GAMES Course 1 Final Exam Solutions

Multiple-Choice

1. (b)
2. (b)
3. (a)
4. (c)
5. (d)
6. (b)
7. (c)
8. (c)
9. (a)
10. (d)

Short Answer Questions

1. (a) 27
   (b) 32.53
   (c) 471.57
2. (a) pN in 1 day, $(pN)^t$ in t days
   (b) $10pN$ in 1 day, $10(pN)^t$ in t days
   (c) $10(2)^5 = 320$
3. (a) 3240
   (b) $2(1 - \frac{1}{2^5}) = 1.94$
4. (a) $2(1.2)^4$, $2(1.2)^{t-1}$
   (b) $2 + 2(1.2) + 2(1.2)^2 + ... + 2(1.2)^9 = \sum_{t=1}^{10} 2(1.2)^{t-1}$
   (c) $2[\frac{(1.2)^{10}-1}{0.2}] = 51.9$ lbs
5. (a) $y_1 = 1 - \frac{x}{6}$
   (b) $y_1 = \frac{1}{9} - \frac{10x}{27}$
6. (a) $P = \dfrac{a - \alpha - 500\tau}{600}$

(b) $P' = -5/6$. An increase in the tax rate would decrease the equilibrium price. Consumers still pay more, $P+\tau$, producers earn Q*P revenue which is lower, and government earns τQ tax revenue.

7. (a) Critical points occur when $x = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

(b)

(a) $f'(x) = \frac{16(2x^2+1) - 4x(16x)}{(2x^2+1)^2} = \frac{16(2x^2+1-4x^2)}{(2x^2+1)^2}$

factor

$\frac{16(2x^2+1-4x^2)}{(2x^2+1)^2} = 0 = \frac{16(-2x^2+1)}{(2x^2+1)^2}$

diff. of squares

$0 = -16(2x^2-1)/(2x^2+1)^2$

$0 = -16(\sqrt{2}x-1)(\sqrt{2}x+1)/(2x^2+1)^2$

critical points: $\sqrt{2}x - 1 = 0$ $\quad$ $\sqrt{2}x + 1 = 0$

(a) $x = 1/\sqrt{2}$ critical points $x = -1/\sqrt{2}$

(b)

	$-\infty$ to $-1/\sqrt{2}$	$-1/\sqrt{2}$ to $1/\sqrt{2}$	$1/\sqrt{2}$ to ∞
-16	−	−	−
$(2x^2+1)^2$	+	+	+
$\sqrt{2}x-1$	−	−	+
$(\sqrt{2})x+1$	−	+	+
$f'(x)$	−	+	−

min (at $-1/\sqrt{2}$), max (at $1/\sqrt{2}$)

(c) Find the second derivative:

$$\begin{aligned}
f''(x) &= \frac{16(-4x)(2x^2+1)^2 - 16(-2x^2+1)(2)(2x^2+1)(4x)}{(2x^2+1)^4} \\
&= \frac{-64x(2x^2+1)^2 - 128x(-2x^2+1)(2x^2+1)}{(2x^2+1)^4} \\
&= -64x(2x^2+1)\frac{(2x^2+1)+2(-2x^2+1)}{(2x^2+1)^4} \\
&= -64x(2x^2+1)\frac{-2x^2+3}{(2x^2+1)^4} \\
&= 64x(2x^2+1)\frac{2x^2-3}{(2x^2+1)^4}
\end{aligned}$$

Observe that the following factors are always positive: $2x^2+1$, $(2x^2+1)^4$ and $2x^2-3$ is always negative, so the sign of $f''(x)$ depends on the factor $64x$.
If$x = 1/2$, then $f''(1/2) < 0$ and $f(1/2)$ is a maximum.
If $x = -1/2$, then $f''(1/2) > 0$ and $f(-1/2)$ is a minimum.

8. (a) $f(60) = 40$ indicates that, for a patient with a weight of 60 kg, the dosage is 40 mg.
   (b) $f0(60) = 6$ indicates that, for a patient with a weight of 60 kg, the dosage should increase by 6 mg for every additional kg above 60 kg.
   (c) For a 65 kg person, the dosage should be $f(60) + f'(60)(65 - 60) = 40 + 6(5) = 70$ mg