On the Possible Existence of Closed Disentropic Systems

Evan Wm. Cameron

Professor Emeritus
Senior Scholar in Screenwriting

Graduate Programmes,
Film & Video and Philosophy

York University

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submitted by:

Evan Cameron
Boston University
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(mailing address:

128 Glenville Avenue
Allston, Massachusetts
02134)

ABSTRACT

I construe the question "Is the existence of a closed physical system whose entropy is decreasing possible?" to be equivalent to the question "Is it possible for an entropically increasing physical system to be coupled, as measurer to measured, with an entropically decreasing physical system such that the entropy of the resulting system decreases?". Wiener's suggestion that the answer is negative, implying the impossibility of finding a counter-example to the 2nd Law of Thermodynamics, is then shown to be incorrect by the contruction of such a counter-example – achieved thru a slight modification of a machine first proposed and rejected by Szilard.

It is then shown, however, thru a proof from n-dimensional geometry, that physical analogies to the counter-example are only possible on the microscopic level (i.e. with systems involving few molecules). While mathematically demonstrating the impossibility of the existence of a macroscopic system with decreasing entropy, therefore, the possible existence of such systems on the microscopic level is mathematically affirmed.

Evan Cameron June 1, 1969

On the Possible Existence of Closed Disentropic Systems*

I shall assume, with Boltzmann, that the direction of time for a closed physical system can be identified with the direction of entropy increase of the system.

The question with which I shall be concerned in this paper is as follows:

(1) Is the existence of a closed physical system whose entropy is decreasing (i.e. whose time is counter-directed to our own) possible?

I shall assume, with Peirce, the logical maxim that a difference which makes no difference is no difference at all. That is, with particular reference to the possible existence of a physical system as specified in (1), if it should be in principle impossible for a measuring device of increasing entropy to detect a disentropic system if it did exist, then such a system could not exist. Having defined 'existence' thusly as a function of two physical systems, the measured and the measuring, question (1) may be restated as follows:

(2) Is it possible for an entropically increasing physical system to be coupled, as measurer to measured, with an entropically decreasing physical system such that the entropy of the resulting coupled system (assumed to be closed) decreases?

If (2) can be answered affirmatively, then in principle evidence for the existence of counter-directed systems could be obtained thru the measuring devices of our system. If the contrary, then in principle such evidence could never be obtained and hence the existence of such counter-directed systems would be impossible.

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* I should like to thank Abner Shimony, under whose constant inspiration, encouragement, and criticism this paper was both conceived and completed.

The import of the respective answers can perhaps best be understood by contrasting 3 extended comments on the possibility of detecting temporally counter-directed physical systems:

"Now, even in a Newtonian system, in which time is perfectly reversible, questions of probability and prediction lead to answers asymmetrical between past and future, because the questions to which they are answers are asymmetrical. If I set up a physical experiment, I bring the system I am considering from the past into the present in such a way that I fix certain quantities and have a reasonable right to assume that certain other quantities have known statistical distributions. I then observe the statistical distribution of the results after a given time. This is not a process which I can reverse. In order to do so, it would be necessary to pick out a fair distribution of systems which, without intervention on our part, would end up with certain statistical limits, and find out what the antecedent conditions were a given time ago. However, for a system starting from an unknown position to end up in any tightly defined statistical range is so rare an occurrence that we may regard it as a miracle, and we cannot base our experimental technique on awaiting and counting miracles. In short, we are directed in time, and our relation to the future is different from our relation to the past. All our questions are conditioned by this asymmetry, and all our answers to these questions are equally conditioned by it.

"A very interesting astronomical question concerning the direction of time comes up in connection with the time of astrophysics, in which we are observing remote heavenly bodies in a single observation, and in which there seems to be no unidirectionalness in the nature of our experiment. Why then does the unidirectional thermodynamics which is based on experimental terrestrial observations stand us in such good stead in astrophysics? The answer is interesting and not too obvious. Our observations of the stars are through the agency of light, of rays or particles emerging from the observed object and perceived by us. We can perceive incoming light, but cannot perceive outgoing light, or at least the perception of outgoing light is not achieved by an experiment as simple and direct as that of incoming light. In the perception of incoming light, we end up with the eye or a photographic plate. We condition these for the reception of images by putting them in a state of insulation for some time past: we dark-condition the eye to avoid after-images, and we wrap our plates in black paper to prevent halation. It is clear that only such an eye and only such plates are of any use to us: if we were given to pre-images, we might as well be blind; and if we had to put our plates in black paper after we use them and develop them before using, photography would be a very difficult art indeed. This being the case, we can see those stars radiating to us and to the whole world; while if there are any stars whose evolution is in a reverse directbn, they will attract radiation from the whole heavens, and even this attraction from us will not be perceptible in any way, in view of the fact that we already know our own past but not our future. Thus the part of the universe which we see must have its past-future relations, as far as the emission of radiation is concerned, concordant with our own. The very fact that we see a star means that its thermodynamics is like our own.

"Indeed, it is a very interesting intellectual experiment to make the fantasy of an intelligent being whose time should run the other way to our own. To such a being, all communication with us would be impossible. Any signal he might send would reach us with a logical stream of consequents from his point of view, antecedents from ours. These antecedents would already be in our experience, and would have served to us as the natural explanation of his signal, without supposing an intelligent being to have sent it. If he drew us a square, we should see the remains of his figure as its precursors, and it would seem to be as fortuitous as the faces we read into mountains and cliffs. The drawing of the square would appear to us as a catastrophe – sudden, indeed, but explainable by natural laws – by which that square would cease to exist. Our counterpart would have exactly similar ideas concerning us. Within any world with which we can communicate, the direction of time is uniform."

(Norbert Wiener, (12), pp. 33-35)

"Let us assume that among the many galaxies there is one within which time goes in a direction opposite to that of our galaxy. We would then have the situation envisaged by Boltzmann. In this situation, some distant part of the universe is on a section of its entropy curve which for us is downgrade; if, however, there were living things in that part of the universe, then their enviornment would for them have all the properties of being on an upgrade of the entropy curve.

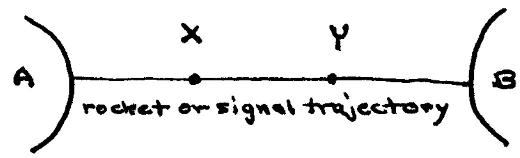
"That such a system is developing in the opposite time direction might be discovered by us from some radiation traveling from the system to us and perhaps exhibiting a shift of spectral lines upon arrival. Because of the great distance, the message would reach us so late that it would merely inform us about the time-direction which the system had aeons ago. We thus cannot acquire knowledge of a system that has, at the present time, a time direction opposite to ours, a limitation unknown to Boltzmann. This holds, at least, when we employ the usual definition of simultaneity according to which light travels in both directions along its path with equal speed. When we abandon this definition, the aeons of time could be transformed into a very short time period. The indeterminacy of simultaneity leaves the time comparison indefinite within wide limits. Furthermore, the radiation traveling from the system to us would, for the system, travel backward in time, that is, would not leave that system but arrive at it. Perhaps the signal could be interpreted by inhabitants of that system as a message from our system telling them that our system develops in the reverse time direction. We have here a connecting light ray which, for each system, is an arriving light ray annihilated in some absorption process; judged from the other system, it is therefore emmitted at its source by an irreversible process going in reverse."

(Hans Reichenbach, (6), pp. 139-140)

"Finally, suppose that we supplement the hypothetical conditions posited by Bridgman by assuming that in addition to his hypothetical human species A whose members are supposed to experience higher entropy states of physical systems as psychologically earlier than lower ones, there is another human species B possessing our actual property of experiencing these same higher entropy states as psychologically later.

Then, as Norbert Wiener has noted, a very serious difficulty would arise for communication between our two species A and B whose psychological time senses are counter directed...

"In amplification of Wiener's statement, consider a situation in which our species A and B have distinct habitats, which are represented respectively by the regions A and B of our diagram.



We can then show that any particle or signal which would be regarded as outgoing by one of the two species would likewise be held to be departing by the other, and any object or message which is incoming in the judgment of either species will also be held to be arriving by the other. For – to take the case of the outgoing influence – suppose that as judged by the members of A, the particle reaches the point Y of its trajectory (see diagram) later than the point X and is therefore held to be departing by the men in A. Then the members of B will conclude that the particle is leaving them as well, since they will judge that it reaches point X after reaching point Y. And thus, if as judged by the A-men, they hurl a rock toward the B region such that the rock comes to rest and remains at rest in B indefinitely, then the B-men, in turn, will judge that a rock, having been at rest in their region all along, suddenly left their habitat and traveled to A, where it was then received by the A-men with ready, open arms. And if the B-men were struck by the discrepancy between the dynamical behavior of that rock and the behavior of other rocks in their habitat – assuming the latter obey the familiar dynamical principles – they might conceivably conclude after a number of such experiences that the dynamically aberrant rocks are linked to the presence elsewhere of temporally counter-directed beings. ((Grünbaum here quotes Reichenbach with approval)) In order to draw this conclusion, however, the B-men would have to assume that the entropy decrease involved in the rock's spontaneous acquisition of kinetic energy from the sand is less probable than the presence elsewhere of temporally counter-directed beings."

(Grünbaum, (3), pp. 228-230)

To Reichenbach and Grünbaum, physical interaction between two entropically counter-directed systems is possible; the crucial question is whether such interaction could be recognized as such by an observer in either system. (And even Wiener, in the final paragraph of the passage quoted, seems to concur in the possibility of such interaction, though holding that observers in either system would never have sufficient reason to recognize the interaction as such.)

Yet, in his initial paragraphs, Wiener comes remarkably close to stating explicitly why such an interaction could in principle be impossible: the activity of measuring could be such that only events whose entropies are similarly directed to that of the measuring device could be measured. If this suggestion were true, Wiener could have avoided the conceptual confusion common to his last paragraph and to those of Reichenbach and Grünbaum after him by strictly developing the consequences of what he himself later notes: that physical interactions differ only in quantity, not quality, whether functioning to communicate or to effect change.

> "If the seventeenth and early eighteenth centuries are the age of clocks, and the later eighteenth and the nineteenth centuries constitute the age of steam engines, the present time is the age of communication. There is in electrical engineering a split which is known in Germany as the split between the technique of strong currents and the technique of weak currents, and which we know as the distinction between power and communication engineering. It is this split which separates the age past from that in which we are now living. Actually, communication engineering can deal with currents of any size whatever and with the movement of engines powerful enough to swing massive gun turrets; what distinguishes it from power engineering is that its main interest is not economy of energy but the accurate reproduction of a signal."
>
> (Wiener(12), p. 39)

The impossibility of perceiving outgoing radiation (eg. light) would then be but a particular instance of the general impossibility of a measuring device detecting any entropically counter-directed physical event. And the conceptual error of Grünbaum (and Reichenbach) could then be indicated as follows. Suppose, in Grünbaum's example, observer A were to throw a rock at observer B. The rock, to A, would be an object which looks, feels, and behaves in the familiar rocklike fashion within A's system. That is, it would be a source of energy directed toward (i.e. perceivable by) A. But, as Wiener has suggested, precisely for this reason, if B were an entropically counter-directed being, such energy would be imperceptible to B. Thus, rather than having suddenly encountered a rock in his system behaving disentropically (and hence being forced to decide whether to accept the behavior of this rock as evidence of A's counter-directed existence), B would have encountered nothing at all. (i.e. Grünbaum would have failed to note that a rock in A's system must have all its properties reversed vis a vis B and his system, not just its mechanical properties. For Grünbaum to hold that B could visually observe in his system A's rock behaving in a mechanically disentropic way would be for Grünbaum to hold that the electromagnetic radiation of the rock (i.e. its light-reflecting property) was not counter-directed in B's system, contrary to Grünbaum's initial premise that the electromagnetic radiation of the rock was directed toward A in A's system such that A could visually perceive it.)

If Wiener's suggestion were true, therefore, communication between two temporally counter-directed systems would not be 'difficult', in Grünbaum's terminology, but rather impossible — for physical interaction of any degree, from communicative to power, would be impossible. Thus, vis a vis either system as given, the other could not exist. And hence question (2) would have a negative answer.

But is Wiener's suggestion true? If it were true, then we ought not to be surprised that no exception has ever been found to the Second Law of Thermodynamics, for none could exist. On the other hand, a counterexample to the 2nd Law would falsify Wiener's suggestion and give an affirmative answer to question (2). (A counterexample to the 2nd Law would be a closed physical system with decreasing entropy; or, in other words, a system from which work could be extracted without an equal or greater amount of work being applied to the system.) But have we any reason to hold that counterexamples to the 2nd Law cannot be found? Those who have examined the question most carefully, and concluded that the answer is 'yes', have usually proceeded as follows: firstly, a purported counterexample is specified which, in its thermodynamic simplicity, is such that, if any more complex counterexample were to exist, then it too would exist; secondly, it is shown that, upon closer analysis, the purported counterexample could not violate the 2nd Law; hence, thirdly, it is concluded by induction that no counterexample could exist.

I shall now specify such a purported counterexample, akin to Maxwell's Demon, first proposed and rejected by Szilard in 1929, and develop the usual information-theoretic arguments brought against its possible existence, preparatory to demonstrating their invalidity.

II

I shall assume with Boltzmann the ergodic hypothesis for statistical mechanics, whereby the possible microscopic states of a closed system of total energy ϕ are considered to be equally probable. The entropy E of such a system in a given macrostate S is then defined to be the product of Boltzmann's constant k and the natural log of the number of possible microscopic states p corresponding to S.

(3) $$E_S = k \ln p$$

The free energy F of such a system (i.e. the amount of energy which can be converted into organized energy, and hence work) is then given by the difference between the total energy and the product of the entropy and the temperature T.

(4) $$F = \phi - T E$$

If we consider a closed system of given energy Φ at constant temperature T, clearly the entropy E and the free energy F vary inversely, [redacted], with the latter having its maximum value whenever E has its minimum value (and, by equation (3), E will have its minimum possible value for any non-degenerate system in a given macrostate S whenever $p = 1$).

Consider now a closed system in a given macrostate S consisting of an ideal, classical, single-molecule gas enclosed within a volume 2 V at a constant temperature T. Consider the volume 2 V to be divided into two halves, V_l and V_r. Since the molecule may be in either, but not both, of the two halves, V_l and V_r, there corresponds to macrostate S two possible microstates: the first when the molecule is in V_l, the second when it is in V_r. Hence, the entropy of the system S is given by:

(4) $$E = k \ln 2 = 0.693\, k$$

Consider now the same closed system S, except for the volume enclosing the gas having been halved from 2 V to V. Corresponding to the sole macrostate of the system S is now a single microstate: the molecule being in V. Hence, the entropy of the system has been decreased to its minimum possible value.

(5) $$E' = k \ln 1 = 0$$

The free energy F of the system S in its initial state is computible, by (3), as:

(6) $$F = \Phi - 0.693\, k\, T$$

Similarly, the free energy F' of the system S in its modified state is computible as :

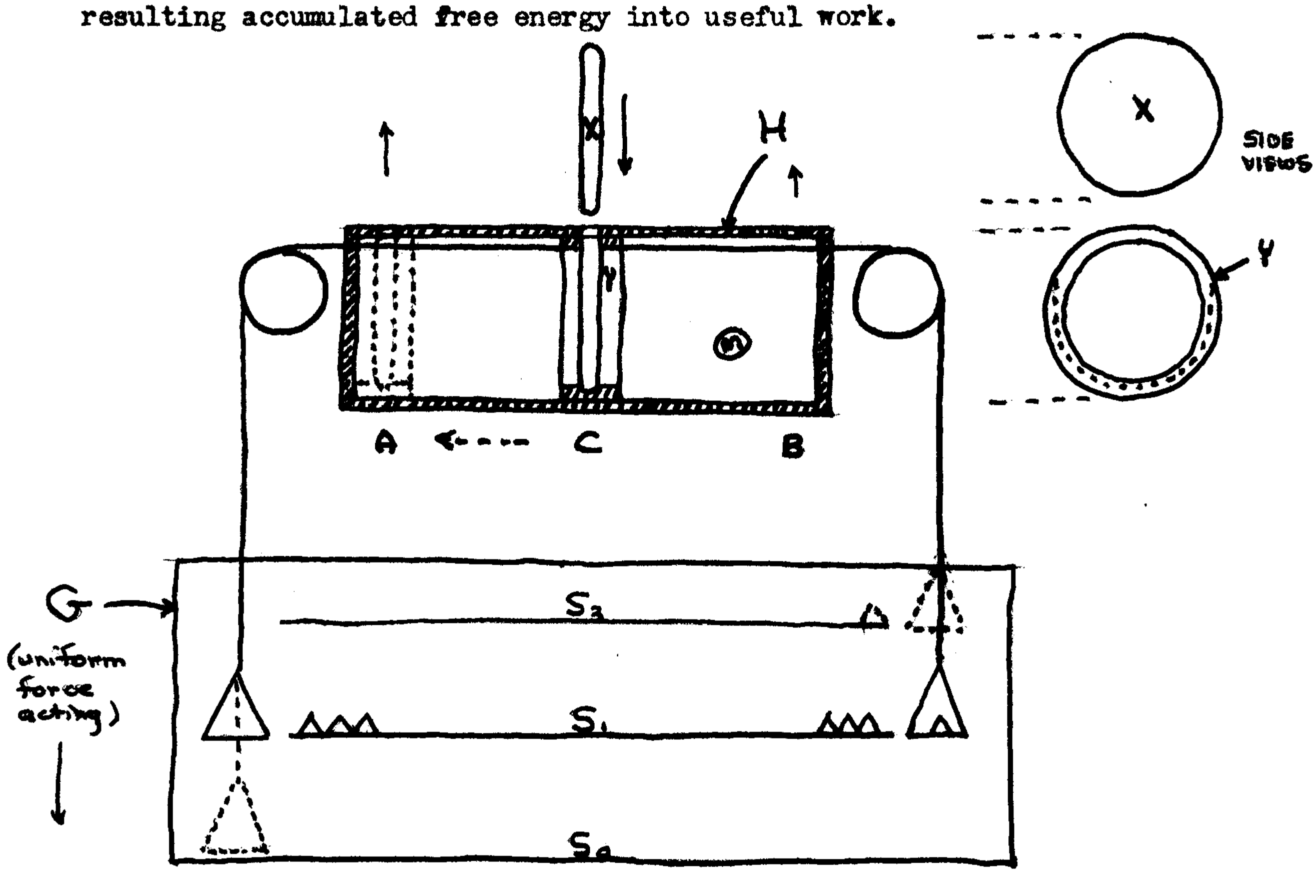
(7) $$F' = \Phi$$

Hence, by halving the volume 2 V of a one-molecule classical gas, we have obtained a maximum decrease in entropy and a corresponding maximum increase in the free energy of the system.

(8) $$F' - F \cong 0.693\ k\ T$$

Thus, if it were possible to halve the volume of such a gas <u>without expending work on the system in the process</u>, it should be possible to obtain 0.693 k T units of energy which might then ideally be converted into organized energy, and thus do work — contrary to the 2nd Law of Thermodynamics.

Consider now a machine purporting to violate the 2nd Law by (a) accomplishing the above without expending work, and then (b) converting the resulting accumulated free energy into useful work.



The closed system H consists of a cylinder containing an ideal, classical, single-molecule gas at a constant temperature T. Within the cylinder is a frictionless piston Y shaped like a washer, having a slot down the middle into which a disk X can be inserted thru the upper wall of the cylinder when the piston is at the midpoint C of the cylinder, and from which the disk can be retrieved when the piston is at either end of the cylinder, A or B. Hence, when the disk is not in the slot in the piston, the piston can be moved readily along the cylinder without doing any work on the gas, for the molecule is free to pass thru the piston at any time. When the disk is in the slot, however, the molecule will exert a constant average pressure on the piston. The piston is connected thru the ends of the cylinder over two frictionless pulleys to two massless trays suspended in a system G in which a uniform force is directed downward, giving weight to the small objects on shelf S_1.

Suppose we center the piston at C and slip the disk in the slot of the piston. The volume of the gas containing the single molecule M will have been halved without exerting work on M; hence, the entropy of H will have been decreased. The repeated impact of the molecule on the piston will slowly force the piston to either A or B, and thus raise one of the trays, D or E, and lower the other. Had we known which of the trays was going to be raised, we could have placed one of the small wieghts from shelf S_1 onto the tray and had it lifted from S_1 to S_2. We could then have taken the weight off the tray onto S_2, slipped the disk out of the piston, centered the piston again at C, and repeated the process. As we know from (8), the maximum work we could have obtained from the molecule in forcing the piston from C to either A or B could not exceed

$$0.693\ k\ T.$$

But since we would have ideally exerted no work in centering the piston and slipping the disk in and out, eventually we could raise an enormous number of little weights from S_1 to S_2, and hence would have done work by means of disorganized thermal energy!

The usual argument advanced against the possibility of such a machine violating the 2nd Law of Thermodynamics comes in two parts. Firstly, suppose we don't know which tray will be raised. (That is, prior to slipping the disk into the piston at C, suppose we don't know on which of the two sides of the piston the molecule is.) If we center the piston, select one of the trays in G at random and place thereon a weight, and then slip in the disk at a random moment, the probability of our having placed the weight on the tray about to rise cannot exceed 1/2. Thus, on the average, for each weight raised from S_1 to S_2, there will be another weight lowered from S_1 to S_0, requiring twice the energy thereafter to raise it to S_2. Hence, without knowing on which side of the piston the molecule is prior to inserting the disk into the centered piston, the average amount of work extractable from the machine will be zero – in accordance with the 2nd Law.

Secondly, suppose we do know which tray will be raised, for we do know on which of the two sides of the centered piston the molecule is prior to inserting the disk. What is the minimum amount of energy we must have expended in deriving this single bit of information?

Shannon has proved that a continuous signal of bandwidth W can be represented completely by specifying its amplitude at 2 W sample points per second. Assuming a stationary ergodic signal source in conjunction with a source of Gaussian noise of power N (where a noise is Gaussian if and only if it contains all frequencies up to W and no higher, and is such that each of the 2 W samples per second which represent it is uncorrelated and independent), Shannon also proved that the maximum number of bits per second C which can be sent with a bandwidth W using a signal power P is:

(9) $$C = W \ln (1 + P/N)$$

Shannon then further demonstrated that the signaling rate can approach as close as one would like to C as given in (9) with as little error as one wishes; hence, C as given in (9) is the effective <u>channel capacity</u> for a continuous channel in which Gaussian noise and an ergodic signal are mixed.

After J. B. Johnson discovered that thermal processes (molecular actions) cause electrical fluctuations (Johnsonian noise), H. Nyquist derived a formula for the <u>maximum</u> noise power a heated resistor can supply:

$$(10) \qquad N = k\,T\,W,$$

where k is Boltzmann's constant, T is the temperature of the resistor in degrees Kelvin, and W is the bandwidth of the noise in cycles per second. Since Johnsonian noise is a Gaussian noise, we may substitute k T W for N in (9), deriving

$$(11) \qquad C = W \ln\left(1 + \frac{P}{k\,T\,W}\right).$$

If we assume P to be given, and then make W very small, C will in turn become very small. On the other hand, if we enlarge W, C does <u>not</u> become arbitrarily large, but rather approaches a limiting value. If P / k T W becomes very small compared with unity, (11) approaches as a limit

$$(12) \qquad C = \frac{1.44\,P}{k\,T}$$

which, in terms of the signal power P, becomes

$$(13) \qquad P = 0.693\,k\,T\,C.$$

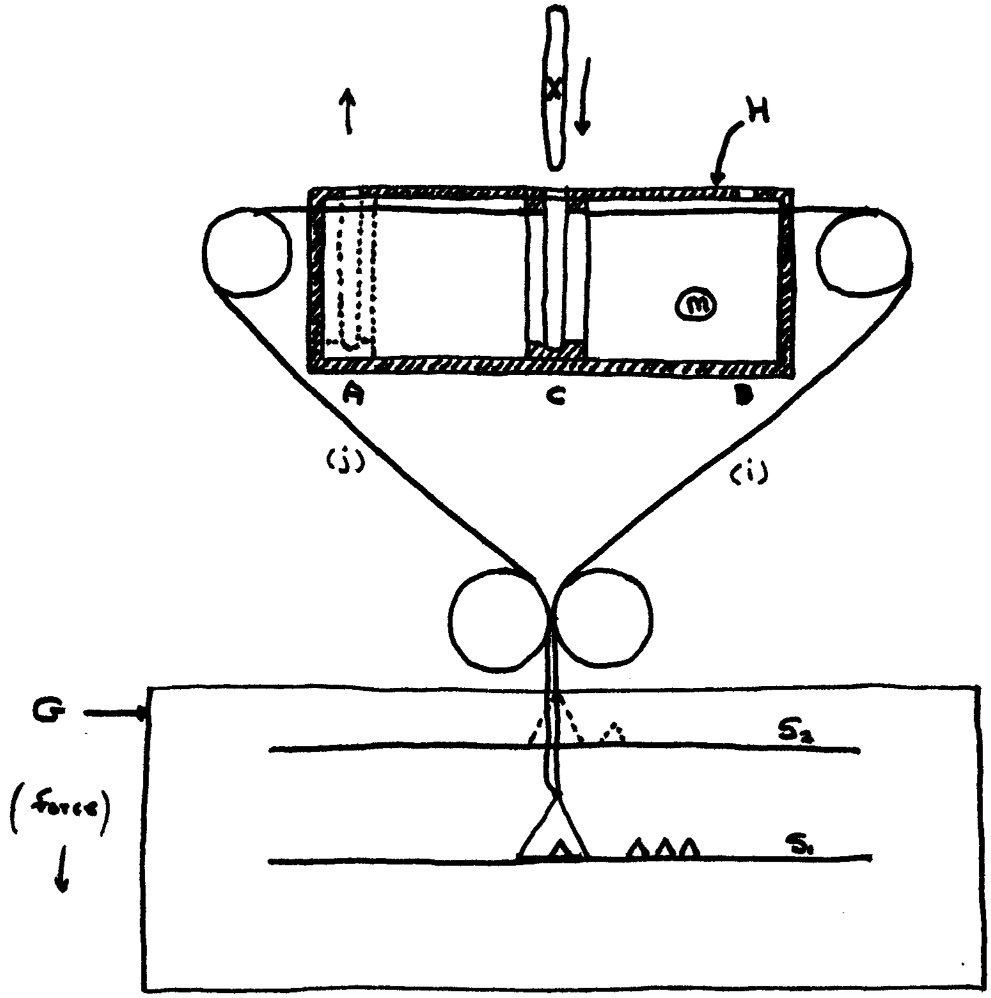
Equation (13) says that, even when using a very wide bandwidth, we need a <u>minimum</u> of 0.693 k T units [redacted] per second to send a message of one bit per second, so that <u>on the average</u> one must supply 0.693 k T units [redacted] of energy for each bit of information transmitted. (And it should be noted that equation (9) holds only for an ideal sort of encoding in which many characters representing many bits of information are encoded together into a long stretch of signal. Most communication systems, therefore, will require much more energy per bit than that indicated in (13).)

In short, the argument goes, to derive the single bit of information concerning on which of the two sides of the centered piston the molecule is located requires a minimum expenditure of 0.693 k T units of energy. Without this information, we cannot derive work from the machine. But, even with this information, the maximum amount of free energy created by one pass of the machine cannot exceed 0.693 k T units of energy, as indicated in (8). Hence, the argument concludes, since even under ideal conditions one could not extract from the machine more work than one would expend on it, the machine must fail to violate the 2nd Law.

I shall now demonstrate the invalidity of the above argument.

III

If, as contended in the first part of the above argument, it were true that we must know on which of the two sides of the centered piston the molecule is located to derive work from the machine, the second part of the argument would be conclusive. But the first part of the argument is invalid: to derive work from the machine, we needn't know on which side of the centered piston the molecule is. By centering the piston and inserting the disk, the entropy of the system is decreased regardless of our unawareness of which side the molecule may be on, and the consequent gain in free energy may be effectively put to work regardless of our unawareness of its direction, as the following modification of the above machine makes clear:



In principle, the machine would operate exactly as before. The piston, however, would be connected by means of four frictionless pulleys to a single massless tray suspended in system G. When the piston is centered, a weight would be placed on the tray and the disk would be inserted in the piston. The piston would then be forced by the molecule either to end A or to end B of the cylinder; but, in either case, the tray would be lifted from S_1 to S_2! If the molecule should be in the right-half of the cylinder, then the piston would move to end A; line (i) would transmit the force to the tray while line (j) would remain slack, not affecting the upward motion of the tray at all. If the molecule should be in the left-half of the cylinder, then the piston would move to end B; line (j) would transmit the force to the tray while line (i) would remain slack, not affecting the upward motion of the tray at all.

The point is that one needn't know anything about the position of the molecule in the cylinder to derive work from the machine. It is enough that one center the piston and insert the disk. By so doing, one would ideally free 0.693 k T units of energy, and would be able to convert it ideally into work regardless of one's unawareness of its direction. (Note: I have not, as yet, argued that the modified machine is either physically possible or impossible. I have simply argued that, if the former machine were to be physically possible, then so would be the modified machine, with its accrued advantages as described.) [1]

Given that the modified machine is ideally possible, why has such a machine neither been constructed nor detected experimentally in nature? The quick answer, of course, is that the above machine involves frictionless pulleys and pistons, massless lines and trays, a disk which requires no work to insert or retract thru holes in a cylinder permitting no energy leakages, etc., and clearly no such objects are available to the working physicist. But this answer, though true on the macroscopic level, misses the point of the ideal machine specified above, as most theoretical physicists since Maxwell have noted. How frictionless our pulleys may or may not be, etc., depends upon our current state of technology. And even if it were true that one could demonstrate that, on the macroscopic level, no actual arrangement of pulleys and pistons could be arranged into a machine which would require less work to overcome the leakages (friction, etc) than the machine could develop in operation, given the ideal possibility of the above machine one could not demonstrate that, on the microscopic level (where our technological limitations are inapplicable), there could not exist suitable analogies to frictionless pulleys and pistons (eg. molecular sieves, etc.) which are such that, in operation in certain combinations, units analogous to the above modified machine could function, thereby decreasing entropy.

Given the ideal modified machine as conceptually rigorous, and assuming the ergodic hypothesis, a proof of the high improbability of detecting (or constructing) such a machine on the macroscopic level is forthcoming.

Consider an ideal classical gas containing $M = 2$ molecules, m and m', in a volume $2V$ ($= V_l$ and V_r). Corresponding to a given macrostate S, there are now 4 possible microstates: m and m' in V_l; m and m' in V_r; m in V_l, m' in V_r; and m' in V_l, m in V_r. Hence, the entropy of the system is:

$$(14) \qquad E = k \ln 2^M = k M \ln 2 = 2 k (0.693) = 1.386 k.$$

If we now halve volume $2V$ (eg. by compressing the gas into volume V_l), there corresponds to ma~~s~~crostate S only one microstate: m and m' in V_l. Hence, the entropy has decreased to a minimum.

$$(15) \qquad E' = k \ln 1 = 0$$

Thus, the <u>maximum</u> free energy which could be made available by halving the volume of a two molecule gas at constant temperature T is

$$(16) \qquad F' - F = 1.386 k T$$

Putting $0.693 k T = \Omega$, the <u>maximum</u> free energy which could be made available by halving the volume of an M-molecule gas at constant temperature T is given by a direct generalization of the above as:

$$(17) \qquad F' - F = M \Omega$$

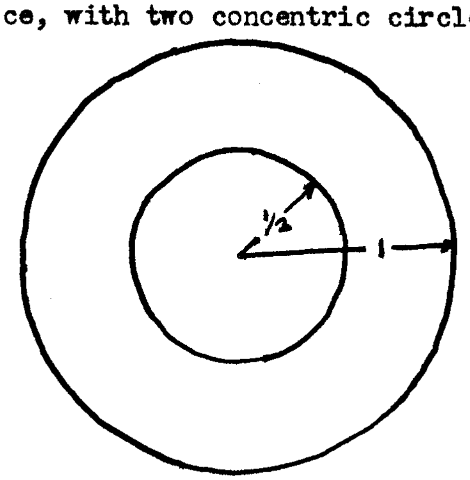
In the case where $M = 1$, the machine described above would effectively halve the gas by each insertion of the disk into the centered piston, for <u>all</u> of the gas (i.e. the single molecule) must then be on one or the other of the two sides of the piston. When $M > 1$, however, a random insertion of the disk into the centered piston would not usually halve the gas. (For example, if $M = 2$, a random insertion of the disk into the centered piston would halve the gas *iff* each of the two molecules were on the same side of the piston (i.e. *iff* the gas were in one of a particular two of the four equally possible microstates, the probability of such being only 1/2).) Before partitioning occurs, for a gas in the modified machine containing M molecules the number of possible microstates is 2^M. The act of partitioning would effectively halve the gas for only *two* of these microstates. Hence, the probability P_h that a random insertion of the disk into the centered piston would halve a gas of M molecules exactly is:

$$P_h = \frac{2}{2^M} = \frac{1}{2^{M-1}} \tag{18}$$

Thus, although the maximum possible free energy available from a gas of M molecules increases in *arithmetic* proportion as M increases (by equation (17)), the probability of achieving this maximum free energy thru the random operation of the machine decreases in *geometric* proportion as M increases (by equation (18)).

We therefore have two functions of M: a linear function, giving the maximum free energy; and an inverse power function, giving the probability of achieving the maximum free energy. Can we determine how these two functions will relate as M increases indefinitely?

Let me use a direct analogy from n-dimensional geometry. Consider a 2-dimensional space, with two concentric circles drawn on it of radii 1 and 1/2.



Since the area of the larger circle is π , the area of the smaller circle is $\pi / 4$, and $\pi - \pi / 4 = 3/4\, \pi$, the area contained between the two circles is 3/4 of the area of the larger circle. Thus, 1/4 of the area of a circle of radius r lies within a circle of radius r/2. Consider now a 3-dimensional space, with two concentric spheres on it of radius 1 and 1/2. The volume of the larger is $4/3\, \pi$, that of the smaller $1/6\, \pi$; hence, one finds that 1/8 of the volume of a sphere of radius r lies within a sphere of radius r/2.

Generalizing, the volume of a hypersphere of M dimensions is proportional to r^M, and, as a consequence, the fraction of the volume which lies in an enclosed hypersphere of radius r/2 is $1/2^M$. In a similar way, one could show the fraction of the volume of a hypersphere of radius r that lies within a radius of (m/n) r . In particular, one would find that, for a space of 1000 dimensions (M = 1000), the fraction of the volume of a hypersphere of radius r lying in an enclosed hypersphere of radius (99/100) r is approximately 1/2500. Thus, in general, in the case of a hypersphere of very high dimensionality, essentially all of the volume lies very near to its surface!

Consider now a gas of M molecules, and consider each molecule as a dimension of a hyperspace. The inverse of the probability P_h of achieving the maximum free energy is 2^{M-1}. Hence, $1/P_h$ is proportional to the volume of a hypersphere of radius 2 in a hyperspace of M - 1 dimensions. But 2 itself is proportional to M - 1 for any given M (i.e. 2 = n(M - 1), where n is some integer). Thus, $1/P_h$ is proportional to the volume of a hypersphere in a space of M - 1 dimensions having a radius proportional to M - 1. Now the maximum free energy $M\Omega$, by equation (17), is also proportional to M - 1, hence proportional to the radius of the hypersphere in the space of M - 1 dimensions whose volume is proportional to $1/P_h$.

Given our modified machine and a gas of 1001 molecules, therefore, (M - 1 = 1000), the probability of achieving even 1/100 of the maximum available free energy by a random partitioning would be roughly 1/2500. (For the volume of that part of the hypersphere whose radius is proportional to 99/100 of the maximum free energy is less than 1/2500 of the total volume of the hypersphere, where the total volume is proportional to the inverse of the probability P_h of halving the gas by a random partitioning. Hence, the probability is about 2499/2500 that one would achieve no more than 1/100 of the maximum free energy by randomly inserting the disk in the centered piston.)

The above argument, one ought to note, pertains to the use of many-molecule gases in the ideal modified machine specified above. Even under these ideal conditions (and for values of M far less than even Avogadro's number), the probability of ever deriving useful work from the machine rapidly approaches zero. Were we, therefore, to consider even the slightest energy leak in our machine due to friction, etc., as would surely needs be the case in any macroscopic situation, the probability of ever deriving useful work would effectively vanish.

The prohibitive improbability of ever achieving a disentropic result from a machine of the above sort on the macroscopic level is evident. (And, even if one were to detect a single disentropic result, the probability of being able to reproduce it again in a subsequent experimental pass would be so vanishingly small compared to the probability of having made an experimental error that one would surely be rationally compelled to ascribe the result to the latter.) But this prohibitive improbability on the macroscopic level ought not to be taken as presumptive evidence of a similar improbability on the microscopic level. Given the conceptual rigor of the ideal machine specified above, the possibility of analogous disentropic processes operative on the microscopic level cannot be disproved.

Indeed, part of the puzzling aspect of those microscopic biological processes which appear to be disentropic in at least some respect (eg. protein synthesis, enzyme mediation), not to speak of the fundamental puzzle of how life itself could emerge from (eg.) a primeval hydrogen soup, may be due to an incautious neglect of the above possibility. We have seen in what sense Wiener's suggestion is true, and in what sense false (and, hence, in what sense question (2) can be answered negatively, and in what sense affirmatively): Given the ideal modified machine specified above, although it is ideally possible for an observing apparatus A to be coupled to a system S such that the system S is observed to be acting disentropically, and such that the entropy of the coupled system A+S decreases, it is not possible for apparatus A to observe the internal workings of S and still maintain the disentropic status of A+S. In other words, S must remain a Black Box to A; for if A were to expend the energy necessary to acquire even the minimum of internal information about S, A could not have expended less energy than S produces disentropically - and hence the entropy of A+S would have increased, not decreased. The appropriateness of this result to the speculation (I think due to Bohr, though documentation escapes me) that perhaps life processes are in principle unobservable, for to observe them would be to destroy precisely that which makes them life processes, is both evident and suggestive.

But that's surely a topic for another day.

Evan Cameron

Evan Cameron
Boston University
June 1, 1969

(128 Glenville Avenue
Allston, Massachusetts
02134)

FOOTNOTE (to page 15)

1. Abner Shimony has privately raised the following objection, which is of sufficient general relevance to deserve immediate comment:

> "I have doubts about the possibility of achieving a cyclic process (as required for a violation of the 2nd Law), for information that the piston has reached the end of the cylinder would have to be transmitted to the mechanism which restores the piston to the center – surely this transmission would cost a bit!"

But, given the calculable average length of time required for the molecule to drive the piston to an end of the cylinder, if one considers that it is highly improbable that a given cycle would require a much larger length of time for its completion, one need only prohibit the restoring mechanism from attempting to perform its function until a time interval substantially greater than the average above has passed to make it highly improbable that the piston would not have reached an end of the cylinder – thereby avoiding the force of the objection.

Of course, one might then object that such a prohibition would require the restoring mechanism to tell time, in some manner or other, which ought surely to cost at least one bit. The crucial question, however, would then become: is it conceivable that a mechanism could take an average amount of time to attempt to perform its function without thereby having expended sufficient energy to have registered the time it took? For if it were so conceivable, one would then only need to conceive of the restoring mechanism as requiring a given average amount of time to attempt to perform its function which is substantially greater than the average length of time required for the molecule to drive the piston to an end of the cylinder to escape the objection. But the complexities of the latter question are beyond the limits of this paper.

REFERENCES

1. D'Abro, A., The Rise of the New Physics, Volumes I & II, Dover Publilications, New York, 1951.

2. Brillouin, Leon, Science and Information Theory, Academic Press, Inc., New York, 1956.

3. Grünbaum, Adolf, Philosophical Problems of Space and Time, Alfred Knopf, New York, 1963.

4. Pierce, J. R., Symbols, Signals, and Noise, Harper Torchbooks, Harper Press, New York, 1966.

5. Raisbeck, Gordon, Information theory: an Introduction for Scientists and Engineers, MIT Press, Cambridge, Mass., 1963.

6. Reichenbach, Hans, The Direction of Time (ed. by Maria Reichenbach), University of California Press, Berkeley and Los Angeles, 1956.

7. Resnick, Robert and David Halliday, Physics: for Students of Science and Engineering, Parts I & II, John Wiley & Sons, Inc., New York, 1960.

8. Shannon, Claude E., and Warren Weaver, The Mathematical Theory of Communication, University of Illinois Press, Urbana, 1964.

9. Singh, Great Ideas in Information Theory, Language, and Cybernetics, Dover Publications, Inc., New York, 1966.

10. Sommerville, D.M.Y., An Introduction to the Geometry of N Dimensions, Dover Publications, Inc., New York, 1960.

11. Weyl, Hermann, Philosophy of Mathematics and Natural Science, Atheneum, New York, 1963.

12. Wiener, Norbert, Cybernetics: 2nd Edition, MIT Press, Cambridge, Mass., 1965.