GAMES Practice Problem Solutions – Optimization

1. (a) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (b) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (c) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (d) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (e) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"

2. $h = 12.5$

3. (a) $\pi = \left(-792.6 + 4n - \frac{n^{3/2}}{20}\right)(1500) - 200n$ take the derivative and solve for $n = 2657.6$
   (b) The optimal quantity of potash would not change because only the constant changed.

4. (a) $\pi = P(Q)Q - zQ$, and taking the implicit derivative we have: $P'(Q)Q + P(Q) - z = 0$
   (b) $P'(Q^*)Q^* + P(Q^*) - z = 0$ becomes $Q' = \dfrac{1}{P''(Q^*)Q^* + 2P'(Q^*)}$

5. (a) $Q^* = \dfrac{\alpha - Q}{2}$
   (b) $\dfrac{\mathrm{d}Q}{\mathrm{d}z} = -1/2$
   (c) $\pi(Q) = (\alpha - Q + s)Q - zQ$ take the derivative w.r.t. Q and solve for Q: $Q^* = \dfrac{\alpha - z + s}{2} = \alpha - z$, solve for s, $s = a - z$

6. (a) Critical points occur when $x = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$
   (b)

(a) $f'(x) = \dfrac{16(2x^2+1) - 4x(16x)}{(2x^2+1)^2} = \dfrac{16(2x^2+1-4x^2)}{(2x^2+1)^2}$

$\dfrac{16(2x^2+1-4x^2)}{(2x^2+1)^2} = 0 = \dfrac{16(-2x^2+1)}{(2x^2+1)^2}$ (factor)

diff. of squares

$0 = -16(2x^2-1)/(2x^2+1)^2$

$0 = -16(\sqrt{2}x-1)(\sqrt{2}x+1)/(2x^2+1)^2$

critical points: $\sqrt{2}x-1=0$ $\quad \sqrt{2}x+1=0$

(a) $x = \frac{1}{\sqrt{2}}$ critical points $x = -1/\sqrt{2}$

(b)

	$-\infty$ to $-1/\sqrt{2}$	$-1/\sqrt{2}$ to $1/\sqrt{2}$	$1/\sqrt{2}$ to ∞
-16	−	−	−
$(2x^2+1)^2$	+	+	+
$\sqrt{2}x-1$	−	−	+
$(\sqrt{2})x+1$	−	+	+
$f'(x)$	−	+	−

min (at $-1/\sqrt{2}$), max (at $1/\sqrt{2}$)

(c)

(d)

7. (a) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (b) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (c) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (d) Q=100. See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (e) Between 19 and 20 units and again between 147 and 148 units of output, Q. See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (f) More output is needed to reach the first breakeven point, profit is maximized at an output of 97 units and profit is less. See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"

8. (a) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (b) See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (c) $Q \approx 72, Q \approx 178$. See "GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel"
   (d) $Q \approx 129$ Profit is \$4,603.82 when $Q \approx 130$. See "GAMES-Optimization-PracticeQuestionSolutions.x for the solution in Excel"

(e) Between 40 and 41 units of output and again when Q=200. See “GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel”

(f) More output is needed to reach the first breakeven point, profit is maximized at an output of $Q \approx 140$ units and profit is only \$2,347.61. See “GAMES-Optimization-PracticeQuestionSolutions.xlsx for the solution in Excel”