Rational Expressions I

SUMMARY KEYWORDS

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SPEAKER

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Hello everyone, Robert j McKeown here and welcome to another ALEKS walkthrough video This one is number six. And we're going to look at rational expressions, which I think are very intelligent, smart sounding kind of expression. Now, the specific what a what a rational expression is, I'll tell you in a few moments when we get into the slides. As an economist, we're often trying to rearrange an expression to get a certain result or to solve for an unknown variable. Like for example, we might want to solve for the price of a product, like gasoline or smartphones, or software products, or we might want to solve for quantities produced similar items. And so it's really, really difficult to get away and not use mathematics, and economics. And I really like today's topic, because it gets into the real nitty gritty of what you're going to be doing. As an economics undergraduate major, you're going to be rearranging expressions, you're going to be trying to show certain results. And that's what today's practice is all about. So becoming a master of this topic is going to pay off for you in every one of your core economics classes, from here to the end of your undergraduate career, and then into grad school, as well. And being good with math is a great job skill, a very employable skill to have. Now, again, what do I need from you, I need you to have your pencil, I need you to have your paper. Or if you're very sophisticated, and you have the money, invest in a tablet that you can write, and work through the problems with me. For this topic, and through this video, remember that the video doesn't cover every important topic in ALEKS, topic six, or every concept in topic six. I'm just trying to get you enough information to make sure that you get started and help you through maybe some of the more difficult problems. So let's get started. Topic six is all about rational expressions. So what is a rational expression? Well, it's when one polynomial, such as X plus five is divided by another polynomial, such as x squared minus nine. And remember, we seen a lot of quadratic equations quadratics quadratic expressions are also a type of polynomial. Now every question in this section involves rational expressions. To start off, why don't we talk about the domain of rational expression? So in an earlier topic, I told you that we must never ever ever divide by zero. And if you look here, it says the domain of any rational expression is all real numbers, except for those that make the denominator equal to zero. Now, why is this important? Let me show you an example. Here, we have x plus five divided by x squared minus nine, I can rewrite that using the difference of squares. Whenever I say something squared minus something else that's squared, I could I can use the difference of squares. For example, I'll rewrite this as x squared minus three squared, three squared is equal to nine. So I'm going to write this as three squared. I know from this rule, that X plus five is divided by x minus three, multiplied by x plus three. And you can prove to yourself at home that x minus three times x plus three is equal to x squared minus three squared or x squared minus nine. Now we know that all real numbers are in the domain, except for those real numbers that make the denominator equal to zero. So using this right here, it's easy to see that if x is equal to three Then we have three plus five, divided by three minus three, multiplied by three plus three, which is eight, over zero over six, which is equal to eight over zero. And I told you that we should never divide by zero, because we never observe a real number like eight divided by zero, so we just don't really know what that is. And so we say it's undefined. We don't know what it is. We can't define what eight divided by zero is. Similarly, if x is equal to negative three, what happens? Well, we get negative three, plus five over negative three minus three times negative three plus three. And this time we get negative two, or excuse me to over zero. So both x is equal to three, and x equal negative three are not in the domain. Here's a ALEKS question. And I've copied and pasted from ALEKS and tells us that find all the values of x that are not in the domain of g. So I can take a look at this thing here, I can do some factoring. So for these questions, the numerator never matters. It doesn't influence the domain. But I could look at this and I could say, well, I noticed that eight times nine is equal to 72. And negative eight minus nine is equal to 17. And when I recognize that, I can factor the numerator. So looking at the denominator, I see my favorite x squared minus two squared is my difference of squares. A very useful formula. It comes up a number of times, including finance. And I can write this as x plus two, and x minus two. Now, I didn't need to do that I, you could probably just look at the original expression and answer this question. But you'll see later questions, we need to be able to do these operations. So I'm going to show you now. And it's pretty obvious here that x equals two and x equals negative two are not in the domain, because when x is equal to two, or x is equal to negative two, the denominator is zero. Here we are an ALEKS. I'm going to put in our answers, we had negative two. And then I'm going to follow the instructions, put in a comma and a two. And I'll click on the check button. And fortunately, we got the right answer. Here's a great question. We have a complex fraction and rational expression but a complex fraction. And when I see a problem like this, I want to start off as my first step to find a lowest common denominator for the numerator. And then I want to do the same thing for the denominator. And after I do that, I'll think about what my next steps are. I don't like to see non common denominators. So that's something that I like to jump on, and fix right away. So starting with the numerator, I've got x squared on one side and 49 on the other. So for this first term, eight over 49, I'm going to multiply it by x squared over x squared, or I'm gonna multiply both the numerator and denominator by x squared. That way, I'm following the rules of algebra x squared. divided by x squared is just equal to one, I'm not changing, I'm not changing the expression in any way when I do this, that's the whole idea behind lowest common denominator. Now looking at the second term in the numerator, I need a 49. I'm going to multiply the top by 49 and the bottom by 49. And now I've got a common a common denominator, I could rewrite that as x squared minus 49. All over 49 x squared. Now, what about the denominator? Let's take a look at the denominator, one over seven, well, I can just multiply that by x. So I'm going to have x over seven x plus one over x, I can multiply the numerator by seven and the denominator by seven. And now I've got x plus seven divided by seven x. Now what's the rule when we're dividing two fractions? Well, the rule the operator is to flip the denominators, numerator and denominator and then multiply it by the numerator, what's the top so I can rewrite this as eight x squared minus 49 multiplied by, well, maybe I'll write it out like this, I'll just rewrite it like that. And this whole thing is going to be multiplied by seven x over x plus seven. That's the rule. We saw that in an earlier video. That's the catch how we deal with division. I like the way that I've written it here, I can clearly see what's being multiplied and what's being divided. And I want to pay particular attention to the division because the division is going to what we say cancel things out. I can make a simplification here. And I can rewrite this expression as eight x squared minus 49 over seven x squared multiplied by seven x over x plus seven. And we can cancel out seven x divided by seven x. And I'm going to be left with eight x squared minus 49 over seven x, x plus seven. So it was this exponent that was cancelled out previously. Not all the seven x's. I went ahead and I input our answer into ALEKS. And you can see that it was the simplest form possible. There wasn't anything more we could really do. There's a 49 there, which is seven squared. But if we rewrote it that way, there's we can't simplify any further because that eight, the eight in front of the x squared is stopping us from doing any more factoring. Our next problem is asking us to solve for w. So we've got a rational expression. And there's three separate terms. Like before, I want to get started by finding a lowest common denominator. So notice that if I call that a and I call that B, A is equal to two B's. So if I multiply the numerator and denominator of this term here by to have the same denominator and I won't be altering the value of that term. I'm just feeling this in so you can see two w minus 12. Now I've got this negative one term here. Well, if I want to give it the same denominator, As the other two terms, two w minus 12, I can multiply it, and its numerator denominator, because of course, negative one is equal to negative one over one, I can multiply its numerator and denominator by two w minus 12. So if I do that, I'm gonna get to w minus five over two w minus 12, you can see very clearly that that's got to be equal to one. And I don't want to forget my negative sign in front. The last term on the left is unchanged, it's still three over two w minus 12. There's a big minus right there. Now the great part is, since the denominator of every single term is the same, I could multiply both sides of this equation by two w minus 12. And if I multiply both sides of this equation by two w minus 12, I'm going to be left with negative three minus two w minus 12 is equal to 14. And you want to be really careful that you don't make a mistake with your signs. Very, very easy mistake to make one that I make fairly regularly. not unusual that I would forget a sign here or there. Given that reality in which I operate, I'm very careful to keep my sign negative on the outside of this bracket, because now I want to bring that negative into the bracket. So I'm going to rewrite this as minus three minus two w plus 12 is equal to 14. Now notice that I've just got a W and a bunch of numbers. And so this is now linear. It's a linear equation or a linear expression. And that means I'm going to get one solution. Now I made a small mistake, I forgot that I forgot a negative sign. I forgot this negative sign partly because I didn't draw it. Clearly, I should have a negative 14 there. Now I want to finish this off and solve for w. So I'm going to have negative two w is equal to negative 14 minus 12 plus three, and I get w is equal to 23 over two