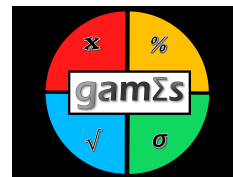


# GAMES Practice Problem Solutions – Introduction to Derivatives



- The input to  $f$  is in litres, while the output of  $f$  is in dollars.
  - The input to  $f'$  is still in litres, while the output of  $f$  is a rate of change of  $f(q)$  with respect to  $q$ , which must be in dollars/litre.
- The input to  $f$  is in kg, while the output of  $f$  is in seconds.
  - The input to  $f'$  is still in kg, while the output of  $f$  is a rate of change of  $f(a)$  with respect to  $a$ , which must be in sec/ml.
- $C$  represents a cost, so it is in dollars.
  - $C'(r)$  represents the rate of change with respect to the interest rate, so it is in dollars per interest rate percentage.
  - If we increase the interest rate on a loan, the total cost to the borrower increases, so the derivative  $C'(r)$  must be positive.
- - The graph appears continuous. Consider  $r = r_0$ . Using the first formula with  $r = r_0$  the equation is  $B = \frac{r_0}{r_0} B_0 = B_0$ . Then, using the first formula with  $r$  approaching  $r_0$  from above, as we get close to  $r = r_0$   $B \approx \frac{r_0}{r_0} B_0 = B_0$ . And we get the same value for  $B$  from both formulas, so  $B$  is continuous
  - For  $r < r_0$ , the graph of  $B$  is upward-sloping line with slope  $\frac{B_0}{r_0}$ . For  $r > r_0$ , the graph of  $B$  looks like  $1/x$ , so it has a negative slope. The graph has a “corner” at  $r = r_0$ , so it cannot be differentiable at that point.
- $f(60) = 40$  indicates that, for a patient with a weight of 60 kg, the dosage is 40 mg.
  - $f'(60) = 6$  indicates that, for a patient with a weight of 60 kg, the dosage should increase by 6 mg for every additional kg above 60 kg.
  - For a 65 kg person, the dosage should be  $f(60) + f'(60)(65 - 60) = 40 + 6(5) = 70$  mg
- The function is continuous everywhere.
  - The function appears not to be differentiable at  $x = -6$  because the graph has a corner there. Taking the limit can verify the result.
- $f'(x) = 4e^x - 18x$
- $f'(x) = 13x^5\sqrt{x} + \frac{35}{2x^4\sqrt{x}}$
- $f'(x) = \frac{21}{2}\sqrt{x} + \frac{7}{2\sqrt{x}} - \frac{5}{x^{3/2}}$
  - 19.73
- $f'(x) = -49t^{-8}$
  - $f'(3) = -49(3^{-8})$ , a small number
- $3e^x$

$$12. f'(x) = 54x^2 + 18x - 12$$

$$13. f'(x) = \frac{4(3x + 2) - 3(4x + 3)}{(3x + 2)^2}$$

$$14. h'(x) = \frac{(3 + 2x)e^x - 2e^x}{(3 + 2x)^2} = \frac{e^x(1 + 2x)}{(3 + 2x)^2}$$

$$15. -\frac{2V^2R}{(R + r)^3}$$



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