Rational Expressions II

SUMMARY KEYWORDS

squared, answer, numerator, negative, equal, expression, multiply, exponent rules, write, factor, denominator, simplify, polynomial, cubed, term, question, quadratic formula, lowest common denominator, square, quadratic equation

SPEAKER

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Here we have a multivariate expression. That means that we've got more than one unknown variable, we've got x, which is an unknown variable y, which is an unknown variable, and W, which is an unknown variable. I like this question, because it's going to bring us back to our exponent rules. Remember our exponents rules, said that if we have x A divided by x B, that's equal to x A minus B. And if we have x A times x B, that's going to be equal to x A plus B. And notice that the base are is the same. To make this look a little bit clear, I can divvy it up so I can write this as four y squared divided by four w, y cubed plus eight, x to the six y to the five divided by four w y to the three. And I can do that just to make it a little bit easier to look at. And if I do that, I get the fours those just divide into one, I'm going to get why six minus three overdub w plus two x to the six y five minus three over w. And simplifying a little bit, I'm going to have y three plus two x six, y two. And that whole thing over who just brought the denominator together. And I can go a little bit further, I could factor out a y two. So I've got y plus two x six over w. Yeah, the question is asking us to simplify. So we want to simplify it as much as possible. So here I am on ALEKS, I'm going to plug in our simplification. So we have y squared, and then a bracket y plus two x to the power of six. And I'll hit write and make sure I get my brackets looking as correct as possible. And then maybe I'll highlight this whole thing, then I'm going to hit this button here and create a fraction for our W. And if I do that, and I click on the check button. Let's see if we get the right answer. And we did so we simplify it as much as possible. And we use those exponent rules that I just showed you. We're asked to simplify the following expression. One thing that makes this expression easier to simplify is that in the numerator, the denominator is already in its lowest common denominator form, that's the three x right here. And similarly in the denominator, it's already in its lowest common denominator form. Since that's that is the case, I'm going to start by putting it from its division form into its multiplicative form. So we're going to have the numerator remaining the same. And it's gonna be multiplied by the flipped denominator. So we're gonna have four x cubed in the numerator and 15 minus five x in the denominator. I see lots of things that we can do here to make this simpler, make this expression simpler. One thing, notice that in this numerator here, what can I do with that? Well, I can use the or write it in the difference of squares formula like that, we'll see what happens. We'll see what happens there. Maybe actually, what I'll do is I'll just take the difference of squares right away. So we've got three minus x three plus x like that. And notice that I've got an x here and three x is over there. So I could just rewrite this as three and four x squared up there. And then with the denominator down here, I've got a five and a 15. So I can factor out five, and I'm left with three minus x. Now we can see that this three minus x here and this three minus x, they're they're going to cancel out. And we're going to be left with three plus x over three times four x squared over five. Well, the last thing I think we can do is we could write this as four x squared, three plus x. So I haven't actually changed the numerator, I've just moved one factor to the left, and then the denominator three times five, that's 15. And I don't think we can simplify it any further. So why don't we put this into ALEKS and see if we have the right answer. So here we are an ALEKS, I'm going to enter the answer that we came up with, I've got four x squared, and create a bracket three plus x. And then I'm going to highlight this whole thing, press this button over here, the square with a line below it, and then another square below that line, and type in 15, click the check button. And we simplify this expression. Let's try some polynomial long division. Now, it's going to be a lot like the long division, you may remember from grade school, I'm going to write three x minus two, this term here, and I'm going to put one of these little tables like that. And I'm going to write the entire numerator inside it. And this is what's called an algorithm. So this is just an algorithm. It doesn't have any, there is no intuition why it works. It just works into an algorithm. It's like, you know, we take algorithms, we take a computer programs, you know, it sucks it, we do it because it works. And so if I want to know how many times the polynomial three x minus two goes into this larger, higher order polynomial, then I start by looking at the first term, that's three x minus two, and I asked myself, how many times is three x go into 12, x cubed, and so on basically saying, well, what's 12 x? three over three x? Well, it's equal to four x squared. So I'm going to write well, I'll write it up here, four x squared. Now I'm going to multiply this four x squared by the first term and the second term. And so I'm going to get and remember we negative negative 12 x cubed minus eight x squared, but that's a minus. And so it's gonna be a plus, because it's minus a minus is a plus. I draw a bar like that, as we always want 12 x cubed minus 12. x cubed is zero. Now I've got this negative 23. or excuse me, I didn't write it correctly, I should write it as x squared like that. plus eight, that's going to give us minus 15 x squared. Now, how many times does three x go into negative 15 X squared? Well, I'll write up here and I'll say negative 15 x divided by three x. Well, that's going to be negative five x So I'll write negative five x up here. Negative five x. multiplied by this thing. No, I better not, I don't want to forget my negative sign. I'm going to get plus 15. x goes to negative and a negative is a plus. That gives me zero. What about the next term, I've got negative five x multiplied by negative two. So I've got a negative and a negative and a negative. So that's gonna give me negative 10 X. Oh, I wrote in the wrong space, excuse me. Negative 10 x. And now I want to know what four x minus 10 x is equal to, well, that's going to be equal to negative six. How many times does three x go into negative six? Well, it goes in there negative two times. So I'll write a negative two up here. Now I've got negative two times three x, so that's plus six x. What about the last term, I've got negative two times negative two minus so a minus a minus and a minus is a minus. So I'm going to have minus four and plus one minus four is equal to minus three. Now, how many times has three x? Go into three x? Well, we're going to say zero times. And this here is it gonna be our remainder. And I'm going to call this thing here, our quotient. So let's plug these answers into ALEKS and see if we got the right answer. Here we are on ALEKS for x squared minus five x minus two. And our remainder is minus three. So I'll click the check button. And let's see if I did it correctly. And I did would have been very easy for me to make a sign error. So I'm a little bit relieved to see the correct button. Correct icon. But we did it. So our next question is asked us to solve for y. Normally, I would like to start with a lowest common denominator. But I'm going to approach this slightly differently. Because I see, I've got this y plus one, and I'd like to move it. So I'm going to multiply both sides of this expression by y plus one. When I do that, we get negative six is equal to negative six times y plus one minus y plus one over y minus one. Why did I do that? Well, now it's very obvious to me what I need to do to find the lowest common denominator here. If I multiply these two terms by y minus one, all the terms will have the same denominator. So why don't I go ahead and do that, when I multiply the numerator and denominator, these two by y minus one, I get negative six y minus one over y minus one is equal to negative six y plus one. And it's y negative one over y negative one minus y plus one, y minus one. And now that they all have the same denominator, I can get rid of them by multiplying this whole expression, both sides by y minus one. So we've got negative six y minus one is equal to negative six y plus one, y minus one minus y plus one. Now might not be obvious how to proceed. If I look at each of the terms, there are no common factors. So we've got a y plus one over here. And a y plus one over here, but there's no y plus one over there. So what I'm going to do is I'm going to open up the brackets, I'm going to multiply through the brackets. And after I do that, I'm going to collect like terms. So let's get started, we've got negative six y, plus one. And then here, I've got negative six, multiplied by, well, that's a difference of squares, I know that's going to be y squared minus one minus y minus one. And there's a hint and the question, which is that we're doing quadratic factoring. And so I'm going to set this question up as a quadratic equation, which means it should be equal to zero. So I'm going to move all the terms to the right hand side. So I haven't haven't finished and we open this bracket. So I've got six y squared plus six minus y minus one. And then I'm going to move these over, so I get plus six y minus one. And I can see right away that I made a mistake, I made a mistake. What mistake did I make, I was going too quickly. And that should have been a plus six. So I want to have a negative six over here. Now let me collect like terms. So there's only one y squared term, so we have negative six y squared. The six minus six is equal to zero. And I'm going to be left with plus five y minus one. And this whole thing, of course, is equal to zero. Now the question is asking us to solve for y. And now that we've got an answer, quadratic form, if we can factor it, or if we use the quadratic equation, we can find the solution for y. Now, we probably suspect, how many solutions are there going to be for a quadratic equation? Typically, there's two, there's going to be two answers for why. So looking at this expression, I want to factor it, I always find it challenging to factor an expression if the coefficient in front of the square term is not one. And in this case, it's negative six. So I can look at I can play around with it, I can do some trial and error to get the right answer. But you also might want to use the quadratic formula. To remind you the quadratic formula says that y is going to be equal to negative v plus minus the square root of v squared, I got rid of that minus sign in front of the B. And I've added the equation right there. So you could use this quadratic formula, and I'll label it to find the answer to the problem, I'm going to factor it by trial and error. And I know that three times two is equal to six. And I also know that three plus two is equal to five. So I'm good at right three, why here like that, I'm going to write to why they're like that. And if I want to get negative one, the only way I can get negative one is to have a plus one and a negative one. And if I want three y plus two y and I've got a negative three y, I'm going to need to have my negative sign there, my plus one there. And that's how you could factor this problem without using the quadratic equation. If you multiply this out, you're going to get the original result. Keep going for the answer. So the answer is, if this thing is equal to zero, or this thing is equal to zero, then that y that gets us to that result is the answer. So negative three y plus one is equal to zero. Well, then y must be equal to one, negative one third. And if we have two y is equal, or excuse me, two y minus one is equal to zero, then y is equal to one half. And so my answers are y is equal to negative one third, or y is equal to one half both these solve, solve the the equation. Well, I can see right away, because I'm plugging it in that I made a mistake over here, it should be y is equal to one third. when y is equal to one third, then we get negative one plus one, which is equal to zero, and y was equal to one half as well. So I almost made a mistake, but I caught myself. Now let's go to ALEKS and make sure that I've done this correctly. So here we are said if there is no solution, click on no solution. But if there's more than one solution, separate them by commas. So I have one over three as one of my answers. I'll provide a comma in there, and one over two, and I'm using my keyboard to navigate along. Now ready for the moment of truth. Let's click the check button and see if I got the answer. Correct. And happily, I did.