A Prescriptive Criterion for Distinguishing Analytic from Synthetic Judgments

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[Presented to the Ottawa Congress on Kant in the Anglo-American and Continental Traditions, University of Ottawa, 13 October 1974. The address and the text of the talk, published in 1976 as pages 391-400 of the *Proceedings of the Ottawa Congress on Kant in the Anglo American and Continental Traditions* held October 10-14, 1974 (Ottawa, Canada: the University of Ottawa Press, 1976), derived from an essay submitted in December, 1964, to fulfil the requirements of a course taught by Professor Morton White of Harvard University on American philosophy, and with successive revisions to the same purpose in May, 1965, to Professor Israel Scheffler of Harvard University for a course on the philosophy of science and yet again in May, 1968, to Professor George D. W. Berry of Boston University for course of directed studies. The revision reproduced here, dated November 1974, is a reformatted version of a penultimate draft of 20 February 1973.]

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10 November 1974

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Kant initiated his 'Copernican Revolution' by suggesting that various of our judgments about experience are necessarily true because they are prescriptively, and not alone descriptively, true of it. (The judgment 'All bodies are extended', for example, is necessarily true of our experience because space, as one of the pure forms of sensible intuition, prescribes that nothing could be experienced as a body except it be spatial, i.e., except it be extended.)

When Kant came to distinguish analytic from synthetic judgments, however, he unfortunately tried to describe the distinctive psychology of the respective judgmental acts rather than to prescribe how to make such judgments.[1,2] This was a mistake; and within recent years the defects peculiar to the descriptive criterion have drawn forceful attacks against the distinction itself, and hence against the entire Kantian enterprise.[3]

My task in this essay will be to specify a prescriptive (i.e., constructive) criterion for distinguishing analytic from synthetic judgments (in the Kantian sense of those predicates) which, while remaining rigorous by contemporary logical standards, will render irrelevant the arguments which have proven fatal to the descriptive criteria previously proposed.

II

I begin by asking: How does one teach (i.e., prescribe) the meaning

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* I wish to thank Morton White, Israel Sheffler, George D.W. Berry, Sidney Axinn, and Richard Aquila for offering criticisms and comments on various earlier drafts of this essay.

of an unfamiliar word or phrase to a pupil who wishes to learn it? Consider an instructor who wishes to impart unambiguously to a pupil an understanding of the meaning of a word or phrase found within a natural language. The normal teaching procedure is for the instructor to confront the pupil with a series of audio-visual events (containing whenever appropriate expressions by the instructor of the word or phrase whose meaning is to be taught) created by the instructor in such a manner as to insure as determinably as seems necessary a working understanding of the word or phrase by the pupil.[4]

In designing this learning situation, let us suppose that the conscientious instructor prepares a listing of imperative sentences prescribing the steps he must follow to establish the audio-visual events with which to confront the pupil. Such a listing of imperative sentences, prescribing a procedure adequate for the teaching of the meaning of a word or phrase S_1 to a pupil P_1, I shall call a

'language game for the teaching of the meaning of S_1 to P_1'
(abbreviated: $G_n(S_1,P_1)$)

From the above I derive the following general definition:

(1) $G_n(S_n,P_n) \stackrel{def}{=}$ a list of ordered imperative sentences, containing at least one instance of S_n to be expressed, which prescribes a procedure adequate for the teaching of the meaning of S_n to P_n

Suppose, for example, that the pupil P_1 does not know the meaning of the word 'dandelion' (S_1), but does know the meaning of the words 'rose', 'daffodil', 'orchid', 'yellow', 'four', 'always', 'flower(s)', and the simple terms and constructions of ordinary English. The instructor

might select $G_1(S_1,P_1)$ as a suitable language game, specified as follows:

(1.1) $G_1(S_1,P_1) \overset{spec}{=}$
(1) Place a rose, a daffodil, an orchid, and a dandelion before the pupil.
(2) Point to each in turn and repeat "rose", "daffodil", "orchid", and "dandelion".
(3) Point to the rose, daffodil, orchid, and dandelion in random order, and observe whether the correct response is elicited from the pupil.
(4) If not, repeat (2) until (3) is satisfied.

But the instructor could as easily select any of a number of language games, possibly including $G_2(S_1,P_1)$ specified as follows:

(1.2) $G_2(S_1,P_1) \overset{spec}{=}$
(1) Place a daffodil and a dandelion before the pupil.
(2) Say to the pupil, "A dandelion is a yellow flower".
(3) Say to the pupil, "The dandelion is that yellow flower which is not the daffodil".

Or $G_3(S_1,P_1)$ specified as follows:

(1.3) $G_3(S_1,P_1) \overset{spec}{=}$
(1) Place a rose, a daffodil, an orchid, and a dandelion before the pupil.
(2) Say to the pupil, "A dandelion is a flower; and, of the four flowers, only one is a dandelion".
(3) Say to the pupil, "A dandelion is yellow, and is that flower which is not the rose, the orchid, or the daffodil". [5]

A writer who is preparing a textbook is often asked by his editor to imagine a reader-in-general possessing certain minimal qualifications of intelligence and literacy, and to design his textbook accordingly. Similarly, an instructor may construct a language game which he takes to be adequate for teaching the meaning of a word or phrase S_a to all pupils possessing certain minimal qualifications of intelligence and literacy

(i.e., adequate for 'teaching the meaning of S_a to a P_n-in-general', which I shall abbreviate as 'teaching S_a in general').

A generalized definition of the phrase

'language game for the teaching of S_n in general'
(abbreviated: $G_n(S_n)$)

may be given as follows:

(2) $G_n(S_n) \stackrel{def}{=}$ a list of ordered imperative sentences, containing at least one instance of S_n to be expressed, which prescribes a procedure adequate for the teaching of the meaning of S_n in general

Suppose, for example, an instructor wishes to design a language game $G_1(S_1)$ adequate in general for the teaching of the meaning of the word 'dandelion' (S_1). He might accept either (1.1), (1.2), or (1.3) as being adequate to the new task. Or he might accept only (1.1), not wishing to assume a knowledge of the meaning of the words 'flower(s)'. 'yellow', 'rose', 'orchid', 'daffodil', 'four', or 'always' on the part of pupils. Or he might wish to expand (1.1) in some fashion, not wishing to assume any knowledge of the English language at all on the part of pupils. Or he might construct a wholly new game of his own design, specifying the content in accordance with whatever level of intelligence and literacy he takes to be characteristic of pupils.

I shall now specify a language game $G_1(S_2)$ which I, as an instructor, consider to be adequate at the present time for teaching the meaning of the phrase

'For any G_a, G_b, S_c, and S_d, $G_b(S_d)$ is a twin of $G_a(S_c)$'
(abbreviated: S_2)

to anyone who has read this paper this far:

(2.1) $G_1(S_2) \overset{spec}{=}$ (1) Say to the pupil, "$G_b(S_d)$ is a twin of $G_a(S_c)$ if and only if the list of imperative sentences constituting the former is identical to that constituting the latter, except for the substitution of S_d for S_c in all and only those sentences where the latter has an instance of S_c to be expressed.

From (2.1) the following definitions can be derived:

(2.2) For any G_a, G_b, S_c, and S_d, $G_b(S_d)$ is a twin of $G_a(S_c)$ $\overset{def}{=}$ the list of imperative sentences constituting $G_b(S_d)$ is identical to that constituting $G_a(S_c)$ except for the substitution of S_d for S_c in all and only those sentences where the latter has an instance of S_c to be expressed

(3) For any G_a, G_b, S_c, and S_d, $G_b(S_d)$ and $G_a(S_c)$ are twins $\overset{def}{=}$ $G_b(S_d)$ is a twin of $G_a(S_c)$, and $G_a^b(S_c^d)$ is a twin of $G_b^a(S_d^c)$

If, for example, S_c were 'unmarried man' and S_d were 'bachelor', then games (4.1) and (4.2) below would be twins by (3).

Having defined the 'twinning' of two language games[8], it is a relatively simple step to a specification of a criterion for the application of the predicate 'are synonymous' to two arbitrary S_n:

(4) For any S_a and S_b, S_a is synonymous with S_b $\overset{spec}{=}$ there exist two language games, $G_e(S_a)$ and $G_f(S_b)$, which are twins by (3)

Specification (4) says that S_a and S_b are synonymous if and only if there exists a language game $G_e(S_a)$ for teaching S_a, and another language game $G_f(S_b)$ for teaching S_b, such that the two games are twins by definition (3). Suppose, for example, I wish to decide whether or not 'bachelor' and 'unmarried man' are synonymous. According to (4), if I can specify at least one language game for 'bachelor' and at least one language game for 'unmarried man' which are twins by (3), then 'bachelor' and 'unmarried

man' are synonymous. Assuming that 'married' and 'man' are unambiguous words, I shall now specify two language games, one for the phrase 'unmarried man' (S_3) and the other for the word 'bachelor' (S_4), as follows:

(4.1) $G_1(S_3)$ $\overset{spec}{=}$ (1) Say to the pupil, "A man who is not married is called an unmarried man".

(4.2) $G_1(S_4)$ $\overset{spec}{=}$ (1) Say to the pupil, "A man who is not married is called a bachelor".

Assuming the adequacy of (4.1) and (4.2), and neglecting therein the change from 'an' to 'a' (such grammatical quirks could be accounted for in an expanded formulation of (2.2) and (3)), $G_1(S_3)$ and $G_1(S_4)$ are twins by (3). Hence, the two conditions of criterion (4) are fulfilled, and 'bachelor' and 'unmarried man' are synonymous.

Before specifying a criterion for distinguishing analytic from synthetic judgments, I must first define a Kantian-Fregean relation[9] which I shall refer to as 'inplication'. (In giving the definition, I presuppose, firstly, that a word or phrase is an ordered set of words; secondly, that a subset of an ordered set is itself ordered; and thirdly, that any set is a subset of itself - e.g., that the phrase 'unmarried man' is itself a subset of the phrase 'unmarried man'.)

(4.3) For any S_a and S_b, S_a is a guardian of S_b $\overset{def}{=}$ Every language game for S_b contains either:

(a) a subset of S_a; or
(b) some word or phrase which is synonymous with a subset of S_a by (4)

(5) For any S_a and S_b, S_a inplies S_b $\overset{def}{=}$ Every class which contains both S_b and all guardians of members of the class also contains [redacted] S_a

Definition (5) is nowhere near as unconstructive as it may appear at first glance. It specifies that one is correct in asserting that 'S_a inplies S_b' if and only if every language game for S_b must contain at least one word or phrase which can be connected to a subset of S_a through a reversed chain of guardian relations (i.e., such that a subset of S_a is a guardian of some S_m, which in turn is a guardian of some S_n, which in turn ... , which in turn is a guardian of the word or phrase contained in the game for S_b); and conversely, that one is correct in asserting that 'S_a does not inply S_b' if and only if there exists at least one language game for S_b which contains no word or phrase which can be connected to a subset of S_a through a reversed chain of guardian relations (i.e., such that, for every word or phrase in the game for S_b, it is <u>not</u> the case that a subset of S_a is a guardian of some S_m, which in turn is a guardian of some S_n, which in turn ... , which in turn is a guardian of the word or phrase in the game for S_b).

Examples such as 'gold watch' and 'gold', or 'little girl' and 'girl', are clearly such that the former(S_a) inplies the latter (S_b) by (5). The latter is a subset of the former, and hence every language game for the latter will contain a subset of the former by (2). Condition (a) of (4.3) is therefore fulfilled, S_a is a guardian of S_b, and hence every class which contains both S_b and all guardians of members of the class must also contain S_a, thus satisfying definition (5).

Similarly, the phrase 'tall unmarried man' inplies the word 'bachelor' by (5). Since 'unmarried man' is a subset of the former, every language game for the latter must contain, by (2), at least one word which is synonymous with a subset of the former (namely, 'bachelor', which is synonymous with 'unmarried man' by (4), (4.1), and (4.2)). Since condition (b) of (4.3)

is therefore fulfilled, 'tall unmarried man' is a guardian of 'bachelor', and hence every class which contains both 'bachelor' and all guardians of members of the class must also contain 'tall unmarried man', thus satisfying definition (5).

But consider the following words and phrases:

S_a	S_b
the linear momentum of a balanced particle system	conserved
12	$288 \div 24$
a lateral face of a prism	a parallelogram

I find that it is possible to specify three language games, one for 'the linear momentum of a balanced particle system' (S_5), a second for '12' (S_6), and a third for 'a lateral face of a prism' (S_7), such that no game entails the inplication of the related S_b by (5):

(5.1) $G_1(S_5)$ $\stackrel{spec}{=}$ (1) Say to the pupil, "A balanced particle system is a system upon which no resulting external force is acting".
(2) Say to the pupil, "The linear momentum of a particle is the vector product of its mass and its velocity".
(3) Say to the pupil, "The linear momentum of a balanced particle system is the vector sum of the momenta of its constituent particles".

(5.2) $G_1(S_6)$ $\stackrel{spec}{=}$ (1) Say to the pupil, "3 + 1 = 4".
(2) Say to the pupil, "4 + 1 = 5".
(3) Say to the pupil, "5 + 1 = 6".
.....
(4) Say to the pupil, "11 + 1 = 12".

(5.3) $G_1(S_7)$ $\stackrel{spec}{=}$ (1) Say to the pupil, "A polygon is any closed broken line in a plane, all segments of which are straight".
(2) Say to the pupil, "A face is any segment of a plane bounded by a polygon".
(3) Say to the pupil, "A polyhedron is any solid bounded by faces".

(4) Say to the pupil, "A prismatic surface is the set of all straight lines which intersect a given polygon and which are parallel to a given line not in the plane of the polygon".
(5) Say to the pupil, "A prism is a polyhedron bounded by a prismatic surface and two parallel planes which intersect each element of the prismatic surface".
(6) Say to the pupil, "A lateral face of a prism is one of the faces which does not lie in either of the two parallel planes bounding the prism".

Although I have assumed a knowledge of the terminology of elementary physics in (5.1), of the meaning of the symbols '+', '=', and the numbers from one to three in (5.2), and of several of the basic definitions of elementary geometry in (5.3), nowhere have I employed a set of words, phrases, or sentences sufficient by definition (5) to inply respectively 'conserved', '288 ÷ 24', or 'a parallelogram', even though each of the latter is truly predicated of its respective S_a. (As illustrated by (5.2) and (5.3), furthermore, it appears likely that for any given mathematical S_a one can specify a language game such that this game does not involve the inplication by (5) of any particular other S_b (exluding, of course, those S_b which are part of the definition of the given S_a). It should be noted below, therefore, that Kant was correct in inferring that mathematical judgments are synthetic.)

I shall now specify a criterion for distinguishing analytic from synthetic judgments. I shall restrict my concerns to a narrowly-defined class of judgments, defining the members of this class of $\mathcal{B}$ judgments as follows:

(6) a $\mathcal{B}$ judgment $\stackrel{\text{def}}{=}$ any judgment of the general form 'Anything which is S_a is S_b'

Given definitions (5) and (6), a criterion for applying the predicate 'is synthetic' to members of the class of $\mathcal{B}$ judgments is as follows:

(7) For any β_c, β_c is synthetic $\overset{spec}{=}$ S_a does not inply S_b by (5) (where S_a and S_b are components of β_c as specified in (6))[b]

Given specification (7), a criterion for applying the predicate 'is analytic' to members of the class of β judgments follows directly:

(8) For any β_c, β_c is analytic $\overset{spec}{=}$ β_c is not synthetic by (7)

The import of criteria (7) and (8), briefly put, is that, given any judgment β_c containing an S_a and an S_b as in (6), if there exists a language game for S_b such that S_a does not inply S_b by (5), then β_c is <u>synthetic</u>; if such a language game does not exist, then β_c is <u>analytic</u>.

Being <u>prescriptive</u> rather than <u>descriptive</u>, criteria (7) and (8) are subject to none of the apparent defects of Kant's original criterion.[10] And the method of extending the criteria to encompass classes of judgments other than β judgments should be apparent. I believe, therefore, that I have designed a prescriptive public decision procedure for distinguishing analytic from synthetic judgments (within Kant's definitions of the predicates) which is neither obviously circular, ambiguous, imprecise, inconsistent, nor inapplicable to judgments expressed in natural languages. And that was the task I set out to do.

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10 November 1974

FOOTNOTES

1. See Immanuel Kant, The Critique of Pure Reason, trans. by Norman Kemp Smith (London: MacMillan & Company Ltd., 1963), p. 48 (A7-B11); and in particular pp. 53-54 (B17): " [When making geometrical judgments] we are required to join in thought a certain predicate to a given concept, and this necessity is inherent in the concepts themselves. But the question is not what we ought to join in thought to the given concept, but what we actually think in it, even if only obscurely." (I shall refer to this book hereafter as CPR.)

2. Kant does, of course, suggest a secondary criterion by which to distinguish analytic from synthetic judgments; namely, that the negations of all and only analytic judgments are self-contradictory. (CPR, p. 49 (B12).) It is fortunate that this is not his principal criterion, however, since, as Church has shown, there can be no generally applicable test of self-contradictoriness, and hence this criterion is ineffectual. (See Alonzo Church, "A Note on the Entscheidungsproblem", Journal of Symbolic Logic I (1936), 40f, 101f.)

3. See, in particular, the writings of the Harvard logical pragmatists, including especially:

   Willard Quine [1], From a Logical Point of View (New York: Harper Torchbooks, 1963).

   Willard Quine [2], Selected Philosophical Studies (booklet of reprints of Quine's previously published essays, available from the Philosophy Department of Harvard University).

   Willard Quine [3], Word and Object (Cambridge, Massachusetts: Massachusetts Institute of Technology Press, 1964).

   Willard Quine [4], "Two Dogmas of Empiricism", reprinted and revised in Quine [1], pp. 20-46.

   Willard Quine [5], "Theory of Reference", reprinted and revised in Quine [1], pp. 130-138.

   Willard Quine [6], "Carnap and Logical Truth", reprinted in Quine [2], article 6.

   Nelson Goodman, "On Likeness of Meaning", reprinted in Semantics and the Philosophy of Language, ed. by Leonard Linsky (Urbana, Illinois: The University of Illinois Press, 1952)

   Morton White [1], Toward Reunion in Philosophy (New York: Athenum Press, 1956).

   Morton White [2], "The Analytic and the Synthetic: an Untenable Dualism", reprinted in Linsky, cited above.

4. Although the above description may appear to be arbitrary, it is meant to mirror formally the teaching procedure followed in those situations in which a teacher <u>does</u> succeed in teaching the meaning of a word or phrase to a pupil.

5. Although language games can be specified to give expected results for some S_n and P_n, it is not the case (or at least not apparently the case) that for <u>every</u> S_n an instructor I_a at T_a can expect to be able to specify a language game whereby he can expect to teach the meaning of S_n to an arbitrary P_a with expected and sure results. Such S_n as 'good', 'virtue', 'God', and 'justice' appear to lead rather charmed and ambiguous lives, and I suspect that their legendary ambiguity is due precisely to the fact that they have not proven susceptible to the rigors of precise language games. To specify, on the other hand, language games for less-ambiguous S_n such as 'photograph', 'black', 'Santa Claus', and 'Sophia Loren' is a task of a much more amenable nature.

6. It is important that the phrase 'to be expressed' in (2.1) and (3) be understood, for otherwise any two language games would be twins which, while retaining their character as language games, permitted blanket substitution of an S_b for an S_a. But blanket substitution is not to be permitted. The restriction 'to be expressed' prohibits (1.2), for example, from being a language game for 'sunflower' under substitution, for substitution is to be permitted by definition only in the latter two of the three sentences constituting the language game (ie., only in the sentences in which 'dandelion' is to be <u>expressed</u>).

7. Although I formally specified (2.1) prior to giving the definition (3) to indicate the manner in which I ought to introduce each word or phrase whose criterion of application is identical to its definition if this essay were to be strictly rigorous (i.e., if I were to <u>teach</u> the meaning of each word or phrase prior to giving its formal definition), the general method should now be apparent. In the remainder of the essay, therefore, as with definitions (1) and (2) above, I shall restrict precision in the interest of concision, and proceed by definition alone.)

8. In this and in all references hereafter in the essay, I shall use the phrase 'language game' to refer to a language game as defined in (2), not (1).

9. By "Kantian relation", I mean an explication of the 'containment' relation used by Kant in his explication of the meaning of the predicates 'is analytic' and 'is synthetic'. (<u>CPR</u>, p. 48 (A7-B11).)

10. It must be remembered that a language game is defined to be a specified list of imperative sentences, not a <u>possible</u> specified list of imperative sentences. The phrase "each language game", therefore, refers to all lists which have been specified, <u>not</u> to any lists possible of specification though not yet actually specified. (And it should be noted below that the phrase in (c) "by conditions (a) and (b) alone" prevents the use of 'inplies' in (c) from making the entire specification (5) circular.

11. The criteria are independent of that notorious family of imprecise words and phrases which includes 'necessary', 'self-contradictory', 'definition', and 'synonymous' (as defined previously to this essay). They are also independent of such extra-linguistic entities as 'meanings' or 'intentions' of words or phrases. They say nothing, therefore, about the notion of 'interchangeability', and are in no way dependent upon it. (See, eg., Quine [4] , pp. 27-32.)

They are not based upon methods of 'verification' of the truth of $\mathcal{B}$ judgments (see, eg., Moreland Perkins and Irving Singer, "Analyticity", The Journal of Philosophy, #48 (1951), pp. 485-497), nor are they conventional in nature, for I have said nothing about either explicit or implicit linguistic conventions and have introduced no 'semantical rules' by which to specify a class of sentences as 'analytic-for-L_n' (and derivatively 'necessary-for-L_n'). (See, eg., Quine [4] , pp. 32-37, and also White [1] , pp. 156-158). I have assumed only that the proper usage of many words can be taught, and that formal games can be specified for the teaching.

(Although Quine has spent a good amount of time portraying the difficulties of a linguist facing a new language (Quine [3] , pp. 30-57), and perhaps is right in saying that no non-behavioristic criterion of analyticity can be justified in such a situation, he does give the linguist an alternative: "He can settle down and learn the native language directly as an infant might" (Quine [3] , p. 47 and pp. 80-81). But that is precisely the difference between prescription and description. My criteria are not made with respect to respect to a linguist observing (and hence describing) a language he does not understand, but with respect to a teacher about to teach (and hence prescribe) a language he does understand. That, therefore, which is a fatal difficulty to the linguist - his inability to distinguish between co-extensive predicates (Quine [3] , pp. 51-54) - is a problem of no great difficulty to the teacher: one can rather easily teach someone the difference between 'rabbit' and 'rabbit's foot', regardless of the peculiar difficulties faced by a linguist in arbitrary observational situations. (Quine [3] , pp. 51-54))

The criteria are not behavioristic in nature, having nothing to do with "producing the condition C and then determining whether response R occurs" (see, eg., Rudolph Carnap, Meaning and Necessity (Chicago: Phoenix Books, University of Chicago Press, 1960), p. 243). They are as far from stating that "If we should be presented with something which wasn't black, we would not call it a raven" (White [1] , p. 145) as they are from being dependent upon a method of Naessian questionnaires (see Arne Naess, "Toward a Theory of Interpretation and Preciseness" reprinted in Linsky, cited above). They therefore escape the flaw common to most behavioristic criteria of similar intent by being able to distinguish between co-extensive predicates: the language game for 'creatures with hearts' and that for 'creatures with kidneys' would not be twins by (3). (Quine's example; see Quine [4] , pp. 21&31). And, lastly, the criteria are not based upon Carnap's alternative, a "method of structure analysis" (i.e., a method based upon a psychological analysis of human cognition and the mental processes), with all its attendant operational difficulties (see Carnap, op.cit., p. 243.).