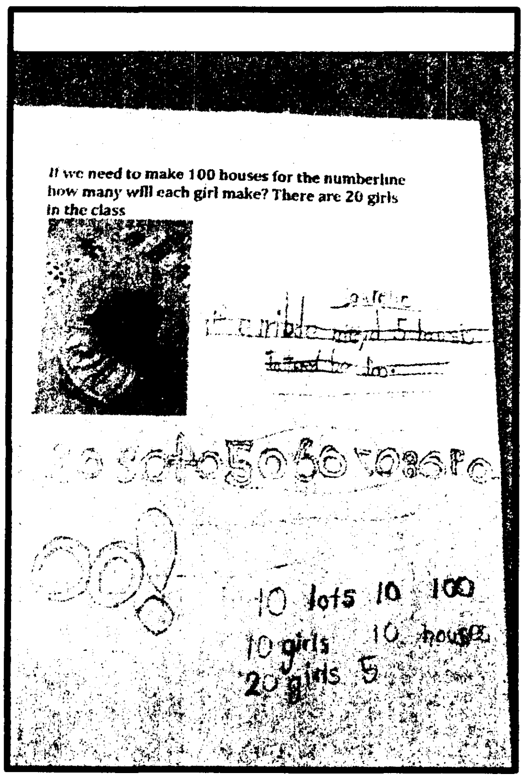
WE HAD TO MAKE 3 STREETS WITH TWO CORNERS TO FIT ALL THE NUMBERS TO 100 IN.

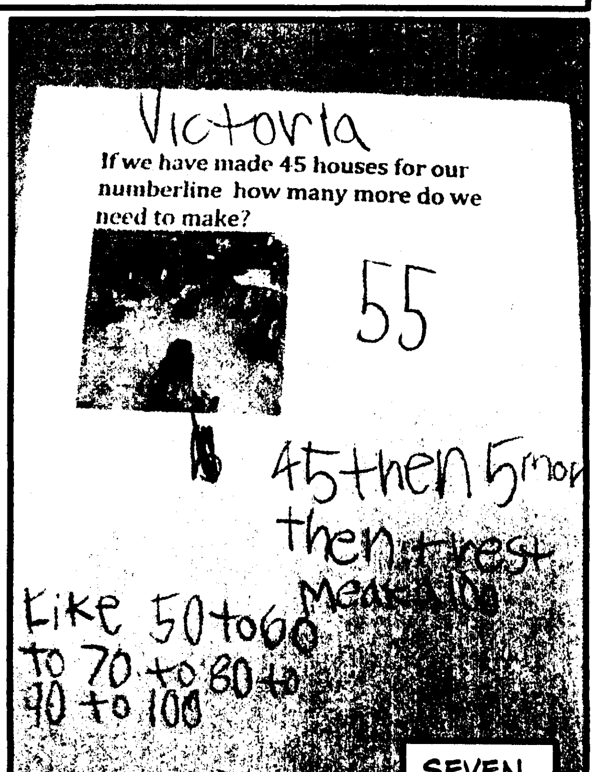
LET'S MAKE OUR OWN NUMBER NEIGHBOURHOOD NOW!

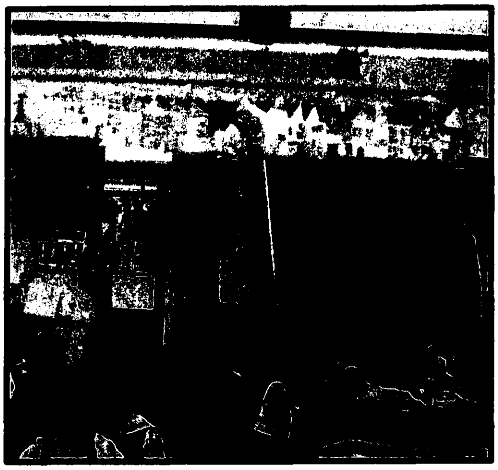
HOW MANY HOUSES SHOULD EACH GIRL MAKE, IF THERE ARE 20 GIRLS IN THE CLASS?

AFTER ONE WEEK WE HAVE MADE 45 HOUSES. HOW MANY MORE DO WE NEED TO MAKE A NUMBERLINE TO 100?

PROBLEM SOLVING:



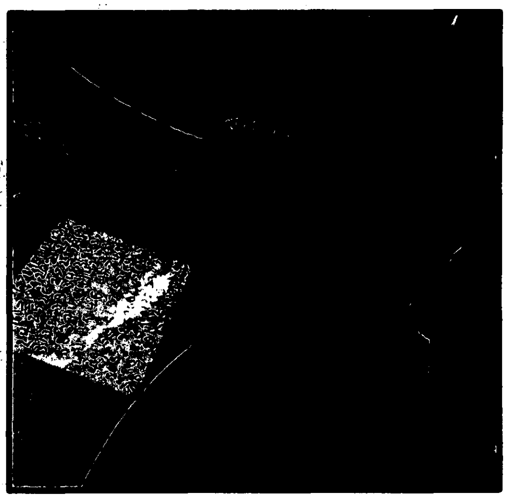




ELLE SOPHIE COUNTS TO 100 WATCHED BY JADZIA.

JADZIA IS INSPIRED TO COUNT AS WELL.

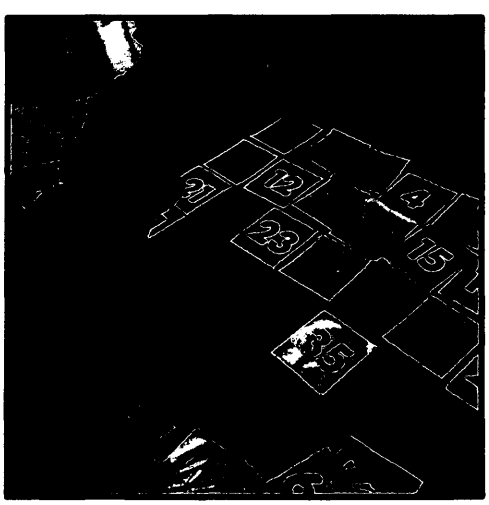
SHE GATHERS A CROWD OF HELPERS AS SHE GOES.

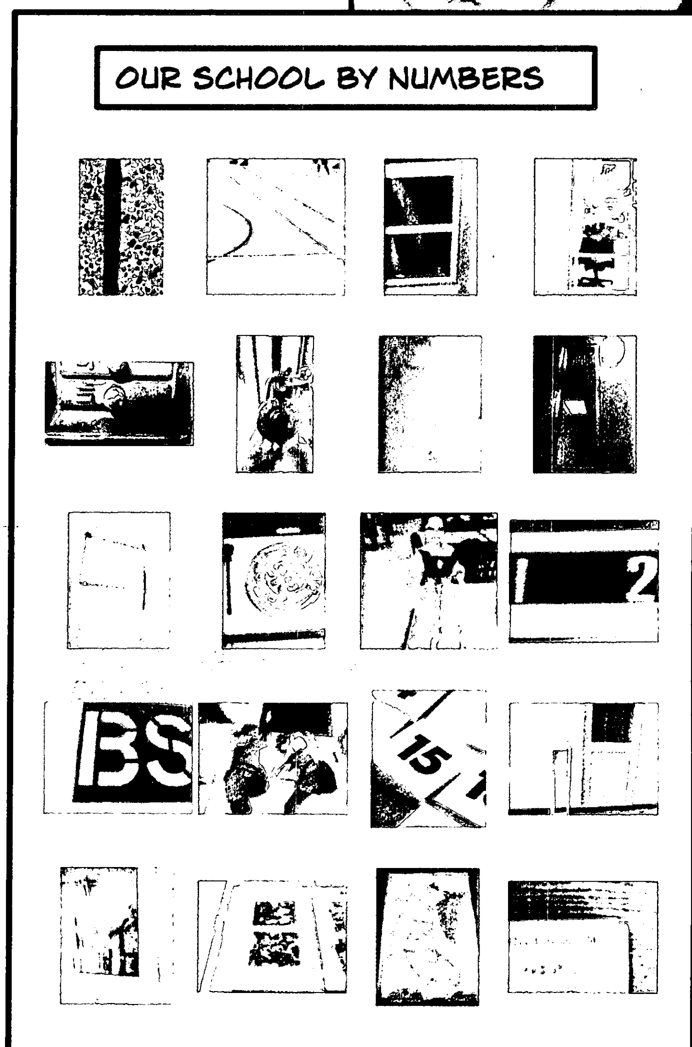


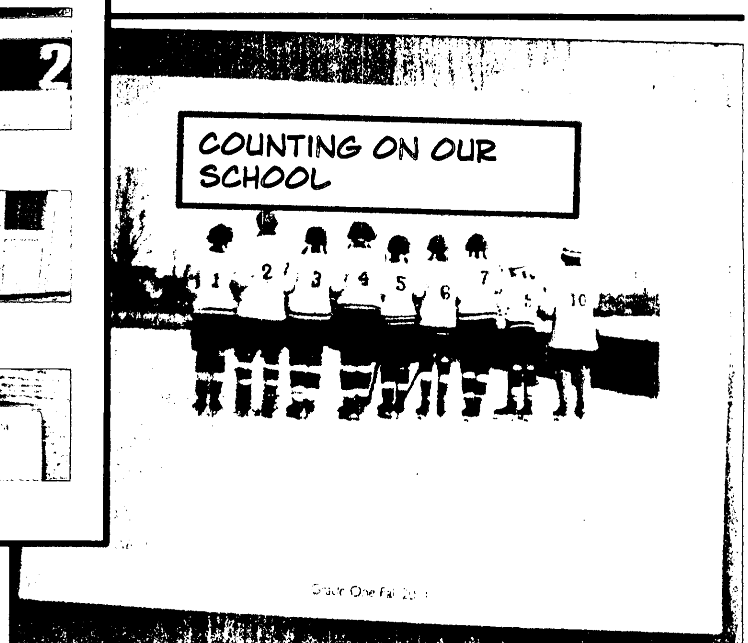
ELIZABETH IS INSPIRED TO WRITE TO 100.

PAIGE AND VICTORIA PLACE CARDS TO 100 ON THE CARPET.

ELIZABETH CONTINUES HER INTEREST IN ORDERING NUMBERS TO 100.







making more manipulatives

NINE

Chapter Five: Number Neighbourhood

Linear Numberlines are seen as important for the numerical estimation development of young children. According to Siegler and Booth,

> Between Kindergarten and second grade patterns of estimates progressed from consistently logarithmic to a mixture of logarithmic and linear to a primarily linear pattern. (2004, p. 428-444)

In very small, indigenous communities (in the Amazon Rainforest for example) a logarithmic mental numberline exists and means that individuals view 5 dots as five times greater than 1 dot but 10 dots are only twice as big as 5 dots. This means they conceptualize larger intervals between lower numbers and smaller intervals between higher numbers, not equal intervals. This intuitive logarithmic numberline plays a role in estimation. A linear understanding, or equal intervals between numbers understanding, is developed through experience at school, is a cultural invention and helps children to increase their abilities in manipulating numbers. Using numberlines is seen as helping with the later concept development of decimals and location of benchmarks, e.g. 5, 10 or ½, to help locate other numerical magnitudes. Proportional reasoning and multiplicative thinking help with the partitioning of the numberlines, e.g. 5 times 20, or 20 divided by 5. When an opportunity to explore numberlines emerged in grade one, I anticipated that this would help in the childrens' understanding of linear numberlines.

Identity project

In our project on identity the children discussed where they lived, the numeral of their house or apartment, and commented on whether they lived at an odd or even numbered house. They delighted in large numbers:

Omira: I live at number 4 Woodvale Crescent. It's an even number.

Georgia: My number is double yours, it's number 8 Hilholm, it's even too.

Sammy: I bet I have the biggest number… 235 Russell Hill Road, but it's odd.

Erica: No, mine's bigger, 541 Cranbrook Road and it's odd as well.

This discussion continued with the children ordering themselves physically on the carpet according to ascending house numbers. They would arrange and rearrange themselves displaying their nominal and ordinal understanding of number and reflecting a developing understanding of magnitude and connections and relationships between numbers.

As I observed the enjoyment, engagement and learning in this kinesthetic activity, I was reminded of a paper numberline, made up of numbered houses, that a previous class had constructed and called their "Number Neighbourhood". After rummaging in a cupboard I found this numberline all jumbled up and in sections of three or four houses laminated together. As I pulled the pieces out the children immediately wanted to assemble the sections in order and reconstruct this numberline from zero to one hundred. They estimated the numberline would fit along the front of the class and make one long street of houses. The children began to order and sort the number sections, lay them on the floor and verbalize their thinking:

Lexi: 72 to 75 goes after 68, because the 60s go before the 70s.

Erica: How much space do I need to fit 81 to 90? That is 9 more houses.

Jadzia: The 90's go near the end somewhere.

Simona: This is 20. It's at the beginning of the numberline.

Laurelle: 74, 75, 76 goes here.

Omira: We have to squeeze 44 in before 45.

Sophie: The 50s should be in the middle somewhere.

Elizabeth: 50 is next to 64, that's not right! We have to squeeze this piece in.

Victoria: We have to move everything along. I am missing 51, I need to make a corner. (One street will not fit across the front of the class.)

Laurelle: Let's count and check we're right all the way from 0 to 100.

We had to make three "streets" with two corners to fit all the numbers to 100 into a numberline. After expressing satisfaction at a job well done, the children voiced their desire to make their own numberline. This numberline creation took two weeks because of the problems the children formulated and the amount of work involved in the artistic representation of houses with paper.

Child-created resources

These experiences also led to the provocation for other child-created resources.

Victoria: Now we have made a numberline, we could start a factory and make all our own math stuff like counting books.

Georgia: We could take photographs and make a book or poster like *City by*

Numbers only *Our School by Numbers*, about our own school.

Jadziah: I can make a chart out of clay to show which way round to write numbers.

After this initial brainstorming session an archival counting book about our school identity, a photographed number chart, clay number plaques, and a math alphabet book, were produced by the class. Having realized that they could create their own numberline they felt empowered to create their own resources and no longer wanted commercially produced items. The class preferred to use their own carefully prepared resources and felt more invested in using them.

Analysis

Enrichment of the numeracy expectations

The girls went beyond the curriculum expectations for numeracy in five ways.

Counting to 100 from different starting points. One of the numeracy expectations for grade one is to understand whole numbers to fifty but in this experience the girls counted to a hundred. Not only did the children count from zero but they began counting at different points on the numberline and verbalized their reasoning, "72 to 75 goes after 68 because the 60s go before the 70s". It is much more difficult and sophisticated to count on from varied points on the numberline than to count up in order from zero. For example when children begin to add two numbers together e.g. 2+2=4 they will count up 1,2,3,4 and it is a later development to count on from 2,3,4.

Measuring and estimating space. The numbers they counted were also in the context of a measurement situation as they worked out spatially whether a section would fit and how the carpet arrangement would look, i.e. a street with two corners. Initially they had estimated one street at the front of the class, but as the work progressed they were able to rethink and adjust this prediction to fit the numberline together.

Beyond unitizing, counting in groups. Establishing a one-to-one correspondence when counting the elements in a set also included moving beyond counting in 1s to counting in groups, an important step and clearly showed their learning about magnitude. For instance they said, "81 to 90, that's 9 more houses". This showed they were thinking in terms of quantity, more and less, a more complex understanding than just ordinality. One-to-one correspondence was displayed as the girls pointed to a group of houses as they counted. Conservation of number and spatial understanding enabled the girls to note the position of numbers on a numberline, such as "20 is at the beginning of the numberline," "The 50s should go in the middle", or "The 90s go near the end".

Intuitive understanding of division and multiplication. Once the children decided to make a numberline to 100, a plan on how to proceed emerged with a question from one of the children. Sloane asked, "How many houses should each girl make, if there are 20 girls in the class and 100 houses?" Every child individually worked out her own strategy for solving this problem. Problem solving strategies to work out how many houses each girl should make required an intuitive understanding of division, 100 divided by 20, concepts not outlined in the numeracy curriculum in grade one. Three general strategies of solving this problem emerged and were shared, as described below.

Some girls drew 20 children or hearts and then shared out the 100 houses between them recording their sharing by a tally system. It was division by sharing equal amounts, "One for you, one for you, etc."

Victoria added 20 (the number of girls in the class) and 20 together to make 40 then another 20 to make 60 and continued adding 20 until she reached 100. Thus she was able to calculate that she had to add 20 together 5 times so each girl would make 5 houses. This repeated addition is a precursor to multiplication. Victoria's manipulation of addition of two digit numbers is called repeated addition and is a developing understanding of multiplication and this shows a complex method of solving the problem.

Laurelle wrote out the decades on her sheet 10, 20, 30 up to 100 and reasoned on her sheet "10 lots of 10 is 100. That means 10 girls would make 10 houses so then 20 girls would make 5 houses each." This is a very sophisticated deductive reasoning example involving an elementary understanding of multiplication "10 lots of 10 is 100". It shows a multi-step procedure, initially counting in decades, recognizing how many decades in 100, and then adjusting this knowledge, " If 10 girls make 10 houses then 20 girls make 5 houses", showing her dexterity with doubling and halving.

Solving subtraction of numbers by addition. After a week of making houses for their numberline the girls had only made 45 houses and needed to know how many more houses to make, they had lost track of how many they had each made.When working out the subtraction problem 100 - 45= 55 the girls intuitively used addition or counting on. Adding numbers beyond 20 took place as they worked out how many more houses than 45 they needed to reach 100. This exceeded the expectations by counting in 10s and then adding a two-digit number 95 and a single digit number 5. Others added on in 5s until they reached 100, which met the expectation outlined in the curriculum. The third strategy was to realize that adding 5 to 45 would make 50 then adding another 50. Adding the two-digit number 50 to 50 again exceeds expectations. The multiple strategies were discussed and ranked for efficacy and speed in a class meeting. The children thought there was a distinction between the quickest method and the easiest. They claimed it was easier and quicker to count by 10s than by 5s, but harder to add 5 to 95 than just counting by 5s all the way to 100. Adding 5 to make 50 was easy and it would be the quickest if you were comfortable adding 50 and 50 but not everybody was. They concluded everyone was best choosing their own method and working in their own time as there was no rush, leaving me wondering why I had asked this question.

As discussed in chapter three, by emphasizing the mathematical processes set out in the curriculum much more content was explored in a deep and meaningful way than outlined in the linear numeracy expectations. The girls used all the mathematical process expectations successfully. Problem solving was evident as they decided how to assign the house-building fairly. They conducted investigations into the relationship between the decades comparing and connecting cardinal values: "The 60s go before the 70s".

Enrichment of the process expectations

Developing reasoning skills were evident as the girls decided their problem solving strategy for how many houses each girl would build. The problems were accessible for each child to solve in her own way. The children were also creating or recognizing and verbalizing the problems to solve. The girls selected and used a variety of concrete and visual learning tools such as the numberline, tallying, simple drawings and eyeballing the space they would need to lay the numberline down. The girls used appropriate computational strategies to investigate mathematical ideas and to solve problems– counting, counting by multiples, drawing, doubling, halving, adding, multiplying and dividing. The girls also made connections among simple mathematical concepts and procedures. Such as solving a subtraction problem with addition: when they were working out 100 – 45 = 55, they chose to count on, adding the number 55 to make 100. The children also related mathematical ideas to situations drawn from everyday contexts such as their homes and house numbers.

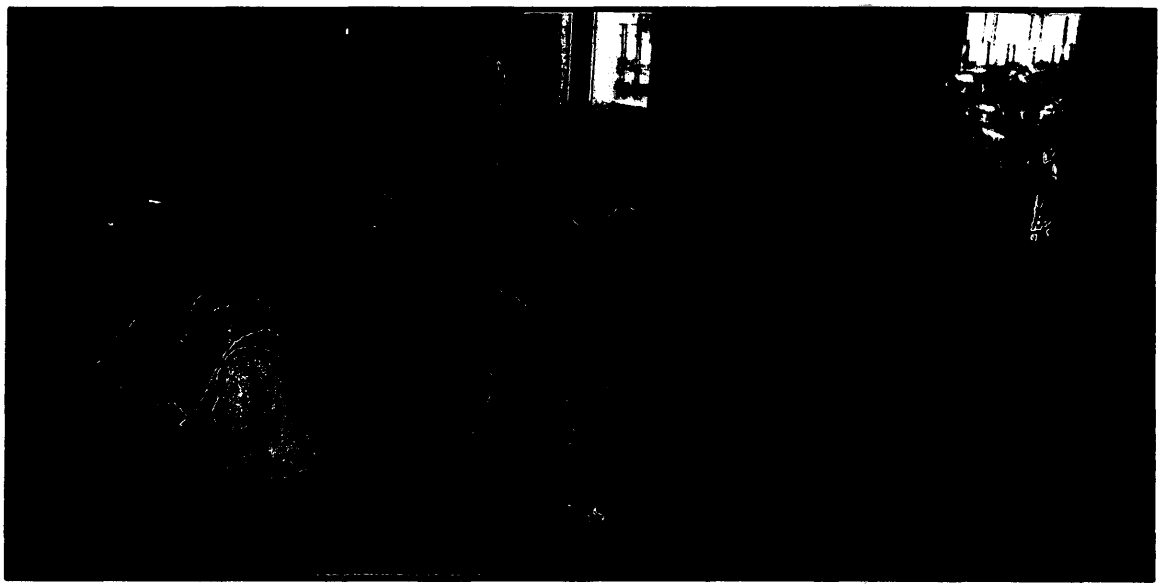
The experiences with the numberline led to important mathematical discoveries and thinking. This was a genuine task with performer–friendly feedback. It was realistic, required judgement and innovation and asked students to "do" the subject instead of reciting, restating or replicating through demonstration what was taught. The task was challenging, multifaceted and non-routine. It required problem clarification, trial and error, adjustment and adaption to the case at hand. The number neighbourhood experience evoked student interest and persistence and seemed apt and challenging to the children and teacher. Mathematical sophistication was not merely documented but the experiences enabled improved performance especially with the consideration of different strategies and recursive elaboration of skills already acquired in counting.

Satisfaction and aesthetics

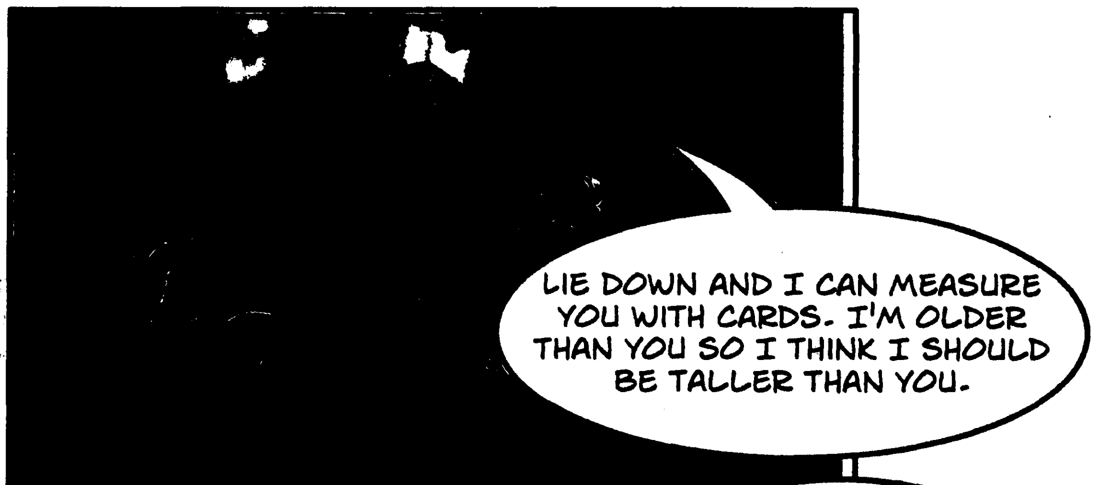
Satisfaction and aesthetics played a role in the creation of numberline houses and streets, and their self-made manipulatives. Their use of the numberline shows their satisfaction and enjoyment in using this mathematical tool as when Elle Sophie begins to count independently and a crowd soon joined her. Elizabeth was inspired to write out numbers to 100 using the numberline and other children used the 100s carpet and 100s chart, referencing their numberline as they did so. There was satisfaction in the collaboration that led to the understanding that problems could have multiple ways of solving them and multiple ways of recording thinking, from pictures to tallies to symbol manipulation. Their idea of a number tool factory was inspired, motivating and engaged them in further mathematical and aesthetic ways, making a clay number tablet, a photographic numeral chart and an archival counting book.

I think the depth of learning came not only from the mathematical content and use of all the process expectations. The depth of learning also came from the children's emotions; their sense of fun in comparing house numbers, surprise about the numberline houses in my cupboard, enjoyment in creating the numberline, satisfaction in the exchanges with others and solving problems. Aesthetics was important in creating and noticing beauty in their math manipulatives factory. Engagement and discoveries and the sensory-emotional responses of the children acted as motivators for in-depth emergent mathematical learning.

PROPORTIONALITY AND PEOPLE

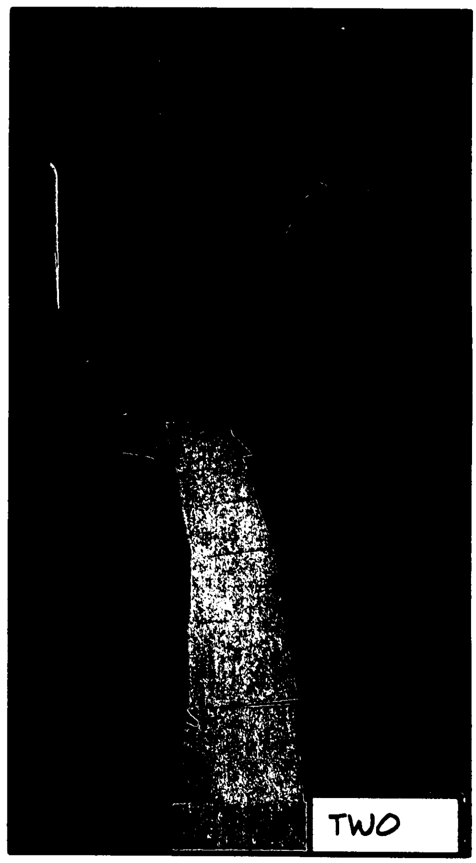


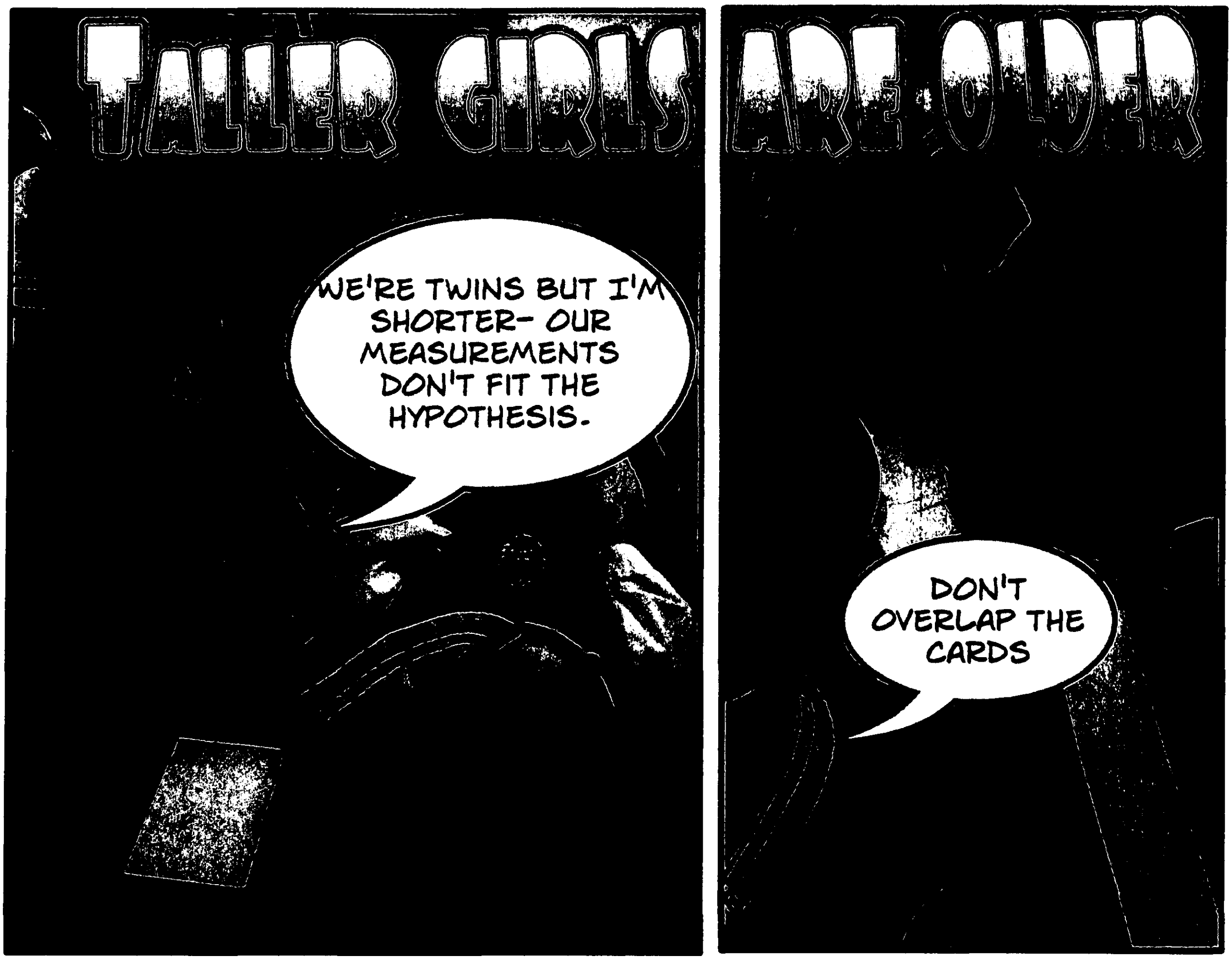
HAPPY BIRTHDAY VICTORIA...HAVE YOU NOTICED YOU ARE SEVEN YEARS OLD AND ONE OF THE TALLEST GIRLS IN THE CLASS! LET'S SEE IF ALL THE SEVEN YEAR OLDS ARE TALLER THAN THE SIX YEAR OLDS.



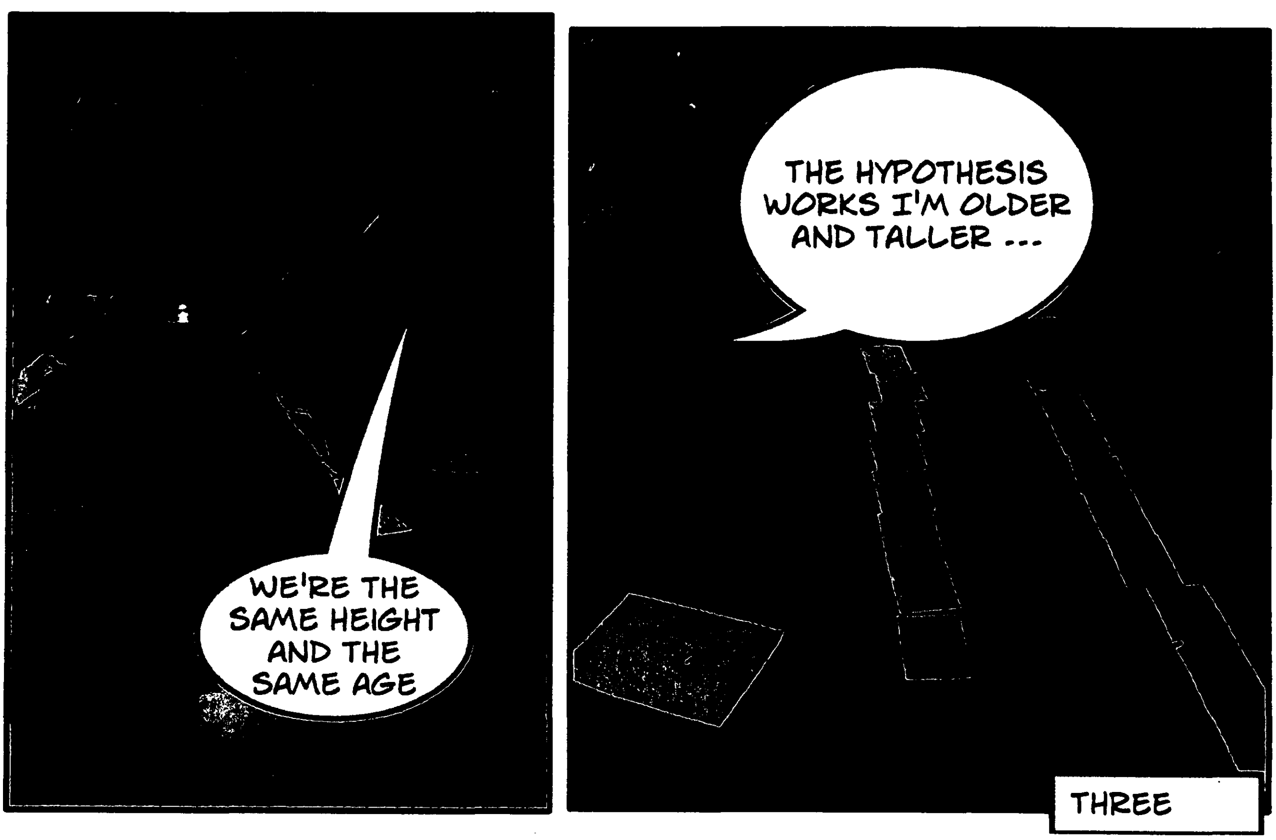
MEASURING LENGTH

THE GIRLS MEASURED THEIR LENGTHS/HEIGHTS USING NON-STANDARD MEASURES, CARDS AND BEGAN TO COMPARE THEIR HEIGHTS AND AGES
HYPOTHESIS: OLDER GIRLS ARE TALLER.

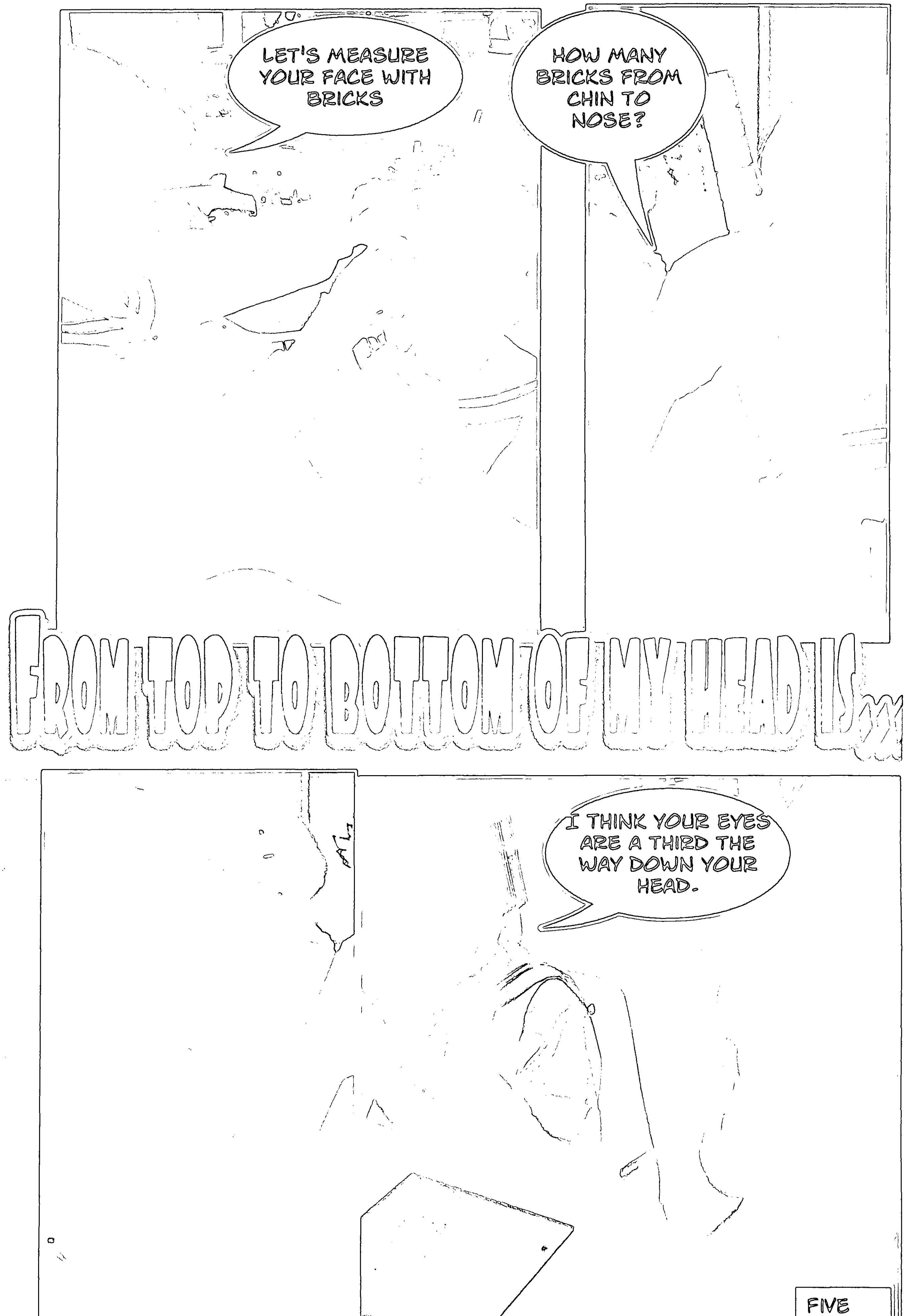




THE GIRLS SHARED THEIR FINDINGS WITH PARTNERS AND THEN WITH THE WHOLE GROUP.
THEIR HYPOTHESIS HELD UP THAT MOST OLDER GIRLS WERE TALLER THAN YOUNGER GIRLS WITH ONLY TWO EXCEPTIONS.

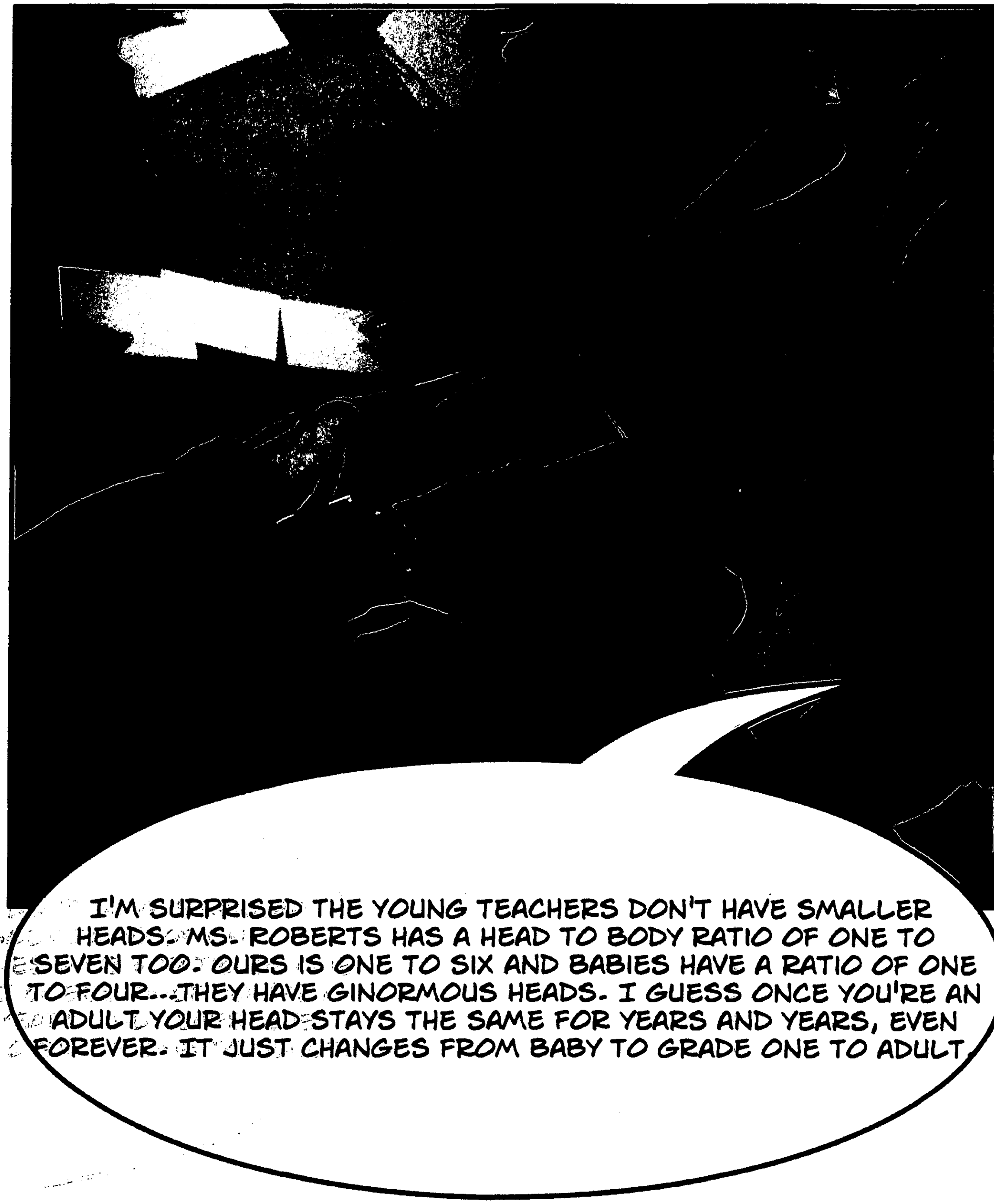



I WONDER HOW LONG MY HEAD IS?
KAREN YOUR HEAD IS ONE AND A QUARTER CARDS LONG. IS MY HEAD ONE AND A HALF? MY WHOLE BODY IS 8 CARDS.
I WONDER HOW LONG MY HEAD IS COMPARED TO MY BODY?


LET'S MEASURE YOUR FACE WITH BRICKS
HOW MANY BRICKS FROM CHIN TO NOSE?
FROM TOP TO BOTTOM OF MY HEAD IS...
I THINK YOUR EYES ARE A THIRD THE WAY DOWN YOUR HEAD.
FIVE


THE PRINCIPAL SHOULD HAVE THE BIGGEST HEAD BECAUSE SHE'S THE BOSS
THIS TEACHER IS THE SAME RATIO AS THE PRINCIPAL
I WONDER IF GROWN-UPS
HAVE THE SAME HEAD TO BODY RATIO ?
MS. HISLOP IS SHORTER THAN THE OTHER TEACHERS. I WONDER IF HER HEAD TO BODY RATIO IS THE SAME?
SIX

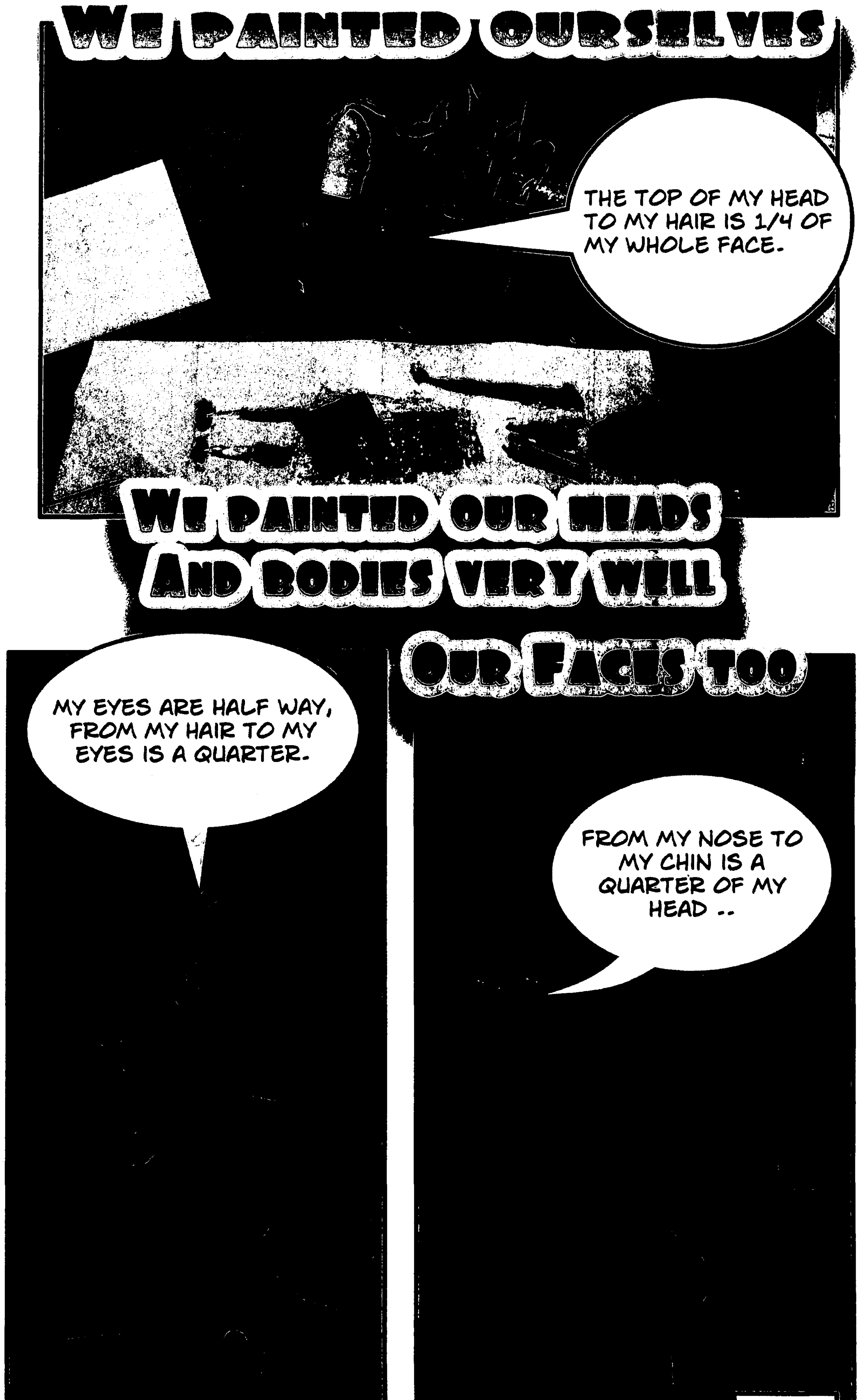

I'M GOING TO CHECK BY MEASURING BOTH SIDES TO SEE IF I GET THE SAME.
LET'S MEASURE THE PRINCIPLE AND VICE -PRINCIPAL AND TEACHERS...THERE HEAD TO BODY RATIO IS 1 : 7
MRS. P.G. IS SHORT BUT HER HEAD TO BODY RATIO IS 1:7 AS WELL.

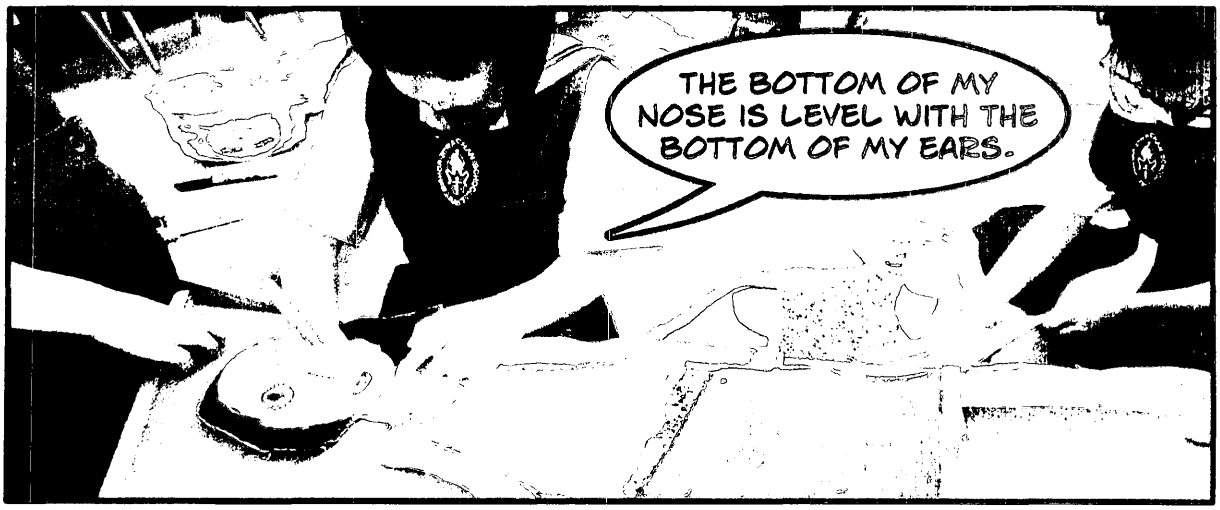


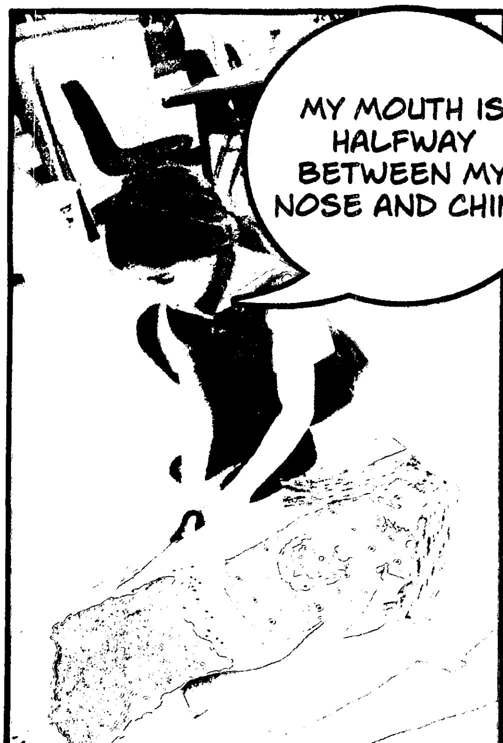
We looked at adults

We measured grade ones

At Home We measured Babies


We painted ourselves
THE TOP OF MY HEAD TO MY HAIR IS 1/4 OF MY WHOLE FACE.
We painted our heads And bodies very well
MY EYES ARE HALF WAY, FROM MY HAIR TO MY EYES IS A QUARTER.
Our Faces too
FROM MY NOSE TO MY CHIN IS A QUARTER OF MY HEAD ..





Our Face Features Are Fractions

Oops! We look a bit thin!

We should have measured width

Chapter Six: Proportionality and People

After celebrating a child's birthday one day, the children began considering their bodies and comparing themselves with each other in terms of height and age wondering about a correlation between the two variables. They wanted to compare age in years and months with height and were gratified when there was in fact a distinct correlation with the youngest girls being shorter than the older members of the class. They accomplished this hypothesis testing by physically lining up, according to age, eyeballing heights, and consulting the birthday list.

The hypothesis that younger children were shorter was discussed by the girls during morning meeting and led to a further inquiry: a desire to apply their hypothesis further and compare measurement of children's heights with that of adults. They decided they would use a different method than eyeballing heights: they planned to use paper cards of the same length as their non-standard unit of measure and for people to lie on the floor to aid the process. There was a cacophony of noise as everyone began to measure:

Erica: How many cards do we need to measure Victoria? She's the tallest in the class.

Victoria: Everybody who guesses 9 cards tall is right.

Maya: No, it's nine and a half.

Kiran: No I think its closer to 9, not to a half.

Even when using non-standard measure, there were ways to reach greater accuracy and consistency, such as keeping the card line straight and not overlapping the units. Elizabeth suddenly asked, "Is my head one in a half? My whole body is 8."

When they are born, babies' heads are extremely large in proportion to the rest of their body. As a child grows up, the head becomes smaller in proportion to the rest of the body. The head of very young children makes up about a quarter of their total height. By the time they are six or seven, the head is about a sixth of their height, and a seventh by adulthood. This progression from a quarter to a sixth to a seventh creates an interesting example of ordering fractions and ratios from larger to smaller as the proportion decreases.

Head to body proportions

Karen helped Elizabeth and they took turns measuring not only their bodies but their heads. I, meanwhile, was thinking about proportions of the head to the body to extend their thinking and wondering about the expression, "one in a half " which I inferred could mean "one and a half." Elizabeth realized her head was longer than one card but when she used two cards, even though one went above her head she maintained that her answer was "one in a half". Did she mean two less a half? In reality it was one and a half. The teacher invited everyone to measure not only heights but also their heads and represent the information with pictures, numbers and words. As their recordings were shared, terminology was discussed and used more precisely in this meaningful context.

The data was collected and arranged into three columns to find generalizations that we could make about head to body proportion. The three columns reflected this data:

A few measured 7 cards long and had a head length of 1 and a bit, a ratio of less than 1:6.

A few measured 8 cards long and had a head length of 1 and a third, a ratio of more than 1:6.

Most girls measured 9 cards and had a head length of one and a half, a ratio of exactly 1:6.

So the ratio of head to body was mostly 1: 6. The class then made predictions about whether what we discovered about grade one girls was applicable to adults and, at their suggestion, babies. For adults they measured the principal, vice-principal, kindergarten and grade one teachers, as well as the office administrators.

Variations in head to body ratio from childhood to adulthood

After all the measuring took place the children gathered with their results.

Teacher: Were grown–ups longer or shorter than children

Karen: Longer.

Teacher: Grade one has a head to body ratio of 1:6. One head would fit six times into the length of your body. What about grown-ups?

Girls: Longer whole bodies and longer heads. Their heads fit seven times into their bodies, they fit more heads in. (They do fit more heads but that means their head to body ratio is smaller: 1/7 is smaller than 1/6.)

Teacher: We found the proportion was 1:7 so their heads are a seventh of their bodies, what are yours?

Karen: They are a sixth of our bodies.

Teacher: Which is bigger a sixth or a seventh?

Karen: A seventh.

Children when initially encountering fractions, often have a common conception that a seventh is bigger then a sixth, as they hear the larger numeral rather than see the fraction notation which is not introduced until a later grade. The vocabulary is confusing: six and one sixth are very similar. Children do not fully understand the fraction notation. Would this provide a good opportunity to reveal the magnitude of fractions, ratios and proportion through project work?

Elizabeth: Babies are smaller, so less than 9 cards in height, I don't know about their heads. I think they're small too.

Teacher: So does everyone agree that grown ups have larger head to body proportion? 1/7 is larger than 1/6?

Class:Yes.

Teacher:(In fact they do not, how can I help them reason this out?)

O.K., we will measure your parents as well as any babies in your home.

Victoria: Can you do cats, too?

Teacher. Maybe, next week. This week we're doing humans. So far adults Can fit more heads inside their body length, 1:7. You can only fit 6 in yours. What do you predict for babies? Can they fit more heads in their body length?

Girls: Yes, they have tiny heads.

Teacher: Do they have bigger heads compared to their bodes?

Girls: No.

I was wondering whether this inquiry was too difficult for the girls to comprehend.

Research on proportion and ratios

I decided to look at the research on proportion and ratios. Baroody says, "Although ratios are not introduced formally until the intermediate grades, informal exploration of ratios can begin at the primary level....symbolic instruction can be linked to children's informal knowledge" (1998, p.12-9). He describes ratios as multiplicative relationships that can be introduced gradually through discussing many situations in which they occur naturally. If I introduced ratios purposefully as a tool for solving this comparison problem and provided concrete experiences this could be an important first step in learning about ratios. Baroody (1998) actually gives the example of drawing well-proportioned faces and bodies so our emergent practices happened upon sophisticated mathematical thinking and concept development appropriate for grade one. Proportions have many authentic applications in later mathematical development, including unit pricing, gas mileage, speed, chemical composition and exchange rates (Heller, Ahlgren, Post, Behr, & Lesh, 1989). It is an important foundation for later mathematical study, especially geometry and algebra. Piaget and Inhelder (1958) argued that preadolescent children are not capable of proportional reasoning. However more recent research in constructivist classrooms suggests that elementary children can indeed reveal this relatively abstract reasoning but not quickly or easily (Noetling, 1980a, 1980b.).

Revising theories about babies' head-to-body ratio

The girls went home and measured their parents and baby siblings and returned triumphantly with their card measurements. At morning meeting parent measurements were shared and confirmed our ratio of 1:7 for adults head to body proportion, which seemed to satisfy the class. The findings on babies' heads were not at all what they expected. The ratio for siblings under a year old was 1:4 and the girls realized that they had been mistaken.

Jadzia: My baby's head is huge! She only fits four heads in her body. A quarter of her body is head, I thought her head would be smaller.

Elle Sophie: My baby Oscar has a huge head, that's why he is so smart and is learning so much.

Claire: It's like the baby budgies, they had a huge head too when they First hatched and couldn't hold it up, just like babies. They had to grow into their heads.

Paige: I don't have a baby sister so I measured my baby doll and she had the same proportion of head to body. One to four.

Teacher: This was not what you predicted. What do you think happens as you grow up from a baby to 6–years-old to an adult?

Laurelle: I think we were wrong about adults having a bigger head to body proportion, our heads get smaller compared to our bodies as we

> grow…its like our teeth. Now we are getting our second teeth they are too big for our head but our heads will grow and our teeth will look the right size, kind of smaller than now. That means an adult's head to body (ratio) is smaller than our head to body (ratio). Fractions are not like numbers… 7 is bigger than 6 but 1/7 is smaller than 1/6.

After investigating at home the children found out the ratio for siblings under a year old was 1:4 and the girls realized that they had been mistaken. My hypothesis is that the difference between 1/6 and 1/7 was not great enough for them to reason out but measuring babies made the ratio more obvious.

Proportions of facial features

I thought representing their findings with paintings of their whole bodies might help consolidate developing ideas: " Let's draw and paint our bodies using what we have learned about proportion and we will see it more clearly." The children collected materials to use to draw a body in proportion for a six-year-old, i.e. with a ratio of 1:6. They drew a head on one piece of paper, cut it out, and then measured out six paper heads long for their bodies. Then they glued their paper head on the first mark and drew a body in proportion to the head. As the children began to add their features, eyes, nose etc., they began to ask questions about proportions of the human head and face.

Simona:	I drew my eyes half way down my head.
Elle Sophie:	I think they should be higher up, maybe a third down the face.
Teacher:	Let's all stop and think about our faces. Could we measure

distances between the top of the head and eyes and then compare that with the length of your whole head? We could measure distances from the top of your head to the bottom of your nose or the bottom of your chin to the top of your lip using unifix cubes?

Laurelle: Let's measure our whole head first with partners.

To consolidate this developing understanding of proportion I gave everyone time to measure their heads and then the relative length of their nose, mouth, eyes etc., in relation to their heads. In essence they compared the relative distance of each feature to the total span of a face. Again the results were collected and compared in the whole group to make generalizations.

The children found out that the proportions of the head can be divided horizontally into four quarters. The first quarter measures from the top of the head down to the hairline. The second quarter measures from the hairline down to the eyes in the middle of the head. Simona was correct in her positioning of the eyes half way. The third quarter contains most of the features. At the top of this section the eyes are usually level with the ears and at the bottom the nose is roughly level with the ear lobes. The final quarter stretches from the base of the nose to the chin with the mouth positioned just above the halfway mark.

The girls continued measuring and painting their heads and bodies to resemble themselves and to put together a collective representation of all the girls in grade one.

The finished paintings revealed important proportions that had not been discussed... width! Many of the portraits were exceptionally thin. We then realized width was an attribute that could have been measured and compared for overall proportions.

Analysis

The girls exceeded expectations by their informal exploration of proportions. Consideration of fractions, ratios and proportions were all difficult and challenging concepts with which to grapple. Mason (in Kaput, Carraher, & Blanton, 2008, p. 57) advocates making use of children's powers to produce algebraic thinking claiming they are not well- tapped in elementary school. The researchers lament,

> The rush to achieve certain topic coverage and procedural skill development and, even more constraining, the low expectations regarding young students' abilities to engage productively in mathematical activities that we would describe as algebraic. (Kaput, Carraher, &Blanton, 2008, p. 2)

This effort to understand body and facial proportions shows otherwise.

Enrichment of numeracy expectations

This investigation into fractions, proportions and ratios was definitely complex and more demanding than the expectations outlined in the curriculum

Algebraic thinking: Ratio, fractions and proportions. There was a combination of the numeracy, algebra and measurement strands in the proportion work. Numeracy was involved in counting measuring units, looking at ranges of numbers to make generalizations and proportions. Reasoning about ratios led to revision of initial predictions through multiple experiences with proportion.

Fractional understanding of one sixth and one seventh. The children went beyond the curriculum investigating not only the stipulated halves and quarters but also sixths and sevenths. I hypothesized that one sixth and one seventh are not such common fractions in a child's environment as halves and quarters. When I asked, "Which is bigger a sixth or a seventh?" the children were thinking of their knowledge of whole numbers, that seven is bigger then six, and it took more experiences measuring babies' head to body ratio before they realized one seventh is actually smaller then one sixth, as noted when Laurelle compares head to body ratio to our teeth to head ratio as we age, applying her discovery to another part of the body. This was a very sophisticated and complex experience with fractions.

Measuring and spatial awareness. Looking at how many cards long each head/body was and how many unifix bricks for the facial features met the expectations in terms of selecting appropriate units of measurement. The children knew to select the smaller unit, the unifix bricks, for the smaller facial measures and the larger unit, the cards, to measure bodies. The breadth of measurement opportunities the children undertook enriched their mathematical experience. Not only did they measure head to body ratio in children but in, adults as well. They then repeated this measuring at home on parents and siblings, including babies. Measurement of facial feature ratios was an extrapolation and reinforcement of their body ratio work. Measuring to work out generalizations about body and facial features is much more sophisticated and required the collection of everyones' data and analysis of the results.

Enrichment of the process expectations

Enrichment was facilitated by use of all the mathematical processes.

Problem solving, reasoning and reflecting. Multiple experiences, considering ratios and fractions led to the reasoning and revision of generalizations. Their initial ratio for a child was solid but at first they did not grasp the decreasing ratio from babies to child to adult. It necessitated the inclusion of babies for them to be able to reason out the differences over time. The documentation showed this:

Teacher: So does everyone agree that grown ups have larger head to body proportion? 1/7 is larger than 1/6?

Class: Yes.

This initial reasoning was revised after more experiences measuring babies at home, and finding out that a baby's big head fitted fewer times into a small baby body, e.g. a quarter of a baby's body is head. They commented on the progression from a baby having a head to body ratio of 1:4 to themselves at 1:6 and adults at 1:7, and realizing that heads looked smaller in relation to bodies as we get older and that 1/7 was smaller than 1/6.They were also making applications of this reasoning to their personal experiences by talking about teeth and budgies' heads.

When the children painted a representation of themselves they were able to see that an important proportion had been omitted, the width of bodies in comparison to height. Although they carefully measured out head to body ratio and face fractions, their representations were incredibly thin and lacked width. They were able to critique their work and reflect on a way to improve it.

Selecting a variety of tools. The children found the appropriate measuring tool, cards or unifix bricks according to the attribute they were measuring and recorded their findings. Using real people of different ages to measure helped them visualize ratio in a concrete way and their painted representations further developed their understanding of proportions.

Connecting. The children were able to make complex connections between head to body proportions in people and budgies. In relating mathematical ideas to situations drawn from everyday context they also included the connection of teeth to head ratio, an important part of a six-year-old's life as they loose their baby teeth and their second teeth appear.

Representing and communicating. The children looked at a range of numbers to generalize about baby, child and adult body proportion and this goes far beyond the expectations. Children are usually asked to consider discrete numbers and compare them, not consider ranges of measurement numbers and compare real data they have collected themselves. The three columns reflected this data collection:

> A few measured 7 cards long and had a head length of 1 and a bit, a ratio of less than 1:6.
>
> A few measured 8 cards long and had a head length of 1 and a third, a ratio of more than 1:6.
>
> Most girls measured 9 cards and had a head length of one and a half, a ratio of exactly 1:6.

Making generalizations from ranges of data is a complex mathematical procedure for a grade one class. They were able to articulate that the most common ratio of head to body in grade one was 1: 6. Interpreting data in this way seems very sophisticated.

Conclusion

As a teacher with many years experience I would never have planned to explore ratio and proportionality, thinking these concepts much too difficult for grade one. However as the opportunity emerged from the children's informal hypothesis-making about two variables, age and height, I felt I had to honour their interest. This exploration took up a great deal of time and at times I was unsure if they would grasp the concepts and extend their partial understandings or if I was wasting their time and confusing them. Their enthusiasm and persistence and enjoyment in the activities convinced me otherwise. Also, my role as a researcher along with the children led me to read Barrody's work on young children investigating body proportions and this relieved my misgivings (1998).

The satisfaction and aesthetics were again important for portraying realistic body representations or self– portraits. Even though the children were dissatisfied with the width of their bodies, they enjoyed their mistake and the skinny appearance of their portraits. They were satisfied with the exploration of variables about themselves and members of their community outside the classroom and the opportunity to share their explorations at home. My image of the child as capable was certainly present but I find whenever I have doubts about a project their sophistication of learning continues to amaze me.

Chapter Seven: Emergent Curriculum Enrichment in Mathematics

The goal of this research was to gain insight into emergent, Reggio-inspired mathematical learning in a grade one class and to address the gap in research on emergent mathematics. I interpreted the principles of the Reggio Emilia experience with reference to the Ontario Mathematics Curriculum in numeracy and process skills for grade one in order to explore if two competing visions could be addressed simultaneously. The question was could I practice the Reggio-inspired processes and address the Ontario Mathematics Curriculum expectations and processes with emergent mathematical practices?

Throughout the first term in grade one the children were engaged in a project to explore their identity and this project connected the disciplines and multiple strands of the mathematics curriculum. The project work and emergent curriculum enabled differentiation of strategies to solve problems so children's intuitive understanding of numeracy was facilitated. The work on enriching numeracy was difficult to isolate from the integrated nature of the projects. However some factors emerged that contributed to an emergent curriculum that enriched expectations in numeracy.

This grade one classroom offered examples of how mathematical understanding is constructed in a Reggio-inspired school. I argue that *not* focusing on the list of expectations in numeracy actually extended complex and sophisticated experiences of these expectations. Mason (2008) talks about enabling all children to use their mathematical powers,

> The issue is not whether these powers exist, but how to make use of them by not obstructing or obscuring the issue, not trying to do the work for them so that they park their powers at the classroom door. Using your own powers is motivating; using your own powers creatively is exhilarating. (2008, p. 89)

The classroom culture was replete with mathematics teaching and learning. Gandini states that " ideas for projects originate in the continuum of the experience of children and teachers as they construct knowledge together following the inquisitive minds of children" (2004, p.23). Mathematics was integrated into the curriculum and life of the grade one children. With a Reggio-inspired experience the mathematics became engaging, joyful, social, complex, satisfying, inquiry–based and authentic.

Summary of Major Findings for Question One

Question One was, what are some of the ways it might be possible to enrich the numeracy expectations for grade one with an emergent curriculum so that student work is more extensive, complex and sophisticated? I will now describe eight key findings.

Prior knowledge and theories

In each of the chapters it was evident that the children came with much prior knowledge about mathematics and for the numeracy expectations to be enriched I needed to know which understandings or conceptions were present in the class, scaffold from there, allow for differentiation with open ended-problems, and facilitate oral communication in mathematics. The girls' illustrations of their theories about mathematics displayed knowledge of most of the expectations and advanced understanding in numeracy beyond the listed curriculum expectations displaying ordinal, cardinal and nominal number understanding.

Regarding their theories about mathematics, teacher scaffolding was necessary to make implicit knowledge of math explicit, through careful questioning, and to support children's theory generation and illustration. Collaborative teacher inquiry into their illustrations helped reveal their understanding about ordinal, nominal and cardinal numbers and quantity. In chapter four the children had an intrinsic desire to quantify the extent of the scatter of the chaff and were already conversant with linear measuring, first employing the string to measure the perimeter of the chaff scatter. They were familiar with non-standard units, using string and cards to measure. In chapter five the children were keen to share their knowledge of their personal addresses in general and their house numbers in particular. Spatial placement of the numberline required a plan of one street that was then revised and arranged into three streets. This experience enabled the children to devise problems, such as, "How many houses should we each make?" and make explicit their differing solutions and strategies. In chapter six, it was important for conceptions about which was a larger proportion, 1/6 or 1/7, to be brought out by teacher-facilitated discussion and questioning. Their prior knowledge of measuring and fractions was made explicit in considering the body and face proportions. The conceptual layers of the problems helped develop deductive reasoning, communication, thinking and meta-cognition.

Each theory of how to solve a problem was honoured and discussed. Again I facilitated the discussion of different problem solving strategies helping the children to articulate their thinking. The role of the teacher is important for as Mason says:

> This is not a recipe for hands-off discovery learning. On the contrary, it is extremely demanding on teachers because they have to be aware not only of their learner's current state, but of appropriate potential mathematical and socio-cultural developments. They do this through enhancing their own awareness. (1998, p.57)

The Reggio experience acknowledges that children have intellectual capabilities and prior knowledge to bring to investigations for theory generation and communication. No matter how magical or charming a child's theory, I have noticed there is always some element of truth or fact embedded in the theory, as with their theories about math.

Multi-strand investigations

In The Ontario Mathematics Curriculum (Ministry of Education, 2005) there are five strands, each with expectations: number sense and numeration, geometry and spatial sense, measurement, data-management and probability, algebra and patterning. However during the course of this research, chapters 3-6 demonstrate the difficulty of separating the strands in meaningful problem solving. In chapter 3, Theories Pertaining to Mathematics, data management and number sense integrate in the sorting and categorizing and interpreting of theories. In chapter 4, Skirting the Chaff with Math, measurement and numeracy integrate. In chapter 5, Number Neighbourhoods, spatial sense and numeracy combine and finally in chapter 6 numeracy and measurement and algebra are represented in the problems solved. It would seem that the experiences were enriched by combining strands rather than isolating them. Mathematics in real life.is essentially about relationships among the different strands.

The Reggio experience also sees learning as connected to a society, a culture, and to real life as an inter-related whole, a project, not discrete disciplines or strands within a discipline. Authentic learning is about relationships among children, parents and teachers and is enriched by the intertwining of knowledge.

Muti-disciplinary investigations

All the experiences emerged from the project on identity as the children went from investigating budgie identity and the feeding characteristics of parakeets (the science of living things), to investigating materials and their properties, to sewing and securing a skirt to a bird-cage. The children discovered how identity can change, and investigated many facets of their own personal identity as well as that of inanimate objects such as elastic in the making of the skirt. In considering characteristics of identity social studies was evident in their consideration of homes, neighbourhoods and the historical school community. Literacy was integrated as they extended their production of self-made math resources and composed a counting book based on the school archives, affording a historical perspective. Art and aesthetics were also an integral part of the identity project as they painted life–sized portraits, created a number poster or represented the parakeets in acrylic and watercolour.

The interconnections made by the children among the subject disciplines, and the relationships and reciprocity among the differing academic domains led to a more enriched experience and deeper learning. Emotional response to learning is also an integral part of the multi-disciplinary project. To enable enriched or deep learning to occur, as the Reggio experience acknowledges, children need to have interest, passion, perseverance, joy and satisfaction to motivate and sustain their learning.

Further problems and actions

The nature of the problems encountered in chapters 3-6 led to student work becoming more extensive, complex and sophisticated than anticipated in the expectations. One facet of these problems was that they included multiple steps and the solving of one problem naturally flowed into the posing of another. In chapter three, theory generation enabled the problem of sorting and categorizing ideas and consideration of similarities and differences. In chapter four, measuring area led to investigation of the best measurement unit to use, rounding up and down of fractional numbers, the differentiation between linear and area measurement as well as the relationship between perimeter and area. In chapter five, multi-steps were evident as the children talked about cardinal numbers of homes and ordered themselves kinesthetically.

There were different steps in ordering and assembling a numberline, in terms of quantity and ordinality, and also solving the problem of spatial arrangement and space needed to reassemble the numberline. These initial experiences enabled the formulation of the problems of how many houses each child should make so the experience was rich in multi-step problems. In chapter 6 a hypothesis about height and age led to solving problems regarding comparisons of multiple variables. Measuring heights led to the problem of what proportion heads were to bodies, how this ratio changed with age, what fractions were present in facial features and how to recreate a proportional portrait.

Invented not procedural solutions

Throughout the identity project there was opportunity for differentiation to accommodate the differing prior knowledge of the children and their individual meaning making processes. The collaborative nature of the class meetings and discussions in combination with the opportunity for risk taking and individual invention of solutions led to a complex and sophisticated development in their mathematical understandings. Theory revision and reflection were enabled and from an instructional perspective,

> The variability of children's knowledge is not a drawback but an asset because the co-existence of discrepant ideas creates the opportunity for cognitive progress. (Sophian as cited in Copley, 1999, p.11)

In chapter three theories of math revealed a variety of individual, invented, unique mathematical illustrations and dictated theories shared among the whole class. In chapter four measurement of area with different, non-standard units and investigation of perimeter allowed personal experimentation and investigation. Further benefits to learning occurred with the whole group discussion of findings, leading to the collaborative generalization that the same perimeter could contain varying areas. In chapter five, a division problem was solved when Laurelle wrote out the decades on her sheet 10, 20, 30 up to 100 and reasoned "10 lots of 10 is 100, that means 10 girls would make 10 houses so then 20 girls would make 5 houses each." This is a very sophisticated deductive reasoning example involving an elementary understanding of multiplication

"10 lots of 10 is 100". Victoria solved the same problem with her invented strategy of repeated addition, which is a developing understanding of multiplication and this shows a complex method of solving the problem. Division by sharing equal amounts was another invented procedure. In each of these three strategies the teacher did not teach a procedure but encouraged real problem solving.

Multiple solutions and strategies

Multiple solutions and strategies were evident in the emergent problems that resulted from the identity project. The mathematical problems in each of the chapters allowed for a great variety of ideas, thinking and mathematical strategies to be used. This increased the complexity and sophistication of the children's work. A great breadth of problem solving strategies was employed in the four chapters. More specifically in chapter three there was evidence of using symbols, thinking tools such as ten frames and t-charts and pictures and words to explain mathematical thinking. In chapter four, trial and error with the string and perimeter was employed as a strategy. In chapter five acting out the order of their house numbers preceded physical ordering of numbers and manipulation of algorithms, counting with multiples and doubling and halving of numbers. In chapter six, the sophisticated understanding that a baby's head was larger in proportion to its body than an adult developed from initial guessing through a series of different experiences so opportunity to revise and reflect was afforded. Flexibility in problem solving was a feature of all the projects. Open-ended problem solving with multiple solutions and strategies led to the children recursively constructing meaning based on their experiences. Initially, young children often construct a partial or approximate understanding of the world around them but with repeated or recursive building of similar or related experiences their understanding often becomes more extensively developed and allows them to make connections with prior learning. I noticed throughout the course of this research that different strategies that seemed circuitous, e.g. measuring the scatter of chaff, led to rich mathematical discoveries.

Satisfaction and aesthetics

Satisfaction and aesthetics played an important role in the building of children's mathematical understanding. Throughout these mathematical experiences the children saw mathematics as useful and necessary for solving problems. There was satisfaction expressed by the children in theory construction as they eagerly illustrated their thinking. The children were impatient to share their theories and the carefully illustrated "thinking pictures" related to each theory. There was validation of their ideas by the whole group as they listened attentively to each other and made suggestions as to the column in which a theory should be placed. There was an aesthetic appreciation for each other's illustrations and how to draw mathematical thinking, from the selection of symbols and also the wording of their theories to convey meaning and beauty. The children made appreciative and thoughtful comments about each other's work.

When the children were solving the problem of the scatter of chaff they expressed a desire to include their acrylic and watercolour budgies in the graphic novel. They noted the work and effort put into the pleasing representations of parakeets and claimed the birds were part of the project and should be included. They also appreciated each other's artwork verbally. The aesthetic appeal of the number neighbourhood emanated from their careful choice of paper and differing designs for each house. They were satisfied by their own ability to produce many of the math tools usually produced commercially such as paper numberlines, counting books, number charts, math alphabet books and posters about math. This math manipulative "factory", as they described it, was empowering and satisfying for them and made the children more invested in using their own creations and inventions. As in the Reggio experience, using the hundred languages of learning or different ways of representing their mathematical thinking afforded an aesthetic element with their self-portraits in paint, their clay numeral tablets, school number poster and photographic counting books.

All process expectations

Each of the experiences rendered the children's work sophisticated and complex by engaging the children in all seven of the mathematical processes of problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing and communicating. The mathematical processes support the acquisition and use of mathematical knowledge and skills and encourage thinking, communication and application. The content of the curriculum became extended, complex and sophisticated when all the mathematical processes were applied through the project work.

Summary of Major Findings for Question Two

Question Two was what are some of the elements that contribute to an emergent curriculum that enriches thinking in Numeracy?

Teacher facilitation

The role of the teacher is essential in emergent mathematics to facilitate an intuitive grasp of concepts and enrich experiences. What are the implications for the role of the teacher in this research? I was carefully observing the children and their ideas, gestures, emotional reponses, interest and actions. When they were discovering the relationship between perimeter and area, I was observing and had to be ready to catch the moment through photographs. Also I was actively listening to help the children communicate their thinking and make their implicit understandings explicit through opportunities to ask questions and discuss at morning meetings. When the children were talking about their homes I was ready to provide a provocation in the form of a numberline to guide but not direct their interests.

I was learning along with the children and prepared to investigate areas outside my comfort zone such as ratios and proportions and original thinking about math by mathematicians. Nurturing the children's developing understanding is very important, to take their theories seriously about math and where it comes from and encourage risk-taking and confidence. I was a partner and researcher along with the children following their lead and providing what they needed to solve problems.

Throughout the emergent math experiences I did not limit the children's learning by restricting their experiences to those listed in the curriculum. The math theory illustrations showed the children already knew much of the content of the curriculum. Therefore direct instruction of the expectations would have had little meaning and possibly have been boring and lacking challenge. The cognitive load required by the expectations seemed low compared to the capabilities of this class. At the same time because of the sophistication and complexity of some of the mathematical concepts, it was necessary for me to keep abreast of recent research in constructivist settings as regards age appropriateness. I researched the work on proportionality as the ideas emerged to ensure the challenge was within their grasp.

Collaborative learning

Following a Reggio-inspired approach enabled the children to debate, discuss and problem solve collaboratively as well as individually. The number neighbourhood began as a collaborative game but led to individual problem solving of how many houses each child should make. In a cyclical fashion, individual solutions were then shared and commented upon to make others aware of different strategies and to enable opportunities to make their learning explicit. Skirting the Chaff with Math started with a group problem discussion at morning meeting and led to whole class trial and error testing of the relationship between perimeter and area. Individuals continued the construction of a skirt for the birdcage but reported back problems with materials and fastening. Individual theory building and illustrations (chap 3) were again taken up by the whole group to consider the perspectives of the whole class and enrich everyone's learning by collaboration. Finally the Proportionality and People chapter started with a whole class hypothesis and testing and cycled through a measurement by pairs exploration to a whole group sorting of measurements to make generalizations. The benefit of the collective was apparent for the advancement of everyone's understandings.

Grounded in the environment and authentic

Emergent learning and problem solving benefited from problems that were grounded in the classroom environment of the children. They were authentic problems and experiences that helped reveal, not hide, the mathematics of daily life. The project approach of the mathematics was enriching and enabled the emergence of mathematics not listed in the expectations such as the investigation into perimeter and area and the investigation of fractions, proportionality and ratios. There was a higher cognitive load and demand when the children were the protagonists in their own learning, as for example, when they decided to divide up how many houses each child should make. The investigation of algorithms went beyond what was expected, for example solving addition with subtraction and working with addition, subtraction, division and multiplication of 2-digit numbers in the number neighbourhood chapter.

Evidence of Reggio-inspired processes in this research

As the team teacher, pedagogista and I read the transcripts of conversations we marked symbols on similar numeracy expectations, process skills or Reggio principles. For example in all chapters we saw evidence of all the Reggio principles; the image of the child as a capable theory builder, problem solver, able to hypothesize and revise theories was evident. We noticed reciprocity and relationships in their meeting transcripts and investigations. The role of the teacher, listening, observing, questioning, documenting and providing provocations and facilitation occurred. Evidence of different languages of learning was visible in describing and illustrating theories, creating manipulatives in collage, symbols, clay, photographs and print. Documentation enabled sorting and categorization, redefining problems, extending problems, acting as a mirror to prior thinking and a lens on the learning process.

Progettazione or flexible planning was important as the teachers discussed how to make the children's implicit understanding explicit and facilitated complex sorting of ideas, supported the childrens' interests with extensions, and created opportunities to change or clarify thinking. Documentation panels and graphic novels acknowledged the value of the children's work and the value of communicating with others about the children's experiences. Documentation panels for chapters three and four highlighted the development of the identity project and the changing ideas of the students as they investigated deeper and thought critically. The panels had multiple audiences: the children, teachers, parents, visiting educators and the community. Also the documentation panels had multiple purposes such as to propel further learning, record a significant moment of learning, display ideas graphically, and inform parents and the community about what and how the children learn mathematics. The sophistication and complexity of the mathematical learning and gradual improvement in making implicit ideas explicit was displayed in the documentation format. The documentation helped the direction and objectives develop and so curriculum emerged from the children's curiosities and interests. The format of the graphic novel was particularly popular with the parents who appreciated the transparency of their child's math learning and viewed the graphic novels as a window into their world of learning.

The image of the child, as intelligent, capable and full of potential was apparent in the research. Each child was seen as intellectually capable of constructing her own mathematical learning, composing and solving problems without specified procedure, with the teacher serving as a facilitator. The children were seen as and encouraged to be mathematicians, just as they are citizens in their own right— not trainee citizens— so they are mathematicians by their experiences in mathematics. Pre-conceived limits were not placed on children's mathematical capabilities, but rather the children were challenged to deal with abstract concepts and symbols and a high cognitive load as these arose within the project. Providing meaningful learning contexts for the children led to high affect, invited creative work, and fused the rational, the affective and the generative.

The role of the teacher, as an observer, listener, learner, nurturer, partner, and researcher was evident in the chapters and experiences described. The image of the child shaped the role of the researcher in learning parallel to the child. I shared in the interest, excitement and joy of learning, scaffolding experiences so that children reached a deeper mathematical understanding. As a teacher I was a facilitator of reciprocity and relationships and encouraged the children to be active participants in their own mathematical learning.

> Student powers are indeed the same ones, in different contexts and in nascent forms, perhaps, that are used in serious mathematics learning. (Mason, 2008, p.2)

The education philosophy of Reggio Emelia is based on reciprocity, and relationships among children, teachers and parents as well as within the community. The mathematical experiences were socially and culturally situated with everyone working collaboratively and co-operatively. The well-being of children, parents and community depended on the well-being of all the protagonists. There was a recursive cycle of reciprocity, exchange and dialogue. Conflict of ideas, discussion and negotiation were common components of this reciprocity. Children relied on one another's competencies and children viewed their school as a collaborative endeavor.

The work on emergent curriculum in the grade one class was one of flexible planning with a dynamic trajectory that could branch off in multiple directions. Preparation and organization of materials, space, thoughts, provocations and occasions for learning were all carefully considered in the emergence of the project identity. As Wien suggests,

> approaching standardized curriculum in other ways, of which emergent curriculum is one, *does* permit us to educate by building connections and studying the connectivity of living systems in a respectful, ethical way. (2008, p. 158)

Further Research

Further research that could use this study as a springboard would be continuing to investigate the implications of a Reggio-inspired pedagogy in grade two or three over time, with the same group or comparing the findings with a more traditional curriculum-based class. The whole inquiry/interest-based, integrated education versus traditional standards-based education could be researched.

Another focus of future research might be a comparative gender study exploring the similarities and differences between male and female learning in constructivist settings. Each year understanding of mathematics and children's thinking expands and thus expands how to teach mathematics more effectively. Experimenting with building on informal knowledge through an emergent curriculum, creating cognitive conflict, raising key questions and facilitating challenging problems could all contribute to the improvement of mathematics teaching.

Conclusion

This research argues that the Reggio-inspired principles and processes support an enriched and sophisticated emergent curriculum that expands numeracy accomplishments far beyond expectations. This research on emergent curriculum is exciting in outlining some of the possibilities in mathematical pedagogy that enrich mathematical thinking and are possible with a set of curriculum expectations. The documentation this research generated in graphic novel form was very popular with parents, educators, the children and community as an entertaining and transparent way to view enriched numeracy learning in action.

My own impressions of the research is that emergent curriculum fosters a much more complex and sophisticated learning of mathematics, specifically but not exclusively numeracy, because of the complexity of the pedagogy. Emergent curriculum seemed to boost confidence, encourage engagement and tenacity in explorations and act as a catalyst for deep and meaningful learning. Implicit understanding became gradually more explicit and as communication of ideas, generalizations, strategies and proofs emerged, the level of mathematics exceeded my most optimal expectations. As an educator I was continually surprised, perplexed and amazed by their intuitive understanding. Following the children's interests not only motivated and nurtured creativity but also inspired and empowered the children and teachers to learn. There was a reciprocal relationship between the playfulness of the emerging ideas, the complexity of the mathematical exploration, and the aesthetics and the satisfaction of the children. In my opinion there was a balance and interplay between the children being playful through the use of movement, dialogue, the hundred languages, the expression of intense emotion, and complex mathematical thinking. Similar to Wheat, Lysaker, and Benson's ideas (2010) of the role of children's play in literacy, this mathematics research enabled the children to infuse the academic environment with elements of their more playful social worlds. Children used play spontaneously, were active and imaginative, joyful, smiling and/or laughing throughout many of the mathematical investigations, as the graphic novels attest. The energy in these mathematics activities was not orchestrated or contrived but facilitated and supported so the children maintained their autonomy and enthusiasm. As a teacher facing a culture of accountability manifested in expectations and assessment, play and emergent curriculum played a welcome and critical role for me in facilitating sophisticated and complex mathematics learning. May we all be inspired by the work of this grade one class, their ideas as mathematicians, their adorned bird cage, their mathematical tool factory and their skinny self-portraits, all evoking learning replete with joy and the wonder of childhood.

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Appendix A Informed consent

Dear Parents

I am currently enrolled in the graduate program in the Faculty of Education at York University. As part of my M. Ed program I am writing a thesis on "Enriching the Mathematics expectations in numeracy through an emergent curriculum". I wish to invite your child to participate in this interesting numeracy research project. I will investigate whether the Reggio Emilia philosophy with its emergent curriculum is a catalyst for the development of children's understanding about numeracy. Do we exceed the expectations?

Conversations will be transcribed, so we can re-call what was said, sample work kept, and these materials will be developed into documentation panels to be shared with the children, parents and other's who come into our classroom. These panels may also be shared with other educators visiting the school, at conferences, workshops or in educational journals. I ask your permission to use your child's words, representations, writings, and image (still or video) in my thesis, and later in presentations to other educators, and possibly in articles submitted for publication to education journals.
The research will take place during regular daily math lessons from September 2011 to December 2011. Your child will be engaged in normal school activities and there are no known risks to your child's participation. The primary purpose for the project is to further my own understanding of learning processes and improve your child's mathematical understanding.

Confidentiality will be provided to the fullest extent possible by law. This research has been reviewed and approved by the Human Participants Review Commitee, York University's Ethics Review Board and conforms to the standards of the Canadian Tri-Council Research Ethics Guidelines.

Since the data (words, images, samples of work) is held in the public place of the classroom and with the educational community, the documentation panels will include children's photos and actual names. I ask your permission to use your child's first name only, to recognize and honour their work. If you prefer your child to remain anonymous in the thesis, you may choose a pseudonym. The data will be kept at school for two years The data will be securely stored in a locked school filing cabinet and destroyed after the retention period.

All research participation is voluntary, and you may withdraw permission at any time for your daughter's materials to be included in the thesis. A participant's decision to not participate will not influence the individual's relationship with the researcher or York University, now or in the future. If the decision is made to stop the minor's participation, all associated data collected will not be used for the thesis.

This research has been reviewed by the Ethics Review Committee, and approved for compliance on research ethics within the context of York Senate Policy on research ethics. You may address any ethical concerns to the Manager of Research Ethics, 5th Floor, York Research Tower (acollins@yorku.cs) or to my supervisor Professor Carol Anne Wien (cawien@edu.yorku.ca)

Sincerely,

Susan Hislop

I am not waiving any of my legal rights by signing this form

Images Yes/No

Drawings Yes/No

Words Yes/No

Video Yes/No

Own First Name Yes / No If No a pseudonym Yes/NO

Relationship to minor?

Susan Hislop	name of researcher and signature
contact: shislop@bss.on.ca Bishop Strachan School	address of researcher
416 425 8896	telephone number
September 2011	date

I have read the information provided and agree to my child's participation in this project

____ name of child

____ signature of parent or guardian

____ relationship to child

____ date

Dear ,

I wish to invite you to share your mathematical ideas and work with your teacher Ms. Hislop, your parents and other teachers. Please may I take your photo when you are working and some video of you talking and share your work in my paper, so I can learn more about mathematics and how to be a better teacher.

Sincerely

Ms. Hislop

Yes/ No

Date__________Signature______________