A More Mathematical Definition of a Function

- The **domain** of a equation is the set of all x's that we can plug into the equation and get back a real number for y. The **range** of an equation is the set of all y's that we can ever get out of the equation.

- A **function** is a **relation or mapping** for which each value from the **domain** is associated with **exactly one value** from the range.

- According to the definition, is g a function?

g is not a function

$$\begin{Bmatrix} 0 \\ 4 \\ 9 \end{Bmatrix} \rightarrow g \rightarrow \begin{Bmatrix} 0 \\ -2 \text{ or } 2 \\ -3 \text{ or } 3 \end{Bmatrix}$$

$g(0) = 0$

$g(4) = +/- 2$ *

$g(9) = +/- 3$

Discrete Functions

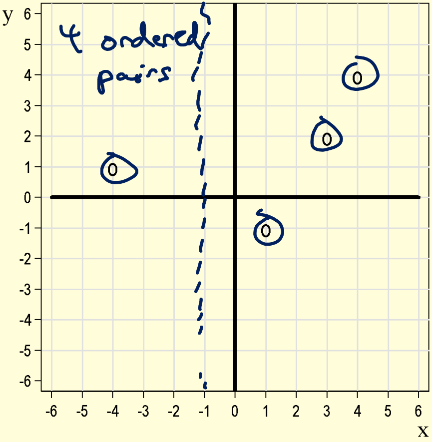
What is a discrete function?

- The domain is countable
- many inputs or x-values have no outputs or y-values

ex. $f(-1) = ??$

Scatterplot type graph

A graph of the discrete function, k

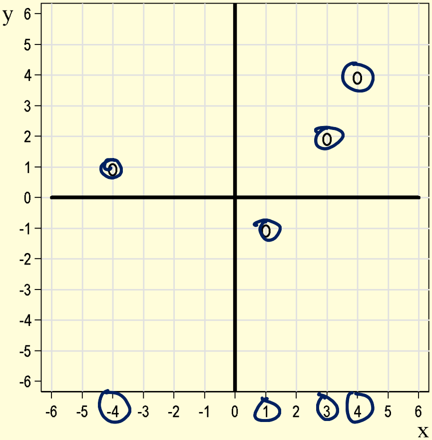


Domain of a Discrete Function

Based on the graph, what is the domain of this discrete function, *k*?

Domain is $\{-4, 1, 3, 4\}$

A graph of the discrete function, *k*

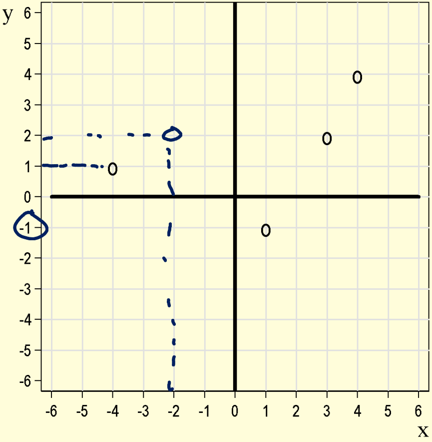


Range of a Discrete Function

Based on the graph, what is the range of this discrete function, k?

Range $\underbrace{\{1, -1, 2, 4\}}$

A graph of the discrete function, k



Application with a Discrete Function

Calculate: $\underbrace{4k(1)} + \underbrace{k(-4)}$

$4k(1) = 4(-1)$

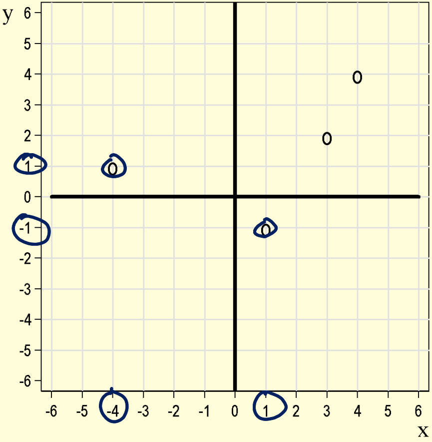
$k(-4) = 1$

$4k(1) + k(-4)$

$= 4(-1) + 1$

$= -3$

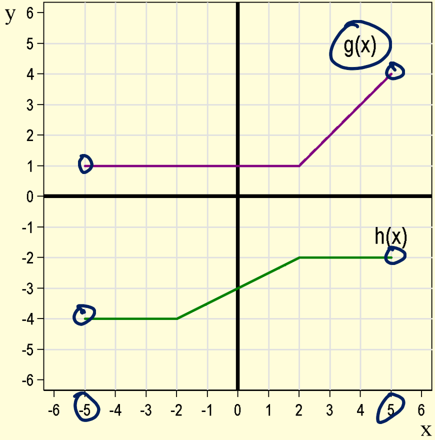
A graph of the discrete function, k



Domain of a Continuous Function

Based on the graph, what are the domains of $g(x)$ and $h(x)$, respectively?

Not countable



g(x)

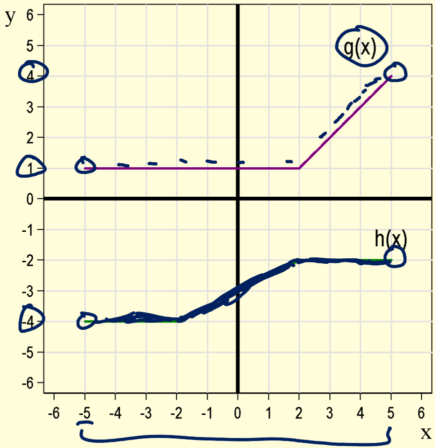
The domain includes any x-value between $-5, 5 \rightarrow [-5, 5]$

$h(x) \rightarrow$ the domain is also $[-5, 5]$

4.00001

Range of a Continuous Function

Based on the graph, what are the ranges of $g(x)$ and $h(x)$, respectively?



g(x)

Range includes any y-value between 1 and 4 → [1, 4]

h(x)

Range is [−4, −2]

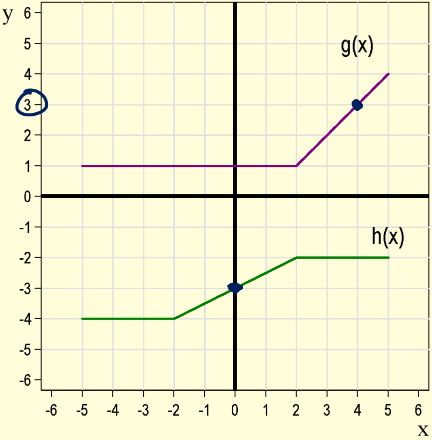
Application of a Continuous Function

Calculate: $2h(0) - g(4)$

$2(-3) - 3$

$= -6 - 3$

$= -9$



Application of a Continuous Function

Let $f(x) = g(x) + h(x)$

Draw $f(x)$ on the diagram

The Vertical Line Test

Domain Range

$$\begin{Bmatrix} 0 \\ 4 \\ 9 \end{Bmatrix} \to g \to \begin{Bmatrix} 0 \\ -2 \text{ or } 2 \\ -3 \text{ or } 3 \end{Bmatrix}.$$

Function

For each point on the x-axis, none of these points have two associated y-values,

A graph of the Discrete Function g

The Vertical Line Test

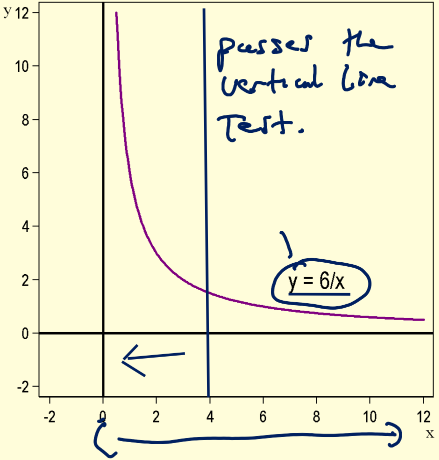
Remember: a function has one y-value for each x-value in the Domain.

$$y = f(1) = \pm 2$$

f not a function!

The vertical line Test: no vertical line should cross the line/curve of a function more than once,

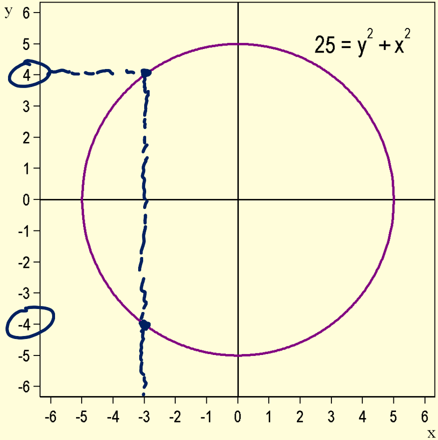
Is this expression a function?



The Vertical Line Test

not a function
⇒ It fails the vertical line test.

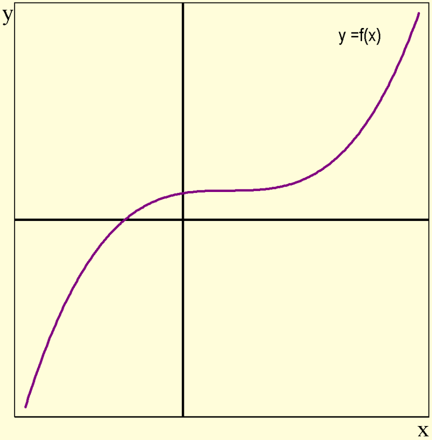
Is this a function?



The Vertical Line Test

Since no vertical line crosses the curve more than once, f(x) is a function.

Is this expression a function?



Find the domain of:

$$f(x) = \frac{1}{x+5}$$

$$\left(f(x) = \sqrt{x-3} \right)$$

x can be very large

$x = 10^{30}$

$f(10^{30}) = \frac{1}{10^{30}+5}$

$x = -5$

$f(-5) = \frac{1}{-5+5}$

$= \frac{1}{0}$

= unknown

= undefined

$f(10) = \sqrt{10-3}$

$= \sqrt{7}$

$f(3) = \sqrt{3-3}$

$= \sqrt{0}$ ✓

$= 0$

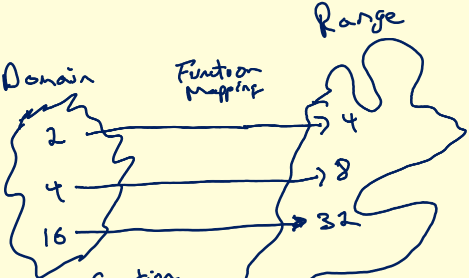
$f(2) = \sqrt{2-3}$

$= \sqrt{-1}$

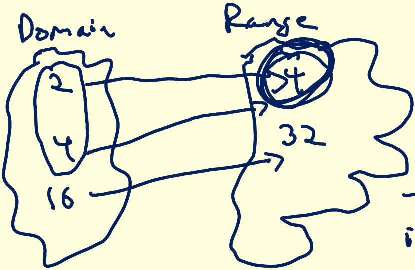
no y-value for $f(2)$

One-to-One Functions

A function is one-to-one if each element of the range corresponds to exactly one element of the domain.



This function is one-to-one



-A function that is not one-to-one.

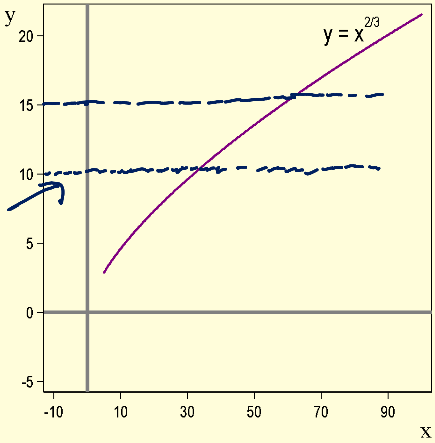
One-to-One Functions

A function is one-to-one if each element of the range corresponds to exactly one element of the domain.

yes

Is this a one-to-one function?

horizontal line

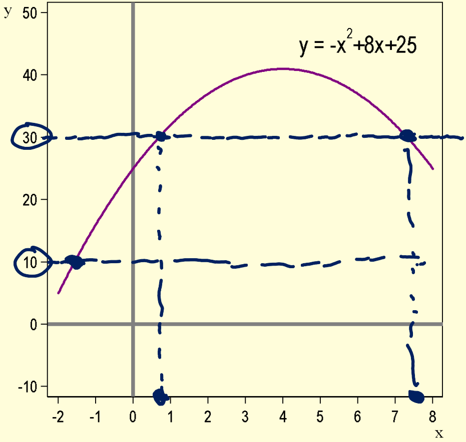


The H-Line Test

no, this is not one-to-one

A function is one-to-one if each element of the range corresponds to exactly one element of the domain.

Is this a one-to-one function?



Inverse and One-to-One Functions

An one-to-one function, $f(x)$, has an inverse function. This inverse function, $f^{-1}(x)$, reverses the mapping!

X values Become Y-values

Y values Become X-values

Example: Consider the following one-to-one discrete function:

$g = \{(-5, 3), (-2, 5), (1, 4), (2, -4)\}$

Find g^{-1}, the inverse of g.

$g(x) \rightarrow$ its inverse is: $g^{-1}(x)$

$g^{-1}(x) = \{(3, -5), (5, -2), (4, 1), (-4, 2)\}$

Functions: Consider the following one-to-one functions:

$h(x) = 3x + 6$

Find the inverse function, h^{-1}

Step 1: is solve for the exogenous variable
(solve for x)

$h(x) = 3x + 6$

$h(x) - 6 = 3x$

$\frac{h(x) - 6}{3} = x$

Step 2: "switch" y and x variables
$h(x)$ and x,

$\frac{x - 6}{3} = h(x) \Rightarrow h^{-1}(x) = \frac{x-6}{3}$

$h(x) = 3x + 6$

$\rightarrow h^{-1}(x) = \frac{x-6}{3}$

$\rightarrow h(x) = 3x + 6$

Special property

$h(h^{-1}(x)) = x$

$h^{-1}(h(x)) = x$

$h^{-1}(\underline{x}) = \frac{x-6}{3}$

$h^{-1}(h(x)) = \frac{(3x+6)-6}{3}$

$= \frac{3x \cancel{+6} \cancel{-6}}{3}$

$= 3x/3 = x$

$h(x) = 3x + 6$

$h(h^{-1}(x)) = 3\left(\frac{x-6}{3}\right) + 6$

$= \frac{3x-18}{3} + 6$

$= \frac{\cancel{3}(x-6)}{\cancel{3}} + 6$

$= x - 6 + 6$

$= x$

Inverse Functions

A function and its inverse have a special property. If $f(x)$ is a function and $f^{-1}(x)$ is its inverse, then:

$f(f^{-1}(x)) = x$

Example: an Inverse Demand Function

supply and demand

The number of sushi meals eaten by customers in a month (Q) depends on its price (P): $Q = 4000 - 2P(Q)$

implicit function

Find the inverse demand function, $P(Q)$ and show that $P(P^{-1}(Q)) = Q$. Illustrates the functions using a graph in Excel.

$$Q = 4000 - 2P(Q)$$

$$Q + 2P(Q) = 4{,}000$$

$$2P(Q) = 4{,}000 - Q$$

$$P(Q) = \frac{4000 - Q}{2}$$

$$P(Q) = \underline{2000 - Q/2}$$

$$P^{-1}(Q) = 4000 - 2Q$$

$$P(P^{-1}(Q)) = 2000 - \left(\frac{4000 - 2Q}{2}\right)$$

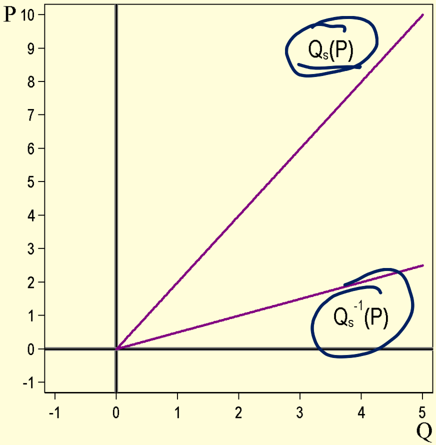
$$P(P^{-1}(Q)) = 2000 - \frac{4000}{2} + \frac{2Q}{2}$$

$$= 2000 - 2000 + Q$$

Inverse Functions

The number of sushi meals eaten by customers in a month (Q) depends on its price (P):
$Q = 4000 - 2P(Q)$

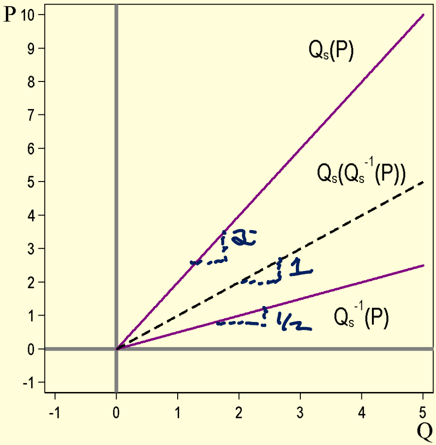
Find the inverse demand function, $P(Q)$ and show that $P(P^{-1}(Q)) = Q$. Illustrates the functions using a graph in Excel.



Inverse Functions

The number of sushi meals eaten by customers in a month (Q) depends on its price (P):
$Q = 4000 - 2P(Q)$

Find the inverse demand function, $P(Q)$ and show that $P(P^{-1}(Q)) = Q$. Illustrates the functions using a graph in Excel.



Example: an Inverse Demand Function

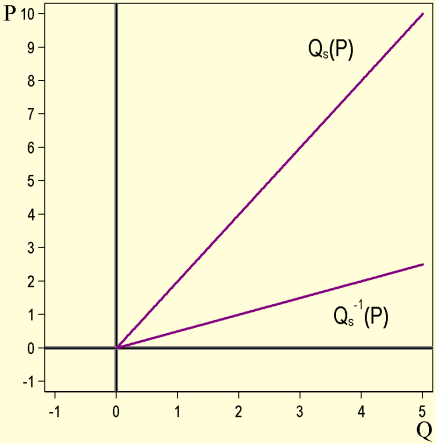
The number of sushi meals eaten by customers in a month (Q) depends on its price (P): $Q = 4000 - 2P(Q)$

Find the inverse demand function, $P(Q)$ and show that $P(P^{-1}(Q)) = Q$. Illustrates the functions using a graph in Excel.

Inverse Functions

The number of sushi meals eaten by customers in a month (Q) depends on its price (P):
$Q = 4000 - 2P(Q)$

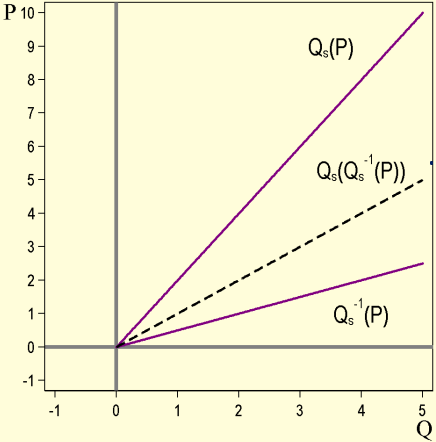
Find the inverse demand function, $P(Q)$ and show that $P(P^{-1}(Q)) = Q$. Illustrate your results using a graph in Excel.



Inverse Functions

The number of sushi meals eaten by customers in a month (Q) depends on its price (P):
$Q = 4000 - 2P(Q)$

Find the inverse demand function, $P(Q)$ and show that $P(P^{-1}(Q)) = Q$. Illustrates the functions using a graph in Excel.



Graphing Solutions to a System of Two Linear Equations

Consider the following supply and demand functions:

$$\begin{aligned} Q_d &= 4000 - 20P \\ Q_s &= 1000 + 30P \end{aligned}$$

Qd = Q = 4000

Using Excel, graph the system for the price (P) and quantity when $Q_d = Q_s$ (this is known as equilibrium in economics). Have price on the y-axis and quantity on the x-axis.

$$Qd = Qs$$

$$4000 - 20P = 1000 + 30P$$

$$4000 - \cancel{20P} + \cancel{20P} - 1000 = 1000 - 1000 + 30P + 20P$$

$$3000 = 50P$$

$$3000/50 = P$$

$$60 = P$$

when P = 60, the Quantity supplied is equal to quantity demanded

$$Q_d = 4000 - 20P$$

$$= 4000 - 20(60)$$

$$= 2{,}800$$

$$Q_s = 1000 + 30(P)$$

$$= 1000 + 30(60)$$

$$= 2{,}800$$

$$\underline{Q_d} = 4000 - 20P$$

$$\underbrace{Q_d - Q_d} + 20p = 4000 - Q_d$$

$$20p = 4000 - Q_d$$

$$P_d = \frac{4000 - Q_d}{20}$$

inverse demand function

$$\underline{Q_s} = 1000 + 30(P)$$

$$Q_s - 1000 = 30p$$

$$30p = Q_s - 1000$$

$$P_s = \frac{Q_s - 1000}{30}$$

inverse supply function

Graphing Solutions to a System of Two Linear Equations

Consider the following revenue function, $R(Q)$, and cost function, $C(Q)$ where Q is the quantity of output.

$$\begin{aligned} R(Q) &= 2Q \\ C(Q) &= 0.01Q^2 \end{aligned}$$

Using Excel, graph the revenue and cost function. Looking at your graph, what quantity of output, Q, maximizes profit?