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Buckling and Post Buckling Behavior for Unsymmetrical Laminates Part II: Stability Characteristics

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Abstract— This is the second of two companion papers that examine the elastic buckling and post buckling behavior of unsymmetric cross-ply laminates. The existence of geometric imperfection and thermal mismatch between plies result in these panels to retain two equilibrium configurations despite being cured in a flat mold. These equilibrium configurations conform to cylindrical shapes of orthogonal and opposite curvatures. Under externally applied load a panel undergoes snap-through behavior from one equilibrium shape to the other. The unified finite element methodology based on Koiter's theory and presented in the first companion paper is extended to predict the required force responsible for snap-through behavior. Accordingly, ABAQUS, ANSYS and LS-DYNA finite element codes are used to predict the critical snap-through forces and their predictions are compared.

Keywords-stability; bistable; snap-through; snap-back; finite element; post buckling

II. Introduction

It was observed that a cured thin unsymmetric cross-ply laminate will possess two equilibrium configurations. These equilibrium configurations conform to two cylindrical shapes of equal and opposite curvatures which was initially reported by Hyer [1]. The cured panel when triggered it undergoes snap-through behavior from one equilibrium shape to the other, hence called bistable. A panel exhibit this post buckling response if it originally possessed large out-of-plane at room temperature caused by thermal stresses in its layers. The effect of thermal stresses and the existence was originally explained and documented by Hamamoto and Hyer [2].

Bistable panels found many applications in morphing and energy harvesting as discussed in Emam and Inman [3]. It is credited to Hyer to present the first analytical solution to predict the out of plane deformations of these panels [4]. Based on the Principal of Minimum Total Potential Energy PMTPE, Hyer developed the extended classical lamination theory (ECLT). Later Hyer's colleagues Dano [5] and Schultz [6] extended the ECLT to predict the requirements to trigger snap-through behavior using shape memory alloys (SMA) and piezoelectric actuators, respectively. Tawfik [7] extended his ABAQUS finite element (FE) methodology to predict the force required to trigger snap-through behavior in panels with various planforms. This methodology was based on Koiter's theory [8] in accounting for geometric imperfections and originally proposed by Tawfik et al. [9, 10]. ABAQUS based FE predictions of snap-through loads and test results reported in [7] were in good agreement. Therefore, the proposed ABAQUS based FE methodology proved to be successful predicting post buckling behavior. As reported by Emam and Inman [3] many researchers worked on the same problem, i.e. triggering snap-through behavior in bistable panels. Meanwhile, the ABAQUS FE methodology proposed by Tawfik [7] proved successful application in predicting mechanical force or piezoelectric actuators requirements. Moreover, this methodology is analytically correct and do not require obtaining any measurement of actual imperfection in the manufactured panel. Therefore, this methodology possesses potential to be used during the design stages of morphing applications.

The current work targeted generalizing the proposed methodology of Tawfik [7] and investigating the applicability in wide variety of FE computer codes. Therefore, this proposed methodology was consistently applied in commonly used FE codes ANSYS and LS-DYNA. The force required to trigger snap-through behavior in thin unsymmetric cross-ply laminates was predicted by ABAQUS, ANSYS and LS-DYNA. Predictions obtained from FE analysis were compared with test values and proved to be consistent and in good agreement.

III. Methodolgy

A. *Problem definition*

Tawfik [7] and Tawfik et al. [9, 10, 11] were the pioneer in investigating the stability of unsymmetric cross-ply laminates using Finite Element Analysis (FEA). Utilizing two back to back non-linear load-displacement analysis they developed a FE based methodology to examine the bistable characteristics of thin unsymmetric cross-ply laminates using ABAQUS, a commercial FE code, and presented their analysis outcome as a load-displacement curve to show that a cured unsymmetric cross-ply laminate is stable in its both equilibrium positions which is termed as the bistable behavior. For different geometries, forces required to trigger snap-through of such bistable laminates were also predicted and compared with test measurements.

This current work is focused on presenting a generalized FE based methodology to capture the bistable characteristics of a unsymmetric cross-ply laminate under thermal load by replicating Tawfik's ABAQUS [9, 10] work in two other commercial FE codes, ANSYS and LS-DYNA, and finally comparing their results with Tawfik's ABAQUS and test results.

B. Description of basic FEA model

The unsymmetric cross-ply laminate used in this problem has a square geometric configuration and it is a thin $[0_2/90_2]_T$ laminate of Hexcel IM7/8551-7 graphite/epoxy prepreg with mechanical properties provided in Table I. Tawfik [7] conducted the convergence study with all the shell elements available in ABAQUS element library and ended with the selection of S4R as the element type and 20×20 as the mesh size for optimal modeling of this problem in ABAQUS. This present work will use the same mesh size and the similar types of elements in two other FE codes. Table II documents the details about the element types used in three different FE codes.

Tawfik [7] mention that investigation of stability characteristics is a subsequent analysis of curing and in FEA bistable behavior of a thermally cured unsymmetric cross-ply laminate can easily be captured by performing an extra non-linear load-displacement analysis right after the curing cycle. Only the boundary conditions and loading need to be changed during this extra step. During the first step, the curing cycle, center point of the laminate geometry is kept fixed in all degrees of freedom and the applied load is a temperature drop with a magnitude of 156 ^{o}C. During the second step, for analyzing stability characteristics, temperature of the cured shape is kept constant at -156 ^{o}C and the center point of the cured shape is kept free only in z-direction while the four corners are kept free in all degrees other than the z-direction. A concentrated force, P, with a value higher than the snap-through load is applied along the z-direction at the center point as displayed in Fig. 1.

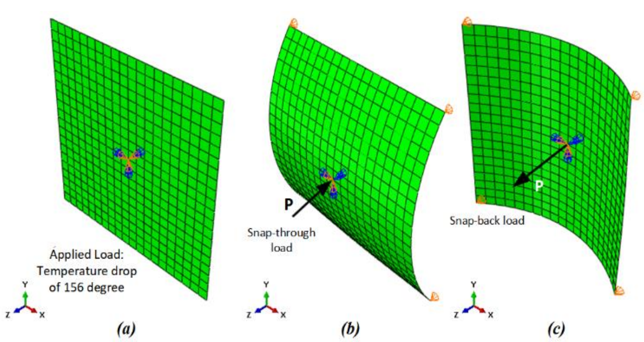

Figure 1. ABAQUS model at different stages of analysis showing the corresponding shapes, boundary conditions and applied load; a) at the beginning of first step b) at the beginning of the second step c) at the end of the second step.

TABLE I. MECHANICAL PROPERTIES OF MATERIAL AS REPORTED IN KIM [12] AND ALSO USED BY TAWFIK [7].

Material	E_{11} **(GPa)**	E_{22} **(GPa)**	G_{12} **(GPa)**	ν_{12}	α_1 **(10^{-6}/^{o}C)**	α_2 **(10^{-6}/^{o}C)**	t **(μm)**
IM7/8551-7 graphite/epoxy prepreg	146.14	8.472	3.879	0.341	0.30	30.98	138.75

TABLE II. ELEMENT TYPES IN FE CODES.

FEA code	**Element name**	**Element type**	**Number of DOF***	**Description**
ABAQUS [13]	S4R	Shell element	6	A four-node general-purpose finite-membrane-strain shell with reduced integration for hourglass control
ANSYS [14]	Shell 181	Shell element	6	A four-node thin to moderately-thick shell with both full and reduced integration schemes
LS-DYNA [15]	Type 16	Shell element	6	A four-node fully integrated thin shell with assumed strain interpolation and available hourglass control option

*Degrees of freedom

Fig. 1 shows the ABAQUS model and its geometric shapes at different stages of the stability analysis. It also illustrates the corresponding boundary conditions and applied loads associated with those stages.

C. Notations ued by Tawfik and Tawfik et al.

Tawfik [7] and Tawfik et al. [11] used some important notations to describe the stability characteristics of a thermally cured bistable unsymmetric cross-ply laminate.

- First equilibrium shape: Laminate's shape when the larger curvature is along the x-axis, Fig. 2a.
- Second equilibrium shape: Laminate's shape when the larger curvature is along the y-axis, Fig. 2b.
- Snap-through: Forcing a laminate to snap from first equilibrium shape to the second equilibrium shape, Fig. 2a.
- Snap-back: Forcing a laminate to snap from the second equilibrium shape to the first equilibrium shape, Fig. 2b.

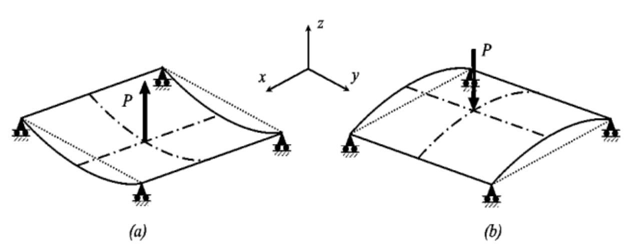

Figure 2. Triggering snap-through in cylindrical panel via concentrated force [7, 11].

D. Snap-through and snap-back analysis with ABAQUS

Tawfik [7] and Tawfik et al. [9, 10] showed that snap-through and snap-back can be captured in ABAQUS using following steps

- Eigenvalue buckling analysis of the perfect geometry using the *BUCKLE keyword to get the eigenmodes.
- Introduce imperfections into the geometry using *IMPERFECTION keyword.
- Non-linear load-displacement analysis of the imperfect geometry using the *STATIC, STABILIZE keyword to get the cured shape.
- A second non-linear load displacement analysis of the cured shape using *STATIC, STABILIZE keyword to capture the snap-through and snap-back.

Among these four steps, first three are for obtaining the cured shape while the fourth step is for snap-through and snap-back of that cured shape [7, 9, 10].

The second non-linear load-displacement analysis in Tawfik's [7, 9] two-step FE methodology was pursued by performing the following sub steps

- Boundary condition of the cured shape was reset such that the center point of the panel was free only in z-direction and the four corner points were free in all degrees other than the z-direction.
- A concentrated force, P, higher than the snap-through load was applied at the center point along the z-direction such that it tended to switch the curved laminate to its second equilibrium shape.
- As the laminate switched to its second equilibrium shape the applied load was removed to make sure that the retained shape was stable.
- After the second equilibrium position was found force, P, was applied again along the z-direction such that it tended to switch the panel to its first equilibrium shape. Boundary conditions were kept unchanged.
- As the laminate switched back to its first equilibrium shape the load was removed once again to make sure that this retained shape was stable.

Further, for modeling of the snap behavior in LS-DYNA and ANSYS Tawfik's two-step methodology will be followed as a guideline.

E. Snap-through and snap-back analysis with LS-DYNA

With LS-DYNA snap behavior of cured bistable unsymmetric laminate can easily be captured as follows

- Eigenvalue buckling analysis of the perfect geometry using the *CONTROL_INPLICIT_EIGENVALUE [15] keyword.
- Introduce imperfections into the geometry using *PERTURBATION_NODE [15] keyword.
- Non-linear load-displacement analysis of the imperfect geometry to capture the cured shape using the *CONTROL_IMPLICIT_STABILIZATION [15] keyword.
- A second non-linear load displacement analysis of the cured shape to capture the snap-through and snap-back using *CONTROL_IMPLICIT_STABILIZATION keyword.

Like ABAQUS the first three steps are for obtaining the cured shape and the fourth one is for snap-through and snap-back of that cured shape.

F. Snap-through and snap-back analysis with ANSYS

Unlike ABAQUS and LS-DYNA solving snap-through and snap-back in ANSYS require following steps

- Static analysis step to get an initial deformation that is going to be used in the Eigen buckling analysis.
- Following is the eigen buckling analysis to generate the 1st eigenmode under thermal loading using any available eigensolver.
- Based on extracted eigenmode shape, the geometry of the model is to be perturbed using a required imperfection size. The command UPCOORD [14] is used to introduce the imperfection.
- The next step is to solve a non-linear analysis of the imperfect geometry using STABILIZE [14] command to get the cured shape.
- A second non-linear load displacement analysis of that cured shape using STABILIZE [14] command.

First four steps are for obtaining the cured shape and the fifth one is for snap-through and snap-back of that cured shape.

G. A unified Finite Element Analysis methodology

Successful implementation of Tawfik's ABAQUS based FE methodology in ANSYS and LS-DYNA can be summarized as follows

Modeling snap-through and snap-back to investigate the stability characteristics of a bistable unsymmetric cross-ply laminate in any FE code requires two back to back analysis

- 1st analysis is for obtaining the cured shape of that unsymmetric cross-ply laminate.
- 2nd analysis is the non-linear load displacement analysis of that cured shape to obtain the snap-through and snap-back.

Tawfik [7] and Tawfik et al. [9, 10,11] first came up with this two-step FE methodology and can be used as a unified guideline for studying stability characteristics of a bistable unsymmetric cross-ply laminate in any FE code, like LS-DYNA or ANSYS.

IV. Results and Discussion

Fig. 3 shows the comparison of load-displacement curves extracted from ABAQUS, ANSYS and LS-DYNA illustrating that they are in good agreement.

Fig. 4 shows the two equilibrium shapes of a bistable laminate extracted from ABAQUS, while Fig. 5 and Fig. 6 display the equilibrium shapes solved by ANSYS and LS-DYNA respectively.

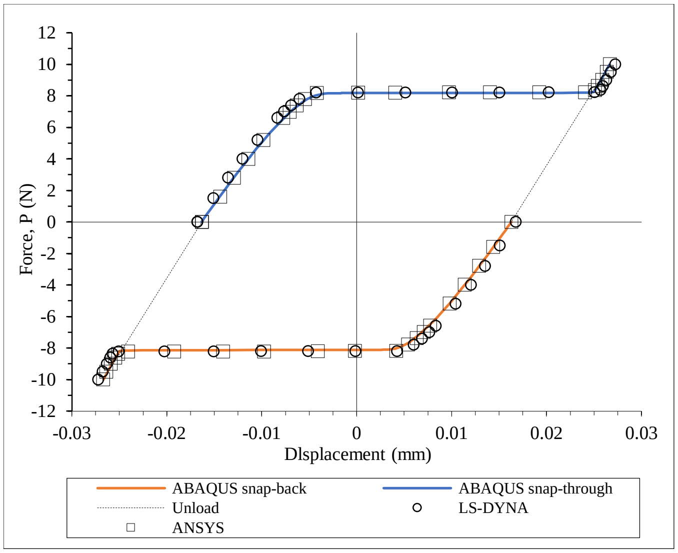

Figure 3. Load-displacement curve for a 150×150 mm^2 cured laminate made of Hexcel IM7/8551-7 graphite/epoxy prepreg with $[0_2/90_2]_T$ stacking.

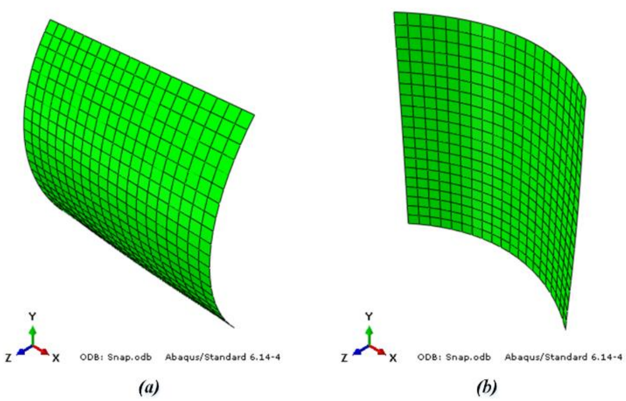

Figure 4. a) First equilibrium shape and b) Second equilibrium shape extracted from ABAQUS.

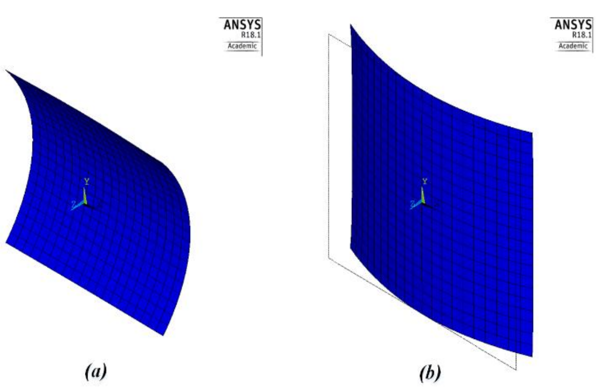

Figure 5. a) First equilibrium shape and b) Second equilibrium shape extracted from ANSYS.

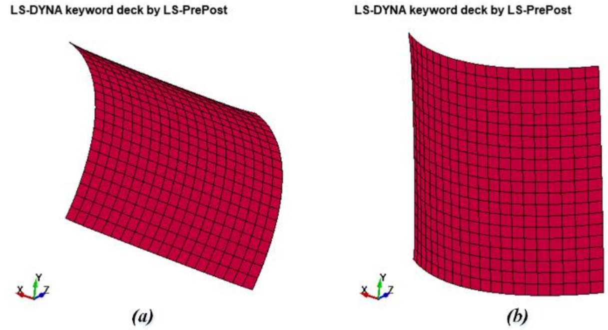

Figure 6. a) First equlibrium shape and b) Second equilibrium shape extracted from LS-DYNA.

TABLE III. EXPERIMENTAL AND FE SNAP-THROUGH LOADS OF SQUARE SPECIMENS

Sidelength [7] (mm × mm)	Test [7] P (N)	ABAQUS [7] P (N)	ABAQUS [7] %	LS-DYNA P (N)	LS-DYNA %	ANSYS P (N)	ANSYS %
151×150	9.61	8.72	-9.29	8.69	-9.50	8.76	-8.84
123×125.5	8.90	8.15	-8.46	8.19	-7.87	8.06	-9.44
100×101	8.09	7.54	-6.76	7.49	-7.29	7.20	-11.00
87×88	5.11	4.93	-3.46	5.09	-0.21	4.82	-5.68
67×67	2.75	2.7	-1.81	2.69	-1.84	2.68	-2.55

TABLE IV. EXPERIMENTAL AND FE SNAP-BACK LOADS OF SQUARE SPECIMENS

Sidelength [7] (mm × mm)	Test [7] P (N)	ABAQUS [7] P (N)	ABAQUS [7] %	LS-DYNA P (N)	LS-DYNA %	ANSYS P(N)	ANSYS %
151×150	10.39	9.42	-9.36	9.49	-8.63	9.48	-8.76
123×125.5	10.06	9.25	-8.02	9.29	-7.61	9.50	-5.57
100×101	8.27	7.75	-6.35	7.69	-6.89	7.68	-7.13
87×88	6.12	5.37	-12.27	5.53	-9.61	5.22	-14.71
67×67	4.45	2.97	-33.26	3.05	-31.37	2.78	-37.53

Tawfik [7] also studied the effect of geometry on the stability characteristics by changing the sidelength of thin $[0_2/90_2]_T$ square laminate and documented the snap-through and snap-back loads comparing the ABAQUS predictions with the test measurements. Table III shows the snap-through loads predicted in this present work by LS-DYNA and ANSYS while table IV shows the snap-back loads. These two tables also illustrate the percentage differences in predictions from these three FE codes while compared with the test measurements, where all the test measurements and ABAQUS results are from Tawfik [7]. ABAQUS, ANSYS and LS-DYNA all are in good agreement with the test measurements which proves the generality of that two-step FE methodology developed by Tawfik [7] and Tawfik et al. [9, 10, 11].

V. CONCLUSION

The current work addressed post buckling behavior in thin unsymmetric cross-ply laminates and stability characteristics. An analytically correct methodology is obtained from literature and applied in commonly used finite element computer codes, ABAQUS, ANSYS and LS-DYNA. Critical forces required to trigger snap-through behaviour in thin unsymmetric cross-ply laminated were compared with test measurement and found to be in good agreement. Therefore, the adopted analytically correct methodology proved generality and recommended for analysis and design of unsymmetric composites morphing applications.

REFERENCES

[1] Hyer, M.W. (1981). “Some Observations on the Cured Shapes of Thin Unsymmetric Laminates”, Vol. 15, 175-194, Journal of Composite Materials.

[2] Hamamoto, A. and Hyer, M.W. (1987) “Non-linear Temperature-curvature Relationships for Unsymmetric Graphite-Epoxy Laminates”, Vol. 23, 919-935, International Journal of Solids and Structures.

[3] Emam SA, Inman DJ. A review on bistable composite laminates for morphing and energy harvesting. Applied Mechanics Reviews. 2015 Nov 1;67(6):060803.

[4] Hyer, M.W. (198l) “Calculations of the Room-temperature Shapes of Unsymmetric Laminates”, Vol. 15, 296-310, Journal of Composite Materials

[5] Dano, M., "SMA-Induced Deformations in General Unsymmetric Laminates", Ph.D. Dissertation, 1997, Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

[6] Schultz, M.R., "Use of Piezoelectric Actuators to Effect Snap-Through Behavior of Unsymmetric Composite Laminates", Ph.D. Dissertation, 2003, Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia

[7] Tawfik S. Stability and morphing characteristics of bi-stable laminates. Ph.D. dissertation, Atlanta, Georgia: School of Aerospace Engineering, Georgia Institute of Technology; 2008

[8] Koiter WT. On the stability of elastic equilibrium, Ph.D. Thesis, 1945, Polytechnic Institute Delft, NASA TTF-10833; 1967.

[9] S. Tawfik, X. Tan, S. Ozbay and E. Armanios, "Modeling of anticlastic stability in elastically tailored composites," in Proceedings of 20th annual technical ASC conference, Drexel University, Philadelphia, Pennsylvania, 2005.

[10] S. Tawfik, X. Tan, S. Ozbay and E. Armanios, "Anticlastic Stability Modeling for Cross-ply Composites," Journal of Composite Materials, vol. 41, no. 11, pp. 1325-1338, 2006.

[11] S. Tawfik, D. S. Dancila and E. Armanios, “Planform Effects Upon the Bistable Response of Cross-Ply Composite Shells,” Composites Part A: Applied Science and Manufacturing, vol. 42, no. 7, pp. 825–833, 2011.

[12] Kim, I., "Development and analysis of elastically tailored composite star shapedbeam sections", Ph.D. Dissertation, School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia; 2003

[13] Simulia, D. S. (2014). Abaqus 6.14 user manual. Providence, RI, USA: DS SIMULIA Corp.

[14] Manual, A. U. S. Version 16.2, 2015. ANSYS Inc., Canonsburg, USA.

[15] Manual, L. D. K. U. S., & Volume, I. (2013). Version 971. Livermore Software Technology Corporation, 7374.