DEVELOPMENT OF A MULTI-PROJECTION APPROACH FOR GLOBAL WEB MAP VISUALIZATION

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Abstract

The popularity of web mapping services such as Google Maps and Microsoft Bing Maps is growing at a rapid rate. However, professional workers and scientific community experience several limitations while using these on-line mapping services. The first major problem is the limited global coverage. The polar regions are only partially shown on the map. The coverage cuts off at latitude of 85° north and south. The second problem is the systematic distortion that increases with latitude. This is most evident when comparing areas of continents. For example, in Google and Bing Maps Greenland appears to be larger than South America, whereas in reality Greenland is 8 times smaller. The third problem is the lack of mathematical rigour for the cartographic projections because the Earth is treated as sphere and not as an ellipsoid. Thus, a better worldwide web mapping system is needed for knowledgeable users and for those who are interested in polar regions. This thesis will present a multi-projection approach for global web map visualization. The multi-projection approach uses different projection types across the globe and for ranges of mapping detail levels. It also minimizes the cartographic distortions depending on map scale/level of detail, regardless of the region being viewed. Thus, for different latitudes, extent of geographic area and zoom-in level the projection (azimuthal, cylindrical or conical) which represent the region with minimal distortion is applied.

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1 Introduction

The popularity of web mapping services, such as Google Maps [1] and Microsoft Bing Maps [2] is growing at a rapid rate. These online mapping services were developed for the general public and are now widely used for a variety of different applications. They offer street maps, satellite imagery, and many additional layers and tools. However, professional workers and scientific community experience many limitations while using these mapping systems. There are three major problems with the popular web mapping services (PWMS) that need to be addressed. The first problem is limited global coverage. The Arctic and Antarctic regions are only partially shown on the map. The coverage cuts off at latitude of 85° north and south [3]. Therefore, Google and Bing maps cannot be used for R&D and other projects in these regions. The second problem is the presence of the systematic distortion that increases with latitude because of the nature of map projection used. Google and Bing Maps use a special variation of Mercator projection known as Web Mercator Projection (WMP). WMP covers the Earth starting from its origin at the equator, and creates a large systematic distortion toward the poles. This is most evident when comparing the areas of continents. For example, in Bing Maps Greenland appears to be larger than South America, whereas in reality Greenland is roughly eight times smaller. The third problem is the lack of mathematical rigour of WMP because the Earth is treated as sphere and not as an ellipsoid. WMP projects World Geodetic System 1984 (WGS84) geodetic latitude and longitude onto the mapping plane using spherical Mercator equations, resulting in loss of conformality and other fundamental properties of spherical/ellipsoidal Mercator map projection [4]. Therefore, the users who plan on using WMP quantitatively will experience additional discrepancies, such as different scale distortions compared to spherical or ellipsoidal Mercator.

While the general public may not recognize or understand map distortions, well-informed professional users are often concerned about these problems and must rely on unique, non-compatible and inefficient systems to mitigate the issues. Organizations, such as Canadian federal government departments that are mandated to provide consistent national map coverage are particularly vulnerable to this problem, because what works in one region of the country must work in another region too. Therefore, an improved worldwide web mapping system must be developed to present geographic information that is low distortion, math based, seamless, and multipurpose. This thesis presents a multi-projection approach to global web map visualization. The multi-projection approach uses different projections across the globe and for a range of detail levels. It also minimizes the distortions depending on map scale/level of detail, regardless of the region being viewed. Thus, for different latitudes, extent of geographic area and zoom-in level, the projection (azimuthal, cylindrical or conic) which represents the region with minimal distortion is applied.

1.1 Motivation

There is increasing worldwide demand for Web mapping and related applications. Professional users are interested in using Web maps for accurate measurements, inspection of northern regions and other scientific work. Yet, current popular web mapping services rely on WMP which has many limitations such as large systematic distortion near the poles and the fact that polar regions are not covered. It is clear that WMP does not satisfy the needs of professional users [5], [6], [17]. Thus, the challenge of current Web mapping technology is how to efficiently and effectively portray geographic information to all users in a clear, minimally distorted and understandable form no matter what location on Earth is being viewed. One possible solution is to use a 3D visualization environment [7], however this is usually computationally heavy and time consuming process. All other approaches rely on the use of map projections and therefore introduce map distortion. The fastest and most popular mapping systems rely on using pre-rendered, pre-projected tiles. However, currently there is no tile based mapping system available that could provide fast, interactive, complete coverage of the Earth with minimal distortion.

This study started as part of a research project titled *SSII-109: Improved Global Web Map Visualization* of the GEOmatics for Informed Decisions (GEOIDE) Network [8], with main participants being York University, University of Laval, University of Calgary, and government and industry partners such as ESRI, Microsoft, NRCan, and others [9]. The main objective of GEOIDE SSII-109 project was to develop and implement an improved worldwide web mapping system to store and present pre-rendered geographic information that is math based, multi-resolution, seamless, low distortion, and multipurpose. Specific objectives of the GEOIDE SSII-109 project included:

1. Provide consistent geographic data visualization across the globe
2. Allow high performance implementations
3. Permit implementations that facilitate data dissemination and sharing
4. Be based on the latest realization of the World Geodetic System 1984 (WGS84)

5. Allow broad implementation across a variety of Internet and geospatial technologies
6. Support visualization of both, raster and vector data.
7. Provide clear, concise and easily understood presentation to all users.

As part of a group effort the project's main and specific objectives were divided between York University, University of Laval, University of Calgary, and other industry and government partners. University of Laval was responsible for designing the most suitable Earth tessellation technique used for web mapping applications. University of Calgary was responsible for creating a software prototype for web map visualization using current Google and Bing maps API's. York University was responsible for researching and analyzing the most suitable map projections for the Earth that would allow minimizing map distortion.

1.2 Objectives of this Research

The main goal of this research is to determine the most appropriate map projection, or a combination of different map projections for a web map visualization system at different scales and levels of detail that would minimize map distortion and provide consistent global coverage. The following is a list of objectives that are designed to meet this goal:

1. Research, implement and test different map projections.
2. Analyze and quantify scale, area and angular distortions of map projections at different geographic latitude and region extent.

3. Select map projections that would be most appropriate candidates based on maximum allowable distortion specified by users while taking into consideration Bing/Google Maps capabilities and limitation.
4. Determine optimal tile size and shape to be used for map visualization system. The tiles are used for storage and retrieval of cartographic data. For example, the imagery at each zoom level can be broken into a set of rectangular map tiles (e.g., 256 x 256 pixel images). When the user moves to a new location or changes zoom level, the map visualization system determines which tiles are needed and retrieves them from the server.
5. Design a tile alignment technique that would permit combining multiple projected tiles together while maintaining minimum distortion.
6. Create a map visualization system prototype that is based on World Geodetic System 1984 (WGS84) to demonstrate the concepts.

1.3 Contributions of this Research

In this thesis I examine the major limitations of WMP, present a multi-projection approach for global web map visualization, and implement a proof-of-concept software prototype. The approach uses different projections for different map scale, latitude and levels of detail, so that any region of the Earth can be viewed seamlessly with minimal distortion. Specifically multi-projection system presents the following advantages:

1. Polar region coverage
2. Seamless coverage of geographic regions with minimal distortion
3. Use of ellipsoidal model

4. Capability of accurate measurements such as map and geodesic distances
5. Improved computational efficiency (pre-rendered tiles)

The proposed scheme provides new insights into visualization of the Earth. The areas of interest can be viewed with minimal distortion, almost equal to their actual representation. Also, the approach emphasizes computational accuracy by using WGS 84 ellipsoidal parameters, and allowing users to extract very accurate measurements of distances and areas.

1.4 Thesis Outline

This thesis is structured as follows:

Chapter 2 discusses general concepts of map projections and map distortions. The three main characteristics of map distortion namely scale, area and shape are explained here. The most common cylindrical, azimuthal and conic projections are examined in terms of their general properties, geographic preference and map distortion.

Chapter 3 focuses on the Web Mapping technology. This chapter discusses the history of interactive web maps, the basic framework of the current popular web mapping service Bing Maps, and alternative approaches for web map visualization of the Earth.

Chapter 4 discusses the concept of multi-projection system, a global web map visualization approach that relies on using a combination of different map projections to view the Earth. The chapter covers the main principles of the approach, describes the idea of individual projection levels, and provides quantitative analysis for selection of most appropriate map projections. Lastly, the principles of inverse projection are briefly discussed.

Chapter 5 describes the design and implementation of proof-of-concept software model developed in this research. The functionality of the software model is also described here.

Chapter 6 evaluates, analyzes and compares the proposed multi-projection approach to Bing Maps in terms of polar region coverage, systematic distortion, mathematical rigour of map projections, and visual cartographic distortion. The results of the analysis are summarised here.

Chapter 7 concludes this thesis. It summarizes the findings of research, discusses the limitations of multi-projection approach, and proposes potential areas of research for the future.

2 Map Projections

In this chapter the concepts of map projections and map distortions are discussed in detail. The three main characteristics of map distortion namely scale, area and shape are explained. The most common cylindrical, azimuthal and conic projections are examined in terms of their general properties, geographic preference and map distortion.

2.1 Introduction

The literature on the subject of map projections is very rich and dates back hundreds of years. Over the time, people have developed many different map projections to suit their specific needs. Most of the widely used projections date from 16th to 19th centuries, and many variations thereof have been developed during the 20th century [10]. The information on these map projections and their equations is easily available and can be found in countless papers, textbooks, online sources [4], [10], [11], [12], [13] and other publications. Therefore, selecting an appropriate map projection is usually not a difficult problem.

There are also multiple sources available that provide classification and selection guidelines for map projections [10], [13], [14], [15], [16]. The most systematic and practical selection guideline is provided by Snyder [10] because he suggests using a hierarchical tree to select the most appropriate map projection depending on the mapping purposes [10], [17], and [18]. In the selection process, Snyder considers map scale, region of the world, projection properties and other characteristics (e.g., special consideration, direction of distortion, etc.). The hierarchy can be converted to knowledge based systems for automatic projection selection and interactive decision making [18], [19], [20], [21]. For the purpose of this research, the selection guidelines are used to pick a number of suitable projections. Then the associated distortions are used to select the best candidates.

The surface of the Earth cannot be represented on the plane without distortion. All map projections distort the reference surface (geodetic reference ellipsoid) in a certain way. A user can select a suitable projection depending on the map scale, region of interest and special characteristics that he/she wants to preserve. One projection cannot satisfy all the requirements for all users. Some users are more interested in preserving the area while others are more interested in preserving the shape. Moreover, one projection cannot maintain minimal distortion in different geographic regions. Therefore, instead of relying on one projection (e.g., WMP) a combination of different projections should be considered in order to visualize the Earth at different scales and geographic regions with the minimal distortion.

2.2 General Concepts

2.2.1 Map Projections: General Concepts

A map projection is a systematic representation of all or part of the surface of a curved body, especially the Earth, on a plane [10]. This process results in distortions in area, scale, and shape. There is no perfect way of going from 3D to 2D, thus some map projections attempt to minimize certain distortion(s) at the expense of others, while other map projections aim to maintain a low level of distortion in all of the above properties simultaneously.

The Earth is not a true sphere but slightly flattened and contains mountains, valleys, and other physical features. Therefore, before the surface of the Earth with all its features can be mapped onto a plane it must be projected onto the ellipsoid, or for mathematical simplicity on a sphere. The chosen ellipsoid or sphere is used as a surface of reference (datum) for the mathematical reduction of geodetic and cartographic data [22]. A map projection establishes a mathematical relationship between a 3D curved surface of reference and 2D mapping plane. The Earth is best represented by an ellipsoid, however, generally the mapping equations of a reference ellipsoid can be more complicated than of a sphere. Therefore, on small scale (e.g., less than 1:50M) maps or in cases in which measurement accuracy is not important, the Earth is usually treated as a sphere. For medium (e.g., between 1:50M and 1:1M) and large scale (e.g., greater than 1:1M) maps, which require high accuracy, it is essential that the Earth be treated as an ellipsoid [4].

A sphere or an ellipsoid is a non-developable surface, which cannot be flattened without shrinking, breaking or stretching. A developable surface is the one that can be flattened onto a plane without a distortion. The three most common types of developable surfaces are plane, cone and cylinder, and their corresponding projections are called azimuthal, conic and cylindrical, respectively. However, not all map projections use a single developable surface. For example, conceptually, the Polyconic projection is based on multiple layered cones. Different projections have unique characteristics and serve different purposes. A number of classification guides exist for map projections [14], [15], [16], [18] which classify map projections according to their methods of derivations, developable surface, global properties, and others. As mentioned earlier, based on the developable surface, map projections can be classified into cylindrical, azimuthal and conic [14]. The developable surface determines the type of distortion pattern that is obtained [19]. Typically, based on the distortion patterns, geometry and other properties cylindrical, azimuthal and conic map projections are used for different regions of the world. For example: cylindrical projections such as Mercator, Plate Carree, and others are generally used for world maps and equatorial regions. Azimuthal projections, such as Azimuthal Stereographic and Azimuthal Equidistant are generally used for mapping polar regions and hemispheres. Conic projections, such as Lambert Conformal Conic, Lambert Equal Area, and others are generally used for mid-latitude regions with greatest East/West extent.

Projections created by geometrically projecting the image of network of parallels and meridians of a reference sphere or ellipsoid on any developable surface are called perspective projections [23]. Depending on the perspective point the projections can be classified as Stereographic, Gnomonic and Orthographic. For Stereographic projection the perspective point is located on the surface of the globe, diametrically opposite to the center point of the projection surface (e.g., if South Pole is the center of the map, the projection point is located at the North Pole). For Gnomonic projection the perspective point is located at the center of the globe. For Orthographic projection the perspective point is located at an infinite distance from the projection surface. However, not all projections are perspective. In a non-perspective projection the perspective center is moved from one location to another to achieve certain properties (e.g., equal area). Non-perspective projections can only be constructed mathematically.

Map projections can be classified in tangent or secant cases. The tangent case means that the projection developable surface touches the reference surface at a point (azimuthal projections) or one standard parallel (conic/cylindrical projections). The secant case means that the projection developable surface cuts through (intersects) the reference surface creating a standard circle (azimuthal projections) or two standard parallels (conic/cylindrical projections) where there is no map distortion. The projection case is usually chosen in a way that minimizes map distortion for a specific area. Map projections can also be classified in terms of their aspect as normal, transverse or oblique. In normal projection, the orientation of projection surface is parallel to the Earth's axis. In transverse projections, the orientation of projection surface is perpendicular to the Earth's spin axis. In oblique projections, the projection surface is oriented in any way that is non-parallel and non-perpendicular to the Earth's axis.

2.2.2 Map Distortions: General Concepts

The fundamental problem of any map projection is that in one way or another it distorts the true ground surface being mapped. The type of map distortion is directly related to the map projection used. Different map projections distort the true ground features in different ways. There are three main geometric characteristics of a map that can be distorted: scale, area and shape. No map projections achieve true scale (k = 1) in the entire projected area. For cylindrical and conic projections the scale is true along standard parallels. For azimuthal projections the scale is true at the point where the plane is tangent to the reference surface.

Different projections have been developed to preserve some but not all the main geometrical characteristics and can be classified based on their distortion characteristics into: equal-area, equidistant, conformal or aphylatic (neither conformal nor equal-area) [24]. Equal-area projections represent the area on the map correctly. A coin placed on the map covers identical area of the Earth as the same coin placed on any other part of the map. Equal-area maps distort scale and shape. Equidistant projections show true scale along every meridian (e.g., Plate Carree projection), or between one or two points and every other point on the map (e.g., Azimuthal Equidistant projection, Two-point Equidistant projection). Conformal projections maintain proper shape, because they maintain angles correctly. Conformality applies on infinitesimal basis, whereas equal-area map projections show areas correctly on a finite basis [10].

The main goal of map projection selection is concerned with reducing distortion [13], [19]. Selection should always lead to the map projection showing the least amount of distortion as compared with other projections suitable for the same application. The amount of distortion generated by a map projection depends not only on the projection characteristics but also on the location, the size, and the shape of the area to be mapped [19]. Therefore, there are numerous criteria that need to be considered when selecting an appropriate map projection. Snyder [10] recommends the selection process to focus on:

1. *Size of the region mapped:* World, hemisphere, continent, ocean or smaller region.
2. *Special property:* conformal, equal-area, equidistant.
3. *Directional extent of region:* predominant east-west, predominant north-south.

4. *General location of regions*: along equator, away from equator.

Consequently, map projection selection consists of choosing the projection for which the area of interest fits best in the projection's area with least distortion [19]. In general, when a user chooses a map projection, the main goal is to minimize map distortion and preserve some of the more important properties at the expense of others. For example: if preserving shape is important and the region of interest is near the equator, a cylindrical, tangent, transverse, and conformal projection such as Mercator can be selected, however the area and distance will be distorted.

2.2.3 Coordinate Systems and Distances

Any surface to be mapped can be described using the general equation [25]:

$$F(X, Y, Z) = 0, \tag{2.1}$$

where

$$\begin{aligned} X &= X(\phi, \lambda), \\ Y &= Y(\phi, \lambda), \\ Z &= Z(\phi, \lambda), \end{aligned} \tag{2.2}$$

are known as the parametric equations. X, Y, Z are Cartesian coordinates, and ϕ, λ are curvilinear coordinates on a particular surface. Parametric equations describe, in a mathematical way, certain curves on the surface (e.g., meridians and parallels) [25]. The two most commonly used reference surfaces for mapping are the sphere and the ellipsoid. The equation for spherical surface is given as [25]:

$$F(X,Y,Z) = X^2 + Y^2 + Z^2 - R^2 = 0. \tag{2.3}$$

The origin of the coordinate system is at the geometric centre of the sphere. The corresponding parametric equations for the sphere are given as [25]:

$$\begin{gathered} X = X(\phi,\lambda) = R\cos\phi\cos\lambda, \\ Y = Y(\phi,\lambda) = R\cos\phi\sin\lambda, \\ Z = Z(\phi,\lambda) = R\sin\phi, \end{gathered} \tag{2.4}$$

where R is the radius, and ϕ and λ are spherical latitude and longitude, respectively. The equation for ellipsoidal surface is given as [25]:

$$F(X,Y,Z) = X^2 + Y^2 + Z^2 - N^2 = 0. \tag{2.5}$$

The parametric equations for the biaxial ellipsoid are [25]:

$$\begin{gathered} X = X(\phi,\lambda) = N\cos\phi\cos\lambda, \\ Y = Y(\phi,\lambda) = N\cos\phi\sin\lambda, \\ Z = Z(\phi,\lambda) = N(1-e^2)\sin\phi, \end{gathered} \tag{2.6}$$

where ϕ and λ are geodetic latitude and longitude respectively, and N is the radius of curvature of the ellipsoid in the prime vertical plane and is expressed as [25]:

$$N = \frac{a}{(1-e^2\sin^2\phi)^{0.5}}, \tag{2.7}$$

where a and b are semi-major and semi-minor axes, and e is the first eccentricity of the ellipsoid:

$$e^2 = \frac{(a^2-b^2)}{a^2}. \tag{2.8}$$

The radius of curvature in the prime vertical N is frequently used in geodesy, and can be described as the distance (shown as $B'D$ on Figure 2.1) from the chosen point on the surface of the ellipsoid along the normal to the ellipsoid to the intersection with the semi-minor axis. The second frequently used radius in geodesy is known as radius of curvature in the meridian M and is expressed as [25]:

$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{1.5}}. \tag{2.9}$$

The radius of curvature in the meridian M does not have a good geometrical interpretation on a figure, however it can be described as the radius of a circle that fits ellipsoidal meridian at the chosen point [26].

The curvilinear coordinates ϕ, λ are used to position points on a particular surface. When that surface is an ellipsoid, representing the shape of the Earth, the curvilinear coordinates are called geodetic coordinates. If the surface is a sphere, representing the shape of the Earth, the curvilinear coordinates are called geographic coordinates. Figure 2.1 shows geodetic ϕ, λ and Cartesian X, Y, Z coordinates.

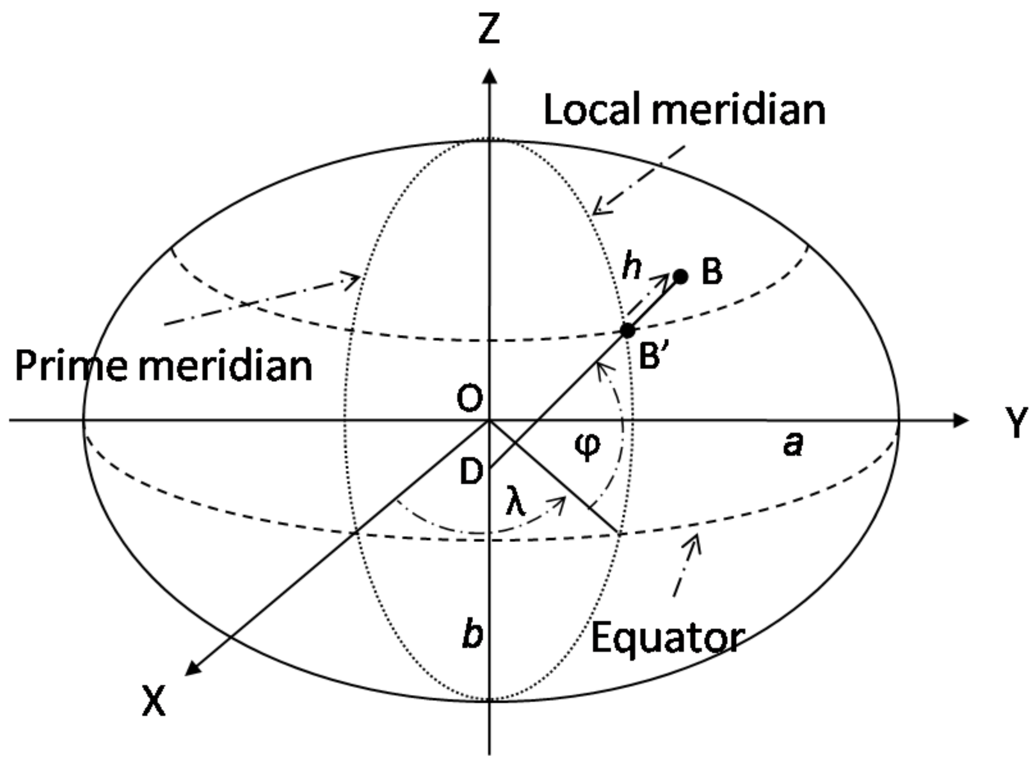

Figure 2.1 Geodetic coordinates and Cartesian coordinates in the same frame of reference [27]

The origin of the Cartesian coordinates is located at the centre of mass of the Earth. The Cartesian Z-axis coincides with the axis of rotation of the ellipsoid. The Cartesian X-axis passes through the point $\phi = 0, \lambda = 0$, where the prime meridian intersects the equator. The Cartesian Y-axis is located 90° eastward along the equator. The geodetic longitude λ is measured East or West from the prime meridian, in the equatorial plane. The geodetic latitude ϕ is measured North or South from the equator, in the local meridian plane. In this thesis the World Geodetic System 1984 (WGS84) ellipsoid is used as a surface of reference for the mapping applications. The WGS84 parameters are given in Table 2.1.

Table 2.1 WGS84 ellipsoidal parameters [28]

Ellipsoid	Semi-major axis a	Semi-minor axis b	Eccentricity e
WGS84	6378137.0 m	6356752.314245 m	1/298.257223563

Originally WGS84 used GRS80 reference ellipsoid; however, since its initial release it has undergone some slight changes, most of which have no practical effect on mapping applications. Therefore, for the purposes of this thesis WGS 84 and GRS80 are equivalent.

Distances in Mapping Applications

The two distances frequently used in mapping applications are map distance and geodesic distance. Map distance is the distance measured on 2D mapping plane and can be calculated from $x,\ y$ coordinates on the map as:

$$l_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{2.10}$$

where l_{12} is the map distance between point 1 $(x_1,\ y_1)$ and point 2 $(x_2,\ y_2)$. Map distance is distorted, depending on the map projection that was used to generate the map. Often times there exists a need to measure undistorted distance on the surface of reference (e.g., ellipsoid). Geodesic is a unique curve c that has the shortest possible length connecting any two points on the ellipsoid S. The geodesic has no curvature in the tangent plane; it is locally straight on the ellipsoid [29]. By definition:

$$min \int_C dS \ \Rightarrow c \tag{2.11}$$

where $\int_C dS$ is the length of curve c between the two fixed points on S. The geodesic is used extensively in implementation of our multi projection system because it represents the true distance on WGS84 ellipsoid, compared to the corresponding map distance, which is generally distorted. The algorithms to calculate geodesic distance vary depending on the surface of reference (e.g., sphere or ellipsoid). In this thesis the geodesic distance is calculated using Vincenty's equations and WGS 84 ellipsoidal parameters in MATLAB [30]. For exact algorithms used, refer to Appendix A.

2.2.4 Mathematics of Map Projections

Map projection equations mathematically relate the surface of reference to the projection surface as shown in the Figure 2.2. An ellipsoidal triangle $P_i P_j P_k$ is mapped onto a plane. The length of projected geodesics S^M is not the same as the length S^E of the original geodesic on the ellipsoid due to map distortion. The ellipsoidal angle ω^E, defined as the angle between the tangents to the two geodesics on the ellipsoid, is not equal to the corresponding projected angle ω^M, defined as the angle between the tangents to the two projected geodesics [29]. The chord length l and plane angle ω^P (angle between the two chords) are used in the computations on the map.

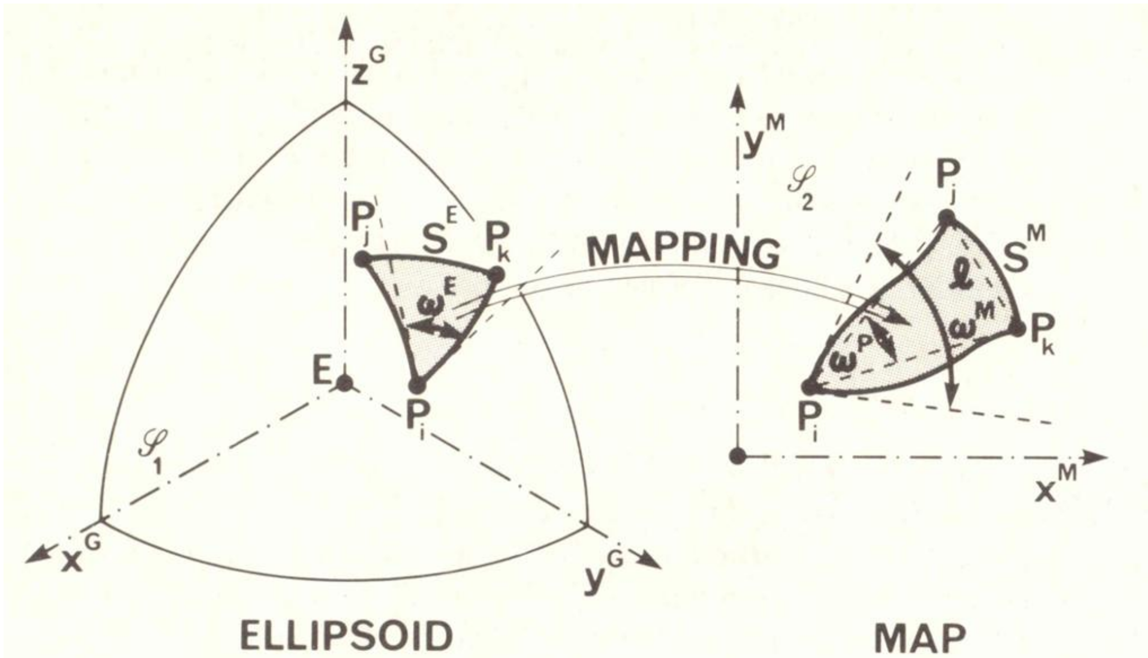

Figure 2.3 Mapping of ellipsoid onto a plane [29]

The length distortion on the mapping plane is related to point scale factor m. Point scale factor is the ratio of infinitesimal distance on the ellipsoidal surface dS^E and the projection surface dS^M [29]:

$$m = \frac{dS^M}{dS^E} \tag{2.12}$$

The infinitesimal distance dS^E on the reference surface can be expressed in terms of differentials dX, dY and dZ:

$$(dS^E)^2 = dX^2 + dY^2 + dZ^2. \tag{2.13}$$

Substituting the differentials of Equation (2.6) into Equation (2.13) and simplifying the terms gives:

$$(dS^E)^2 = Ed\phi^2 + 2Fd\phi d\lambda + Gd\lambda^2, \tag{2.14}$$

where E, F, and G are the Gaussian fundamental quantities related to the reference surface and are expressed as [25]:

$$\begin{aligned} E &= \left(\frac{\partial X}{\partial \phi}\right)^2 + \left(\frac{\partial Y}{\partial \phi}\right)^2 + \left(\frac{\partial Z}{\partial \phi}\right)^2, \\ F &= \frac{\partial X^2}{\partial \phi \partial \lambda} + \frac{\partial Y^2}{\partial \phi \partial \lambda} + \frac{\partial Z^2}{\partial \phi \partial \lambda}, \\ G &= \left(\frac{\partial X}{\partial \lambda}\right)^2 + \left(\frac{\partial Y}{\partial \lambda}\right)^2 + \left(\frac{\partial Z}{\partial \lambda}\right)^2. \end{aligned} \tag{2.15}$$

The Gaussian fundamental quantities are a means of describing the geometrical properties of the surface to be mapped [25]. For the ellipsoid (from Equations (2.6) and (2.15)) [31]:

$$E = M^2, \qquad F = 0, \qquad G = N^2 \cos^2 \phi. \tag{2.16}$$

For the sphere (from Equations (2.4) and (2.15) [31]:

$$E = R^2, \qquad F = 0, \qquad G = R^2 \cos^2 \phi. \tag{2.17}$$

The infinitesimal distance along the meridian dS_λ^E and parallel dS_ϕ^E can be found from Equation (2.14) by holding $d\lambda = 0$ in the first case, and $d\phi = 0$ in the second case:

$$dS_\lambda^E = \sqrt{E} d\phi = M d\phi, \tag{2.18}$$

$$dS_\phi^E = \sqrt{G} d\lambda = N \cos \phi \, d\lambda. \tag{2.19}$$

The angle ω^E between meridians and parallels on reference surface can be expressed as [25]:

$$\cos\omega^{E} = \frac{F}{\sqrt{EG}}. \tag{2.20}$$

The geodetic azimuth α can be written in terms of dS_{λ}^{E} and dS_{ϕ}^{E} [31]:

$$\tan\alpha = \frac{dS_{\phi}^{E}}{dS_{\lambda}^{E}} = \frac{\sqrt{G}d\lambda}{\sqrt{E}d\phi}\ . \tag{2.21}$$

The infinitesimal distance on the projection surface dS^{M} can be expressed as [31]:

$$(dS^{M})^2 = dx^2 + dy^2 = ed\phi^2 + 2gd\phi d\lambda + fd\lambda^2, \tag{2.22}$$

where e, g and f are Gaussian fundamental quantities for the projection surface [31]:

$$\begin{gathered} e = \left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \phi}\right)^2, \\ f = \frac{\partial x^2}{\partial \phi \partial \lambda} + \frac{\partial y^2}{\partial \phi \partial \lambda}, \\ g = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2. \end{gathered} \tag{2.23}$$

The angle ω^{M} between meridians and parallels on the projected surface is [31]:

$$\cos\omega^{\mathrm{M}} = \frac{f}{\sqrt{eg}}. \tag{2.24}$$

By substituting Equations (2.14) and (2.22) into Equation (2.12) the point scale factor m can be written as [31]:

$$m^2 = \left(\frac{e}{E}\right)\cos^2\alpha + \frac{f}{\sqrt{EG}} 2\sin\alpha\cos\alpha + \left(\frac{g}{G}\right)\ \sin^2\alpha. \tag{2.25}$$

The general expression for the meridian scale factor h can be calculated from Equation (2.25) by setting azimuth equal 0° or 180° [31]:

$$h = \frac{\sqrt{e}}{\sqrt{E}}. \quad (2.26)$$

The general expression for parallel scale factor k can be calculated from Equation (2.25) by setting azimuth equal 90° or 270° [31]:

$$k = \frac{\sqrt{g}}{\sqrt{G}}. \quad (2.27)$$

For calculations of scale, angular and area distortions of map projections that use sphere or ellipsoid as a reference surface refer to Appendix B, where the equations are derived from the general expressions discussed above.

Tissot's Indicatrix

In 1859 and 1881, Nicolas Auguste Tissot published a classic analysis of the distortion which occurs on a map projection [10]. Any two lines intersecting on the reference surface (e.g., ellipsoid) intersect at the same or a different angle on the map. At almost every point on the ellipsoid there are two lines that intersect at a right angle in some direction, which are also shown at right angles on the map. These lines might not necessarily be a meridian and a parallel. All the other intersections at that point on the ellipsoid will not intersect at the same angle on the map (unless the map is conformal). Tissot's Indicatrix shows this relationship graphically. An infinitely small circle on the ellipsoid projects as an infinitely small ellipse on the map [10]. If the projection is conformal the ellipse will become a circle. Tissot's Indicatrix can be derived mathematically from the point scale factor m. Equation (2.25) can be written as [32]:

$$m^2 = E^{'} \cos^2\alpha + 2F^{'} \sin\alpha\cos\alpha + G^{'}\ \sin^2\alpha, \tag{2.28}$$

where: $E^{'} = e/E$, $F^{'} = f/\sqrt{EG}$, and $G^{'} = g/G$. Equation (2.28) is a positive-definite quadratic form that forms an ellipse with the direction of maximum point scale factor being associated with the major axis and the minimum point scale factor being associated with the minor axis as shown in Figure 2.4 [29].

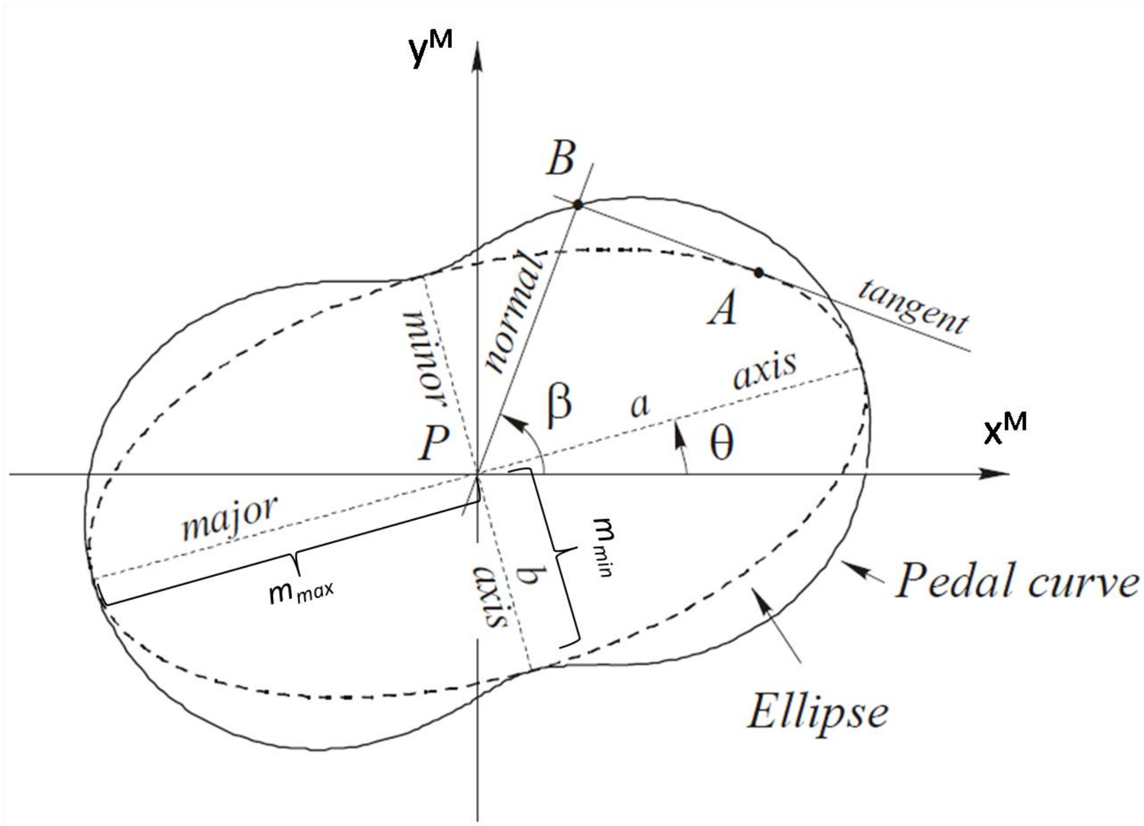

Figure 2.4 Pedal curve and Tissot's Indicatrix ellipse [32]

This ellipse is known as Tissot’s Indicatrix. A is a point on an ellipse. The tangent to the ellipse at A intersects a normal to the tangent passing through P at B. As A moves around the ellipse, the locus of all points B is the pedal curve of the ellipse. The distance

$PB = m^2$ for the angle β. The semi-major axis a and semi-minor axis b of the Tissot's Indicatrix can also be expressed in terms of Gaussian fundamental quantities [32]:

$$a = \sqrt{m_{max}^2} = \sqrt{0.5(E' + G' + W)},$$
$$b = \sqrt{m_{min}^2} = \sqrt{0.5(E' + G' - W)}, \tag{2.29}$$

where $W = \sqrt{(E' - G')^2 + 4(F')^2}$. Tissot's Indicatrix is very useful for showing general visual impression of map distortion and also for quantifying distortion of scale and angle precisely.

2.3 Cylindrical Projections

Cylindrical projections are created by projecting the Earth's surface onto a tangent or secant cylinder. In cylindrical projections, meridians are equally spaced straight lines, and parallels are straight lines intersecting meridians at 90° angle. Cylindrical projections are generally used for world maps and equatorial regions. One of the most famous cylindrical projections is Mercator. Presented by Gerardus Mercator in 1569 Mercator projection has been widely used in navigation because it is conformal and can show loxodromes as straight lines. Figure 2.5 shows Mercator projection with one standard parallel centered at the equator.

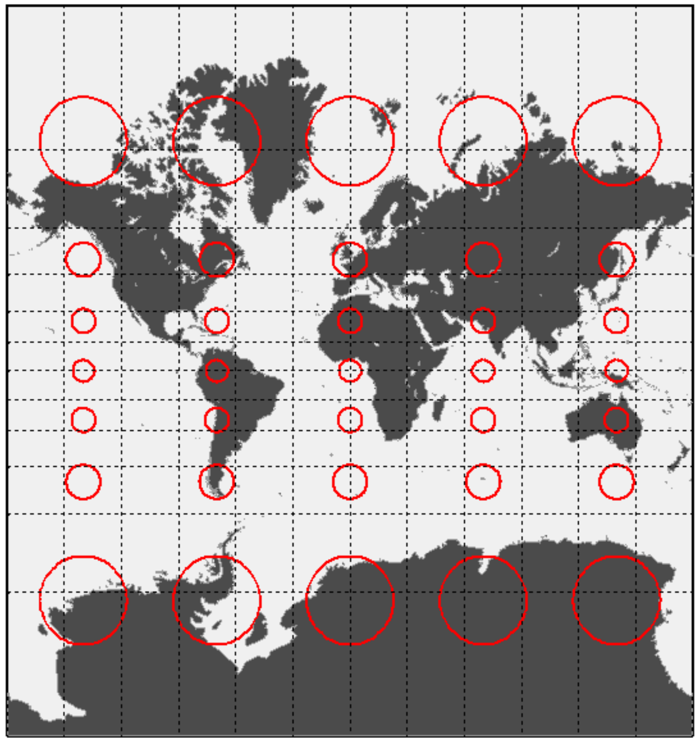

Figure 2.5 Mercator projection with Tissot's Ellipses of distortion. The size of distortion is proportional to latitude

Cylindrical projections can cover equatorial areas of greatest East/West extent with relatively small distortion. For example: Mercator projection with one standard parallel at the equator can cover the geographic area extending 16 degrees in latitude with less than 2% area distortion. Please refer to Appendix C for more details. In 2005 Google introduced its web mapping service – Google Maps. Google Maps uses Web Mercator Projection (WMP) a modified version of Mercator projection. Although WMP closely resembles Mercator projection it doesn't preserve some of its fundamental characteristics. For example WMP is non-conformal [4]. More on WMP in Section 3.2

2.4 Azimuthal Projections

The azimuthal projections are formed onto a plane which is usually tangent to a point on the reference surface (sphere or ellipsoid). Depending on where the tangent point is located, the projections can have polar, equatorial or oblique aspect. The general characteristic of azimuthal projections is that the azimuth from the center of the projection to every other point on a map is shown correctly. Another characteristic of azimuthal projections which applies to all spherical forms is that all great circles (circles whose planes pass through the center of the sphere) passing through the center of projection are shown as straight lines. Therefore the distance from the center point to any other point is shown as straight line and is the shortest path.

There are three main classes of azimuthal projections according to their distortion characteristics: the conformal, the equal-area, and the equidistant. The Azimuthal Stereographic projection is conformal for sphere or ellipsoid, but the ellipsoidal form is not truly perspective [10]. The Lambert Azimuthal Equal-Area and Azimuthal Equidistant are not true perspective projections. Figure 2.6 shows Azimuthal Stereographic projection of the Earth centered at North Pole.

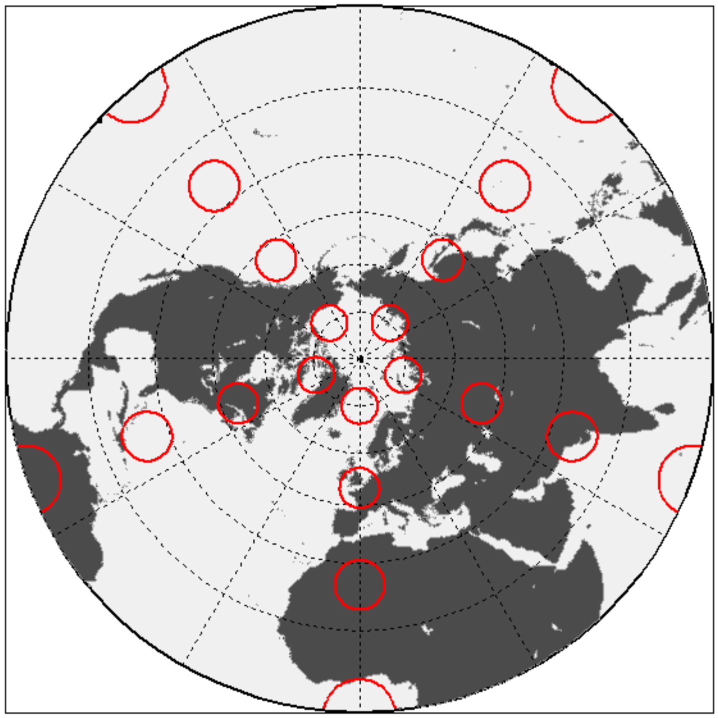

Figure 2.6 Polar aspect of azimuthal Stereographic projection with Tissot's Ellipses of distortion

Azimuthal projections can be used to minimize map distortion in a circular region, such as Antarctica, but not for areas with predominant length in one direction [10]. Tsoulos et al. [33] provide detailed analysis of most common cylindrical, azimuthal and conic projections for the Arctic and sub-Arctic Regions in their paper *Choosing a Suitable Projection for Navigation in the Arctic*. According to their research, the most suitable projections for Arctic Region (north of the 68th parallel) are Azimuthal Polar Equidistant projection and the Azimuthal Polar Stereographic projection based on minimization and distribution of distortion, the shape of great circles, and meridian representation (whether the projection shows straight meridians and, if possible straight parallels perpendicular to meridians).

2.5 Conic Projections

Simple forms of conic projections can be created by placing a cone tangent to the reference surface (sphere or ellipsoid). Just like cylindrical map projections, conic map projections can have one or two standard parallels depending on whether the case is tangent or secant. Normal conic projections have arcs of concentric circles for parallels of latitude and equally spaced straight radii of this circles for meridians [10]. There are three main classes of conic projections according to distortion characteristics which are: the equidistant, the conformal and the equal-area. Figure 2.7 shows the Lambert Conformal Conic Projection with one standard parallel centered at the equator.

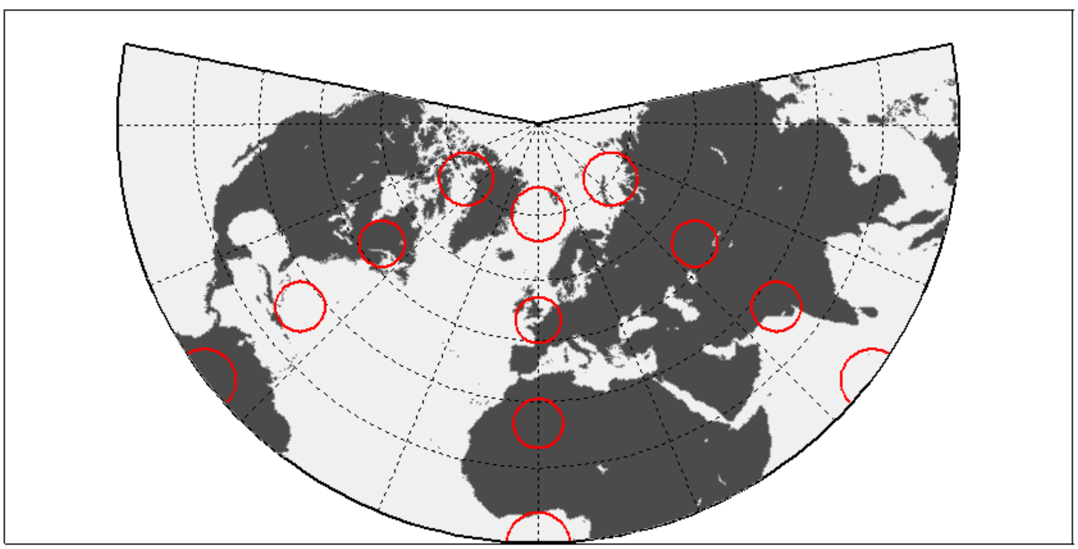

Figure 2.7 Lambert Conformal Conic projection with Tissot's Ellipses of distortion

Conic projections are mainly used to map mid-latitude regions of greatest East/West extent with minimal distortion. In order to minimize map distortion a secant case should be applied. In general, standard parallels should be so selected that they enclose about two-thirds of the latitudinal extent of the area to be represented [34]. For example, the first standard parallel is 1/6 of the total mapping extent in north-south direction, from the southern edge of the mapping area, and second standard parallel is 1/6 from northern edge of the mapping area. This should not, however, be treated as a rule because the selection will partly depend upon the purpose of the map and the area to be mapped. Kavraisky [33] provides a more systematic approach for selecting standard parallels, later published by Bugayevskiy & Snyder [11] and Maling [13]. The latitudes φ_1 and φ_2 of the standard parallels are selected by using Equations (2.30) and (2.31).

$$\varphi_1 = \varphi_s + \Delta\varphi, \tag{2.30}$$

$$\varphi_2 = \varphi_N - \Delta\varphi, \tag{2.31}$$

where

$$\Delta\varphi = \frac{\varphi_N - \varphi_s}{K}, \tag{2.32}$$

φ_s and φ_N are the bounding parallels of the map.

The use of constant K leads to the choice of the suitable standard parallels for conical projections. This constant may vary according to the shape of the area to be mapped as follows (empirical values):

a. Small extent in latitude but large extend in longitude: K = 7

b. Rectangular outline with longer axis north-south: K = 5

c. Circular or elliptical outline: K = 4

d. Square outline: K = 3

According to Tsoulos et al. [33], the most suitable projections for sub-Arctic Regions (50° – 75°) are the Lambert Conic Conformal projection and the Conic Equidistant projection based on minimization and distribution of distortion, the shape of Great Circles, and Meridian representation.

2.6 Summary

In this chapter the concepts of map projections and map distortions were explained in detail. There are three main characteristics of a map that can be distorted namely scale, area and shape. The main goal of map projection selection is concerned with minimization of map distortion. There are many existing map projections that can be selected depending on the specific purpose. Different guidelines for classification and selection of map projections were examined. Lastly, the most common cylindrical, azimuthal and conic projections were analyzed in terms of their general properties, geographic preference and map distortion.

3 Global Web Map Visualization

This chapter focuses on the Web Mapping technology. The chapter begins with a brief summary of the history of interactive web maps, starting from the earliest Web Mapping prototypes. Next, the design architecture of popular web mapping service Bing Maps and its use of Web Mercator Projection (WMP) is analyzed, followed by discussion on alternative approaches to global web map visualization. The chapter ends with a quick summary.

3.1 History of Interactive Web Maps

The history of interactive web maps closely resembles the development of the World Wide Web (WWW). In June 1993, Xerox PARC Map Viewer was released by Xerox Palo Alto Research Center, United States. It was one of the earliest prototypes of static Web mapping, which enabled users to display graphical images with the ability to create new documents based on the user input. Map Viewer allowed users to toggle the display of national boundaries and rivers, change projection, change scale and even add place markers [35]. Soon, other mapping services were released and followed similar design. These servers required a lot of manual labour for maintenance and update.

In 1995, Dr. Susan Huse, a researcher at University of California, Berkeley, developed GRASSLinks, a web interface on top of GRASS (Geographic Resources Analysis Support System), an open source GIS package originally developed by the US Army

Corps of Engineers [36]. GRASSLinks allowed users to view, pan and zoom any GRASS dataset. A unique feature of GRASSLinks was the ability to obtain information on clicked location. Also, in 1995, the U.S. Census Bureau launched TIGER Map Server, a web interface for its large TIGER (Topologically Integrated Geographic Encoding and Referencing system) dataset [37]. The TIGER Map Server generated map “on-the-fly” and made it possible for users to zoom in and out, scroll about, and toggle on/off many of the geographic entities in the dataset [38].

In 1996 GeoSystems Global Corporation (later renamed MapQuest) launched the first consumer-focused interactive mapping site on the Web, MapQuest.com [39]. MapQuest captured the attention of the Internet consumers and business market by greatly improving the usability and quality of web mapping. Most of the web mapping services developed from 1995 through 2004 suffered from two major flows: a slow performance and a complicated/intimidating user interface [40].

In 2005 Google transformed the mapping world with the release of Google Maps which used Web Mercator Projection (WMP) and took advantage of Ajax (Asynchronous JavaScript and XML) technology bringing significant improvement in performance [41]. Google then decided to make the Application Programming Interface (API) for their Maps application public [40]. This allowed third-party developers to mix in their own content, integrating map services into existing websites. Yahoo! and MapQuest followed suit. The result was an explosion of custom mapping applications.

3.2 Popular Web Mapping Services

The following section describes the basic framework of the popular web mapping service Bing Maps provided by Microsoft Corporation. The design architecture of Bing Maps is very similar to the design architecture of Google Maps. The section is divided into three subsections: Web Mercator Projection (WMP), Bing Maps Tile System, and Major Limitation of WMP. The first subsection explains in detail WMP, a projection used by Google and Bing Maps. The second subsection describes the design of Bing Maps tile system. The tile system uses pre-rendered tiles for quick retrieval and display. Lastly, the third subsection covers in detail the major limitations of WMP, which were first introduced in Chapter One.

3.2.1 Web Mercator Projection (WMP)

Popular Web mapping services, such as Google Maps and Microsoft Bing Maps, use Web Mercator Projection (WMP) to map the Earth. The Web Mercator Projection, also known as Popular Visualization Pseudo Mercator, maps WGS84 geodetic latitude and longitude using the equations of spherical Mercator projection. WMP looks very similar to spherical or ellipsoidal Mercator projection that is why it is hard to visually tell the difference between WMP and spherical or ellipsoidal Mercator. According to Bing Services, the Mercator projection was chosen because of two main properties (listed below). These properties outweigh significant scale and area distortion introduced by the projection [42].

1. “It’s a conformal projection, which means that it preserves the shape of relatively small objects. This is especially important when showing aerial imagery, because

we want to avoid distorting the shape of buildings. Square buildings should appear square, not rectangular” [42].

2. “It’s a cylindrical projection, which means that north and south are always straight up and down, and west and east are always straight left and right” [42].

Bing Services also states that “To simplify the calculations, we use the spherical form of this projection, not the ellipsoidal form. Since the projection is used only for map display, and not for displaying coordinates numerically, we don’t need the extra precision of an ellipsoidal projection. The spherical projection causes approximately 0.33% scale distortion in the Y direction, which is not visually noticeable.” However, the use of spherical Mercator equations to map WGS 84 geodetic latitude and longitude results in a loss of some of the fundamental properties of Mercator projections such as conformality.

WMP is computationally faster than ellipsoidal Mercator projection because it uses the same equations as in spherical Mercator projection [6]. Given geodetic latitude φ and longitude λ, the equations to derive the projected x and y coordinates of WMP are computed as follows [4]:

$$R = a, \tag{3.1}$$

$$x = R\,(\lambda - \lambda_0), \tag{3.2}$$

$$y = R\,ln[tan(\pi/4 + \phi/2)], \tag{3.3}$$

where a – semi-major axis, and *R* – radius of Earth.

According to the European Petroleum Survey Group (ESPG) Coordinate Transformations Manual [4], the scale factors in meridian (h) and scale factors in parallels (k) for WMP are given as:

$$h = \frac{R}{M \cos \varphi}, \tag{3.4}$$

$$k = \frac{R}{N \cos \varphi}, \tag{3.5}$$

where the radius of curvature of the meridian M and the radius of curvature of the prime vertical N can be calculated from Equations (2.7) and (2.9). The difference between h and k scale factors demonstrates that the Web Mercator Projection is not conformal.

3.2.2 Bing Maps Tile System

Google Maps and Microsoft Bing Maps have a unique tiling scheme. The map is pre-rendered at different levels of detail, making user interactions very fast and responsive. The user can view small to large scale map by changing level of detail. When a user zooms in, the level of detail is incremented and when a user zooms out, the level of detail is decremented. The first level of detail covers the entire world and is represented by four tiles. The second level of detail shows a portion of the world and is represented by a total of 16 tiles, however only four tiles are shown to the user. The third level of detail is represented by 64 tiles and so on [42]. Figure 3.1 shows Microsoft Bing Maps levels of detail. Each level is represented by different number of tiles, each tile having a unique quadkey value. The length of a quadkey value equals the level of detail of the corresponding tile. The quadkey of any tile starts with the quadkey of its parent tile.

Figure 3.1 Bing Map tile structure and hierarchy for the first 3 levels of detail. The number indicates a unique quadkey value for the tile. [42].

The geodetic coordinates are converted into x_p, y_p pixel coordinates for each level of detail as follows:

$$x_p = 256\left[\frac{\lambda + 180}{360}\right]2^{level},$$
$$y_p = 256\left[0.5 - log\left(\frac{1 + \sin\phi}{(1 - \sin\phi)4\pi}\right)\right]2^{level}, \tag{3.6}$$

where ϕ and λ are geodetic latitude and longitude, respectively. Pixel coordinates start from (0, 0) at the top left corner and end at pixel coordinates limit x_{limit}, y_{limit} at the bottom right corner. The limit changes depending on level of detail. Figure 3.2 shows pixel coordinates at level of detail 3. At this level of detail, the coordinates range from (0, 0) to (2047, 2047). The pixel coordinates limit x_{limit}, y_{limit} for any level of detail can be found using Equation (3.7).

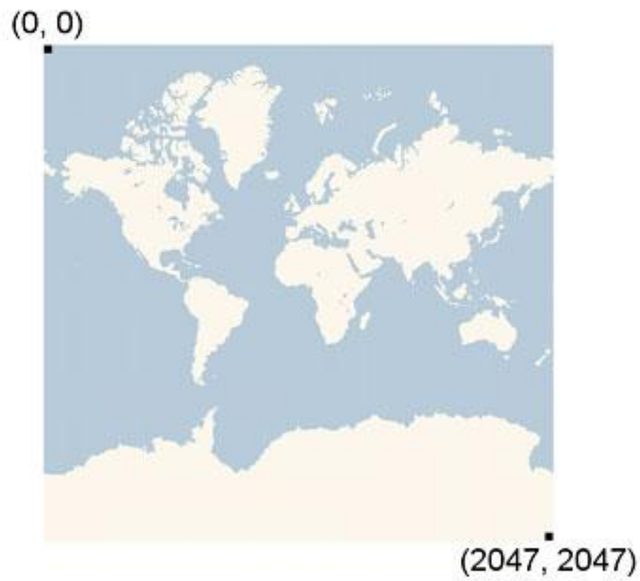

Figure 3.2 Bing Maps pixel coordinates at level of detail 3 [42]. The top left corner indicates origin and the bottom right corner indicates end limits.

$$(x_{limit}, y_{limit}) = \left(256 \times 2^{level} - 1, 256 \times 2^{level} - 1\right). \qquad (3.7)$$

Each tile is given a specific X_T, Y_T tile coordinates ranging from (0, 0) at the top left corner to (2^{level}–1, 2^{level}–1) at the bottom right corner. For example, at level 3 the tile coordinates range from (0, 0) to (7, 7) as shown in Figure 3.3. Each X_T, Y_T tile coordinates cover a range of x_p, y_p pixel coordinates. Given a pair of x_p, y_p pixel coordinates, a web server can easily determine the X_T, Y_T tile coordinates containing that pixel, and pull the right tile out of the system:

$$X_T = floor\left(\frac{x_p}{256}\right),$$
$$Y_T = floor\left(\frac{y_p}{256}\right), \qquad (3.8)$$

where $floor(x)$ returns the largest integer less than or equal to x. (e.g., $floor(2.7) = 2$)

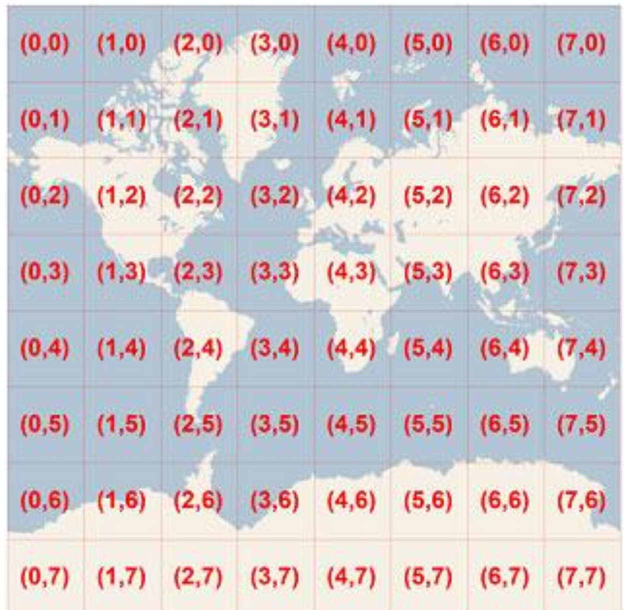

Figure 3.3 Bing Maps tile coordinates for level of detail 3 [42]. The numbers indicate a unique tile coordinate value for each tile.

The existing tiling scheme allows web servers to quickly process user input and create a smooth browsing experience.

3.2.3 Major Limitations of Web Mercator Projection

There are 3 major limitations that are introduced by the Web Mercator Projection.

1. Lack of polar coverage.
2. Large systematic distortion.
3. Additional discrepancies because of the use of spherical equations instead of ellipsoidal equations to map WGS 84 geodetic coordinates.

The first major limitation of WMP is the inability to show polar regions. North and South Poles are not shown on the map. The *y* coordinates of the projection become infinite at the poles. WMP is truncated in a way to preserve 1:1 aspect ratio, meaning that map extent from West/East must correspond to the map extent from South/North. The maximum latitude therefore must correspond to the following *y* coordinates:

$$y = \pm\frac{width}{2} = \pi a. \tag{3.9}$$

The inverse equation for WMP latitude is given below, where *e* is a base of natural logarithm:

$$\phi = \frac{\pi}{2} - 2atan\left(e^{\frac{-y}{a}}\right). \tag{3.10}$$

The maximum latitude can be calculated by substituting *y* from Equation (3.9) into Equation (3.10):

$$\begin{aligned}\phi = \frac{\pi}{2} - 2atan\left(e^{\frac{-\pi a}{a}}\right) = \frac{\pi}{2} - 2\,atan(e^{-\pi}) \\ = 85.051129°.\end{aligned} \tag{3.11}$$

Therefore, WMP is suitable for global use between 85° south and 85° north latitude, and becomes impractical for users interested in polar regions. The second major limitation of WMP is systematic distortion that increases with latitude. At 85° latitude the scale distortion of WMP is 1147% and the area distortion is 13165%. At latitude of 70°, the web map server needs roughly three and a half times more space to store the same geographical area as at the equator. Also, the scale factor has to be constantly recalculated to account for scale distortion.

The third major limitation of WMP is the use of spherical equations to map WGS 84 geodetic latitude and longitude instead of the ellipsoidal equations. This creates three problems:

1. Distance difference in y coordinates between WMP and ellipsoidal Mercator
2. Different scale distortion between WMP and spherical/ellipsoidal Mercator
3. Angular distortion of WMP.

The equations to calculate x and y coordinates of ellipsoidal Mercator are given as [10]:

$$x = a\,(\lambda - \lambda_0), \tag{3.12}$$

$$y = a\,ln\left[tan(\pi/4 + \phi/2)\left(\frac{1 - e\sin\phi}{1 + e\sin\phi}\right)^{e/2}\right], \tag{3.13}$$

where a is the semi-major axis of ellipsoid, and e is the eccentricity.

While there is no difference in x coordinates between WMP and ellipsoidal Mercator, the difference in y coordinates is more than 40 km at latitude of 70°. Figure 3.4 shows difference of y coordinates between Web Mercator Projection and ellipsoidal Mercator.

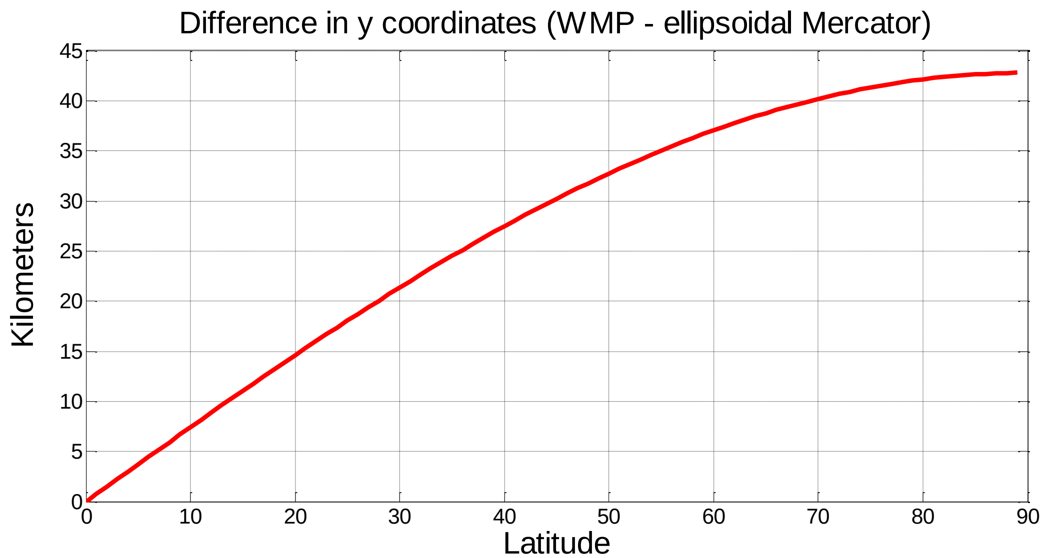

Figure 3.4 The difference in y coordinates between WMP and ellipsoidal Mercator vs. latitude. The difference is 0 at the equator and increases with latitude.

The difference in y coordinates between the two projections exceeds 20 km at latitude of 30° and exceeds 40 km at latitude above 70°. This is significant, especially when considering larger scale maps. The users who are unaware of it can experience serious problems, if they believe that their measurements are represented on ellipsoidal Mercator.

The second problem is the different scale distortion between WMP and spherical/ellipsoidal Mercator. Even though WMP uses spherical Mercator equations, the scale distortion between WMP and spherical Mercator is different, and has to be calculated separately. The scale distortion equations for WMP are more complicated than those of spherical Mercator and thus are more computationally heavy. The scale factor for spherical Mercator is calculated as [10]:

$$h = k = \frac{1}{\cos\varphi}. \tag{3.14}$$

The scale factor for ellipsoidal Mercator is calculated as:

$$h = k = \frac{(1 - e^2 \sin^2\varphi)^{0.5}}{\cos\varphi}. \tag{3.15}$$

Scale factor equations for spherical Mercator projection (3.14) and ellipsoidal Mercator projection (3.15) require less computational effort than scale factor equations for Web Mercator Projection (3.4) and (3.5). One notable advantage of WMP is its computational efficiency. However if scale factor at a point is added to the computation, then the WMP is much slower than the spherical Mercator and even a little slower than the ellipsoidal Mercator [6]. Yet, the scale must always be known in order to conduct accurate measurements of distance and areas on WMP because it changes with latitude.

The third problem is the angular distortion introduced by WMP. Angular distortion in WMP results directly from mapping WGS 84 latitude and longitude using spherical Mercator equations. Angular distortion in WMP is a setback because both spherical and ellipsoidal Mercator projections are conformal which means that they preserve angles. Figure 3.5 shows the behaviour of angular distortion for WMP at different latitudes.

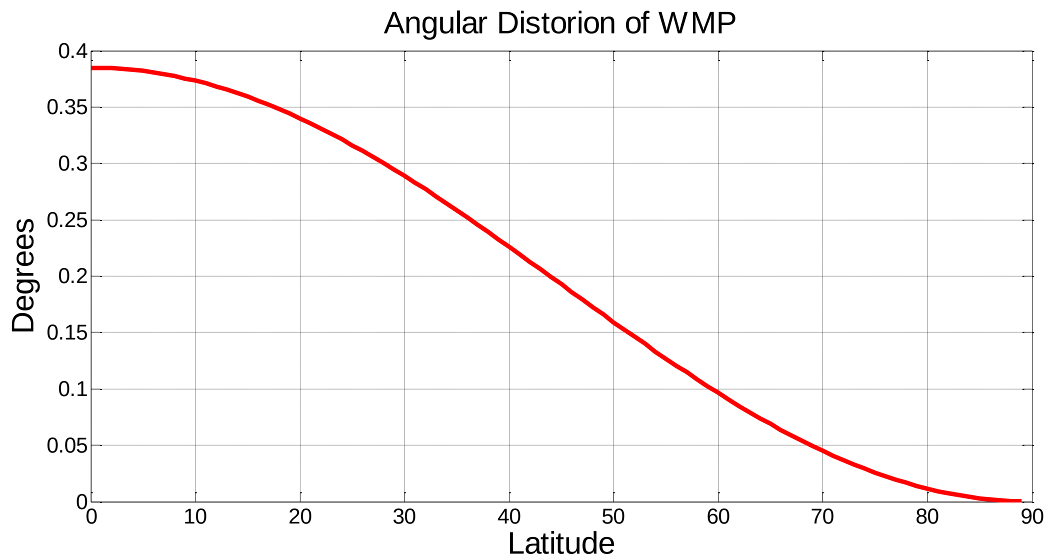

Figure 3.5 Angular Distortion of WMP vs. latitude. The angular distortion decreases with higher latitude in a non-linear manner.

The maximum angular distortion occurs at the equator and corresponds to 0.38º. The angular distortion decreases with increasing latitude until it approaches 0º toward the poles. Because of the angular distortion, WMP cannot be considered a conformal projection. The Mercator projection is used for navigation purposes because it shows loxodromes as straight lines. On WMP a straight line does not have a constant true azimuth therefore loxodromes are not straight [6]. Figure 3.6 shows the loxodrome at 35° grid azimuth.

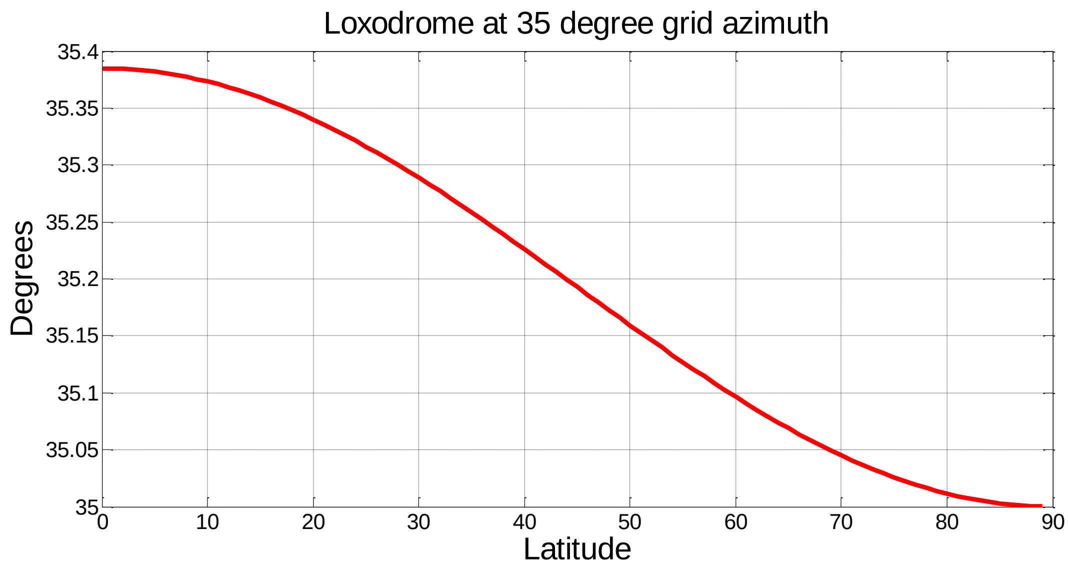

Figure 3.6 WMP loxodrome at 35° grid azimuth vs. latitude. On a conformal map the loxodrome would be represented by a straight horizontal line centered at 35°.

The general distortion pattern in Figure 3.6 is similar to distortion pattern in Figure 3.5. Therefore loxodromes get distorted due to the angular distortion of WMP. Based on these angular distortion patterns, WMP might not be very useful for precise navigation.

3.3 Other Approaches to Global Web Map Visualization

Many interesting approaches exist for web map visualization of the Earth. There are many mapping systems that only use one specific map projection for the entire region. They typically select a map projection that best suits their region of interest. For example, Toporama maps of Canada use only Lambert Conformal Conic projection with standard parallels at 49° and 77° for its purposes [43]. There are other approaches for the Earth's visualization that use special tessellation techniques, such as Hierarchical Triangular Mesh to subdivide the surface of a sphere into spherical triangles of similar, but not identical, shapes and sizes [44] and then unfold these triangles to view the area of interest. A Buckminster Fuller's Dymaxion map is such an example. In Dymaxion map, the sphere is projected on the surface of an icosahedron, which can be unfolded, flattened and viewed in two dimensions. These approaches present the problem of continuity. Figure 3.7 shows Buckminster Fuller's Dymaxion map.

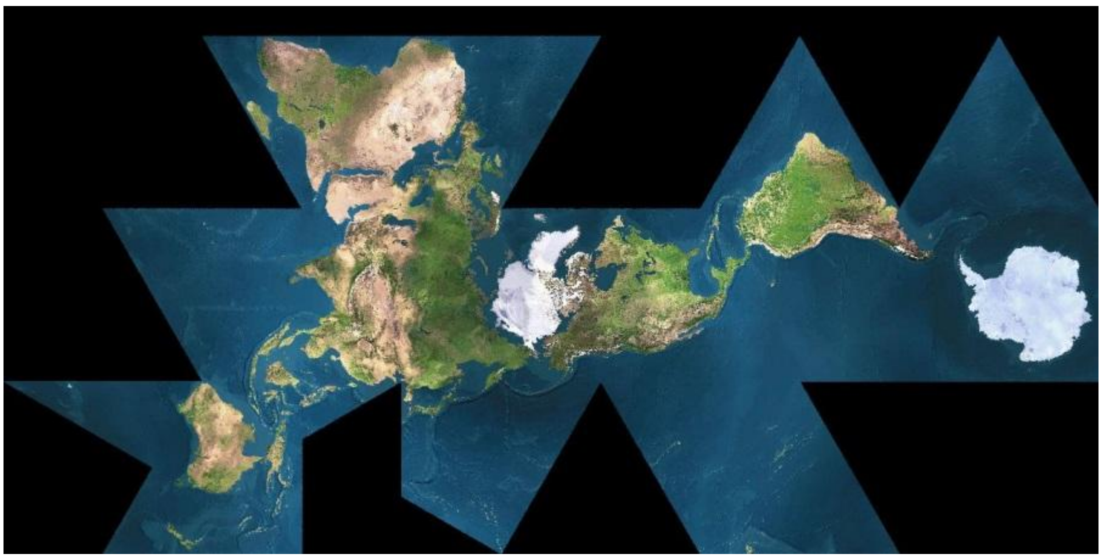

Figure 3.7 Buckminster Fuller's Dymaxion Map [45]

One of the most recent approaches on the subject of global web map visualization is the *Adaptive Composite Map Projections* presented by Bernhard Jenny from Oregon State University [17]. Jenny proposes a very unique way of minimizing map distortion at smaller scales. Adaptive Composite Map Projections approach minimizes map distortion at small scales by dynamically changing the projection as the point of view changes [17]. The main benefit of composite projection, as compared to popular web mapping services, is that areas are displayed with little distortion which is an important factor for small and medium scales. The approach uses Java Script and HTML 5 Canvas to do re-projections on the fly. There are however a number of limitations with composite projections. First, the data is re-projected on the fly, which means that for slower computers the process might be computationally heavy. Second, according to Jenny "the biggest weakness of the proposed composite projection might be the fact that existing tile-based web mapping systems have to be redesigned; components for storing, visualizing, transferring and caching vector map data have to be developed" [17].

Composite projections can be morphed into Mercator projection at larger scales, if compatibility with existing mapping services is required [17]. However the morphing doesn't solve the problem of lack of polar coverage for Mercator projection at larger scales. The polar regions are very important for many research and development projects, so they must be mapped. Also, the data layers for large scale maps usually involve more data points than data layers for small scale maps. Therefore re-projection on the fly at larger scales may be a slow and computationally heavy process, especially for older computers. The main conceptual difference between Adaptive Composite Map Projections approach and the Multi-Projection System is the use of reprojection on the fly versus tile based methods, respectively.

Then there are 3D Globes. This type of visualization is very realistic and allows users to rotate the Earth in any direction, for example, Google Earth, NASA WorldWind, ESRI ArcGlobe and others. However users need to download and install the software before they can use it. For the most part these 3D environments are computationally heavy for mobile applications, and cannot guarantee reliable measurements. For example, a

Google employee states that ”Google makes no claims as to the accuracy of the coordinates in Google Earth. These are provided for entertainment only and should not be used for any navigational or other purpose requiring any accuracy whatsoever” [46].

3.4 Summary

This chapter discussed the design architecture of popular web mapping service Bing Maps focusing on its use of Web Mercator Projection (WMP). The major limitations of WMP such as systematic distortion and lack of polar coverage were explained. Lastly, it is concluded that one projection cannot satisfy the requirements of all professional users therefore a multi-projection scheme is proposed.

4 Multi-Projection System

In this chapter I will present the Multi-Projection System (MPS), a global web map visualization approach that relies on using a combination of different projections to view the Earth. This chapter describes the main principles of MPS and explains how it is used for global web map visualization. The main goal of this approach is to minimize map distortion regardless of the geographical region and level of detail. To achieve this goal, MPS uses four unique projection levels, each corresponding to a range of Bing Maps levels of detail. The chapter explains the concept of individual projection levels and illustrates how these projection levels are subdivided to minimize map distortion. Lastly, the chapter covers back projection, a procedure used to convert x, y map coordinates into geodetic φ and λ. The geodetic coordinates are used for accurate distance measurements, user orientation and transition between projection levels.

4.1 Introduction and Principles

Two very important elements were considered in developing a multi-projection approach to web mapping. First, the mapping system must cover polar regions. Second, the mapping system must minimize map distortion for different geographic regions and zoom level. These requirements are impossible to achieve with one projection (without doing re-projection on the fly), therefore a system based on a combination of different projections is developed.

MPS minimizes map distortion in area, scale, and shape by using a combination of projections across the globe and for ranges of mapping detail levels. Thus, for a specific zoom level or geographic region the most appropriate projection is applied. For small scales, the Earth is treated as a sphere, and therefore spherical projection equations are applied. For medium and large scales which require higher accuracy, the Earth is treated as an ellipsoid, and therefore WGS 84 ellipsoidal projection equations are applied.

The multi-projection scheme is based on the idea that minimum distortion for any geographic region can be achieved by determining in advance the type of map properties that should be preserved and then finding the most appropriate map projection for that geographic region. For example, if preserving shape is important then, the azimuthal Stereographic projection can be used to minimize map distortion in circular polar regions, and the cylindrical Mercator projection can be used to minimize map distortion in equatorial regions with the greatest East/West extent. Therefore, in order to keep small map distortion different map projections should be used when transitioning to different geographical regions rather than using one projection for the entire Earth.

To be able to minimize map distortion at different geographic regions and levels of detail, MPS is subdivided into four unique projection levels. These four projection levels cover Bing Maps zoom levels 1 – 19. Each projection level covers a range of Bing Map zoom levels and minimizes map distortion for that specific range of zoom levels. Some projection levels use only one projection while others use a combination of different projections. Four projection levels allow separating total coverage into maps of the world, hemisphere (small scale), medium scale and large scale, and therefore making it easier to find the most appropriate projections for these maps. For higher accuracy requirements additional projection levels can be added, however in this research only four projection levels are considered. Projection Level 1 covers Bing Maps level of detail 1. This projection level uses Hammer equal-area projection for worldview map. Projection Level 2 covers Bing Maps levels of detail 2 - 4. This projection level uses Azimuthal Stereographic projection for maps of hemisphere and continents (small scale). Projection Level 3 covers Bing Maps levels of detail 5 – 7. This projection level is used for maps transitioning from small to medium scale. It uses a combination of narrow zones using azimuthal, conic and cylindrical projections. The projected zones have small overall distortion and maintain proper geodesic distance (ellipsoidal distance [30]) from the center of one zone to the center of another zone. Lastly, Projection Level 4 covers Bing Map levels of detail 8 and above. This projection level is used for map transitioning from medium to large scale. It uses a combination of 1° x 1° tiles projected using Stereographic Azimuthal Projection to form a total mapping area of 3° x 3°. Table 4.1 summarizes MPS details such as: projection level, corresponding zoom levels, projection type and maximum scale distortion.

Table 4.1 Multi-Projection System design details and corresponding Bing Maps zoom levels.

Microsoft Bing Maps	**Multi-Projection System**		
Zoom Levels	**Projection Level**	**Map Projection**	**Max Scale Distortion (%)**
1	1	Hammer	136
2 – 4	2	Lambert Azimuthal Equal Area	6.4
5 – 7	3	Azimuthal (1), Conic (7), Cylindrical (1)	1.6
8 +	4	Azimuthal Stereographic 3° x 3°	0.03

Minimization of scale distortion is used as the main criterion for selecting each projection level, however minimization of area and angular distortion are also considered. The specific tiling scheme must also be considered for fast loading and visualization; however, the tiling scheme is beyond the scope of this research. Popular web mapping services use pre-rendered square tiles. In the future the new tiles will have to be designed for multi-projection system to accommodate each projection level. The new tile scheme will be based on popular web mapping hierarchy, however the shape of tiles might be different.

4.2 Projection Levels

Each projection level consists of a unique projection or a combination of different projections, and covers a range of Bing Maps levels of detail. There are a total of 19 Bing Maps levels of detail, which are covered by four projection levels [47]. This section describes Projection Levels 1 – 4 in detail. It includes the types of projections that they use and explains the selection criteria for each level.

4.2.1 First Projection Level: World Map

This projection layer corresponds to Bing Maps zoom level 1. It is used for world map and replaces Web Mercator Projection. There are numerous alternatives that can be used for world projection instead of WMP. In general, the equal-area property is more preferred than conformality for small scale maps, because users tend to be more interested in measuring areas than angles [17]. For uninterrupted equal-area projections, Snyder [10] recommends Eckert IV or VI, Hammer, Mollweide, McBryde or McBryde-

Thomas variations, Boggs Eumorphic or Sinusoidal projections. At 85° latitude, Hammer projection has the maximum scale distortion of 136% compared to 539% scale distortion of Eckert IV projection and 1147% scale distortion of WMP. Figure 4.1 shows Eckert IV projection of the Earth and Figure 4.2 shows Hammer projection of the Earth.

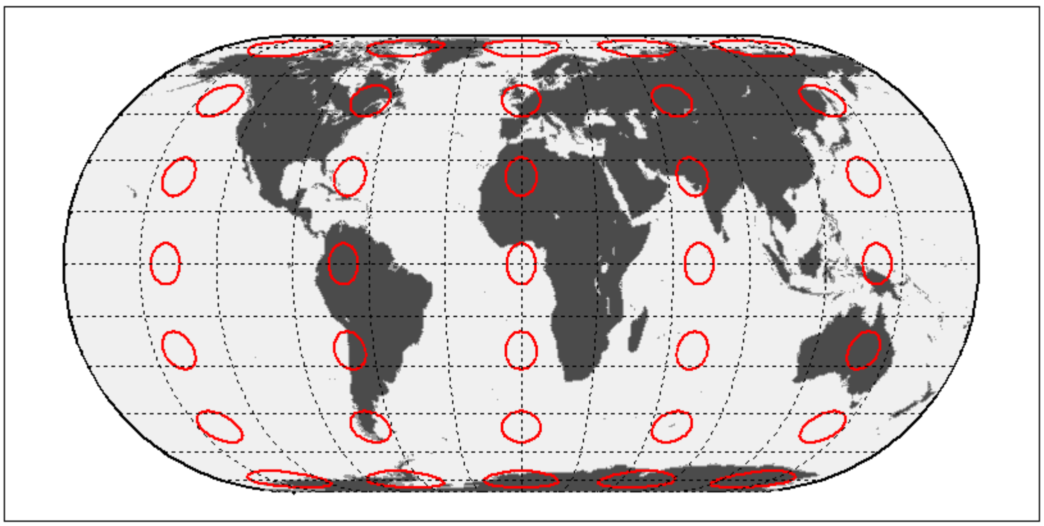

Figure 4.1 Eckert IV projection of the Earth with Tissot's Ellipses of distortion.

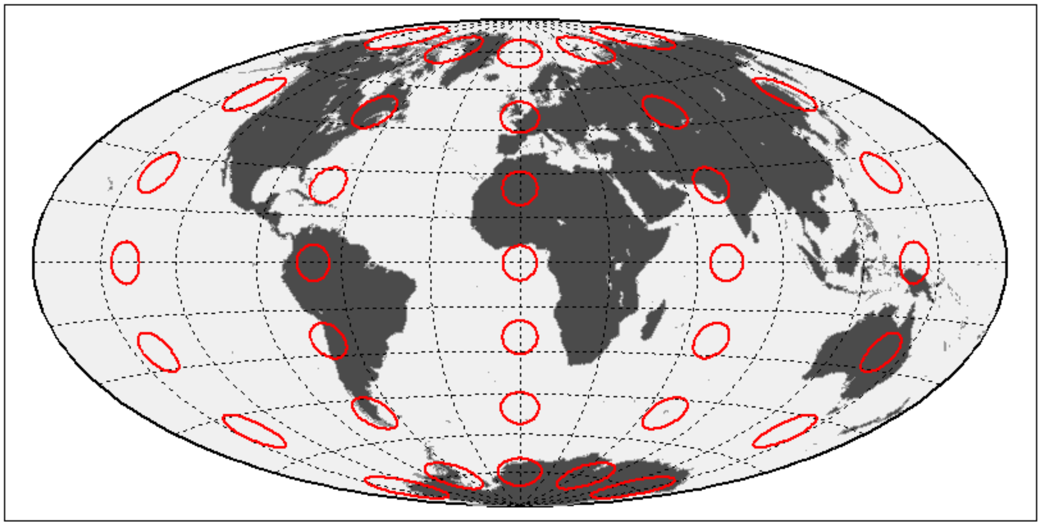

Figure 4.2 Hammer projection of the Earth with Tissot's Ellipses of distortion.

From Figure 4.1 and Figure 4.2 the Hammer projection has less distortion toward the poles compared to Eckert IV projection. The equations for Hammer projection are given as [48]:

$$W = 0.5, \tag{4.1}$$

$$D = \frac{2}{(1 + cos\,\varphi\, cos[W(\lambda - \lambda_0)])}, \tag{4.2}$$

$$x = R\left(\frac{D^{0.5}}{W}\right) cos\,\varphi \sin[W(\lambda - \lambda_0)], \tag{4.3}$$

$$y = RD^{0.5}\, sin\,\varphi, \tag{4.4}$$

where W is a constant, *R* is radius of Earth, λ_0 is the central meridian and *D* is intermediate variable used in calculations. The inverse equations of Hammer projection can be derived by using intermediate variable *z*:

$$z = \sqrt{1 - \left(\frac{1}{4}x\right)^2 - \left(\frac{1}{2}y\right)^2}. \tag{4.5}$$

Then longitude λ and latitude φ can be calculated as follows:

$$\lambda = 2\, tan^{-1}\left[\frac{zx}{2(2z^2 - 1)}\right], \tag{4.6}$$

$$\varphi = \sin^{-1}(zy). \tag{4.7}$$

For the first projection level of the multi-projection system, the Hammer projection is selected primarily because of its relatively small scale distortion, simple direct/inverse equations and ellipsoidal look.

4.2.2 Second Projection Level: Maps of Hemisphere

The second projection level corresponds to Bing Maps levels of detail 2 – 4 inclusive. This level is used at the hemisphere and continent scale. For maps of hemisphere Snyder [10] recommends Lambert Azimuthal Equal-Area, Stereographic Azimuthal, Equidistant Azimuthal or Orthographic projection. The Orthographic projection is not considered because it is neither equal-area, nor equidistant nor conformal. It also has the largest distortion of the four projections. For the second layer of multi-projection system the Lambert Azimuthal Equal-Area projection is selected because it has the smallest scale distortion out of four proposed projections. Lambert Azimuthal Equidistant projection is also a good candidate. Figure 4.3 and Figure 4.4 compare scale distortion of Azimuthal Stereographic, Lambert Azimuthal Equal-Area, and Lambert Azimuthal Equidistant projections. Figure 4.5 compares area distortion of Stereographic Azimuthal Projection and Lambert Azimuthal Equidistant Projection. The projection centers are selected at 0° latitude and 0° longitude. For example in Figure 4.3 (left) the scale distortion at 20° from projection origin is about 3%.

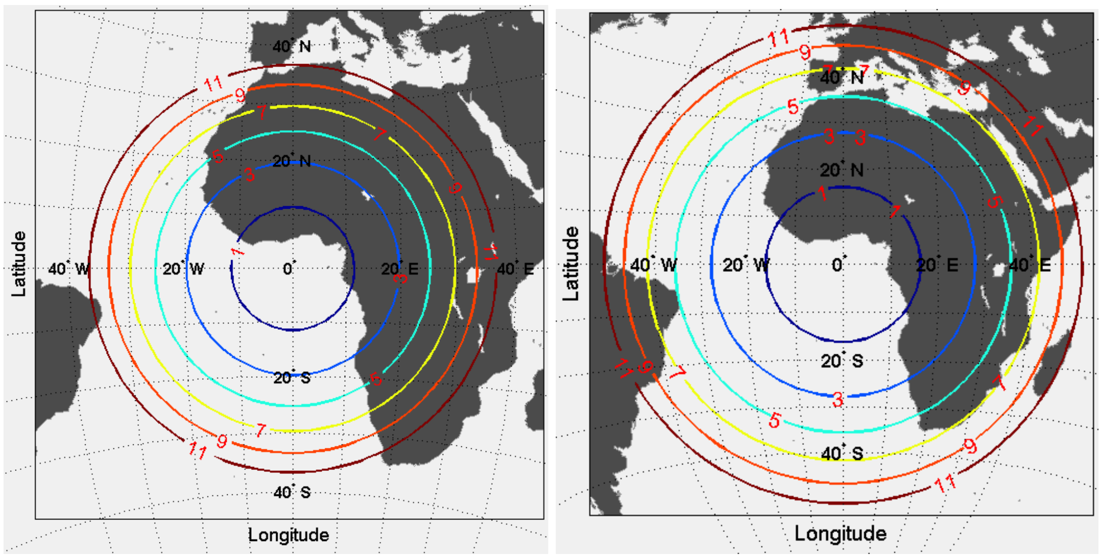

Figure 4.3 Scale distortion (%) of Azimuthal Stereographic Projection (left) and Lambert Azimuthal Equal-Area Projection (right). The circles indicate contours of equal distortion, with labels indicating distortion in percent.

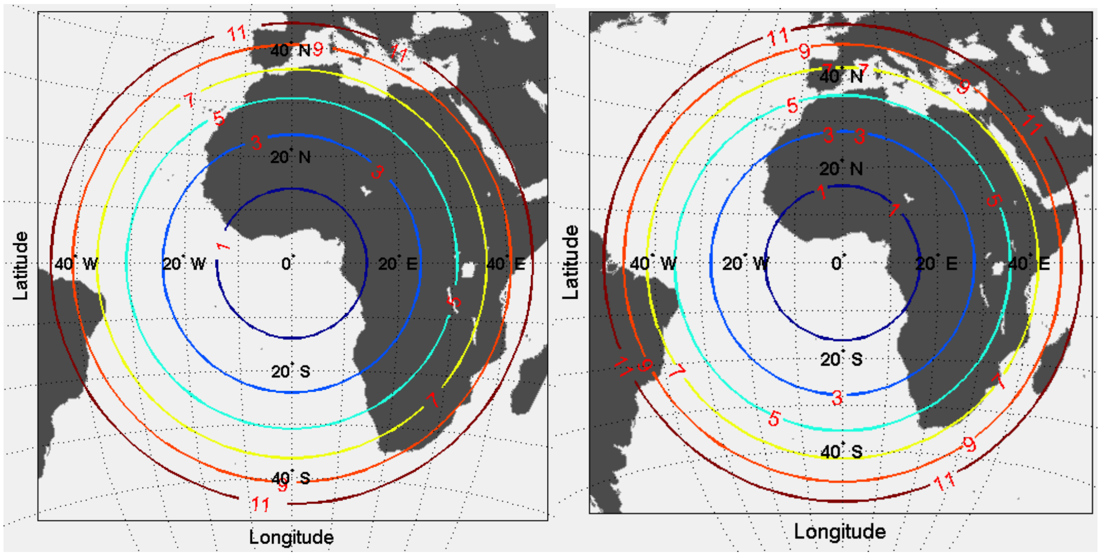

Figure 4.4 Scale distortion (%) of Lambert Azimuthal Equidistant Projection (left) and Lambert Azimuthal Equal-Area Projection (right). The circles indicate contours of equal distortion, with labels indicating distortion in percent.

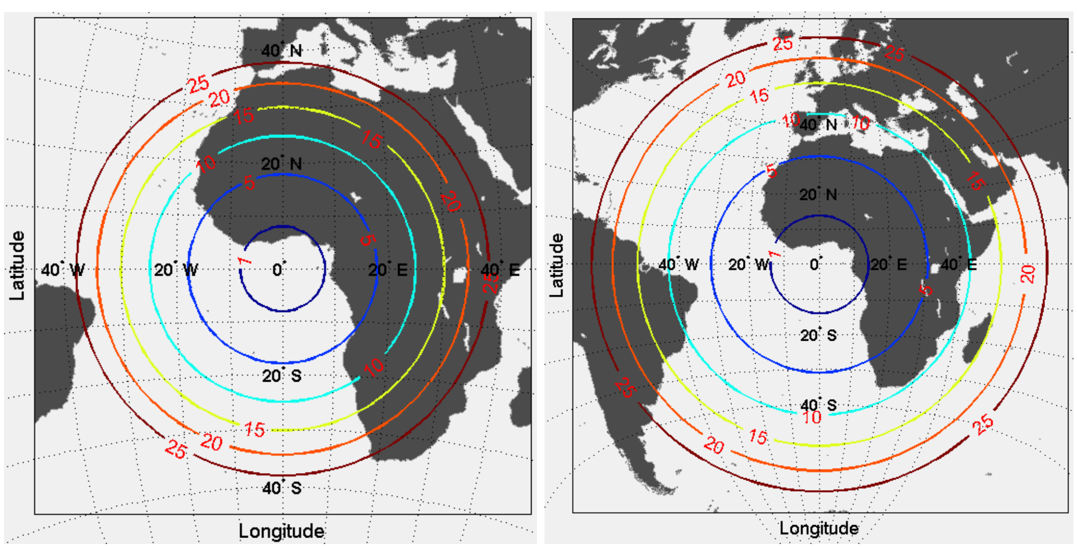

Figure 4.5 Area distortion (%) of Azimuthal Stereographic Projection (left) and Lambert Azimuthal Equidistant Projection (right). The circles indicate contours of equal distortion, with labels indicating distortion in percent.

From Figure 4.3 (right) a user can see that the area corresponding to the radius of 40 degrees can be covered with scale distortion of less than 7%. Therefore the origins of each Lambert Azimuthal Equal-Area projection can be separated by as much as 80° while maintaining less than 7 % scale distortion. The proposed location of projection centers are: W 120°, W 80°, 0°, E 60°, E 120°, E 180° in longitude, and S 90°, S 45°, 0°, N 45°, N 90° in latitude. The forward ellipsoidal equations for Lambert Azimuthal Equal-Area projection (oblique aspect) are given as follows [10]:

$$x = BD \cos\beta \sin(\lambda - \lambda_0), \tag{4.8}$$

$$y = \left(\frac{B}{D}\right)[\cos\beta_1 \sin\beta - \sin\beta_1 \cos\beta \cos(\lambda - \lambda_0)], \tag{4.9}$$

where λ_0 is the central meridian. *B, D,* and β are intermediate variables that are calculated as:

$$B = R_q \left\{\frac{2}{[1 + \sin\beta_1 \sin\beta - \cos\beta_1 \cos\beta \cos(\lambda - \lambda_0)]}\right\}^{0.5}, \tag{4.10}$$

$$D = \frac{am_1}{(R_q \cos\beta_1)}, \tag{4.11}$$

$$\beta = \sin^{-1}\left(\frac{q}{q_p}\right), \tag{4.12}$$

where *a* is the semi-major axis of the ellipsoid and *e* is its eccentricity. R_q, *q*, and *m* are intermediate variables that are calculated as:

$$R_q = a\left(\frac{q_p}{2}\right)^{0.5}, \tag{4.13}$$

$$q = (1 - e^2)\left\{\frac{\sin\phi}{(1 - e^2\sin^2\phi)} - \left[\frac{1}{(2e)}\right]\ln\left[\frac{(1 - e\sin\phi)}{(1 + e\sin\phi)}\right]\right\}, \tag{4.14}$$

$$m = \frac{\cos\phi}{(1 - e^2\sin^2\phi)^{0.5}}. \tag{4.15}$$

β_1 is found from Equation (4.12) substituting q_1 for q. The values of q_1 and q_p are found from Equation (4.14) using $\phi = \phi_1$ to find q_1, and $\phi = 90°$ to find q_p. The value of m_1 is found from Equation (4.15) substituting ϕ_1 for ϕ. Note that ϕ_1 and λ_0 are geodetic latitude and longitude of map origin, respectively. The inverse ellipsoidal equations for Lambert Azimuthal Equal-Area projection (oblique aspect) can be calculated as follows [10]:

$$\begin{aligned}\phi = \phi + \frac{(1 - e^2\sin^2\phi)^2}{2\cos\phi}\Bigg[\frac{q}{1 - e^2} - \frac{\sin\phi}{1 - e^2\sin^2\phi} \\ + \frac{1}{2e}ln\left(\frac{1 - e\sin\phi}{1 + e\sin\phi}\right)\Bigg],\end{aligned} \tag{4.16}$$

$$\lambda = \lambda_0 + \tan^{-1}[x\sin c_e/(D\rho\cos\beta_1\cos c_e - D^2 y\sin\beta_1\sin c_e)], \tag{4.17}$$

where the intermediate variable q is calculated as:

$$q = q_p[\cos c_e\sin\beta_1 + (Dy\sin c_e\cos\beta_1/\rho)]. \tag{4.18}$$

The intermediate variables ρ and c_e are calculated as follows:

$$\rho = [(x/D)^2 + (Dy)^2]^{0.5}, \tag{4.19}$$

$$c_e = 2\sin^{-1}(\rho/2R_q), \tag{4.20}$$

and the variables D, R_q, q_p, and β_1 are found from Equations (4.11), (4.13), (4.14) and (4.12) respectively. If $\rho = 0$, then $q = q_p\sin\beta_1$, and $\lambda = \lambda_0$. Equation (4.16) requires iteration by successive substitution, using $\sin^{-1}(q/2)$ as the first trial ϕ on the right-hand side. Therefore, for the second projection level of the multi-projection system, the Lambert Azimuthal Equal-Area projection is selected because it has the smallest scale distortion compared to other azimuthal projections, and because it preserves the area.

4.2.3 Third Projection Level: Medium Scale Maps

The third projection level corresponds to Bing Maps levels of detail 5 – 7 inclusive and consists of a combination of azimuthal, conic and cylindrical projections. The Earth is subdivided into multiple zones, covering roughly the same geographical area. The zones are projected using different projection depending on their central latitude. The Northern hemisphere consists of one 10° azimuthal zone projected using Stereographic Azimuthal projection centered at the North Pole, one 10° cylindrical zone projected using Mercator projection centered at the Equator and seven 10° conic zones projected using Conic Equidistant Projection with projection centers located between the North Pole and the Equator. The Southern hemisphere is identical to northern hemisphere however, the projection order is reversed.

The zones maintain area distortion less than 3.1%, scale distortion less than 1.6%, and angular distortion less than 0.1°. For more information please refer to Appendix A. The Conic Equidistant projections use the Kavraisky [33] analytical criterion for selecting standard parallels. Equations (2.30), (2.31) and (2.32) are used to find the optimal standard parallels with constant $K = 7$. The projected zones are connected together using geodesic distance from the center of one zone to the center of another zone. For the Northern hemisphere the projection zone centers are located at latitudes of 0°, 15°, 25°, 35°, 45°, 55°, 65°, 75°, 90° for a total of nine projection zones. For the South hemisphere, the same distribution is applied. Figure 4.6 shows the global coastline given in geodetic ϕ, λ (WGS84) projected using a combination of different projections.

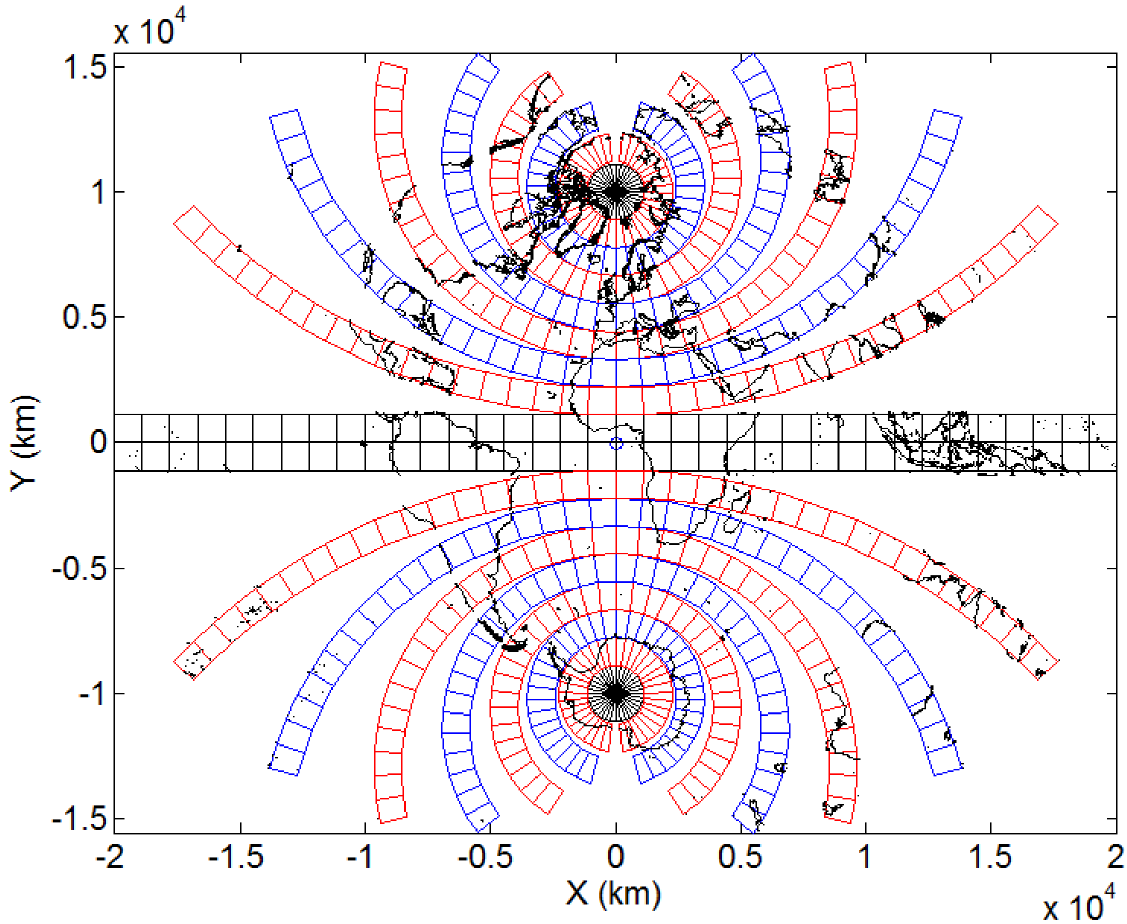

Figure 4.6 Global coastline and graticule projected into 18 different zones using conic (red, blue), azimuthal (black: poles) or cylindrical (black: equator) projections. Grid is 10° x 10°.

The use of a different map projection for different geographic regions results in the gaps between projected zones. The solution to this problem is to constrain the window in the middle of the region of interest so that the gaps are not visible. When the user is interested in other areas, the map server can simply roll the zones in a cycloid sense to match the area of interest. Figure 4.7 shows how the Earth's Northern hemisphere is subdivided into 9 zones that have identical North/South geographic extent. The central meridian corresponds to λ = 0° and x = 0. The user can only see the map within the window of observation (black square).

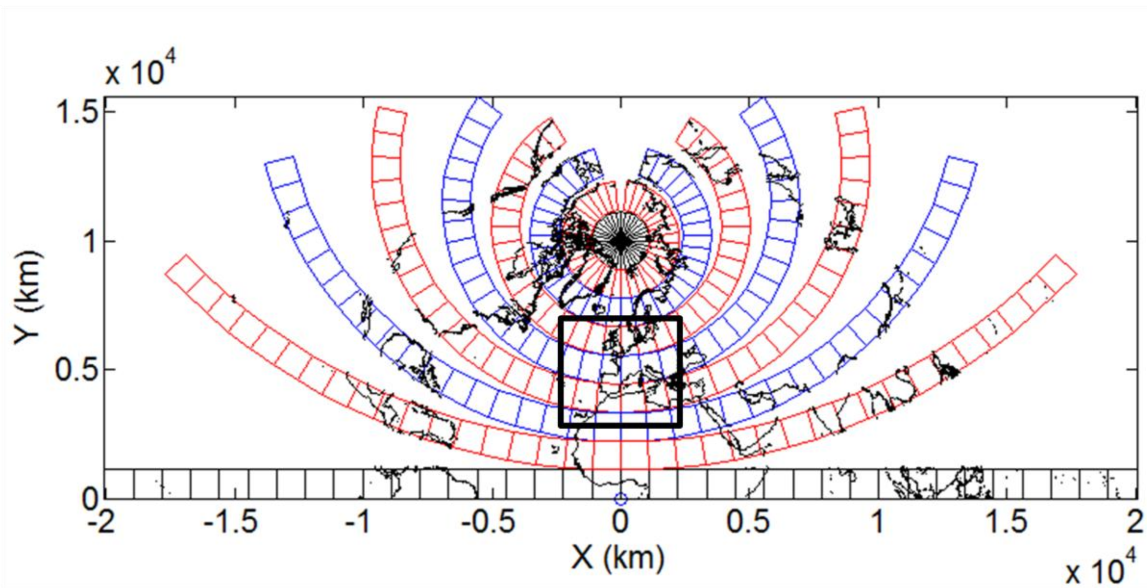

Figure 4.7 Earth's Northern Hemisphere composed of 9 projection zones. User's window of observation is shown as a black square. Central meridian is at 0° longitude.

As the user zooms in/out and moves around the map, the zones automatically roll in a cycloid mode so that the gaps between zones are not visible on the screen. Figure 4.8 shows how the map of Earth's Northern hemisphere is adjusted to a central meridian corresponding to 100° longitude.

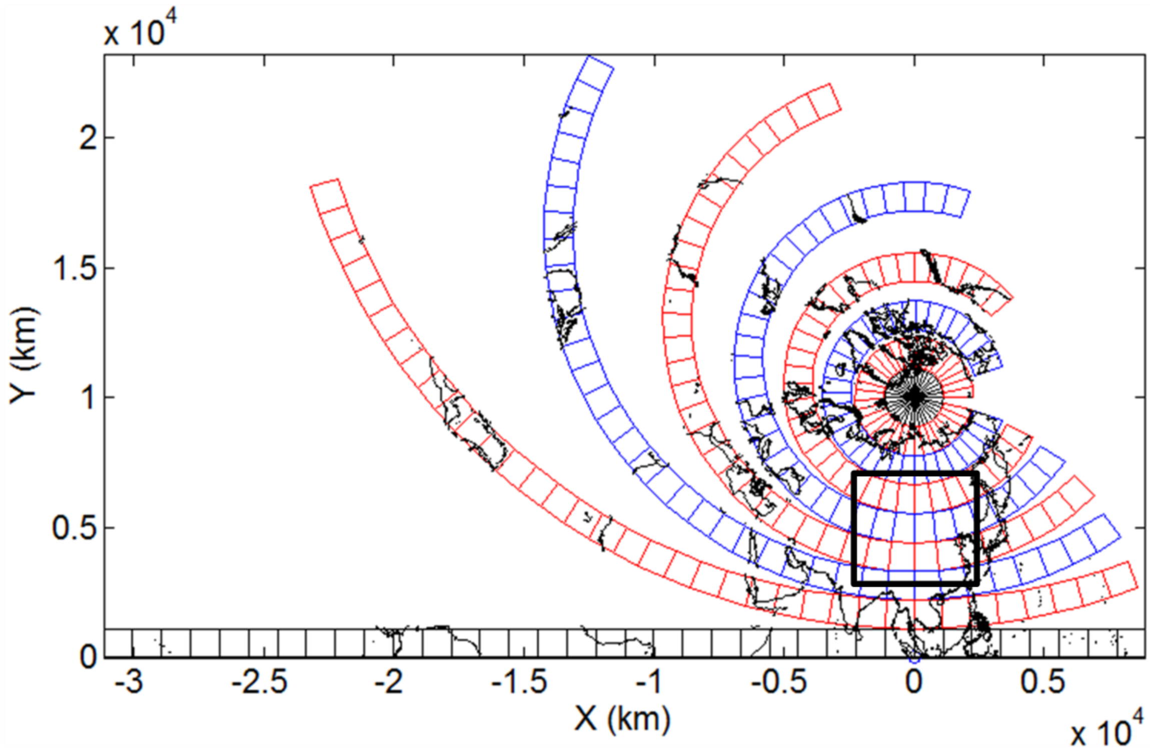

Figure 4.8 Rotated Earth's Northern Hemisphere composed of 9 projection zones. Central meridian is at 100° longitude. Window of observation (black square).

The advantage of this approach is that it allows minimization of map distortion within the window of observation by using a combination of projection zones. The same zones are then used to minimize map distortion in a different geographic region by following a simple rolling. The window of observation is constrained based on the map scale/level of detail so that the gaps between zones are not visible. The gaps are not visible because their size is smaller than the pixel size at that level of detail. For example, at Bing Maps zoom level 11, the pixel size at the equator is 76.44 meters, and the maximum gap between MPS projection zones is about 25 meters.

4.2.4 Fourth Projection Level: Large Scale Maps

The fourth projection level is used for large scale maps. It corresponds to Bing Maps zoom levels of 8 and above. For this projection level, the Azimuthal Stereographic, Azimuthal Orthographic, Azimuthal Equidistant and Lambert Azimuthal Equal Area projections with WGS 84 ellipsoidal equations are examined. The scale, angular and area distortion of each projection are tested in 1° x 1° area at three latitudes: 0°, 45° and 80°. The results for maximum scale, area and angular distortion are summarized in Table 4.2.

Table 4.2 Maximum scale, area and angular distortion of different azimuthal projections

Azimuthal Projection	**Max Scale Distortion (%)**	**Max Angular Distortion [dms]**	**Max Area Distortion (%)**
Stereographic	0.02	N/A	0.03
Orthographic	0.02	0° 1' 28.40"	0.03
Equidistant	0.01	0° 0' 20.94"	0.01
Lambert Equal-Area	0.01	0° 0' 32.01"	N/A

From the values above, the Stereographic, Equidistant and Lambert Equal-Area Azimuthal projections are good candidates. The Stereographic Azimuthal projection and Equidistant Azimuthal projection are also the most suitable projections for minimizing map distortions in the polar regions [33]. Therefore for the fourth projection level, the Stereographic Azimuthal projection is selected. The fourth projection level consists of nine 1° x 1° tiles each projected separately using Azimuthal Stereographic Projection. The tiles are aligned together to form an overall area of 3° x 3°. Figure 4.9 shows the Canadian road network data (GRS80, scale 1:50K) [49], [50] re-projected using Azimuthal Stereographic Projection. The global map center is selected at latitude of 52.5° N and longitude of 105.5° W. Each 1° x 1° area is represented by a different colour.

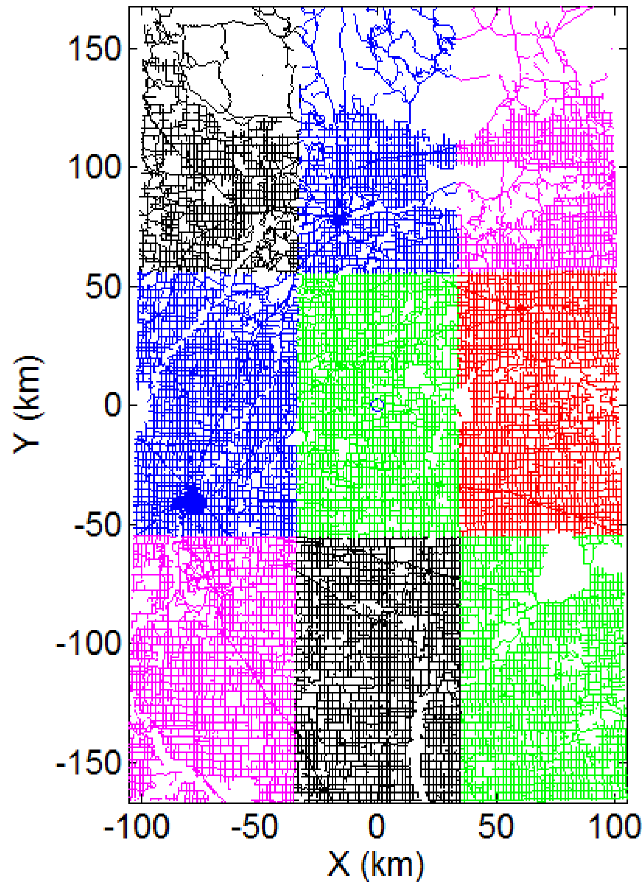

Figure 4.9 Canadian Road Network composed of nine 1° x 1° tiles (represented by different colours), covering a total area of 3° x 3°. Map center is N 52.5°, W 105.5°, located near Saskatoon, Saskatchewan.

A unique tile alignment procedure has been developed to connect/match multiple tiles together. Each tile is projected individually with respect to its local center. Then, multiple projected tiles are assembled with respect to the global map center using geodesic distance. This is a two-step process where each tile is translated and rotated with respect the global map center. First, the projected tile is translated by the geodesic distance from the global map center to the local center of the tile. The translation distance is calculated using equations for geodesic distance and WGS 84 parameters [30]. Second, the projected tile is rotated by the angle of meridian convergence between global map center and local center of the tile using 2D rotation equations. The equations for 2D clockwise rotation matrix are given as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\,\gamma & -sin\,\gamma \\ sin\,\gamma & cos\,\gamma \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \tag{4.21}$$

where (x', y') are the coordinates of a point (x, y) after rotation and γ is the rotation angle. The meridian convergence angle γ is calculated as follows [51]:

$$\gamma = (\lambda_{cm} - \lambda)\sin\phi_0, \tag{4.22}$$

where λ_{cm} is the longitude of the central meridian, λ is the longitude through the point, and ϕ_0 is the latitude of the center of the zone. Figure 4.10 illustrates the procedure for tile alignment. The center of tile A corresponds to global map center. The center of tile B corresponds to local center. Tile B is aligned with respect to tile A. First, tile B is translated by the geodesic distance from center of tile B to center of tile A. Second, tile B is rotated by meridian convergence angle at the center of tile B with respect to the center of tile A.

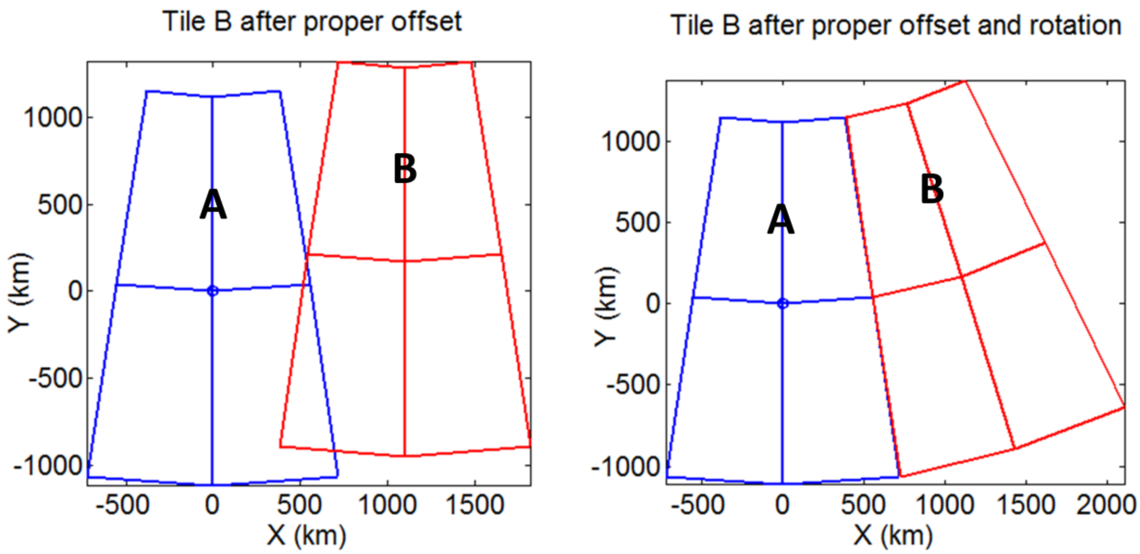

Figure 4.10 Two step procedure for tile alignment which includes offset (left) and rotation (right).

This approach minimizes map distortion because each tile maintains the geodesic distance from the center of the map. Even though the distortion for the entire map is reduced there is still however a systematic distortion which increases when moving away from the individual tile center. This is a normal behaviour of any Stereographic Azimuthal projection. There are also gaps that are produced on the edges between tiles. Generally the gaps depend on the latitude and the size of covered geographical area. Figure 4.11 shows the gaps created as a result of using six 20° x 20° tiles aligned together. Each tile is represented by different colour.

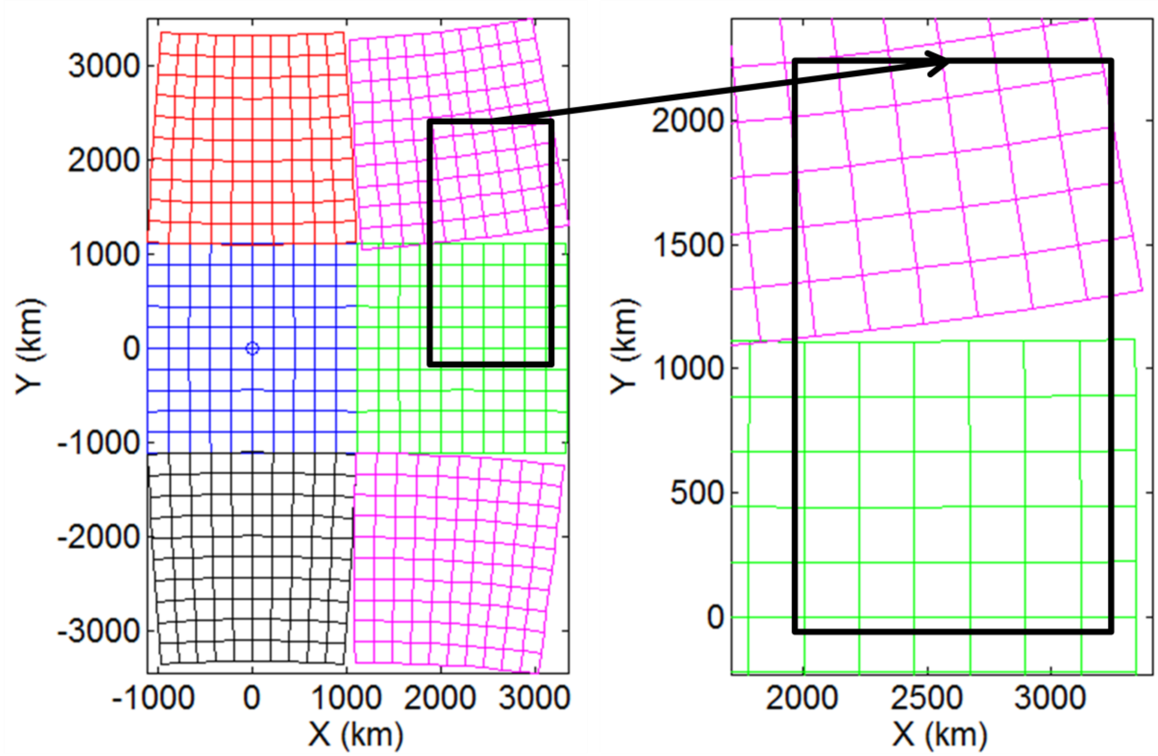

Figure 4.11 Six 20° x 20° tiles (left) are aligned together with respect to global map center (0, 0) resulting in map gaps (right). The maximum gap is about 150 km. The total area of the map is about 4000 km x 6000 km.

Since the total area in the figure above is very large, the size of the gaps also becomes large. However the gap size can be significantly reduced by reducing the size of projected tiles. In general the size of gaps for 1° x 1° tiles which roughly cover the area of 330 km by 330 km at the equator does not exceed 25 meters. Also, if the gap is smaller than pixel size at specific level of detail it will not be visible. One advantage of using 1° x 1° tiles is that the tiles can be rasterized and put into the cache system. Whenever the user is interested in specific area the tiles will be pulled from the system and assembled to form the map of the area of interest.

4.3 Inverse Projection

Inverse projection is the process of going from x and y map coordinates to latitude φ and longitude λ. Inverse projection steps change depending on projection level. Generally, for the first and second projection levels the process of inverse projection is straight forward because there is only one projection per level. Since the latitude and longitude of projection center is known, the inverse equations are used to obtain φ and λ from x and y coordinates. Inverse equations for Hammer projection (Equations (4.6), (4.7)) are used to get φ and λ for Projection Layer 1, and inverse equations for Stereographic Azimuthal projection (Equations (4.16), (4.17)) are used to get φ and λ for Projection Level 2. For the third and fourth projection layers the process is a little more complicated. The third projection layer consists of three different projections: Stereographic Azimuthal, Cylindrical Mercator and Equidistant Conic. Therefore, the first step is to identify which projection zone the x and y coordinates belong to. This is done by comparing the y coordinate against y_{min} and y_{max} limits of each projection zone. When y coordinate falls within the limits of a certain zone, such that $y_{min} \geq y > y_{max}$, that zone is identified. The second step is to offset y coordinate by geodesic distance from the center of identified projection zone (e.g., φ = 35°, λ = 0°) to map origin (φ = 0°, λ = 0°). For Northern hemisphere the new *y* coordinate y_{new} can be found from equation:

$$y_{new} = y_{old} - s, \tag{4.23}$$

where y_{old} is the old y coordinate and s is the geodesic distance between map origin and the center of projection zone. For Southern hemisphere, Equation (4.23) stays the same but the sign is reversed. The third step is to use inverse projection equations for that specific projection zone with the required parameters to obtain φ and λ. For example, if the zone is identified as Conic Equidistant projection zone #3 (third conic zone from the Equator), the inverse equations for Conic Equidistant projection will be applied with the values of standard parallels corresponding to the zone #3. Figure 4.12 illustrates the first two steps of inverse projection process for Projection Level 3. Here, only one Conic Equidistant projection zone is shown.

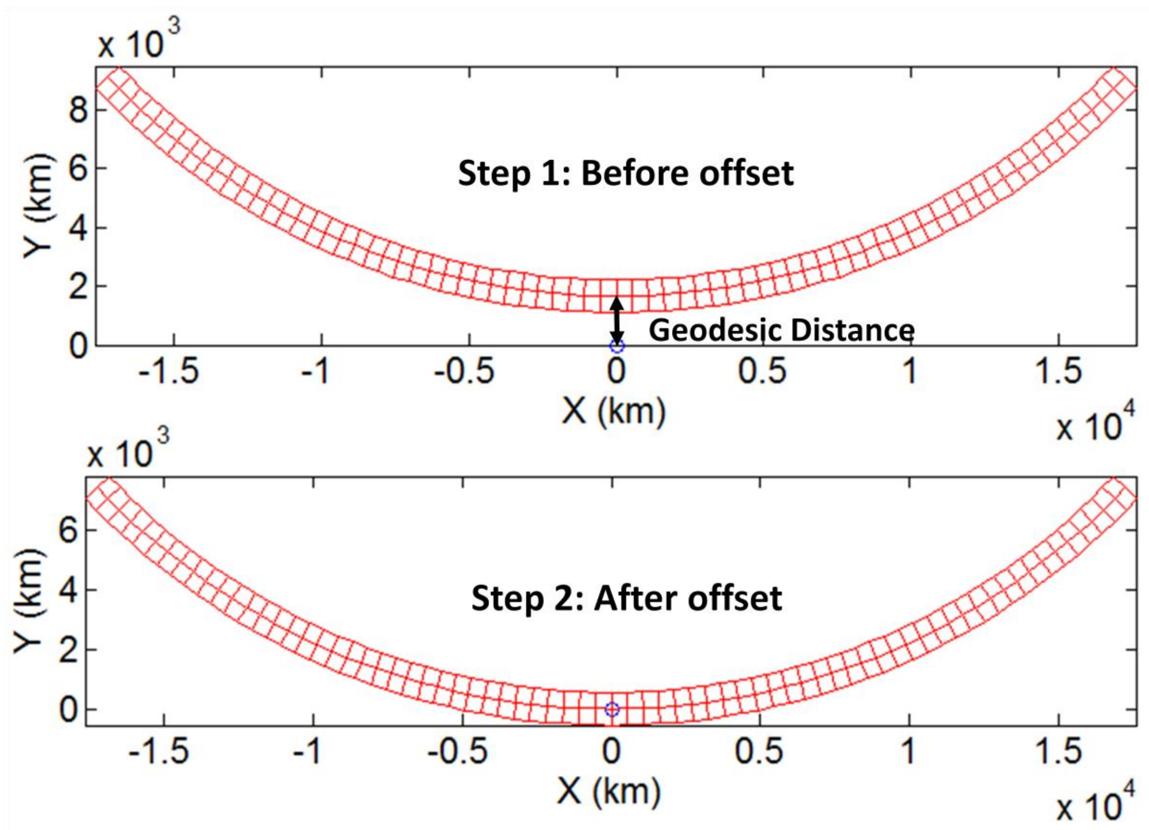

Figure 4.12 The first two steps of inverse projection procedure for Projection Layer 3. First, the projection zone is identified (e.g. conic zone #3). Second, the projection zone is offset to its original position.

The forth projection level consists of Stereographic Azimuthal 1° x 1° tiles; however, the tiles are offset by the geodesic distance from the center of the map and rotated by the meridian convergence angle from the center of the map to the local center of the tile. Therefore, before the inverse projection equations can be applied, the tile with x, y coordinates must be rotated and translated to its original location. Thus, depending on the projection level different steps are applied to get φ and λ from x and y coordinates.

4.4 Summary

In this chapter I developed the Multi-Projection System (MPS) and discussed its concepts in detail. The main goal of MPS is to minimize map distortion regardless of the geographical region and level of detail. To achieve this goal MPS uses four unique Projection Levels, which cover a range of Bing Maps zoom levels and minimize map distortion for that range of zoom levels. The most suitable map projection or combination of different projections was selected for each projection level, based on maximum scale distortion. For the first projection level, the Hammer projection was selected. For the second projection level, the Lambert Azimuthal Equal-Area projection was selected. For the third projection level, a combination of Stereographic Azimuthal, Mercator and Conic Equidistant projections was selected. For the fourth projection level the Stereographic Azimuthal projection was selected.

5 Implementation of Software Prototype

5.1 Introduction

In this chapter I design and implement a proof-of-concept software prototype using MATLAB programming language and vector data to demonstrate the concept of multi-projection system. The functionality of the software is also discussed. The prototype is limited to a certain amount of vector data used for different geographic areas for computational purposes. The first objective of the prototype is to demonstrate how different projection levels can be smoothly integrated together to minimize overall map distortion. The second objective of prototype is to compare cartographic distortion between WMP and Multi-Projection System. The design of the software is done using MATLAB Graphic User Interface (GUI). The code for individual functions and scripts is also written in MATLAB. Figure 5.1 shows the basic GUI for Multi-Projection System prototype.

The GUI design is relatively straight forward. The map window takes up the majority of GUI's space. The panel below allows users to move around the map and control the zoom level. The panel on the right provides users with basic mapping information and allows them to make distance measurements.

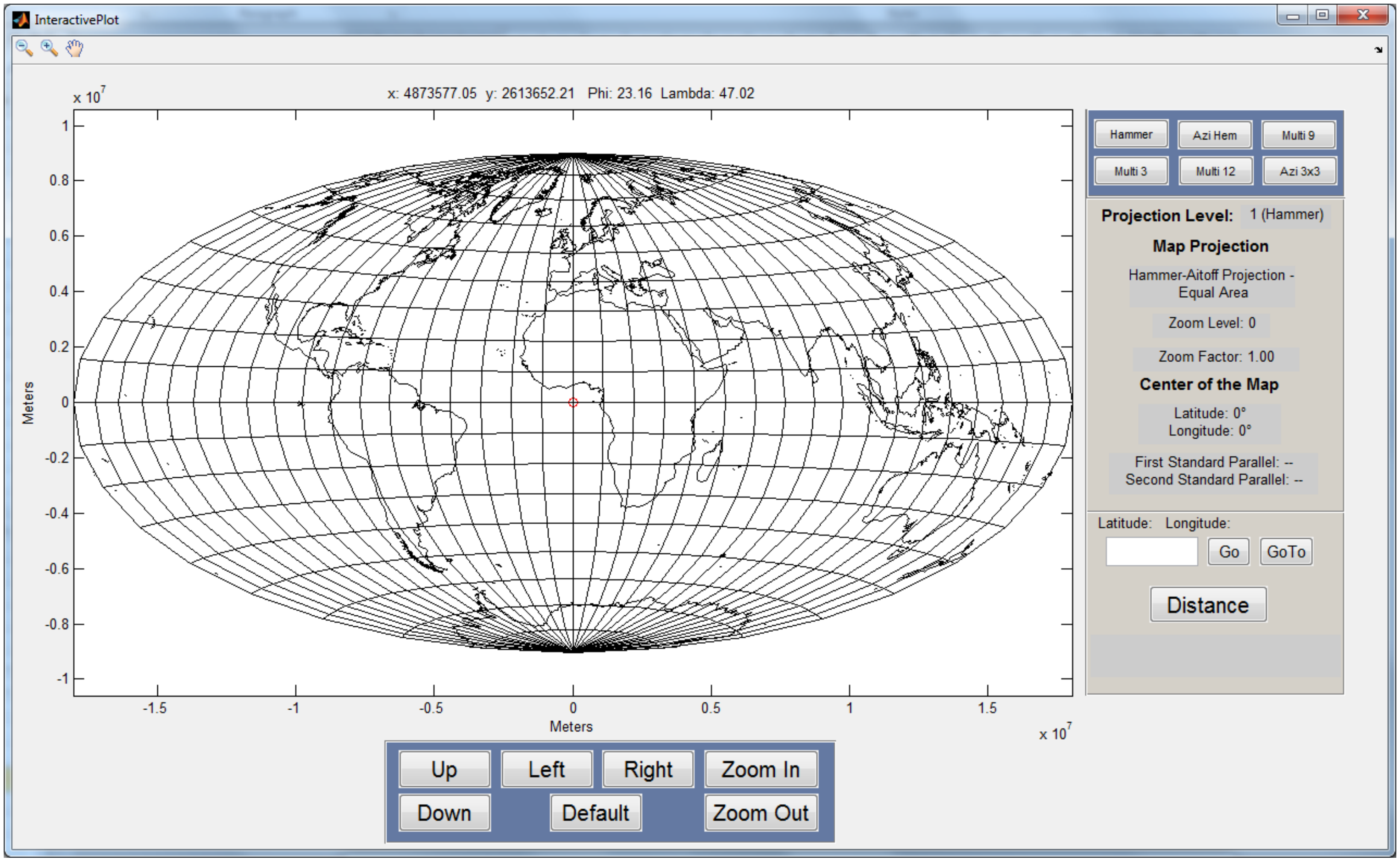

Figure 5.1 Multi-Projection System GUI showing Hammer projection, navigation buttons at the bottom and information panel on the right.

5.2 Functionality

The software is designed to allow users to smoothly transition from one projection level to another, when changing map projections. The software allows the user to move around the map, zoom in/out, measure geodesic distance, measure map distance (for comparison purposes), go to specific location on the map and other functionalities. The prototype has an information panel on the right which shows general information, such as projection level, name of map projection used, zoom level, zoom factor (the ratio of x axis at zoom level 0 to x axis at zoom level n), current center of the map, and location of standard parallels where applicable.

By pressing **Left**, **Right**, **Up** and **Down** buttons, the user can navigate the map in certain increments. The increments are set to 30°, 15° and 5° for the first, second and third projection layers, respectively. Clicking **Default** returns the user to a default latitude and longitude of 0°. The user can also go to the latitude and longitude of preference by typing it in the window next to **Go**, or by finding and clicking the location on the map screen using **GoTo** button. **Zoom In** and **Zoom Out** buttons let users zoom in or zoom out of the map roughly by a factor of 2. Every time the user presses **Zoom In** the zoom level is incremented by one and every time the user presses **Zoom Out** the zoom level is decremented by one. When the zoom level reaches a certain level, the projection level changes. For example, when zoom level reaches 5, the projection level changes from 2 to 3. The **Distance** button allows the user to measure both geodesic, and map distance. Map distance is calculated on the mapping plane using x, y coordinates of the selected points. Geodesic distance is calculated on the surface of WGS84 ellipsoid using φ, λ coordinates. The advantage of having both map and geodesic distances is that they can be compared with each other for analysis of map distortion. The current version of the software allows users to measure geodesic distance only on WGS84 ellipsoid.

The software has the functionality to display map coordinates of mouse cursor. When the user moves the mouse cursor around the map screen the software shows cursor location in the form of x, y coordinates and corresponding φ, λ coordinates. Both forms of coordinates are very helpful for navigation purposes. Figure 5.2 shows four separate windows of the multi-projection system prototype. Each window corresponds to a different projection level indicated by the number in upper left corner. As the user zooms in/out projection level automatically changes. When the user zooms in, more detail is added to the map. When the user zooms out the detail is reduced. The starting projection level is level 1, which shows the Earth's coastline projected using Hammer projection. The second projection level shows the Earth's coastline and country boundaries using Stereographic Azimuthal Projection, etc.

The ability to measure accurate distances and areas is important to professional users. This software provides distance measurement capabilities where the distance is calculated between two points on the map or ellipsoid. The area measurements are beyond the scope of this thesis. Figure 5.3 shows map and geodesic distance at zoom level 2 measured using software prototype. Map distance shown = 3363661.43 m, ellipsoidal distance shown = 3342421.46 m showing an average distortion of 0.64%.

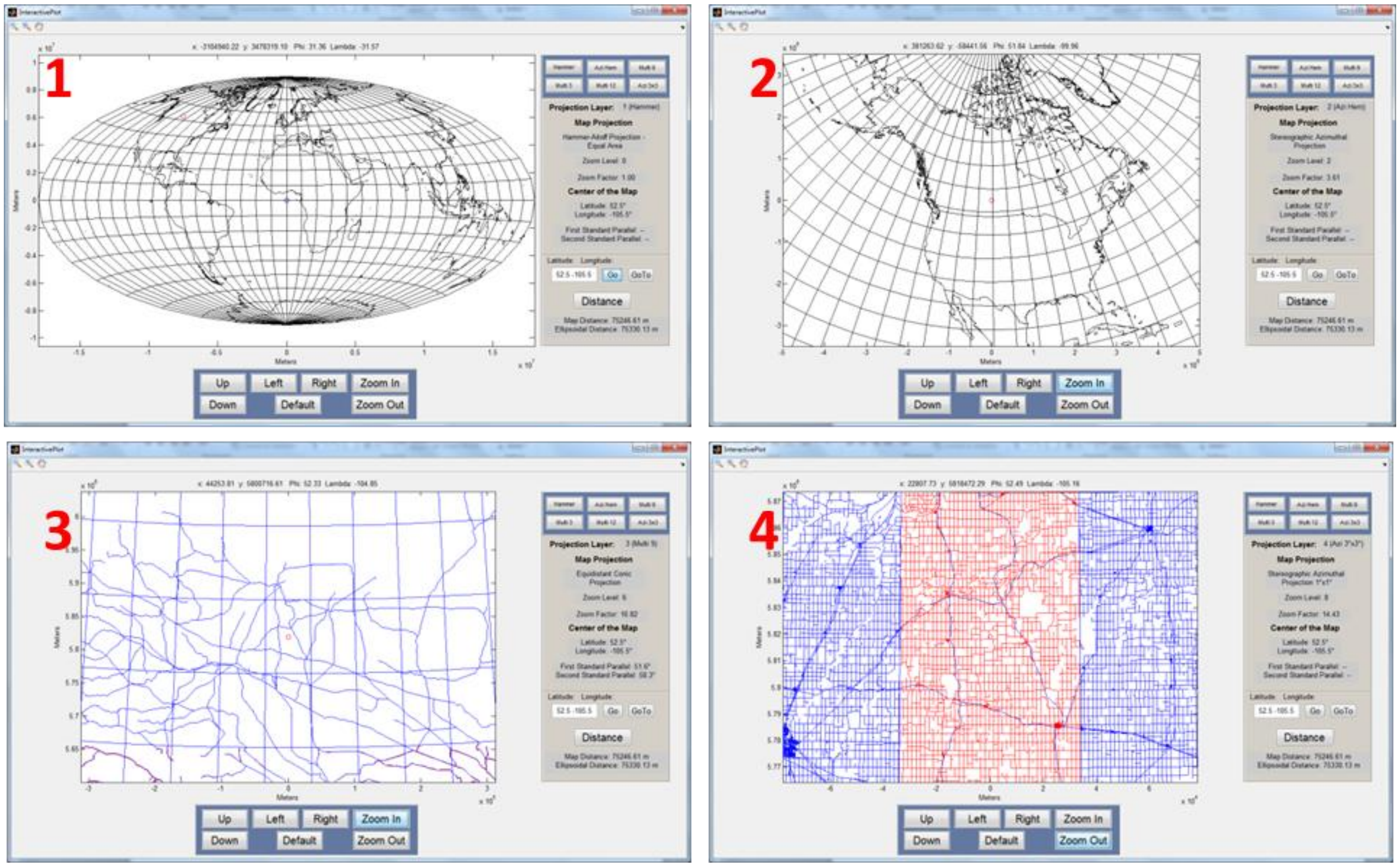

Figure 5.2 Four Projection Levels (indicated by number) of Multi-Projection System. The projection level changes when the user zooms in or zooms out.

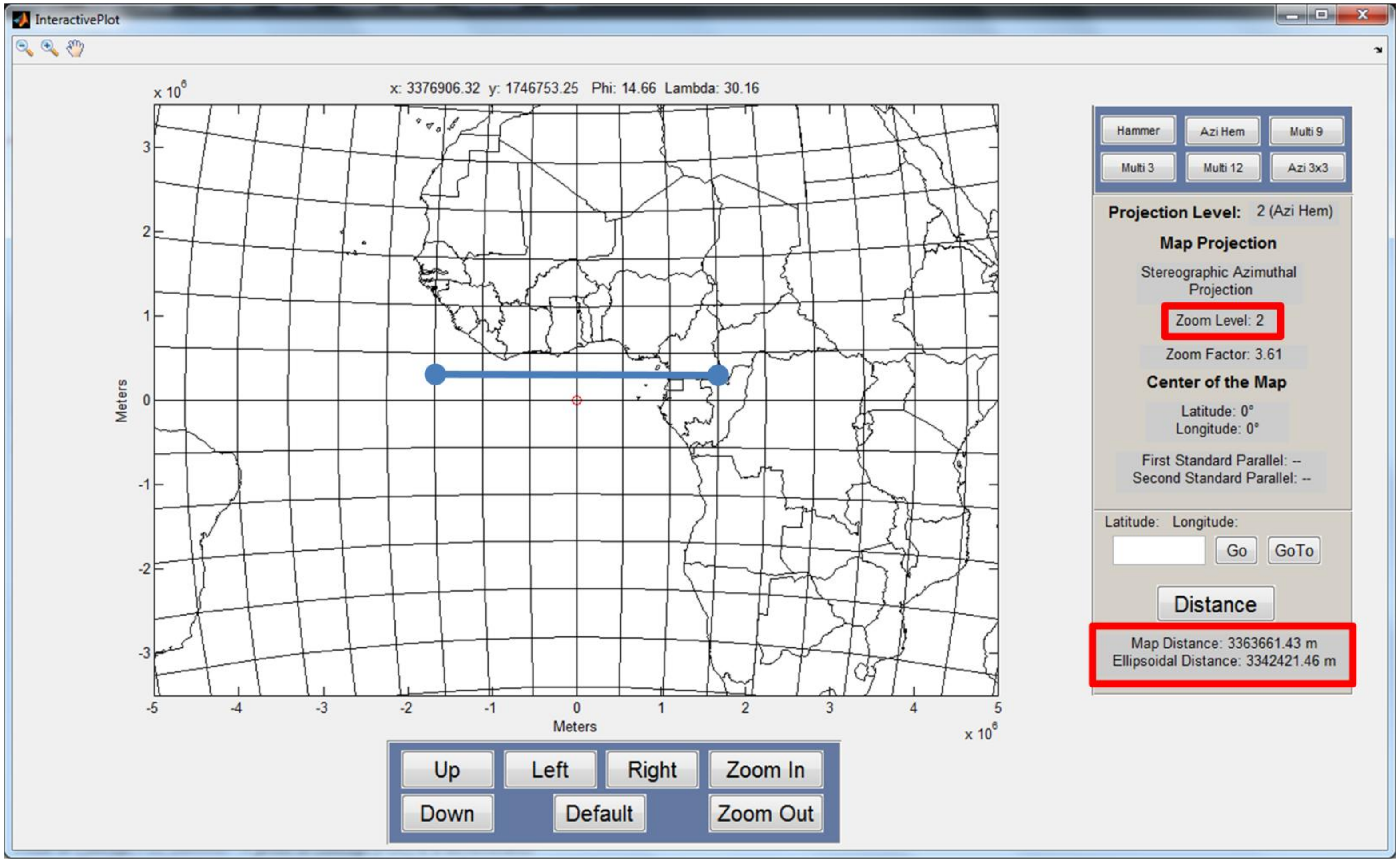

Figure 5.3 The measurement (blue line) of map and geodesic distance (red box), and zoom level (red box) over Africa.

5.3 Re-projection of Raster Data

Although the software prototype was implemented using vector data only, the procedure for re-projection of raster data is also considered here. Re-projection is the process of transforming an image from one projection system to another [52]. The data are re-projected differently depending on whether they are in vector or raster format. Re-projection of vector data is straight forward and involves two steps. First, the map coordinates of a pixel are converted back to corresponding geodetic (latitude, longitude) co-ordinates using the inverse projection equations. Second, the geodetic coordinates of the selected pixel are converted into the desired projection by applying the mathematical equations of that projection. The re-projection of raster data is more complicated because of the structure of raster imagery. The raster imagery consists of rows and columns of pixels with different intensity values. Therefore when raster imagery is re-projected, the original raster grid must be transformed into a new raster grid. The cells of the new grid are filled with the corresponding cells from the original image. However, since the cells of two raster grids do not usually coincide with each other, the interpolation or resampling must be performed to estimate the pixel values in the empty cells. The three most commonly used interpolation/resampling methods are nearest neighbor, bi-linear, and cubic interpolation [52]. Figure 5.4 shows the satellite imagery of the Earth, originally displayed in Plate Carree projection. The original image is resampled from 10020 by 5010 pixels to 1800 by 900 pixels to reduce the computational effort of re-projection. Figure 5.5 shows the re-projected image of the Earth from Figure 5.4. The image is displayed in Hammer projection and includes 10° vector grid overlaid on top.

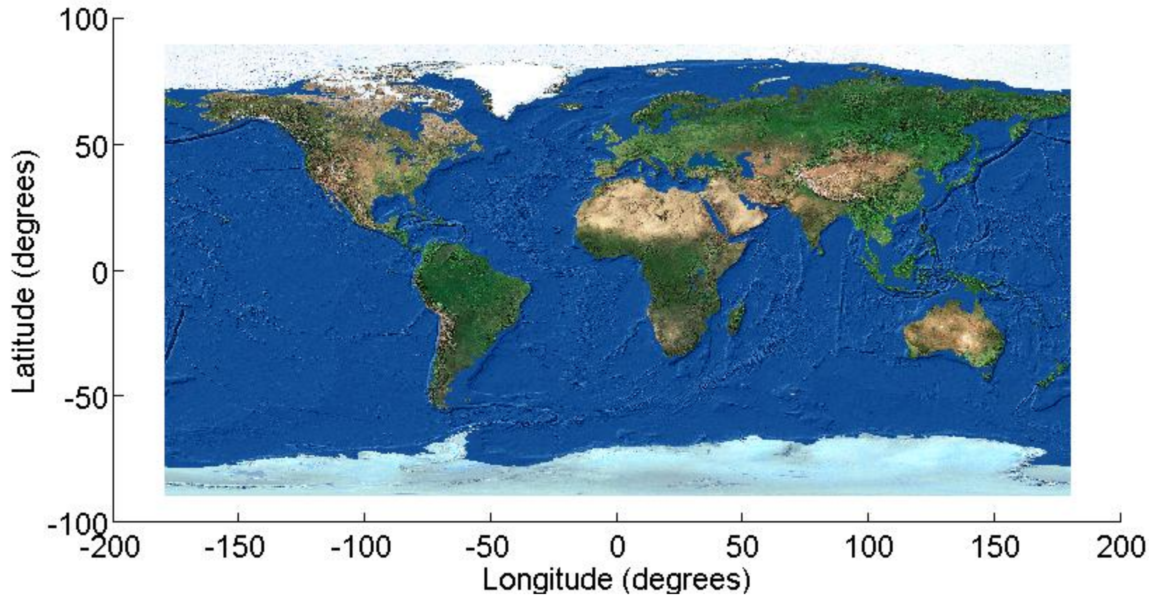

Figure 5.4 Georeferenced image of the Earth displayed in Plate Carree projection [53]. The axes indicate geographic latitude and longitude.

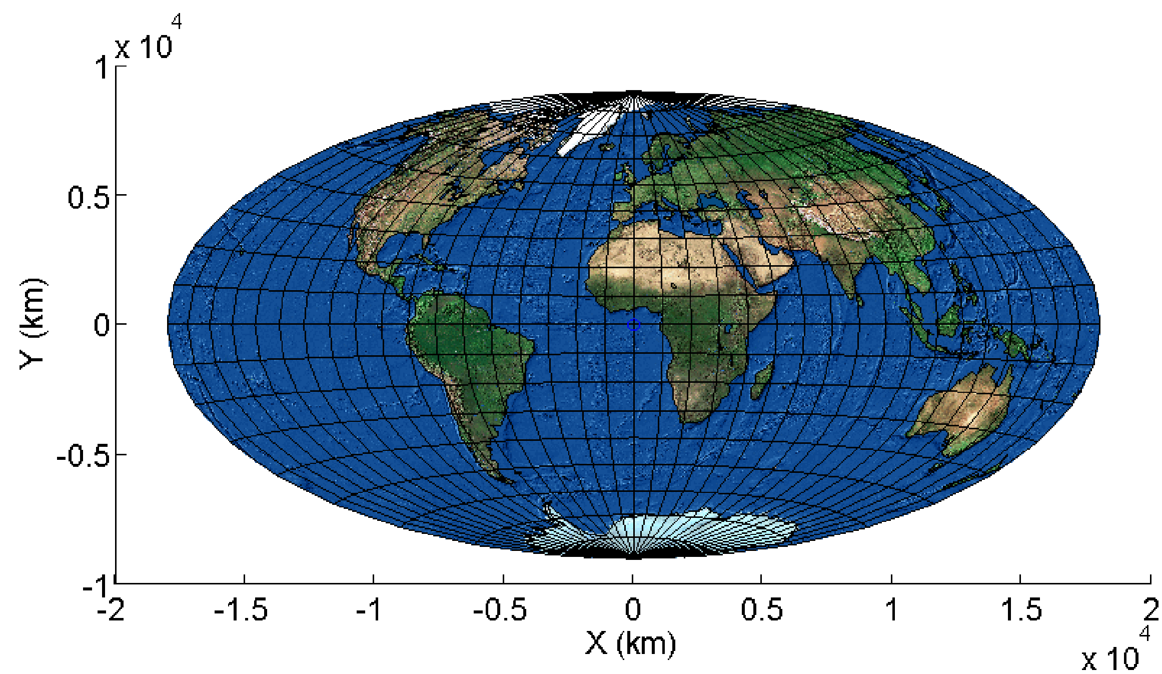

Figure 5.5 Re-projected image of the Earth displayed in Hammer projection including 10° graticule. The axes show *x* and *y* map coordinates.

The raster data are not implemented into the software model. Generally, re-projection of raster data is computationally slower, due to the extra effort required for interpolation.

5.4 Summary

In this chapter I designed and implemented a software prototype to demonstrate the concept of using multiple projections for global map visualization. The model's GUI and code were developed in MATLAB programming language, following the design specifications outline in Chapter 3. The software allows users to navigate the map, zoom in/out, and make accurate distance measurements. The chapter finishes by discussing procedures and examples for the re-projection of raster data.

6 Results and Analysis

This chapter conducts an extensive analysis of the multi-projection approach. Fist, the polar region coverage of multi-projection system is analyzed. Second, overall map distortions are analyzed. Third, the use of ellipsoidal equations is analyzed as compared to their spherical counterparts. Lastly, the multi-projection system is compared to Bing Maps at different levels of detail and geographic regions in terms of map coverage and map distortion.

6.1 Analysis of Polar Region Coverage

The original limitations of the Popular Web Mapping Services (PWMS) have been discussed throughout this thesis and are explained in detail in Section 3.2.3. These limitations were the main motivation of our research and the development of multi-projection approach. In general, the limitations can be summarized as lack of polar coverage, systematic distortion and use of spherical equations instead of ellipsoidal. Therefore, it is important to examine our multi-projection system in the context of these limitations and to analyze how effectively the software deals with them.

Lack of polar coverage is the first major limitations of PWMS. The use of WMP limits the coverage of the poles to ±85° in latitude. The multi-projection approach is able to deal with this limitation by using a combination of different projections. By using multiple projections the poles can be fully covered with minimal distortion at any projection level above 1. Since the first projection level is used for the entire world, it cannot show all regions in undistorted manner. The Hammer projection, used in first level of detail, is able to cover poles better than WMP due to its lower systematic distortion. Figure 6.1 (top) shows Antarctica (black rectangle) in the multi-projection system at projection level 1. Figure 6.1 (bottom) shows Antarctica (black rectangle) in Bing Maps at zoom level 1. The polar regions on Figure 6.1 (top) are covered with less distortion.

Figure 6.2 shows coverage of Antarctica at projection levels 2 and 3. Projection level 2 shows the entire continent composed of one projection. Projection level 3 shows Antarctica composed of one Stereographic Azimuthal projection layer joined together with Conic Equidistant projection layers. Note that for projection level 3, the level of detail ranges from 5 to 7. The window of observation is indicated by the black box in Figure 6.2 (right), so only the area shown within the window of observation will be displayed to the user.

Projection level 4 covers any geographic region, including polar regions at levels of detail 8 and above. Since the tiles are 1° x 1°, they can be used to cover any geographic region with small distortion. The multi-projection approach is able to successfully cover regions above ±85° in latitude. Please refer to Table 4.1 for correspondence between MPS projection levels and Bing Maps zoom levels.

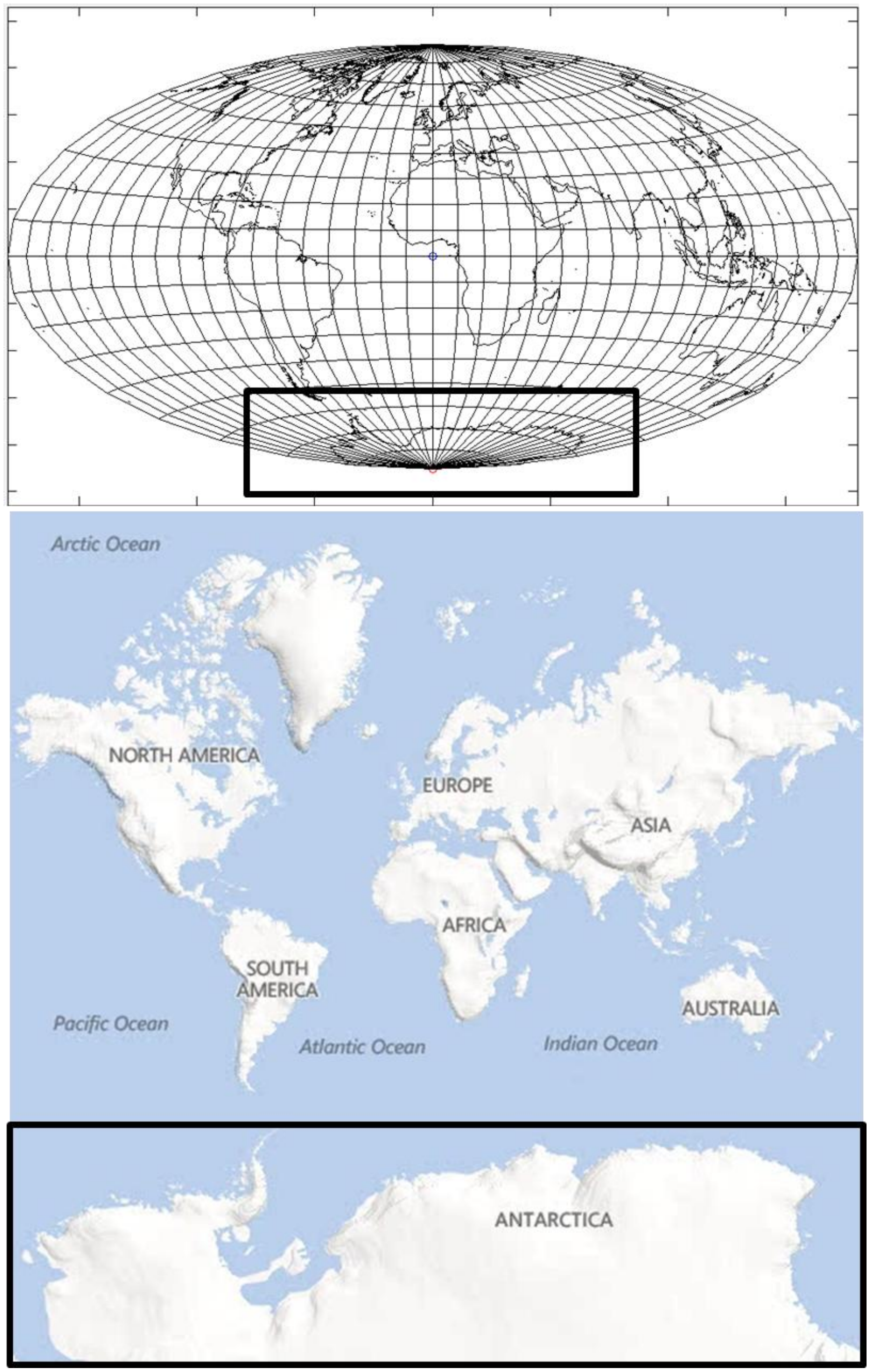

Figure 6.1 Antarctica coverage of Multi-Projection System at Projection level 1 (top) and Bing Maps at zoom level 1 (bottom) [2]

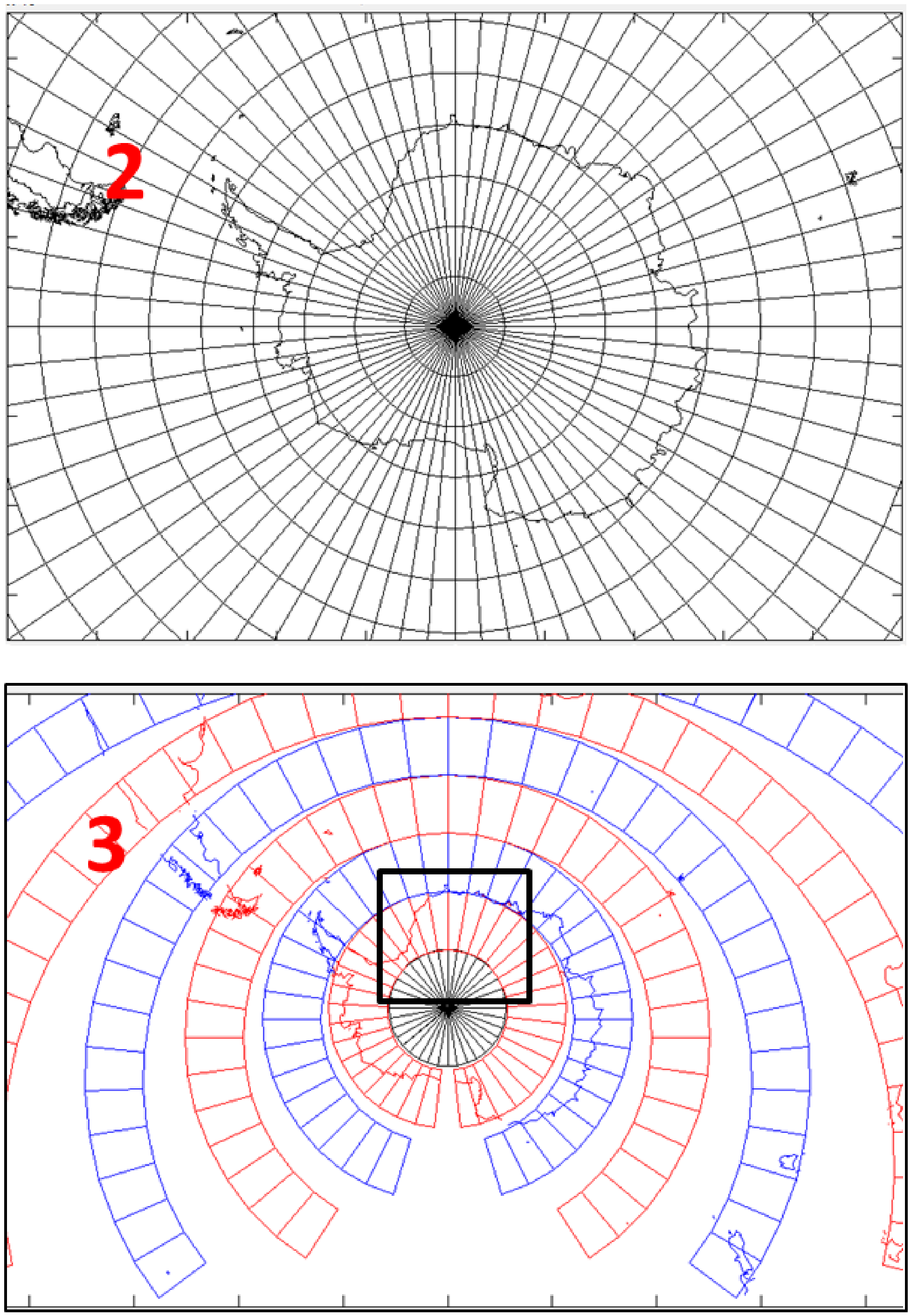

Figure 6.2 Antarctica coverage of MPS at projection levels 2 (top) and 3 (bottom)

6.2 Analysis of Systematic Distortion

The second major limitation of PWMS is large systematic distortion. This section provides quantitative analysis of area, scale, and angular distortion of each projection level and compares them to the distortions of WMP.

Table 6.1 Maximum scale, angular and area distortion for each projection level of Multi-Projection System and Bing Maps WMP.

Projection	Max Scale Distortion (%)	Max Angular Distortion [dms]	Max Area Distortion (%)
MPS Level 1	136	34° 23’ 53”	0
MPS Level 2	6.4	7° 7’ 24”	0
MPS Level 3	1.6	0° 7’ 27”	3.1
MPS Level 4	0.02	0	0.03
Bing Maps WMP	1147	0° 22’ 48”	13165

The maximum scale, angular and area distortion of MPS Level 1 is measured at 85° latitude (Figure 5.2 (1), Figure 6.1 (top)). The maximum scale and area distortion for WMP is measured at 85° latitude (Figure 6.1 (bottom)). The maximum angular distortion for WMP occurs at the equator. For MPS level 2 projection center intervals are separated by a maximum of 80°, therefore maximum distortion occurs at 40° from individual projection center (Figure 4.3 (right), Figure 5.2 (2), Figure 6.2 (top)). For distortion computations, the projection origin is set at the equator. For MPS level 3 each (conic, cylindrical, azimuthal) 10° zone is considered individually. The maximum area, scale and angular distortion for entire projection level are a combination of maximum area, scale and angular distortion of each zone (Figure 4.6, Figure 5.2 (3)). For MPS level 4, maximum distortions are measured for 3° x 3° area at latitudes of 0°, 45° and 80°. The largest distortion occurs at latitude of 45° and is indicated in the table above (Figure 4.9, Figure 5.2 (4)).

The scale and area distortions of Web Mercator Projections do not change with level of detail. For example, at latitude 85° and level of detail 1, the scale distortion is 1147%, at latitude 85° and level of detail 12 the scale distortion is still 1147%. To adjust for large systematic distortions, Bing Maps have to constantly recalculate and adjust map scale.

Based on the data in Table 6.1, the MPS significantly reduces scale and area map distortion as compared to WMP. At projection levels 2 – 4, the angular distortion of MPS is smaller than angular distortion of WMP. Note that at projection level 1 MPS has larger angular distortion than WMP; however, projection level 1 is only used for world map view and the view compromises well between area, scale, and angular distortion.

6.3 Analysis of Mathematical Rigour for Cartographic Projections

WMP uses spherical equations of Mercator projection to map WGS 84 geodetic latitude and longitude instead of ellipsoidal equations. The resulting problems were extensively discussed in Section 3.2.3.

MPS uses ellipsoidal equations for projection levels 2 – 4. For the first projection level, spherical equations of Hammer projection are used. At small scales, the use of spherical or ellipsoidal equations does not make any difference, because the size of the pixel is larger than the difference between the two. At medium and large scales, the use of ellipsoidal equations provides a better approximation of the Earth surface which results in more accurate measurements and representations of area. That is why MPS uses ellipsoidal equations for projection levels greater than 1. Figure 6.3 shows the difference between ellipsoidal and spherical equations for Azimuthal Stereographic projection. The projection center is identical in both cases. At smaller scale Figure 6.3 (top) the difference between ellipsoidal and spherical equations is not very noticeable, however, at larger scales Figure 6.3 (bottom) the difference becomes apparent (e.g., 1.2 km as indicated in the figure).

One notable set back of use of ellipsoidal projection equations as compared to the spherical projection equations tends to be the increased computational effort, since the ellipsoidal equations are usually more complicated. This should not be a problem because technology is constantly evolving. Powerful supercomputers can be used in tile generating process, working with large quantities of data at a time.

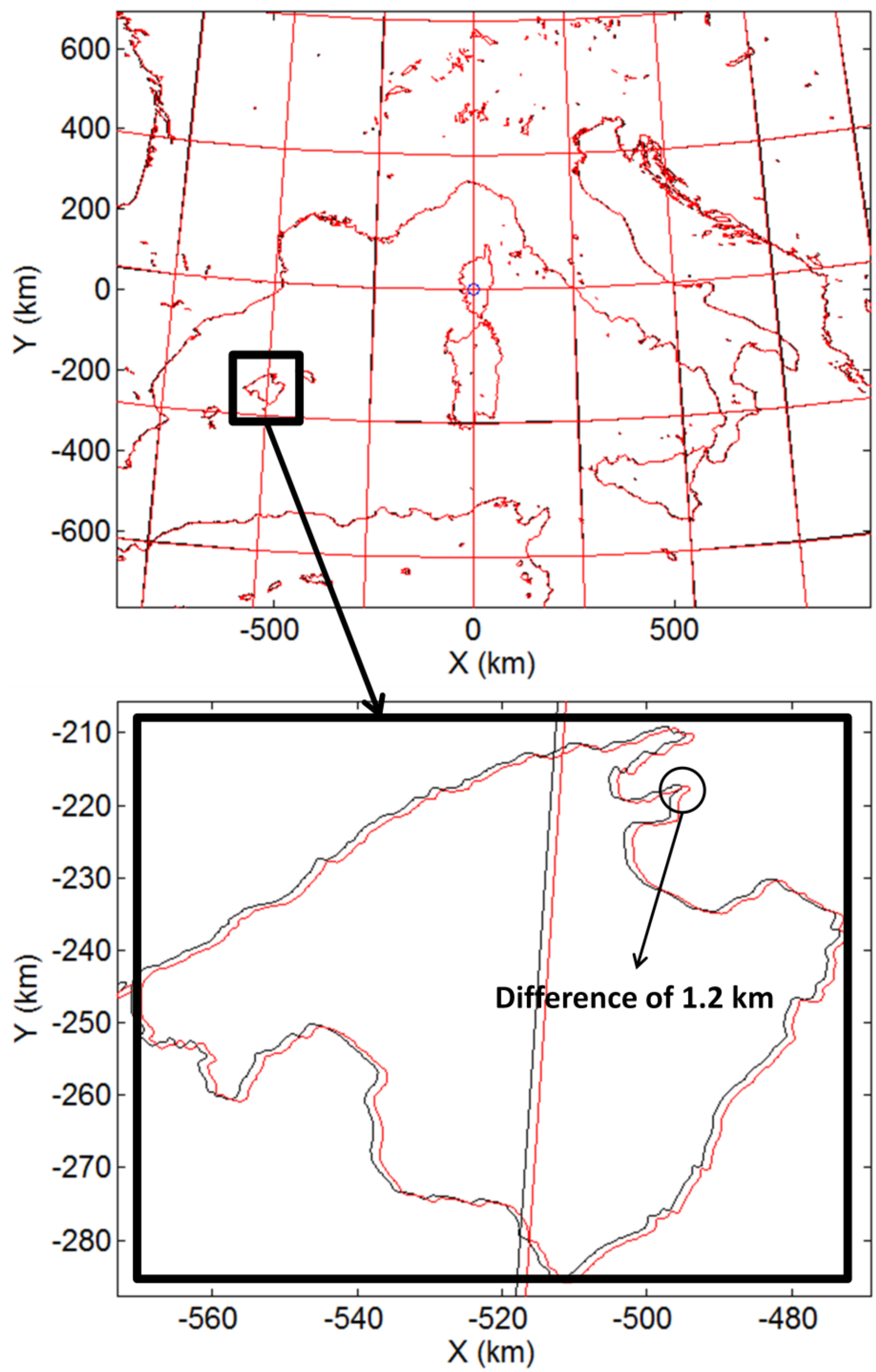

Figure 6.3 Comparison of ellipsoidal (black) and spherical (red) equations for Azimuthal Stereographic projection. The projection center is at N 42°, E 9°. The grid is 3° x 3°.

6.4 Comparison of MPS to Bing Maps

This section compares MPS to Bing Maps in terms of visual cartographic distortion. Regions in Europe and North America are selected at different latitudes for comparison purposes. The comparison mostly focuses on mid-latitudes and northern latitudes where larger distortion occurs in Bing Maps. Different levels of detail are also considered.

First, North America is examined. Figure 6.4 compares North America between Bing Maps and MPS at level of detail 2. The center of projection is set at latitude of N 49°. MPS is indicated by black coastline outline, and is overlaid on top of Bing Maps. The grid for MPS is set at 5° x 5°. The most obvious deviations between Bing Maps and MPS happen away from the projection center, near North Pole. This is reasonable since the largest distortion for MPS at this level of detail occurs furthest away from the projection center, and the largest distortion for Bing Maps happens closer to polar regions, where Bing Maps are unable to represent the area correctly.

Figure 6.5 compares lower portion of North America at zoom level 3, centered at N 40° latitude. At this level of detail and latitude the difference between Bing Maps and MPS is noticeable but not substantial. For example, near the edges the difference between two maps exceeds 200 km.

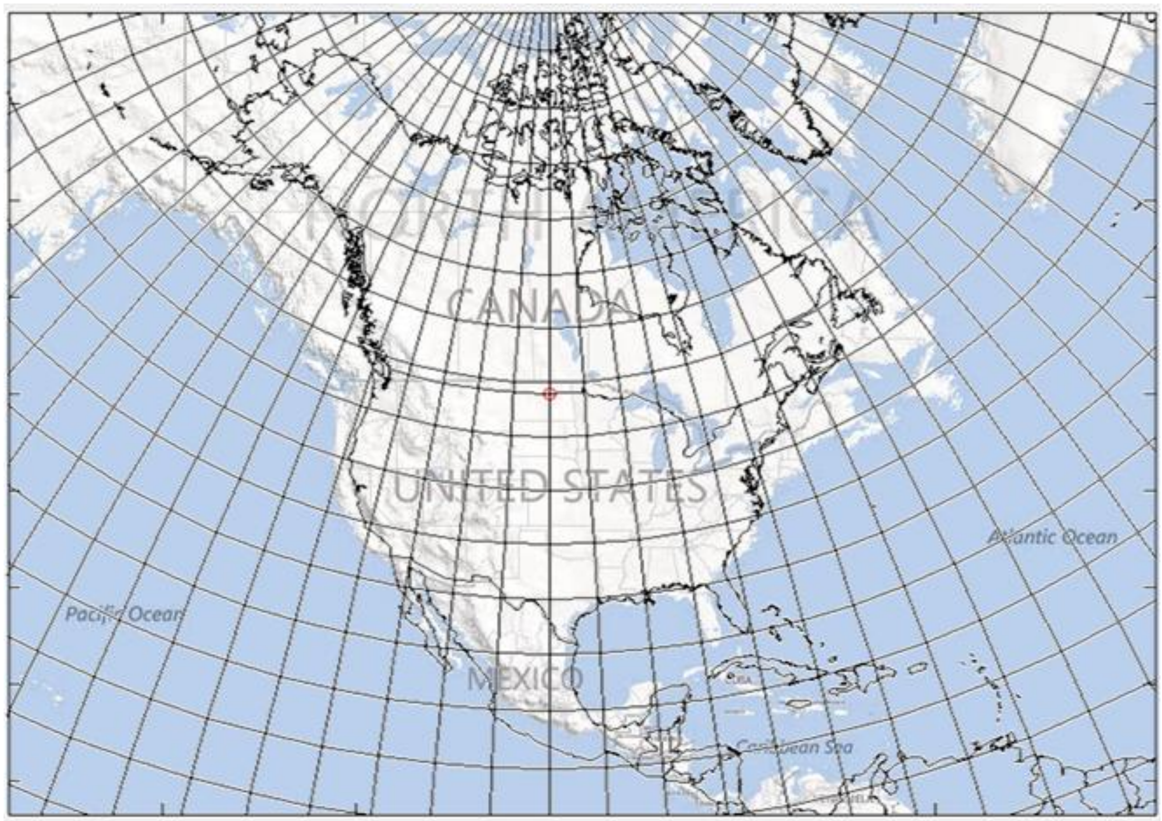

Figure 6.4 Bing Maps [2] vs. MPS. The projection center is at N 49°, W 100°. Zoom level 2. The grid is 5° x 5°. The difference in size and orientation of Greenland is very noticeable. Black lines indicate MPS.

Figure 6.6 compares a portion of North America at zoom level 3, centered at N 60° latitude. In Figure 6.6 the deviation between Bing Maps and MPS is dramatic, especially toward North Pole. The difference between parts of Alaska on both maps exceeds 1500 km. In MPS Greenland and northern Canadian islands look reasonably scaled with proper orientation. It is clear that MPS accounts for meridian convergence while Bing Maps do not.

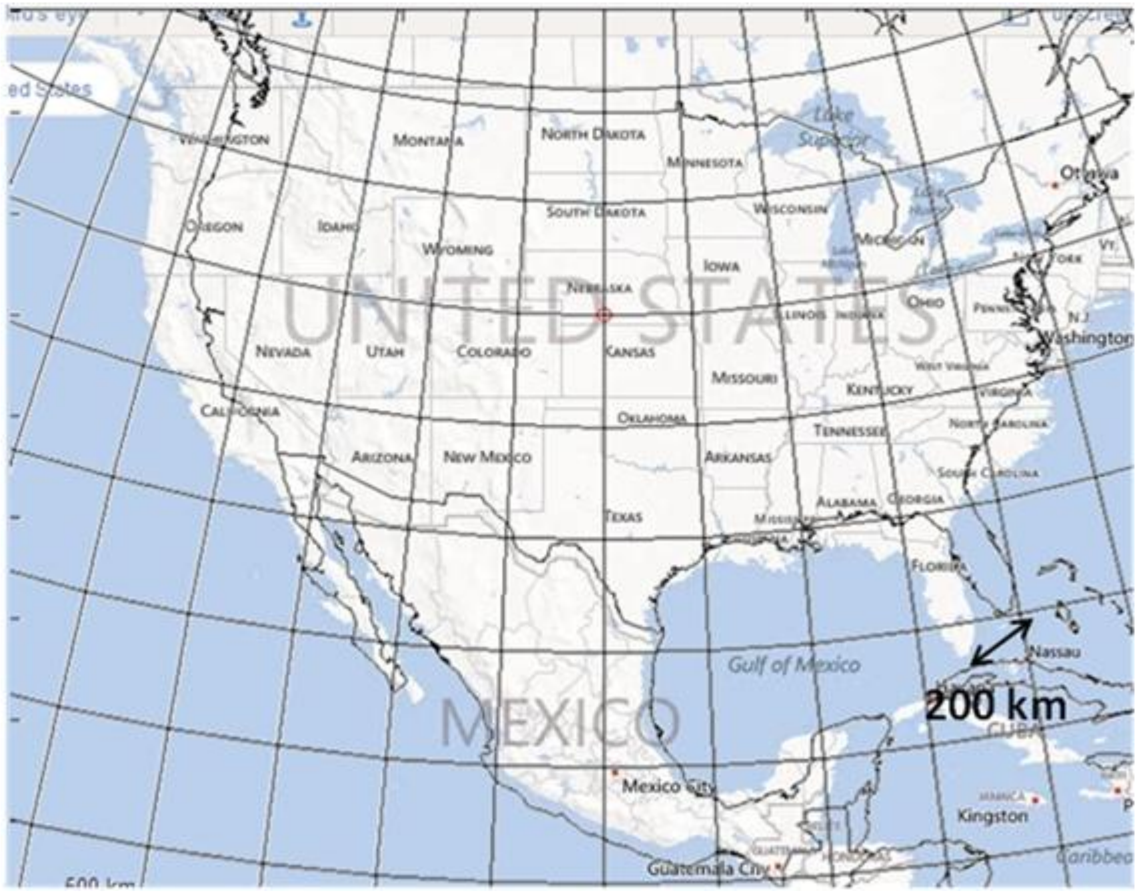

Figure 6.5 Bing Maps [2] vs. MPS. The projection center is at N 40°, W 100°. Zoom level 3. The grid is 5° x 5°. On the edges the difference between two maps exceeds 200 km (e.g., Florida). Black lines indicate MPS.

Figure 6.7 shows a portion of northern Canadian islands in Nunavut at zoom level 5 centered at N 75° latitude. Zoom level 5 corresponds to MPS projection level 3, and at N 75° Lambert Conic Equidistant projection zone is used to cover the area. The orientations of islands differ between the two mapping systems, but the shape of islands is preserved relatively well. At the edges the difference between islands exceeds 80 km.

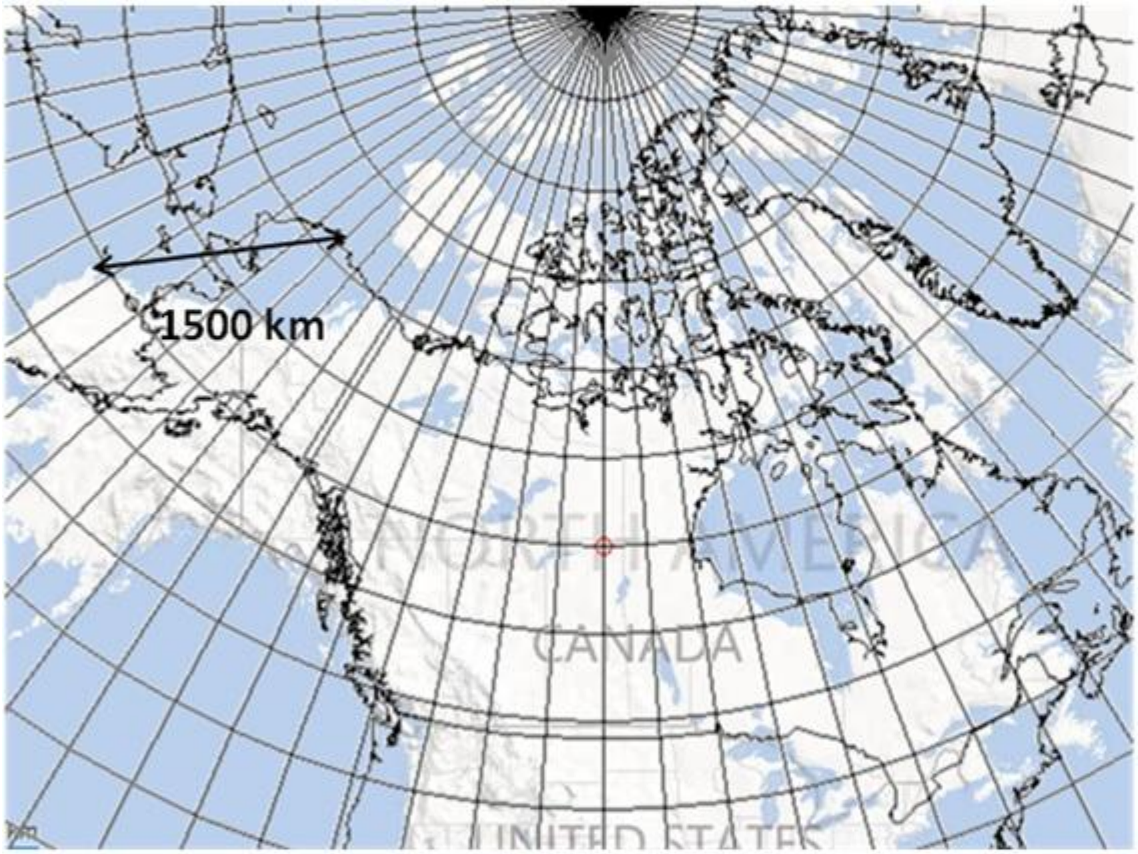

Figure 6.6 Bing Maps [2] vs. MPS. The projection center is at N 60°, W 105°. Zoom level 3. The grid is 5° x 5°. In MPS meridians converge toward North Pole, whereas in Bing Maps meridians are parallel. Black lines indicate MPS.

Figure 6.8 shows a part of Saskatoon, Saskatchewan at zoom level 12, centered at N 52.05° latitude. Zoom level 12 corresponds to MPS projection level 4. There appears to be no visible difference between Bing Maps and MPS at this level of detail, because at higher levels of detail the geographic area covered is relatively small. However, Bing Maps have to constantly readjust the scale bar to account for scale distortion.

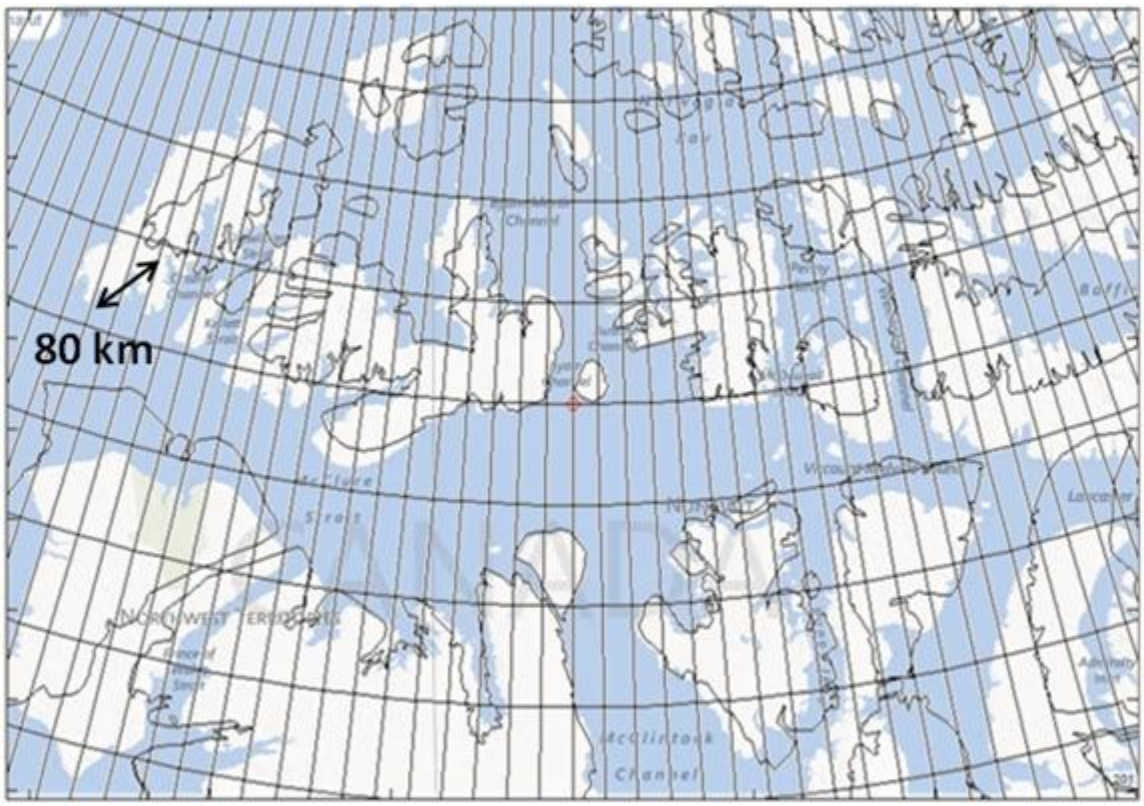

Figure 6.7 Bing Maps [2] vs. MPS. The projection center is at N 75°, W 105°. Zoom level 5. The grid is 1° x 1°. The meridian convergence is very noticeable at high latitudes. Black lines indicate MPS.

Next, Europe is examined. Figure 6.9 and Figure 6.10 focus on parts of Europe centered close to N 40° latitude. Zoom levels 3 and 5 are examined corresponding to MPS projection levels 2 and 3 respectively. The visual difference between the two mapping systems is more significant in Figure 6.9 at lower zoom level. The MPS in Figure 6.10 consists of two Lambert Conic Equidistant bands with a small overlap.

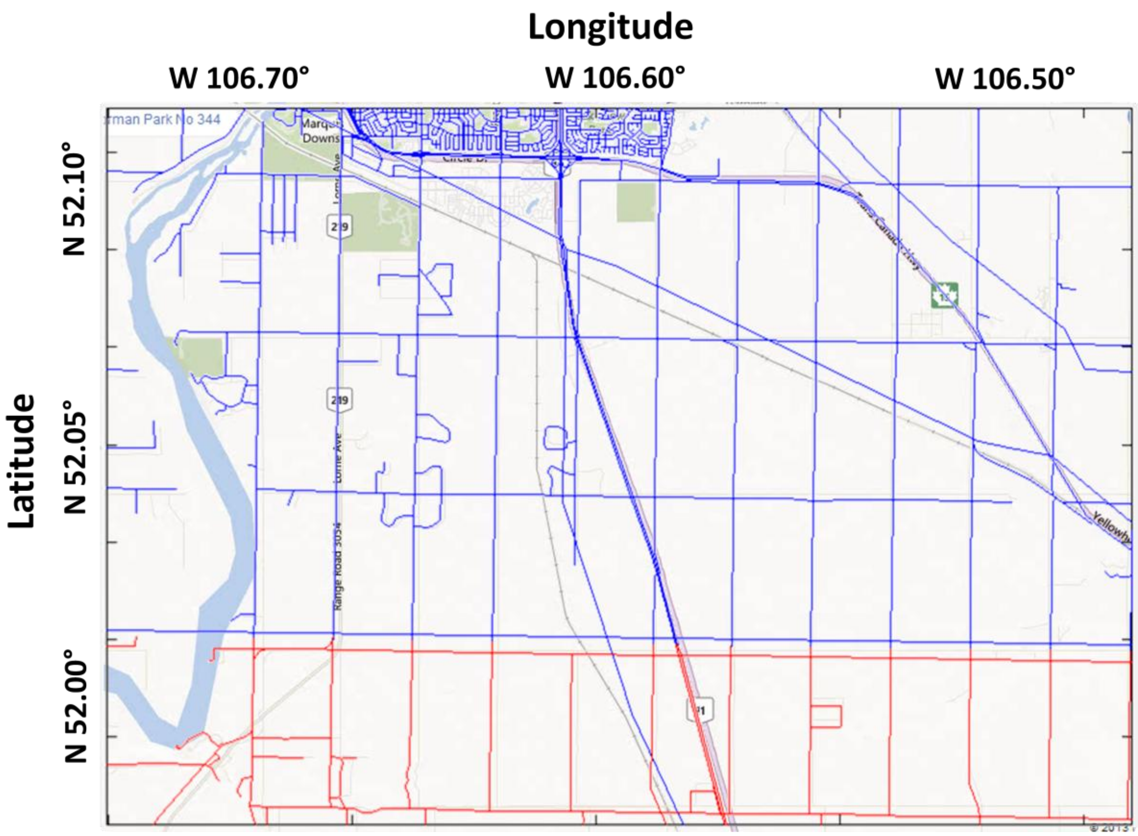

Figure 6.8 Bing Maps [2] vs. MPS. Part of Saskatoon, Saskatchewan. Level of detail 12. The size of distortion is less noticeable at higher levels of detail. Blue and red lines indicate MPS.

Therefore, based on the figures examined in this section, two conclusions can be reached. First, the largest discrepancies between Bing Maps and MPS occur at smallest scales (e.g. maps of hemispheres and continents). Second, the discrepancies near the poles are larger than discrepancies at lower latitude at the same level of detail. Overall to smaller scales the cartographic difference between the two mapping systems is very visible, and at larger scales the difference is not as visible.

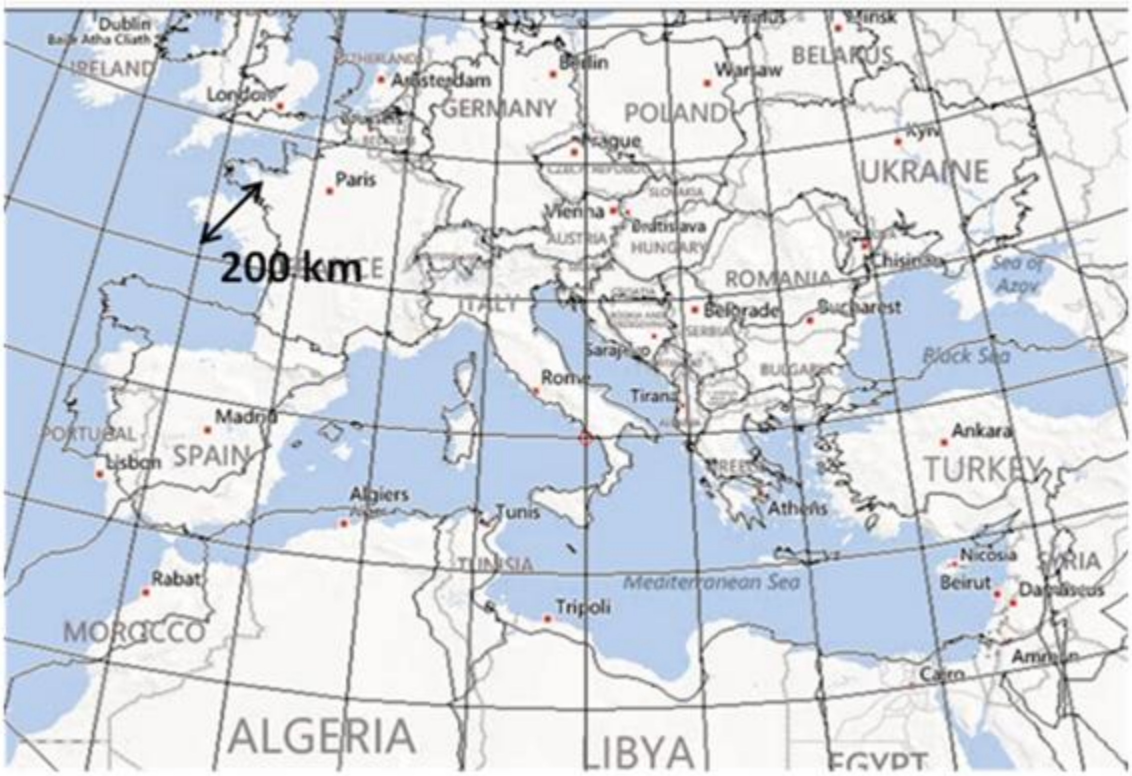

Figure 6.9 Bing Maps [2] vs. MPS. The projection center is at N 40°, E 15°. Zoom level 3. The grid is 5° x 5°. On edges the difference between the two maps exceeds 200 km. Black lines indicate MPS.

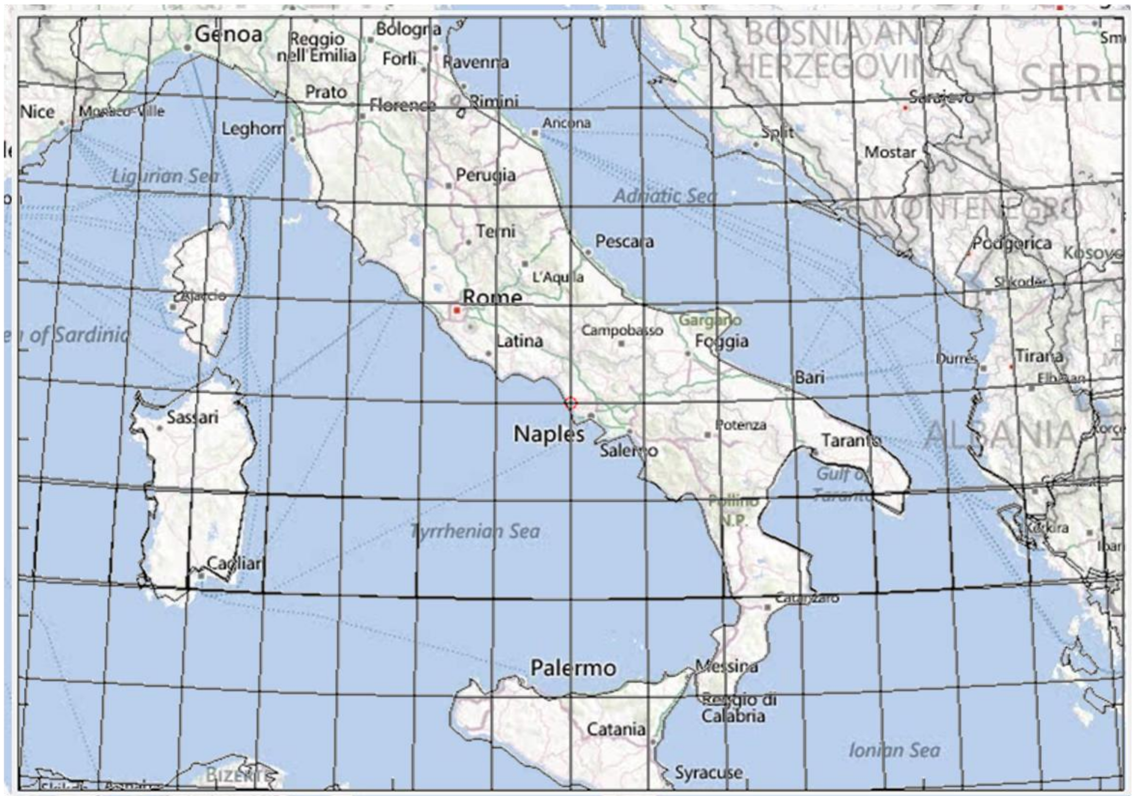

Figure 6.10 Bing Maps [2] vs. MPS. The projection center is at N 41°, E 14°. Zoom level 5. The grid is 1° x 1°. Italy is represented by two different conic layers with 2° overlap. Black lines indicate MPS.

6.5 Summary

This chapter provided analysis of Multi-Projection System in terms of polar region coverage, systematic distortion, and the use of ellipsoidal equations. Then, the Multi-Projection System was compared to Bing Maps in terms of visual cartographic distortion. From the analysis of Multi-Projection System it can be concluded that the MPS effectively covers polar regions, and significantly reduces overall map distortion as compared to Bing Maps. The differences between MPS and Bing Maps are more noticeable at lower levels of detail.

7 Conclusions and Recommendations

Current popular web mapping services, such as Google and Bing Maps rely on Web Mercator Projection (WMP) which lacks polar coverage, grossly distorts the area, and cannot satisfy the needs of professional users [5], [6]. A closer examination of WMP determined that WMP is only suitable for the use in the world between latitude of S 85° and N 85°. The reason for this is because *y* coordinates of WMP become infinite at the poles, and also because the projection must be truncated to preserve 1 to 1 aspect ratio. At latitude of N 85° the scale distortion of WMP is 1147% and the area distortion is 13165%. At latitude of N 70° the web map server that uses WMP needs roughly three and a half times more space to store the same geographical area as at the equator. To account for large distortions, Google and Bing Maps constantly recalculate and readjust the scale bar, and therefore do not give an accurate representation of scale for the entire map.

WMP projects World Geodetic System 1984 (WGS84) geodetic latitude and longitude onto the mapping plane using spherical Mercator equations. This results in 0.38° angular distortion at the equator (relative value of 0.0066), and makes WMP a non-conformal projection. The distance difference in *y* coordinates between WMP and ellipsoidal Mercator exceeds 40 km at latitude above 70°. The users who are unaware of this difference can experience serious problems, especially if they believe that their measurements are done on ellipsoidal Mercator. Moreover, distortions in map projections may lead to errors in perception of geometric properties or spatial patterns, as, for instance, when population density is shown on a nonequal-area map. These perceptual errors can then influence decisions made based on the mapped information, or, more insidiously, our ability to recall accurate information for geographic areas and patterns [54]. Therefore WMP cannot satisfy the needs of all professional users.

A Multi-Projection System (MPS) was designed and developed to overcome major limitations of WMP. A software model of MPS was implemented in MATLAB programming language to demonstrate the proof-of-concept. MPS minimizes map distortions by using a combination of different map projection types across the globe and for a range of levels of detail. The projections are selected primarily based on the criterion of minimal scale distortion. The MPS divides 19 Bing Maps levels of detail into four projection levels (Table 4.1). The first projection level corresponds to Bing Maps zoom level 1 and uses Hammer projection. The second projection level corresponds to Bing Maps zoom levels 2 – 4 and uses Lambert Azimuthal Equal-Area projection. The third projection level corresponds to Bing Maps zoom levels 5 – 7 and consists of eighteen 10° zones that use Stereographic Azimuthal (2), Conic Equidistant (14) and cylindrical Mercator (2) projections. The fourth projection level corresponds to Bing Maps zoom levels 8 and above. The fourth projection level consists of nine 1° x 1° tiles projected using Azimuthal Stereographic Projection and covers a total area of 3° x 3°.

The Multi-Projection System is able to reduce WMP maximum scale distortion from 1147 % to 136 %, 6.4 %, 1.6 % and 0.03 % at projection levels 1, 2, 3 and 4 respectively. Moreover, the scale distortion at any projection level stays constant, therefore the scale bar does not need to be recalculated and readjusted every time the latitude changes.

MPS also effectively covers regions above ±85° latitude at any level of detail and allows users to make accurate distance measurements on map and ellipsoidal surfaces. Therefore the new system can be used for R&D projects in Northern Canada or Antarctica.

7.1 Limitations and Recommendations

There are some limitations of multi-projection approach that need to be addressed and resolved. The limitations open the door for potential future research. The main limitations are:

1. The continuity between projection zones.
2. The challenge of importing new raster or vector data into the system.
3. The current design of WMP tiles, which includes large sets of data already stored in WMP, does not support MPS. Therefore WMP tile structure and hierarchy might need to be redesigned to suit the new multi-projection approach.

The first limitation of MPS is that the projected zones and projected tiles at Projection Levels 3 and 4 are not continuous because they are projected individually and aligned together based on geodesic distance in order to minimize distortion. The process results in creation of map gaps that are especially visible at lower zoom levels, when map scale is small. One solution to this problem is to constrain user's window of visualization and to make sure that the pixel size of the map is always smaller than the gap size between projected zones. Therefore, it is possible to achieve visible continuity between projection zones at certain levels of detail, by rotating projected zones to match user's window of visualization. However, a procedure for effective rotation of pre-rendered zones has not been developed yet. Also, even though the projected zones appear to be continuous for a certain window of observation, in reality they are still separated by small distance. This might be an issue for applications that rely on actual continuity of data rather than on the visual continuity.

The second limitation of MPS is the challenge of importing new vector or raster data into the system. Since there are four projection levels, the process of transfer of data between projection levels for MPS is usually more complicated than it is for the corresponding single projection system. For example, whenever new data are imported, they must be re-projected at different projection levels so that they can be smoothly viewed at all zoom levels. This process requires more time and computational effort. One possible solution is to use supercomputers to re-project large sets of data.

The third limitation of MPS is that the current design of WMP tiles does not support MPS. This means that the large sets of data tiles already stored in WMP will not be applicable for MPS. Today's popular web mapping services use only one projection and rely on square tiles with specific tile hierarchy. Since MPS uses a combination of different projections a new tile structure and hierarchy will need to be developed that support different projection levels. The new tiles might not be of equal size and shape. Lastly, large sets of data will need to be re-projected to cover the world at different zoom levels. This can be done using supercomputers.

7.2 Conclusion

The popularity of web mapping has created a multibillion dollar global industry and continuous to grow. To support the growth and use of Web maps they must be constantly improved. The Multi-Projection System provides new insights and capabilities into global web map visualization, by offering low level distorted view of the Earth for different geographic regions and levels of detail, and by providing accurate measurement capabilities to professional users. The results of this research can be implemented by the industry for the benefits of everyday users. A plan can be developed to promote the necessary collaboration between the research team and major web mapping companies for technology transfer and commercialization. By improving Web maps of the world this project has the potential to provide many benefits to private, social and public sectors.

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Appendix A: Vincenty's Equations for Calculating Geodesic Distance

The *inverse* problem on ellipsoid is defined as: given latitude and longitude of points P_1 and P_2 on ellipsoid compute the forward and reverse azimuths α_{12} and α_{21}, and the geodesic distance c. The computation of the forward and reverse azimuths α_{12} and α_{21} is not discussed here. The geodesic distance s can be found by solving the *inverse* problem using Vincenty's equations [55]. Given ellipsoidal parameters a, e, and given latitude and longitude ϕ_1, λ_1 and ϕ_2, λ_2 of P_1 and P_2 respectively:

1. Compute parametric latitude ψ_1 and ψ_2 of P_1 and P_2 from:

$$\tan\psi = (1 - f)\tan\phi. \tag{A.1}$$

2. Compute the longitude difference $\Delta\lambda$ on the ellipsoid:

$$\Delta\lambda = \lambda_2 - \lambda_1. \tag{A.2}$$

3. Compute the longitude difference $\Delta\omega$ on the auxiliary sphere between $P_1^{'}$ to $P_2^{'}$ by iteration using the following sequence of equations until there is negligible change in $\Delta\omega$. Note that σ should be computed using atan2 function after evaluating $\sin\sigma = \sqrt{\sin^2\sigma}$ and $\cos\sigma$. This will give $-180° < \sigma \le 180°$.

$$\sin^2\sigma = (\cos\psi_2 \sin\Delta\omega)^2 + (\cos\psi_1 \sin\psi_2 - \sin\psi_1 \cos\psi_2 \cos\Delta\omega)^2, \tag{A.3}$$

$$\cos\sigma = \sin\psi_1 \sin\psi_2 + \cos\psi_1 \cos\psi_2 \cos\Delta\omega, \qquad \text{(A.4)}$$

$$\tan\sigma = \frac{\sin\sigma}{\cos\sigma}, \qquad \text{(A.5)}$$

$$\sin\alpha_E = \frac{\cos\psi_1 \cos\psi_2 \sin\Delta\omega}{\sin\sigma}, \qquad \text{(A.6)}$$

$$\cos 2\sigma_m = \cos\sigma - \frac{2\sin\psi_1 \sin\psi_2}{\cos^2\alpha_E}, \qquad \text{(A.7)}$$

$$C = \frac{f}{16}\cos^2\alpha_E \left(4 + f(4 - 3\cos^2\alpha_E)\right), \qquad \text{(A.8)}$$

$$\Delta\omega = \Delta\lambda + (1 - C) f \sin\alpha_E \{\sigma + C\sin\sigma [\cos 2\sigma_m + C\cos\sigma(-1 + 2\cos^2 2\sigma_m)]\}. \qquad \text{(A.9)}$$

The first approximation for $\Delta\omega$ in this iterative solution can be taken as $\Delta\omega \cong \Delta\lambda$.

4. Compute the parametric latitude of the geodesic vertex ψ_0 from:

$$\cos\psi_0 = \sin\alpha_E. \qquad \text{(A.10)}$$

5. Compute the geodesic constant u^2 from:

$$u^2 = e'^2 \sin^2\psi_0. \qquad \text{(A.11)}$$

where e'^2 is the second eccentricity squared and is computed as $e'^2 = \frac{e^2}{1-e^2}$

6. Compute Vincenty's constants A' and B' from:

$$A' = 1 + \frac{u^2}{16384}\left(4096 + u^2\left(-768 + u^2(320 - 175u^2)\right)\right). \qquad \text{(A.12)}$$

$$B' = \frac{u^2}{1024}\left(256 + u^2\left(-128 + u^2(74 - 47u^2)\right)\right). \qquad \text{(A.13)}$$

7. Compute geodesic distance c from:

$$\Delta\sigma = B' \sin\sigma \left\{\cos 2\sigma_m + \frac{1}{4} B' \left[\cos\sigma \left(2\cos^2 2\sigma_m - 1\right) - \right.\right.$$
$$\left.\left. \frac{1}{6} B' \cos 2\sigma_m \left(-3 + 4\sin^2\sigma_m\right)\left(-3 + 4\cos^2 2\sigma_m\right)\right]\right\}, \tag{A.14}$$

$$c = bA(\sigma - \Delta\sigma). \tag{A.15}$$

The geodesic distance c is calculated using WGS 84 parameters in MATLAB [30].

Appendix B: Calculation of Distortions for General Spherical and Ellipsoidal Map Projection Equations

The values for scale, angular and area distortion can be calculated when the surface of reference for map projection is either a sphere or on ellipsoid. The complexity of calculations depends on the individual map projection and surface of reference. The calculations are the simplest when applied to regular cylindrical, conic or polar azimuthal projections on the sphere [10]. The general cases of distortion calculation on the sphere and ellipsoid are discussed below.

The scale, angular and area distortions for map projections that use sphere as a surface of reference are calculated from general equations of scale, angular, and area distortions discussed in Section 2.2.4. Therefore, the meridian h and parallel k scale factors for the sphere are obtained from Equations (2.26) and (2.27), using the values of Gaussian fundamental quantities for the sphere (Equation (2.17)):

$$h = \frac{\sqrt{e}}{\sqrt{E}} = \left(\frac{1}{R}\right)\left[\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2\right]^{0.5}, \tag{B.1}$$

$$k = \frac{\sqrt{g}}{\sqrt{G}} = \left(\frac{1}{R\cos\varphi}\right)\left[\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dy}{d\lambda}\right)^2\right]^{0.5}. \tag{B.2}$$

The maximum angular distortion ω^M for the sphere is obtained from Equation (2.24):

$$\cos\omega^M = \frac{f}{\sqrt{eg}} = \frac{f}{R^2 hk\cos\varphi}. \tag{B.3}$$

The area scale factor J' for the sphere is calculated as [31]:

$$J' = \frac{\sqrt{eg}\sin\omega^M}{\sqrt{EG}\sin\omega^E} = hk\sin\Omega. \tag{B.4}$$

The angle ω^E between meridians and parallels on the sphere is 90°, thus $\sin\omega^E = 1$.

The distortions for map projections that use ellipsoid as a reference surface are calculated from the general equations of distortions discussed in Section 2.2.4. Therefore, the meridian h and parallel k scale factors for the ellipsoid are obtained from the Equations (2.26) and (2.27) using the values of Gaussian fundamental quantities for the ellipsoid (Equation (2.16)):

$$h = \frac{\sqrt{e}}{\sqrt{E}} = \left[\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dx}{d\phi}\right)^2\right]^{0.5}\left[\frac{(1-e^2\sin^2\phi)^{1.5}}{[a(1-e^2)]}\right], \tag{B.5}$$

$$k = \frac{\sqrt{g}}{\sqrt{G}} = \left[\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dx}{d\lambda}\right)^2\right]^{0.5}\left[\frac{(1-e^2\sin^2\phi)^{0.5}}{[a\cos\phi]}\right]. \tag{B.6}$$

The maximum angular distortion ω^M for the ellipsoid is obtained from Equation (2.24):

$$\cos\omega^M = \frac{f}{\sqrt{eg}} = \frac{f}{NMhk\cos\varphi}. \tag{B.7}$$

The area scale factor J' for the ellipsoid is obtained from (B.4) using the scale factor values for ellipsoid.

Appendix C: Sample Distortion for Cylindrical, Azimuthal and Conic Projections.

Table C.1 Area, Scale and Angle distortion of Mercator projection centered at equator with one standard parallel also at equator. Coverage is 10° in latitude.

Distortion of Mercator projection. Origin at ϕ = 0°.					
Latitude (*ϕ*°)	**Area (*J'*)**	**Area %**	**Scale (*k*)**	**Scale(*h*)**	**Angle (ω °)**
10	1.031	3.088	1.015	1.015	0.000
9	1.025	2.492	1.012	1.012	0.000
8	1.020	1.962	1.010	1.010	0.000
7	1.015	1.498	1.007	1.007	0.000
6	1.011	1.097	1.005	1.005	0.000
5	1.008	0.760	1.004	1.004	0.000
4	1.005	0.486	1.002	1.002	0.000
3	1.003	0.273	1.001	1.001	0.000
2	1.001	0.121	1.001	1.001	0.000
1	1.000	0.030	1.000	1.000	0.000
0	**1.000**	**0.000**	**1.000**	**1.000**	**0.000**

Table C.2 Area, Scale and Angle distortion of Azimuthal Stereographic Projection centered at North Pole. Coverage is 10° in latitude.

Distortion of Azimuthal Stereographic projection. Origin at ϕ = 90°.					
Latitude (*ϕ*°)	**Area (*J'*)**	**Area %**	**Scale (*k*)**	**Scale(*h*)**	**Angle (ω °)**
90	**1.000**	**0.000**	**1.000**	**1.000**	**0.000**
89	1.000	0.015	1.000	1.000	0.000
88	1.001	0.061	1.000	1.000	0.000

87	1.001	0.137	1.001	1.001	0.000
86	1.002	0.244	1.001	1.001	0.000
85	1.004	0.382	1.002	1.002	0.000
84	1.006	0.550	1.003	1.003	0.000
83	1.007	0.750	1.004	1.004	0.000
82	1.010	0.980	1.005	1.005	0.000
81	1.012	1.243	1.006	1.006	0.000
80	1.015	1.537	1.008	1.008	0.000

Table C.3 Area, Scale and Angle distortion of Conic Equidistant Projection with origin at ϕ = 25° and two standard parallels. First standard parallel = 21.66° and second standard parallel = 28.33°. Coverage is 10° in latitude.

Distortion of Conic Equidistant Projection. Origin at ϕ = 25°.					
Latitude (ϕ°)	**Area (*J'*)**	**Area %**	**Scale (*k*)**	**Scale(*h*)**	**Angle (ω °)**
30	1.002	0.217	1.002	1.000	0.124
29	1.001	0.076	1.001	1.000	0.043
28	1.000	0.033	1.000	1.000	0.019
27	0.999	0.109	0.999	1.000	0.062
26	0.998	0.154	0.998	1.000	0.088
25	**0.998**	**0.168**	**0.998**	**1.000**	**0.096**
24	0.998	0.152	0.998	1.000	0.087
23	0.999	0.107	0.999	1.000	0.061
22	1.000	0.031	1.000	1.000	0.018
21	1.001	0.073	1.001	1.000	0.042
20	1.002	0.205	1.002	1.000	0.118