GAMES Practice Problems – Optimization

1. Create diagrams for the following functions in Excel and identify the maximum and minimums. Use your judgment to decide on the appropriate values for x which give a complete picture of the expression.

   (a) $z(x) = \dfrac{18}{2x^2 + 2}$

   (b) $w(x) = 3\,(x+1)^4 - 3$

   (c) $v(x) = \dfrac{2}{1 + 2x^4}$

   (d) $p(x) = \dfrac{-8}{4 + x^2}$

   (e) $H(x) = 2 - \sqrt{2 - x}$

2. Suppose Toronto, which once had the knickname *Hog Town*, can produce pork bellies, p, at a rate of $p(h) = -2 + h - \frac{h^2}{25}$ where h represents hours worked. Determine how many hours worked would maximize production.

3. Suppose the Fraser Valley in British Columbia yields $y(n) = -792.6 + 4n - \frac{n^{3/2}}{20}$ tonnes of lettuce. n represents potash fertilizer produced in Saskatchewan. The price of a tonne of lettuce is \$1500 and the cost of potash is \$200.

   (a) Determine how much potash would maximize profits.

   (b) Determine how your answer change would change if the yield function was $y(n) = -1292.6 + 4n - \frac{n^{3/2}}{20}$.

4. A monopolist has inverse demand function, $P(Q)$ where P is price and Q is output. The cost per unit of output is z.

   (a) Write down the profit function and use it to find the profit maximizing first-order condition.

   (b) Assume the first-order condition is $P(Q^*) + Q^*P'(Q^*) = z$ and recognize that the profit maximizing quantity, Q^*, is an implicit function of z. Use implicit differentiation to show how Q^* is affected by changes in cost per unit, z.

5. Suppose now the inverse demand function is $P(Q) = \alpha - Q$ and the cost per unit of output is z.

   (a) Find the profit maximizing output, Q*, and the associated monopoly profit, $\pi(Q^*)$

   (b) Find $\frac{\partial d\pi}{\partial dz}$ to illustrate how monopoly profit reacts to a change in cost per unit, z

   (c) Suppose the government wants the monopoly to produce $a - z$ units of output. What subsidy per unit of output, s, must the government provide to elicit $Q^* = \alpha - z$?

6. $$f\,(x) = \frac{32x}{2x^2 + 1}$$

   (a) Find all critical points. Assume the domain includes $-\infty \leq x \leq \infty$.

(b) Use the first derivative test and a sign diagram to evaluate all critical points.

(c) Use the second derivative test to verify your result in part 6b.

(d) Create a graph in Excel and identify any global maximum and minimum when $x \in \{-10, 10\}$. Does your result verify the earlier tests?

7. Consider a firm with the cost function $C(Q) = 0.02q^3 - 3q^2 + 120q + 1000$ That can sell its output at a price of \$120.

   (a) Use Excel or another spreadsheet program to calculate revenue, total cost, and profit for $Q \in \{0, 150\}$.

   (b) Create one graph illustrating revenue, cost and profit. What shape is revenue? Format the graph appropriately.

   (c) Studying your graph, how many units of output must be produced to earn a profit of \$5,000?

   (d) Which level of output maximizes profits?

   (e) Which level of output Earns a profit of zero dollars. In business, this level of production or sales is known as the breakeven point.

   (f) Suppose now the sale price decreases to \$100. Manipulate your spreadsheet to illustrate how the fallen price impacts optimal profit and output.

8. Consider the monopolists problem with $C(Q) = \frac{q^3}{500} - \frac{q^2}{3} + 50q + 4000$ and an inverse demand function of $p(Q) = 150 - \frac{Q}{3}$

   (a) Use Excel or another spreadsheet program to calculate revenue, total cost, and profit for $Q \in \{0, 215\}$.

   (b) Create one graph illustrating revenue, cost and profit. What shape is revenue?

   (c) Studying your graph, how many units of output must be produced to earn a profit of \$2,500?

   (d) Which level of output maximizes profits?

   (e) Which level of output Earns a profit of zero dollars. In business, this level of production or sales is known as the breakeven point.

   (f) Suppose the inverse demand function becomes $p(Q) = 150 - \frac{Q}{2}$. What happens to profit and the breakeven points?