

# ESSAYS ON INEQUALITY AND PRODUCTIVITY GROWTH DECOMPOSITION.

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## ABSTRACT

This dissertation focuses on two aspects of globalization: how it impacts inequality and productivity growth. The first two chapters are concerned with inequality, trade and FDI and are theoretical in nature. The third chapter is an empirical study of productivity growth decomposition for Chile.

Chapter 1 considers trade and foreign direct investment between two symmetric countries, where firms differ across their productivity and share their profit with workers who are assumed to be homogenous. These firms serve in a monopolistically competitive output market and can supply to both foreign and domestic markets. A firm can choose to supply in the foreign market via either trade or foreign direct investment. I find that, as countries open up new channels to access foreign market, they increase their inequality and welfare. In the trade and FDI equilibrium inequality decreases with a decrease in tariff and an increase in fixed cost in investing in the foreign economy. I also find that, an access to better technology increases inequality and increase in rents at first increases inequality and then starts to decrease in higher range.

Chapter 2 relaxes the country symmetry assumption and allows for only trade between north and south. A prospective entrant in the north has a better chance of obtaining higher productivity relative to a prospective entrant in the south. As long as some firms in the north cannot earn export status, i.e. the trading partners are not too asymmetric, the model predicts both countries experience an increase in inequality from globalization but the south's inequality increases more. I also find that a bilateral symmetric decrease in trade costs decreases inequality more in the south. However the north is better off maintaining lower tariff than the south as it generates relatively lower inequality than the symmetric reduction of tariffs for the north. A symmetric decrease in fixed cost to export has a similar effect as a bilateral symmetric tariff reduction.

Chapter 3 decomposes the aggregate productivity growth for Chile using plant-level data from 1979 to 1996. Chile got integrated into the world economy in early 70's, during which most of their businesses were denationalized as well. These features make Chile an interesting country to study the productivity growth due to both inter-industry and intra-industry reallocation,

thus combining elements of both neoclassical and new trade theories. I find that neither type of reallocation played an important role in the productivity growth in Chile for this period. Instead I find that technological progress contributed to the majority of this productivity growth.

To my loving daughter, Rumaisa Akbar, and my father, Fazle Elahi Akbar.

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# Chapter 1

# Heterogenous Firms, Trade/FDI and Inequality

## 1.1 Introduction

Often trade economists argue about the welfare gains from opening up to trade and ignore other effects on the economy such as the effect on inequality. Most trade models predict a welfare gain but at the expense of what? It is true that aggregate welfare is one of the most important tools in the study of an economy, but social inequality plays an important role as well. Is it possible to have welfare gain from open markets without creating any inequality or is inequality inevitable? These are some questions trade economists are trying to address in the present day.

To address the above questions this paper develops a theoretical model to study inequality in a heterogenous firms setup. These firms share their profit with workers who are assumed to be homogenous. In a Melitz environment globalization causes an expansion in production by firms that serve the foreign market and a contraction in production by firms serving the domestic market only. This expansion/contraction leads to increase/decrease of market share for firms and their employment. Hence total labor payment of firms that have access to the foreign market increases, since they are hiring more workers at the given wage rate. On the other hand firms serving only the domestic market decrease their total labor payment. This causes a distortion in the economy leading to inequality.

Think of an economy that has two channels to access the foreign market, trade and FDI, similar to Helpman, Yeapel and Meltiz. Now let firms share some of their rent as wage paid to their workers, these workers who assumed to be homogenous in terms of their productivity. Hence workers from a high productivity firm will always earn a higher wage. Note that the workers are not getting a higher wage because they are more productive, rather they may be fortunate to be hired by a high productivity firm. These workers have no choice over their employer; which is a stringent assumption, but this isolates the fact that inequality is a consequence from the redistribution mechanism of Melitz environment only. However any sorting in the labor market, as in other models that deals with inequality in the Melitz framework, will increase inequality. The economy described above gives a well-defined employment distribution that can be used to construct the Lorenz curve to study the economy separately at three different states: (1) autarky, (2) trade equilibrium and (3) trade and FDI equilibrium. In the end I conduct a thorough study on the effects of change in tariff and fixed cost in investment on inequality.

The paper finds that as countries open up for trade inequality increases; this further increases as firms get an additional channel to access foreign market via FDI. As countries open up new channels to access foreign market, they increase competition that leads to reallocation of resources (here labor) to more productive firms. A firm with higher productivity hires more workers, pay higher wage since they earn higher profit. Hence any reallocation of resource leads to an increase in the wage of workers coming from exiting and shrinking firms. On the other hand the wage of workers who stayed back with shrinking firms remain the same, but their total labor payment will go down with firms market share. This leads to a higher inequality in the economy.

I find that any decrease in tariff or increase in fixed cost in investment leads to lower inequality even though it increases welfare. Any decrease in tariff creates exporting opportunities to some highly productive domestic and least productive investing firms. The reallocation of resource in this case goes from less productive domestic firms to more productive exporters and from more productive investors to less productive exporters. This causes an increase in the concentration around the mean for the wage distribution, hence it decreases inequality. On the contrary, an increase in fixed cost to invest makes exporting market more profitable to some

least productive investing firms. As these firms self select themselves as exporters labor gets reallocated to less productive firms as it decreases the level of competition in the market. This mechanism expands the mass of exporters relative to investors and domestic producers only; hence again, relatively more labor earn close to the mean wage of the economy that results in lower inequality. However any situation that leads to higher competition in market causes the welfare per worker to go up. This is a familiar dynamic in the Melitz environment.

The paper also finds that, access to better technology makes the local market relatively more competitive. This forces some least productive firms to exit the markets. On the other hand their market share is occupied by expanding firms. Note that, investors observe a higher increase in market share relative to exporters and exporters observe a bigger market share than domestic producers only. In this way a bigger portion of the population moves away from the average wage of the economy and increases inequality in both economy. However, this still improves aggregate welfare.

The literature about welfare gain from open markets started from Ricardo and continues onto models of heterogenous firms. This increase in welfare usually comes from increased competition in the domestic market. As countries open up to foreign firms they let high productivity exporters and investors access their market. These high productivity firms in turn lowers aggregate price and pushes the least productive firms out of the industry. This result was first demonstrated by Melitz (2003), then by Helpman Melitz and Yeaple (2003), Demidova (2006) and other extension of the Melitz model. In one way or another it is also possible to show that a decrease in tariff will increases welfare as well. This result holds as long as we have symmetric countries; however, country asymmetry may lead to a decrease in welfare as shown by Demidova (2006). A further examination of the welfare gain analysis is done by Melitz and Redding (2013), where they compare the homogenous firm setup to a heterogenous firm setting. They were able to show that by endogenous selection process, heterogenous firms setting has this additional channel to obtain aggregate welfare gain. However none of these studies looks at the inequality aspect, hence this chapter considers inequality as well as welfare.

Recently some studies looked at the inequality aspect of globalization with heterogenous firms. These studies introduced labor market friction, which lead to higher inequality in the economy from globalization. This kind of result is proved in Egger and Kreckemeier (2009) as

well as in Helpman, Itskhoki and Redding (2008), Danziger (2014) and Pupato (2014). They consider heterogenous workers, who had different ability or marginal product of labor and/or effort. The first paper from the list above took a measure of fair wage that has two parts in it, one firm-specific and another industry-specific. I drop the industry-specific part and consider only the firm-specific component of the wage defined by them. This opens up a firm's rent-sharing mechanism with a homogenous labor force. Labor is not mobile and they are employed randomly over the pool of unemployed labor<sup>1</sup>. Since workers have no decision to make over their employer, the firms cannot sort workers. Egger and Kreickemeier predict within firm wage inequality, mine on the other hand creates inequality across firms and industries. The most of the other literature consider some sort of friction/imperfect labor market that induces higher inequality form open markets. These studies consider setups more general than my model since I do not have any labor market matching or sorting. However my specification shows that globalization can create inequality from a very natural phenomena; the efficient redistribution of resources in Melitz environment. A matching or sorting in the labor market will only make inequality worse, hence globalization will always create some inequality in this framework.

It is important to determine the role of rent in the determination of wage in labor economics. Both theory and an extensive collection of empirical work finds the existence of rent sharing behavior by firms. Essentially this means that larger firms offer higher wage. Wage can still be a function of firms' productivity, however modeling wage from rents will exclude individual characteristics of workers. This behavior is documented in many papers such as Budd and Slaughter (2004), Christofides and Oswald (1992), Hildreth and Oswald (1997), Blanchflower, Oswald and Sanfey (1996), Budd, Konings and Slaughter (2005). For example, Christofides and Oswald (1992) studied 600 labor contracts and found that firms' profit from the previous year can effect current wage. They also estimated the profit elasticity of pay to be 0.006. Budd, Konings and Slaughter (2005) tested if rents are shared across the border. They found it does and the elasticity of wage to profit per worker is 0.03, which explains 20% of the observed

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<sup>1</sup>Globalization can contract/expand market share for firms, depending on how productive the firm is. This in turn creates some unemployment by contracting firms and equal number of employment opportunity by expanding firms. Hence this model cannot generate unemployment. However, it is possible to imagine a portion of the population remains unemployed. Hence it becomes exogenous in the model.



variation in wage. The rent-sharing behavior of the firm is one of the channels that generates wage differential across firms. Hence it is essential to explore this mechanism that leads to inequality from open markets.

All these studies of inequality from globalization have considered wage as some function of workers ability. But in theory we can see that wage can be some function of firms rent as well. A large firm usually has higher productivity, hires more workers and pays higher wage. This higher wage can come from either hiring high performing workers, higher rents or from both. Considering homogenous workers will close down the channel through which a worker can effect his or her wage. This creates a labor market with a wage profile instead of one single wage, since workers cannot move across firms and their employment will be firms decision alone. In this way workers are stopped from joining one firm to create one symmetric wage. Hence the paper shows that inequality is a very natural phenomenon in the Melitz environment. Sorting or matching of labor in the labor market will only make the inequality worse. As a result, opening up an economy will always create inequality.

The above studies of inequality from globalization only consider the trade equilibrium. What happens from opening up new channels to access the foreign market in this setup? As far as I know, this still remains an unexplored aspect of studying inequality from globalization. In the general equilibrium set up, if firms can access foreign market via FDI as well then inequality will increase even more. In this paper I have shown that FDI will further reallocate the resource to high productive firms. This increases the wage profile for those who work for high productive investors and lower the wage profile offered by shrinking exporters and domestic producers only. Hence opening for FDI will further increase inequality in the economy.

## 1.2 Model

The model is based on Melitz (2003). There are two countries indexed by  $i$  and  $j$  with  $L$  workers each. Workers are homogenous and have no choice to move across firms. Each country produces two types of goods: differentiated intermediate goods and homogenous final output. Intermediary goods are traded in the open market, where as homogenous final output (not traded in open market) is used to pay all of the sunk fixed costs and consumption. Firms have

access to foreign market via trade and FDI. In the following section I will give more specific description of the demand side.

### 1.2.1 Demand

The consumers consume the final output  $Y$ , which is an aggregate of intermediate goods. Their preference of intermediate goods is given by the standard CES utility function and  $\sigma > 1$  is the constant elasticity of substitution:

$$U = Y = \left[ \int_{\phi \in \Omega} q(\phi_i)^{\frac{\sigma-1}{\sigma}} d\phi_i \right]^{\frac{\sigma}{\sigma-1}} \quad 1$$

$\Omega$  is the mass of available variety. Every individual has  $l = 1$  endowment of labor that they supply inelastically. They have no choice over wage, even though I assume homogenous workers, hence they accept any wage that is offered by the firm to them. As a result a firm with productivity  $\phi$  offers the following wage,  $w(\phi) = \phi^\theta$  for  $\theta \in (0, 1)$  where  $\phi$  is the firms productivity and  $\theta$  is the rent sharing parameter in this model.

The wage is usually, in standard labor theory, some function of workers ability and portion of firms rent ( $w = f(MPL, ability, \frac{\pi}{l})$ ). The homogenous workers assumption imply that they all have same ability. In theory if that is the case workers will always move to a firm that offers the highest wage. In this way they will drive down the wage to one single level for all firms. However this is prohibited in this paper, simply because they do not have any choice when it comes to employment. It is solely firms decision to employ workers. It is possible to construct a model where workers can make this sort of decision, this creates an imperfect labor market, that leads to a higher inequality. The whole purpose of the model is to show that rent sharing mechanism leads to income inequality with opening up to trade and investment, even though they observe aggregate welfare gain. Note that,  $\theta = 0$  imply Melitz model with unit wage and I cannot consider  $\theta = 1$  since this leads to a marginal cost of 1 for all firm. Since price is a constant markup over marginal cost, all firms charge the same price which is their mark up. Hence firms share all rent with her workers and in the equilibrium all firm make same profit. Anything in between is a special case of Hartmut and Udo's paper, since I do not consider the industry-specific contributor to the wage. Under this condition an individual earns  $\phi^\theta$  and has a demand for specific variety as:

$$q(\phi_i) = Rp(\phi_i)^{-\sigma} P^{\sigma-1} \quad (1.1)$$

$P$  is the aggregate price index corresponding to the final output. I will normalize the aggregate price index to be 1 and  $R = \int_{\phi \in \Omega} r(\phi) d\phi$  is the aggregate expenditure.

### 1.2.2 Production

Each firm trying to enter the market undertakes a sunk fixed cost of  $f_e$ . Once they pay the cost of  $f_e$ , they get to draw their productivity from a distribution  $g(\phi)$ . After observing their productivity a firm can stay in the market and produce or exit. Firms going to production pays another fixed cost of  $f$ . All of the costs are paid in terms of the final output  $Y^2$ . Firms compete in a monopolistically competitive market and require labor ( $l(\phi) = q(\phi)/\phi$ ) input to produce intermediate goods. The price charged by the firm in a monopolistically competitive market is given by the following function:

$$p(\phi_i) = \frac{\sigma}{\sigma-1} \phi_i^{\theta-1} \quad (1.2)$$

$\phi_i^{\theta-1}$  is the marginal cost and  $\frac{\sigma}{\sigma-1}$  is the constant mark up from the CES utility. Note that the limiting case of  $\theta = 1$  leads to a marginal cost of 1. So the price charged by a firm no longer depends on the productivity and all firms make same profit; hence we lose firm heterogeneity in equilibrium. Firms revenue is given by the following equation:

$$r(\phi_i) = R\phi_i^\varepsilon \left[ \frac{\sigma-1}{\sigma} \right]^{\sigma-1} \quad (1.3)$$

Revenue is an increasing function of firm's productivity, and  $\varepsilon$  is defined as  $(1-\theta)(\sigma-1)$ . The profit of the firm with observed  $\phi$  is given by the following equation:

$$\begin{aligned} \pi(\phi_i) &= r(\phi_i) - l(\phi_i)\phi_i^\theta - f \\ &= \frac{r(\phi_i)}{\sigma} - f \end{aligned} \quad (1.4)$$

---

<sup>2</sup>Note that normalizing price index corresponding to the final output, will fix the entree barrier at fixed level. This will be consistent with Meltiz literature.

Since profit is increasing in its argument, the marginal firm has productivity  $\phi_i^*$  such that  $\pi(\phi_i^*) = 0$ . This identifies the producing firms from exiting firms and therefore the distribution in equilibrium is given by  $\mu(\phi_i) = \frac{g(\phi_i)}{1-G(\phi_i^*)}$  for  $\phi_i \in (\phi_i^*, \infty)$ . The aggregate price is given by the following equation:

$$P^{1-\sigma} = M \int_{\phi_i^*}^{\infty} p(\phi_i)^{1-\sigma} \frac{g(\phi_i)}{1-G(\phi_i^*)} d\phi_i \quad (1.5)$$

Where  $M$  is the mass of firms active in a country, that is defined by the labor market clearing condition  $L = M \int_{\phi_i^*}^{\infty} l(\phi_i) \frac{g(\phi_i)}{1-G(\phi_i^*)} d\phi_i$ . This condition balances aggregate supply and demand for labor and pins down the mass of active firms to be:

$$M_a = \frac{L}{(\sigma-1)f} \left[ \left( \frac{1}{\phi^*} \right)^\varepsilon \int_{\phi_i^*}^{\infty} \phi_i^{\varepsilon-\theta} \frac{g(\phi_i)}{1-G(\phi_i^*)} d\phi_i \right]^{-1} \quad (1.6)$$

Go to appendix for complete derivation. Note that the price index from equation (1.5) corresponds to the homogenous final output, hence the price index represents CPI for the basket  $Y$ . The aggregate productivity in this economy is then given by the following equation.

$$\tilde{\phi}_i = \left[ \int_{\phi_i^*}^{\infty} \phi_i^\varepsilon \frac{g(\phi_i)}{1-G(\phi_i^*)} d\phi_i \right]^{\frac{1}{\varepsilon}} \quad (1.7)$$

This is the productivity index observed by the consumers. This index considers all the variety that is available to consumers. Hence the good can come from both foreign or domestic producers. However consumers cannot access foreign goods since they are still in a closed economy. Producers on the other hand face a different index<sup>3</sup>.

$$\tilde{\phi}_{ie} = \left[ \int_{\phi_i^*}^{\infty} \phi_i^{\varepsilon-\theta} \frac{g(\phi_i)}{1-G(\phi_i^*)} d\phi_i \right]^{\frac{1}{\varepsilon-\theta}} \quad (1.8)$$

Note that producers observe less average productivity, since a part of it goes to rent by the rent sharing parameter  $\theta$ . Contrasting to the previous productivity index, this index considers only the mass of variety produced inside an economy. At autarky the mass of variety produced

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<sup>3</sup>Since the relative employment compared to average firm will be  $\frac{l(\phi)}{l(\phi^*)} = \left(\frac{\phi}{\phi^*}\right)^{\varepsilon-\theta}$ , the average in the producer index will be raised to  $\varepsilon - \theta$ .

and consumed inside an economy is same, since no foreign firms can access the domestic market of both countries<sup>4</sup>. These two indices jointly define the average wage of the economy as  $E(w) = \frac{\tilde{\phi}_i^\varepsilon}{\tilde{\phi}_{ie}^{\varepsilon-\theta}}$ <sup>5</sup>. This can be obtained from wage distribution, that is discussed in the subsequent sections, as well and these two methods give rise to same average wage.

In country i given any two observed value of the productivity ( $\phi_1 < \phi_2$ ) parameter I can write the equations for relative output, revenue, price and wage by the following equations.

$$\frac{q(\phi_{i2})}{q(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\varepsilon-\theta+1} \quad (1.9a)$$

$$\frac{r(\phi_{i2})}{r(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^\varepsilon \quad (1.9b)$$

$$\frac{w(\phi_{i2})}{w(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^\theta \quad (1.9c)$$

$$\frac{p(\phi_{i2})}{p(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\theta-1} \quad (1.9d)$$

$$\frac{l(\phi_{i2})}{l(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\varepsilon-\theta} \quad (1.9e)$$

so a firm with higher productivity produces more output, earns a higher revenue, offers a higher wage, employs relatively more workers and charges a lower price.

### 1.3 Equilibrium in Autarky

Firms draw their productivity  $\phi_i$  from a distribution  $g(\phi_i)$ . Since only the firms making positive profit can stay in the market, firms value function is then given by  $\max\{0, \frac{\pi(\phi_i)}{\delta}\}$ . Here ' $\delta$ ' is the external shock that a firm has to exit in future<sup>6</sup>. Hence the free entry condition (FEC) will be:

$$[1 - G(\phi_i)] \frac{\pi(\tilde{\phi}_i)}{\delta} = f_e \quad (1.10)$$

---

<sup>4</sup>This will not be the case at trade and FDI equilibrium, since now a mass of foreign firms can compete in the global market. This will be discussed in details in the following sections.

<sup>5</sup>Note that  $\tilde{\phi}_i^\theta \neq \int_{\phi^*}^{\infty} \phi^\theta \mu(\phi) d\phi$ . It will be the ratio of productivity index coming from consumption and production.

<sup>6</sup>In a steady state equilibrium, the mass of exiting firms will be replaced by the mass of new entrees each period.

$f_e$  is the sunk fixed cost to enter the market. This condition implies that firms expected future profit balance the fixed cost to enter the market. After the prospective firm pays the cost  $f_e$  it gets to draw the productivity parameter for the firm ' $\phi_i$ ' from the PDF  $g(\phi_i)$ . On the other hand the Zero Cutoff profit identifies average profit from the marginal firm. Recall that the profit function for a firm with average productivity is given by,

$$\pi(\tilde{\phi}_i) = \frac{r(\tilde{\phi}_i)}{\sigma} - f$$

Note that the revenue of the average firm can be expressed as the ratio of the cutoff to average productivity. The equation (1.9b) implies that  $r(\tilde{\phi}_i) = \sigma f \left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^\varepsilon$ . The zero cutoff profit (ZCP) is then given by the following equation:

$$\begin{aligned} \pi(\tilde{\phi}_i) &= f \left[ \left( \frac{\tilde{\phi}_i}{\phi_i^*} \right)^\varepsilon - 1 \right] \\ &= f k(\phi_i^*) \end{aligned} \tag{1.11}$$

where  $k(\phi_i) = \left(\frac{\tilde{\phi}_i}{\phi_i}\right)^\varepsilon - 1$ , Equation (1.10) and (1.11) jointly identify the domestic cutoff  $\phi_i^*$  by solving the following equilibrium condition.

$$[1 - G(\phi_i^*)] f k(\phi_i^*) = \delta f_e \tag{1.12}$$

LHS of the equation is the present discounted profit of the firm upon drawing from the CDF  $G(\cdot)$ . Let us define  $j(\phi) = [1 - G(\phi)]k(\phi)$ . For the rest of the paper I will consider Pareto distribution, where  $g(\phi) = \frac{\alpha}{\phi^{\alpha+1}} \forall \phi > 1$  and for  $\alpha > \varepsilon$ . This assumption implies that  $j(\phi) = \frac{1}{\phi^\alpha} \left[ \frac{\varepsilon}{\alpha - \varepsilon} \right]$ . So,  $j(\phi)$  is decreasing in it's argument as  $j'(\phi) < 0$  and we have a unique solution from the system.

### 1.3.1 Distribution of employment at Autarky

**Claim 1** *Given the active mass of firms  $M$ , a firm draws  $\phi$  from  $g(\phi)$  and employs  $l(\phi)$ . So the distribution of employment will be:*

$$e(\phi_i) = \frac{M\mu(\phi_i)l(\phi_i)}{L} \quad \forall \in (\phi_i^*, \infty) \quad (1.13)$$

*The Pareto assumption implies the equilibrium distribution to be Pareto as well,*

*$\mu(\phi_i) = \frac{\alpha\phi_i^{*\alpha}}{\phi_i^{\alpha+1}}$  for  $\phi_i > \phi_i^*$ , and employment distribution simplifies to:*

$$e(\phi_i) = \frac{1}{\phi_{ie}^{\varepsilon-\theta}} \frac{\alpha\phi_i^{*\alpha}}{\phi_i^{\alpha+\theta+1-\varepsilon}} \quad \forall \in (\phi_i^*, \infty) \quad (1.14)$$

The employment distribution takes the form of Pareto distribution with location parameter being  $\frac{\alpha\phi_i^{*\alpha}}{\phi_{ie}^{\varepsilon-\theta}}$  and the shape parameter being  $\alpha + \theta + 1 - \varepsilon$  ( See appendix for prove). Since we have the equation for wage ( $w(\phi) = \phi^\theta$ ), the wage distribution function can be obtained by applying a random variable transformation technique. This distribution is then given by the following expression.

$$y_i(w) = \frac{1}{\phi_{ie}^{\varepsilon-\theta}} \frac{\alpha}{\theta} \phi_i^{*\alpha} w^{\frac{\varepsilon-\theta-\alpha}{\theta}-1} \quad \forall w \in (\phi_i^{*\theta}, \infty) \quad (1.15)$$

This is the wage weighted by the employment of the firm of every observed  $\phi$ . Note that this still remains a Pareto distribution, but with different shape and location parameter. (See appendix)

The CDF and average wage can be constructed from the PDF of wage. Again by standard statistical theory the CDF and average is given by ' $Y_i(w) = 1 - \left(\frac{w}{\phi_i^{*\theta}}\right)^{\varepsilon-\theta-\alpha}$ ', and ' $E(w) = \phi_i^{*\theta}$ '. These statistics help construct the Lorenz curve at autarky to be:

$$L(Y_i) = 1 - [1 - Y_i]^{\frac{\varepsilon-\alpha}{\varepsilon-\alpha-\theta}} \quad (1.16)$$

The Lorenz curve Shows some level of inequality. However given Pareto distribution this was expected. This is the base line level of inequality that I compare with inequality from other state of the economy. (For details go to appendix)

## 1.4 Trade economy

In this section countries open up for trade and firms can engage in both domestic production and exporting activity. I assume country symmetry, hence I can drop the country index. Exporting firms have to undertake two additional costs; a per unit cost of tariff  $\tau > 1$  and a fixed cost to export to be  $f_x$ , which is bigger than  $f\tau^{1-\sigma}$ . Now the export price, revenue and profit will have the following expression.

$$p_x(\phi_x) = \tau p(\phi) \quad (1.17)$$

$$r_x(\phi_x) = R\phi^\varepsilon \left[ \tau^{-1} \frac{\sigma - 1}{\sigma} P \right]^{\sigma-1} = \tau^{1-\sigma} r(\phi_x) \quad (1.18)$$

$$\pi_x(\phi_x) = \frac{r_x(\phi_x)}{\sigma} - f_x = \frac{\tau^{1-\sigma} r(\phi_x)}{\sigma} - f_x \quad (1.19)$$

Note that the market conditions are symmetric across countries, hence the aggregate price index is same across countries and  $r_i(\phi) = r_j(\phi)$  for any two countries  $i$  and  $j$ . Again by the property of the profit function I can find the cutoff for exporter  $\phi_x^*$  such that  $\pi_x(\phi_x^*) = 0$ . This implies that  $\tau^{1-\sigma} r(\phi_x^*) = \sigma f_x$ . The cutoff for domestic and export market is tied by  $\phi_x^* = \phi^* \left[ \frac{f_x}{f} \tau^{\sigma-1} \right]^{\frac{1}{\varepsilon}}$ . The consumers can now access foreign goods as these countries opened up for trade. As a result the expression for average productivity from consumption goods is given by,  $\tilde{\phi}_t^\varepsilon = \frac{1}{M_t} [M \tilde{\phi}_d^\varepsilon + M P_x \tau^{-\varepsilon} \tilde{\phi}_x^\varepsilon]^\frac{7}{}$ . This is a similar index defined by Melitz in his 2003 paper. Note that,  $P_x = \frac{1-G(\phi_x^*)}{1-G(\phi^*)}$  is the probability of some one being an exporter given than it survives the domestic competition. This productivity index considers both foreign and domestic goods, since now firms have exposure to foreign market.

### 1.4.1 Trade economy equilibrium

The FEC and ZCP together gives the following expression in trade economy:

$$\frac{f}{\delta} j(\phi^*) + \frac{f_x}{\delta} j(\phi_x^*) = f_e \quad (1.20)$$

---

<sup>7</sup>Here  $M_t = M(1+p_x)$  and the mass of firms inside a country is determined by the market clearing condition.  $\tilde{\phi}_d^\varepsilon$  and  $\tilde{\phi}_x^\varepsilon$  are the average coming from domestic producers and exporters only respectively.



The interpretation remains the same, however due to the additional term in LHS the domestic cutoff becomes higher. This happens due to the increased competition coming from foreign market. More productive exporters enters the market with lower price and drives the less productive domestic producers with higher price out from the market; a well known mechanism in Melitz literature.

### Mass of firms

The labor market clearing condition at trade has an additional term from employment of exporters and is given by:  $L = M \int_{\phi^*}^{\infty} l(\phi) \frac{g(\phi)}{1-G(\phi^*)} d\phi + MP_x \int_{\phi_x^*}^{\infty} l_x(\phi) \frac{g(\phi)}{1-G(\phi_x^*)} d\phi$ . By using the similar analogy as before, the mass of firms at trade economy is:

$$M = \frac{L}{\sigma - 1} \left[ \frac{f}{\phi^{*\varepsilon}} \int_{\phi^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi^*)} d\phi + \frac{P_x f_x}{\tau \phi_x^{*\varepsilon}} \int_{\phi_x^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_x^*)} d\phi \right]^{-1} \quad (1.21)$$

Please refer to appendix for complete derivation. Note that the active mass of firms from a country is lower than autarky, but now countries have access to foreign market. So the total number of firms serving any market is  $M_t = M(1 + P_x) > M_a$  =mass at autarky. Hence total variety available to consumer increases.

#### 1.4.2 Distribution of employment and Lorenz curve at trade economy

Employment distribution depends on aggregate productivity index faced by both consumers and producers. Aggregate productivity index faced by consumers is similar to the productivity index defined by Melitz (2003), but productivity index faced by producers considers the aggregate productivity of all the output produced in a country for both domestic consumption and exporting activities. This index can be expressed in two ways: a) consider the productivity of all firms producing for domestic market ( $\phi \in (\phi_d^*, \infty)$ ) and exporters only ( $\phi \in (\phi_x^*, \infty)$ ), b) consider productivity of firms producing for domestic market only ( $\phi \in (\phi_d^*, \phi_x^*)$ ) and the productivity of firms that serve both domestic and export market ( $\phi \in (\phi_x^*, \infty)$ ). The second way disaggregates the index in two groups, a) contribution coming from domestic producers only

and b) contribution coming from firms that both export and supply in domestic market. Hence the productivity index using the latter method can be expressed as follows,

$$\tilde{\phi}_{te}^{\varepsilon-\theta} = \frac{1}{M_t} [(1 - P_x)M \left(x\tilde{\phi}_d\right)^{\varepsilon-\theta} + P_x M (1 + \tau^{\theta-\varepsilon}) \left(\tilde{\phi}_x\right)^{\varepsilon-\theta}] \quad (1.22)$$

where  $\left(x\tilde{\phi}_d\right)^{\varepsilon-\theta} = \int_{\phi_d^*}^{\phi_x^*} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_d^*)} d\phi$  and  $\left(\tilde{\phi}_x\right)^{\varepsilon-\theta} = \int_{\phi_x^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_x^*)} d\phi$ . The first index,  $\left(x\tilde{\phi}_d\right)^{\varepsilon-\theta}$ , refers to the firms that serve only domestic market and the second index,  $\left(\tilde{\phi}_x\right)^{\varepsilon-\theta}$ , refers to the firms that serve both domestic and export market. Note that the relative mass of the firms that are contributing to the productivity index is multiplied. However, the contribution to aggregate productivity for exporters is discounted by the tariff and/or transportation cost  $\tau$ , that is lost in transit; but the same mass  $P_x M$  of firms do not pay this tariff while serving the domestic market. Hence exporters effective productivity is different, depending where they sell their product. After I define the aggregate productivity index in this manner, the employment distribution can be derived from it as shown in equation (1.14).

$$e(\phi) = \begin{cases} \frac{1}{\tilde{\phi}_{te}^{\varepsilon-\theta}} \frac{1-P_x}{1+P_x} \frac{\alpha\phi_d^{*\alpha}}{\phi^{\alpha+1-\varepsilon+\theta}} & \forall \phi \in (\phi_d^*, \phi_x^*) \\ \frac{1}{\tilde{\phi}_{te}^{\varepsilon-\theta}} \frac{P_x}{1+P_x} (1 + \tau^{\theta-\varepsilon}) \frac{\alpha\phi_x^{*\alpha}}{\phi^{\alpha+1-\varepsilon+\theta}} & \forall \phi \in (\phi_x^*, \infty) \end{cases} \quad (1.23)$$

**Proof.** Note that  $\int_{\phi_d^*}^{\phi_x^*} \frac{\alpha\phi_d^{*\alpha}}{\phi^{\alpha+1-\varepsilon+\theta}} = \left(x\tilde{\phi}_d\right)^{\varepsilon-\theta}$  and  $\int_{\phi_x^*}^{\infty} \frac{\alpha\phi_x^{*\alpha}}{\phi^{\alpha+1-\varepsilon+\theta}} = \left(\tilde{\phi}_x\right)^{\varepsilon-\theta}$ . Hence once we

integrate over the interval I get  $e(\phi_d^* < \phi \leq \infty) = 1$  and  $e'(\cdot) < 0$ , so this is our employment distribution at trade. ■

The employment distribution in equation (1.23) has two parts, a) employment in firms that produce for domestic market only and b) employment in firms that produce for both export and domestic markets. Note that at the cutoff point for exporters there is an increase in employment, as they find additional demand in the foreign market. To satisfy this additional demand exporters hire more workers, that are absorbed from the exiting domestic firms and existing domestic firms that are shrinking. Trade does not effect the shape of wage equation, it just shifts it to the right. By using the wage equation I can find the wage distribution in trade economy as the following:

$$y(w_t) = \begin{cases} \frac{1}{\tilde{\phi}_{te}^{\varepsilon-\theta}} \frac{1-P_x}{1+P_x} \frac{\alpha}{\theta} \phi_d^{*\alpha} w_t^{\frac{\varepsilon-\theta-\alpha}{\theta}-1} & \forall w_t \in (\phi_d^{*\theta}, \phi_x^{*\theta}) \\ \frac{1}{\tilde{\phi}_{te}^{\varepsilon-\theta}} \frac{P_x}{1+P_x} (1 + \tau^{\theta-\varepsilon}) \frac{\alpha}{\theta} \phi_x^{*\alpha} w_t^{\frac{\varepsilon-\theta-\alpha}{\theta}-1} & \forall w_t \in (\phi_x^{*\theta}, \infty) \end{cases} \quad (1.24)$$

The steps to find this density is similar to the case of autarky. The only difference is that I apply the transformation technique to distribution of employment in both sections separately. Since exporters are serving two markets, they employs additional labor to produce some extra units to serve foreign market. Hence the weighted payment to this segment of the workers is higher even though the firms offer the same wage. This extra amount of labor is less efficient since they face tariff. Together the density has a jump at  $\phi_x^{*\theta}$ . This jump creates a distortion in the labor market. Please refer to the figure 1.1<sup>8</sup>:

FIGURE 1.1 : "PDF of wage in trade and autarky"

Trade reallocates some workers from least paid exiting domestic firms to high paying exporting firms. Note that the firms who earn export status already has some employment for domestic production, hence the new workers joining them earns same as old workers. This reallocation offers higher wage to only one segment of the population. This makes the tail of the distribution fat relative to autarky. As a result, now more worker earn relatively more.

To find the Lorenz curve I need to know the CDF ( $Y(w_t)$ ) that can be obtained by standard statistical method and the average wage  $E(w_t)$ . The expected wage is given by,  $E(w_t) = \int w_t y(w_t) dw_t = \frac{\tilde{\phi}_t^\varepsilon}{\tilde{\phi}_{te}^{\varepsilon-\theta}} = \frac{\tilde{\theta}}{\phi_{te}}$ , that is discontinuous at the cutoff for marginal firm who earns exporting status .

**Claim 2** Given  $\tau$  and  $f_x$  the Lorenz curve has the following expression from opening up for trade:

$$L(Y_t) = \begin{cases} \frac{1}{E(w_t)} \gamma_{11}^t \left( 1 - (1 - Y_t \times \gamma_{12}^t)^{\frac{\varepsilon-\alpha}{\varepsilon-\alpha-\theta}} \right) & \forall Y_t \in (0, Y_t(\phi_x^{*\theta})) \\ L(Y_t(\phi_x^{*\theta})) + \frac{1}{E(w_t)} \gamma_{21}^t & \\ \times \left( 1 - (1 - (Y_t - Y_t(\phi_x^{*\theta})) \gamma_{22}^t)^{\frac{\varepsilon-\alpha}{\varepsilon-\alpha-\theta}} \right) & \forall Y_t \in (Y_t(\phi_x^{*\theta}), 1) \end{cases} \quad (1.25)$$

<sup>8</sup>The figure is generated by using the following values for the parameter.  $L = 100$ ,  $\theta = .5$ ,  $\sigma = 4$ ,  $\alpha = 5$ ,  $f = 3$ ,  $f_x = 5$ ,  $f_i = 11$ ,  $f_e = 2$ ,  $\delta = .2$ ,  $\tau = 1.3$ .

Where,  $\gamma_{11}^t = \frac{1}{\tilde{\gamma}_{te}^{\varepsilon-\theta}} \frac{1-P_x}{1+P_x} \frac{\alpha}{\alpha-\varepsilon} \phi_d^{*\varepsilon}$ ,  $\gamma_{12}^t = \left(\frac{\tilde{\phi}_{te}}{\phi_d^*}\right)^{\varepsilon-\theta} \frac{1+P_x}{1-P_x} \frac{\alpha+\theta-\varepsilon}{\alpha}$ ,  $\gamma_{21}^t = \frac{1}{\tilde{\gamma}_{te}^{\varepsilon-\theta}} \frac{P_x}{1+P_x} (1+\tau^{\theta-\varepsilon}) \frac{\alpha}{\alpha-\varepsilon} \phi_x^{*\varepsilon}$   
and  $\gamma_{22}^t = \left(\frac{\tilde{\phi}_{te}}{\phi_x^*}\right)^{\varepsilon-\theta} \frac{1+P_x}{P_x} \frac{1}{1+\tau^{\theta-\varepsilon}} \frac{\alpha+\theta-\varepsilon}{\alpha}$ . Note that the Lorenz curve has a kink at  $Y_t(\phi_x^{*\theta}) = \frac{\alpha-\varepsilon}{\alpha+\theta-\varepsilon} \gamma_{11}^t [1 - \left(\frac{\phi_x^*}{\phi_d^*}\right)^{\varepsilon-\theta-\alpha}]$ , this comes from the fact that the employment at the marginal firm observes a raise. Due to this kink I have two segments in the Lorenz Curve. Now the relative wage of exporters is higher than domestic producer only.

**Proposition 3** *If wage is an increasing function ( $w = \phi^\theta$ ) of firms productivity with rent sharing parameter of  $\theta \in (0, 1)$ , Under Melitz environment opening up to trade increases wage inequality.*

Please refer to the figure 1.2<sup>9</sup>:

FIGURE1.2:” lorenz curve at autarky and trade”

The mass of firms who can export suddenly finds additional profit that can be earned by exporting. To satisfy this additional demand of their goods in foreign market they hire some workers, who used to work for exiting small firms. These small firms exit due to increased competition. However the group of newly hired workers, who lost their jobs from exiting firms, earn same as some old workers hired by the same firm with exporting capability. This distortion increase the employment at the cutoff for exporters and increases the weight at this cutoff and above. For example consider any  $\phi \geq \phi_x^*$ , then employment by this firm jumps due to additional production. This leads to a higher average wage at open market. The average wage of exporters increase more than the average wage of domestic producer. This distortion in the average wage across sectors (domestic and export market producers) leads to higher inequality for both trading countries<sup>10</sup>.

<sup>9</sup>An additional simulation of Gini at autarky and trade confirms that  $G_a = 0.1099 < G_t = 0.1157$ , where they are Gini at autarky and trade respectively.

The model parameters for simulation will be discussed later.

<sup>10</sup>This result aligns with the existing literature that studies the link between inequality and globalization. For example, Egger and Kreickemeier (2009) as well as in Helpman, Itskhoki and Redding (2008), Danziger (2014) and Pupato (2014)

## 1.5 FDI economy

Firms investing in foreign market have to under take an additional sunk fixed cost of  $f_I > f_x \tau^{\sigma-1} > f > f_e$  (An assumption to separate investors from exporters). However since I consider horizontal FDI with proximity concentration, now goods will not have to bear transportation cost. With proximity concentration firms invest only when the profit from investing more then compensates from avoiding transportation cost to serve foreign market. Note that the symmetry assumption implies similar demand condition across countries. This essentially implies that the price asked by investors in foreign market is same as their domestic competitors with same level of productivity. Given this I can find the expression for price, revenue and profit by the following expressions:

$$p_I(\phi_I) = p(\phi_I) \quad (1.26)$$

$$r_I(\phi_I) = r(\phi_I) \quad (1.27)$$

$$\pi_I(\phi_I) = \frac{r(\phi_I)}{\sigma} - f_I \quad (1.28)$$

Since I used the proximity concentration, the cutoff for investors is pinned down where the difference in profit for investors to exporters is zero; by the expression  $\pi_I(\phi_I^*) - \pi_x(\phi_I^*) = 0$ . This implies that  $(1 - \tau^{1-\sigma})r(\phi_I^*) = \sigma(f_I - f_x)$  and the FDI and domestic cutoffs are tied by,  $\phi_I^* = \phi^* \left[ \frac{f_I - f_x}{f} \frac{1}{1 - \tau^{1-\sigma}} \right]^{\frac{1}{\varepsilon}}$ . The aggregate productivity index faced by consumers now has three terms, along with their relative mass in the economy.

$$\tilde{\phi}_{tI}^\varepsilon = \frac{1}{M_I} [M (x' \tilde{\phi}_{d'})^\varepsilon + M(P_{x'} - P_I) \tau^{-\varepsilon} (I \tilde{\phi}_{x'})^\varepsilon + M P_I \tilde{\phi}_I^\varepsilon] \quad (1.29)$$

A point to note here is that the mass of foreign firms do not increase due to this additional channel as these investors were exporters before they had access to this channel of FDI. Hence the total variety available to consumer now is  $M_I = (1 + P_{x'})M$ . Firms probability of being an exporter depending on if it survives the domestic competition or not has a similar expression as before; however, now the domestic and export cutoff is higher due to increased competition and the expression uses the new cutoffs described in FDI economy.

### 1.5.1 Equilibrium in FDI economy

Once the firms decision is characterized I can combine the FEC and ZCP together to have the following expression of equilibrium with both trade and FDI:

$$\frac{f}{\delta}j(\phi_{d'}^*) + \frac{f_x}{\delta}j(\phi_{x'}^*) + \frac{f_I - f_x}{\delta}j(\phi_I^*) = f_e \quad (1.30)$$

Note that, now I have an additional term on top of trade economy equilibrium. This implies higher competition in domestic market. This increased competition applies the reallocation of resources to higher productive firms and push out the least productive firms out of business. High productive investors charge a lower price relative to exporters and domestic producers in a country. Once they enter the market least productive domestic firm cannot compete since they charge a higher price. This drives them out of the market and foreign competitors expand their market share by occupying this excess demand.

#### Mass of firms

The labor market clearing condition in FDI economy has an addition term as it will employ some additional workers for investing firms and is given by:  $L = M \int_{\phi^*}^{\infty} l(\phi) \frac{g(\phi)}{1-G(\phi^*)} d\phi + M(P_x - P_I) \int_{\phi_x^*}^{\infty} l_x(\phi) \frac{g(\phi)}{1-G(\phi_x^*)} d\phi + MP_I \int_{\phi_I^*}^{\infty} [l_I(\phi) - l_x(\phi)] \frac{g(\phi)}{1-G(\phi_I^*)} d\phi$ . The mass of firms that survive the increased competition due to FDI gets lower than trade equilibrium and is given by,

$$M = \frac{L}{\sigma - 1} \left[ \frac{f}{\phi^{*\varepsilon}} \int_{\phi^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi^*)} d\phi + \frac{P_x - P_I}{\tau} \frac{f_x}{\phi_x^{*\varepsilon}} \int_{\phi_x^*}^{\phi_I^*} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_x^*)} d\phi + P_I(1 - \tau^{-\sigma}) \frac{f_I - f_x}{\phi_I^{*\varepsilon}} \int_{\phi_I^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_I^*)} d\phi \right]^{-1} \quad (1.31)$$

Please refer to the appendix for detail derivation. However the total variety available is  $M_I = M(1 + P_{x'})$  and  $M_I \leq M_t > M_a =$  mass of firms active in autarky. Note that  $P_I = \frac{1-G(\phi_I^*)}{1-G(\phi^*)}$  is the conditional probability of some one being an investor given that it survives the domestic market competition.

## 1.5.2 Employment distribution at FDI economy

Again we need to define the aggregate productivity index such that, we have contribution coming from three different sources: such as a) firms producing for domestic market only, b) firms producing for both domestic and export market, c) firms producing for both domestic and FDI market. Hence the index will be :

$$\tilde{\phi}_{tIe}^{\varepsilon-\theta} = \frac{1}{M_I} [M(1-P_{x'}) (x' \tilde{\phi}_{d'})^{\varepsilon-\theta} + M(P_{x'} - P_I)(1+\tau^{\theta-\varepsilon}) (I \tilde{\phi}_{x'})^{\varepsilon-\theta} + 2P_I M (\tilde{\phi}_I)^{\varepsilon-\theta}] \quad (1.32)$$

Note that here  $(x' \tilde{\phi}_{d'})^{\varepsilon-\theta} = \int_{\phi_{d'}^*}^{\phi_{x'}^*} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_{d'}^*)} d\phi$ ,  $(I \tilde{\phi}_{x'})^{\varepsilon-\theta} = \int_{\phi_{x'}^*}^{\phi_I^*} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_{x'}^*)} d\phi$  and  $(\tilde{\phi}_I)^{\varepsilon-\theta} = \int_{\phi_I^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_I^*)} d\phi$ <sup>11</sup>. The mass of active firms in this economy is lower than trade economy, and the three indices are the aggregate productivity index of domestic firms only, exporters and domestic firms only and the last one is for investors and domestic firms only. After defining the appropriate averages, the employment distribution can be derived using similar steps as trade economy.

$$e(\phi) = \begin{cases} \frac{1}{\tilde{\phi}_{tIe}^{\varepsilon-\theta}} \frac{1-P_{x'}}{1+P_{x'}} \frac{\alpha \phi_{d'}^{*\alpha}}{\phi^{\alpha+1-(\varepsilon-\theta)}} & \forall \phi \in (\phi_{d'}^*, \phi_{x'}^*) \\ \frac{1}{\tilde{\phi}_{tIe}^{\varepsilon-\theta}} \frac{P_{x'}-P_I}{1+P_{x'}} (1+\tau^{\theta-\varepsilon}) \frac{\alpha \phi_{x'}^{*\alpha}}{\phi^{\alpha+1-(\varepsilon-\theta)}} & \forall \phi \in (\phi_{x'}^*, \phi_I^*) \\ \frac{1}{\tilde{\phi}_{tIe}^{\varepsilon-\theta}} \frac{2P_I}{1+P_{x'}} \frac{\alpha \phi_I^{*\alpha}}{\phi^{\alpha+1-(\varepsilon-\theta)}} & \forall \phi \in (\phi_I^*, \infty) \end{cases} \quad (1.33)$$

Note that the mass of firms working as investors are smaller than mass of domestic producers only. On the other hand, the mass of exporters can be smaller or bigger to mass of investors depending on the parameter values. For example, if tariff is low enough then the mass of exporters is higher. However this small mass of investors employ relatively higher number of workers compare to exporters, and exporters employ relatively more workers than domestic producer only. Both exporters and investors observe an increase in the employment, using similar mechanism for trade economy. These jumps are coming from the fact that they face additional demand other than domestic demand from foreign market. Exporters face higher

<sup>11</sup>Note that  $\phi_{d'}^*$  and  $\phi_{x'}^*$  refers to domestic and export cutoff with FDI in the economy.

marginal cost due to tariff and investors avoid this per unit cost but have to undertake a higher fixed cost to invest. This decreases the contribution of productivity of worker (marginal product of labor), who works for firms with exporting status, hence the jump for exporters is less than investors. Note that investors do not have such productivity loss from serving foreign market, since they set up new plants in foreign market and avoid paying tariff  $\tau$ .

Following the same steps as before, the wage distribution at trade economy can be constructed from the employment distribution. Now to obtain the PDF, I apply the Jacobian transformation to three different parts of the employment distribution separately. The wage equation again shifts to the right due to increased competition in the market. After the calculation the wage distribution takes the following expression:

$$y(w_{tI}) \begin{cases} \frac{1}{\tilde{\phi}_{tIe}^{\varepsilon-\theta}} \frac{1-P_{x'}}{1+P_{x'}} \frac{\alpha}{\theta} \phi_{d'}^{*\alpha} w_{tI}^{\frac{\varepsilon-\theta-\alpha}{\theta}-1} & \forall w_{tI} \in (\phi_{d'}^{*\theta}, \phi_{x'}^{*\theta}) \\ \frac{1}{\tilde{\phi}_{tIe}^{\varepsilon-\theta}} \frac{P_{x'}-P_I}{1+P_{x'}} (1+\tau^{\theta-\varepsilon}) \frac{\alpha}{\theta} \phi_{x'}^{*\alpha} w_{tI}^{\frac{\varepsilon-\theta-\alpha}{\theta}-1} & \forall w_{tI} \in (\phi_{x'}^{*\theta}, \phi_I^{*\theta}) \\ \frac{1}{\tilde{\phi}_{tIe}^{\varepsilon-\theta}} \frac{2P_I}{1+P_{x'}} \frac{\alpha}{\theta} \phi_I^{*\alpha} w_{tI}^{\frac{\varepsilon-\theta-\alpha}{\theta}-1} & \forall w_{tI} \in (\phi_I^{*\theta}, \infty) \end{cases} \quad (1.34)$$

Please refer to the figure 1.3<sup>12</sup>:

FIGURE1.3:”PDF of wage at autarky trade and FDI”

The figure shows that the wage distribution has two jumps for the marginal firms for exporters and investors. Some highly productive exporters are able to invest in the foreign market. This creates additional demand for these new investor’s goods in the foreign market since they charge lower price than before. To satisfy this demand investors shrink the export and domestic market by increasing competition. The increased competition forces some firms to exit and the surviving firms to contract. This in turn releases some workers from their previous employer and reallocates them to some highly paid investors. Similar kind of result is revealed from the lorenz curve as well.

To find the Lorenz curve I need to know the CDF ( $Y(w_{tI})$ ) and the average wage  $E(w_{tI})$ . The CDF can be obtained by standard statistical theory and the expression for Expected wage at FDI economy is  $E(w_{tI})$  is derived from the weighted wage distribution as well. The average wage has contribution coming from three parts; (1)domestic producers only, (2) exporters and

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<sup>12</sup>I cannot consider  $\tau > 1.3$  since this will lead to export cutoff being higher than FDI cutoff given  $f_I = 11$ .



domestic market producers and (3) investors and domestic market producers. As countries open new channels to access the foreign market the average wage increases. Now investors employ more workers since their foreign market activity is more efficient. This leads to a higher employment above the cutoff for investment ( $\phi \geq \phi_I^*$ ). The exporting sector and domestic sector shrinks in terms of employment. In the process the average wage of investment sector increases more than the average of exporting and domestic sector.

**Claim 4** Given  $\tau$ ,  $f_x$  and  $f_I$  the Lorenz curve has the following expression from to opening up for trade and investment:

$$L(Y_{tI}) = \begin{cases} \frac{1}{E(w_{tI})} \gamma_{11}^I \left( 1 - (1 - Y_{tI} \times \gamma_{12}^I)^{\frac{\varepsilon - \alpha}{\varepsilon - \alpha - \theta}} \right) & \forall Y_{tI} \in (0, Y_{tI}(\phi_{x'}^{*\theta})) \\ L(Y_{tI}(\phi_{x'}^{*\theta})) + \frac{1}{E(w_{tI})} \gamma_{21}^I \times \\ \left( 1 - (1 - (Y_{tI} - Y_{tI}(\phi_{x'}^{*\theta})) \gamma_{22}^I)^{\frac{\varepsilon - \alpha}{\varepsilon - \alpha - \theta}} \right) & \forall Y_{tI} \in (Y_{tI}(\phi_{x'}^{*\theta}), Y_{tI}(\phi_I^{*\theta})) \\ L(Y_{tI}(\phi_I^{*\theta})) + \frac{1}{E(w_{tI})} \gamma_{31}^I \times \\ \left( 1 - (1 - (Y_{tI} - Y_{tI}(\phi_I^{*\theta})) \gamma_{32}^I)^{\frac{\varepsilon - \alpha}{\varepsilon - \alpha - \theta}} \right) & \forall Y_{tI} \in (Y_{tI}(\phi_I^{*\theta}), 1) \end{cases} \quad (1.35)$$

$$\text{Where, } \gamma_{11}^I = \frac{\phi_{d'}^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \frac{1 - P_{x'}}{1 + P_{x'}} \frac{\alpha}{\alpha - \varepsilon}, \quad \gamma_{12}^I = \left( \frac{\tilde{\phi}_{tIe}}{\phi_{d'}^{*\varepsilon}} \right)^{\varepsilon - \theta} \frac{1 + P_{x'}}{1 - P_{x'}} \frac{\alpha + \theta - \varepsilon}{\alpha}, \quad \gamma_{21}^I = \frac{\phi_{x'}^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \frac{P_{x'} - P_I}{1 + P_{x'}} (1 + \tau^{\theta - \varepsilon}) \frac{\alpha}{\alpha - \varepsilon}, \\ \gamma_{22}^I = \left( \frac{\tilde{\phi}_{tIe}}{\phi_{x'}^{*\varepsilon}} \right)^{\varepsilon - \theta} \frac{1 + P_{x'}}{P_{x'} - P_I} \frac{1}{1 + \tau^{\theta - \varepsilon}} \frac{\alpha + \theta - \varepsilon}{\alpha}, \quad \gamma_{31}^I = \frac{\phi_I^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \frac{2P_I}{1 + P_{x'}} \frac{\alpha}{\alpha - \varepsilon} \quad \text{and} \quad \gamma_{32}^I = \left( \frac{\tilde{\phi}_{tIe}}{\phi_I^{*\varepsilon}} \right)^{\varepsilon - \theta} \frac{1 + P_{x'}}{2P_I} \frac{\alpha + \theta - \varepsilon}{\alpha}.$$

Note that the Lorenz curve has two kinks at investment economy: the First one is at exporters cutoff,  $Y_{tI}(\phi_{x'}^{*\theta}) = \left( \frac{\phi_{d'}^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \right)^{\varepsilon - \theta} \frac{1 - P_{x'}}{1 + P_{x'}} \frac{\alpha}{\alpha + \theta - \varepsilon} \left[ 1 - \left( \frac{\phi_{d'}^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \right)^{\varepsilon - \theta - \alpha} \right]$ , and the second one is at the investors cutoff  $Y_{tI}(\phi_I^{*\theta}) = Y_{tI}(\phi_{x'}^{*\theta}) + \left( \frac{\phi_{x'}^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \right)^{\varepsilon - \theta} \frac{P_{x'} - P_I}{1 + P_{x'}} (1 + \tau^{\theta - \varepsilon}) \frac{\alpha}{\alpha + \theta - \varepsilon} \left[ 1 - \left( \frac{\phi_I^{*\varepsilon}}{\tilde{\phi}_{tIe}^{\varepsilon - \theta}} \right)^{\varepsilon - \theta - \alpha} \right]$ . These kinks are there due to the distortion created in the economy by opening up to foreign market.

**Proposition 5** If wage is an increasing function ( $w = \phi^\theta$ ) of firms productivity with rent sharing parameter of  $\theta \in (0, 1)$ , Under Melitz environment opening up to trade and FDI further increases wage inequality.

The mass of firms who can invest suddenly finds investing in a foreign market is more profitable than exporting. Moreover the effective productivity of workers are now no longer be less due to no tariff in place, hence these new investing firms can ask for a lower price compared to what they used to ask as exporters. This effect creates additional demand for

their output. To satisfy this additional demand of their good in foreign market they hire some workers. These workers come from exiting and contracting firms from domestic market and export market. Increased competition increases the cutoff for domestic market and investors increases competition in foreign market for exporters, hence some least productive exporters exits the export market and start producing for domestic market. Now these two groups of people are hired by the investors to satisfy their extra demand in the foreign market. Additional to the jump of employment for all exporters there is another jump in the level of employment in the investors as well. However this newly hired workers by investors earn higher wage than before (when their firm was exporter). This causes an additional distortion in the average wage of the economy, that leads to increase in social inequality. Please refer to the figure 1.4<sup>13</sup>:

FIGURE1.4:"Lorenz curve at autarky, trade and FDI"

## 1.6 Comparative static analysis

This section studies two aspects of globalization: a symmetric bilateral tariff reduction by trading partners and a symmetric decrease in fixed cost to invest. This analysis is conducted using computer simulations<sup>14</sup>. The model parameters for these task are given below;

$L=100, \theta=.4, \sigma=3.8, \alpha=3.3, f=3, f_x=5.5, f_I=12, f_e=2, \delta=.2, \tau=1.15$  and  $l=1$ . Note that  $\tau$  and  $f_I$  will take different values for comparative static analysis.

### 1.6.1 Trade liberalization "decrease in tariff"

For this exercise I solve the entire model with two level of tariffs, a)  $\tau = 1.1$  and b)  $\tau = 1.2$ . Please go to figure 1.5 and 1.6.

FIGURE1.5:"Effect of change in tariff to domestic cutoff"

FIGURE1.6:"Effect of change in tariff to export and FDI cutoff"

Given the parameter values if tariff goes above 1.32, domestic cutoff starts to rise. This violates the assumptions related to the fixed costs in three different states of the economy.

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<sup>13</sup>The Gini at FDI was simulated as well for comparison.  $G_I = 0.1821 > G_t = 0.1157 > G_a = 0.1099$ .

<sup>14</sup>The parameter values that I assumed during the simulation is taken from Demidova (2006) and Helpman, Yeaple and Melitz (2004) paper.

Hence figure 1.5 plots tariff from 1.1 to 1.32 and from figure 1.6, it is understood that above 1.32 of tariff I am no longer able to separate exporters from investors. Hence for this section, the analysis takes place in the tolerable range.

A decrease in tariff has two opposing effects on the domestic market competition: a) some highly productive domestic producers have additional profit opportunity from exporting and b) some low productive investors find exporting is profitable than investing. The first effect increases competition in domestic market and pushes up the cutoff, where as the second effect decreases competition in foreign market for exporters and increases the FDI cutoff. This additional exporters further increase competition that leads to higher domestic cutoff for both countries. These entree and exit governs the direction of inequality created by trade liberalization. The mass of firms in domestic market decreases, mass of exporters increases and finally the mass of investors decreases. People leaving from least productive exiting firms (both from domestic and investment market) decreases inequality and people joining to high productive firms increases inequality. It turns out that due to this the inequality goes down, hence the first effect dominates. The simulation generates the PDF of wage at FDI economy for two levels of tariff. Please refer to the figure 1.7.

FIGURE1.7:"Effect of change in tariff to PDF of wage at FDI economy "

From the figure, decrease in tariff increases the income of workers going from domestic exiting firms to exporting firms and decreases income of workers going from exiting investing firms to exporting firms. Workers coming from exiting domestic and investing firms decrease inequality. In this process the majority of the population who got displaced from the center of the distribution due to higher tariff goes back to the center. As a result the Lorenz curve displays a decrease in wage inequality. Please refer to the figure 1.8:

FIGURE1.8:" Effect of change in tariff to lorenz curve of FDI"

### 1.6.2 A decrease in fixed cost to invest $f_I$

Following same kind of idea, I took two different levels of  $f_I$  and solve the entire model. The fixed cost  $f_I$  varies between 11 and 16. However in this case there is a lower bound to fixed cost that is applicable. Below that level I cannot separate the investors from exporters. Please refer to figure 1.9 and 1.10.

Figure1.9: "Effect of change in fixed cost to invest to domestic cutoff "

Figure1.10: "Effect of change in fixed cost to invest to export and FDI cutoff "

Figure 1.10 confirms that as fixed cost to invest gets smaller the cutoff for investment gets smaller. However the export cutoff is unaffected by this change, hence There is a point ( $f_I < 7.2$ ) below that I cannot separate exporters from investors. The rest of the analysis takes place in tolerable range for the model.

The decrease in fixed cost to invest has two opposing effects like before: a) open up new investment opportunity for some highly productive exporters, hence increase the competition in foreign market and b) increased competition in foreign market pushes some exporters out of the market, hence decrease competition in foreign market. However the simulation result confirms that the first effect dominates and leads to an increase in domestic cutoff for both countries. The change in mass of domestic firms is ambiguous, where as the mass of exporters decrease and mass of investors increase. Please refer to the figure 1.11:

FIGURE1.11: "Effect of change in fixed cost to invest to PDF of wage at FDI economy"

As the transition takes, place more workers from less paid jobs lose their jobs for high paid jobs in investing firms. This effect increases inequality, on the other hand people losing jobs from least productive exiting domestic firms decrease inequality. The first effect dominates in this case and majority of the population moves away from the average wage of the economy. Please refer to the figure 1.12:

FIGURE:1.12"Effect of change in fixed cost to invest to Lorenz curve at FDI"

### 1.6.3 The effect of Technological Improvement

Technological improvement in this set up with Pareto distribution implies a decrease in the shape parameter of Pareto distribution<sup>15</sup>. As technology improves the local market of both economies become more competitive. This puts some of the least productive domestic firms out of the economy and increases the market share of all existing firms. However, investors market share increases the most, followed by exporters and lastly surviving domestic firms only. Please refer to figure 1.13:

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<sup>15</sup>The shape parameter,  $\alpha$ , varies from 3.3 to 3.1.

FIGURE:1.13"PDF of wage and technological progress"

As countries get access to better technology, investors increase their average wage relatively more than exporters and exporters average increase relatively more than the entire economy. In this manner the majority of the population move away from the average wage and the countries observe a higher inequality. Please refer to figure 1.14:

FIGURE:1.14"Lorenz Curve and technological progress"

However, there seems to be two opposing effect working from this change. Some least productive domestic firms release some workers, which decreases inequality. On the other hand, when they are hired by high productive exporters and investors that increases inequality. The simulation suggests that the second effect dominates and from the transition both countries observe an increase in the level of inequality.

#### 1.6.4 The effect of rent-sharing on inequality

Rent sharing ( $\theta$ ) varies from 0.1 to 0.9. Given the parameter values of the model, the equilibrium condition breaks down outside of this range. For this section, I will let this parameter to vary in the tolerable range.  $\theta$  has two different effects on inequality: for relatively low  $\theta$  inequality rises and for high  $\theta$  it decreases. Please refer to figure 1.15.

Figure:1.15"Inequality and rent sharing"

Note that from 0.1 to 0.5 inequality increases then it decreases up to 0.8 and then it is not that responsive from 0.8 to 0.9. The interpretation of the first interval (0.1-0.5) is trivial, as it offers higher wage to workers and by this workers from high productive firms earn higher wage. As a result, the inequality increases. However, after 0.5 firms share majority of their profit as rent. This lower their profit and they can no longer compete in the export and investment market. As these foreign firms decrease their activity in other country, they decrease their market share. This market is occupied by the domestic producers only. Hence workers lose jobs from high productive exporters and investors and hired by low productive domestic producers only. In this way for high rents inequality starts to fall. This effect starts to slow down after 0.8 and any further increase in  $\theta$  does not change inequality.

### 1.6.5 Welfare analysis

The aggregate welfare per worker has the following expression:

$$\frac{W}{L} = E(w) \tag{1.36}$$

Since aggregate price index is normalized to one the welfare only depend on the average wage of the economy. Note that an increased competition due to opening of market for trade and FDI increases the cutoff for domestic market. As the cutoff increases the minimum productivity require to serve the domestic market increases. This results in higher wage and higher welfare per worker.

## 1.7 Conclusion

This paper develops a model to study wage inequality created by opening up to foreign competitors in a heterogenous firms setup. It shows that as countries open more channels to access foreign market they observe an increase in welfare and wage inequality. No sorting/matching in the labor market is not a very realistic assumption but at least in theory we have this room to experiment with some extreme environments that indicates inequality is unavoidable. It shows that inequality is a very natural phenomenon created by trade and FDI. Under sorting/matching inequality will be higher than no sorting/matching, since it adds more sources to increase the inequality. Workers displacement from the autarky equilibrium redistributes population in such a way to increase welfare that, it results in creating some wage inequality. As more productive exporters and investors come to compete, least productive domestic producers exit the market. Hence these workers from exiting firms start earning high wage as workers from exporting and investing firms. The displacement of these workers generates wage inequality very naturally due to trade and FDI.

The paper also studies the impact of decrease in fixed cost to invest and tariff on employment and wage inequality. They have opposite effect on wage inequality; as a decrease in tariff redistributes the worker back to the center of the distribution and a decrease in fixed cost to invest puts them back at the tail of the distribution. A decrease in tariff decrease inequality, on

the other hand a decrease in fixed cost to invest has the opposite effect. Lastly, an access to better technology increases the inequality along with the aggregate welfare.

Figure 1.1: PDF of wage in autarky and trade.

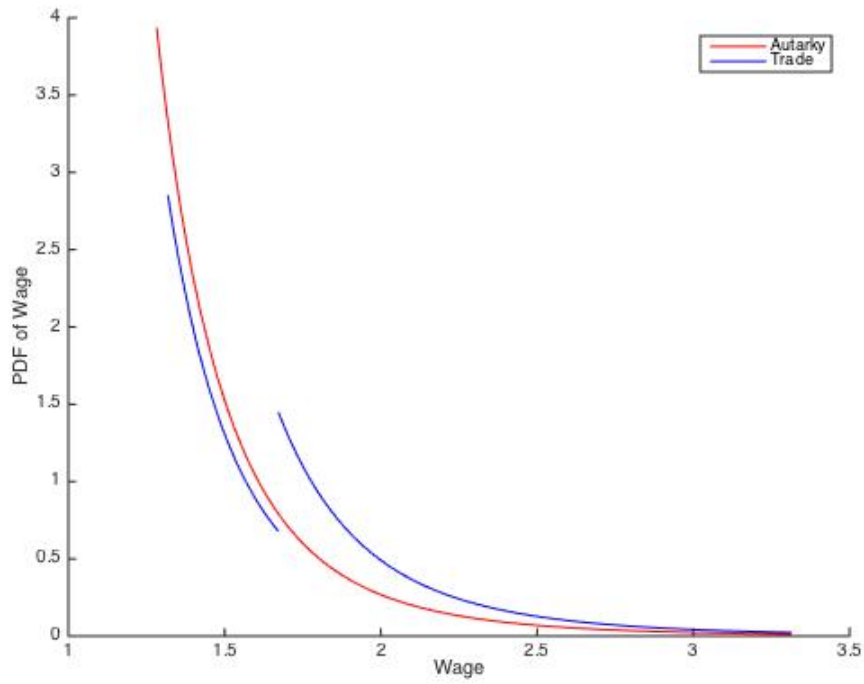


Figure 1.2: Lorenz Curve at autarky and trade.

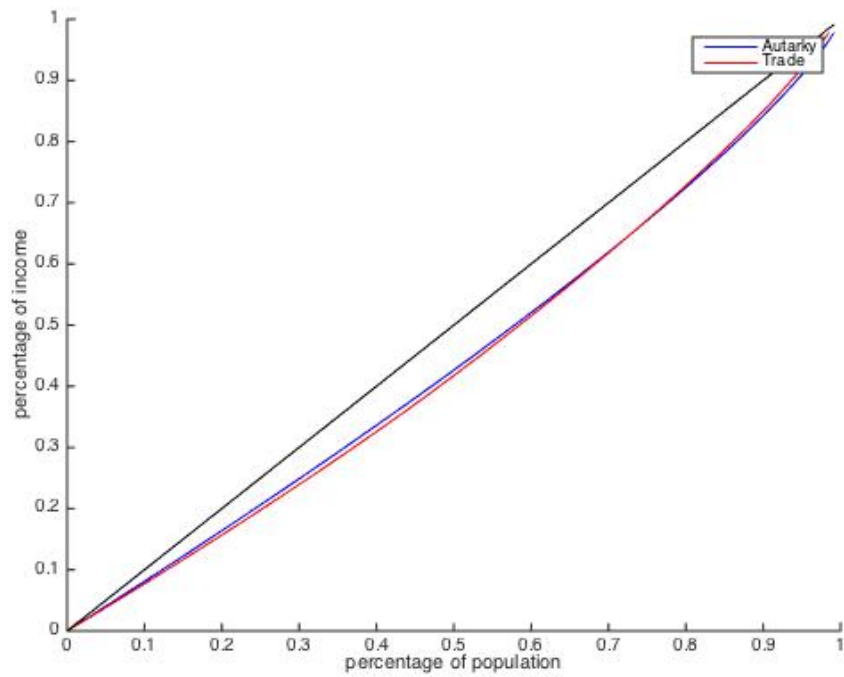




Figure 1.3: PDF of wage in trade and FDI.

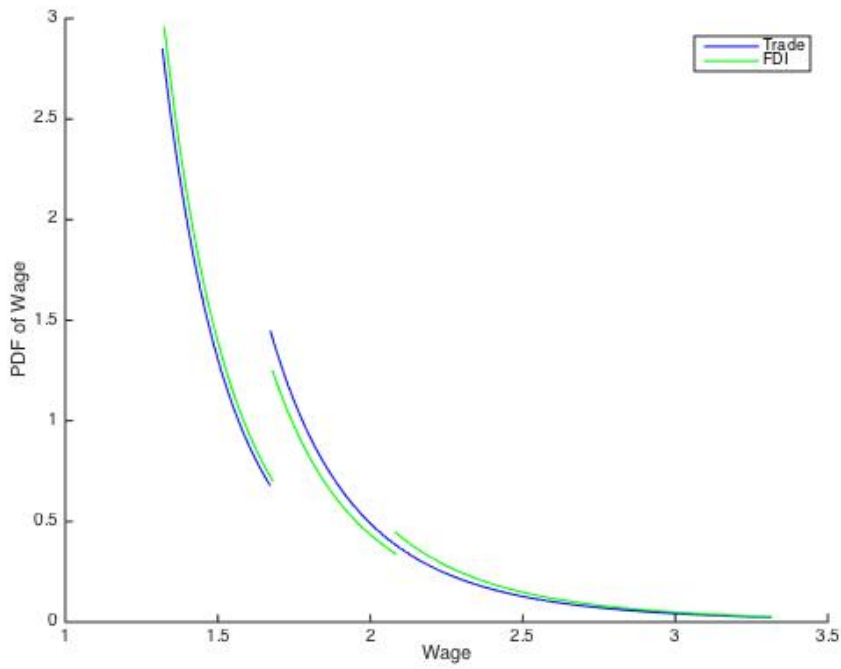


Figure 1.4: Lorenz Curve at trade and FDI.

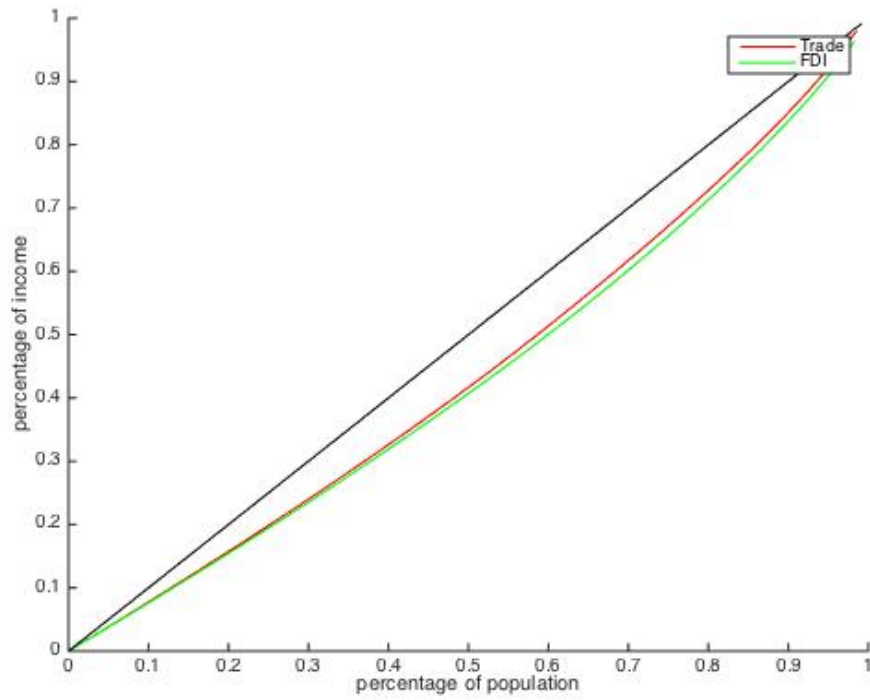


Figure 1.5: Effect of change in tariff to domestic cutoff.

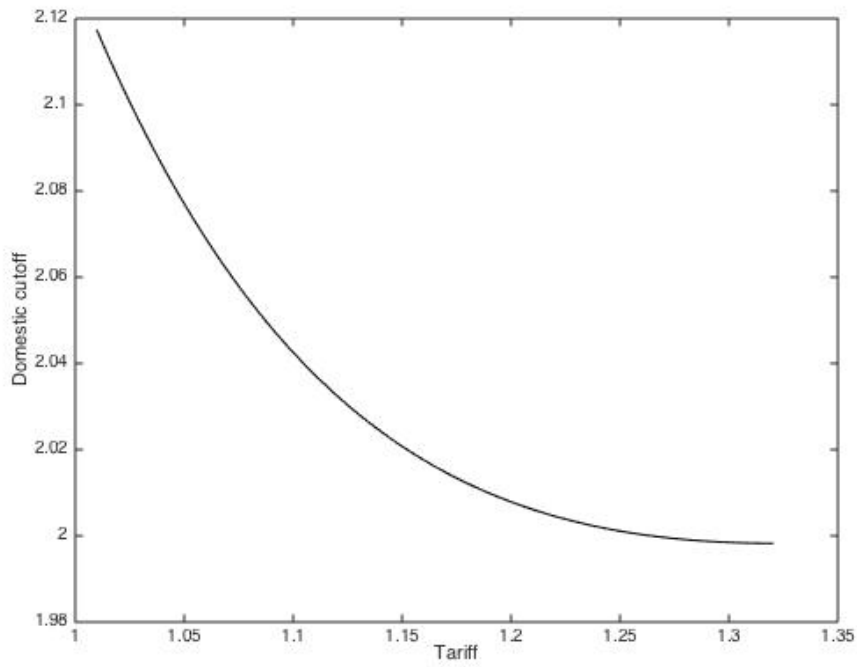


Figure 1.6: Effect of change in tariff to export and FDI cutoff.

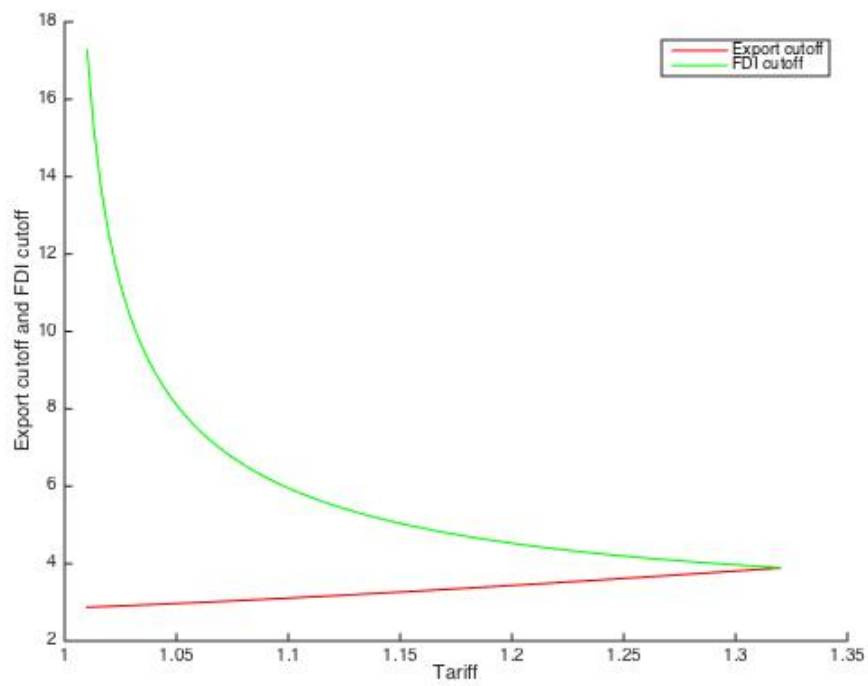


Figure 1.7: Effect of change in tariff to PDF of wage at FDI economy.

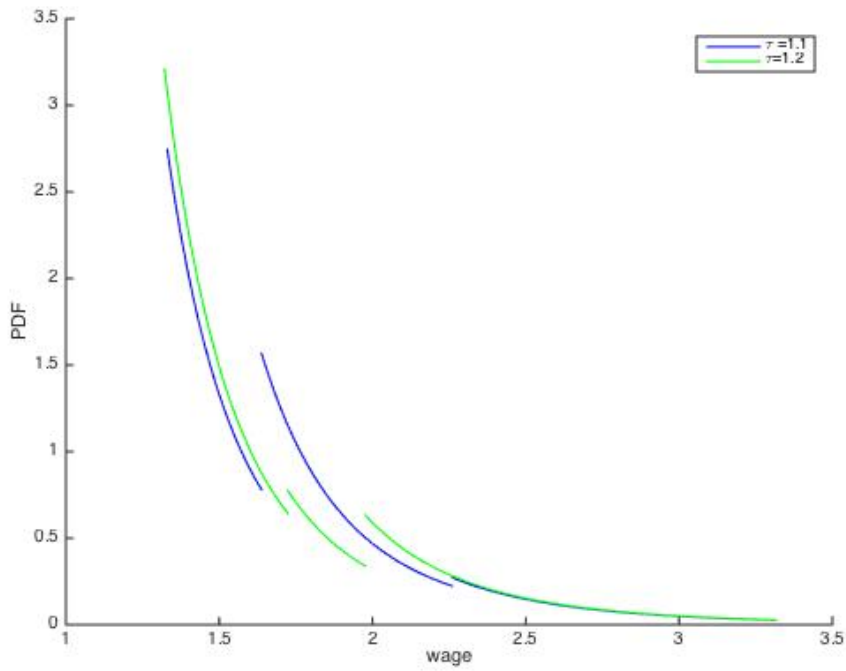


Figure 1.8: Effect of change in tariff to Lorenz curve of FDI economy.

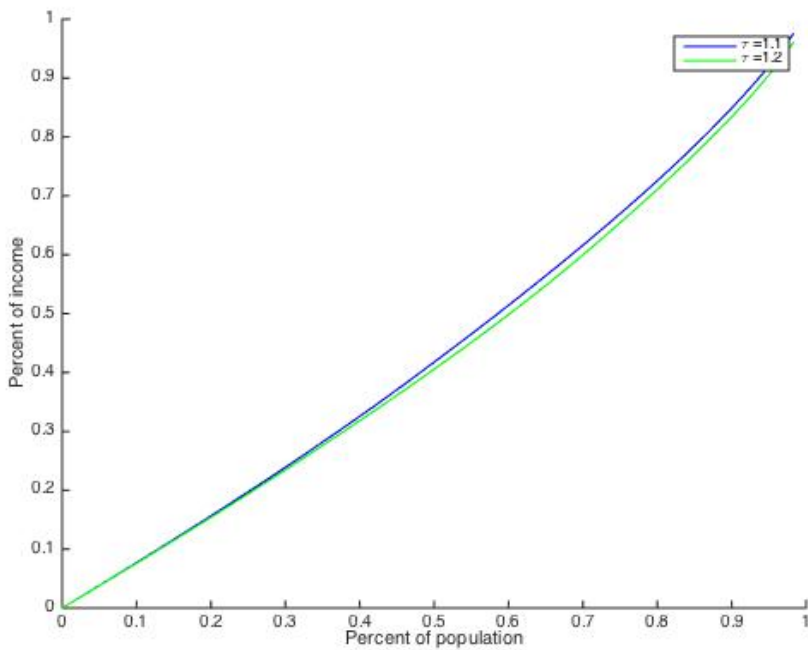


Figure 1.9: Effect of Change in fixed cost to invest to domestic cutoff.

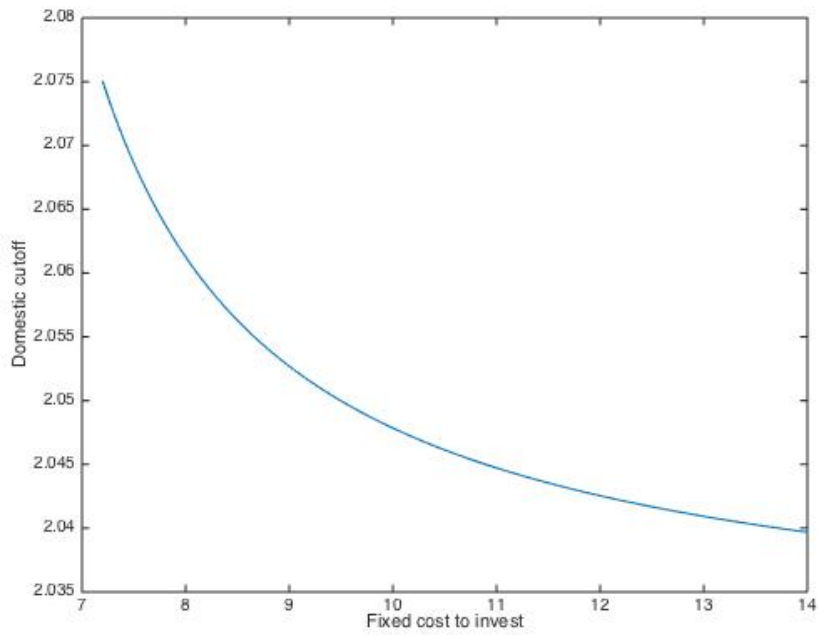


Figure 1.10: Effect of change in fixed cost to invest to export and FDI cutoff.

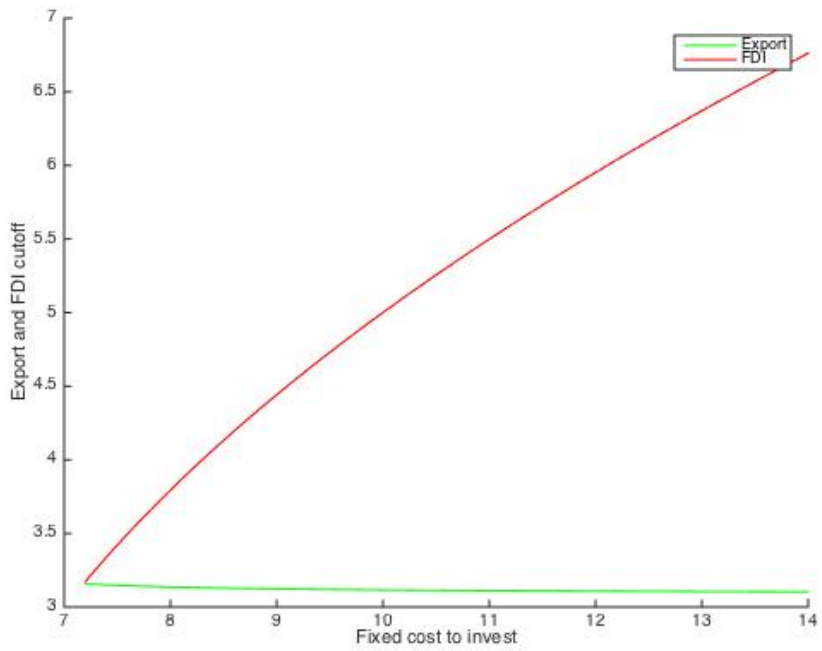


Figure 1.11: Effect of change in fixed cost to invest to PDF of wage at FDI economy.

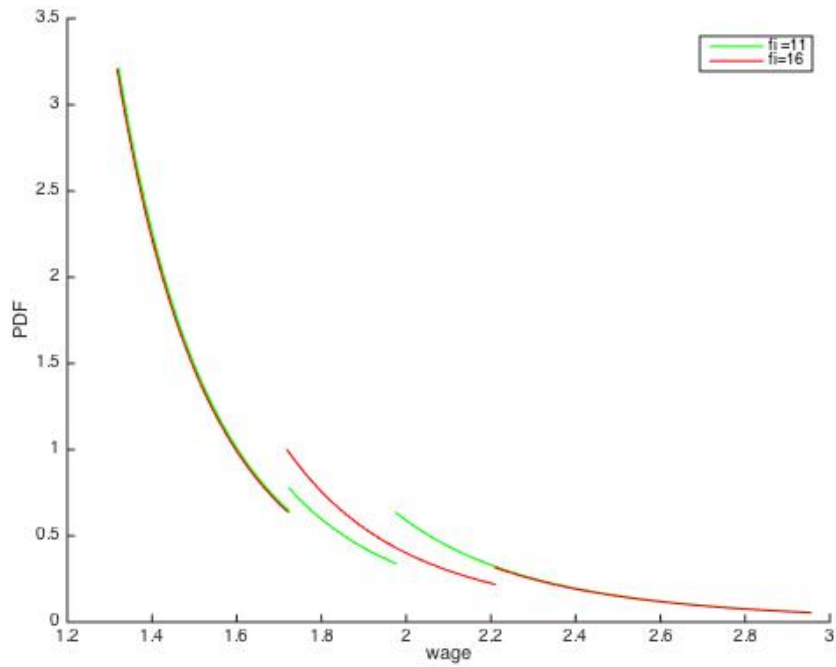


Figure 1.12: Effect of change in fixed cost to invest to Lorenz curve at FDI economy.

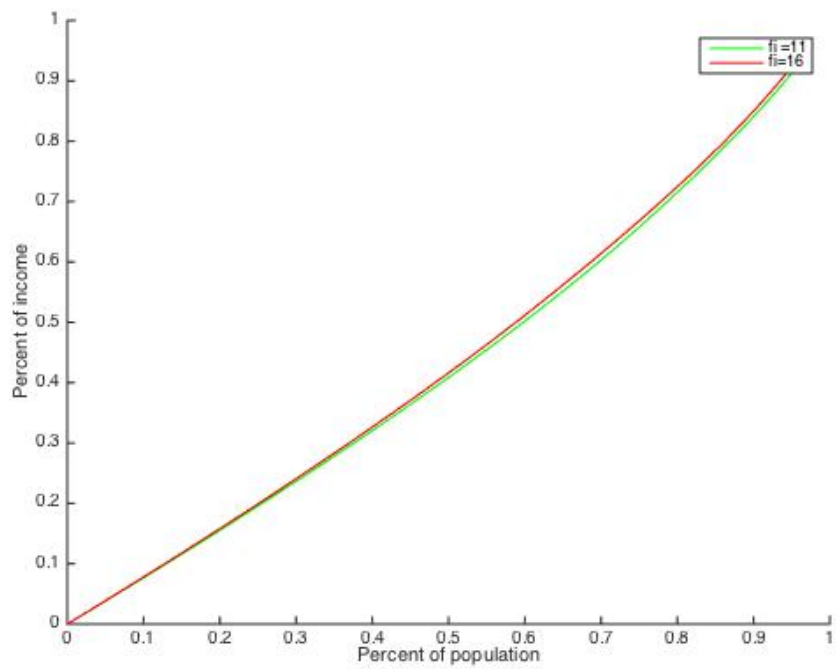


Figure 1.13: PDF of wage and technological Progress.

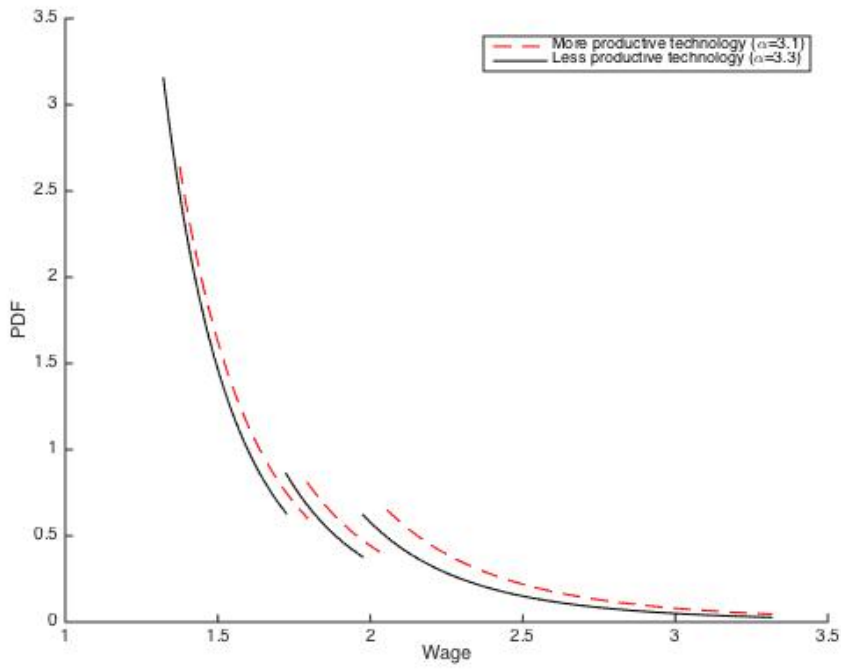


Figure 1.14: Lorenz Curve and technological progress.

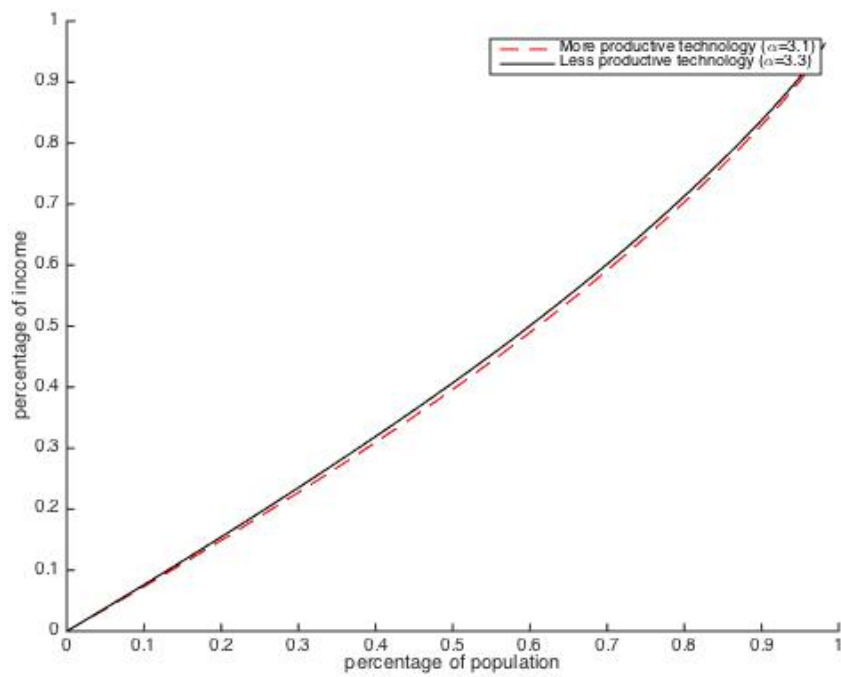
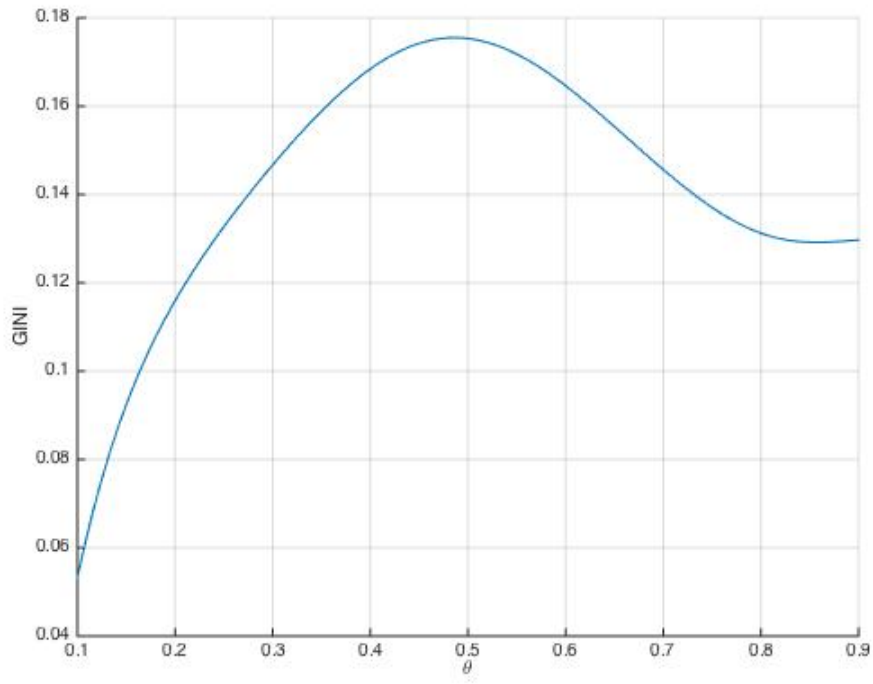


Figure 1.15: Inequality and rent sharing.



## Chapter 2

# Heterogenous Firms, Trade and Inequality across Asymmetric Countries

### 2.1 Introduction

These days trade economists are trying to understand the link between globalization and inequality. A variety of theoretical model in trade literature shows that globalization will increase the aggregate welfare of an economy. But if globalization causes inequality as well, then it is important to address how inequality is raised structurally from opening up an economy. The theoretical literature of this topic considers trade between symmetric countries and the effect it has on inequality. However the question still remains open, when it comes to globalization and inequality across asymmetric countries. What happens to inequality and welfare if the average productivity of active firms across country is different? Which direction does the inequality go for these countries and who wins or loses?

To answer these questions, imagine two economies with heterogenous firms and monopolistically competitive output market. These firms share their profit with workers. However, firms in



north exhibits a higher average productivity relative to south<sup>1</sup>. These workers are homogenous in terms of their productivity. Hence workers from a high productivity firm earn a higher wage, simply because they work in a bigger and more productive firm. Workers have no choice over their employer, as a result it generates a wage profile instead of one single wage. This profile along with the employment distribution leads to the Lorenz curve of the economy at different states, such as autarky and trade. Country asymmetry opens up the door to study the economy across north and south as well. At the end I conducted a thorough study of the effects changes in tariff and fixed cost on exports and inequality.

The paper considers a labor market with no sorting mechanism, where the firms share rents. This behavior is well documented in the labor economics literature. For example, Budd and Slaughter in their 2004 paper considered 1,014 Canadian manufacturing firm-union contract. They found that American industry profitability can effect the Canadian wage outcome and there seems to be international rent sharing across borders. Christofides and Oswald in their 1992 paper studied 600 labor contracts from Canadian economy between 1978 and 1984. They found that wages are an increasing function of employers past profits. They also estimate the profit elasticity of pay to be 0.006 from the data. Also, Hildreth and Oswald (1997) estimated the long run elasticity of wage with respect to profit per worker to be slightly below 0.02. Blanchflower, Oswald and Sanfey (1996), Budd and Slaughter (2005) also find supporting evidence that rent share takes place. The rent sharing behavior of the firm is one of the channels that generates wage differentials across firms. Hence it is essential to explore this mechanism that leads to inequality from open market.

The paper finds that the inequality is higher in the north in autarky. South has less high productive firms relative to north in autarky. Hence the market share of more productive firms is bigger in north, since they make their local market more competitive. As a result, these high productive firms in north have a higher labor payment. At the same time, local market in north is more competitive relatively to south. So on an average the aggregate productivity is higher, since the least productive firms have to have a higher productivity to survive in north. Since wage is firms' rent only, firms from north pay higher wage to more workers relative to south. All of these together leads to a higher inequality in north at autarky.

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<sup>1</sup>This setup is similar to Demidova (2008) asymmetric country model.

The paper also finds that a shift from autarky to trade equilibrium increases inequality relatively more in south than north. As countries open up to trade, there is increased competition that leads to reallocation of the resources (labor) to more productive firms. A firm with high productivity hires more workers, pays higher wage since they earn higher profit. Hence any reallocation of resources leads to an increase in the wage of workers coming from exiting and shrinking firms. At the same time wage of workers who stayed back with shrinking firms remains the same, but their total labor payment goes down with firms' market share. This leads to higher inequality for both countries.

The above effect is applicable for both countries, however south is relatively more affected by the redistribution. Globalization opens up exporting opportunities to more firms in north relative to the south. As a result the market share of these firms in north grows even bigger than south's high productive firms. This in turn redistributes more workers to high productive firms in north. In this process the majority of the population works for firms that has bigger market share in north. But in south relatively more workers stay back with the firms that has both less productivity and smaller market share. Hence south observes higher inequality from globalization relative to north.

Lastly, the paper finds that a symmetric decrease in tariff or fixed cost to export leads to a lower inequality and higher welfare. These changes create exporting opportunities to some highly productive domestic firms for both countries. The reallocation of resources in this case goes from less productive domestic firms to more productive exporters. Hence this creates a situation where larger share of workers have higher wage. This in turn decreases the inequality from the transition, since now exporters go through an expansion. It leads to a higher wage for workers from exiting and shrinking firms, since their new employers expand and earn more profit and in the end share more rent. The paper also studies the effect of unilateral tariff reduction by the south. The direction of change in inequality remains the same in this case however, it reveals that north can observe a lowest possible inequality by imposing lower tariff relative to its less productive trading partner.

The paper compares and contrasts the result of inequality with welfare analysis. The welfare effect of the model agrees with the existing literature of heterogeneous firms. The literature about welfare gain from trade started from Ricardo and continued to Melitz and extensions of his work

on heterogenous firms. This increase in welfare usually comes from increased competition in the domestic market with a heterogenous firms setup. As countries open up to foreign firms they let high productive exporters access their market. This high productive market in turn lowers aggregate price and pushes the least productive firms out of the industry. This result was demonstrated first by Melitz (2003), then by Helpman Melitz and Yeaple (2003), Demidova (2008) and other extensions of Melitz. It is also possible to show that as countries decrease tariff this welfare increases even more. This result is well known as long as we have symmetric countries. However country asymmetry may lead to a decrease in welfare, as shown by Demidova (2008), if trading countries are too asymmetric. A further examination of the welfare gain analysis is done by Melitz and Redding (2013), where they compare the homogenous firm setup to a heterogenous firm setting. They were able to show that by endogenous selection process, heterogenous firms setting has this additional channel to obtain aggregate welfare gain. This chapter contrasts the result of welfare gain at different state to inequality at different state.

To introduce country asymmetry this paper builds on Demidova's 2008 paper. Here a firm trying to enter the market in a north has a better chance of observing a higher productivity compared to a firm in south. This asymmetry induces in a different level of competition across countries. She found that if trading countries are not too asymmetric then they both go through a welfare improvement. However if they are too asymmetric then south can observe a decrease in welfare from opening up for trade. Her paper offers some interesting insight of trade with country asymmetry, but does not talk about inequality. The reallocation of resources in this economy is similar to the case of Melitz (2003). The reallocation of resources along with a mechanism to share profit of firms with their workers creates a labor market with no matching or sorting to study inequality. Similar to Demidova's model, I also consider countries that are not too asymmetric. Too much asymmetry results in a situation where all the existing firms in the north start to export<sup>2</sup>. As long as country asymmetry is in a tolerable range, both countries go through an increase in welfare and north can benefit more when I considered inequality.

Aside from the extensive research on welfare and globalization under firm heterogeneity, some papers have considered the inequality aspect of globalization. These studies used het-

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<sup>2</sup>Allowing for this jeopardizes the existence of the model equilibrium.

erogenous firms environment with some labor matching mechanism to study inequality. These market sorting in turn generates inequality in the economy. This kind of result is proved in Egger and Kreickemeier (2009) as well as in Helpman, Itskhoki and Redding (2008), Danziger (2014) and Pupato (2014). They consider heterogenous workers, who had different ability or marginal product of labor and/or effort. The paper by Egger and Kreickemeier took a measure of fair wage that has two parts in it: firm specific and industry specific. This paper considers only the firm specific part of wage, since this will close down the sorting of labor in the market. At the same time workers have no choice over their wage that results in a labor market with no sorting or matching mechanism. The paper by Egger and Kreickemeier listed above found inequality increased within the firm due to globalization, this research on the other hand finds globalization creates inequality across firms and industries. The rest has some sort of friction that results in higher inequality form open market as well. These studies are in a more general setup than in my model since I do not have any labor market sorting or matching mechanism. However this specification shows that globalization can create inequality from a very natural phenomena, such as the efficient redistribution of resources in Melitz environment. Sorting or matching in labor market will only make inequality worse, hence globalization will always create some inequality in asymmetric country setup as well.

All of the studies about inequality listed above have considered wage as some function of workers ability or effort. But, in theory wage can be some function of firm's rent as well. This mechanism is considered across symmetric countries in Akbar (2014) with FDI and trade. A large firm usually has higher productivity, hires more workers and pays higher wage. This higher wage can come from either hiring high performing workers and higher rents or from both. Considering homogenous workers close down the channel through which a worker can effect his wage. This creates a labor market with a wage profile instead of one single wage; since these workers cannot move across firms and their employment is firms' decision alone. In this way workers are prohibited from going to one single firm and in equilibrium end up creating symmetric wage. Hence the paper shows that inequality is a very natural phenomenon in heterogenous firm setup. Sorting/matching in the labor market only make it worse. As a result opening up an economy always creates inequality.

Also these studies of inequality consider a symmetric country setup only. As a result some

interesting dynamics of inequality is lost from the discussion. Allowing for country asymmetry (such as Demidova (2008)) shows, the direction inequality goes across countries as they open up for trade. It helps to understand the difference between mechanism across asymmetric countries that leads to higher inequality. I find that globalization increases inequality, but north will observe higher increase in inequality compared to north. But both of the trading partners observe welfare improvement. I also find that trade liberalization reduces inequality but increases welfare.

## 2.2 The Model

The model is based on Demidova (2008). There are two countries indexed by  $i$  and  $j$  with  $L$  workers each. Workers are homogenous and have no choice to move across firms. Each country produces two types of goods: differentiated intermediate goods and homogenous final output. Intermediary goods are traded in the open market, but homogenous final output (not traded in open market) is used to pay all of the sunk fixed costs and consumption.

### 2.2.1 Demand side

The consumers consume the final output  $Y$  that uses intermediate goods as inputs. Their preference is given by the standard CES utility function and  $\sigma > 1$  is the constant elasticity of substitution:

$$U_i = Y_i = \left[ \int_{\phi_i \in \Omega_i} q_i(\phi_i)^{\frac{\sigma-1}{\sigma}} d\phi_i \right]^{\frac{\sigma}{\sigma-1}} \quad (2.1)$$

$\Omega_i$  pins down the mass of available variety. Every individual has  $l$  endowment of labor that they supply inelastically. Workers have no choice over wage, even though they are homogenous, and they accept any wage that is offered by the firm to them. As a result, the economy generates the following wage given the productivity of the firm is  $\phi_i$ ,

$$w_i(\phi_i) = \phi_i^\theta \quad \theta \in (0, 1) \quad (2.2)$$

Where,  $\theta$  is the rent sharing parameter in this model. The wage is usually, in standard labor

theory, some function of marginal product of labor and portion of firms rent ( $w = f(MPL, \frac{\pi}{l})$ ). The homogenous workers assumption implies that they all have same ability. In theory if that is the case workers always move to a firm that offers the highest wage. In this way they drive down the wage to one single level for all firms.

However this does not happen in this model, simply because they do not have any choice when it comes to employment. It is solely firms decision to employ workers. It is possible to construct a model where workers can make this sort of decisions, this only creates a sorting in the labor market, that leads to a higher inequality. The whole purpose of the model is to show that rent sharing mechanism leads to income inequality with opening up to trade, even though countries observe aggregate welfare gain. Note that  $\theta = 0$ , imply unit wage like Melitz (2003) and I cannot consider  $\theta = 1$  since this leads to a marginal cost of 1 for all firms<sup>3</sup>. Since price is a constant markup over marginal cost, all firms charge the same price if  $\theta = 1$ ; which is their mark up. Hence firms share all rent with her workers. Anything in between leads to a special case of Hartmut and Udo's paper, since I do not consider the industry specific contributor to the wage. Under this condition an individual earns  $l\phi^\theta$  and has a demand for specific variety to be:

$$q_i(\phi_i) = R_i p_i(\phi_i)^{-\sigma} P_i^{\sigma-1} \quad (2.3)$$

$P_i$  is the aggregate price index corresponding to the final output. This index is normalized to one and  $R_i = \int_{\phi_i \in \Omega} r_i(\phi_i) d\phi_i$  is the aggregate expenditure. Note that, I cannot normalize the price index in both countries since the aggregate price index is not same across countries.

### 2.2.2 Supply side

Each firm trying to enter the market have to undertake a sunk cost. Once they pay the sunk cost of  $f_e$ , they draw their productivity from a distribution  $g_i(\phi_i)$ . After observing their productivity a firm can either stay in the market and produce or exit. Firms going to production pays another fixed cost of  $f$ . All of the costs are paid in terms of the final output  $Y_i$ . Firms compete in a monopolistically competitive market and require labor ( $l_i(\phi_i) = q_i(\phi_i)/\phi_i$ ) input to produce

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<sup>3</sup>Marginal cost will come from firm's problem. This will be discussed in the flowing section.

intermediate goods. The price charged by the firm is given by the following function that arises from equalizing marginal benefit and marginal cost of the monopolist:

$$p_i(\phi_i) = \frac{\sigma}{\sigma - 1} \phi_i^{\theta - 1} \quad (2.4)$$

$\phi_i^{\theta - 1}$  is the marginal cost and  $\frac{\sigma}{\sigma - 1}$  is the constant mark up from the CES utility. Note that when  $\theta = 1$ , marginal cost becomes 1. So the price charged by a firm no longer depends on the productivity and in equilibrium firm heterogeneity is lost. Firms revenue is given by the following equation,

$$r_i(\phi_i) = R_i \phi_i^\varepsilon \left[ \frac{\sigma - 1}{\sigma} \right]^{\sigma - 1} \quad (2.5)$$

Here,  $\varepsilon = (1 - \theta)(\sigma - 1)$ . The profit of the firm with observed  $\phi_i$  is given by the following equation:

$$\begin{aligned} \pi_i(\phi_i) &= r_i(\phi_i) - l_i(\phi_i) \phi_i^\theta - f \\ &= \frac{r_i(\phi_i)}{\sigma} - f \end{aligned} \quad (2.6)$$

Since profit is increasing in its argument, there exists  $\phi_i^*$  such that  $\pi_i(\phi_i^*) = 0$ . This identifies producing firms to exiting firms and therefore, the distribution in equilibrium is given by  $\mu_i(\phi_i) = \frac{g_i(\phi_i)}{1 - G_i(\phi_i^*)}$  for  $\phi_i \in (\phi_i^*, \infty)$ . The aggregate price index in an economy is then given by:

$$P_i^{1 - \sigma} = M_i \int_{\phi_i^*}^{\infty} p_i(\phi_i)^{1 - \sigma} \frac{g_i(\phi_i)}{1 - G_i(\phi_i^*)} d\phi_i \quad (2.7)$$

where,  $M_i$  is the mass of firms active in country  $i$ . The mass of firms active in this economy comes from the labor market clearing condition, that is  $L = M_{ia} \int_{\phi_i^*}^{\infty} l_i(\phi_i) \frac{g_i(\phi_i)}{1 - G_i(\phi_i^*)} d\phi_i$ . This condition equalizes aggregate demand and supply of labor. Hence the mass of firm in this economy is given by the following expression<sup>4</sup>.

$$M_{ia} = \frac{L}{(\sigma - 1)f} \left[ \left( \frac{1}{\phi_i^*} \right)^\varepsilon \int_{\phi_i^*}^{\infty} \phi_i^{\varepsilon - \theta} \frac{g(\phi_i)}{1 - G(\phi_i^*)} d\phi_i \right]^{-1} \quad (2.8)$$

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<sup>4</sup>The mass calculation in this model is similar to Akbar (2014).

Note that the price index from equation (2.7) defined above is the price of homogenous final output. The aggregate productivity is,

$$\tilde{\phi}_i = \left[ \int_{\phi_i^*}^{\infty} \phi_i^\varepsilon \frac{g_i(\phi_i)}{1 - G_i(\phi_i^*)} d\phi_i \right]^{\frac{1}{\varepsilon}} \quad (2.9)$$

This productivity index is observed by the consumers. Another way to look at it as the average productivity used to produce all available variety to consumers in a country. These supply of goods can come from both domestic and foreign producers. However, producers on the other hand face a different index<sup>5</sup>.

$$\tilde{\phi}_{ie} = \left[ \int_{\phi_i^*}^{\infty} \phi_i^{\varepsilon - \theta} \frac{g_i(\phi_i)}{1 - G_i(\phi_i^*)} d\phi_i \right]^{\frac{1}{\varepsilon - \theta}} \quad (2.10)$$

Note that producers observe less average productivity, since a part of it goes to rent. This productivity index takes an average over all variety produced by firms inside a country. So same goods lead to different measure of productivity for producers and consumers. At autarky the available variety consumed and produced is same, but this is not the case at trade equilibrium. This will be discussed in details in following section for trade equilibrium. These two indices jointly define the average wage of the economy as  $\tilde{\phi}_{ie}^\theta = \frac{\tilde{\phi}_i^\varepsilon}{\tilde{\phi}_{ie}^{\varepsilon - \theta}}$ <sup>6</sup>.

In a country i given any two observed value of the productivity ( $\phi_{i1} < \phi_{i2}$ ) parameter the

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<sup>5</sup>Since the relative employment compared to average firm is  $\frac{l(\phi)}{l(\bar{\phi})} = \left(\frac{\phi}{\bar{\phi}}\right)^{\varepsilon - \theta}$ . The average in the producer index will be raised to  $\varepsilon - \theta$ . This will simply the calculation.

<sup>6</sup>Note that  $\tilde{\phi}_i^\theta \neq \int_{\phi_i^*}^{\infty} \phi^\theta \mu(\phi) d\phi$ . It will be the ratio of productivity index coming from consumption and production or use expected value method once PDF of wage is defined.



equations for relative output, revenue, price and wage is given by the following,

$$\frac{q(\phi_{i2})}{q(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\varepsilon-\theta+1} \quad (2.11a)$$

$$\frac{r(\phi_{i2})}{r(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\varepsilon} \quad (2.11b)$$

$$\frac{w(\phi_{i2})}{w(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\theta} \quad (2.11c)$$

$$\frac{p(\phi_{i2})}{p(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\theta-1} \quad (2.11d)$$

$$\frac{l(\phi_{i2})}{l(\phi_{i1})} = \left(\frac{\phi_{i2}}{\phi_{i1}}\right)^{\varepsilon-\theta} \quad (2.11e)$$

So a firm with higher productivity produces more output, earns higher revenue, offers a higher wage to it's workers, employs relatively more workers and charges a lower price.

## 2.3 Equilibrium in Autarky

Firms draw their productivity  $\phi_i$  from a distribution  $g_i(\phi_i)$ . Since only the firms making positive profit stay in the market, firms value function is given by  $\max\{0, \frac{\pi_i(\phi_i)}{\delta}\}$ . Here  $\delta$  is the external shock that a firm has to exit in future<sup>7</sup>. Hence the free entry condition (FEC) is,

$$[1 - G_i(\phi_i)] \frac{\pi_i(\tilde{\phi}_i)}{\delta} = f_e \quad (2.12)$$

Where,  $f_e$  is the sunk cost to enter the market. This condition implies that firms expected future profit balance the fixed cost to enter the market. After the prospective firm under takes the cost  $f_e$ , it gets to draw the productivity parameter for the firm  $\phi_i$  from the PDF  $g_i(\phi_i)$ .

Zero Cutoff profit, on the other hand, defines the relation between the average profit to cutoff level, that is derived from the definition of average profit.

$$\pi_i(\tilde{\phi}_i) = \frac{r_i(\tilde{\phi}_i)}{\sigma} - f$$

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<sup>7</sup>The mass of exiting firms will be replaced by the mass of new entrees each period. This mechanism is well understood from Melitz model.

Now equation (2.11b) implies that  $r_i(\tilde{\phi}_i) = \sigma f\left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^\varepsilon$ . The zero cutoff profit (ZCP) is given by the following equation:

$$\begin{aligned}\pi_i(\tilde{\phi}_i) &= f\left[\left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^\varepsilon - 1\right] \\ &= f k_i(\phi_i^*)\end{aligned}\tag{2.13}$$

Note that,  $k_i(\phi_i^*) = \left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^\varepsilon - 1$ . Equation (2.12) and (2.13) jointly identifies the domestic cutoff  $\phi_i^*$  in a closed economy.

$$[1 - G_i(\phi_i^*)] \frac{f k_i(\phi_i^*)}{\delta} = f_e\tag{2.14}$$

LHS of the equation is the present discounted profit of the firm upon drawing from the PDF  $G_i(\cdot)$ . Lets define  $J_i(\phi_i) = [1 - G_i(\phi_i)]k_i(\phi_i)$ . For the rest of the paper let us consider Pareto distribution, where  $g_i(\phi_i) = \frac{\alpha_i}{\phi_i^{\alpha_i+1}} \forall \phi_i > 1$  and for  $\alpha_i > \varepsilon$  for  $\forall i, j$ . This assumption implies that  $J_i(\phi_i) = \frac{1}{\phi_i^{\alpha_i}} \left[\frac{\varepsilon}{\alpha_i - \varepsilon}\right]$ , hence it is obvious that  $J_i'(\phi) < 0$  and  $J_i''(\phi) > 0$ . But the RHS of equation (2.14) has only a constant. This implies that LHS intersects once and the solution to the equilibrium is unique.

Let us assume that  $\alpha_i < \alpha_j$ , hence country  $i$  can be thought of the north and  $j$  is then south. This implies that the domestic cutoff at  $i$  is higher than  $j$  ( $\phi_i^* > \phi_j^*$ ), since  $\phi_c^* = \left[\frac{f}{\delta f_e} \left(\frac{\varepsilon}{\alpha_c - \varepsilon}\right)\right]^{\frac{1}{\alpha_c}}$  for  $\forall c \in i, j$ .

### 2.3.1 Distribution of employment at Autarky

**Claim 6** *Given the active mass of firms  $M_i$ , a firm draws  $\phi_i$  from  $g_i(\phi_i)$  and employs  $l_i(\phi_i)$ . Hence, the distribution of employment in this economy is:*

$$e_i(\phi_i) = \frac{M_i \mu_i(\phi_i) l_i(\phi_i)}{L} \quad \forall \phi_i \in (\phi_i^*, \infty)\tag{2.15}$$

The Pareto distribution implies that,  $\mu_i(\phi_i) = \frac{\alpha_i \phi_i^{*\alpha_i}}{\phi_i^{\alpha_i+1}}$  for  $\phi_i > \phi_i^*$  and the employment

distribution simplifies to;

$$e_i(\phi_i) = \frac{1}{\tilde{\gamma}_{\varepsilon-\theta}} \frac{\alpha_i \phi_i^{*\alpha_i}}{\phi_i^{\alpha_i+\theta+1-\varepsilon}} \quad \forall \phi_i \in (\phi_i^*, \infty) \quad (2.16)$$

The employment distribution takes the form of Pareto distribution with location parameter  $(\frac{\alpha_i \phi_i^{*\alpha_i}}{\tilde{\gamma}_{\varepsilon-\theta}})$  and the shape parameter  $(\alpha_i + \theta + 1 - \varepsilon)$ . ( See appendix for prove). The wage equation  $(w_i(\phi_i) = \phi_i^\theta)$  and employment distribution leads to the weighted wage distribution of the economy.

$$y_i(w_i) = \frac{1}{\tilde{\gamma}_{\varepsilon-\theta}} \frac{\alpha_i}{\theta} \phi_i^{*\alpha_i} w_i^{\frac{\varepsilon-\theta-\alpha_i}{\theta}-1} \quad \forall w_i \in (\phi_i^{*\theta}, \infty) \quad (2.17)$$

This is the wage distribution weighted by the employment of the firm for every observed  $\phi_i$ . Note that this still remain a Pareto distribution, but with different shape and location parameter. (See appendix). This wage distribution is different across countries since they arise from asymmetric productivity distribution. However, every thing remains the same other than shape parameter of the wage distribution since  $\alpha_i < \alpha_j$ .

Figure 2.1: PDF of wage distribution across country at autarky.

Since domestic cutoff for north is higher, the PDF for north starts at a higher wage value  $(w_i(\phi_i^{*\theta}) > w_j(\phi_j^{*\theta}))$  than the other country and also lies below the distribution of south.

To construct the Lorenz curve, I use the CDF  $(Y_i(w_i) = \left[ 1 - \left( \frac{w_i^\theta}{\phi_i^*} \right)^{\varepsilon-\theta-\alpha_i} \right])$  and average wage  $(E(w_i) = \tilde{\phi}_i^\theta = \int w_i y_i(w_i) dw_i)$  of the economy, hence the Lorenz curve is:

$$L(Y_i) = 1 - [1 - Y_i]^{\frac{\varepsilon-\alpha_i}{\varepsilon-\alpha_i-\theta}} \quad (2.18)$$

Figure 2.2: Lorenz curve at autarky across country.

**Claim 7** *If wage is an increasing function ( $w = \phi^\theta$ ) of firms productivity with rent sharing parameter of  $\theta \in (0, 1)$ , then at autarky north observes higher inequality compare to south.*

The Lorenz curve Shows that north has more inequality than the south at autarky. This inequality is observed because the high productive firms in north is relatively bigger than the south  $j$ . The resources in north is used more efficiently than the other country. Hence same level of productive (such as  $\phi$ ) firm in different countries observe that at north a firm has access to bigger market share and employ relatively more workers to satisfy the demand. This leads to a situation where, relatively bigger share of the population in north compare to south works for relatively high productive firms. But at the same time this share of the population is not the majority of the population in north as well as. This causes higher level of inequality in north relative to south<sup>8</sup>.

## 2.4 Trade economy

Now firms can engage in exporting activity. Exporting firms have to undertake two additional costs. Firms face a fixed cost  $f_x > f \tau^{1-\sigma}$  ( An assumption to separate exporters from domestic producers only) and transportation cost or tariff of  $\tau > 1$ . All the sunk costs and fixed costs are paid in terms of the final output  $Y_i$  and  $Y_j$ . However now I can no longer normalize the price index of both countries, since demand conditions are different across countries. If I normalize only one of the price index then firms from the other country pays lower or higher fixed costs. For example, suppose I normalize  $P_i = 1$ , this implies that  $P_j > 1$  since  $G_i \succ_{hr} G_j$ . This makes fixed cost expensive in country  $j$  and the fixed barrier to enter a market no longer stays fixed as Melitz paper.

To handle this problem I assume that firms' in country  $j$  (south) face a parameter  $c \in (0, 1)$  such that  $Y_i P_i = c Y_j P_j^9$ . One peculiar result of the Melitz model is that, more productive firms charge lower price. It is due to this result the aggregate price index in country  $i$  is lower than the other. The active mass of firms in country  $i$  have higher aggregate productivity than the other country, this leads to the dispersion in the price index. Since  $P_i = 1$  by normalization, one obvious choice of the contributor is  $P_j^{-1}$ . This assumption makes the theory coherent and

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<sup>8</sup>A simulation for the Gini coefficient confirms the result, where Gini at autarky for north " $\alpha_i = 3.1$ " = 0.1235 > 0.1099 = Gini at autarky for south " $\alpha_j = 3.3$ "

<sup>9</sup>The parameter  $c$  makes the model consistent with Melitz literature. Without this assumption the interpretation of the model becomes void.

consistent with the literature of firm heterogeneity<sup>10</sup>. Firms export price, revenue and profit is expressed by:

$$p_{xi}(\phi_{xi}) = \tau p_i(\phi_i) \quad (2.19)$$

$$r_{xi}(\phi_{xi}) = R_j \phi_{xi}^\varepsilon \left[ \tau^{-1} \frac{\sigma - 1}{\sigma} P_j \right]^{\sigma-1} = \tau^{1-\sigma} r_j(\phi_{xi}) \quad (2.20)$$

$$\pi_{xi}(\phi_{xi}) = \frac{r_{xi}(\phi_{xi})}{\sigma} - f_x = \frac{\tau^{1-\sigma} r_j(\phi_{xi})}{\sigma} - f_x \quad (2.21)$$

Asymmetry across country implies that, revenue of a firm with same level of productivity is not same across countries ( $r_i(\phi) \neq r_j(\phi)$ ). Total profit of the firm given it observes  $\phi_i$  is,  $\pi_{total}(\phi_i) = \max[0, \pi_i(\phi_i)] + \max[0, \pi_{xi}(\phi_i)]$ . The profit function establishes the connection between export and domestic cutoff across countries. The relation between the cutoffs is  $\phi_{xi} = \phi_j A$  where  $A = \left[ \tau^{\sigma-1} \frac{f_x}{f} \right]^{\frac{1}{\varepsilon}}$  for  $\forall i$  and  $j$ <sup>11</sup>. See appendix for complete derivation. Since local markets marginal firm determines the marginal foreign exporters and the shape parameter determines the domestic cutoff for both countries, too asymmetry across countries may lead to a situation where all of domestic firms in north becomes exporters. Generally this is not the case in data and the equilibrium condition falls apart, hence I ignore the case of too asymmetry and assume that  $\frac{\phi_{xi}^*}{\phi_i^*} = \frac{\phi_j^* A}{\phi_i^*} > 1$ .

Similar to the autarky case, this economy faces two types of aggregate productivity index. Index faced by consumers is  $\tilde{\phi}_{ti}^\varepsilon = \frac{1}{M_{ti}} [M_i \tilde{\phi}_{di}^\varepsilon + M_j P_{jx} \tau^{-\varepsilon} \tilde{\phi}_{xj}^\varepsilon]$  and the index faced by producers is discussed in the following section. Note that, this index is constructed from available variety consumed by the end consumers. So for example, consumers have all the goods available from domestic producers and some foreign goods available from exporters. Hence the total mass of variety available to consumers is  $M_{ti} = M_i + M_j P_{jx}$  and  $P_{jx} = \frac{1-G_j(\phi_{xj}^*)}{1-G_j(\phi_j^*)}$ <sup>12</sup>. This index is similar to the aggregate index defined by Demidova's 2008 paper.

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<sup>10</sup>An alternative method of addressing this problem is to impose trade balance condition by allowing a transfer of  $D$  by developing country. Hence the trade balance condition is:  $M_{xi} \int_{\phi_i^*}^{\infty} q_{xi}(\phi_i) \frac{g_i(\phi_i)}{1-G_i(\phi_i)} d\phi_i = M_{xj} \int_{\phi_j^*}^{\infty} q_{xj}(\phi_j) \frac{g_j(\phi_j)}{1-G_j(\phi_j)} d\phi_j + D$ . This will normalize the aggregate price index in both economy. However, this method will turn the model into partial equilibrium and welfare analysis is no longer applicable.

<sup>11</sup>Note that  $A > 1$ .

<sup>12</sup> $M_i$  is determined from the labor market clearing condition of country "i".

$\tilde{\phi}_{di}^\varepsilon$  and  $\tilde{\phi}_{xi}^\varepsilon$  are the averages coming from domestic producers and exporters only respectively.

### 2.4.1 Trade economy equilibrium

The FEC and ZCP together leads to the following expression in trade economy,

$$\frac{f}{\delta} J_i(\phi_i^*) + \frac{f_x}{\delta} J_i(\phi_{xi}^*) = f_e \quad for : i, j \quad (2.22)$$

The interpretation remains the same, however due to the additional term in LHS, the economy has a higher cutoff for existing domestic firm. This happens due to the increased competition coming from foreign market. More productive exporters enters the market with lower price and drives the less productive domestic producers with higher price out from the market. This mechanism is well known in Meltiz literature. With Pareto distribution this condition becomes  $\frac{f}{\phi_i^{*\alpha_i}} + \frac{f_x}{(A\phi_j^*)^{\alpha_i}} = \delta f_e [\frac{\alpha_i - \varepsilon}{\varepsilon}]$  for country  $i$  and  $j$ . Hence, these two conditions jointly define the domestic cutoff for both countries in the  $(\phi_i \times \phi_j)$  space. Note that, these equations relates the domestic cutoff across countries.

$$\begin{aligned} \phi_i &= \left[ \frac{\delta f_e}{f} \frac{\alpha_i - \varepsilon}{\varepsilon} - \frac{f_x}{f} \left( \frac{1}{A\phi_j^*} \right)^{\alpha_i} \right]^{\frac{1}{\alpha_i}} = B(\phi_j) \\ \phi_i &= \frac{1}{A} \left[ \frac{\delta f_e}{f_x} \frac{\alpha_i - \varepsilon}{\varepsilon} - \frac{f}{f_x} \left( \frac{1}{\phi_j^*} \right)^{\alpha_j} \right]^{\frac{1}{\alpha_j}} = D(\phi_j) \end{aligned} \quad (2.23)$$

**Claim 8** *As long as the export cutoff is bigger than the domestic cutoff for north ( $\phi_{xi}^* > \phi_i^*$ ), hence the model follows all the assumptions, there exists a unique solution  $(\phi_i^*, \phi_j^*)$  and  $\phi_{xi}^* > \phi_i^* > \phi_j^*$ .*

See appendix for full derivation. Since export cutoffs are related by the domestic cutoff of other country and  $A > 1$ , the above inequality follows.

#### Mass of firms

The mass of firms active in the economy can be identified by using the labor market clearing condition. Now the exporters employ more worker to satisfy the additional demand and the condition becomes:  $L = M_i \int_{\phi_{di}^*}^{\infty} l_i(\phi_i) \frac{g_i(\phi_i)}{1 - G_i(\phi_{di}^*)} d\phi_i + M_i P_{ix} \int_{\phi_{xi}^*}^{\infty} l_{xi}(\phi_i) \frac{g_i(\phi_i)}{1 - G_i(\phi_{xi}^*)} d\phi_i$ . By using the optimal pricing rule by monopolist and equation (2.11b) the mass of firms in the economy can be expressed as:

$$M_i = \frac{L}{\sigma - 1} \left[ \frac{f}{\phi_{di}^{*\varepsilon}} \int_{\phi_{di}^*}^{\infty} \phi_i^{\varepsilon - \theta} \frac{g_i(\phi_i)}{1 - G_i(\phi_{di}^*)} d\phi_i + \frac{P_{ix} f_x}{\tau \phi_{xi}^{*\varepsilon}} \int_{\phi_{xi}^*}^{\infty} \phi_i^{\varepsilon - \theta} \frac{g_i(\phi_i)}{1 - G_i(\phi_{xi}^*)} d\phi_i \right]^{-1} \quad (2.24)$$

Note that the active firms from a country is lower than autarky. However now both economies have some foreign firms serving their local markets. So the total number of firms serving each market is  $M_{ti} = M_i + M_j P_{jx}$ , for any  $i \neq j$ , and available variety to consumer increases.

## 2.4.2 Distribution of employment and Lorenz curve at trade economy

Lets look at the aggregate productivity faced by producers, since it gives rise to employment distribution in trade economy. This index looks at the total production in a country and takes the average for their output. For example at country  $i$ , this index calculates average productivity from the total output produced for their domestic market and output produced for export market. Hence it uses  $M_i$  mass of variety produced inside a country for domestic market and  $M_i P_{ix}$  for mass of exporters who serve in south. This index can be expressed in two ways: a) the productivity of all firms producing for domestic market ( $\phi_i \in (\phi_{di}^*, \infty)$ ) and exporters only ( $\phi_i \in (\phi_{xi}^*, \infty)$ ), b) productivity of firms producing for domestic market only ( $\phi_i \in (\phi_{di}^*, \phi_{xi}^*)$ ) and the productivity of firms that serve both domestic and exporting market ( $\phi_i \in (\phi_{xi}^*, \infty)$ ). The second method disaggregates the index in two groups: a) contribution coming from domestic producers only and b) contribution coming from exporters and domestic producers. Hence the productivity index by the second way is,

$$\tilde{\phi}_{tei}^{\varepsilon - \theta} = \frac{1}{M_{tei}} [(1 - P_{ix}) M_i \left( {}^{xi} \tilde{\phi}_{di} \right)^{\varepsilon - \theta} + P_{ix} M_i (1 + \tau^{\theta - \varepsilon}) \left( \tilde{\phi}_{xi} \right)^{\varepsilon - \theta}] \quad (2.25)$$

Where,  $\left( {}^{xi} \tilde{\phi}_{di} \right)^{\varepsilon - \theta} = \int_{\phi_{di}^*}^{\phi_{xi}^*} \phi_i^{\varepsilon - \theta} \frac{g_i(\phi_i)}{1 - G_i(\phi_{di}^*)} d\phi_i$  and  $\left( \tilde{\phi}_{xi} \right)^{\varepsilon - \theta} = \int_{\phi_{xi}^*}^{\infty} \phi_i^{\varepsilon - \theta} \frac{g_i(\phi_i)}{1 - G_i(\phi_{xi}^*)} d\phi_i$ . The first index,  $\left( {}^{xi} \tilde{\phi}_{di} \right)^{\varepsilon - \theta}$ , refers to the firms that serve only domestic market and the second index,  $\left( \tilde{\phi}_{xi} \right)^{\varepsilon - \theta}$ , refers to the firms that serve both domestic and export market. Note that, the relative mass of firms that are contributing to the productivity index is multiplied and the total variety produced in country  $i$  is  $M_{tei} = M_i(1 + P_{ix})$ . However when exported, exporters

lose some goods in transit due to tariff and/or transportation cost  $\tau$ . These firms also serve in domestic market during which they do not pay this per unit cost. Hence the exporters aggregate index has two groups: a) when serving domestic market only the index becomes  $\left(\tilde{\phi}_{xi}\right)^{\varepsilon-\theta}$  and b) when serving export market only the index is discounted by tariff and is  $\left(\frac{\tilde{\phi}_{xi}}{\tau}\right)^{\varepsilon-\theta}$ . However, the mass of firms in this group remains same  $P_{ix}M_i$ . Now the employment distribution in this economy is similar to equation (2.16).

$$e_i(\phi_i) = \begin{cases} \frac{1}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{1-P_{ix}}{1+P_{ix}} \frac{\alpha_i \phi_{di}^{*\alpha_i}}{\phi_i^{\alpha_i+1-\varepsilon+\theta}} & \forall \phi_i \in (\phi_{di}^*, \phi_{xi}^*) \\ \frac{1}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{P_{ix}}{1+P_{ix}} (1 + \tau^{\theta-\varepsilon}) \frac{\alpha_i \phi_{xi}^{*\alpha_i}}{\phi_i^{\alpha_i+1-\varepsilon+\theta}} & \forall \phi_i \in (\phi_{xi}^*, \infty) \end{cases} \quad (2.26)$$

**Proof.** Note that  $\int_{\phi_{di}^*}^{\phi_{xi}^*} \frac{\alpha_i \phi_{di}^{*\alpha_i}}{\phi_i^{\alpha_i+1-\varepsilon+\theta}} = \left(x_i \tilde{\phi}_{di}\right)^{\varepsilon-\theta}$  and  $\int_{\phi_{xi}^*}^{\infty} \frac{\alpha_i \phi_{xi}^{*\alpha_i}}{\phi_i^{\alpha_i+1-\varepsilon+\theta}} = \left(\tilde{\phi}_{xi}\right)^{\varepsilon-\theta}$  as defined

earlier. Hence the integral over  $\phi \in (\phi_x^*, \infty)$  the entire range of  $\phi$  adds up to unity ( $e_i(\phi = \infty) = 1$ ) and  $e_i'(\cdot) < 0$ , so this is our employment distribution at trade. ■

The employment distribution in equation (2.26) has two parts: a) employment in domestic producers only and b) employment in export and domestic producers. Note that, the marginal exporters faces an increase in employment. Exporters find additional demand in the foreign market. To satisfy this additional demand exporters hire more workers, that are released from the exiting domestic firms and existing domestic firms that are shrinking. Trade does not effect the shape of wage equation it just shifts it to the right. The employment distribution leads to wage distribution at trade economy as well<sup>13</sup>.

$$y_i(w_{ti}) = \begin{cases} \frac{1}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{1-P_{ix}}{1+P_{ix}} \frac{\alpha_i}{\theta} \phi_{di}^{*\alpha_i} w_{ti}^{\frac{\varepsilon-\theta-\alpha_i}{\theta}-1} & \forall w_{ti} \in (\phi_{di}^{*\theta}, \phi_{xi}^{*\theta}) \\ \frac{1}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{P_{ix}}{1+P_{ix}} (1 + \tau^{\theta-\varepsilon}) \frac{\alpha_i}{\theta} \phi_{xi}^{*\alpha_i} w_{ti}^{\frac{\varepsilon-\theta-\alpha_i}{\theta}-1} & \forall w_{ti} \in (\phi_{xi}^{*\theta}, \infty) \end{cases} \quad (2.27)$$

Since exporters are serving two markets, they employ additional labor to produce some extra units to serve foreign market. Hence the weighted payment to this segment of the workers is higher even though the firms offer the same wage. This extra amount of labor is less efficient since they face tariff. Hence the density has a jump at  $\phi_{xi}^{*\theta}$ . This jump creates a distortion in

<sup>13</sup>The steps to find this density is similar to the case of autarky. The only difference is that now I apply the transformation technique to distribution of employment in both sections of employment distribution separately.



the labor market. Please refer to the figure 2.3 <sup>14</sup>:

Figure 2.3 : PDF of wage in trade for asymmetric countries.

Inside a country trade reallocates some workers from least paid exiting domestic firms to high paying exporting firms. Note that the firms who earn export status already have some employment for domestic production, hence the new workers joining them earn same as old workers. This reallocation offers higher wage to only one segment of the population. This makes the tail of the distribution fat, hence the percentile of the highest earning population comes from higher cutoff than before.

Across country open market gives exporting opportunities to relatively low productive firms in north ( $\phi_{xj}^* > \phi_{xi}^*$ ), since higher competition in north market. Exporting firms in north have bigger market share since they have smaller mass of firms active inside the economy. Opening up to trade results in a even bigger market share since they are relatively more productive than exporters in south  $j$ . On the other hand firms that survives the domestic market competition in north, shrinks relatively more compare to south. This implies that domestic firms in north releases relatively more workers. This is why in north, workers serving in domestic market lose more than the other country's workers in their domestic market, but workers in export market wins in north. However the disparity of wage in south is higher since some least productive firms are active in this economy that cannot survive the market competition in north ( $\phi_j^* < \phi_i^*$ ).

**Claim 9** Given  $\tau$  and  $f_x$  the Lorenz curve has the following expression from opening up for trade:

$$L_i(Y_{ti}) = \begin{cases} \frac{1}{E(w_{ti})} \lambda_{11}^i \left( 1 - (1 - Y_{ti} \times \lambda_{12}^i)^{\frac{\varepsilon - \alpha_i}{\varepsilon - \alpha_i - \theta}} \right) & \forall Y_{ti} \\ & \in (0, Y_{ti}(\phi_{xi}^{*\theta})) \\ L(Y_{ti}(\phi_{xi}^{*\theta})) + \frac{1}{E(w_{ti})} \lambda_{21}^i & \\ \times \left( 1 - \left( 1 - (Y_{ti} - Y_{ti}(\phi_{xi}^{*\theta})) \lambda_{22}^i \right)^{\frac{\varepsilon - \alpha_i}{\varepsilon - \alpha_i - \theta}} \right) & \forall Y_{ti} \\ & \in (Y_{ti}(\phi_{xi}^{*\theta}), 1) \end{cases} \quad (2.28)$$

<sup>14</sup>The figure is generated by using the following values for the parameter.  $L = 100$ ,  $\theta = .4$ ,  $\sigma = 4$ ,  $\alpha = 5$ ,  $f = 3$ ,  $f_x = 5$ ,  $f_i = 11$ ,  $f_e = 2$ ,  $\delta = .2$ ,  $\tau = 1.3$ .

Where  $\lambda_{11}^i = \frac{1}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{1-P_{ix}}{1+P_{ix}} \frac{\alpha_i}{\alpha_i-\varepsilon} \phi_{di}^{*\varepsilon}$ ,  $\lambda_{12}^i = \left(\frac{\tilde{\phi}_{tei}}{\phi_{di}^*}\right)^{\varepsilon-\theta} \frac{1+P_{ix}}{1-P_{ix}} \frac{\alpha_i+\theta-\varepsilon}{\alpha_i}$ ,  
 $\lambda_{21}^i = \frac{1}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{P_{ix}}{1+P_{ix}} (1+\tau^{\theta-\varepsilon}) \frac{\alpha_i}{\alpha_i-\varepsilon} \phi_{xi}^{*\varepsilon}$  and  $\lambda_{22}^i = \left(\frac{\tilde{\phi}_{tei}}{\phi_{xi}^*}\right)^{\varepsilon-\theta} \frac{1+P_{ix}}{P_{ix}} \frac{1}{1+\tau^{\theta-\varepsilon}} \frac{\alpha_i+\theta-\varepsilon}{\alpha_i}$ . Note that the Lorenz curve has a kink at  $Y_{ti}(\phi_{xi}^{*\theta}) = \frac{\phi_{di}^{*\varepsilon}}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} \frac{1-P_{ix}}{1+P_{ix}} \frac{\alpha_i}{\alpha_i+\theta-\varepsilon} [1 - \left(\frac{\phi_{xi}^*}{\phi_{di}^*}\right)^{\varepsilon-\theta-\alpha_i}]$ <sup>15</sup>. Due to this kink the Lorenz Curve of these economies have two segments. Now the relative wage of exporters is higher than domestic producer only.

**Proposition 10** *If wage is an increasing function ( $w = \phi^\theta$ ) of firms productivity with rent sharing parameter of  $\theta \in (0,1)$  and model parameters are such that some firms cannot access the foreign market in north ( $\phi_{xi}^* > \phi_{di}^*$ ); then opening up to trade relatively increase more wage inequality in the south "j" than north "i".*

Please refer to the figure 2.4<sup>16</sup>:

Figure 2.4: Lorenz curve at trade across asymmetric countries.

The mass of firms who can export suddenly finds additional profit that can be earned by exporting. To satisfy this additional demand of their goods in foreign market they hire some workers, who used to work for exiting small firms. These small firms exit due to increased competition. However the group of newly hired workers, who lost their jobs from exiting firms, earn same as some old workers hired by the same firms with exporting capability. This distortion increases the employment for exporters and increases the weight at this cutoff and above. For example; take any  $\phi_c \geq \phi_{xc}^*$  for  $\forall c \in i, j$  then, employment by this firm jumps due to additional production. This leads to a higher average wage at open market. The average wage of exporters increases more than the average wage of domestic producers. This distortion in the average wage across sectors (domestic and export market producers) leads to the higher inequality in a country.

<sup>15</sup>To find the Lorenze curve we need to know the CDF ( $Y_i(w_{ti})$  that can be obtained by standard statistical method) and the average wage  $E(w_{ti})$ . The expected wage will be  $E(w_{ti}) = \int w_{ti} y_i(w_{ti}) dw_{ti} = \frac{\tilde{\phi}_{tei}^{\varepsilon}}{\tilde{\phi}_{tei}^{\varepsilon-\theta}} = \tilde{\phi}_{tei}^\theta$

<sup>16</sup>An additional simulation of Gini at trade across country confirms that, after trade inequality will increase more in south relative to north,  $G_{trade,i} = 0.1301 < G_{trade,j} = 0.1319$ , where they are Gini at trade for country "i" and "j" respectively.

south "j" observes that their wage distribution has higher spread than the other country. This spread comes from the fact that, country "j" has less competition followed by a lower domestic cutoff at open economy setup. Hence the average wage coming from the domestic producers only in this country is relatively lower than the north "i". On the other hand, exporters in north pay a higher average wage than the other country's exports. Exporters in north observe a bigger market share increase than south and domestic producers in south observe a less reduction of market share relative to north. It seems that the first effect dominates and from open market south observes a higher inequality.

## 2.5 Comparative static analysis

The goal of this section is to study the effects of change in tariff and fixed cost to export. But tariff can change unilaterally or bilaterally. Hence the discussion goes into three different sections: a) symmetric bilateral tariff reduction, b) unilateral tariff reduction by south and c) a symmetric decrease in fixed cost to export. To study the comparative statics, I rely on computer simulations<sup>17</sup>. The model parameters are as follows:  $L=100$ ,  $\theta=.4$ ,  $\sigma=3.8$ ,  $\alpha_i=3.1$ ,  $\alpha_j=3.3$ ,  $f=3$ ,  $f_x=5.5$ ,  $f_e=2$ ,  $\delta=.2$ ,  $\tau=1.1$  and  $l=1$ . Note that  $\tau$  and  $f_x$  will take different values for comparative static analysis.

### 2.5.1 A symmetric bilateral tariff reduction

For this exercise, I solve the entire model with two level of tariffs, a)  $\tau = 1.1$  and b)  $\tau = 1.3$ . Note that this effect can be looked at from two different perspectives, a) inside a country and b) across countries. Please go to figure 2.5 and 2.6.

Figure 2.5: PDF of wage at symmetric tariff reduction for north.

Figure 2.6: PDF of wage at symmetric tariff reduction for south.

A decrease in tariff opens up the export market to some highly productive domestic firms from both countries. This in turn increases the competition in both markets followed by an

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<sup>17</sup>The parameter values that I assumed during the simulation is taken from Demidova (2008) and Helpman, Yeaple and Melitz (2004) paper.

increase in domestic cutoff and decrease in export cutoff. Hence from a symmetric decrease in tariff, both countries observe a shrink in the mass of firms available in an economy. However the firms with export status (both old and new) expand more in both countries, where as the domestic firms that survive the increased competition expands relatively less to their own exporters in north but contracts in south. This induces in a more efficient way of using labor inside both economies. Hence the tail of the weighted wage distribution for both countries become fatter relative to high tariff case.

Figure 2.7: Lorenz at trade liberalization ( $1.1 < \tau < 1.3$ ) north  $\alpha_i = 3.1$ .

Figure 2.8: Lorenz at trade liberalization ( $1.1 < \tau < 1.3$ ) south  $\alpha_i = 3.3$ .

As tariff decreases some workers are released from exiting domestic firms. These workers are absorbed by the expanding exporters in both countries and expanding domestic producers in north. Since existing domestic firms in south goes through a contraction of market share, they release some workers as well. These workers are absorbed by their exporters spontaneously. Hence in both countries now more workers have higher wage compared to high tariff case. This induces in a lower inequality in both countries as more people converge to the average wage. An additional simulation result of GINI coefficient confirms the result. One interesting result revealed from this simulation is that, north observes a relatively higher reduction in inequality than the other country. Note that a decrease in GINI coefficient at north is 0.0079, where as south observed a decrease of 0.005<sup>18</sup>.

Figure 2.9: PDF of wage at trade ( $\tau = 1.1$ ) across asymmetric country.

Figure 2.10: PDF of wage at trade ( $\tau = 1.3$ ) across asymmetric country.

At high tariff some highly productive exporting firms in north have relatively higher market share compare to south, but some low productive domestic firms at north have relatively lower market share compare to other country. As tariff decreases, more least productive domestic firms at north are forced to exit due to high competition and only the firms that have relatively

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<sup>18</sup>The GINI coefficients from the simulation is given in the following matrix,

| <i>GINI</i>  | North $\alpha_i = 3.1$ | South $\alpha_j = 3.3$ |
|--------------|------------------------|------------------------|
| $\tau = 1.1$ | 0.1389                 | 0.126                  |
| $\tau = 1.3$ | 0.1468                 | 0.1311                 |

more market share survive the competition and expand. Note that, these exiting firms in north had lower market share than least productive domestic firms in south. Hence, a symmetric tariff reduction redistributes labor in a more efficient manner for north. The exporters on the other hand expand more from tariff reduction in north.

Figure 2.11:Lorenz at trade ( $\tau = 1.1$ ) across asymmetric country.

Figure 2.12:Lorenz at trade ( $\tau = 1.3$ ) across asymmetric country.

Decrease in tariff, decrease inequality in this setup. But north observes a higher decrease in inequality than south. However north still have a higher inequality relative to her less north trading partner. But symmetric tariff reduction benefits north more.

## 2.5.2 Unilateral tariff reduction of south

This section considers a case where the north imposes lower tariff and south unilaterally matches the tariff rate of north. For this exercise, I will fix the tariff rate for north at  $\tau_i = 1.1$ . But south goes through a tariff reduction from  $\tau_j = 1.3$  to  $\tau_j = 1.1$ .

Figure 2.13: PDF of wage for north at unilateral tariff reduction and  $\tau_i = 1.1$ .

Figure 2.14: PDF of wage for south at unilateral tariff reduction and  $\tau_i = 1.1$ .

As south decrease their tariff more firms in north gets access to the export market. These new exporters from north increases the competition in south's local market and forces the least productive firms to exit in south. The workers coming from exiting domestic firms is absorbed by expanding domestic and exporting firms in south. Hence both export and surviving domestic firms go through an increase in market share, where export market goes through a bigger expansion than domestic market. This increased competition in south in turn increases pressure on exporters of north. Hence all the exporters go through a contraction of market share in north. Now exporters of north increases competition in their local market. Since exporters from north find a decrease in the demand for their goods in south, they sell these in the local market and avoid tariff. Hence the goods become cheaper. These cheap goods drive the competition up and earns larger market share for firms with export status. It seems that the first effect dominates for exporters, hence both exporters and domestic producers go through a contraction in north.

Figure 2.15: Lorenz at trade liberalization ( $1.1 < \tau_j < 1.3$ ) north  $\alpha_i = 3.1$ .

Figure 2.16: Lorenz at trade liberalization ( $1.1 < \tau_j < 1.3$ ) south  $\alpha_i = 3.3$ .

As south decrease tariff, both countries observe a lower inequality. This mechanism is symmetric to previous case. This change in the environment redistributes labor in such a way that relatively more workers have access to higher wage. As a result the inequality decreases inside both countries<sup>19</sup>.

Figure 2.17: PDF of wage at trade ( $\tau_j = 1.1$  and  $\tau_i = 1.1$ ) across countries.

Figure 2.18: PDF of wage at trade ( $\tau_j = 1.3$  and  $\tau_i = 1.1$ ) across countries.

A higher tariff by south restricts the export market entry. Hence the marginal exporter from north is relatively high productive due to high tariff. At this tariff, the exporters in the north occupies a bigger market share relative to exporters of south. But north imposes relatively low tariff. This makes the local market of north easy to access. As a result, exporters from south can exploit the local market of north and increase the competition. Hence the domestic producers in north shrink in mass and more productive firms sustain the competition. However, both domestic and export market in north enjoys a bigger market share.

As south decreases tariff, more exporters enter the local market of south and decrease the market share of all exporters in north. These new exporters increase the competition in the local market of south. Increased competition forces the least productive firms out of the economy. Now, this displaces some workers and causes the domestic and export firms to expand in south. At the same time, this reduces the market share for existing exporters from north. Hence the dispersion of market share for exporters across country reduces. The reduction of export market share by north now turns to increase competition in their local market. This shrinks the market share for domestic producer only in south. However domestic producers go through an expansion in south.

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<sup>19</sup>The GINI coefficient across country by unilateral tariff reduction by developing country

| <i>GINI</i>    | Developed $\alpha_i = 3.1$ | Developing $\alpha_j = 3.3$ |
|----------------|----------------------------|-----------------------------|
| $\tau_i = 1.1$ |                            |                             |
| $\tau_j = 1.1$ | 0.1301                     | 0.1238                      |
| $\tau_i = 1.1$ |                            |                             |
| $\tau_j = 1.3$ | 0.1416                     | 0.1303                      |

Figure 2.19: Lorenz at trade ( $\tau_j = 1.1$  and  $\tau_i = 1.1$ ) across asymmetric countries.

Figure 2.20: Lorenz at trade ( $\tau_j = 1.3$  and  $\tau_i = 1.1$ ) across asymmetric countries.

When south imposes a high tariff both country observe a higher inequality in the economy. However north observes a higher inequality, since now GINI coefficient at trade for north is 0.1416 but for south is 0.1303. As the unilateral tariff reduction takes place, both country observe a lower inequality in the economy. But now north has a GINI coefficient of 0.1301 and south has 0.1238. This decrease in tariff reduction by south lowers the inequality differential across countries. north is better off maintaining a lower level of tariff as it generates the lowest level of inequality compare to symmetric decrease of bilateral tariff.

### 2.5.3 A decrease in fixed cost to export $f_x$

Following same kind of idea, let us consider two different levels of  $f_x$  and solve the entire model. The fixed cost to export  $f_x$ , will vary from 5.5 to 7.5. Again at first I will look at the country level and then compare them across countries.

Figure 2.21: PDF of wage ( $7.5 < f_x < 5.5$ ) at north ai=3.1

Figure 2.22: PDF of wage ( $7.5 < f_x < 5.5$ ) at south ai=3.3

A decrease in fixed cost to export opens up exporting opportunities to some highly productive domestic firms. This decreases the export cutoff. These new exporters along with old exporters observe an increase in their market share and expand. On the other hand, some least productive domestic firms are forced to exit the market from increased competition. They release some workers in the economy. These workers are absorbed by the expanding surviving firms (both domestic and exporting firms).

Figure2.23: Lorenz curve at trade liberalization ( $7.5 < f_x < 5.5$ ) for north ai=3.1

Figure2.24: Lorenz curve at trade liberalization ( $7.5 < f_x < 5.5$ ) for south ai=3.3

Since all the surviving firms from decrease in fixed cost to export go through an expansion, now all the workers are better off since more workers have higher wage compare to higher fixed cost to export (7.5). This decreases inequality in both countries. The simulation of GINI

coefficient confirms the result<sup>20</sup>. Similar to symmetric tariff reduction, north observes a higher reduction in inequality( $\Delta GINI_i = 0.0098$ ) relatively to south( $\Delta GINI_j = 0.0076$ ).

Figure 2.25: PDF of wage at trade ( $f_x=5.5$ ) across countries

Figure 2.26: PDF of wage at trade ( $f_x=7.5$ ) across countries

As fixed cost to export decrease, exporters in north goes through a bigger expansion than south. On the other hand relatively domestic producers in south expands more relative to most domestic firms of north.

Figure2.27: Lorenz at trade ( $f_x=5.5$ ) across asymmetric countries.

Figure2.28: Lorenz at trade ( $f_x=7.5$ ) across asymmetric countries.

A decrease in  $f_x$  reduces inequality in both countries. However north has higher inequality in both high and low  $f_x$  value.

## 2.6 Welfare analysis

The aggregate welfare per worker in north has the following expression:

$$\left(\frac{W}{L}\right)_i = E(w_i) \tag{2.29}$$

Since aggregate price index is normalized to one, the welfare only depends on the average wage of the economy. Note that an increased competition due to opening of market for trade increases the cutoff for domestic market. As the cutoff increases the minimum productivity require to serve the domestic market increases. This results in higher wage and higher welfare per worker in north "i".

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| <i>GINI</i>               | Developed country $a_i = 3.1$ | Developing country $a_j = 3.3$ |
|---------------------------|-------------------------------|--------------------------------|
| <sup>20</sup> $f_x = 5.5$ | 0.1389                        | 0.126                          |
| $f_x = 7.5$               | 0.1487                        | 0.1336                         |



On the other hand, the price index  $P_j$  in south is greater than one. This imply that the welfare per worker in south is:

$$\left(\frac{W}{L}\right)_j = \frac{E(w_j)}{P_j} \quad (2.30)$$

Note that, an increased competition will increase the domestic cutoff. Now the average wage in the economy increases with the increased competition. Aggregate price index, on the other hand, will decrease with an increase in domestic cutoff. Hence Any effect that causes in higher competition, such as trade liberalization, will increase the welfare per worker in south as well .

As globalization takes place, countries open their market to all. This gives incentive for high productive firms to export and increase the competition. Now globalization, according to this paper, can happen by the following four processes: a) countries go from autarky to trade, b) countries decrease tariff symmetrically, c) south unilaterally decrease tariff to north's level and d) a symmetric decrease of fixed cost to export. Note that, all of them increase competition and results in a higher welfare per worker for both countries. However, all of the procedures above does not result in a lower inequality. For example, as countries go from autarky to trade the inequality goes up in both countries.

## 2.7 Conclusion

This paper develops a model to study wage inequality across asymmetric countries created by opening up to trade in a heterogenous firms setup. It shows that trade will increase inequality relatively more in south to north. Workers displacement from the autarky equilibrium to trade will redistribute population in such a way to increase welfare that, it will result in creating some wage inequality. This obviously will increase inequality inside both economy. On the other hand the welfare will increase in both country as well.

A symmetric decrease in tariff and fixed cost to invest has similar effect on the economy. These decrease will increase the mass of exporters and increase competition in both countries. The exporters from both countries will go through an expansion and employ more labor that is released from the domestic producers. This reallocation will end up providing a higher labor payment to a bigger share of the population. Hence the inequality will decrease in both countries. However inequality will relatively decrease more in north than south, even though

north will have a higher inequality than the other. On the other hand both country will still observe a welfare gain from this transition. Hence north will always benefit more from opening up for trade than south.

A unilateral tariff reduction by the south will reduce the inequality for both countries as before. However, from the perspective of north it is better to impose a lower tariff no matter what the south does. This ensures the higher involvement of exporters from south and keeps the competition higher. As a result, the economy employes labor more efficiently and keeps the inequality level as low as possible. However, unilateral tariff reduction will increase some competition as well. This leads to a higher welfare per worker.

Figure 2.1: PDF of wage at autarky across asymmetric countries.

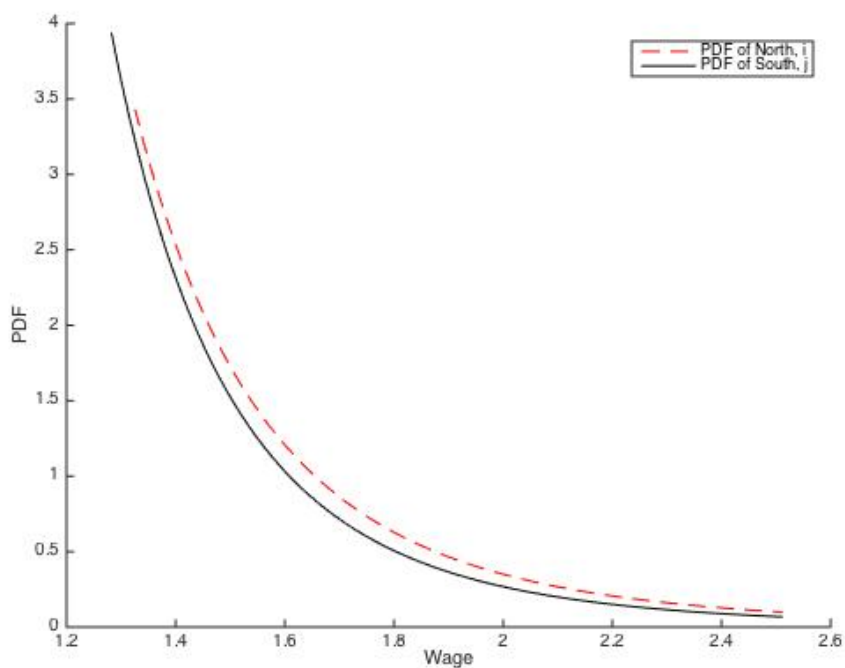


Figure 2.2: Lorenz curve at autarky across asymmetric countries.

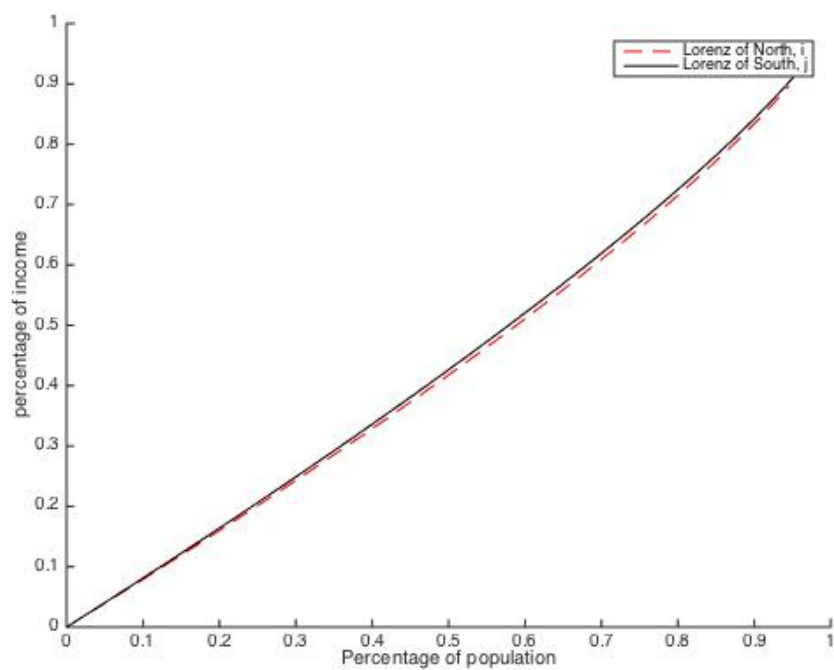


Figure 2.3: PDF of wage at trade across asymmetric countries.

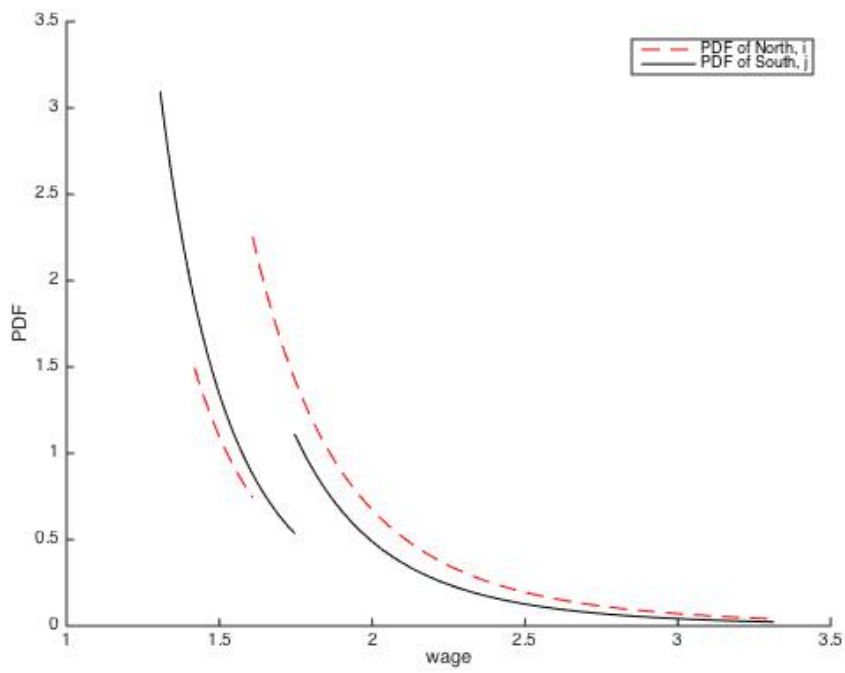


Figure 2.4: Lorenz curve at trade across asymmetric countries.

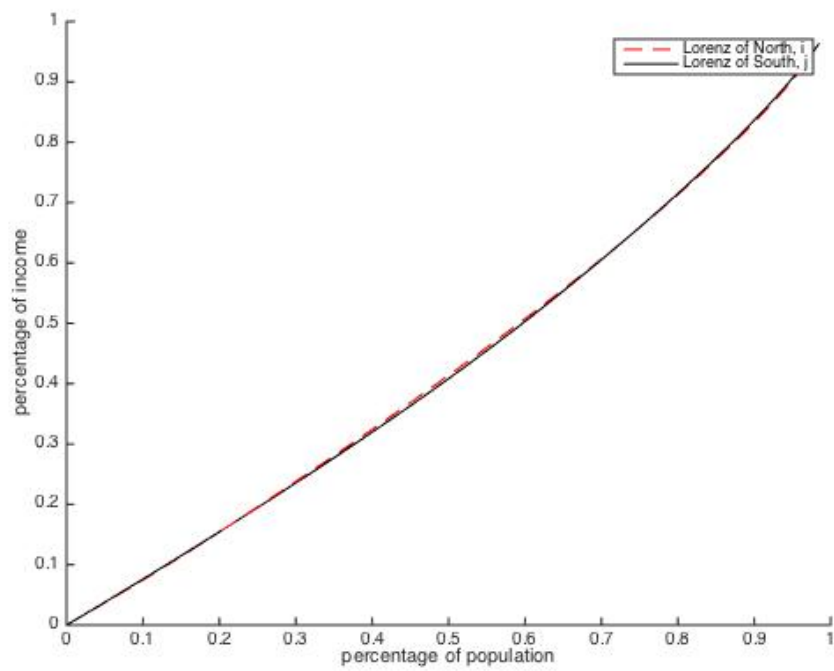


Figure 2.5: PDF of wage at symmetric tariff reduction for North.

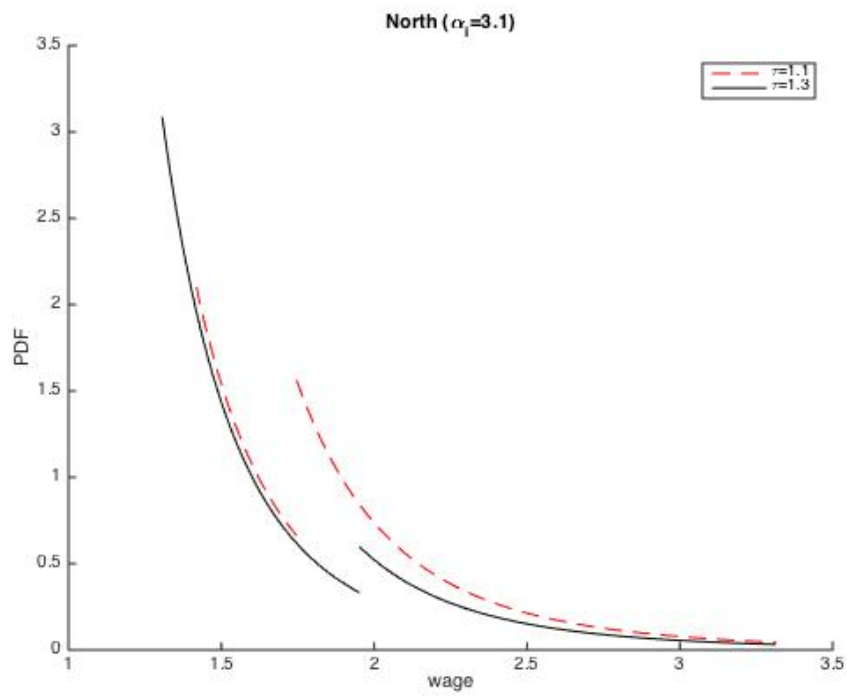


Figure 2.6: PDF of wage at symmetric tariff reduction for South.

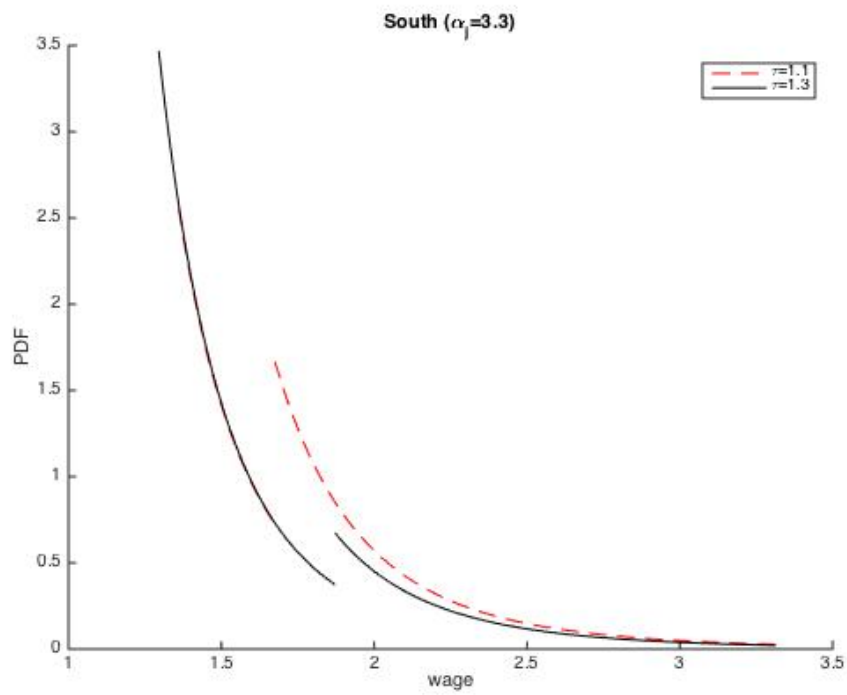


Figure 2.7: Lorenz at trade liberalization ( $1.1 < \tau < 1.3$ ) North ( $\alpha_i = 3.1$ ).

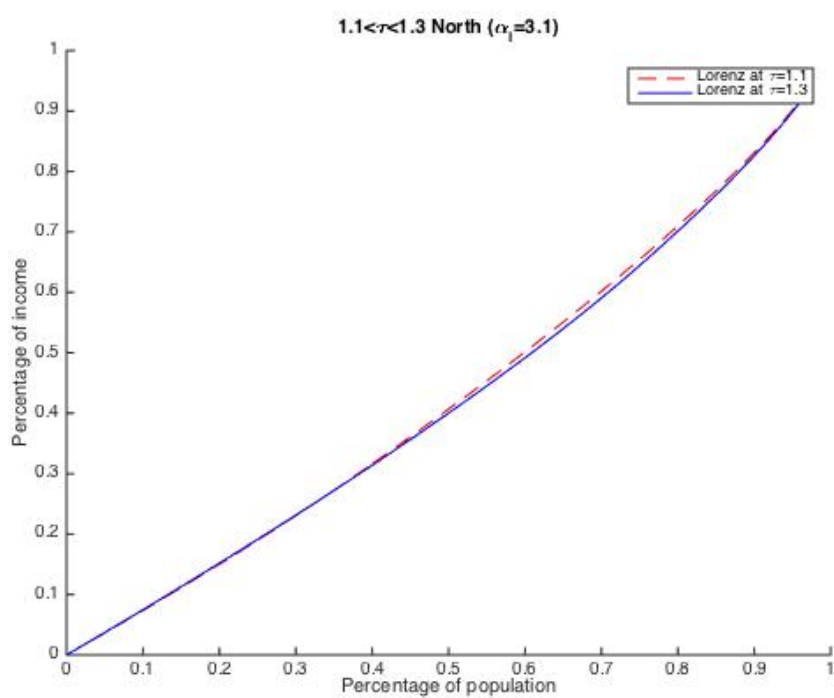


Figure 2.8: Lorenz at trade liberalization ( $1.1 < \tau < 1.3$ ) South ( $\alpha_j = 3.3$ ).

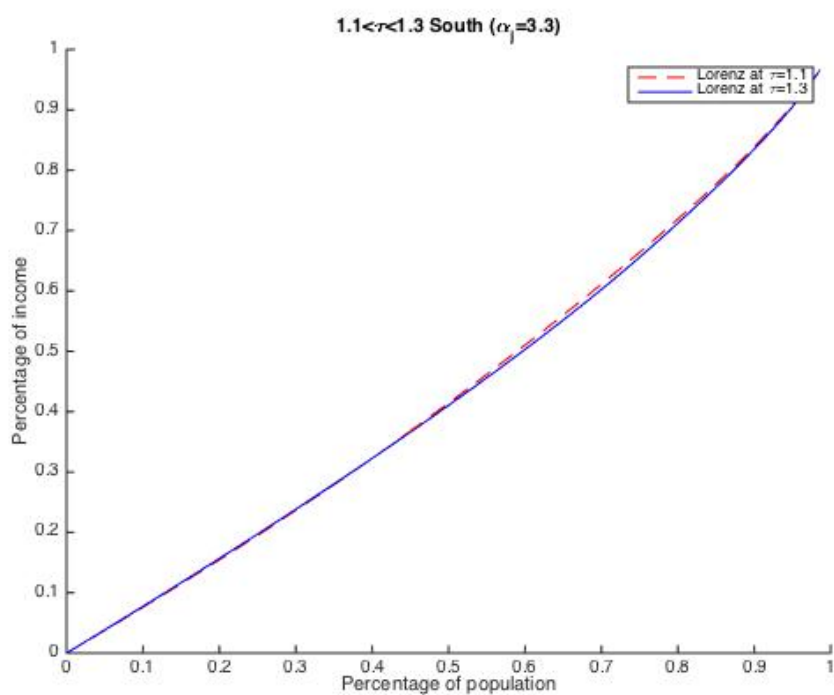


Figure 2.9: PDF of wage at trade ( $\tau=1.1$ ) across asymmetric countries.

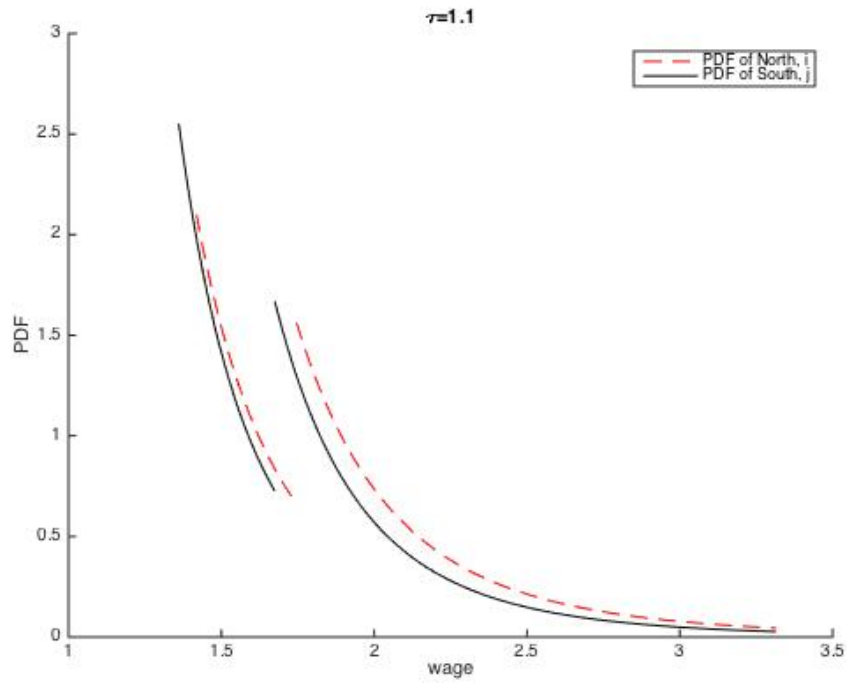


Figure 2.10: PDF of wage at trade ( $\tau=1.3$ ) across asymmetric countries.

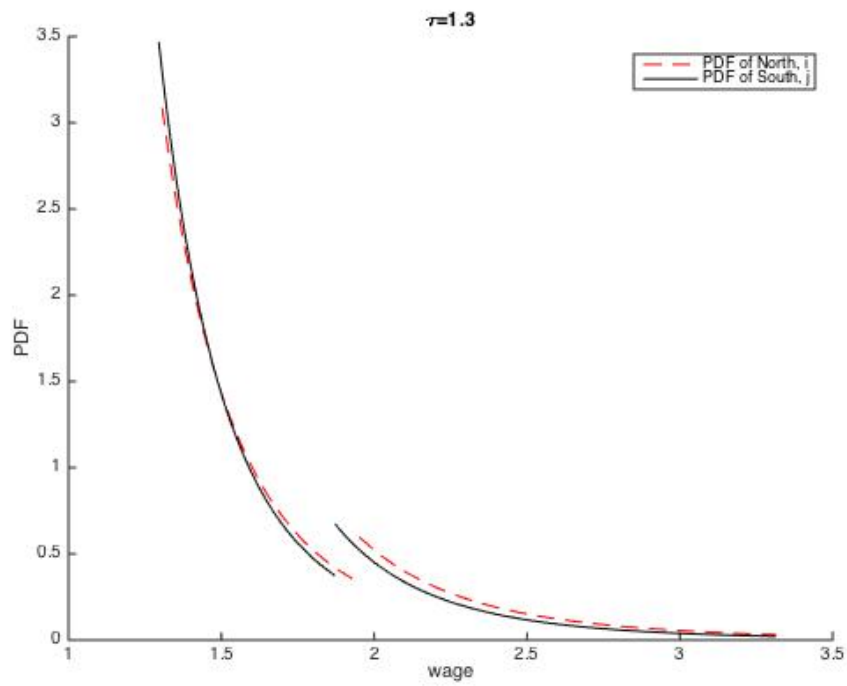


Figure 2.11: Lorenz at trade ( $\tau=1.1$ ) across asymmetric countries.

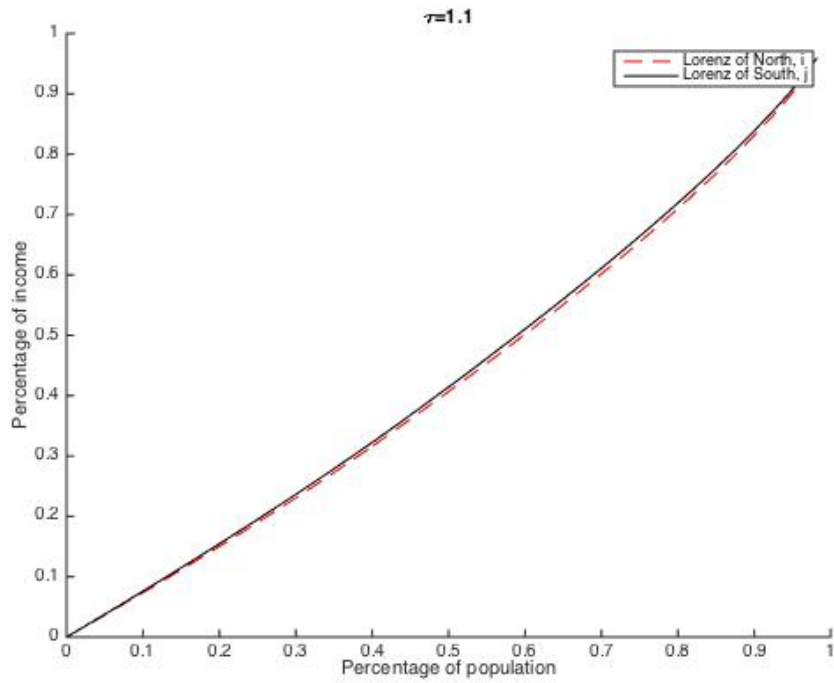


Figure 2.12: Lorenz at trade ( $\tau=1.3$ ) across asymmetric countries.

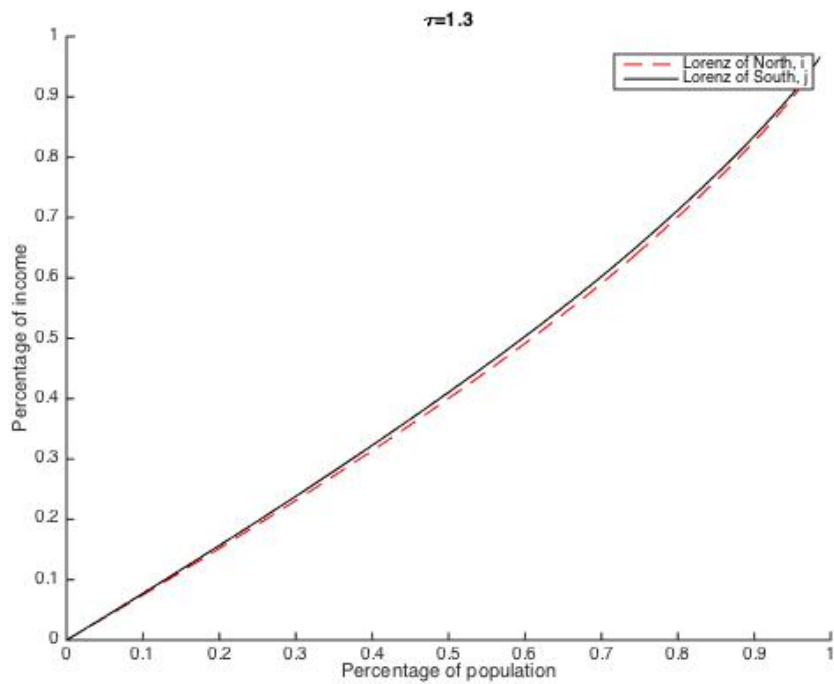




Figure 2.13: PDF of wage for North at unilateral tariff reduction and  $\tau_i=1.1$

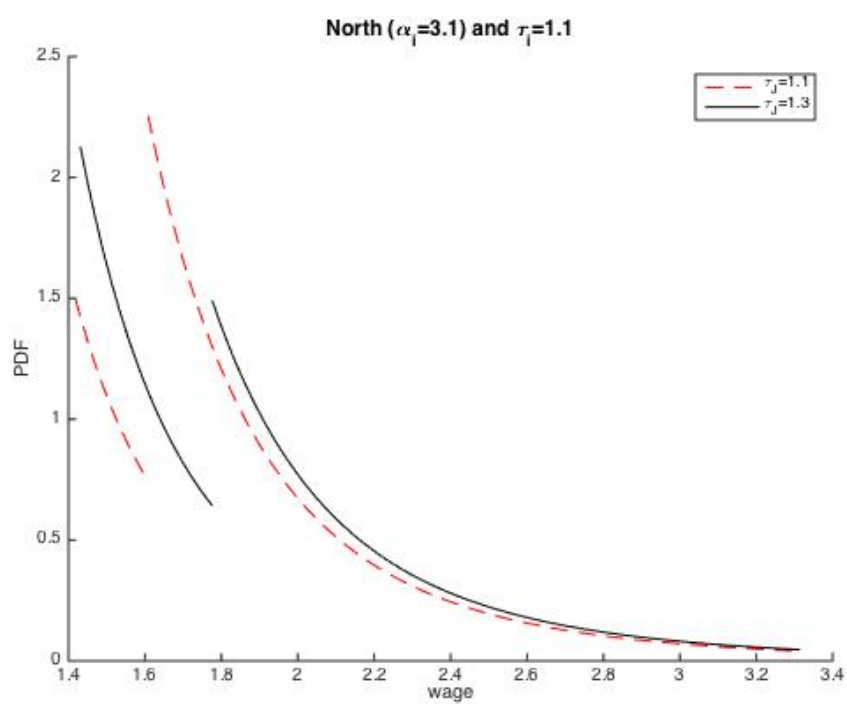


Figure 2.14: PDF of wage for South at unilateral tariff reduction but  $\tau_i=1.1$

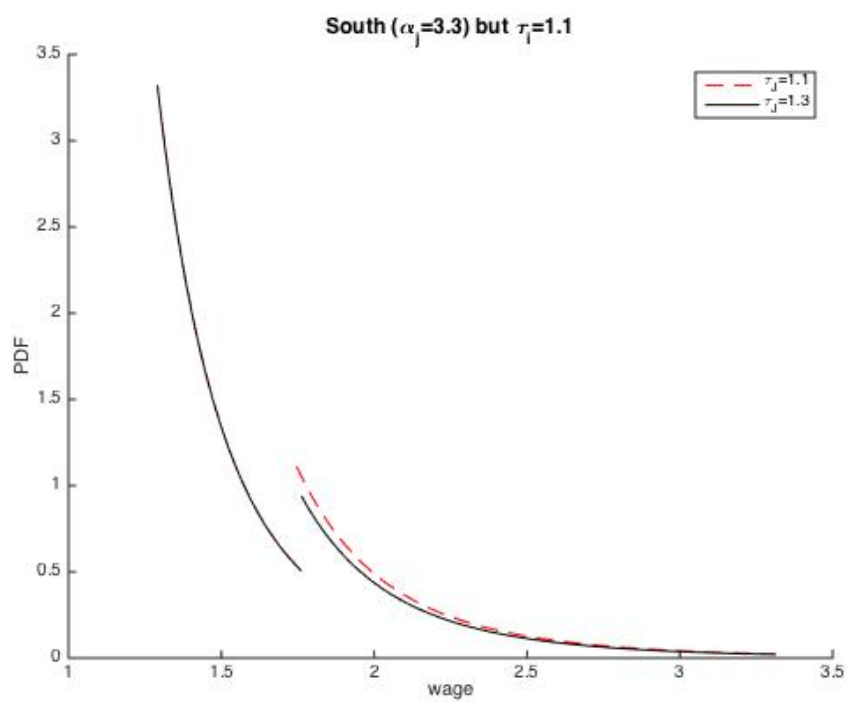


Figure 2.15: Lorenz at trade liberalization ( $1.1 < \tau_j < 1.3$ ) North ( $\alpha_i = 3.1$ )

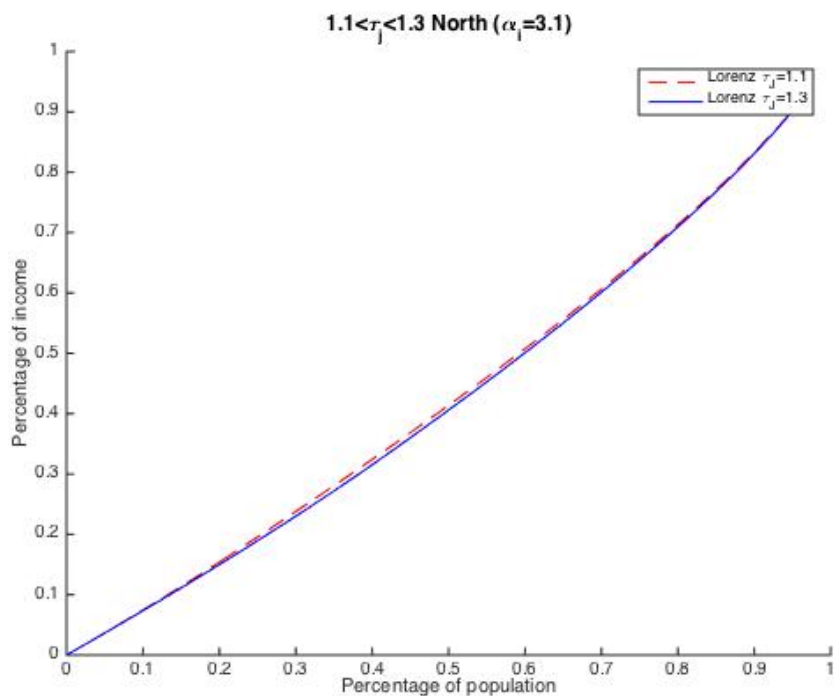


Figure 2.16: Lorenz at trade liberalization ( $1.1 < \tau_j < 1.3$ ) South ( $\alpha_i = 3.3$ )

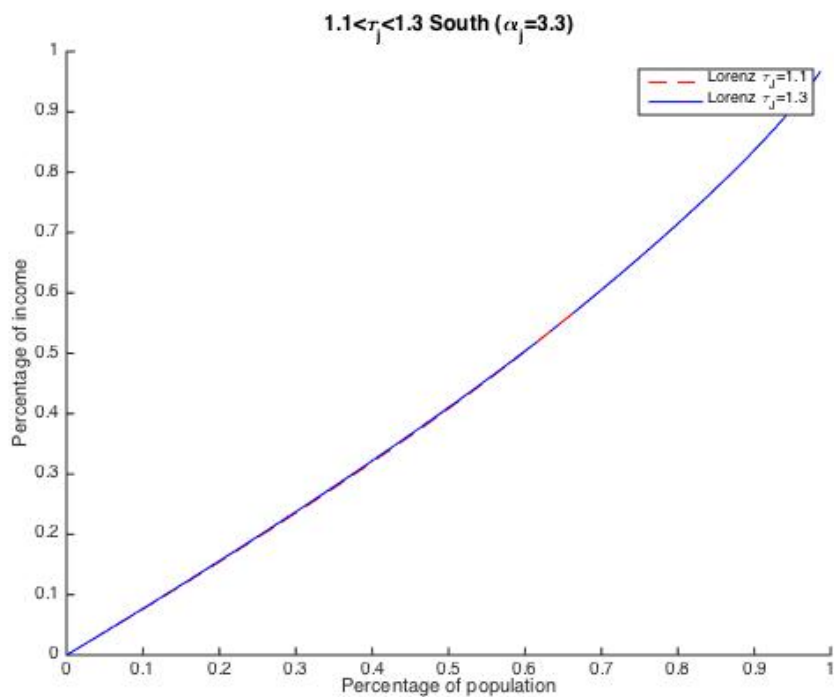


Figure 2.17: PDF of wage at trade ( $\tau_j=1.1$  and  $\tau_i=1.1$ ) across asymmetric countries.

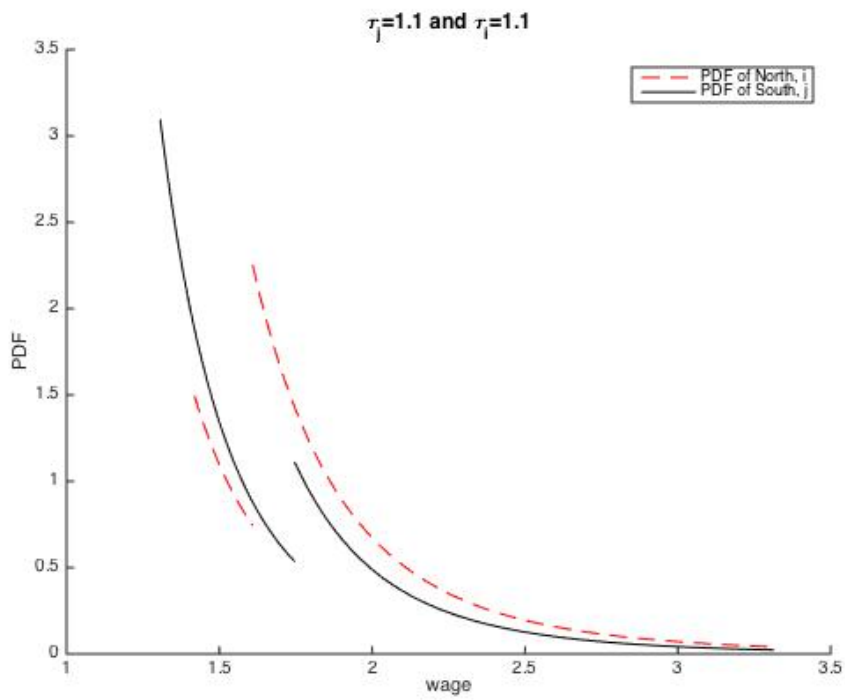


Figure 2.18: PDF of wage at trade ( $\tau_j=1.3$  and  $\tau_i=1.1$ ) across asymmetric countries.

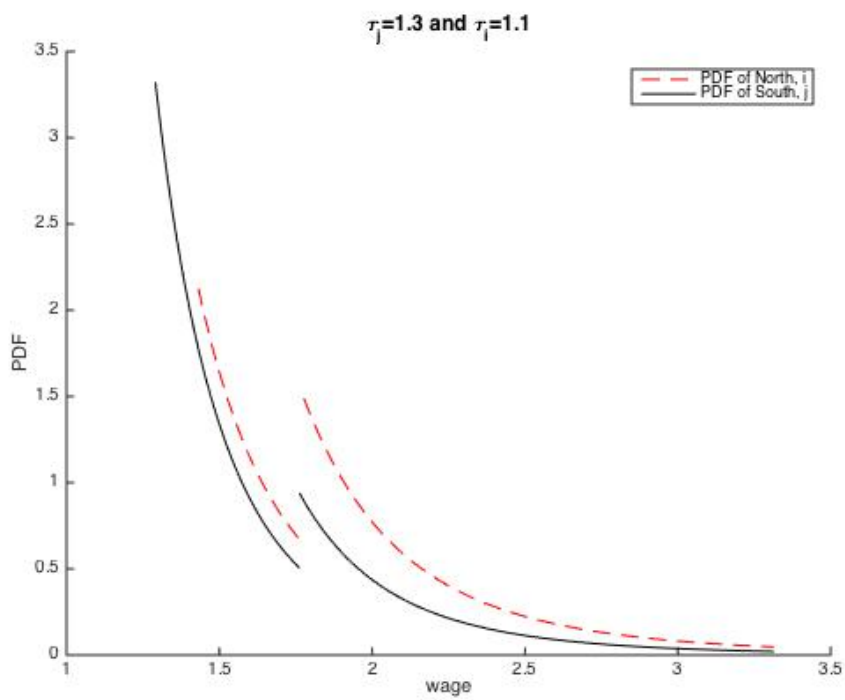


Figure 2.19: Lorenz at trade ( $\tau_j=1.1$  and  $\tau_i=1.1$ ) across asymmetric countries.

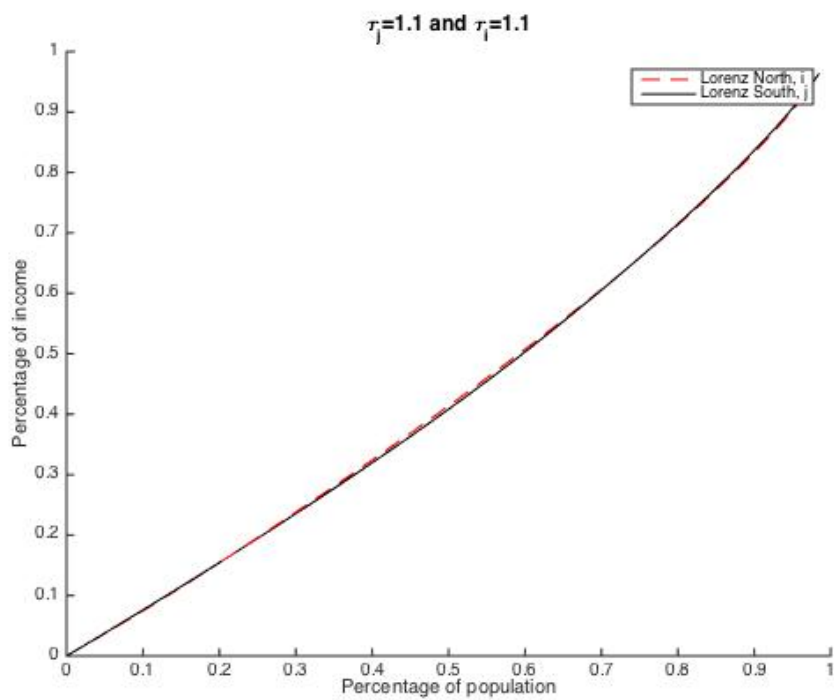


Figure 2.20: Lorenz at trade ( $\tau_j=1.3$  and  $\tau_i=1.1$ ) across asymmetric countries.

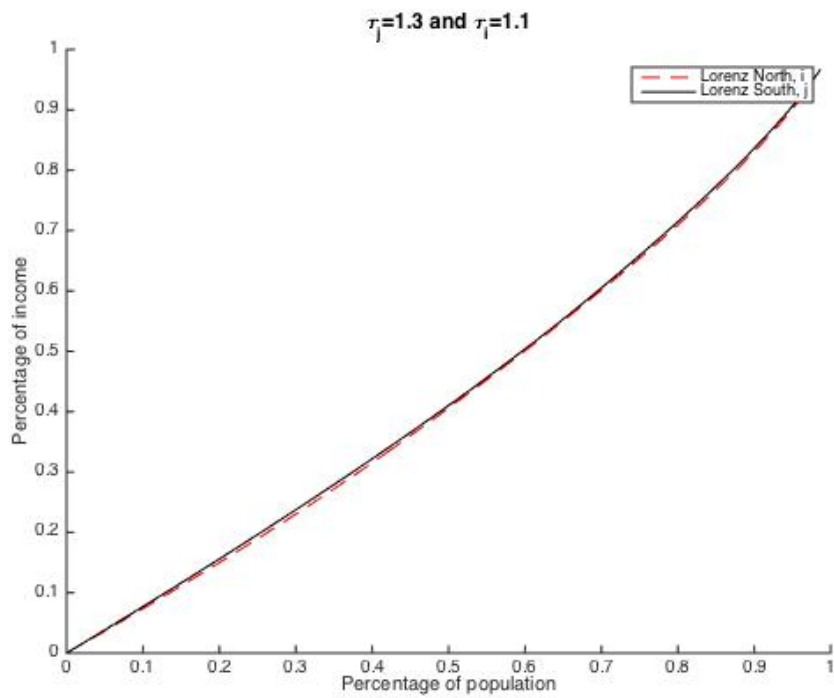


Figure 2.21: PDF of wage at trade ( $7.5 < f_x < 5.5$ ) for North  $\alpha_i = 3.1$

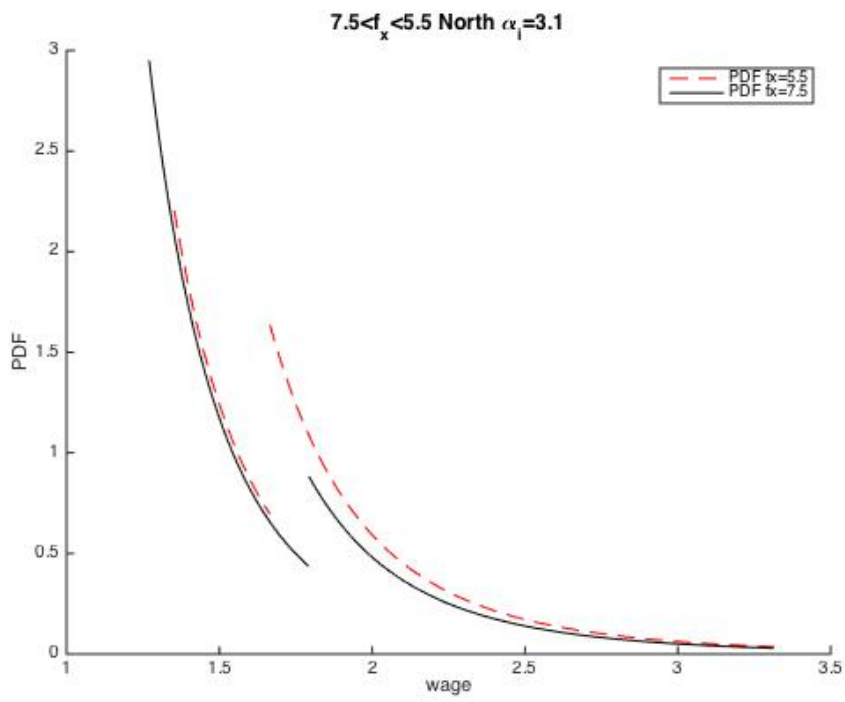


Figure 2.22: PDF of wage at trade ( $7.5 < f_x < 5.5$ ) for South  $\alpha_j = 3.3$

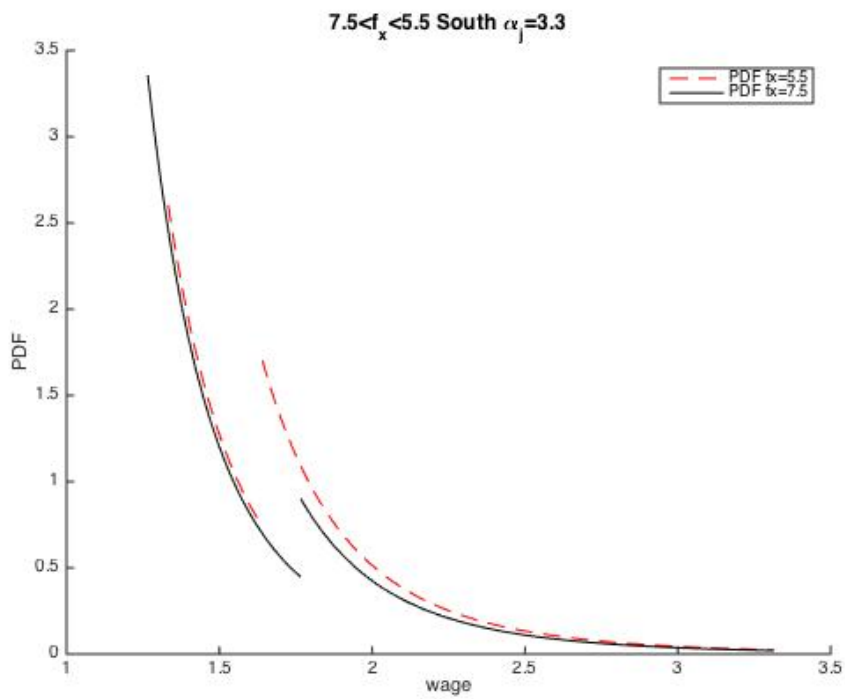


Figure 2.23: Lorenz at trade liberalization ( $7.5 < f_x < 5.5$ ) for North  $\alpha_i = 3.1$ .

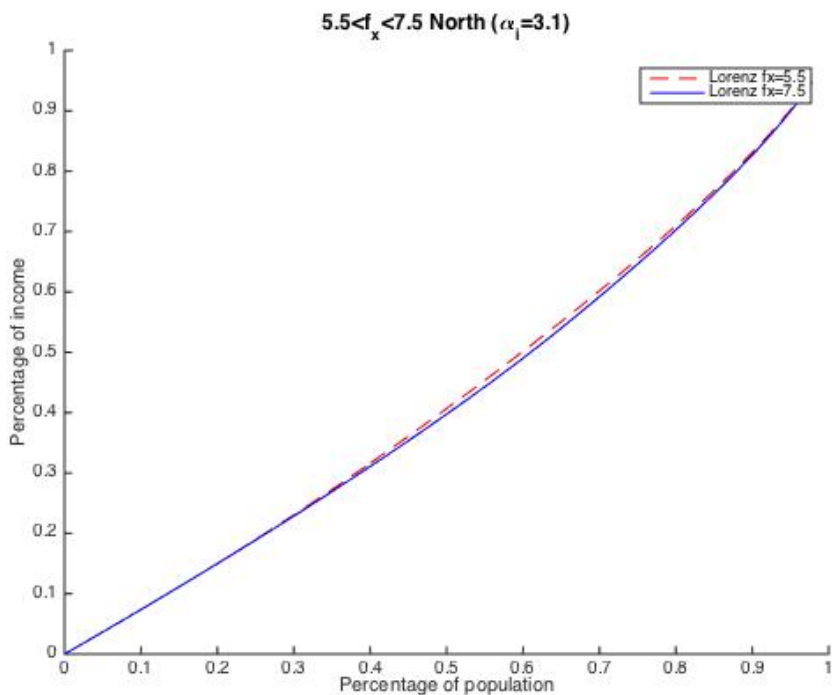


Figure 2.24: Lorenz at trade liberalization ( $7.5 < f_x < 5.5$ ) for South  $\alpha_j = 3.3$ .

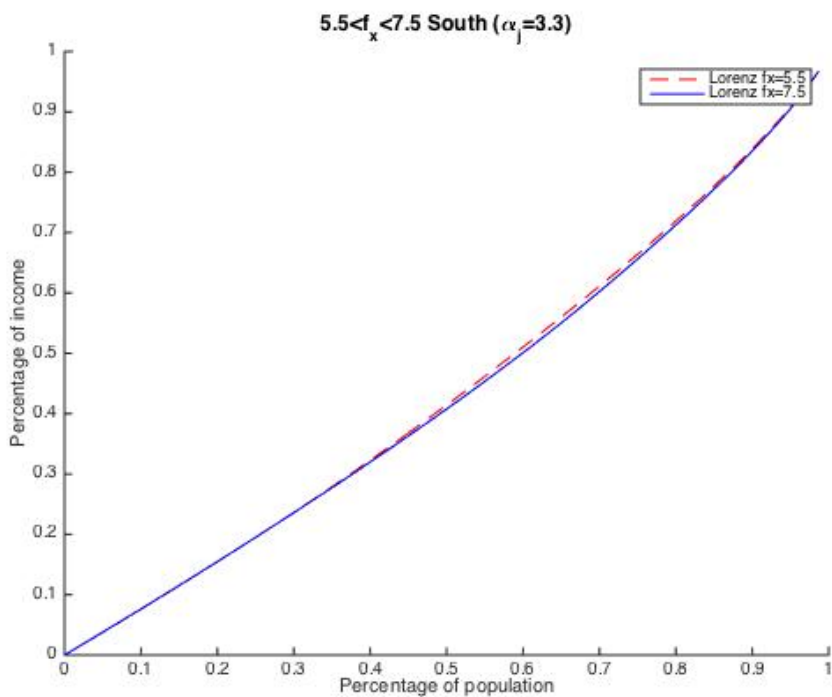


Figure 2.25: PDF of wage at trade ( $f_x=5.5$ ) across asymmetric countries.

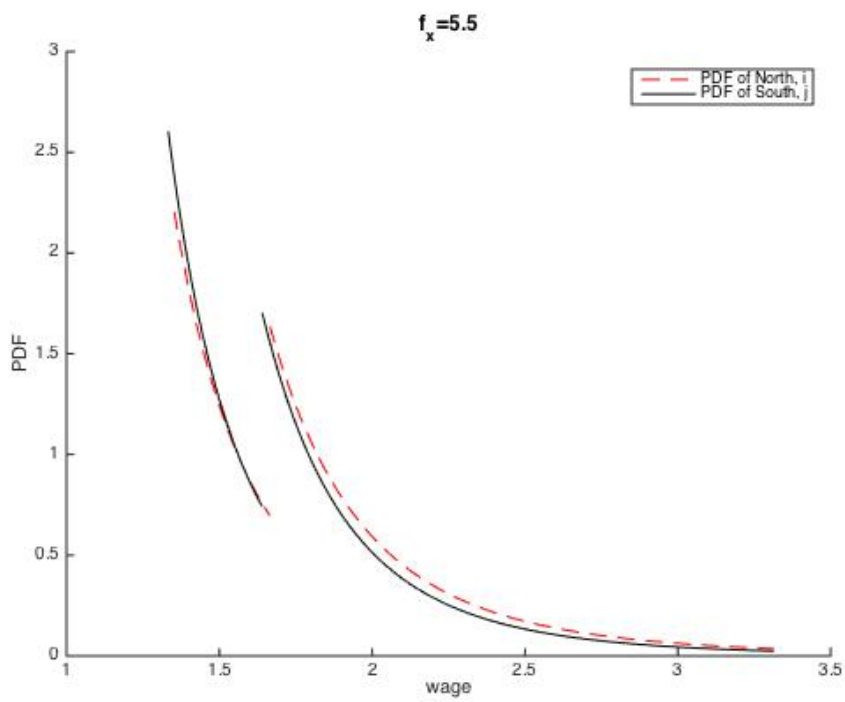


Figure 2.26: PDF of wage at trade ( $f_x=7.5$ ) across asymmetric countries.

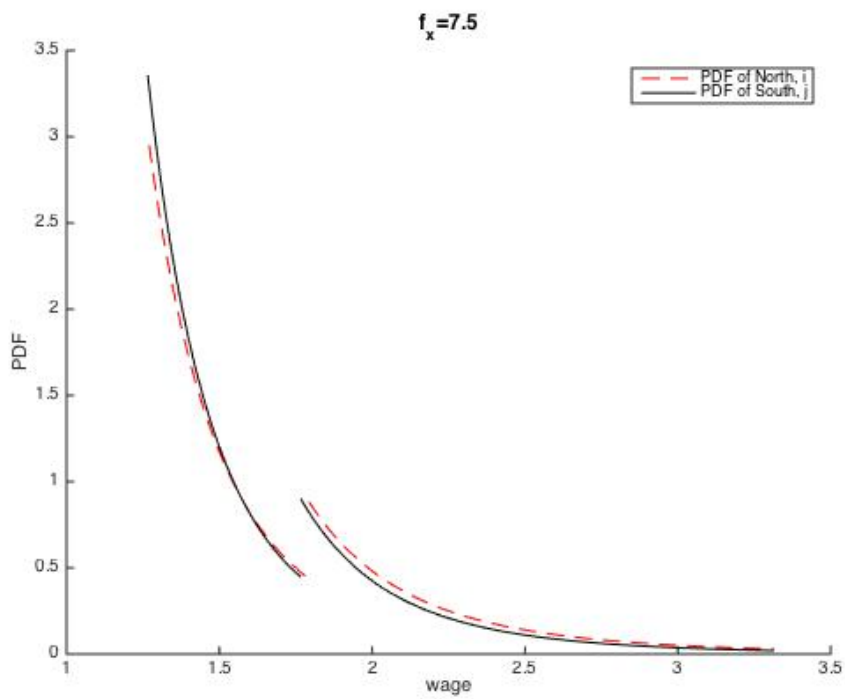


Figure 2.27: Lorenz at trade ( $f_x=5.5$ ) across asymmetric countries.

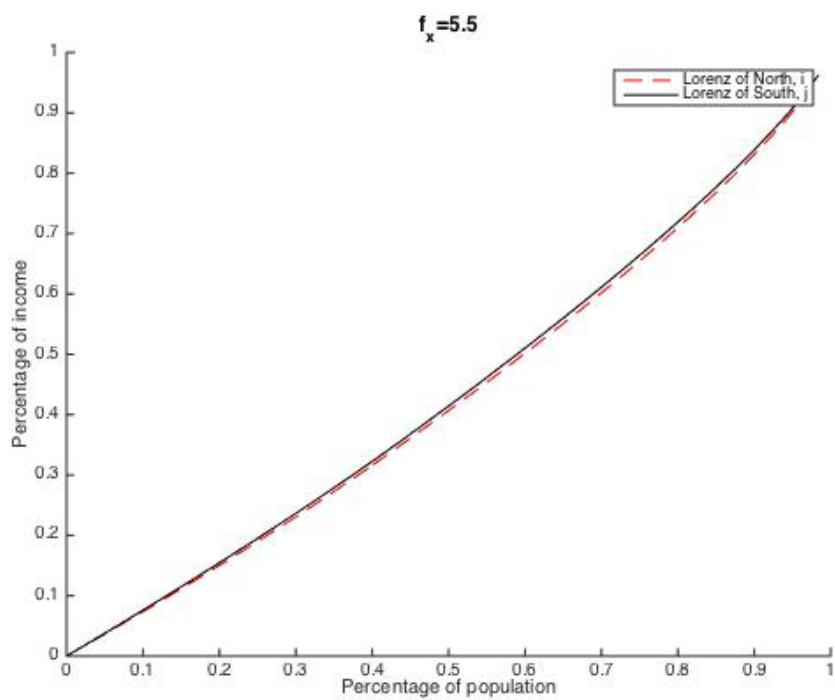
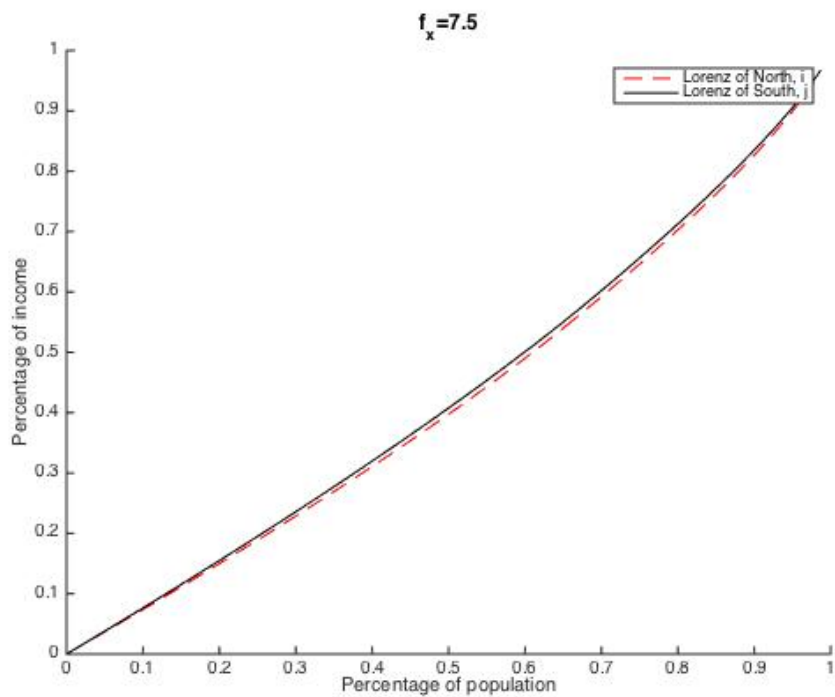


Figure 2.28: Lorenz at trade ( $f_x=7.5$ ) across asymmetric countries.





## Chapter 3

# Understanding the Sources of Aggregate Productivity Growth. The Case of Chilean Plants 79-96

### 3.1 Introduction

Both new and neoclassical trade theory talk about productivity growth due to globalization. Neoclassical theories depend on the fact that some industries have comparative advantage over others; hence countries specialize in industries in which they can produce output more efficiently. However lately trade economists realized that countries can also trade similar goods. They have showed the importance of intra industry reallocation and therefore interest shifted to reallocation of resources across firms and/or plants inside an industry. Both theories talk about reallocation of resources that lead to aggregate productivity growth. In this chapter I ask which of these theories contribute more to explaining the evolution of productivity growth for Chile. Are there other forces that can explain the productivity growth for Chile?

Aggregate productivity is the weighted average of all the firms or plants in an economy, where these weights are defined by their market share. Empirically it has been shown that these producers vary a lot in terms of their productivity, even when we narrowly define their sectors. This composition of productivity can change due to the change in composition of

active firms, entry firms and lastly exiting firms. Many studies showed that this composition of firms is an important factor in the evolution of productivity; such as Foster, Haltiwanger and Krizan (2001); Bartelsman, Haltiwanger and Scarpetta (2013). These studies have suggested a productivity decomposition into four different components; productivity distribution shifts among active firms, market share reallocation among active firms and obviously entry and exit.

The four component decomposition was first studied by Baily, Hulten and Campbell (1992); hereafter BHC. They tracked a firm/plant over time and then decomposed the productivity growth coming from surviving, entry and exiting firm. Then the contribution from surviving firms further breaks down to within firm increase in share and across firm productivity growth. Similar method has been adopted by Griliches and Regev (1995), here after GR, and Foster, Haltiwanger and Krizan (2001), here after FHK. In particular they constructed some reference level of productivity,  $\Phi^{ref}$ , and then took the weighted average of the difference from this reference level and firm's productivity. Hence, in this context, the aggregate productivity became  $\sum_i \text{sale share}_i \times (\text{productivity}_i - \Phi^{ref})$ . GR took the average of two consecutive aggregate productivities,  $\bar{\Phi} = (\Phi_1 + \Phi_2)/2$ , to be the reference level. FHK on the other hand took the aggregate productivity of first period to be the reference level. Note that BHC always estimates that entry effect as positive; however, this is not the case with GR and FHK as it has to be at least larger than the reference level of productivity to have a positive impact on the growth of aggregate productivity. This problem has been further addressed by Melitz and Polanec (2014), hereafter MP, by considering the reference level to be the average productivity of continuing firms only in an industry.

The decomposition of aggregate productivity growth for Chilean economy was conducted by Pavcnik (2002) for the time period of 1979-1986. She decomposed the growth into two components. These components represent within firm productivity growth and across firm reallocation of resources. In this way the weighted average of the aggregate productivity can be decomposed into two components, unweighted average productivity and a covariance term respectively, was first discussed by Olley and Pakes (1996). A positive second term implies that a productive firm gains more in market share. Pavcnik found that from 1979-1986 aggregate TFP grew 19%: 6.6% due to within plant productivity improvement and 12.7% due to reallocation of resources from less to more productive firms. Bergoëing & Repetto (2006), henceforth BR,

decomposed the aggregate growth of Chilean manufacturing plants for 1980-2001. They applied a four component decomposition, similar to BHC. They found that 96.7% of the growth took place due to reallocation of resources. Firm entry played an important role as well.

In this paper, I will reopen the case study for Chile for the time period of 1979-1996. Chile is an interesting case for trade economists, since it went through a massive tariff reduction during the early 70s. Their industries were mostly state owned before 70s, but after 1973 they became privatized and had no protection. These effects exposed Chilean firms to face high level of competition as Chile got integrated to the world economy. These features make Chile a good case study to analyze the trade models. However, Chilean economy went through a high inflation rate through out the 70s and up to my period of interest. The average inflation from 1979 to 1996 was 19.45 percent<sup>1</sup>. To address this, I normalize all the prices in 1980 Chilean Pesos. Lastly, the Latin American crisis in 1982 had the largest impact on Chilean economy. This crisis is observed in all the estimates of TFP growth discussed in this paper.

I applied a six component decomposition north by Lewrick, Mohler and Weder (2014) henceforth LMW. The First one comes from the neoclassical trade models and accounts for reallocation of resources to the industry with comparative advantage. This term is called inter industry effect; hence forth IIE. The second term is the technology effect, TE, that represents the external technological shocks that arises from exposure to high competition and/or knowledge spillover<sup>2</sup>. The next two terms, within firm share effect and within firm productivity effect, jointly defines within firm effect from new trade theory; these two effects that defines within firm effect are noted as WFSE and WFPE respectively. WFSE accounts for growth of firm's sale share and WFPE accounts for the productivity growth of these firms. Firm entry effect, FEE, accounts for growth in TFP coming from firms who enter with above average productivity. Firm exit effect, FXE, on the other hand accounts for growth coming from firms who exit with below average productivity.

In the literature, FEE and FXE went through quite substantial developments. FEE for example, is considered to be positive only if the entry firm has higher than some reference level

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<sup>1</sup>The data on CPI is collected from IMF data base from 1978-1997. From this Inflation information is constructed.

<sup>2</sup>These are only two possible explanations for the growth of TE. In theory it is anything else that is not captured by other components of the decomposition.

productivity that has been discussed by GR and FHK. MP and LMW proposed that FEE should have a positive effect on the growth of aggregate TFP only if the entering firm is more productive than the average productivity of the surviving firms in that industry; and FXE should have positive impact only if the exiting firm is less productive than the average productivity of the surviving firms of that industry. In this way the construction of the counterfactual level of productivity is more relevant and informative.

Another aspect of this decomposition involves estimating the productivity. For this paper I considered two kinds of productivity estimates; value added per worker and total factor productivity, TFP. Since these variables are in log form, value added per worker is simply the difference between value added and labor input ( $VA - L$ ). To estimate TFP, on the other hand, one has to estimate the Cobb Douglas production function. I used four methods to estimate this production function; ordinary least square (OLS), Olley and Pakes (OP), Wooldridge (WOP) and lastly Wooldridge with fixed effect (WOPfe)<sup>3</sup>. These methods are discussed in the following paragraph.

Value added per worker, exploits the fact that Value added is the differences between output and materials. Hence, it predicts that productivity is just the difference between value added and labor input. The second one is a simple linear fit to the data. It is evident from the literature that OLS gives us biased estimate, but it serves as a good bench mark model to compare our results with other estimators. In their seminal paper, Olley and Pakes (1996), they exploited the fact that investments and capital stocks of a plant are correlated. Hence, they use a higher order polynomial of capital and investment as a control function and a two stage estimation to correct for the endogeneity problem. Wooldridge (2009), WOP from above estimators, used generalized method of moment to estimate this production function with control function in one stage. Stoyanov, Zubanov and Lee (2015), hence forth SZL (2015), in their working paper found that, WOP estimation still suffers from firm specific shocks that harms the estimation of production function. They added firm specific fixed effects, WOPfe from above, to absorb these shocks in the GMM setting of WOP.

This paper finds that, for Chile it was not firms productivity growth rather it was the general

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<sup>3</sup>Pavcnik (2002) used OP regression and LMW (2014) used a method by Akerberg, Caves and Frazer (2006), Lavinsohn and Petrin (2003) along with OP. I excluded Lavinsohn and Petrin, since pooled regression reports very low returns to scale compare to OP, WOP, WOPfe and even OLS.

technological progress that resulted in the aggregate productivity growth. The decomposition of TFP evolution from 1979-96 using OP estimator, to compare the results with Pavcnik (2002), reports a firms productivity growth that refers to WFPE from LMW of 7.63%. Where as, TE accounts for 84.23% of TFP growth. Firm share effect on the other hand, had a negative impact over the entire time frame. Pavcnik found that the reallocation of resources corresponds to 12.7%, I found that WFSE from OP estimation to align the comparison with her method is -15.59% from 1979 to 1986. For my time frame WFSE accounts for a growth of -20.95%. Hence WFSE and WFPE, jointly within firm effect from new trade theory, is responsible for a decrease of 13.32% of TFP growth. Pavcnik used a two component decomposition similar to OP, one that does not consider the technology effect. This effect captures the growth of industry average productivity over time. This growth is caused by external reasons such as exposure to world competition, knowledge spillover, etc. As her decomposition does not account for this external technological effect, this entire growth is absorbed by within firm productivity growth in her decomposition.

The paper also finds that, firm entry effect is almost negligible. However, data on entering firms are mostly incomplete. Hence, these effects may be biased. Firm exit effect has a negative impact on the TFP growth. This implies that the exiting firms are more productive than the average TFP of surviving firms. Lastly, IIE covaries a lot with the TFP growth. However, it can have both positive and negative impact on TFP evolution depending on what method is used to estimate the TFP.

In the next section I will lay out the theoretical breakdown of the decomposition, followed by the estimation procedure for the production function and TFP. Section 4 will discuss the data set. After that I will go through the decomposition of aggregate growth coming from all five methods to estimate the productivity. Then I will discuss the results from the decomposition and present concluding remarks.

### **3.2 The sources of productivity growth**

International trade literature identifies three different sources of productivity growth: inter-industry effect, intra-industry effect and lastly technology effect. Inter-industry effect is iden-

tified by neoclassical trade theory, for example Ricardian and Heckscher-Ohlin model. This generates reallocation from less productive industries to more productive industries, hence use the resource more efficiently. As a result countries specialize in producing goods in which they have comparative advantage. Intra-industry effect inspired the foundation of new trade theory, that arises from firm level heterogeneity. This effect shifts the production to more productive firms in the same industry. Lastly, the traditional technological growth can arise from research, knowledge spill over or exposure to high competition; and is identified as technology effect. This paper uses the decomposition frame work north by LMW to study the productivity growth for Chilean data from 1979-1996.

All methods start with a definition of aggregate productivity at time  $t$  to be the weighted average of firms' productivity. The weights correspond to firms sale share relative to the total sale by manufacturing industry. Hence, aggregate productivity at time  $t$  can be expressed as:

$$\Phi_t = \sum_{j=1}^J \sum_{i=1}^{N_{jt}} s_{ijt} \varphi_{ijt} \quad (3.1)$$

where,  $N_{jt}$  is the number of active firms in industry  $j$  at time  $t$  and  $J$  is the number of industries.  $s_{ijt}$  is the firm's sale share relative to the total yearly sale. Summing over all the firms in an industry gives the industry sale share ( $S_{jt}$ ) relative to the total sale in a year. Hence  $S_{jt} = \sum_i s_{ijt}$  and summing over all industry adds up to unity,  $\sum_j S_{jt} = 1$ . The expression for TFP in equation (3.1) can be decomposed into two components, similar to OP decomposition, as:

$$\Phi_t = \sum_{j=1}^J S_{jt} \bar{\varphi}_{jt} + \sum_{j=1}^J \sum_{i=1}^{N_{jt}} S_{jt} \tilde{s}_{ijt} \tilde{\varphi}_{ijt} \quad (3.2)$$

where,  $\tilde{s}_{ijt} = \frac{s_{ijt}}{S_{jt}} - \bar{s}_{jt}$  and  $\tilde{\varphi}_{ijt} = \varphi_{ijt} - \bar{\varphi}_{jt}$ <sup>4</sup>.  $\bar{s}_{jt}$  and  $\bar{\varphi}_{jt}$  are the industry average sale share and industry average TFP respectively. The term  $\tilde{s}_{ijt}$  represents the deviation of firm's sale share from the average sale share of the industry it belongs to. Similar interpretation is applicable for the next term  $\tilde{\varphi}_{ijt}$ , here the deviation is between firm's TFP and their industry

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<sup>4</sup>This method of decomposition is similar to the method of OP (1996).

average TFP. Hence, the second term in equation (3.2) is the covariance between firms sale share and TFP. The first term is the industry level average TFP<sup>5</sup>. Consider two consecutive time periods 0 and t, then the TFP evolution over time,  $\Phi_t - \Phi_0$ , is the time difference of equation (3.2). Then the aggregate TFP growth can further decomposed into the following six components<sup>6</sup>.

$$\Phi_t - \Phi_0 = \left\{ \begin{array}{l} \underbrace{\sum_{j \in J} (S_{jt} - S_{j0}) \Delta \Phi_{jt}}_{\text{Inter-industry Effect}} + \underbrace{\sum_{j \in J} S_{j0} (\bar{\varphi}_{jt}^C - \bar{\varphi}_{j0}^C)}_{\text{Technology Effect}} \\ \underbrace{\sum_{j \in J} \sum_{i \in C} S_{j0} \tilde{s}_{ijt} (\tilde{\varphi}_{ijt} - \tilde{\varphi}_{ij0})}_{\text{Within-firm productivity effect}} + \underbrace{\sum_{j \in J} \sum_{i \in C} S_{j0} \tilde{\varphi}_{ij0} (\tilde{s}_{ijt} - \tilde{s}_{ij0})}_{\text{Within-firm share effect}} \\ \underbrace{\sum_{j \in J} S_{j0} s_{jt}^E (\bar{\varphi}_{jt}^E - \bar{\varphi}_{jt}^C) + \sum_{j \in J} \sum_{i \in E} S_{j0} \tilde{s}_{ijt} \tilde{\varphi}_{ijt}}_{\text{Firm entry effect}} \\ \underbrace{\sum_{j \in J} S_{j0} s_{j0}^X (\bar{\varphi}_{j0}^X - \bar{\varphi}_{j0}^C) + \sum_{j \in J} \sum_{i \in E} S_{j0} \tilde{s}_{ij0} \tilde{\varphi}_{ij0}}_{\text{Firm exit effect}} \end{array} \right. \quad (3.3)$$

Note that firms are separated by continuing, entering and exiting status. For each category, I define the industry average as  $\bar{\varphi}_{jt}^\gamma = (\sum_{i \in \gamma} \varphi_{ijt}) / N_{jt}^\gamma$  for  $\gamma = [C, E, X]$ .  $\Delta \Phi_{jt}$  is the deviation of industry TFP from average industry TFP<sup>7</sup>. As discussed previously, inter industry effect captures the reallocation effect across industries with comparative advantage. This effect is expected to be positive whenever a country's production moves to the industry with comparative advantage. As a result, the economy uses it's resources more efficiently and produces output with comparative advantage.

A possible reason for a growth in technology effect can arise from knowledge spill over due

<sup>5</sup>This definition of Aggregate TFP is used by Olley and Pakes, Nina Pavcnik and many other trade economists.

<sup>6</sup>This method is borrowed from LMW (2014) paper. The porve of the decomposition can be found in their appendix.

<sup>7</sup>Industry TFP is defined as,  $\Phi_{jt} = \sum_i (s_{ijt} / S_{jt}) \varphi_{ijt}$ .

to globalization or external competition<sup>8</sup>. Exposure to the world competition, such as Chile, makes the local markets competitive. This effect can drive up the average productivity of the economy. Technology effect accounts for the increase in the average TFP of continuing firms in an industry. Thus, this effect holds the industry share constant at the base year and accounts for the productivity improvement in an industry from continuing firms only. Note that, Pavcnik's decomposition does not isolate TE; as a result the contribution of this technological progress is absorbed in both reallocation and firm's productivity growth. This is why, both productivity growth effect and reallocation effect seems to be relatively larger than LMW method.

Intra-industry effect has three components: a) within firm effect, b) firm entry effect and c) firm exit effect. Within firm effect then breaks down into two parts: within firm productivity effect and within firm share effect. These four effects capture the effect of globalization on aggregate growth described by new trade theory. Most productive firms are expected to self identify as exporters and go through an expansion. This effect is captured by within firm share effect. Whenever a firm with above average productivity goes through an expansion, WFSE will be positive. Within firm productivity effect captures the productivity growth of these firms. Very small evidence is found in support of WFPE. Exiting and entering firms TFPs are compared to the average productivity of the continuing firms in that industry. Hence an entering firm increases the TFP evolution, if it is more productive than the average of continuing firms of that industry. Similarly, an exiting firm increases the TFP growth only if it is less productive than the average of continuing firms in that industry.

### **3.2.1 Estimation of Productivity**

As discussed previously, this paper considers two measures of productivity; value added per worker and TFP. Value added per worker is fairly simple and a good starting point to view the data. It exploits the definition of value added, which is the difference between firm's output and material input. Since these variables are all expressed in natural log, value added per worker simple becomes the difference between value added and labor input. Hence, the expression for value added per worker, hence forth VAL, becomes  $VAL = \text{value added} - \text{labor input}$ . This

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<sup>8</sup>The technology effect can capture the common productivity growth in an industry that can come from any external effect.



method is expected to over estimate productivity, as it ignores the growth of capital that may contribute to the growth of VAL.

Estimation of the total factor productivity, on the other hand, begins with identifying the production function. Suppose firm  $i$  produces output  $Q_{ijt}$  at time  $t$  and serves in the industry  $j$ . A firm has three factor inputs: a) labor ( $L_{ijt}$ ), b) capital ( $K_{ijt}$ ) and c) intermediates ( $M_{ijt}$ ). Suppose they all face a Cobb-Douglas production function,  $Q_{ijt} = \varphi_{ijt} L_{ijt}^{\alpha_l} K_{ijt}^{\alpha_k} M_{ijt}^{\alpha_m}$ ; and the TFP is identified by  $\varphi_{ijt}$ . Taking natural log to this production function leads to the basic estimation equation:  $q_{ijt} = \alpha_0 + \alpha_l l_{ijt} + \alpha_k k_{ijt} + \alpha_m m_{ijt} + \omega_{ijt} + \varepsilon_{ijt}$ , where I assume that  $\varphi_{ijt} = \exp(\alpha_0 + \omega_{ijt} + \varepsilon_{ijt})$ . Note that  $\varepsilon_{ijt}$  is assumed to be iid normal, which is a common assumption.  $\alpha_0$  is the time invariant measure of average productivity across all plants and  $\omega_{ijt}$  is the plant specific productivity level. At aggregate level firm's output may vary due to industry-year specific effect. Hence, for pooled regression I include these fixed effects in the model and it becomes

$$q_{ijt} = \alpha_0 + \alpha_l l_{ijt} + \alpha_k k_{ijt} + \alpha_m m_{ijt} + \omega_{ijt} + \delta_{jt} + \varepsilon_{ijt} \quad (3.4)$$

Where  $\delta_{jt}$  is the industry-year fixed effects. For industry level analysis I added yearly fixed effect instead of industry-year fixed effects. The estimation equation at industry level then becomes,

$$q_{ijt} = \alpha_0 + \alpha_l l_{ijt} + \alpha_k k_{ijt} + \alpha_m m_{ijt} + \omega_{ijt} + \delta_t + \varepsilon_{ijt} \quad (3.5)$$

where,  $\delta_t$  is a vector of yearly fixed effects. I use four different methods to estimate the production function. OLS is the simplest one imaginable that fits a linear line to the data and does not worry about the fact that the model suffers from endogeneity problem. OP uses a control function to control the unobserved variation and a two step estimation method to correct for endogeneity bias. WOP is an extension of OP that uses a one step GMM estimation method to simultaneously estimate the coefficient from the problem addressed by OP. The last estimator, WOPfe, incorporates a firm specific fixed effect to control for any unobserved variation in output. SZL (2015) used the one step GMM approach of WOP with firm level fixed effects. A variety of estimators are used to study the case of Chile, as Chile went through a lot

of restructuring from early 70s till the end of the time frame. As a result, I can compare and contrast the decomposition effect across different methods. At last TFP of the plant can be estimated by the difference between firm's output and estimated output. Hence, the expression is given by,  $TFP_{ijt} = q_{ijt} - \hat{\alpha}_l l_{ijt} - \hat{\alpha}_k k_{ijt} - \hat{\alpha}_m m_{ijt}$ <sup>9</sup>.

### 3.3 Data

The plant level data has been collected from statistical agencies, Instituto Nacional de Estadística. This data has been extensively used in trade literature. The data set is an unbalanced panel of 10,927 plants over the period of 1979 to 1996 and has total 86,168 data points of manufacturing sector only. These plants compete in 22 different industries, those are identified by three digit industry code. The data set identifies these plants with at least 6 or more workers by unique id. These firms are uniquely identified to the industry they belong to and are followed when they change industry as well. Plants changing industry are identified as exiting firms from the industry they used to serve and than entering firms in the new industry where they will serve. The data set consists of plant level factor input information, such as skilled and unskilled labor, investments in machinery, land, real estate, fixed asset, total asset etc. On the other hand it has information on firms sales, inventories, profit/loss, gross output etc. Using the data set, variables such as capital, labor, intermediate inputs and output are produced<sup>10</sup>.

Table 1: Summary statistics of the data set

| Year                                | 1979 | 1982 | 1985 | 1988 | 1991 | 1994 | 1996 |
|-------------------------------------|------|------|------|------|------|------|------|
| Number of Firms                     | 5814 | 4484 | 4333 | 4498 | 4765 | 5082 | 5466 |
| Capital/Employee <sup>1</sup>       | 57   | 75   | 55   | 39   | 35   | 36   | 42   |
| Intermediates/Employee <sup>1</sup> | .85  | 1    | 4    | 8    | 13   | 19   | 25   |
| Investment/Employee <sup>2</sup>    | 51   | 57   | 41   | 45   | 132  | 83   | 120  |
| Output/Employee <sup>3</sup>        | 1    | 2    | 7    | 13   | 22   | 32   | 43   |
| Employee <sup>3</sup>               | 311  | 230  | 266  | 343  | 392  | 425  | 423  |

<sup>9</sup>Note that, the TFP growth in equation (3.3) is an approximation as the measure of TFP is in natural log instead of levels.

<sup>10</sup>The description of variable generation for this paper can be found in data appendix.

Notes: These are sample averages of the selected year. <sup>1</sup>:Capital/Employee Intermediates/Employee are in Chilean Peso thousands. <sup>2</sup> :In Chilean Peso. <sup>3</sup> :In thousands.

The table shows summary statistics for Chilean firms. Over the period of study capital per employee reached the lowest at 1991. It shows a decline after 1982 then again starts to pick up in 1996. Intermediate input per employee shows a steady increase over time and investment per employee fluctuates more. It is important to remember that during this period the average inflation was 19.45%. Hence, the depreciation of capital was very high as well. Output per worker seems to have a steady growth. A lot of the firms lost their business. The total percentage change in the number of firm over the period is approximately -4%.

### 3.4 Results

In this section, I will start by presenting the TFP estimation and then go into the six components decomposition for Chilean aggregate productivity growth.

#### 3.4.1 TFP Estimates

The table below uses a pooled regression that refers to the model described in equation (3.4). This estimate the production function using four different methods: OLS, OP, WOP and lastly WOPfe. The pooled regression includes Industry-year fixed effects in the models. They absorb any industry level or yearly shocks that affected firm’s sale.

Table 2: Production function estimate (pooled regression)

|                  | OLS   | OP    | WOP   | WOPfe |
|------------------|-------|-------|-------|-------|
| Labor            | 0.273 | 0.231 | 0.211 | 0.205 |
| Capital          | 0.072 | 0.094 | 0.081 | 0.039 |
| Intermediates    | 0.744 | 0.731 | 0.767 | 0.752 |
| Returns to scale | 1.089 | 1.056 | 1.059 | 0.996 |

OLS has highest returns to scale relative to other estimators. Among the return on factor inputs, labor and intermediate inputs have similar contribution to production across different estimators. The return on capital varies a lot, relative to other factor returns, across different estimators and has very small impact on production. The highest estimated returns on factor

input across all methods are: labor from OLS of 0.273, capital from OP of 0.094 and lastly intermediates from WOP of 0.767. All of the estimators other than WOPfe, which exhibits decreasing returns to scale, exhibit increasing returns to scale.

Table 3 reports the summary statistics of estimated TFP for four different methods. The table shows that, WOPfe has the highest mean among the all four estimators. OLS and OP, on the other hand, predicts similar mean TFP. WOP reports the smallest possible mean.

Table 3: Summary Statistics of estimated TFP

|                    | OLS   | OP    | WOP   | WOPfe |
|--------------------|-------|-------|-------|-------|
| Mean               | 1.468 | 1.401 | 1.226 | 1.875 |
| Standard Deviation | 0.725 | 0.859 | 0.761 | 0.939 |

It is also quite clear that, WOPfe predicts most variation among them. Interestingly, OLS predicts smallest variation from the mean. OLS and WOP reports vary similar variation.

### 3.4.2 Aggregate Productivity Growth Decomposition

Five different estimators for aggregate productivity growth give a diverse picture of the Chilean economy. The range of productivity estimate varies from a maximum growth of 317.25% from VAL and a minimum of 69.56% using WOPfe. However, this is expected as VAL over estimates the productivity growth since it does not consider the growth of capital at all. The rest of the estimators report: 80.62% using OP, 75.94% using OLS and lastly 70.49% from WOP. These four estimators of TFP report reasonable predictions regarding it's growth. Figure 3.1 reports the productivity level predicted by all five estimators and estimation by OP, OLS and WOPfe and WOP are dominated by VAL. As discussed previously, VAL is expected to over estimate the growth as it does not consider the effect of capital contribution to productivity evolution for the firms. When restricted up to 1986, the growth predicted by OP for six component decomposition is around 20.74%. However Pavcnik (2002) found this growth to be 19.3%; this paper finds a similar growth of 19.58%, when applied the two component decomposition.

Figure 3.2 reports the change in growth rate of productivity over time using all five methods. OP, OLS, WOP and WOPfe shows relatively small changes compared to VAL estimators. The Latin American crisis of 1982 slowed down this growth for all four methods other than VAL, that exhibits a huge fall in the productivity growth a year before. Other than that, all the

productivity reports one more fall in growth rate in 1986.

Figure 3.3.1-3.3.5 decompose the growth of productivity into six components, such that the reported aggregate growth resembles the size of relative growth compare to other years<sup>11</sup>. Hence the length of each bar represents how large the growth was relative to other years and each component represents the percentage contributed to that growth. As a consequence, for each year the total contribution coming from all six components adds up to 100%. The decomposition for all the five methods seems to have some similar patterns. For example, TE has a very strong positive and WFSE has a negative impact on Productivity growth for all five methods. FXE is negative for all of them other than WOP, this implies that on an average the exiting firms were more productive than the average of the industries they belong to. The data records 3991 entry firms, but these firms lack factor input data that makes the estimation impossible. This effects all the estimators, hence FEE plays almost an insignificant role in the growth of Chilean economy. IIE has asymmetric effect on the TFP growth, hence I will discuss it separately along with other effects in details. Lastly, WFPE plays a marginal positive impact on the evolution of TFP.

IIE has negative impact in only WOPfe estimator that predicts a decrease of 1.73%. Hence, this decomposition predicts that resources have been reallocated to less productive industries due to globalization. The most dominant impact of IIE is observed with VAL estimator, that estimates a growth of 12.19%. Among the other reasonable estimators the highest estimated inter industry reallocation comes from WOP estimator to be 10.42%. Given that Chile observe a TFP growth around 70%, the impact of IIE seems to be notable small.

Secondly, WFSE has negative impact form all the estimators. However it is important to discuss within firm effect, that is the total effect from WFSE and WFPE, since this refers to reallocation across firms in an industry as described by trade models with firm heterogeneity. WFPE seems to have a positive impact for most of them, but relatively smaller than WFSE in absolute value<sup>12</sup>. As a result, within firm reallocation effect is negative and implies that resources have shifted to less productive firms over time.

FEE and FXE are also the components north by new trade theory, that account for negative

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<sup>11</sup>Figure 3.3.1-3.3.5 reports the decomposition results from Table 4-8 from Table Appendix.

<sup>12</sup>Growth in firm's productivity was also predicted by Pavnick (2002).

impact on productivity growth for all the estimators other than VAL. FXE seems to have negative impact on the TFP growth, as exiting firms on an average are more productive than the averages of their industry TFP. FEE lacks information on firm entry and as a result it has negligible effect on the growth. Among them FXE dominates the contribution coming from entry and exit. As a result, jointly they decrease the growth of TFP evolution. This implies that both classical and new trade theory plays a minimal role in the evolution of productivity for Chilean economy.

Lastly, the most important component of this growth for Chile came from TE. As discussed previously, this component reports positive growth for all five methods of productivity estimation. Leaving VAL aside, since it predicts an unreasonable positive growth, this component reports a growth of 74.56% from OLS, 83.63% from WOPfe, 72.48% from WOP and lastly 84.23% from OP. It is possible to imagine a situation, where entry firms are relatively more productive than the average productivity of the industries they belong to. These firms in the consecutive period may pushed the TE up as continuing firms of that industry. Hence, the lack of data on entry firm's factor input may lead to over estimate of TE.

The six component decomposition is able to identify the technological progress that is observed as industry growth in this method, not as firms productivity growth in Pavcnik's decomposition, is the answer to understand the divergence of results from her's to this decomposition method. As this growth is absorbed by within firm productivity growth, it seems that better plants got bigger as well by the two component decomposition. To study this, I restricted the six component decomposition up to year 1986 and constructed a two component decomposition, similar to her work, for OP estimation of TFP to compare them. Table 8 from Appendix B reports the decomposition results for two component decomposition. The first component identifies the reallocation effect, that accounts for reallocation across firms, and second component refers to firm's productivity growth.

Pavcnik's decomposition reports a 6.6% increase in firm's productivity, which is similar to WFPE in this paper, and a 12.7% increase in reallocation across firm (that resembles WFSE in this paper). The parallel components from this paper representing Pavcnik's decomposition reports 10% increase in WFPE and 15.59% decrease in WFSE. Again TE dominates in this time frame and reports 32.54% contribution to the TFP evolution. When restricted up to 1986,

all the components north from classical and new trade theory reports a negative impact on growth. For example, IIE decreases 5.36%, within firm effect jointly reports 5.59% decrease and lastly FEE & FXE had a marginal negative impact of 0.08%. Hence this is obvious that, 20.74% increase in TFP growth is solely caused by TE.

### 3.5 Conclusion

The paper studies the aggregate productivity growth decomposition for the case of Chile from 1979-96. It uses two definition of productivity, value added per worker and total factor productivity. Then, four different methods where used to estimate TFP; these are OLS, OP, WOP and lastly WOPfe. Right after the reformation in early 70s, Chile decentralized her industries and plants faced world competition for the first time. This makes Chile a very interesting case study for trade economists.

Previously, Pavcnik studied the Chilean economy and found that within firm productivity improvement was the dominant factor in TFP growth. Hence, new trade theory was able to describe the bigger picture. However this paper finds that, The TFP growth observed in the data is mostly coming from the growth in technological progress. The exposure to high competition forces the aggregate productivity of the economy to rise. This in turn shifts the production away from relatively high productive firms, decreasing the within firm share effect over the period. This implies that, the intra-industry effect described by the new trade theory did not play that big of a role from the restructuring of Chilean economy as described in the previous studies.

Notably, neither IIE nor FEE or FXE contributed to the growth of TFP either. It may be the case that, lack of data on firm entry failed to report the entry of high productive firms. These firms may have contributed to the average productivity growth of industries they belong to.

## Tables

Table 4. Decomposition of aggregate growth by VAL

| year | IIE    | TE    | WFPE   | WFSE   | FEE   | FXE   | Total |
|------|--------|-------|--------|--------|-------|-------|-------|
| 1980 | 0.061  | 0.379 | 0.024  | -0.074 | 0.016 | 0.028 | 0.378 |
| 1981 | -0.043 | 0.219 | -0.017 | -0.128 | 0.016 | 0.008 | 0.038 |
| 1982 | 0.128  | 0.071 | 0.159  | -0.033 | 0.008 | 0.040 | 0.293 |
| 1983 | 0.074  | 0.299 | 0.102  | -0.077 | 0.027 | 0.015 | 0.410 |
| 1984 | -0.039 | 0.313 | 0.082  | -0.152 | 0.023 | 0.014 | 0.213 |
| 1985 | 0.041  | 0.242 | 0.069  | -0.027 | 0.007 | 0.011 | 0.321 |
| 1986 | 0.021  | 0.101 | -0.011 | -0.037 | 0.008 | 0.004 | 0.078 |
| 1987 | -0.046 | 0.198 | 0.057  | -0.070 | 0.044 | 0.005 | 0.177 |
| 1988 | 0.079  | 0.221 | 0.045  | -0.126 | 0.128 | 0.013 | 0.333 |
| 1989 | -0.019 | 0.237 | 0.023  | 0.001  | 0.046 | 0.031 | 0.258 |
| 1990 | -0.067 | 0.237 | 0.020  | -0.004 | 0.009 | 0.008 | 0.187 |
| 1991 | -0.046 | 0.188 | 0.032  | -0.078 | 0.023 | 0.013 | 0.106 |
| 1992 | -0.004 | 0.243 | -0.062 | -0.074 | 0.036 | 0.011 | 0.128 |
| 1993 | -0.025 | 0.159 | 0.028  | -0.055 | 0.015 | 0.023 | 0.099 |
| 1994 | 0.007  | 0.146 | 0.005  | -0.028 | 0.014 | 0.014 | 0.129 |
| 1995 | -0.012 | 0.089 | 0.043  | 0.011  | 0.028 | 0.014 | 0.144 |
| 1996 | 0.011  | 0.096 | 0.033  | -0.096 | 0.000 |       |       |

Table 5. Decomposition of aggregate growth by OLS

| year | IIE    | TE    | WFPE   | WFSE   | FEE   | FXE    | Total  |
|------|--------|-------|--------|--------|-------|--------|--------|
| 1980 | 0.022  | 0.057 | -0.007 | -0.020 | 0.003 | 0.002  | 0.053  |
| 1981 | -0.004 | 0.053 | 0.033  | -0.027 | 0.001 | 0.000  | 0.055  |
| 1982 | -0.016 | 0.025 | 0.065  | -0.016 | 0.000 | 0.003  | 0.055  |
| 1983 | 0.000  | 0.013 | 0.001  | -0.023 | 0.000 | 0.002  | -0.010 |
| 1984 | 0.013  | 0.058 | 0.019  | -0.031 | 0.000 | 0.002  | 0.058  |
| 1985 | -0.002 | 0.043 | -0.006 | -0.005 | 0.000 | 0.001  | 0.030  |
| 1986 | -0.005 | 0.019 | -0.015 | -0.007 | 0.000 | 0.001  | -0.009 |
| 1987 | 0.027  | 0.070 | 0.015  | -0.008 | 0.000 | 0.001  | 0.103  |
| 1988 | 0.035  | 0.027 | 0.010  | 0.007  | 0.000 | -0.001 | 0.079  |
| 1989 | 0.038  | 0.051 | -0.005 | -0.004 | 0.000 | 0.003  | 0.076  |
| 1990 | -0.004 | 0.073 | 0.017  | -0.015 | 0.000 | 0.000  | 0.070  |
| 1991 | -0.031 | 0.058 | -0.013 | -0.010 | 0.000 | 0.001  | 0.003  |
| 1992 | 0.021  | 0.055 | -0.018 | -0.010 | 0.000 | 0.001  | 0.048  |
| 1993 | -0.016 | 0.040 | -0.012 | -0.002 | 0.000 | 0.000  | 0.009  |
| 1994 | 0.000  | 0.034 | -0.012 | -0.006 | 0.000 | 0.000  | 0.016  |
| 1995 | 0.022  | 0.039 | 0.023  | 0.002  | 0.000 | 0.000  | 0.086  |
| 1996 | -0.005 | 0.030 | -0.001 | -0.004 | 0.000 |        |        |



Table 6. Decomposition from OP estimation.

| year | IIE    | TE    | WFPE   | WFSE   | FEE   | FXE   | Total  |
|------|--------|-------|--------|--------|-------|-------|--------|
| 1980 | 0.014  | 0.073 | -0.009 | -0.036 | 0.003 | 0.002 | 0.043  |
| 1981 | 0.003  | 0.060 | 0.030  | -0.024 | 0.001 | 0.001 | 0.070  |
| 1982 | -0.069 | 0.011 | 0.050  | -0.024 | 0.001 | 0.004 | -0.035 |
| 1983 | 0.000  | 0.026 | 0.017  | -0.018 | 0.000 | 0.003 | 0.023  |
| 1984 | 0.026  | 0.063 | 0.009  | -0.032 | 0.000 | 0.002 | 0.064  |
| 1985 | -0.009 | 0.061 | 0.001  | -0.009 | 0.000 | 0.002 | 0.041  |
| 1986 | -0.018 | 0.031 | 0.002  | -0.013 | 0.000 | 0.001 | 0.002  |
| 1987 | 0.042  | 0.052 | 0.030  | -0.008 | 0.000 | 0.002 | 0.115  |
| 1988 | 0.025  | 0.046 | -0.003 | 0.007  | 0.000 | 0.002 | 0.073  |
| 1989 | 0.031  | 0.059 | -0.020 | -0.010 | 0.000 | 0.003 | 0.057  |
| 1990 | 0.013  | 0.070 | 0.001  | -0.013 | 0.000 | 0.000 | 0.072  |
| 1991 | 0.004  | 0.068 | 0.008  | -0.008 | 0.000 | 0.001 | 0.071  |
| 1992 | 0.018  | 0.068 | -0.028 | -0.012 | 0.000 | 0.000 | 0.046  |
| 1993 | 0.005  | 0.050 | -0.008 | 0.000  | 0.000 | 0.001 | 0.047  |
| 1994 | -0.011 | 0.035 | -0.010 | -0.006 | 0.000 | 0.000 | 0.008  |
| 1995 | 0.021  | 0.035 | 0.017  | 0.000  | 0.000 | 0.000 | 0.074  |
| 1996 | -0.005 | 0.033 | -0.011 | -0.003 | 0.000 |       |        |

Table 7. Decomposition of aggregate growth using WOP estimator

| year | IIE    | TE    | WFPE   | WFSE   | FEE   | FXE    | Total  |
|------|--------|-------|--------|--------|-------|--------|--------|
| 1980 | -0.024 | 0.052 | -0.008 | -0.023 | 0.003 | -0.002 | -0.002 |
| 1981 | 0.073  | 0.054 | 0.018  | -0.019 | 0.001 | 0.000  | 0.127  |
| 1982 | -0.148 | 0.019 | 0.061  | -0.008 | 0.000 | -0.002 | -0.078 |
| 1983 | -0.051 | 0.000 | 0.011  | -0.020 | 0.000 | -0.002 | -0.064 |
| 1984 | 0.003  | 0.064 | 0.014  | -0.032 | 0.000 | -0.002 | 0.046  |
| 1985 | 0.018  | 0.052 | -0.016 | -0.006 | 0.000 | -0.001 | 0.047  |
| 1986 | 0.003  | 0.019 | -0.010 | -0.010 | 0.000 | 0.000  | 0.003  |
| 1987 | -0.005 | 0.078 | 0.006  | -0.009 | 0.000 | -0.001 | 0.069  |
| 1988 | -0.043 | 0.038 | -0.001 | -0.004 | 0.000 | 0.002  | -0.008 |
| 1989 | 0.000  | 0.042 | 0.001  | -0.008 | 0.000 | -0.004 | 0.031  |
| 1990 | 0.046  | 0.067 | -0.004 | -0.011 | 0.000 | 0.000  | 0.098  |
| 1991 | 0.087  | 0.052 | -0.001 | -0.009 | 0.000 | 0.000  | 0.129  |
| 1992 | 0.043  | 0.056 | -0.014 | -0.010 | 0.000 | 0.003  | 0.077  |
| 1993 | 0.120  | 0.042 | -0.027 | -0.005 | 0.000 | 0.001  | 0.131  |
| 1994 | -0.045 | 0.023 | -0.001 | -0.006 | 0.000 | 0.000  | -0.028 |
| 1995 | -0.052 | 0.047 | 0.016  | 0.003  | 0.000 | 0.001  | 0.015  |
| 1996 | 0.078  | 0.021 | -0.003 | -0.005 | 0.000 |        |        |

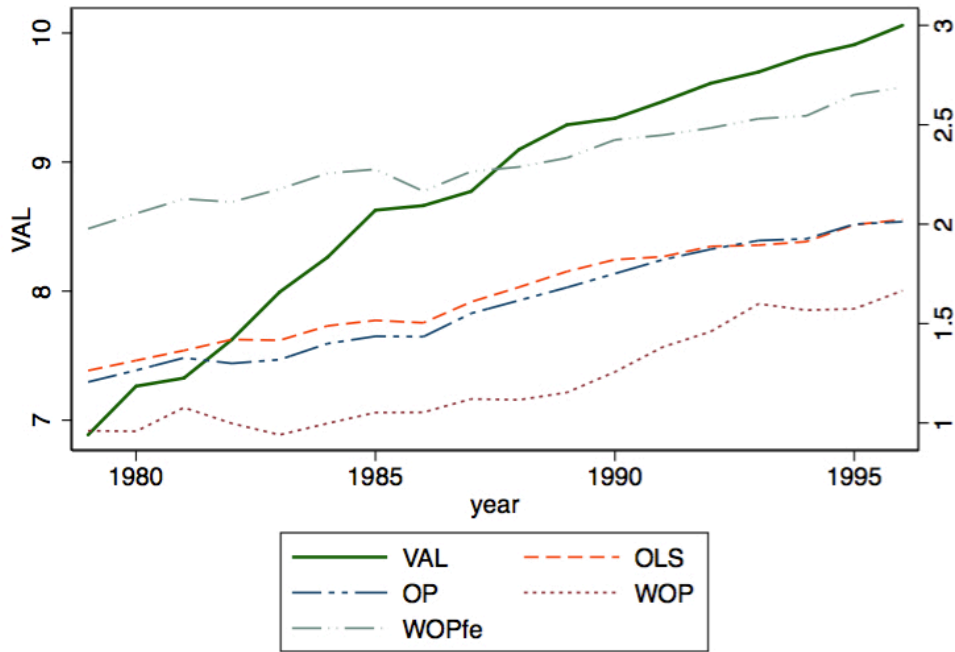
Table 8. Decomposition of aggregate growth by WOPfe

| year | IIE    | TE    | WFPE   | WFSE   | FEE   | FXE   | Total  |
|------|--------|-------|--------|--------|-------|-------|--------|
| 1980 | 0.014  | 0.068 | 0.015  | -0.037 | 0.006 | 0.003 | 0.062  |
| 1981 | 0.027  | 0.063 | 0.014  | -0.022 | 0.002 | 0.001 | 0.084  |
| 1982 | -0.057 | 0.005 | 0.040  | -0.009 | 0.000 | 0.011 | -0.032 |
| 1983 | 0.033  | 0.026 | 0.014  | -0.009 | 0.000 | 0.006 | 0.058  |
| 1984 | 0.027  | 0.063 | 0.006  | -0.026 | 0.000 | 0.004 | 0.066  |
| 1985 | -0.013 | 0.058 | -0.001 | -0.006 | 0.000 | 0.002 | 0.035  |
| 1986 | -0.084 | 0.021 | -0.001 | 0.007  | 0.000 | 0.000 | -0.057 |
| 1987 | 0.017  | 0.060 | 0.032  | -0.004 | 0.000 | 0.002 | 0.103  |
| 1988 | 0.002  | 0.048 | -0.020 | 0.006  | 0.000 | 0.007 | 0.030  |
| 1989 | 0.022  | 0.061 | -0.011 | -0.012 | 0.000 | 0.006 | 0.054  |
| 1990 | 0.000  | 0.067 | 0.014  | -0.006 | 0.000 | 0.000 | 0.074  |
| 1991 | 0.004  | 0.067 | 0.012  | -0.009 | 0.000 | 0.004 | 0.070  |
| 1992 | 0.017  | 0.072 | -0.028 | -0.008 | 0.000 | 0.000 | 0.053  |
| 1993 | -0.001 | 0.048 | 0.001  | 0.004  | 0.000 | 0.003 | 0.049  |
| 1994 | -0.046 | 0.036 | -0.015 | 0.003  | 0.000 | 0.001 | -0.023 |
| 1995 | 0.052  | 0.040 | 0.018  | 0.008  | 0.000 | 0.003 | 0.114  |
| 1996 | -0.032 | 0.034 | -0.004 | 0.001  | 0.000 |       |        |

Table 9. Two components decomposition of OP estimation 1979-89

| year | RE_op  | PGE_op | agg_growth |
|------|--------|--------|------------|
| 1980 | -0.027 | 0.048  | 0.021      |
| 1981 | -0.025 | 0.088  | 0.063      |
| 1982 | -0.106 | 0.075  | -0.031     |
| 1983 | -0.016 | 0.042  | 0.026      |
| 1984 | -0.002 | 0.073  | 0.071      |
| 1985 | -0.022 | 0.068  | 0.045      |
| 1986 | -0.022 | 0.024  | 0.001      |
| 1987 | 0.033  | 0.090  | 0.123      |
| 1988 | 0.050  | 0.040  | 0.091      |
| 1989 | 0.010  | 0.049  | 0.059      |

Figure 3.1: Aggregate growth of Chile 1979-96



NOTE: VAL is plotted on the left Y axis and the rest are (OLS, OP, WOP, WOPfe) on the right Y axis.

Figure 3.2: Aggregate growth rate 1979-96.

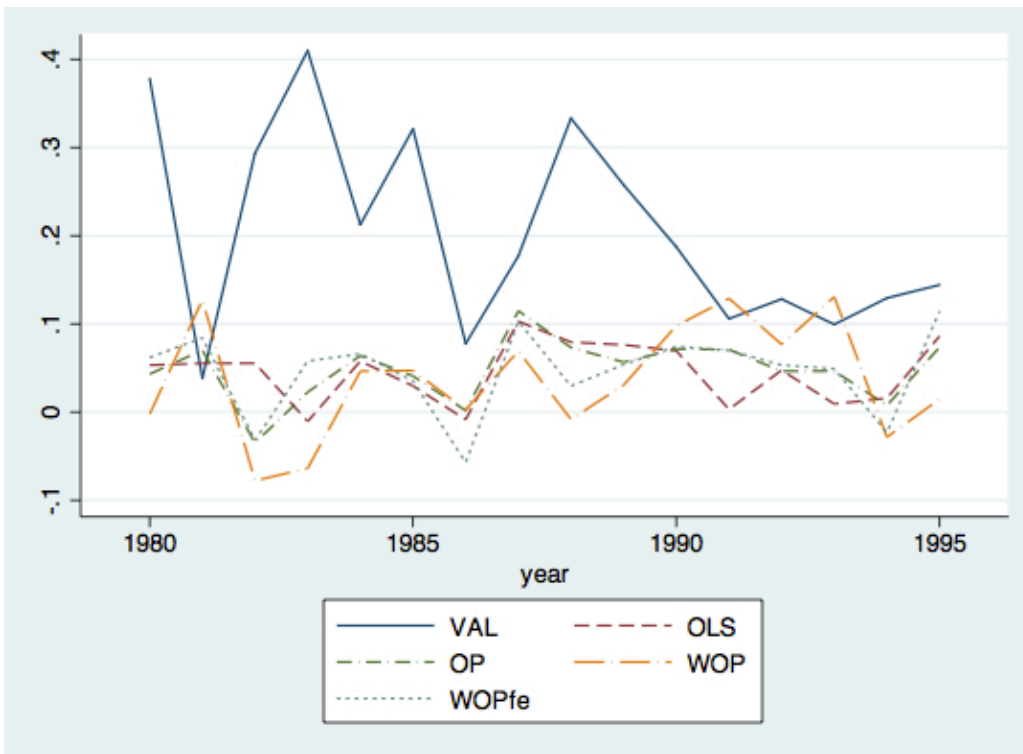


Figure 3.3.1: Six components of growth from VAL.

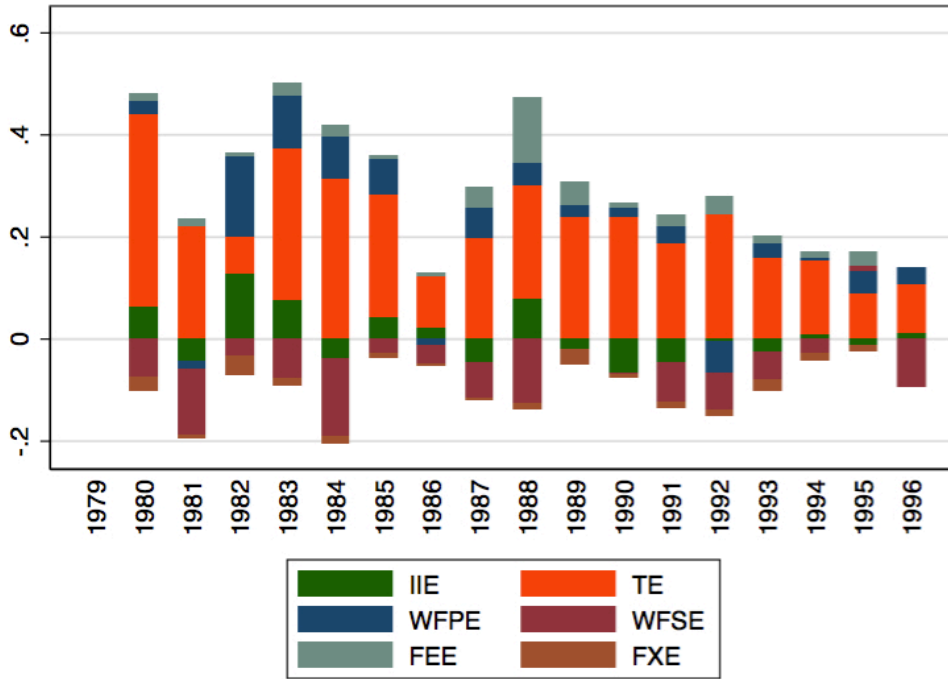


Figure 3.3.2: Six components of growth from OLS.

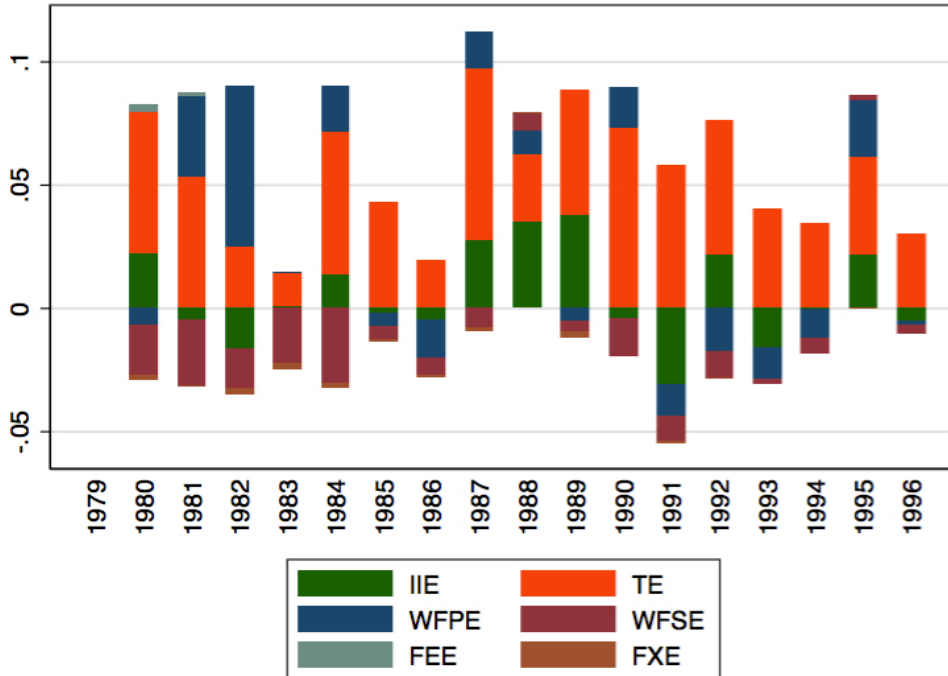


Figure 3.3.3: Six components of growth from OP.

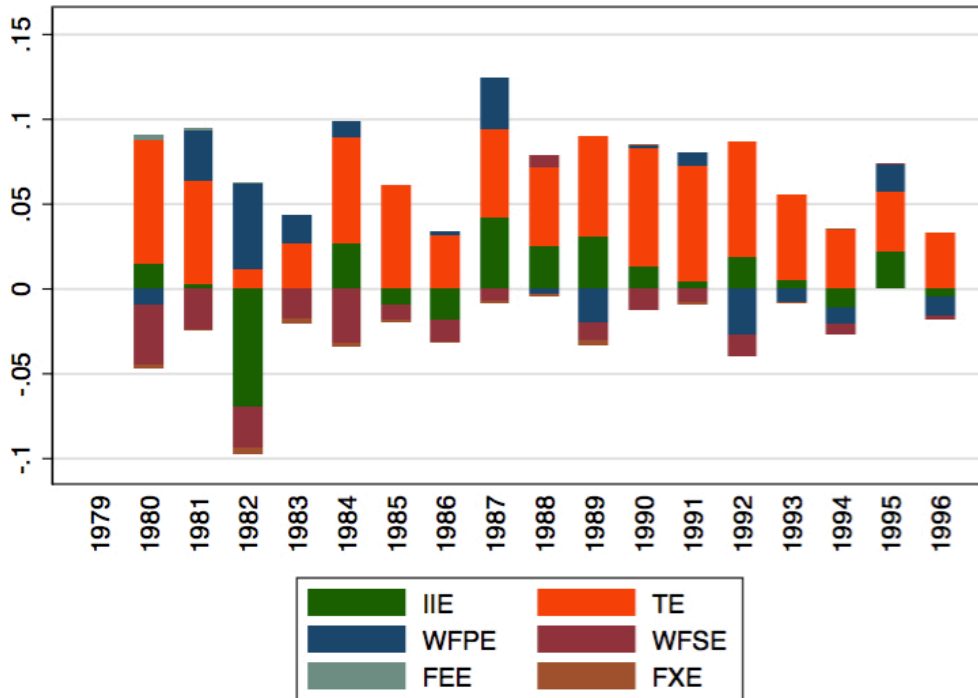


Figure 3.3.4: Six components of growth from WOP.

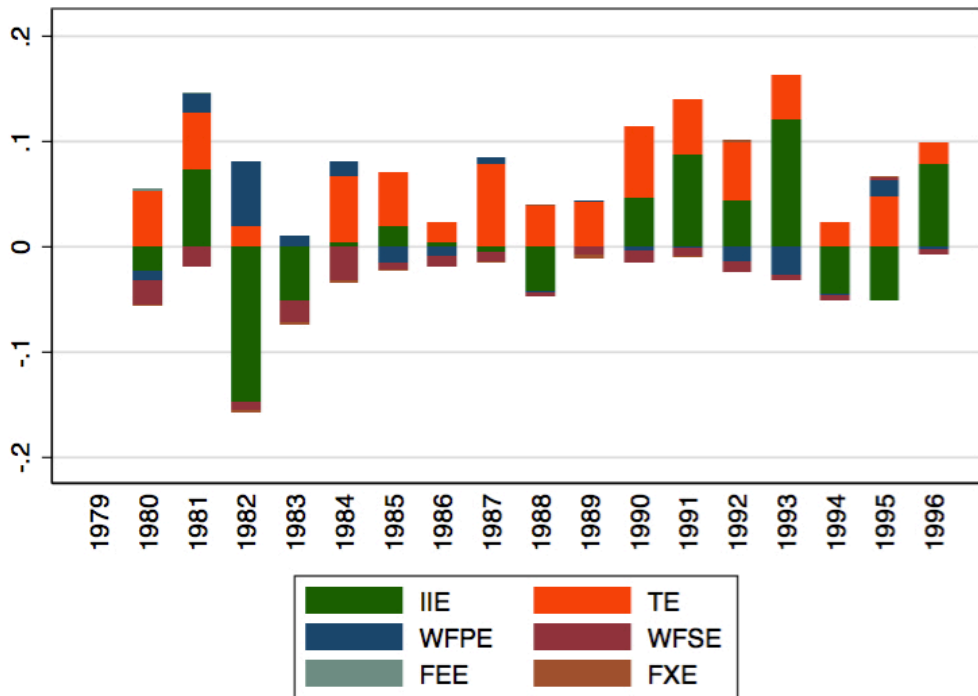
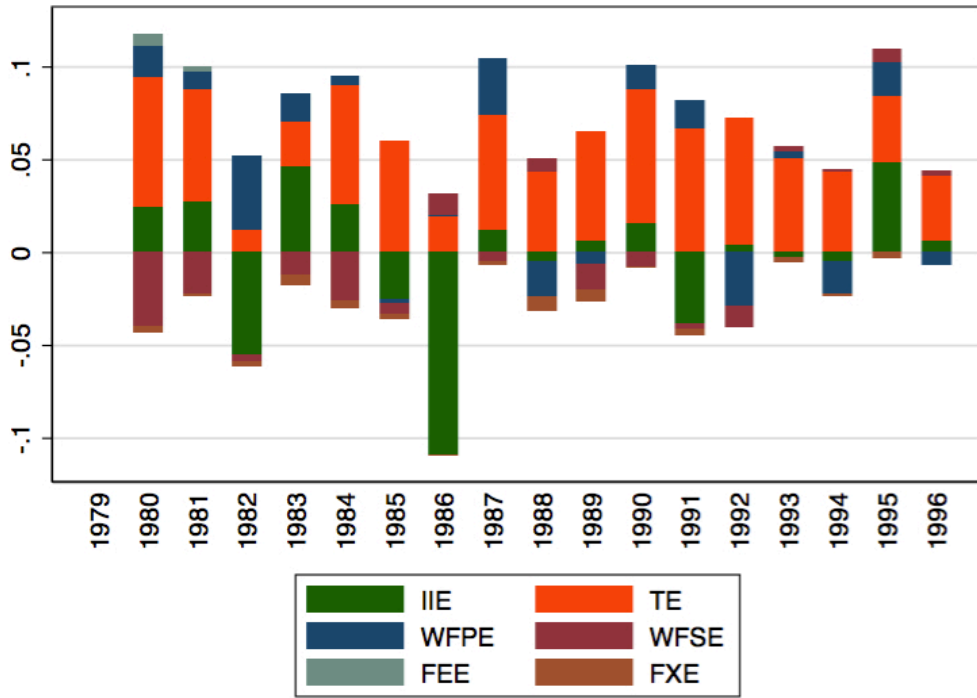


Figure 3.3.5: Six components of growth from WOPfe.



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## Appendix A: Chapter 1

### A.1 Derivation of mass of firms.

Use the expression for labor supply of firms given it observes  $\phi$ . Hence the labor market clearing condition becomes  $L = M \int_{\phi^*}^{\infty} \frac{q(\phi)}{\phi} \frac{g(\phi)}{1-G(\phi^*)} d\phi$ . Now use the optimal pricing rule by monopolist to express  $\phi^{-1} = p(\phi) \frac{\sigma-1}{\sigma} \phi^\theta$ ; and use this to obtain the left hand side of the expression to be:  $M \int_{\phi^*}^{\infty} r(\phi) \frac{\sigma-1}{\sigma} \phi^\theta \frac{g(\phi)}{1-G(\phi^*)} d\phi$ . By using equation (1.9b), hence by taking the ratio of revenues for firms productivity to domestic cutoff productivity implies  $r(\phi) = \left(\frac{\phi}{\phi^*}\right)^\varepsilon \sigma f$ . Lastly by rearranging the terms, the expression for mass of firms becomes:

$$M_a = \frac{L}{(\sigma-1)f} \left[ \left(\frac{1}{\phi^*}\right)^\varepsilon \int_{\phi^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi^*)} d\phi \right]^{-1}$$

### A.2 Derivation of employment distribution.

Take employment ratio of any firm with the average firm to find,(1.9e);

$$\frac{l(\phi)}{l(\tilde{\phi})} = \left(\frac{\phi}{\tilde{\phi}}\right)^\varepsilon$$

Now take the numerator of employment distribution and use the above relation in it:  $M \int_{\forall \phi \geq \phi^*} l(\phi)g(\phi)d\phi = Ml(\tilde{\phi}) = L$ . Note that  $\tilde{\phi}^\varepsilon = \int_{\forall \phi \geq \phi^*} \phi^\varepsilon g(\phi)d\phi$ , hence I get  $\frac{M \int_{\phi^*}^{\infty} g(\phi)l(\phi)d\phi}{L} = 1$  and this proves that it is the density of employment. By using Pareto assumption and this I can find that.

$$e(\phi) = \frac{Ml(\tilde{\phi})}{L} \frac{1}{\tilde{\phi}^\varepsilon} \frac{\alpha \phi^{*\alpha}}{\phi^{\alpha+1-\varepsilon}} \quad \forall \phi \in (\phi^*, \infty)$$

where  $\alpha$  is the shape parameter of the Pareto distribution. Hence we get  $e(\phi) = \frac{1}{\tilde{\phi}^\varepsilon} \frac{\alpha \phi^{*\alpha}}{\phi^{\alpha+1-\varepsilon}}$   $\forall \phi \in (\phi^*, \infty)$ . Note that the distribution of employment does not depend on the employment level or endowment of labor. However it is solely defined by the density of productivity.

A Pareto distribution for the productivity density  $g(\phi) = \frac{\alpha \phi^\alpha}{\phi^{\alpha+1}}$  for  $\alpha > \theta + \varepsilon$  to find the distribution function to be:

$$e(\phi) = \frac{1}{\phi^\varepsilon} \frac{\alpha \phi^{*\alpha}}{\phi^{\alpha+1-\varepsilon}} \quad \forall \in (\phi^*, \infty) \quad (\text{A.1})$$

### A.3 Derivation of weighted wage distribution.

Take the wage equation and invert it to find

$$\begin{aligned} w^{\frac{1}{\theta}} &= \phi \\ d\phi &= \frac{1}{\theta} w^{\frac{1}{\theta}-1} dw \end{aligned}$$

Now apply the Jacobean approach of random variable transformation technique  $y(w) = \frac{1}{\phi^\varepsilon} \alpha \phi^{*\alpha} w^{\frac{\varepsilon-\alpha-1}{\theta}} \frac{1}{\theta} w^{\frac{1}{\theta}-1}$  to find the PDF of wage to be  $y(w) = \frac{1}{\phi^\varepsilon} \frac{\alpha}{\theta} \phi^{*\alpha} w^{\frac{\varepsilon-\alpha}{\theta}-1}$  for all  $w \in (\phi^{*\theta}, \infty)$ .

### A.4 Derivation of Lorenz curve.

First use the PDF of wage to convert it to CDF and find the expected wage from it. Once CDF is attained, isolate wage (w) to the left had side and find the expression for wage as  $w=f(Y)$ . Now we can apply the standard statistical method to find the Lorenz curve.

From the definition of Lorenz curve we get :

$$L(Y) = \frac{1}{E(w)} \frac{1}{\phi^\varepsilon} \frac{\alpha}{\theta} \phi^{*\alpha} \int_{\phi^{*\theta}}^{w(Y)} x^{\frac{\varepsilon-\alpha}{\theta}} dx$$

This will simplify to the Lorenz curve at autarky. For trade and FDI equilibrium, apply same method to corresponding segments of wage distribution.

### A.5 Derivation of mass of firms at trade economy.

For trade economy exporters require additional workers to serve export market and is given by:  $l_x(\phi) = \frac{q_x(\phi)}{\phi} = \frac{r_x(\phi)}{\tau} \frac{\sigma-1}{\sigma} \phi^{-\theta}$ ; by using the optimal pricing rule of monopolist in export market second equality can be obtained. Again by equation (1.9b), the revenue can be expressed as  $r_x(\phi) = \left(\frac{\phi}{\phi_x^*}\right)^\varepsilon \sigma f_x$ . Use this to determine the labor demanded for exporting activity,  $\frac{MP_x(\sigma-1)}{\tau} \frac{f_x}{\phi_x^{*\varepsilon}} \int_{\phi_x^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_x^*)} d\phi_i$ .

Labor demand for domestic market production only, on the other hand, remains the same as autarky economy. Hence labor market clearing condition becomes:

$$L = M(\sigma - 1) \left[ \frac{f}{\phi^{*\varepsilon}} \int_{\phi^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi^*)} d\phi + \frac{P_x}{\tau} \frac{f_x}{\phi_x^{*\varepsilon}} \int_{\phi_x^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi^*)} d\phi \right]$$

By rearranging this condition the expression for mass of firm will be given by equation (1.21).

### A.5 Derivation of mass of firms at FDI economy.

Investing firms require relatively more workers when they obtain FDI status. As a result, the additional labor demand of a firm with FDI status is given by:  $l_I(\phi) - l_x(\phi) = \frac{q_I(\phi) - q_x(\phi)}{\phi}$ . Note that,  $q_x(\phi) = \tau^{-\sigma} q(\phi)$  and  $q_I(\phi) = q(\phi)$ . Hence,  $q_I(\phi) - q_x(\phi) = (1 - \tau^{-\sigma})q_I(\phi)$ . By using optimal pricing rule for monopolist in the investment market simplifies the expression to be:  $l_I(\phi) - l_x(\phi) = (1 - \tau^{-\sigma})r_I(\phi) \frac{\sigma-1}{\sigma} \phi^{-\theta}$ . Lastly, use equation (1.9b) to find the expression for revenue,  $r_I(\phi) = \left(\frac{\phi}{\phi_I^*}\right)^\varepsilon \sigma(f_I - f_x)$ , to use in the labor demand function by investing firms. Hence the total labor demand by all the investing firms in an economy is given by:  $MP_I(1 - \tau^{-\sigma})(\sigma - 1) \frac{f_I - f_x}{\phi_I^{*\varepsilon}} \int_{\phi_I^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_I^*)} d\phi$ .

Labor demand by exporting firms for exporting activity only is given by:

$\frac{M(P_x - P_I)(\sigma - 1)}{\tau} \frac{f_x}{\phi_x^{*\varepsilon}} \int_{\phi_x^*}^{\phi_I^*} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_x^*)} d\phi$ . Note that, a mass of exporters converted to investors now, as a result the mass decreases. Labor demand by domestic producers only still remains the same as autarky economy. Lastly, the labor market clearing condition implies:

$$L = M(\sigma - 1) \left[ \frac{f}{\phi^{*\varepsilon}} \int_{\phi^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi^*)} d\phi + \frac{P_x - P_I}{\tau} \frac{f_x}{\phi_x^{*\varepsilon}} \int_{\phi_x^*}^{\phi_I^*} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_x^*)} d\phi \right. \\ \left. + P_I(1 - \tau^{-\sigma}) \frac{f_I - f_x}{\phi_I^{*\varepsilon}} \int_{\phi_I^*}^{\infty} \phi^{\varepsilon-\theta} \frac{g(\phi)}{1-G(\phi_I^*)} d\phi \right]$$

Rearranging this leads to equation(1.31).

## Appendix B: Chapter 2

### B.1 Relation between export and domestic cutoff across asymmetric countries

Take the ratio of revenue at domestic cutoff across countries.

$$\frac{r_i(\phi_i^*)}{r_j(\phi_j^*)} = \left[ \frac{\phi_i^*}{\phi_j^*} \right]^\varepsilon = \left[ \frac{P_j}{P_i} \right]^{\sigma-1} \frac{R_j}{R_i} \quad (3.6)$$

note that  $r_i(\phi_i^*) = \sigma f$  for  $i$  and  $j$ . Now lets take the ratio of export to domestic cutoff in a country.

$$\frac{r_{xi}(\phi_{xi}^*)}{r_i(\phi_i^*)} = \left[ \frac{\phi_{ix}^*}{\phi_i^*} \right]^\varepsilon = \left[ \frac{P_i}{P_j} \right]^{\sigma-1} \frac{R_i}{R_j} \tau^{\sigma-1} \frac{f_x}{f} \quad (3.7)$$

and lastly lets look at the ratio of export cutoff across country.

$$\frac{r_{xi}(\phi_{xi}^*)}{r_{xj}(\phi_{xj}^*)} = \left[ \frac{\phi_{xi}^*}{\phi_{xj}^*} \right]^\varepsilon = \left[ \frac{P_i}{P_j} \right]^{\sigma-1} \frac{R_i}{R_j} \quad (3.8)$$

The equations imply the following relation between the export and domestic cutoff across country.

$$\phi_{xi} = \phi_j A \quad \forall i, j \quad (3.9)$$

Where  $A = \left[ \tau^{\sigma-1} \frac{f_x}{f} \right]^{\frac{1}{\varepsilon}} > 1$ .

## B.2 Derivation of existence and uniqueness of equilibrium

I will find the slopes of equation (2.23) lines to compare them.

$$\begin{aligned} B'(\phi_j) &= -\frac{f_x}{f} \frac{1}{A^{\alpha_i}} \left( \frac{\phi_j}{\phi_i} \right)^{-\alpha_i-1} < 0 \\ D'(\phi_j) &= -\frac{f}{f_x} A^{\alpha_j} \left( \frac{\phi_j}{\phi_i} \right)^{-\alpha_j-1} < 0 \end{aligned}$$

Note that they have negative slope, hence now I can compare their absolute values ( $|B'(\phi_j)| \leq |D'(\phi_j)|$ ). This can be simplified to  $\left( \frac{f_x}{f} \right)^2 A^{-(\alpha_i+\alpha_j)} \leq \left( \frac{\phi_i}{\phi_j} \right)^{\alpha_j-\alpha_i}$ .

Now LHS can be simplified to  $\left( \frac{f_x}{f} \right)^{\frac{2\varepsilon-(\alpha_i+\alpha_j)}{\varepsilon}} \tau^{(1-\sigma)\frac{\alpha_i+\alpha_j}{\varepsilon}}$ . Note that  $\frac{f_x}{f} > 1$  and  $2\varepsilon - (\alpha_i + \alpha_j) < 0$ . This imply that  $\left( \frac{f_x}{f} \right)^{\frac{2\varepsilon-(\alpha_i+\alpha_j)}{\varepsilon}} \in (0, 1)$ . On the other hand  $\tau > 1$  and  $(1 - \sigma) < 0$ . This imply that  $\tau^{(1-\sigma)\frac{\alpha_i+\alpha_j}{\varepsilon}} \in (0, 1)$ . So we have  $LHS < 1$ .

At any intersection of two curves, RHS imply that  $\left( \frac{\phi_i^*}{\phi_j^*} \right) > 1$ , since I have country "i" hazard rate dominates the productivity distribution of country "j". But this imply that  $\alpha_j - \alpha_i > 0$ . Hence RHS at the intersection will be grater than 1. So at any intersection the  $B(\phi_j)$  will be falter than  $D(\phi_j)$ . Since they both have negative slope we have a unique solution as well.

## Appendix C: Chapter 3

### C Data

The following data description is useful for this paper. All variables are expressed in log.

**Output** is calculated from difference of gross output to building machinery and vehicles produced for own use. from the table the expression for output in log levels is,  $y = \ln(\text{groutput} - \text{prodbld} - \text{prodmach} - \text{prodveh})$ .

**Intermediates** is constructed from four variables from the table. These are total intermediate purchase, electricity bought, final inventory of raw material and lastly initial inventory of raw material. The expression for the variable is given by,  $m = \ln(\text{totipurc} + \text{elecval} - (\text{finvrm} - \text{iinvrm}))$ .

**Labor** input has only two components, skilled and unskilled workers. Their log of sum will yield the labor input. The expression for this input is  $n = \ln(\text{sklab} + \text{unsklab})$ .

**Capital** is log of  $\text{tnk80\_new}$ , that is the real capital stock in PPP adjusted thousands of 1980 Chilean Pesos. The expression for this is self explanatory,  $k = \ln(\text{tnk80\_new})$ .

**Investment** is the summation of three variables and they are real gross capital investment in building, real gross capital investment in machinery and real gross capital investment in vehicles. The construction looks like,  $i = \ln(\text{rinvcapb} + \text{rinvcapm} + \text{rinvcapv})$ .

**Value added** is the log of variable  $\text{valadded}$  and the expression is  $va = \ln(\text{valadded})$ .