GAMES Practice Problem Solutions – Introduction to Derivatives

1. (a) The input to f is in litres, while the output of f is in dollars.

   (b) The input to f' is still in litres, while the output of f is a rate of change of $f(q)$ with respect to q, which must be in dollars/litre.

2. (a) The input to f is in kg, while the output of f is in seconds.

   (b) The input to f' is still in kg, while the output of f is a rate of change of $f(a)$ with respect to a, which must be in sec/ml.

3. (a) C represents a cost, so it is in dollars.

   (b) $C'(r)$ represents the rate of change with respect to the interest rate, so it is in dollars per interest rate percentage.

   (c) If we increase the interest rate on a loan, the total cost to the borrower increases, so the derivative $C'(r)$ must be positive.

4. (a)

   (b) The graph appears continuous. Consider $r = r0$. Using the first formula with $r = r0$ the equation is $B = \frac{r_0}{r_0}B_0 = B_0$. Then, using the first formula with r approaching r_0 from above, as we get close to $r = r0$ $B \approx \frac{r_0}{r_0}B_0 = B_0$. And we get the same value for B from both formulas, so B is continuous

   (c) For $r < r_0$, the graph of B is upward-sloping line with slope $\frac{B_0}{r_0}$. For $r > r_0$, the graph of B looks like $1/x$, so it has a negative slope. The graph has a "corner" at $r = r_0$, so it cannot be differentiable at that point.

5. (a) $f(60) = 40$ indicates that, for a patient with a weight of 60 kg, the dosage is 40 mg.

   (b) $f0(60) = 6$ indicates that, for a patient with a weight of 60 kg, the dosage should increase by 6 mg for every additional kg above 60 kg.

   (c) For a 65 kg person, the dosage should be $f(60) + f'(60)(65 - 60) = 40 + 6(5) = 70$ mg

6. (a) The function is continuous everywhere.

   (b) The function appears not to be differentiable at $x = -6$ because the graph has a corner there. Taking the limit can verify the result.

7. $f'(x) = 4e^x - 18x$

8. $f'(x) = 13x^5\sqrt{x} + \dfrac{35}{2x^4\sqrt{x}}$

9. (a) $f'(x) = \frac{21}{2}\sqrt{x} + \frac{7}{2\sqrt{x}} - \frac{5}{x^{3/2}}$

   (b) 19.73

10. (a) $f'(x) = -49t^{-8}$

    (b) $f'(3) = -49(3^{-8})$, a small number

11. $3e^x$

12. $f'(x) = 54x^2 + 18x - 12$

13. $f'(x) = \dfrac{4(3x+2) - 3(4x+3)}{(3x+2)^2}$

14. $h'(x) = \dfrac{(3+2x)\,e^x - 2e^x}{(3+2x)^2} = \dfrac{e^x\,(1+2x)}{(3+2x)^2}$

15. $-\dfrac{2V^2R}{(R+r)^3}$