# SUPPORTING PRODUCTIVE STRUGGLE IN DEVELOPMENTAL MATHEMATICS

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**Abstract**

This dissertation highlights the potential of productive struggle in addressing the issues of teaching and learning in developmental mathematics. This research presents a course designed to support productive struggle, empirical findings on students' experiences and conceptions, and my own experience supporting students' struggles.

The design of the course is oriented towards supporting productive struggle by engaging students with tasks that elicit uncertainty. Instruction was delayed, providing an opportunity to promote self-explanation as students explained and questioned their thinking with a partner. As the course instructor, I asked purposeful questions during students' engagement with the tasks to show students that struggle is a necessary part of learning. This environment is in stark contrast to skill-and-drill instruction often found in developmental mathematics classrooms.

Empirical findings suggest that students experienced and conceptualized struggle and productive struggle in various ways. Significant to the findings was the connection to deep approaches to learning, persevering, positive affective structures, and habits of mind. Through phenomenography, semi-structured interviews were conducted, data was collected, and students' experiences and conceptions were analyzed. The findings bring focus to the affective nature of learning, a facet infrequently explored in developmental mathematics. More importantly, these findings starkly contrast with students' reliance on rote memorization often reported in developmental mathematics classrooms.

I engaged in the Discipline of Noticing to investigate my experience of supporting productive struggle. The methodology presented in this study acts as a form of professional development that simultaneously produces research for others to test in their own practice. This systematic inquiry

into my practice contributes to the underrepresented area of self-based methodologies to understand instructors' learning in mathematics teacher professional development. Deliberately honing my skill of noticing enhances the choices that can come to mind in my future practice of supporting productive struggle.

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# Chapter One – Introduction

## Researcher's Background

Helliwell (2019, p.2) wrote, "Behind every piece of research lies at least one human being. Sometimes there is a personal motivation for research activity, other times it is a professional one." Helliwell's quote sums up the forces from which my research emerged. Central to this dissertation is the need to better understand and improve my teaching. Thus, sharing how I came to this research problem provides a basis for considering what I learned from this process.

I first started teaching developmental mathematics during my Master's in Applied Mathematics. Hired by two colleges in Ontario, I taught math courses to business, engineering, and architecture students. Without any pedagogical training, I taught the way I experienced math classrooms as a student: through a traditional lecture format. Students sat in rows and columns, listened, and copied notes as I presented the material.

My lessons comprised a sequence of examples worked out on the board in front of the class. I believed that students could mimic how I worked through problems, as I articulated my thinking during examples in front of the class. I thought presenting various examples targeting different misconceptions would warn students what to watch out for, thus removing obstacles they may encounter. My conception of the lecture was shared with authors who viewed the lecture as a stage, and I played the role of an actor (Charlton, 2006; Hodgson, 1997). I believed the flair with which I presented would capture students' attention and motivate them to work on mathematics. Jokes were told, different colors were used during examples to distinguish between operations, hand motions pointed out key features, and I altered the tones of my voice to captivate the audience. In my mind, I could not imagine why a student would not attend my class, and after attending my class, why would they not do the assigned homework to be ready for the test?

Effort, in my view, was the key to academic achievement. I believed students who scored high on tests put in time and effort. Those who did not achieve the grade could have benefited from more practice. The correlation between effort and achievement was primarily based on my personal experience. When I prepared and put in the time, I got good grades. Through this view, I did not consider the effort required to struggle with mathematics, seek help, or understand mathematics more deeply.

Through repetition, I felt I was perfecting my teaching. Students' tests revealed more common misconceptions, which I converted into more worked examples to show students what to watch out for. The repetition habituated my actions to automatically address certain misconceptions and critical elements for each mathematical topic. However, a contradiction to my beliefs was beginning to propagate. If my teaching was improving, why were students not more engaged? Also, why did a growing number of students with consistent effort (put in the required time) fail? I realized I was missing something, but I did not know what.

I sought answers by continuing my education, this time in the field of education. With the recommendation from my Master's supervisor, I enrolled in a stand-alone graduate diploma in mathematics education. A remark from my supervisor, "It would be good for your development to understand the educational point of view," was initially interpreted as advice for me to succeed in courses from the Faculty of Education. Little did I know it would completely change my views of teaching, especially how effort is directed.

Exposure to educational literature started to change my practice. Theories of mathematical understanding, affect, mindsets, perseverance, engagement, and diversity made me reconsider how I taught. For instance, I allotted more class time for students to show their work and understanding. I no longer assumed that students' efforts could be productive from sitting in my class and taking notes of what I was directing them to do.

Examples were still worked out at the beginning of class, but students were allotted time to try examples independently and ask questions on mathematical concepts. Watching students solve problems gave me a better grasp of how they directed their efforts to understand topics, providing opportunities for me to answer or address their concerns. However, I still dominated the time, showing examples I thought students should know. One reason for my reliance was to meet the demands of a condensed mathematics curriculum.

My doctoral studies inspired me to take on a student-centered approach. However, I was experiencing obstacles in implementing research in my developmental mathematics classrooms. For example, works by Glaserseld (1991) and Piaget (1975) convinced me to provide opportunities for students to construct mathematical knowledge. However, engaging students in learning experiences required more class time, which was limited due to a dense curriculum. Students had limited time to understand mathematical ideas since each class introduced a new topic. There was also a growing number of interactions with students who held negative views towards math or had only experienced passive mathematics learning environments. Students' experiences made them resistant to different teaching actions. For these reasons, I turned to the literature to seek possible solutions, from which I found research on productive struggle. Implementing the literature within the constraints of developmental mathematics (defined in the next section) became the motivation for this dissertation.

**Research Context**

There is serious concern that developmental mathematics is a barrier to educational opportunity (Bonham & Boylan, 2011; Zientek, Lane, Sechelski, & Shupp, 2022). *Developmental mathematics* is a sequence of required courses for students needing to build their knowledge in mathematics. A large number of students are placed in developmental mathematics courses each year, but most fail to complete these courses, restricting them from further educational advancement (Fong, Melguizo, & Prather, 2015; Zientek, Fong, & Phelps, 2019). Many students enter college weak in mathematics content knowledge and lacking the skills needed for academic success (Bailey, 2009; Bickerstaff & Edgecombe, 2019; Merseth, 2011). However, in these courses designed to support students' learning, research has found that students lack an understanding of mathematical concepts, and instructors rely heavily on transmission models of teaching (Grubb et al., 2011; Stigler, Givvin, & Thompson, 2010a; Weiss, Scrivener, Slaughter, & Cohen, 2021). As a result, students are forced to withdraw from their programs because they do not meet the requirements (Fong et al., 2015). The purpose of developmental mathematics was to prepare students for the next stage in their careers, yet it has become a "graveyard of dreams and aspirations" (Merseth, 2011, p.32).

Productive struggle supports understanding of mathematical concepts as students grapple with challenging tasks. *Struggle* refers to students' efforts to figure out something that is not immediately apparent (Hiebert & Grouws, 2007). *Productive struggle* is the positive outcome of struggle when students' efforts deepen their understanding of mathematics (Warshauer, 2015a). Hiebert and Grouws (2007) claim that for students to understand mathematics, it is necessary for them to struggle with concepts, procedures, and ideas. *Supporting productive struggle* is a pedagogical approach encouraging students to engage deeply with challenging mathematical concepts, allowing them to actively grapple with problems and obstacles (Warshauer, 2015b). The NCTM (2014) considers supporting productive struggle one of the core elements of quality mathematics teaching, as seeing struggle as a necessary part of learning prepares students to succeed in college, career, and life.

Supporting productive struggle has the potential to address classroom issues in developmental mathematics. Supporting productive struggle contrasts with instruction typically found in developmental mathematics, discouraging activity in the classroom where students are expected to memorize rules and procedures (Grubb, 1999; Weiss et al., 2021; Wladis et al., 2020). This prevalent mode of instruction has resulted in poor success rates where only half of developmental mathematics students in the U.S. complete the sequence of courses (Cox, 2015; Fong et al., 2015; National Academy of Sciences, 2019). Canadian research on developmental mathematics is less extensive, but a study conducted by the College Student Achievement Program reported poor developmental mathematics success rates and recommended that colleges enhance their faculty development and teacher training programs to improve student learning outcomes (CSAP/PREC, 2015). Research has also reported that students are anxious about math (e.g., Jameson & Fusco, 2014; Maciejewski, Tortora, & Bragelman, 2021), lack motivation and belief that they can succeed in math (e.g., Zientek et al., 2019), and are only able to call upon memorized rules and procedures (e.g., Stigler et al., 2010a; Wladis et al., 2019). As a result, researchers (e.g., Benken, Ramirez, Li, & Wetendorf, 2015; Kozeracki, 2005; Zientek et al., 2019) have stressed that developmental mathematics instruction should support students to gain a more positive view towards learning mathematics. This view is further stressed by Zientek et al. (2019), as they found that students' self-efficacy was negatively impacted due to a history of mathematics failure. The authors recommended that lessons provide tasks at an appropriate level of challenge to cultivate new experiences. Zientek et al.'s call echoes the National Council of Teachers of Mathematics' (NCTM, 2014) advice on how productive struggle should be supported in K-12 classrooms to build positive experiences with mathematics. However, few studies (e.g., Bickerstaff & Edgecombe, 2019; Edwards & Beattie, 2016) have focused on supporting productive struggle of developmental mathematics students. These reasons emphasize the need for further investigation into the impact supporting productive struggle has on the developmental mathematics classroom experience.

Supporting productive struggle can be a much-needed shift for developmental mathematics instruction. Little research exists on the quality of developmental mathematics instruction (Mesa, Celis, & Lande, 2014; Nabors & Zientek, 2023). The sparse body of research has pointed out that instructors are under tremendous pressure to cover content so their students can succeed (Mesa, Celis, et al., 2014; Mesa, Shultz, & Jackson, 2020). One outcome of this pressure is an over-reliance on drill-and-practice procedures that fail to engage students in meaning-making activities (Cox, 2015; Grubb et al., 2011; Weiss et al., 2021). Without meaning, students orient toward memorizing rules and applying them with little understanding (Grubb & Gabriner, 2013; Wladis et al., 2019). This often results in students incorrectly applying procedures because they rely on what they can recall (Stigler et al., 2010a; Wladis et al., 2019). Furthermore, only being able to recall rules makes students inflexible to apply their knowledge when needed in future courses (Cox, 2015; Stigler et al., 2010a; Wladis et al., 2019). This instructional approach contradicts research-recommended practices to support productive struggle, where they grapple with key mathematical ideas through problems that do not have immediately apparent solutions (Baker, Jessup, Jacobs, Empson, & Case, 2020; Hiebert & Grouws, 2007; Kapur, 2016; Warshauer, 2015b). Despite research recommendations, the question remains of how instructors implement these supports within the pressures of developmental mathematics.

Effective professional development supports instructors' learning through ongoing opportunities to reflect on their decisions and actions (Desimone, 2009; Guskey, 2000). Most developmental mathematics instructors do not have any pedagogical background (Bailey, 2009; Bickerstaff & Edgecombe, 2019; Grubb & Gabriner, 2013), and colleges address this gap by offering professional development. However, the opportunities are typically in the form of one-shot workshops focusing on best practices (Bailey, Smith Jaggars, & Jenkins, 2015; Edwards, Sandoval, & McNamara, 2015). Even if instructors accept practices like productive struggle, supporting students' learning requires them to know how to act moment by moment in the classroom (Mason & Davis, 2013). For example, instructors' decisions, actions, and the timing of the response during students' struggles. That is why the Carnegie Community College Pathways Faculty Support Program (Edwards et al., 2015) designed continuous learning to support instructor change, and part of that learning includes how instructors support students' productive struggle. It is not a surprise that the prevailing models of one-stop workshops have done little to influence instructional change (Bailey et al., 2015). One-stop workshops often neglect the years faculty bring on what they consider teaching (Cohen, 1990). Additionally, workshops tend to promote best practices rather than explore in-the-moment decisions instructors make in practice (Edwards et al., 2015). As a result, faculty are less likely to accept new instructional strategies (Cohen, 1990; Mesa et al., 2020). Change comes from oneself (Mason, 2002) and not by transmitting knowledge to faculty in workshops. Thus, there is a need for effective professional development that motivates faculty to seek different possibilities in their instruction.

A solution to professional development of developmental mathematics instructors can be found in the work of mathematics teacher professional development. This field is interested in how mathematics teachers learn and develop (Chapman, 2017; Jaworski, 1998; Matos, Powell, Sztajin, Ejersbo, & Hovermill, 2009). Researchers (e.g., Attard, 2017; Mason, 2002; Schön, 1983) have found that meaningful change can occur by reflecting on one's own teaching. This is because the change comes from within, compared to an outsider suggesting changes during a professional development opportunity. More importantly, this dissertation offers an underrepresented perspective of myself as a developmental mathematics instructor researching one's own learning. Tinberg, Duffy, and Mino (2007) found that developmental mathematics instructors rarely have the opportunity to explore this view to change practice. This critical view contributes to research currently clarifying the knowledge, skills, and beliefs for educators to succeed in the classroom (Chapman, 2021). The field has typically focused on the teaching of prospective teachers or instructors of advanced mathematics content in higher education (Goos & Beswick, 2021), and Chapman (2021) encourages research-based evidence from other groups because the current body is not yet sufficient. Mason's Discipline of Noticing (2002) offers a robust methodology to investigate professional development through one's experience. Enacting Mason's methodology in this dissertation contributes a much-needed voice of one's own learning and development to the field of mathematics teacher professional development (Chapman, 2008; Matos et al., 2009).

**Research Questions**

This dissertation investigates the impact of struggle and productive struggle by better understanding students' experiences and conceptions, and my experience as the instructor. The following research questions embody these purposes: *How do struggle and productive struggle impact the classroom experience in developmental mathematics?*

a. In what ways do students experience and conceptualize struggle and productive struggle?
b. How do their experiences and conceptions of struggle and productive struggle impact their learning?

c. What accounts of struggle and productive struggle did I notice as the instructor of the course?

d. What did I learn from my accounts of noticing struggle and productive struggle?

**Significance**

Supporting productive struggle can address the distressing picture Stigler, Givvin, and Thompson (2010a; 2010b) reported of developmental mathematics students' fragile and weakly connected understanding of mathematical concepts. Well-documented in grades K-12 is that productive struggle promotes learning mathematics with deep understanding (Baker et al., 2020; Manalo & Kapur, 2018; Murawska, 2018; Murdoch, English, Hintz, & Tyson, 2020; Warshauer, 2015a; Zeybek, 2016). Research on the potential impact of supporting productive struggle in college developmental mathematics is sparse. More importantly, this study investigates the impact that productive struggle may have on prevalent non-cognitive issues faced by students, such as poor relationships with math and lack of motivation that factor into poor grades (Fong et al., 2015; Gula, Hoessler, & Maciejewski, 2015; Zientek et al., 2019). Many productive struggle studies based on K-12 mathematics have focused on cognitive aspects (e.g., cognitive demand during tasks), leading to Murdoch et al. (2020) calling for more attention to the affective dimension (e.g., uncertainty, confusion, or difficulty felt by students). This study extends the literature into college developmental mathematics and responds to Murdoch et al.'s (2020) call by amalgamating the literature on productive struggle with research on approaches to learning, perseverance, affect, and habits of mind. Developmental mathematics students' experiences and conceptions of these impacts are significant, as it is evidence that students found meaning in their learning (Marton, Dall'Alba, & Beaty, 1993), amended their efforts in challenging situations (Middleton, Tallman, Hatfield, & Davis, 2015), described positive affective structures (DeBellis & Goldin, 2006), and developed habits that extend beyond a developmental mathematics course (Cuoco, Goldenberg, & Mark, 1996).

Self-study can contribute to knowledge for mathematics teacher professional development. Researching one's own practice encourages instructors to deepen their awareness of their instructional actions, decisions, and judgments in consideration of students' learning (Jaworski, 1998). Mason (2002) states that making explicit teaching processes in a systematic manner contributes knowledge that can be looked at critically. In turn, a public conjecturing atmosphere is undertaken "in order to generate response in others, to test for resonance, to trigger other perspectives" (Mason, 2002, p.163). This study contributes a much-needed voice of the instructor in the developmental mathematics setting (Mesa et al., 2020; Nabors & Zientek, 2023) to the conjecturing atmosphere. It is reasonable to expect the specific setting of a developmental mathematics classroom to produce different perspectives of teaching and learning than K-12 grades and higher education in universities (Mesa, Wladis, & Watkins, 2014). This self-study accounts for how I, the course instructor, became more aware of supporting productive struggle in developmental mathematics, where many (e.g., Cox, 2015; Grubb et al., 2011; Merseth, 2011) have expressed concerns about the quality of instruction. These accounts from my practice can contribute to a basis for designing professional development interventions and further research on mathematics teacher professional development (Herbst & Chazan, 2023; Simon & Tzur, 1999).

## Format of Dissertation

This dissertation is outlined in the following manner. In Chapter 2 – Relevant Literature, further arguments are made on why productive struggle should be considered in developmental mathematics, how students can be impacted by productive struggle, and how I, as the course instructor, can learn from my own practice. The specific lens through which I will argue my claims is described in the Conceptual Framework in Chapter 3. In Chapter 4 – Methodology and Methods, phenomenography (Marton & Säljö, 1976), the chosen methodology to investigate students' experiences, is described, along with how the Discipline of Noticing (Mason, 2002) embodies learning from my experience, as the instructor supporting students' struggles. Chapter 4 includes the design of the course supporting productive struggle. This study contains two parts to investigate productive struggle in developmental mathematics. The first part brings forth students' experiences and conceptions of struggle and productive struggle from the course. The second part focuses on my experience supporting productive struggle. The various ways the students experienced and conceptualized struggle and productive struggle and what I noticed and learned are presented in the findings (Chapter 5). How the findings connect to literature is presented in the discussion in Chapter 6. The dissertation concludes with the limitations, suggestions, and avenues for future research.

# Chapter Two – Relevant Literature

The literature reviewed in this section describes the classroom issues in developmental mathematics. Supporting productive struggle can be a solution to reported issues, as it positively impacts students' experiences of learning mathematics. Furthermore, the literature calls for the need to improve developmental mathematics instruction (Edwards et al., 2015; Mesa, 2012; Severs, 2017). Research (e.g., Attard, 2017; Mason, 2002) suggests that a meaningful way to improve instruction is through a systematic inquiry into one's practice. This form of professional learning enhances the quality of instruction while producing research from an insider's perspective (Mason, 2002).

## College Developmental Mathematics in Canada

It is necessary to discuss how literature defines developmental mathematics, its aims, and its role in Canada, specifically in Ontario. Developmental education aims to help students deemed underprepared for their degree/diploma courses achieve their academic goals (Boylan, 1999). Developmental mathematics is one part of the field of developmental education and was previously known as remedial mathematics (Arendale, 2005). The term *remedial*, which implied faulty study habits and a need to raise competency, was replaced with *developmental* to emphasize building on what students know (Arendale, 2005). Developmental mathematics aims to build students' knowledge through a sequence of courses that range from numeracy and arithmetic to pre-calculus topics such as functions. Meaning many students are of adult age having to retake mathematical concepts they previously encountered in K-12 mathematics (CSAP/PREC, 2015). Most students enter these sequences of courses based on an assessment that is part of their college admissions requirement (CSAP/PREC, 2015). The assessment determines whether students require extra course(s) to 1) develop mathematical content knowledge necessary for their career path after college or 2) develop mathematical knowledge required for their first mathematics course in their diploma/degree (e.g., calculus, mathematics for finance, statistics) (CSAP/PREC, 2015). In other words, students enter developmental mathematics based on an assessment, and then they have to take course(s) that prepare them for mathematics specific to their diploma/degree.

Canadian developmental mathematics is typically situated in the college system, which is different from the higher education system in the United States. Canadian provinces and territories have two systems of higher education: colleges and universities (Jones, 2009). Canadian colleges vary in their role within each provincial education system in Canada (Jones, 2009). Some colleges are similar to community colleges' role in the United States. For example, Ontario colleges focus on technical/vocational training, but colleges and universities were designed as parallel pathways; thus, few programs prepared students for university transfer (Jones, 2009). The pathways from Ontario colleges to universities are growing due to demand, but the transfer from college to university remains low (2.03%) compared to the entire post-secondary population based on a recent study (Zarifa, Sano, & Hillier, 2020). Another difference is that it is atypical for a college mathematics instructor in Canada to be expected to conduct research (Maciejewski & Matthews, 2010). This is part of the reason that research in developmental mathematics is burgeoning in the United States, but research out of Canada is sparse. In Canada, funding has treated colleges as teaching institutions without research requirements, while universities have assumed a core research function (Panacci, 2014). As a result, Canada's college education system has led to very few studies focusing on teaching and learning in developmental mathematics.

### ***Issues in Developmental Mathematics***

The effectiveness of developmental mathematics has become a hot debate topic (Bahr, 2008; Bailey et al., 2015; Boylan & Bonham, 2007; Fong et al., 2015; National Academy of Sciences, 2019). Only half of U.S. students complete the sequence if they are assessed to require two developmental mathematics courses (Fong et al., 2015). The same study found that the success rate drops to 27% if a student has to complete four developmental mathematics courses. The issue extends to Canada, as one-third of developmental mathematics students are in jeopardy of not completing their program due to poor academic achievement in their first developmental mathematics course (CSAP/PREC, 2015).

Issues in developmental mathematics permeate the classroom, where students often rely on rote memorization of mathematical procedures (Bickerstaff & Edgecombe, 2019; Wheland, Konet, & Butler, 2003). Students believe that they have learned mathematics when they can recall procedures on a test (Safford-Ramus, 2008). This belief is reinforced by teaching in developmental mathematics, where it is typical for lessons to be presented as step-by-step actions that students must memorize without promoting the connections between topics (Bickerstaff & Edgecombe, 2019; Stigler et al., 2010b). This form of teaching can devoid mathematics learners of meaning and emotion, and from seeing how mathematics applies to their career path (White-Fredette, 2010). As a result, some students view mathematics only as courses they need to pass to graduate (Stephens, 1988).

Developmental mathematics instructors often lack pedagogical training and rely on transmitting information (Bailey et al., 2015; Bickerstaff & Edgecombe, 2019; Grubb & Gabriner, 2013). Instructors over-rely on content-focused knowledge transmission delivery models in classrooms (Bailey et al., 2015; Bickerstaff, Raphael, Zamora, & Leong, 2019; Grubb, 1999; Kember & Gow, 1994; Miles, 2000). The delivery tends to focus on factual and procedural knowledge instead of mathematical understanding, such as presenting worked examples for students to copy and memorize (Mesa, 2011). Grubb and Kalman (1994) describe this common experience resulting from the deficit model of developmental mathematics when students are evaluated by what they cannot do. Entrance assessments reinforce that students need to make up knowledge through these courses, and skill-and-drill instruction, when students have "almost no choices and in which they are recipients of another's knowledge convey the sense that they are ignorant" (Grubb & Kalman, 1994, p.71). Instructors open to learner-centered approaches often doubt that the approach works for students they perceive as weakly prepared (Edwards et al., 2015; Grubb & Gabriner, 2013). These doubts are partly due to the background and employment status of the faculty. Although most faculty have a Doctorate or Master's degree in mathematics, they often do not have any pedagogical training (Bickerstaff & Edgecombe, 2019; Grubb, 1999; Maciejewski & Matthews, 2010). Pedagogical learning opportunities are often forsaken as 76% of the faculty comprises adjunct and part-time faculty with heavy teaching loads spread over multiple campuses (Center for Community College Student Engagement, 2014; Maciejewski & Matthews, 2010). For these reasons, the lack of pedagogical training has often led to an over-reliance on transmission modes of teaching, compounding classroom issues in developmental mathematics.

### Productive Struggle for Developmental Mathematics

Supporting productive struggle can tackle teaching and learning issues in developmental mathematics. Hiebert and Grouws (2007) state that struggle is necessary for learning mathematics, as it provides students with opportunities to reconfigure mathematical concepts to deepen understanding. The authors define understanding as the mental connections among mathematical facts, ideas, and procedures. By struggle, Hiebert and Grouws (2007) mean that students expend effort to make sense of mathematics that is not immediately apparent. This definition is important to the context of developmental mathematics, as students attribute failure to a lack of effort but often procrastinate as a defense mechanism to not feel so bad if they do fail (Boylan & Nolting, 2011). Students then do not persist due to their previous failures, and eventually withdraw from their courses or programs (Bahr, 2012; Boylan & Nolting, 2011). In contrast, conceptualizations of productive struggle share an emphasis that students dive deeper into understanding mathematical concepts by engaging in challenging situations (Granberg, 2016; Hiebert & Grouws, 2007; Warshauer, 2015a). For example, the National Council of Teachers of Mathematics (NCTM) states that productive struggle encourages students to dive "more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct answers" (NCTM, 2014, p.48). Seeking correct answers is a prevalent issue in developmental mathematics (Stigler et al., 2010b; Wladis et al., 2019). For instance, students often rely on calculators and phones to look up correct solutions, detracting from reasoning with mathematical ideas (Boylan & Nolting, 2011). Students may have spent years working on mathematics without focusing on reasoning, believing mathematics means memorizing rules, procedures, and formulas (Stigler et al., 2010a; Wladis et al., 2019). Unable to understand the connections between procedures, students have little to fall back on when they cannot recall rules and formulas (Stigler et al., 2010b). Permatasari (2016) reports students understood the connections within mathematical concepts and were more able to apply their learning in future contexts as they learned to struggle productively. The presented differences indicate that there is a need to focus on supporting productive struggle when addressing challenges in developmental mathematics.

Teaching issues in developmental mathematics can be addressed by embracing the vital role teachers play in supporting productive struggle in the classroom. Research suggests that teachers play an essential role in building a classroom environment that values struggle, making students aware that learning occurs from struggle, and encouraging students to deepen their understanding of mathematical ideas (Murawska, 2018; C. Townsend, Slavit, & Mcduffie, 2018; Warshauer, 2015a). In typical developmental mathematics instruction, encouraging students to deepen their understanding is lacking (Mesa, 2011). Indeed, Grubb (2013) describes typical developmental mathematics instruction as “remedial pedagogy” (p.49), where teachers emphasize correct answers, rules, and procedures through drill and practice. Lynch et al. (2018) recommend that teachers help students consider the underlying mathematical principles to support productive struggle, contrasting with remedial pedagogy. Murawska (2018) suggests that collaborative tasks can encourage students to struggle productively, which differs from remedial pedagogy’s drill-and-practice routine. The NCTM (2014) suggests that the teacher ask questions to guide and scaffold students’ thinking and help students realize that confusion and errors are a natural part of learning during these tasks. This viewpoint is contrary to the focus on correct answers and following rules described by the instructor. Thus, remedial pedagogy can be addressed when instructors shift to supporting productive struggle.

Supporting productive struggle has yet to make a widespread impact in developmental mathematics. Research on productive struggle in K-12 mathematics is extensive (e.g., Kapur, 2008; C. Townsend et al., 2018; Warshauer, 2015a), including studies on how prospective teachers support and notice productive struggle (e.g., Warshauer, Starkey, Herrera, & Smith, 2019; Zeybek, 2016). In higher education, studies have used a similar term, productive failure, to design lessons with complex problems and challenge students to solve and investigate their solutions (e.g., Kapur

& Bielaczyc, 2012; Mazziotti, Loibl, & Rummel, 2015; Westermann & Rummel, 2012). Supporting productive struggle includes the design of challenging tasks that elicit students' struggle, and teaching actions to support students to struggle productively (Warshauer, 2015a). Based on the arguments previously made, research in developmental mathematics is much needed. A few programs, such as City University of New York's (CUNY) Start (Bickerstaff & Edgecombe, 2019) and Carnegie Math Pathways (Edwards & Beattie, 2016; Silva & White, 2013), apply the construct of productive struggle to reform the design of their developmental mathematics courses. In CUNY Start, instructors are provided a curricular document and professional development to support them in making in-the-moment instructional decisions to emphasize number relationships, elicit student talk and discussion through question, and draw students' attention to study skills (Bickerstaff & Edgecombe, 2019). Students are "active participants in their own learning, given time to think and struggle, and encouraged to speak and respond to each other" (Scrivener et al., 2018, p.29). CUNY Start's doubling of students' graduation rates (Bickerstaff & Edgecombe, 2019) suggests that supporting productive struggle in developmental mathematics deserves more attention.

### *The Impact of Productive Struggle on Students' Learning*

Supporting developmental mathematics students to struggle productively can significantly impact students' learning experience. Murdoch et al. (2020) call for greater attention to the affective aspect, arguing that struggle is not purely cognitive but rather existential. The affective domain characterizes the emotional aspects of mathematical learning (DeBellis & Goldin, 2006; McLeod & Adams, 1989). Research on the deep interplay between cognitive and affective dimensions of learning mathematics (DeBellis & Goldin, 2006; Di Martino & Zan, 2011; Hannula, 2002) fuels the need for researchers to not only focus on cognition. This need is further emphasized in the area of developmental mathematics, which lacks studies focusing on the affective domain of learning (Bonham & Boylan, 2011; Zientek et al., 2019). Thus, this section brings together the literature on productive struggle with deep approaches to learning (Marton & Säljö, 1976), affect (DeBellis & Goldin, 2006), perseverance (Middleton et al., 2015), and habits of mind (Cuoco et al., 1996) to consider the interaction between affective and cognitive domains during students' struggles.

**Approaches to Learning.** The typical developmental mathematics learning environment lends itself to students adopting a surface approach to learning. Developmental mathematics instruction that typically transmits knowledge to students was described earlier in this chapter (Grubb et al., 2011). Trigwell, Prosser, and Waterhouse's (1999) large-scale quantitative study reported that students were more likely to adopt a surface approach when teaching focused on transmitting information. Approaches to learning refer to the ways in which students process their academic tasks (Marton & Säljö, 1976). Students adopting a surface approach tend to focus on concrete and literal aspects of the tasks rather than on the meaning (Biggs, 1991; Kember, Biggs, & Leung, 2004; Marton & Säljö, 1997). Students with a deep approach seek to understand the underlying meaning of the learning task (Biggs, 1991; Kember et al., 2004; Marton & Säljö, 1997). Research on approaches to learning (e.g., Byrne, Flood, & Willis, 2004; Entwistle & Tait, 1995; Papinczak, 2009) found a significantly positive correlation between students who may prefer the transmission model of teaching to surface approaches. Another factor is workload, and developmental mathematics students have to take extra courses each semester (Edgecombe, Cormier, Bickerstaff, & Barragan, 2013). The high workload may force students to cope by relying on surface approaches, such as memorizing (Trigwell, Ellis, & Han, 2012). Unfortunately, these factors suggest that it may be typical for developmental mathematics students to adopt surface approaches to learning.

The conditions that support productive struggle can foster deep approaches to learning. First, productive struggle is often supported by engaging students in challenging but not overwhelming mathematical tasks, in which it is not immediately apparent what solution strategies are required (Edwards & Beattie, 2016; Lynch et al., 2018; Murawska, 2018; Permatasari, 2016). Teachers support students' struggles during tasks by encouraging students to engage in critical thinking about the problems, instead of focusing on the correct answer (Warshauer, 2015b). Trigwell, Prosser, and Taylor (1994) consider this teaching approach a student-focused strategy, in which the teacher makes time for students to question ideas. This approach to teaching is known to foster deep approaches to learning (Trigwell et al., 1999). Secondly, productive struggle is often supported in a collaborative learning environment (C. Townsend et al., 2018). Booth and James (2001) found that this form of engagement and interaction can significantly improve the classroom experience, in turn supporting deeper approaches to learning. For these reasons, creating conditions to support productive struggle in developmental mathematics may foster more deep approaches.

Students who productively struggle approach learning in a deeper manner. "The ultimate goal of productive struggle is to encourage students to make meaning of mathematical content for themselves" (Edwards & Beattie, 2016, p.31). Engaging in meaningful learning is the essence of deep approaches to learning (Marton et al., 1993). Students who are productively struggling are motivated to persist and are not discouraged by mistakes because they value struggle as part of the learning process (Warshauer, 2015b). Understanding that making mistakes is connected to the learning process is a way of seeing. This way of seeing promotes students to perceive themselves as more capable (Marton et al., 1993), as they attribute mistakes as unavoidable rather than marks of failure. Seeing oneself as a more capable person implies a fundamental change from a conception that things just happen, and according to Marton, Dall'Alba, and Beaty (1993), it is a conception that connects to deep approaches to learning. In other words, the research suggests that students who productively struggle also approach learning in a deeper manner.

**Perseverance.** Bringing the literature of perseverance and productive struggle together offers affordances to qualify impacts on students' learning. Perseverance has often been used as a characterization of productive struggle (e.g., Abrahamson & Kapur, 2018; Hughes, Carney, Champion, & Yundt, 2021; Olanoff, Johnson, & Spitzer, 2021), but few have explored the differences and similarities. Perseverance is overcoming difficulties in learning mathematics by assessing one's course of action (DiNapoli, 2019; Middleton et al., 2015). The idea that difficulties and obstacles are opportunities for students to make deeper connections with mathematical ideas has been widely discussed, evidenced in Piaget's (1975) theory of disequilibrium, thinking mathematically (Mason, Burton, & Stacey, 1982), and overcoming epistemological obstacles (Sierpińska, 1994). For example, Mason, Burton, and Stacey (1982) describe being stuck as an honorable state, and productive courses of action include taking time to ponder and trying several approaches. Amalgamating the literature opens up room for productive struggle to be described through perseverance, when students see "effort in mathematics as worthwhile and viewing oneself as capable of and effective at learning and doing mathematics are related to one's understanding and skills" (Bass & Ball, 2015, p.5). Of note in Bass and Ball's definition is students' perception of effort because it marks the difference between productive and unproductive struggle. Students who struggle productively value effort in understanding what they can do and what still remains to be done in mathematics (Warshauer, 2015b). Conversely, students' struggles are unproductive when they no longer see worth in their efforts and give up on the task (Warshauer, 2015a). The literature on perseverance also compares the construct to persistence (DiNapoli, 2018). Persistence is the "voluntary continuation of a goal-directed action in spite of obstacles, difficulties, or discouragement" (Peterson & Seligman, 2004, p.229). The contrast between perseverance and persistence marks another difference in the course of action between productive and unproductive struggle. Persistence can characterize students who expend effort in vain by repeating the same strategy (DiNapoli, 2019), an unproductive struggle (Permatasari, 2016; Warshauer, 2015a). However, students who persevere amend their plan of attack when facing difficulties (Middleton et al., 2015). Amending one's attack is exemplified when students consider alternative strategies to address their struggle and evaluate their progress, an indicator of productive struggle (Warshauer, 2015b). Thus, connecting the literature on productive struggle with perseverance provides a more comprehensive description of the difference between unproductive and productive struggle.

Perseverance is not innate, meaning explicit study is required to determine how perseverance can be supported during moments of students' struggle (DiNapoli & Miller, 2022). The NCTM (2014) stated that mathematical learning environments need to help students improve their perseverance over time. Research on productive struggle has indirectly supported the importance of students' persevering through moments of struggle in problem solving (Baker et al., 2020; Zeybek, 2016). Bass and Ball (2015) characterized these moments as students' actions to initially engage in an activity despite uncertainty, self-regulatory actions to remain productively engaged despite struggle, and acting with confidence during problem solving even without a clear pathway to a solution. Productive struggle indirectly informs perseverance because both involve the learner encountering mathematical obstacles (DiNapoli & Miller, 2022). This indirect relationship helps characterize perseverance. For instance, Gresalfi, Martin, Hand, and Greeno (2009) claim that struggle is only productive when the student is accountable for their progress. Hence, perseverance can be characterized as students recognizing the direction of their efforts as they solve problems. DiNapoli and Miller (2022) investigated this indirect relationship by examining the impact of providing a structure of thinking (i.e., scaffolding) on students' perseverance. Their statistical analysis compared students who worked with scaffolded tasks to non-scaffolded tasks, and found that students demonstrated more perseverance over the five-week course when they worked through scaffolds embedded in challenging mathematical tasks. Studies such as DiNapoli and Miller's (2022) are rare, however, the results confirm that perseverance can be improved within mathematical tasks, warranting further work.

Developmental mathematics students' perseverance during tasks has yet to be understood. Studies have mentioned that students need to persevere to succeed in developmental mathematics (e.g., Merseth, 2011), and many describe perseverance and persistence synonymously (e.g., Benken et al., 2015; Code, Merchant, Maciejewski, Thomas, & Lo, 2016; Mesa, 2012). Many large-scale quantitative studies, like Bahr's (2013), define persistence when students stay in the program semester by semester compared to dropping out. Authors from the Carnegie Pathways programs refer to perseverance as *productive persistence*, describing students putting forth effort during challenges using effective strategies (Dolle, Gomez, Russell, & Bryk, 2013). Productive persistence is central to Carnegie's Pathways programs, and students in their program can complete developmental mathematics in one semester and succeed at double the national rate (Hoang, Huang, & Sulcer, 2017), suggesting perseverance deserves attention.

**Affect.** There is a need to characterize developmental mathematics students' affect in the classroom. The affective domain includes beliefs, attitudes, emotions, and values (DeBellis &

Goldin, 2006; Zan, Brown, Evans, & Hannula, 2006). Research has shown that affective factors influence students' performance in developmental mathematics (Bonham & Boylan, 2011; CSAP/PREC, 2015; Fong et al., 2015). Quantitative studies have focused on the correlation between affect and grades, while accounting for demographics (e.g., gender) and external factors (e.g., work outside of school) (e.g., Guy, Cornick, & Beckford, 2015; Mireles, Offer, Ward, & Dochen, 2011; Zientek, Fong, & Phelps, 2019). Less focus has been on students' affect as they work through coursework, "our knowledge of students and their attitudes toward learning is sorely lacking … our understanding of how students move from initial assessments to developmental courses to college-level and advanced courses is also thin" (Grubb & Cox, 2005, p.95). Grubb and Cox's quote still holds true as research has acknowledged that affective characteristics may inhibit students abilities to fully engage with mathematics, but less research exists on how affect impacts students' placement and development through the sequence of courses (Maciejewski et al., 2021). Stigler, Givvin, and Thompson (2010a) conducted one of the few studies to answer the call by investigating what students understand about the mathematical concepts being taught. The study uncovered students' beliefs about mathematics (e.g., math is just steps) and tendencies to apply computational procedures inappropriately. The authors stressed the importance of reawakening students' disposition to understand mathematics. DeBellis and Goldin (2006) view this disposition as an interaction between the affective and cognitive domains. The authors' theoretical framework characterizes states of affect interacting with cognitive processes that develop into global affective structures. For example, the student's belief that math is just steps may have developed because they were successful in the past when they memorized steps. The success brings elation and joy, reaffirming their belief. Multiple successes develop into a global structure of a stable belief that math is just steps. Although DeBellis and Goldin's work is not based on developmental mathematics, connecting their research allows the impact on students' learning to be explained through cognitive-affective interactions.

The affective dimension of learning is inherent in students' struggle to make sense of mathematics. Struggle is a process in which a student engages effortfully in mathematics that is not apparent (Hiebert & Grouws, 2007). Dewey (1922) describes this process as a learner wrestling with ideas in a way that interrupts the otherwise smooth flow of the learner's experience. This discontinuity in experience incites a state of doubt, uncertainty, confusion, or difficulty which is felt (Dewey, 1910). These affective moments signify that one may know that the old ideas no longer suffice but have yet to find a new way forward; in other words, struggling to make sense of the new and old (Murdoch et al., 2020). These moments can lead to productive struggle, as a learner effortfully tries to understand mathematics. If not supported, these moments can be unproductive, causing frustration that may result in the learner giving up their efforts (Hiebert & Grouws, 2007). The process of struggle described by research emphasizes an interconnection between affect (uncertainty or doubt) and cognition (giving up or expending effort to understand). Meaning DeBellis and Goldin's (2006) theoretical framework can be applied to describe how global affective structures develop from affective and cognitive interactions during moments of struggle. Therefore, amalgamating research on affect with productive struggle affords a more explicit description of moments of struggle.

**Habits of Mind.** Habits of mind indicate that productive struggle can be applied beyond mathematics and into students' careers. Mathematical habits of mind refer to how students learn and adopt some of the ways mathematicians think about problems (Cuoco et al., 1996). Watson (2008) states, "To do maths includes holding nagging questions in mind while carrying on with life, and not expecting answers to be found, problems to be solved, within the confines of a particular room or timescale" (p.6). Watson adds that students should learn to use mathematical tools and ways of working so that they can be used to learn more tools and ways later on. These tools and ways are embodied in Cuoco, Goldenberg, and Mark's (1996) mathematical habits of mind. Watson's quote also adeptly resonates with proponents of productive struggle. Although Watson does not refer to productive struggle per se, one can explore the connections of productive struggle with mathematical habits of mind to develop tools and ways that students can apply beyond the mathematics classroom.

Amalgamating the literature on habits of mind with productive struggle provides a broader picture of how students expend effort to understand mathematics. Examining the interplay of productive struggle and habits of mind gains insight into the holistic nature of students' learning experiences. For example, Costa and Kallick (2008) describe habits such as perseverance and thinking flexibly as positive attitudes during setbacks or confusion. These attitudes echo Warshauer's (2015b) claim that dispositions toward challenging tasks need to be strengthened to effectively support productive struggle. Hence, students need to exercise and refine their mathematical habits of mind to struggle productively. Cuoco, Goldenberg, and Mark (1996) describe many habits, such as sniffing out patterns, experimenting, describing, and conjecturing, that students can use to navigate complex problems. Reinforcing mathematical habits of mind supports students to become more aware, echoing Granberg's (2016) claim that productive struggle is supported as students become increasingly aware of the relevant concepts and components of mathematical concepts. Thus, combining the literature underscores the importance of students becoming mindful problem solvers during moments of struggle, a habit that prepares students for future study and employment (Watson, 2008).

In summary, the impact of productive struggle on students' learning experience can be expanded through how affective factors interact with the cognitive domain of learning. The existing body of literature connects productive struggle with deep approaches to learning, perseverance, positive affective relationships with mathematics, and mathematical habits of mind. The amalgamation of the literature is significant, as it expands on the different ways developmental mathematics students can benefit from learning to struggle productively.

**Inquiries into One's Practice as Effective Professional Development**

Professional development is about becoming more aware of one's practice (Mason, 2002). Mason adds that effective professional development questions and changes habitual reactions through a personal inquiry to broaden sensitivities to notice and to act. Such inquiry is lacking in professional development opportunities offered in colleges (Edwards et al., 2015). One reason is that typical professional development involves one-shot workshops that lack meaning without ongoing opportunities to improve instruction (Bailey et al., 2015; Boylan, 2002; Huber, 2008). Professional development is any form of structured activities in the workplace to improve teaching (Dall'Alba & Sandberg, 2006). The less-than-desired state of college professional development has motivated researchers (e.g., Edwards et al., 2015; Mireles, 2010; Severs, 2017) to design more effective professional development programs. Inquiries into one's practice fulfill the aims of what many researchers (e.g., Bates & Morgan, 2018; Clarke & Hollingsworth, 2002; Desimone, 2009; Guskey, 2002; Huber, 2008; Hunzicker, 2011) consider effective professional development, because instructors become more aware of the various aspects (e.g., pedagogy, students' needs, teaching habits) for teaching in developmental mathematics. Habitual reactions in one's teaching are interrogated and challenged, cultivating a mindset of ongoing learning (e.g., Attard, 2017; Mason, 2002). Mason (2002) states that sensitivities are broadened, enabling one to respond more mindfully in the future. Sharing one's sensitivities sheds light for researchers to understand the nature of teaching during various moments of their development (Simon & Tzur, 1999).

Inquiries into one's practice may support developmental mathematics instructors in addressing skill-and-drill instruction. Developmental mathematics instructors tend to rely on skill-and-drill pedagogy, breaking down mathematical content into smaller discrete skills, such as using the formula to calculate the slope of a linear line (Grubb et al., 2011; Stigler et al., 2010b; Wladis et al., 2020). Students are drilled until they have mastered them (e.g., pass a test). The approach is contained in the assumption that the instructor's main responsibility is to implement the curriculum (Grubb & Kalman, 1994). In this view, students are defined in terms of what they are unable to do, and the only way to change this view is to score well on a test (Grubb & Kalman, 1994). It is also the responsibility of the student to reassemble individual skills into complex competencies and apply such skills in specific applications (Grubb et al., 2011; Stigler et al., 2010a). However, this approach violates the maxims of effective teaching to adult learners in which tasks are meaningful and flexible (Knowles, 1984; Rodrigues, 2012). Becoming more aware of one's practice involves systematically interrogating one's routines, such as skill-and-drill pedagogy. The effectiveness of such routines in fostering student learning is questioned, and one seeks to consider alternatives they were previously unaware of (Mason, 2002). Awareness is enhanced through others' insights and perspectives (Mason, 2002), further challenging the effectiveness of relying on pedagogy such as skill-and-drill instruction.

Becoming more aware of one's practice as a developmental mathematics instructor involves learning the knowledge necessary to teach effectively. Previously mentioned was that instructors were often hired without any pedagogical training or experience (Bickerstaff & Edgecombe, 2019; Grubb, 1999; Maciejewski & Matthews, 2010). Such findings suggest that instructors are often only equipped with mathematical knowledge. However, mathematical knowledge is not enough, as Begle (1972, 1979) showed little or no correlation between teachers' college credits in mathematics and their students' performance. Effective professional development efforts (Edwards et al., 2015; Hodara, 2011; Severs, 2017) aim to address pedagogy, which relates to Shulman's (1986) pedagogical content knowledge, the knowledge teachers have to teach particular mathematical concepts. Awareness of more pedagogical possibilities (e.g., supporting productive struggle) may shift the instructor's reliance on skill-and-drill pedagogy. However, instructors need to know more than pedagogical content knowledge to teach developmental mathematics. For instance, pedagogical content knowledge focuses on the teacher's knowledge of children's thinking and strategies. Developmental mathematics students are adults learning topics from K-12 mathematics, pointing to the need to know adults' thinking and strategies. The knowledge developmental mathematics instructors need is further complicated as instructors are themselves adult learners, with deeply rooted perceptions about teaching and the abilities of their students (Mesa, 2012; Mesa, Celis, et al., 2014). Inquiring about one's developmental mathematics practice has the potential to address different aspects of knowledge mentioned. As one questions their own practice, they become more sensitive to their own learning while simultaneously enhancing sensitivities to notice how students learn (Mason, 2002).

Inquiries into one's practice, such as the Discipline of Noticing, enhance how instructors teach in future classroom moments (Mason, 2002). Mason's Discipline of Noticing is a set of practices to intentionally sensitize one to notice moments in the classroom. Ball and Bass (2000) assert that teachers need to know how to respond quickly to unpredictable daily classroom occurrences. This assertion can be extended to how productive struggle is supported in developmental mathematics classroom moments. This is evidenced in Bickerstaff and

Edgecombe's (2019) report on CUNY's professional development program. In the program, developmental mathematics instructors were encouraged to implement instructional approaches, such as supporting productive struggle. An important insight emerging from interviews with instructors was the discomfort they experienced supporting productive struggle during classroom moments (Bickerstaff & Edgecombe, 2019). Mason and Davis (2013) claim that for teachers to respond mindfully moment by moment in practice requires an education of awareness. The Discipline of Noticing educates such awareness, as broadening the possibilities to act makes it more likely for mindful action to come to mind, compared to relying on limited habitual reactions (Mason, 2002).

Inquiries into a developmental mathematics instructor's practice have the potential to contribute to the knowledge of mathematics teacher professional development. Insight can be gained into instructors' awareness of their approaches and decisions (Simon & Tzur, 1999). Such insights can inform how developmental mathematics should be taught, how the curriculum could be structured, and how students can be supported in their developmental mathematics courses (Mesa, Wladis, et al., 2014). The insights can also inform how productive struggle is supported in a developmental mathematics classroom. For example, in Carnegie's Pathways program, where developmental mathematics students are supported in engaging in productive struggle, instructors' insights have led to recommendations for all faculty to make explicit students' ways of thinking and encourage students to explain solution strategies (Edwards & Beattie, 2016). This knowledge is salient to equip future instructors with strategies, challenges, and possible solutions that are relevant and effective for teaching developmental mathematics. Such contributions echo that of Wagner et al.'s (2007) study, which revealed challenges specific to university instructors' own mathematics teaching and how the findings inform "educators, educational researchers, curriculum designers who wish to develop support structures for mathematics teachers undergoing changes in their teaching practices" (p.264). By systematically investigating one's practice and sharing insights, mathematics teacher professional learning can be enriched.

### *Professional Learning Occurring in Inquiries of One's Practice*

There is a need to detail inquiries into one's practice and the learning that can occur. Professional learning can be enhanced by systematically reflecting on one's practice (Mason, 2002). Professional learning refers to learning during professional development opportunities (Darling-Hammond, Hyler, & Gardner, 2017), such as inquiries into one's practice. Inquiries into one's practice require ongoing systematic reflection, as it may take a while to discover and figure out appropriate ways to change one's approach in the classroom (Huber, 2008; Tinberg et al., 2007). Systematic reflection refers to the ordered ways of recollecting, rethinking, and analyzing events (Cochran-Smith & Donnell, 2012). Systematic reflection is involved in moving professional learning from wanting to change to knowing how to act differently in the future (Mason & Spence, 1999). This is important as instructors' personal beliefs and experiences guide how teachers teach (Guskey, 1989). These beliefs are relatively stable, making them hard to change even if instructors seek improvement (Sweeney, 2003; Tatto, 1999). To bring about change in the classroom practices of teachers, researchers (e.g., Attard, 2017; Cochran-Smith & Lytle, 1999; Dewey, 1933; Guskey, 2002; Mason, 2002; Rodgers, 2002) often recommend the use of systematic reflection. Systematic reflection enhances ways to deeply understand one's experience (Dewey, 1933; Schön, 1983). The practices are rigorous and disciplined, involve interacting with others, and value personal and intellectual growth of self and others (Hayden, Moore-Russo, & Marino, 2013; Rodgers, 2002). For example, prospective mathematics teachers in Stockero's (2008) study analyzed their classroom events to describe, explain, theorize, confront, and restructure teaching decisions with peers. The teaching changes from the study suggest that systematic reflection can transfer skills learned in teacher education programs to the practice of teaching mathematics. The potential of transferring professional learning into developmental mathematics classrooms is an enhancement to one-stop professional development without opportunities to reflect.

Professional learning takes place during systematic reflection when teachers discern details not previously noticed (Mason, 2010). As stated earlier, it is hard to change teaching habits (Attard, 2007). For this reason, a significant but not overwhelming disturbance is typically required to change behavior (Festinger, 1957; Piaget, 1975; Sierpińska, 1987). Learning from one's teaching can then be seen as fresh action arising from a disturbance compared to reworking habits automatically (Mason, 2010). Novice teachers often rely on habits in response to disturbance (Hayden et al., 2013). Their efforts to notice and reflect are often stymied as attention is used up to manage classrooms, plan lessons, deliver content, and manage curriculum (Rodgers, 2002). Learning can be seen as novices gain more knowledge, proficiency, and experience, freeing attention to notice and taking responsibility for students' learning (Hayden et al., 2013; Mason, 2002). In comparison, expert teachers are aware to notice and reflect on appropriate actions to respond (Mason, 2002; Schön, 1983). Studies (e.g., Attard, 2017; Hayden et al., 2013; Lyons, 2010; Rodgers, 2002) have shown that this expertise can be developed through systematic reflection. A particular form of systematic reflection is Mason's Discipline of Noticing (2002), a collection of techniques to notice in the moment of practice, and systematically seek fresh possibilities of action rather than acting out of habit. Several studies have evidenced professional learning by engaging with the Discipline of Noticing (e.g., Barnes & Solomon, 2013; Breen, McCluskey, Meehan, O'Donovan, & O'Shea, 2014). Although the Discipline of Noticing has been cited almost two thousand times, it has not yet been widely utilized as a form of professional development, and no studies have applied it in the context of developmental mathematics. Thus, there is room to investigate how a developmental mathematics instructor applies the techniques from the Discipline of Noticing to learn professionally.

### *Professional Learning as a Form of Research*

Systematic reflection of one's practice can be a form of research (Cochran-Smith & Lytle, 2012; Holly, 2012; Mason, 2002; Pinnegar & Hamilton, 2009; Saunders & Somekh, 2012). For example, Mason's Discipline of Noticing (2002) exemplifies how a set of practices to systematically become more aware aligns with qualitative research processes. Qualitative research can be seen as a systematic inquiry that contains a research question, a review of the literature to define the objects of study, methods to answer the research question, collection and analysis of data, and communication of the findings to the research community (Lichtman, 2013; Marshall & Rossman, 2014). In systematic reflections like the Discipline of Noticing and self-study, meaningful research questions are asked about one's teaching for the purpose of learning ways to improve practice (Rodgers, 2002). Data can be autobiographical, reflections after class, or what is noticed moment by moment in teaching (e.g., Grant & Butler, 2018; LaBoskey, 2004; Mason, 2002; Pinnegar & Hamilton, 2009). Analyzed are one's instructional choices, and professional learning occurs as one seeks alternative possibilities to become more aware of their practice (Attard, 2017; Loughran, 2002; Mason, 2002). The claims are shared publicly to seek review and critique, providing valuable insights into the intricacies of teaching and learning in a specific instructional setting (Pinnegar & Hamilton, 2009; Tinberg et al., 2007). Thus, there is literature that supports systematic learning from one's practice as qualitative research.

This form of research is unique as it offers an inside-out perspective of professional learning (Cochran-Smith & Lytle, 1993). The perspective provided is one of an insider and how they come to learn about teaching. The insight differs from the outside perspective, recommending what teachers should know to teach mathematics. For instance, teachers overhelping students is a well-documented issue (Errico, Leone, & Poggi, 2010). This issue is particularly important to supporting productive struggle, as many teachers are quick to jump in, limiting students' struggles (NCTM, 2014). Unique to the insider perspective is that it offers research on how teachers become aware that they are overhelping, and how the realization changes their practice. In developmental mathematics, this is a rare perspective. For example, discussed earlier were the issues such as skill-and-drill instruction (e.g., Grubb et al., 2011). Rarely found in research is how an instructor becomes aware that they are relying on skill-and-drill instruction and the reason for their reliance. Also, despite the wealth of studies on what teachers should do to support productive struggle (e.g., NCTM, 2014; Warshauer et al., 2019), lacking are valuable insights from enacting such classroom practices. Thus, an instructor's perspective from the inside uniquely contributes to research in developmental mathematics and supporting productive struggle.

Researching from the inside blurs traditional roles. For instance, self-based research blurs the roles of researcher and instructor as they engage in research activities while instructing mathematics (Cochran-Smith & Lytle, 2001). This stance is rarely found in developmental mathematics, as instructors are typically not responsible for engaging in research (Maciejewski & Matthews, 2010). Just as rare is the blurring of roles to support productive struggle. Personal experiences, perspectives, and subjectivity are brought forth for other researchers to draw on their own experiences (Mason, 2002). This view recognizes that the researcher is an active participant within the context they are studying, and not a detached observer often seen in positivist paradigms

(Cochran-Smith & Lytle, 1993). By blurring the roles, inquiry, reflection, and presenting findings publicly are not seen as add-ons to work because “without such introspection and collaboration, teaching becomes more labor intensive, not to mention less rewarding, because it is less informed” (Tinberg et al., 2007, p.28). The site of research also blurs with the classroom, changing the traditional belief that knowledge is held by administrators and curriculum designers and transported into the classroom (Cochran-Smith & Lytle, 1993). Instead, the instructors are seen as knowers, and their experiences often reveal the discrepancies between curriculum intentions and what occurs (Cochran-Smith & Lytle, 2012; Huber, 2008). Consequently, researching from the inside challenges the one-size-fits-all assumption held by developmental mathematics curriculum designers (CSAP/PREC, 2015).

This review highlights the well-documented issues of teaching and learning in developmental mathematics. Literature suggests that supporting productive struggle may be a solution to these issues. One reason is the possible impacts on students’ experiences, evidenced by the connection of productive struggle to deep approaches to learning (Marton et al., 1993), perseverance (Middleton et al., 2015), positive affect (DeBellis & Goldin, 2006), and good habits of mind (Cuoco et al., 1996). The instructor plays a vital role in creating the necessary conditions for students’ productive struggle (Warshauer, 2015a). However, supporting productive struggle in developmental mathematics requires a shift from typical instruction. Such shifts require effective professional development involving systematic inquiries into one’s practice, such as the Discipline of Noticing (Mason, 2002). The literature suggests systematic inquiries of one’s practice occur when the roles of researcher and instructor are blurred (Ball, 1995; Cochran-Smith & Lytle, 1993). The blurring of such roles simultaneously produces professional learning and findings for others to test in their own practice (Mason, 2002).

# Chapter Three – Conceptual Framework

The purpose of this dissertation is to investigate struggle and productive struggle in developmental mathematics. The concept of productive struggle guides the research design, aiming to elicit students' experiences of struggle in a developmental mathematics classroom. This study examines both the students' experience of struggle and productive struggle, as well as my own experience as an instructor supporting their struggles. Experience is viewed through a phenomenological stance, through how people interpret and attribute meaning to a phenomenon (Heidegger, 1962; Husserl, 1970; Marton, 1986; Mason, 1998; Merleau-Ponty, 2002). Experience can be described and characterized through variations (Marton, 1986) in students' struggles and productive struggles, as well as different accounts I noticed supporting productive struggle (Mason, 2002). To analyze how students interpret and attribute meaning to their struggles and productive struggles, I draw on the constructs of deep approaches to learning (Marton et al., 1993), perseverance (Middleton et al., 2015), positive affective structures (DeBellis & Goldin, 2006), and good habits of mind (Cuoco et al., 1996). The Discipline of Noticing (Mason, 2002) offers a framework to examine my experience supporting productive struggle in developmental mathematics. The set of practices serves as a form of effective professional development and blurs the boundaries of research and practice.

## My Position

At the core of this dissertation is my personal experience as a developmental mathematics instructor. The challenges I encountered as an instructor motivated me to seek research on supporting productive struggle. While my initial motivation was to improve my instruction, it was the difficulties students faced that compelled me to delve deeper into the dynamics of teaching and learning. Gaining insights into how students perceive and conceptualize struggle and productive struggle enhances my future practice.

A recurring scenario involves students approaching me, expressing their lack of confidence in mathematics, often before the first class of the course. I have come to acknowledge the years of feelings towards math and misconceptions developmental mathematics students bring into the classroom. Such experiences guide this study to know more about students' perspectives through both the affective and cognitive domains of learning.

By interrogating my experience, I can better relate to the learners and provide more effective support. I also gain a deeper appreciation of the richness of students' and my lived experience. Thus, the objectives of Mason's Discipline of Noticing (2002) relate to my own.

**Productive Struggle**

The concept of productive struggle is central to this dissertation's understanding of lived experience in the developmental mathematics classroom. Recall that *struggle* is associated with expending effort to make sense of mathematics that is not immediately apparent (Hiebert & Grouws, 2007). There is no struggle when the solution is obvious. *Productive struggle* leads to a deeper understanding of mathematical ideas (Warshauer, 2015a), and unproductive struggle is when students communicate that their effort leads to giving up on the task and feelings of hopelessness or anger (Hiebert & Grouws, 2007; Warshauer, 2015a). The potential for students to struggle productively and understand mathematics is a shift from recalling rules and formulas commonly reported in developmental mathematics (Stigler et al., 2010b).

*Supporting productive struggle* means to implement challenging tasks to elicit students' struggles with pedagogical actions intended to promote productive struggle (Kapur & Bielaczyc, 2012; Murawska, 2018; C. Townsend et al., 2018). The tasks should elicit uncertainty (Kapur & Bielaczyc, 2012; Zaslavsky, 2005), linking the idea that tasks are not immediately apparent and students need to expend effort to understand mathematics. Uncertainty means perplexity, confusion, and doubt caused by logical inconsistencies or contradictions, motivating the learner to (re)consider what to do and the reasons behind the mathematical idea (Dewey, 1938; Festinger, 1957; Piaget, 1975; Sierpińska, 1994; Zaslavsky, 2005). Opportunities for students to articulate their reasons are a feature of supporting productive struggle that differs from the idea that learning is memorizing information and repeating what is demonstrated (Hiebert & Grouws, 2007). It is recommended that the instructor chooses the appropriate actions during the tasks to support students' efforts toward productive struggle (Murawska, 2018; C. Townsend et al., 2018). Warshauer (2015b) outlined appropriate actions to support productive struggle. Specifically, asking purposeful questions to help students reflect on the source of their struggle in understanding, encouraging students to engage in the process of thinking about the mathematical ideas and not focus on the correct solution, providing ample time for students to struggle, and showing students that struggle is a necessary part of learning (Warshauer, 2015b). Baker et al. (2020) recommended that instructors reflect on their efforts to support productive struggle, and noted that the pedagogical method might be unfamiliar and takes time to become a classroom norm. Thus, supporting productive struggle can be identified within tasks that provide opportunities for students to articulate their understanding with instruction intended to support students' struggles.

### Impacts on Students' Learning

The claim that productive struggle significantly impacts students' experiences gains theoretical grounding by examining students' articulated learning experiences. This section explicitly outlines how productive struggle becomes evident through students' approaches to learning, perseverance, affect, and habits of mind. These claims draw attention to the affective domain of learning (Murdoch et al., 2020), particularly as students struggle through challenging tasks.

The affective domain refers to the beliefs, attitudes, emotions, and values pertaining to mathematics learning (DeBellis & Goldin, 2006; McLeod, 1992). Research on the affective domain in mathematics learning emphasizes the interplay between affective and cognitive factors (Evans, 2000). For instance, emotions influence a student's decision between giving up or expending effort to understand mathematical concepts within a challenging task. Through an analysis of the impacts on learning, the evidence sheds light on how students interpret and attribute meaning to productive struggle through both the affective and cognitive domains.

**Approaches to Learning.** Productive struggle can be evidenced through deep approaches to learning. Students finding meaning in their learning was discussed in the previous chapter as a connection between productive struggle and deep approaches to learning. Marton, Dall'Alba, and Beaty (1993) evidenced deep approaches to learning through students' conceptions that the purpose of learning was to understand, see something in a different way, and change as a person. The important link between the conceptions that relate to deep approaches to learning is that students describe grasping the meaning of a phenomenon (Marton et al., 1993), such as productive struggle. For instance, the first conception is evident when students state they are grasping new ideas or understanding. Students talking about changing their ways of thinking about a phenomenon is evidence of the conception, seeing something in a different way (Marton et al., 1993). Statements on how students changed as a person evidences the last conception (Marton et al., 1993). The conceptions can evidence productive struggle. When students talk about gaining meaning from much-discussed productive struggle indicators, such as considering alternative strategies, expending effort to make sense of their work, understanding what to do, and realizing confusion and errors are a natural part of learning (Warshauer, 2015b), they are describing deep approaches to learning.

Deep approaches to learning offer an extension to higher education for productive struggle. Research on approaches to learning extend beyond the cognitive as researchers (e.g., Entwistle & Tait, 1995; J. T. E. Richardson, 2005; Trigwell et al., 2012) have found that it impacts students' engagement, motivation, and orientation to studying in higher education. Murphy's (2016) study based on New Zealand higher education, including vocational education, suggests that students should adopt deep approaches to learning to succeed in mathematics. Well-documented in the literature is the connection of deep approaches to learning to success in higher education (e.g., Abaté & Cantone, 2005; Inglis & Alcock, 2012; Richardson, 2005). Few studies focus on developmental mathematics, but the connection provides an extension for productive struggle mainly situated in K-12 mathematics.

Barriers to success in developmental mathematics relate to surface approaches to learning. Previously discussed were the high workloads students face (Edgecombe et al., 2013) and drill-and-skills forms of teaching (Grubb et al., 2011). Trigwell et al. (1999) found that such factors harbor surface approaches to learning. Indeed, developmental mathematics students complain that courses are not very interesting because they sit down, take notes, and memorize for exam after exam, losing meaning in their learning (Fink, 2003; Jorgensen, 2010; Merseth, 2011; Stigler et al., 2010a). Conceptions that learning is to increase knowledge, memorize and reproduce, and apply were described by Marton, Dall'Alba, and Beaty (1993) as surface approaches to learning. The conceptions can be considered unproductive struggles when students' efforts to memorize and reproduce do not lead to a deeper understanding of mathematics.

**Perseverance.** Perseverance has often been used synonymously with productive struggle (e.g., Abrahamson & Kapur, 2018; Hughes et al., 2021; NCTM, 2014), but their relationship requires explicit study (DiNapoli & Miller, 2022). Productive struggle involves students expending effort through challenging situations (Warshauer, 2015a). Perseverance reflects a student's willingness to sustain effort, try different strategies, and persist in the face of difficulties (Middleton et al., 2015), characteristics of productive engagement with the task. Students who persevere in the face of challenges view obstacles as opportunities for learning and improving (Bass & Ball, 2015). This view aligns closely with Warshauer's (2015a) recommendation that students see struggle as an important part of mathematics. Students who persevere amend their attack, which involves reflecting on the problem solving process (Middleton et al., 2015). Students amending their attempts in moments of struggle can refine their understanding, as they identify misconceptions or mistakes and adjust their approach. Thus, the ways students amend their attack during moments of struggle deserves attention.

Perseverance can be evidenced through Middleton et al.'s (2015) definition as "the continuance of effort, carried out in a thorough and diligent manner, towards some perceived goal while overcoming unforeseen difficulties, obstacles, or discouragement along the way by amending one's plan of attack" (p.4). Meaning students who persevere demonstrate interest in mathematical tasks, see the reason for learning mathematics while pursuing challenging goals, and are willing to put in effort while weighing alternative actions to achieve their goals. Merseth (2011) reported that developmental mathematics students lack the motivation to persevere. Hence, it is significant to examine the impact supporting productive struggle has on students' perseverance.

Mentioned in the previous chapter was how productive struggle and perseverance were similarly conceptualized. For instance, students who persevere see effort as worthwhile and amend their plans when facing difficulty (Bass & Ball, 2015; Middleton et al., 2015), connecting to Warshauer's (2015b) conception that students who struggle productively consider alternative strategies and value effort in understanding what they can do and still remains to be done. Thus, how students apply their effort during their struggles can be conceptualized through Middleton et al.'s (2015) definition of perseverance. The conception is important as perseverance is often mentioned in productive struggle (e.g., Baker et al., 2020; Zeybek, 2016), but this indirect relationship requires further examination in developmental mathematics.

**Affect.** Claims of productive struggle can be made in terms of affect. Murdoch et al. (2020) called for more attention to the affective domain in productive struggle. Quantitative studies have also correlated positive affective factors to developmental mathematics success (e.g., Zientek et al., 2019), but less is known about students' affect as they work through problems in the classroom. Students' belief that they can tackle challenging problems and overcome obstacles as they improve their mathematical understanding is crucial to productive struggle (Murawska, 2018; Warshauer, 2015b). DeBellis and Goldin's (1999, 2006) work provides a theoretical framework to characterize how such beliefs are formed during learners' problem solving processes. The authors' framework describes affective pathways, sequences of feeling that interact with cognitive configurations that manifest into global structures. An example of a pathway is a student approaching a challenging task with curiosity and experiencing bewilderment when they realize the solution is not immediately apparent. They may be frustrated when they reach an impasse, but in rethinking their strategies, they may feel pleasure in finding new approaches. Multiple experiences of positive pathways may lead to global affective structures of meta-affect, mathematical integrity, and mathematical intimacy that relate positive emotions to learning mathematics (DeBellis & Goldin, 2006). In contrast, experiences with negative affective pathways may turn to despair or avoidance in learning mathematics. Characterizing productive struggle through these global structures provides a representation of the critical role affect plays as students engage in challenging tasks.

DeBellis and Goldin's framework can be used to analyze students' descriptions of struggle. Students' articulations of their attitudes, beliefs, emotions, and values as they grapple with mathematics can be analyzed to describe the affective structures formed (DeBellis & Goldin, 2006). The framework provides a structured approach to understanding students' affective experiences and perceptions of productive struggle in mathematics. Representations can be reported on how students made sense of their struggles and how feelings influenced their approach to the problems. These representations are an important contribution to developmental mathematics, as they provide much-needed evidence to investigate "the relationship between the cognitive and affective factors influencing students' success in developmental mathematics" (Bonham & Boylan, 2011, p.6). The contribution also allows research to characterize productive struggle from an affective lens, beyond the often-used description that productive struggle has been achieved when cognitive demand is maintained during the task (Murdoch et al., 2020).

The global structures of meta-affect, mathematical intimacy, and mathematical integrity evidence the deep influence emotions play in the learning of mathematics (DeBellis & Goldin, 2006). Meta-affect is an individual's thinking about the direction of one's feelings (DeBellis & Goldin, 2006). For instance, a student is aware of their frustration because they reached an impasse. This awareness may positively influence cognition if they are willing to expend effort to consider alternative strategies. On the other hand, a negative structure may be a decision to give up or a belief that they cannot solve the problem. Supporting students' awareness of one's thinking is often discussed in productive struggle (e.g., Kapur, 2016; Warshauer, 2015a). Evidence of students' meta-affect emphasizes the need for students to be aware of their feelings as they engage in struggle.

Mathematical intimacy is an individual's deeply-rooted emotional engagement with mathematics that builds the individual's meaning and purpose for learning (DeBellis & Goldin, 2006). Intimate behaviors can be identified through observable verbal or nonverbal actions. For example, expressing relief and appreciation from overcoming a topic that a student has struggled with in the past. Intimacy explains why it may be challenging to reconsider a long-held misconception, as the emotional dynamics solidify an individual's belief that the misconception is true. Warshauer (2015a) found that discussing misconceptions supports productive struggle because students have to "think harder" (p.380) to find the right steps. Mathematical intimacy adds to the complexity of overcoming a long-held misconception as deeply-rooted emotional engagement influences a student's decision to think harder.

Mathematical integrity is an individual's honesty about one's ability (DeBellis & Goldin, 2006). Mathematical integrity can be identified by an individual's articulation that they are not confident with a topic like fractions. Willing to expend effort to overcome their doubts with fractions is an example of how positive action can emerge from mathematical integrity. Researchers (e.g., Hall & Ponton, 2005; Wheland et al., 2003; Zientek et al., 2019) have reported developmental mathematics students' ineptitude in assessing one's ability to succeed. Such quantitative studies often divide affect into factors, such as attitudes, self-efficacy, and mathematics anxiety, and correlate each to student success. Meta-affect, mathematical intimacy, and mathematical integrity can be seen as representations of students' emotional ability, and such evidence adds to the complexity that is difficult to measure quantitatively. These affective structures also contribute to the complexity of productive struggle, offering evidence of the importance of students' emotional awareness.

**Habits of Mind.** Claims that productive struggle extends beyond mathematics can be made through habits of mind. Recall that mathematical habits of mind refer to thinking in the ways mathematicians tackle problems (Cuoco et al., 1996; Schoenfeld, 1992; Watson, 2008). Research suggests that good habits of mind extend beyond the mathematics classroom, as they are essential dispositions that motivate one to learn continuously, bringing the model of school to life (Banks & Barlex, 2020; Costa & Kallick, 2000; Cuoco et al., 1996; Watson, 2008). Developing habits to tackle future problems is a goal shared by productive struggle research (e.g., C. Townsend et al., 2018) and Ontario colleges whose mission is to prepare students for workplace needs (Ontariocolleges.ca, 2022). Thus, contributions to these objectives can be made by analyzing students' struggles through mathematical habits of mind.

Specifically, mathematical habits of mind can be indicators of productive struggle. Reviewed in the literature was the close connection between indicators of productive struggle and good habits of mind. For example, Warshauer (2015a) described students evaluating their work as a particular indicator of productive struggle. The indicator is evidenced by a well-documented mathematical habit of mind that students should be describers (Cuoco et al., 1996). Cuoco, Goldenberg, and Mark (1996) explain that describing is evidenced through the ability of students to say what they mean, struggle with the problem of describing the phenomenon, convince their classmates of a particular argument, and write down their thoughts, conjectures, and questions. The indicator of productive struggle that students need to consider strategies and representations (Warshauer, 2015a) is embodied through a mathematical habit of mind of pattern sniffing (Cuoco et al., 1996). Mathematicians are pattern sniffers, and students should foster a delight in finding patterns (Cuoco et al., 1996). The habit of finding patterns extends to students' daily lives to sniff out patterns from everyday situations. Cuoco, Goldenberg, and Mark (1996) describe a repertoire of habits, such as tinkering and conjecturing, that can be evidence of productive struggle. Since habits of mind serve as indicators of routines students can apply beyond the mathematics classroom (Costa & Kallick, 2000; Mark, Cuoco, Goldenberg, & Sword, 2010), similar claims for productive struggle can be made. This connection is a salient claim for developmental mathematics, which prepares students for their college courses and subsequent careers (Arendale, 2005).

In summary, developmental mathematics students' experiences of productive struggle can be cultivated through challenging tasks and evidenced through impacts on their learning. Claims can be made that students found meaning in their learning (Marton et al., 1993) by viewing productive struggle through the lens of deep approaches to learning. Evidencing productive struggle through perseverance demonstrates that students see effort as worthwhile and amend plans when facing challenges (Middleton et al., 2015). Analyzing the findings through DeBellis and Goldin's (2006) theory of affect provides evidence that productive struggle can be claimed through an affective-cognitive domain, contributing to research that has mainly focused on the cognitive domain. Finally, evidence of habits of mind indicates that productive struggle can extend beyond the mathematics classroom.

## The Discipline of Noticing as Effective Professional Development

Instructors can gain awareness of the knowledge they need to effectively teach developmental mathematics through the Discipline of Noticing (Mason, 2002). According to Mason, honing one's skill of noticing means paying deliberate attention to and making sense of the complex and dynamic aspects of students' thinking and one's instructional practices. Noticing students' thinking requires sensitizing oneself to notice specifics in the classroom. For example, "I don't know" may be often used by students, but it signals ambiguity. Mason (2002) claims that being attuned to asking students to clarify fosters an appreciation of "what they are trying to say and help them improve the clarity of their descriptions" (p.68). Instructors also gain a methodology to examine their teaching choices critically. Many decisions are made in the moment automatically because one's habits developed over years (Mason, 2002). Mason's Discipline of Noticing fosters one to be more sensitive to choices by recognizing some typical situations about to unfold and being aware of alternative actions. These practices help instructors refine choices and make decisions more mindfully and flexibly. Systematically noticing one's practice enhances instructors' knowledge of their strengths and areas for improvement, such as words, actions, and behaviors to be more sensitive to. This knowledge informs future instructional decisions and supports continuous professional learning to seek more possibilities from one's practice (Mason, 2002).

Seeking fresh possibilities from one's practice supports productive struggle. Researchers have identified that getting started is a common struggle for students (C. Townsend et al., 2018; Warshauer, 2015b). The Discipline of Noticing hones one skill to notice signs of struggle, and by interrogating one's decisions in the moment, one can further distinguish between productive, unproductive, or no struggle and enhance the choices that can be made in future moments. For example, providing time is a recommended strategy to support productive struggle (Warshauer, 2015b). However, when time should be provided requires a sensitivity to the context of the moment in the classroom. Seeking fresh possibilities intentionally considers one's decision to provide time, and one learns about alternative choices to make in those moments (Mason, 2002). As one hones the skill to notice and seek fresh possibilities, one becomes more attuned to students' struggles and expands the choices to respond mindfully and flexibly to support productive struggle.

In the Discipline of Noticing (Mason, 2002), seeking fresh possibilities begins by collecting accounts-of practice. An account-of is a description of an incident noticed as objectively as possible, minimizing evaluation, judgment, and explanation (Mason, 2002). "To notice is to make a distinction, …, to distinguish some 'thing' from its surroundings" (Mason, 2002, p.33). However, this may not be conscious; thus marking, when one is able to initiate mention of what has been noticed, is important. The marking of an account (e.g., 'a student has been staring at the question for some time without writing anything down') is evidence that one noticed something important about the incident, and makes it more likely for "further access, reflection, and reconstruction in the future" (Mason, 2002, p.33). For instance, upon reflection, the account described may have caught the instructor's attention because it contrasts with the instructor's experiences of students working on problems productively. Collecting accounts-of provides a collection of incidents to reenter and seek fresh possibilities.

Seeking fresh possibilities requires one to stand back and reenter accounts-of (Mason, 2002). Standing back is to separate oneself from the act of teaching so that one can reflect upon actions (Mason, 2002, 2020). Reentering accounts is to reflect on the accounts-of with the intention to see the account differently (Mason, 2002). One can consider what was not noticed. For instance, different possibilities that brought forth the student's action rather than interpreting that the student is stuck. The aim of standing back is to reenter the incident to see aspects one was unaware of at the time (Mason, 2002). During teaching moments, one may only see one possibility and act immediately (Mason, 2020). For example, the instructor may interpret the student's action as being stuck, and act to guide the student through the question without considering alternative choices.

By standing back and reentering accounts-of to seek different aspects, one can see more than just a single possibility.

Fresh possibilities can be sought through common threads from several accounts (Mason, 2002). Mason (2002) states that looking back over a variety of accounts-of makes it much easier to detect common threads that may not have been apparent at the time. For instance, multiple accounts of intervening when seeing students stuck may reveal the instructor's habit of responding in the same way. Mason (2002) also recommends discussing accounts and themes with others, as threads that one was not aware of may be offered. Through common threads and discussions with others, habits one was unaware of at the time may be revealed, allowing alternative possibilities to act to be considered.

Seeking fresh possibilities produces new actions that one was previously unaware of (Mason, 2002). Collecting accounts-of, standing back, reentering, and common threads allows one to see more possibilities from one's experience (Mason, 2002). For instance, standing back and reentering the example may reveal that the instructor intervened immediately during the account. A habit may be seen from multiple incidents of intervening whenever the instructor sees that students are stuck. Seeking a fresh possibility may be considering the student's point of view (Chapman, 1999; Mason, 2002), who may assume that they will eventually get help. The assumption reveals that the instructor's action may limit students' ability to learn to struggle productively. Considering how one can act differently (Chapman, 1999; Mason, 2002), the instructor may gain a new awareness to afford the student time to seek help, or work with the student to identify sources of struggle. Thus, the process of seeking fresh possibilities produces new actions that were not apparent from the initial account.

In summary, new actions emerging from seeking fresh possibilities from one's experience is a form of effective professional development. Instructors gain awareness of the knowledge they need to teach developmental mathematics. They also enhance their ways of supporting productive struggle, as they are more sensitive to noticing struggle and have an expanded repertoire of actions to respond to students' moments of struggle.

**Blurring Boundaries of Research and Practice**

Professional learning can become research by blurring the boundaries between research and practice (A. Hargreaves, 1996; Hedges, 2010; Mason, 2002; Wilson, 1995). Chapman et al. (2020) reported a need for practitioner research that utilizes methodologies that privilege the self. Similarly, Helliwell (2019) reported a shortage of self-based methodologies published in mathematics education journals. Researchers (e.g., Ball, 1995; Cochran-Smith & Lytle, 1993) state that such methodologies exist when the boundaries between research and practice are blurred. Blurring boundaries opens a space to produce professional learning and research simultaneously (Ball, 1995). There is no agreed-upon definition of blurring the boundaries between research and practice. Although there is no agreed-upon definition, research has described professional learning emerging as the boundaries blur (e.g., Attard, 2007, 2017; Ball, 1995; Chak, 2006; Metz & Simmt, 2015; Wilson, 1995). Thus, a contribution to the field of mathematics teacher professional development (Chapman, 2021; Jaworski, 2021; Matos et al., 2009) can be made by adopting a methodology for professional learning becoming research, where the products for both research and practice emerge within the blurred space. Mason (2002) provides such a methodology of professional learning becoming a form of research for the claim of blurring boundaries to be made.

Professional learning becomes research starting with a research question (Mason, 2002). Not every question is a research question. For instance, 'Why is the student frustrated?' is unrefined. There is a lot to probe (e.g., Is this the instructor's observation? How is frustration defined and in what context?) before something precise and detailed can be investigated. Mason (2002) states that a research question identifies objects to study and the methodology of inquiry. The question, 'What accounts of struggle and productive struggle are noticed by the instructor in developmental mathematics?' identifies struggle and productive struggle as the object of study in developmental mathematics. Noticing is the methodology of inquiry, systematically interrogating what the instructor is attending to, producing new possibilities that inform one's future action (Mason, 2002). Hence, professional learning starts to become a form of research by asking a research question about one's own practice.

Professional learning becomes research by reporting identifiable data (Mason, 2002). Mason (2002) states that data becomes identifiable when constructed by someone taking a research stance, intending to analyze it. Thus, accounts-of moments in the instructor's practice can become data when the instructor intends to analyze the data into themes of similarities and differences, and seek fresh possibilities from the themes. Mason (2002) states that data is supposed to reassure readers by enabling them to test the method themselves. The accounts-of describes what the instructor was sensitive to in one's context (e.g., supporting productive struggle in developmental mathematics). Others can test if they experience similar sensitivities in their own context (Mason, 2002). When others identify with the data, also known as resonance, the claims emerging from the data can be supported (Mason, 2002). The claims may need modification if the data does not resonate (Mason, 2002). For these reasons, making accounts-of identifiable allows others to resonate with the data, providing evidence that supports or modifies a research claim.

Professional learning becomes research through robust analysis (Mason, 2002). A robust analysis can categorize the instructor's sensitivities so the reader can recognize them (Mason, 2002). The analysis aids in convincing the reader of the interpretations and claims of the findings. For instance, there can be a claim that the instructor experienced supporting productive struggle in different ways. Accounts-of can be categorized and grouped into themes of similarities and differences to convince the reader of the claims. Such is the purpose of a phenomenographic analysis (Marton, 1976; Marton & Säljö, 1976). Using an established method like phenomenography to sort the accounts-of noticing strengthens the analysis (Tight, 2016). Sin (2010) states that the robustness of the phenomenographic analysis occurs by being explicit with the details of the analysis process, for example, describing how accounts were sorted and clear arguments on the distinction between categories. Mason (2002) states that robustness allows readers to resonate with the findings, as they can test the analysis method and offer critique. Thus, a phenomenographic analysis convinces the reader of the distinctions in the instructor's experience, providing a robust analysis for the reader to test and offer critique.

Professional learning produces knowing from experiencing what it is like (Mason, 2002). The research product simultaneously contributes to teaching and research. For instance, the product contributes to mathematics teacher professional development research (Chapman, 2021; Matos et al., 2009), as knowledge is gained on supporting oneself to teach mathematics. At the same time, knowledge is produced on supporting productive struggle in developmental mathematics. The products answer the questions that initiated the research inquiry (Mason, 2002). It aims to benefit others' practice of supporting productive struggle by offering moves (e.g., seeking fresh possibilities) to enhance future practice. It can also benefit researchers as it portrays the complex interrelationships of different aspects of the instructor's learning (Simon & Tzur, 1999). Others can test if they see similar or different interpretations from the analysis and the possibilities offered by the research in their own practice (Mason, 2002). Meaning that the product of knowing from experience offers activities and interpretations for others to test in their practice.

Professional learning can become research through a research question, identifiable data, robust analysis, and the product of knowing from experiencing what it is like (Mason, 2002) to support productive struggle in developmental mathematics. The research question has to identify objects to study (e.g., productive struggle) and the methodology of inquiry (e.g., the Discipline of Noticing). The data (e.g., accounts-of) has to be identifiable so evidence for and against a claim can be made. A robust analysis (e.g., phenomenography) allows others to critically examine the findings of similar and different accounts-of experience. The product of knowing from experiencing what it is like answers the research questions. It allows others to test the product in their own practice. By making the research components explicit, research products and the professional development of one's practice are produced simultaneously (Mason, 2002).

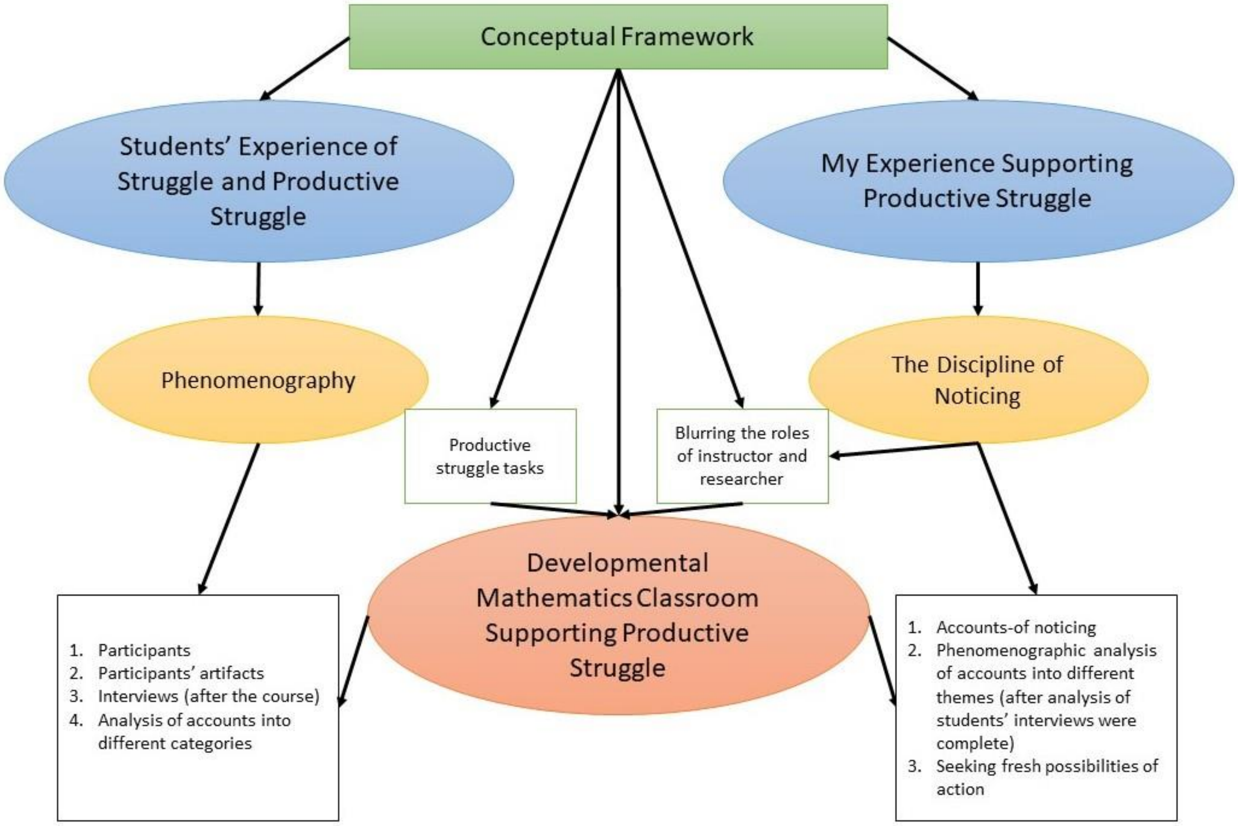
This chapter provides a theoretical basis for how productive struggle can be investigated in developmental mathematics. Supporting productive struggle means to implement challenging tasks to elicit students' struggles. Claims that students experienced productive struggle can be evidenced through approaches to learning, perseverance, affect, and habits of mind. The Discipline of Noticing outlines the process of how one can seek fresh possibilities in one's own practice. Seeking fresh possibilities simultaneously produces professional learning and research from experiencing what it is like to support productive struggle.

# Chapter Four – Methodology and Methods

Consistent with my position and this study's conceptual framework to investigate experience, struggle and productive struggle is described by how one interprets and attributes meaning to their learning. Thus, this study's methodologies and methods were approaches to elicit students and my experience. This study has two objectives: first, to investigate students' experiences, and second, to describe my professional learning from supporting productive struggle. This chapter presents the methodology and methods to address the research questions. Figure 1 displays how the conceptual framework informs the methodology and methods for both parts of this dissertation. The methodology and methods include phenomenography, the Discipline of Noticing, the design of the developmental mathematics classroom to support productive struggle, data collection, and analysis. The research questions and subquestions are: *How do struggle and productive struggle impact the classroom experience in developmental mathematics?*

a. In what ways do students experience and conceptualize struggle and productive struggle?
b. How do their experiences and conceptions of struggle and productive struggle impact their learning?
c. What accounts of struggle and productive struggle did I notice as the instructor of the course?
d. What did I learn from my accounts of noticing struggle and productive struggle?

**Figure 1**
*Methodology and Methods*




## Phenomenography

Phenomenography was chosen as the methodology to investiage students' experiences because it allows descriptions of the qualitatively different ways a phenomenon, like productive struggle, can be experienced and conceptualized (Marton, 1986, 1988). The methodology was selected due to its well-established roots in learning experiences, especially in higher education (e.g., Entwistle, 1997; Marton & Booth, 1997). One of the most influential studies identified two distinct approaches to learning (described earlier) based on how 30 first-year students of education or educational psychology read a newspaper article on curriculum reform (Marton & Säljö, 1976). Phenomenography grew out of concerns to improve learning through student conceptions (Åkerlind, 2018; Entwistle, 1997a; Marton, 1986; Marton & Säljö, 1976), and has led to research such as categorizing the relationship between experienced emotions and outcomes of learning (Trigwell et al., 2012), and approaches and conceptions of learning mathematics (Murphy, 2016). Phenomenography is a methodology developed within higher education by higher education researchers (Tight, 2016), fitting the context of this study in developmental mathematics. Hence, phenomenography was chosen because it aligns with the context of this study to investigate struggle and productive struggle in the developmental mathematics classroom.

Phenomenography aligns with this study's focus on students' experiences of learning. Experience in phenomenography is seen as the relationship between the learner and their world, not as two distinct entities (Pang, 2003). Phenomenographic researchers will argue that a phenomenon, such as productive struggle, can not be studied separately from the people who experience it (Sin, 2010). The main assumption is that there are a number of qualitatively different ways to experience each phenomenon (Bruce, 1994; Marton, 1986). How a learner discerns and focuses on simultaneous characteristics is a specific way of experiencing that phenomenon (Marton & Booth, 1997). However, discernment occurs as the learner experiences variations of that phenomenon (Marton, 2015). For example, a student may experience struggle as getting stuck and finding a way to overcome it compared to giving up due to getting stuck. Thus, phenomenography explores student experiences by revealing the various meanings, similarities, and differences associated with the phenomenon (Svensson, 1997).

The worth of applying phenomenography is to examine how people experience or conceptualize different parts of a phenomenon. Even if people experience the same parts, they may not be aware of them (Yates, Partridge, & Bruce, 2012). Meaning people may focus on different aspects, prioritize different factors, or have varying levels of awareness of different elements within the same phenomenon. This difference is emphasized in phenomenography by portraying the meaning of an experience and the structure of that experience (Marton & Pong, 2005). The two aspects are intertwined (Marton & Booth, 1997). The structure of the experience is composed of what the subject attends to, how they describe the phenomenon's parts to each other and to its cohesive whole (Marton & Booth, 1997). For instance, students may describe struggle as getting stuck or making mistakes. These discernments are what Marton and Booth (1997) referred to as the structure of that experience. Phenomenography goes beyond a simple description expressed by people, as the meaning conveys the richness and complexity of people's experiences. For example, the meaning of the experience could be students identifying that getting unstuck and learning from mistakes are productive struggles that support their mathematics learning. Awareness to describe the structure of a phenomenon and its unique meaning to an individual reveals the different experiences and conceptions of a phenomenon, like productive struggle. Marton and Häggström (2017) explain that this awareness makes teaching more powerful, as it offers the different ways learning situations can be handled.

The focus on students' varying conceptions and experiences of struggle and productive struggle is why phenomenography was chosen over other methodologies. Phenomenography is most often compared to phenomenology as researchers have argued that phenomenography is encompassed in the works of Husserl (Cibangu & Hepworth, 2016; Marton, 1981; Needleman, 1968; Richardson, 2005). This study adopts aspects applied in phenomenology, such as horizontalization within the phenomenographical method of analysis. Phenomenography was specifically chosen for this study because it concentrates on differing ways struggle and productive struggle can be experienced. Phenomenology was never intended to be a one-way or one-size-fits-all methodology, explaining the growing trend of different phenomenology methodologies (Cibangu, 2022). Some forms, such as the Discipline of Noticing, focus on first-person experience

(Mason, 2002, 2020), while other forms aim to make shared (rather than different) accounts of experiences (Käufer & Chemero, 2021). Phenomenological methodologies can be adopted to find differences, but phenomenography already exists. Hence, it was chosen. Grounded theory (Glaser & Strauss, 1967) aims to construct a theory to show action and change through an action-oriented research methodology (Kinnunen & Simon, 2012). This study aims to describe the various ways students experience and conceptualize struggle and productive struggle rather than generating a theory of struggle and productive struggle. Thus, the analysis aims to uncover distinctions between categories rather than uncovering a core category that describes or explains the whole phenomenon used in most grounded theory methodologies (Corbin & Strauss, 1990). Ethnography (Atkinson, 1991; Hammersley, 2006; D. H. Hargreaves, 1967) was not chosen because this study does not aim to provide a descriptive account of the culture of developmental mathematics students' experiences of struggle and productive struggle. Some ethnographic methodologies require the researcher to be embedded in the culture day-by-day and be skeptical towards statements made by participants (Richardson, 1999). Phenomenography used in this study takes the participants' descriptions as-is to form various conceptions of struggle and productive struggle. The main difference between phenomenography and the aforementioned methodologies is the focus on the variation in conceptions of a phenomenon. The methodologies share the goal of understanding lived experience where findings emerge from an analysis process that is inductive and iterative (Cibangu & Hepworth, 2016; Kinnunen & Simon, 2012; Richardson, 2005). As varying students' descriptions offer a unique way to describe the experience of struggle and productive struggle, and phenomenography provides such a methodology, it was most appropriate to answer this study's research questions.

Phenomenographic researchers usually collect interview data (the method adopted in this study to collect the students' experiences), but other data collection methods are possible (Kinnunen & Simon, 2012). Participants are asked to describe how they experienced or conceptualized the phenomenon in their own words. The analysis is an iterative process to form categories of similarities and differences emerging from participants' accounts. The first phase is for the researcher to familiarize themselves with the data by reading through the transcripts a couple of times to get an overview. The second phase is capturing quotes of interest for the questions being investigated. The selected quotes make up what phenomenographers call the 'pool of meanings' (Åkerlind, 2005; Marton, 1994). Categories emerge from the pool, grouped according to the critical attributes and distinguishing features between groups (Åkerlind, 2005). Next, categories are refined and redefined until a final set, referred to as the 'outcome space,' of qualitatively different categories emerges (Kinnunen & Simon, 2012; Marton, 1986). The phenomenographic categories reported in the findings of this study report the qualitatively different ways students experienced and conceptualized struggle and productive struggle in the classroom.

### *Quality of Phenomenographic Research*

The quality of phenomenographic research can face criticism regarding transferability, objectivity, and data reliability, but these issues can be addressed (Sin, 2010). These criticisms stem from phenomenography's origins, in which Marton (1981) argued against the positivist tradition of understanding students' experiences. One marker of quality for qualitative research is the extent to which the findings can be used or applied in other contexts (Mason, 2002; Miyata & Kai, 2009; Sin, 2010). In phenomenography, this transferability can be enhanced when the researcher provides sufficient information for users to make their own judgments (Miyata & Kai, 2009). In doing so, the worth of the phenomenographic findings is qualified by how others apply the findings in their own contexts, shifting from the positivist tradition in which quality is seen through generalizability. For example, the experiences and conceptions of struggle and productive struggle reported in this study findings do not claim to represent the population of developmental mathematics students. Instead, the findings report that students can experience and conceptualize struggle in various ways. Other practitioners can apply the findings to better design, notice, and respond to students' struggles in their own classrooms.

Objectivity is an issue when the researcher's influences are not addressed (Lincoln & Egon, 1985). Sin (2010) recommends reflexivity, where the researcher documents fully and explicitly each stage of the research process so that readers can make a judgment. Sin (2010) adds that phenomenographic interviews explicitly deal with objectivity through the expressive approach in which the participants' perspective is the focus. This study on productive struggle emerged from my need to improve my instruction (referred to in the introduction and my position in the conceptual framework). Hence, my developmental mathematics classroom was chosen as the setting. Further described was how I aimed to implement the literature on productive struggle within the 'constraints of developmental mathematics,' referencing my previous experience with a dense curriculum and passive modes of learning. My intent to implement productive struggle is incomplete without the students' experience, thus, phenomenography was chosen for this study. The interview questions focus on students' views, asking them to articulate their experiences. The pool of meanings shifts my attention from individual subjects to ways the quotes answer the research question. The focus is on what participants said, aiming to sort their comments into qualitatively different categories. Finally, the findings claim that the categories are qualitatively different.

The issue of reliability, the extent to which the findings of a study can be replicated, is to re-examine a phenomenon after an interval with the view of making a fresh appraisal in qualitative research (Sin, 2010). Inconsistencies in the outcomes from a previous analysis of the phenomenon are not seen as a failure in reliability. To address reliability in phenomenography, Sin (2010) recommends that the researcher document and explain clearly their research process so others can make a judgment about the analysis. Bruce (1994) suggests that researchers borrow reduction methods used in phenomenology. First, the researcher puts aside preconceptions about the phenomenon. For example, awareness to put aside the conceptions of productive struggle I have read from literature, such as students struggle to get started or carry out a process (Warshauer, 2015a). Secondly, the researcher needs to describe as objectively as possible rather than explain the experiences reported by theorizing or using judgment and evaluation, but the claim is not that this is an objective analysis. In phenomenography, this is achieved by focusing on the participants' words rather than my interpretation of what they said. For example, reporting students' stating they got 'stuck,' instead of the category students struggle to carry out a process often reported in research (e.g., Warshauer, 2015b). Finally, horizontalization treats all descriptions or experiences as having equal value (Friesen, Henriksson, & Saevi, 2013). In this step, the researcher needs to be receptive to every statement of the participants' experiences (Eddles-Hirsch, 2015; Moustakas, 1994). By explicitly reporting the analysis, others can test the method for themselves, achieving reliability.

## The Discipline of Noticing

The Discipline of Noticing educates awareness and informs action in teaching, learning, and professional development (Mason, 2011). Mason describes the Discipline of Noticing as "a collection of practices both for living in, and hence learning from, experience, and for informing future practices" (2002, p. 29). The practices are reminiscent of the NCTM's (2010) recommendations for professional development to build teachers' knowledge of mathematics and their ability to use it in practice, teachers' capacity to notice, analyze and respond to students' thinking, and teachers' ability to analyze instruction. Furthermore, Mason (2002, 2020) argues that a systematic methodology to work on oneself is a form of research. This form of inquiry provides a unique view of one's behavior that is notoriously difficult to replicate through experiments and measurements (Mason, 2020). Thus, adopting the Discipline of Noticing for this study provides a philosophically well-founded methodology of inquiry to investigate one's experience supporting developmental mathematics students' productive struggle.

This dissertation concerns what and how I am learning as an instructor in developmental mathematics. Described in the introduction was how I started teaching developmental mathematics, equipped only with how I experienced mathematics as a student. It was through discrepancies from my experience that I sought different ways to teach developmental mathematics. However, at that time, I did not have the research background to analyze how I came to be a different mathematics instructor. The Discipline of Noticing (Mason, 2002) provides a methodology for me to describe what I learned and share how learning emerges from the inside. The Discipline of Noticing brings forth my position to improve future action and realize fresh possibilities through what I have done or am doing.

The Discipline of Noticing (Mason, 2002) fulfills personal and social forces to improve my practice. Discussed in the previous paragraph was my personal motivation. My conceptual framing of seeking fresh possibilities is an interpretation of how the Discipline of Noticing can be used as a form of professional development that becomes research. Seeking fresh possibilities fulfills social forces, as I am required to produce research as a Ph.D. student and, as an instructor, enhance student learning by improving my teaching. Mason (2002) states that these forces drive change and development. These forces collide with everyday experiences and can be uncomfortable, frustrating, or surprising. For example, a conflict may arise in implementing research-recommended practice to support productive struggle in the developmental mathematics classroom. Being sensitive to the ebbs and flows allows me to accept changes and consider what is possible at any given moment (Mason, 2002).

The Discipline of Noticing can be combined with other self-based methodologies to enhance one's learning. Self-based methodologies privilege the self (Chapman et al., 2020) to handle the complexities of learning to teach from the insider's perspective (Helliwell, 2019). There is a shortage of self-based papers within mathematics education journals, but self-based study is a well-established genre of research within the broader teacher education community (Helliwell, 2019). To seek fresh possibilities from one's practice, one has to reenter moments and consider alternative ways of acting. Narrative inquiry is well suited for this role as meaningful episodes are organized to allow the narrator to interpret their experiences (Connelly & Clandinin, 1990). For example, Mason (2002) suggests adopting Chapman's (1999) narrative approach of rewriting the story of accounts to trigger a different possible action. Further possibilities in exploring the self become an examination of the social and cultural aspects of personal experience (Ellis, 2004). Autoethnography blurs the distinction between personal and cultural in understanding personal experience, through which a wider cultural context is also understood (Roth, 2005). The issues with the developmental mathematics system in Canada, which can be considered a culture, was the motivation for this study. My personal experience of supporting productive struggle is continually shaped through the interactions with the actors of the developmental mathematics system, classrooms, and curriculum. Such interactions are bound to have conflicts and contradictions, but seeking possibilities in these contradictions creates conditions for change (Helliwell, 2019). This change is not limited to my practice, but intertwined with students' changes (explored in the other part of this study), contributing insights into how the culture of developmental mathematics can also be changed.

### *Validating with Others*

The research findings within the Discipline of Noticing are validated through how my future action is informed (fresh possibilities), and how readers are alerted to something they can test out in their own experience (Mason, 2002). Thus, presented for readers is a way of working that instructors, especially in developmental mathematics, can use to educate awareness. It does not make a case for absolute truth or assertions about other people. This form of validation seeks resonance with others sought through multiple stages within the Discipline of Noticing (Mason, 2002). Accounts-of are recorded in a manner that others may reenter the account and seek fresh possibilities for themselves. A sensitivity to notice can only be "deemed insignificant or ineffective for certain people at certain times in certain situations" (Mason, 2002, p.189). Thus, if someone else does not recognize the account, it does not mean the moment is false. It may be that the moment offered is inappropriate for that time. However, the researcher has to be sensitive to continual failure to achieve resonance, as that may indicate that the researcher is deluding themselves (Mason, 2002). One approach used in this dissertation to achieve resonance was offering moments to *critical friends* to seek differing opinions, opening up a discussion or negotiation of meanings, and questioning the claims (Schuck & Russell, 2005). A critical friend is "a trusted person who asks provocative questions, provides data to be examined through another lens, and offers critique of a person's work as a friend" (Costa & Kallick, 1993, p.50). I will describe later how my supervisor was well-suited to be my critical friend during this study. A critical friend is considered foundational in self-based methodologies (Hamilton & Pinnegar, 2013), as it provides an alternative lens on one's work through fresh eyes to reach outcomes otherwise impossible from working independently (Baskerville & Goldblatt, 2009). The critical friend strengthens the resonance-seeking approach to validity crucial to the Discipline of Noticing (Mason, 2002).

Resonance-seeking is not simply recognizing or not recognizing someone else's account (Mason, 2002). Mason (2002) suggests that the most desirable form of validation comes from spontaneous utterances that someone has noticed something freshly and meaningfully for themselves. The more spontaneous the resonance, the more evidence that the resonance is real and not imagined (Mason, 2002). Failure to obtain resonance may mean that the noticing done by the researcher is of their own construction. It may also mean that the task to notice is ineffective or others are not prepared to resonate with the account at this moment. Resonance-seeking in this dissertation is achieved through feedback on findings from critical friends and connections to literature. For example, discussing how my findings connect to the literature on professional development establishes resonance with other researchers' views.

Rigor is viewed as the marker of quality in qualitative research (Krefting, 1991; Sandelowski, 1993; Smith & McGannon, 2018). In the Discipline of Noticing, rigor is addressed through "the subjective (personal observation) to achieve objectivity in resonance and confirmation with others" (Mason, 2002, p.192). Mason (2002) views knowledge as action and acting effectively addressed in a subjective manner. Meaning that acting effectively in the future is a product of investigating one's experience, which is inherently subjective. In this study, effective action is produced by seeking fresh possibilities from accounts noticed so that more choices can come to mind when called upon. Effective action is not evaluated by external criteria like statistical analysis but by my learning (seeking fresh possibilities).

Seeking multiple interpretations separates the researcher from solipsism, the danger of seeing only a single perspective (Mason, 2002). One remains flexible by intentionally seeking alternatives or variations, even if interpretations are conflicting (Mason, 2002). Furthermore, accounts are offered as conjectures, so coming to a single consensus is not the justification for action (Mason, 2002). In this study, fresh possibilities are sought through noticing, moments are discussed with critical friends, and literature is reviewed to evidence the relevance of the research data. For example, I may notice that I have the habit of helping a student in the same way when they reach an impasse. Critical friends may view the account as overhelping. The literature on productive struggle offers a possible action: teachers should be mindful of giving time for students to manage their struggles and not step in too soon or too much (Warshauer, 2015a). Thus, seeking multiple interpretations helps seek validation through resonance.

### *Using a Critical Friend to Support the Discipline of Noticing*

A critical friend supports the practices in the Discipline of Noticing. Although Mason does not specifically use the term ‘critical friend’ in the Discipline of Noticing, the process of learning from one’s practice and seeking resonance with others can be fulfilled through a critical friend. The act of engaging with a critical friend involves an exchange of insights, feedback, and perspectives (Crowe & Berry, 2007; Loughran & Brubaker, 2015; Schuck & Russell, 2005), which mirrors the process of seeking fresh possibilities advocated by the Discipline of Noticing. For example, a critical friend may offer an alternative perspective on an account-of that was previously unnoticed. However, the role of the critical friend is not only to offer one’s view, but to work with the individual to better align their purpose, practice, and learning outcomes (Crowe & Berry, 2007; Schuck & Russell, 2005). This study describes seeking fresh possibilities as a form of professional learning and research offered to others. Thus, the insights from a critical friend also provide feedback on how one can move from an account-of to a form of research. This could be offering insights that pertain to research. For example, the critical friend may offer that the account-of relates to overhelping because of the ongoing research on teachers overhelping students (Errico et al., 2010; Towers & Proulx, 2013; Warshauer, 2015a). In doing so, the critical friend supports how the account-of resonates with others.

During the analysis process, my supervisor assumed the role of a critical friend (Costa & Kallick, 1993; Schuck & Russell, 2005). For instance, I recorded an account of a student expressing frustration when I denied verifying if their answer was correct. My supervisor suggested that there may be a mismatch of conceptions of productive struggle. My supervisor's interpretation is evidence that the data was identifiable. More importantly, a possibility that I was not aware of was offered. Thus, the possibilities offered by a critical friend were vital to my claim to seek fresh possibilities from my experience.

With an extensive research background and shared commitment for me to complete my dissertation, my supervisor was uniquely positioned to assume the role of a critical friend. Knowledgeable of both the Discipline of Noticing and phenomenography, coupled with a genuine investment in my academic journey, formed a basis of trust required for successful critical friendships (Baskerville & Goldblatt, 2009; Costa & Kallick, 1993). This trust was necessary to balance challenge and support during my analysis and presentation of findings. As a novice researcher, carrying out a rigorous analysis was new to me. I was unaware that I was interpreting, explaining, or justifying actions in some accounts. Such practice is what Mason (2002) describes as accounting-for, which when done prematurely limits sensitivities to consider alternatives. For example, I first assumed a student was stuck because they responded “I don’t know” to my prompt. I proceeded to help the student under that assumption. My supervisor challenged my action and considered it overhelping. It was the challenge that made me reconsider my assumptions. Not only was the interaction significant for me to seek fresh possibilities from my practice, but teacher overhelping is often discussed in the literature (e.g., Towers & Proulx, 2013; Warshauer, 2015a). This connection to literature is evidence that the account resonates with much-discussed teaching issues. Thus, insights from my supervisor as a critical friend supported my professional learning and helped my findings achieve resonance significant to current literature.

### *Using Phenomenography within the Discipline of Noticing*

Mason (2002) described how phenomenography could enhance the seeking of fresh possibilities in the Discipline of Noticing. In this position, learning consists of extending variations of which we are aware (Marton & Booth, 1997). In doing so,

> one becomes more sensitive to making distinctions, developing awareness of connections and inter-relations amongst those distinctions, broadening the range of resources one calls upon and the tasks one undertakes in order to pursue aims and goals which are more precisely articulated, and increasing the scope and nature of possibilities and potential we recognize, because our present moment is extended. (Mason, 2002, p.231)

In performing a phenomenographical method of analysis of my accounts of noticing struggle, I become more sensitive to the distinctions between the accounts. I broaden the range to seek fresh possibilities of accounts as I can reflect on a particular account or a theme emerging from the analysis. New actions can be triggered from a similar account occurring in the future and from a theme or distinctions between themes, increasing the nature of possibilities. As phenomenography focuses on the experience of my past self as the instructor, the analysis provides me another way to stand back and analyze the accounts in a fresh way before reentering accounts to seek fresh possibilities (Mason, 2002). It also provides a robust analysis of data required to produce qualitative research (Mason, 2002).

In summary, phenomenography and the Discipline of Noticing align with this study's investigation of experiences of struggle and productive struggle. Phenomenography answers the research questions about students' experiences and conceptions of struggle and productive struggle. The second part, the instructor's experiences, is investigated through the Discipline of Noticing. Applying phenomenography's method of analysis within the Discipline of Noticing expands the ways I can seek fresh possibilities from my experience. My conception of students' efforts was a theme emerging from my research background. Discussed was how my conception evolved as I was exposed to literature and altered my teaching actions. My position influences how I interpret the data (described in the conceptual framework). However, it is through the interrogation of my previously held conceptions that an in-depth understanding of experience emerges.

**Supporting Productive Struggle in a Developmental Mathematics Course**

The first phase was designing a developmental mathematics course that supports productive struggle. The details of the course, participants, my role as an instructor researching my practice, and how productive struggle was supported are described in this section.

The developmental mathematics course ran for 14 weeks from September to December 2019 at a college in Ontario. Students in this course were enrolled in diplomas such as Heating, Refrigeration, and Air Conditioning Technician (HRAC), Electrical Engineering Technician (EET), and Technology Foundations (TF). All students in this course were in the first semester of their program. The course was the first of either two (HRAC students), three (TF students), or four (EET students) developmental math courses to be completed in their diploma. There were five sections of the same course, but this study focuses on one section where I was the instructor. Two classes were scheduled weekly on Tuesday and Thursday, slotted for an hour and fifty minutes each. The learning outcomes were: (1) perform arithmetic operations with real numbers, including those in engineering and scientific notation; (2) apply appropriate mathematical laws and principles to perform fundamental algebraic operations; (3) solve linear equations and rearrange formulas; (4) factor algebraic expressions; (5) apply the concepts of ratio, proportion, and variation; (6) convert between binary, octal, and hexadecimal numbers; (7) use an appropriate scientific calculator to perform all required computations; and (8) apply studied mathematical concepts and methods to analyze and solve a variety of scientific and technical problems.

The topics in chronological order of delivery included operations with integers, fractions, order of operations, calculations with approximate numbers, scientific and engineering notation, percent, computer arithmetic, roots and radicals, exponents, simplifying algebraic expressions, formula rearrangement, ratios and proportions, and factoring using special products. The topics and the delivery order were fixed, and tests and quizzes were standardized among all the sections. Instructors had the freedom to select their method of delivery.

The course's evaluation involved two written tests worth 25% each, four in-class quizzes (10% each), and eight online quizzes that comprised the other 10%. Students had to achieve 50%

or higher in this course before moving on to the next developmental mathematics course. The eight online quizzes throughout the semester were preset by an instructor who previously taught the course on Pearson's MyMathLab. Although I created two in-class quizzes, they had to follow a template so they could be distributed to all sections. The only adjustments I could make were the variables and values within the questions, not the content or the type of question. Thus, the tests and quizzes were not designed to measure productive struggle. Tests were delivered on weeks 7 and 14, and in-class quizzes on weeks 3, 5, 9, and 12.

The rigid structure of the course can be seen as a limitation in supporting students' productive struggle. Since I had no control over the topics to cover, order of delivery, and type of questions on assessments, the question of how to design assessments supporting productive struggle in developmental mathematics is beyond the scope of this study. However, the findings are relevant to how productive struggle can be supported within the constraints of a traditional developmental mathematics course.

***Participants***

The initial enrollment for the course was thirty-seven. After the enrollment deadline in week 2, the class settled at thirty-three students. All students enrolled in the developmental mathematics course were invited to participate in this study. Based on an entrance assessment, the students of this course were deemed to require this course to develop the mathematical knowledge required for their diploma. Students in this course had five to seven courses in their first semester.

A colleague read a script that I prepared and collected consent forms during week five while I was not in the room. The consent forms were returned to me after the final grades were submitted in December. To ensure that students' grades for the course were not affected by their decision to participate in this study, I did not know who provided consent to participate until after the course was completed and the final grades had been submitted to the college. Students had the option of whether or not to be part of the research study. Thirteen students agreed to participate in the study. In December, all of them were invited for interviews. Of the thirteen participants, five responded to the email. All five participants were interviewed between January and March 2020. Pseudonyms were used for all participants. 'They' was used as a pronoun to describe the participants since genders were not identified. I was not their instructor during the subsequent developmental mathematics course during the semester that the interviews took place.

### *Blurring the Role of Instructor and Researcher*

Blurring the roles of instructor and researcher afforded me a unique insight into noticing classroom moments of students' struggles. This synergy of my experience from both roles has two immediate implications. As an instructor for the past decade, I have gained personal experience teaching developmental mathematics courses. For example, how students in my past classroom negotiated the demands of their program while studying developmental mathematics. Hence, my role provides a closer connection to students than an outside researcher. Secondly, as a graduate student in a faculty of education, I engage with educational literature and have insights into pragmatic theories of teaching and learning. The literature on productive struggle, understanding, and affect may conflict with or agree with my classroom experiences. These clashes may be surprising or confusing, marking a discrepancy from what I have experienced. Noticing can emerge from what was unexpected (Mason, 2002), providing the data that is the basis of this study. Awareness of these conflicts also acknowledges my beliefs and biases that need to be set aside but not abandoned in the phenomenographic analysis (Bruce, 1994). I do not claim that I know more because of my position from the inside. Instead, it is a different position, and being more aware of this position is one of the aims of this study. As mentioned earlier, this position is rare as it is atypical for Canadian college instructors to partake in research (Maciejewski & Matthews, 2010). Furthermore, large-scale quantitative studies (e.g., Fong et al., 2015; Zientek et al., 2019) are limited to exploring the developmental classroom from the outside. For this reason, blurring the roles contributes a different perspective.

***Productive Struggle Tasks***

The design of lessons followed recommendations from literature on productive struggle to provide opportunities for students to expend effort to understand mathematical ideas (Hiebert & Grouws, 2007; Warshauer, 2015b). For instance, students should struggle with mathematical tasks to experience breakthroughs emerging from struggle and confusion (Murawska, 2018; NCTM, 2014; C. Townsend et al., 2018; Warshauer, 2015b). During these tasks, opportunities to engage with the learning content in a self-determined way are necessary for students to become aware of their struggles (Hiebert & Grouws, 2007; Westermann & Rummel, 2012). Kapur’s (2008) design of productive failure encompasses such goals through two phases: students first collaboratively grapple with problems without receiving instructional support, then compare and contrast solutions and approaches.

Instruction was delayed, pointing out to students that delaying can act as a trigger to uncover their struggles and learn in a self-determined fashion (Roll, Holmes, Day, & Bonn, 2012; Westermann & Rummel, 2012). Delaying instruction meant that students worked on questions at the start of the class, and explanations were provided if requested. This is compared to starting the class with a lecture explaining examples. While the instruction was delayed, students were encouraged to solve questions in pairs but could choose to work independently.

Students were asked to play roles with their partners to promote self-explanation. Self-explanation can help students explicitly relate to each step, be aware, and question their own understanding (Alcock, 2018; Rittle-Johnson, Loehr, & Durkin, 2017). Each partner assumed a role to promote conversation that focused on questioning each other's understanding. One student adopted the role of a thinker, and the other a questioner. The thinker verbalized their thinking process (self-explanation). The questioner was asked to listen carefully to the thinker's verbalization, and to pose questions clarifying unclear explanations or questioning an incorrect step rather than simply providing the answer. These scripted roles have been shown to promote cognitive and metacognitive processes and have been used in high school and university classrooms (Berg, 1993; Westermann & Rummel, 2012). The partners switch roles when solving the next question. Hence, the partner roles provided opportunities for students to share struggles and probe deeper into their thinking with each other.

My actions focused on supporting students to struggle productively during the task. Warshauer's (2015b) four recommendations guided my actions: (1) Asking purposeful questions to help students reflect on the source of their struggle in understanding. For example, students were asked to describe confusions, moments of impasse (e.g., why they felt they were stuck), and share how they felt during moments of struggle with their partners, the instructor, and class discussions. (2) Encouraging students to engage in the process of thinking about mathematical ideas and not focusing on the correct solution. For instance, students were encouraged to consider alternative solutions and convince their partner that their solution was correct rather than ask for verification. (3) Providing ample time for students to struggle (e.g., delayed instruction, pausing for students to think, walk away and return later). (4) Showing students that struggle is a necessary part of learning. The fourth recommendation was exemplified in the explicit labeling of struggle and productive struggle in the classroom, as students were unfamiliar with struggle and productive struggle. Examples such as falling down as a necessary component of learning to ride a bike were told. Also, I explicitly related students' moments of impasse and feelings of discouragement as struggle and described that their efforts to come up with different strategies during those moments were examples of productive struggle. Mason (2002) states that having a name for complex ideas helps trigger awareness of the idea, permitting access to the possibility of responding differently.

Three exceptions did not follow the proposed design. One exception was the two in-class tests. Two review sessions before each test were planned differently. Because the test (standardized for all sections) mainly focused on procedure, I decided to mimic the test environment for students in the class before a test. I revealed several questions at a time and used a time limit to challenge the students. Questions were displayed on the projector. Students had to put their notes away and turn off their phones. Three to four questions would appear, and I informed the students that they had ten minutes. Afterward, they had another ten minutes to discuss the questions they were unsure of with each other. One other class differed in design based on how students reacted to a topic that they were unfamiliar with. Scientific and engineering notation in week 4 was new to students, and students immediately voiced that they did not know how to start. As a result, I performed a mini-lecture for 20 minutes to explain the concepts of accuracy and precision in rounding, and work through some examples on the whiteboard in front of the class. Two weeks later, I anticipated that computer arithmetic was another new topic for students. The class began with a mini-lecture going through conversions between binary, octal, decimal, and hexadecimal numbers.

**The Questions.** Questions designed to elicit uncertainty provided opportunities for students to expend effort to understand mathematical ideas. Through "some perplexity, confusion, or doubt" (Dewey, 1933, p.15), tasks that elicit uncertainty can foster mathematical understanding and identify dispositions towards and beliefs about mathematics (Zaslavsky, 2005). In other words, students struggle with questions that are not immediately apparent to deepen their mathematical understanding. Students worked on twelve to twenty questions in each class. The questions were based on the topic that was outlined in the curriculum. For example, the first class included fifteen questions on fractions. I explained that the questions could be done in any order, that the goal was not to complete all questions during class time, and encouraged students to work with their peers. The questions were based on Zaslavsky's (2005) examples to create uncertainty through contradicting statements, incompatibility with prior knowledge, difficulty verifying solutions, or lack of necessary knowledge.

Specifically, questions targeting misconceptions provided opportunities for students to question their prior knowledge. The contradictions may trigger a rethinking of critical conceptual features (Kapur & Bielaczyc, 2012). The following are examples of multiple-choice questions assigned to students in this study to draw attention to misconceptions in exponents:

1. $\left(\frac{-24t^6}{8t^3}\right)^5$
   a. $=-3t^3$
   b. $=-3t^{15}$
   c. $=-\frac{243}{t^{-15}}$
   d. $=243t^{15}$

2. $14a^4b^6+a^6b^3$
   a. $=15a^{10}b^9$
   b. $=14a^{24}b^{18}$
   c. $=14a^{10}b^9$
   d. $=15a^{24}b^{18}$
   e. None of the above

During the delayed instruction, students had opportunities to apply the rules they knew. As they shared their reasons, their partner may question how and why certain operations were performed. If the student circled what they thought looked right, uncertainty might arise when they lack the reasons to defend their claim. The uncertainty may provide opportunities for students or partners to question their prior knowledge of the concept. Meaning questions targeting misconceptions aimed to elicit uncertainty from students' prior knowledge, providing opportunities for struggles to be shared.

Incorrect worked examples were also provided for opportunities to compete with students' prior claims (Zaslavsky, 2005). Through the search for contradicting statements, students may reconsider what they believe are mathematical truths. The following question is an example of an incorrect worked example assigned during the course.

$$\begin{aligned}
&\left(\frac{35m^7n^{-4}}{26}\right)\left(\frac{56m}{13m^{-3}n^5}\right)^{-2}\\
&=\left(\frac{35m^7}{26n^4}\right)\left(\frac{56m^4}{13n^5}\right)^{-2}\\
&=\left(\frac{35m^7}{26n^4}\right)\left(\frac{3136m^8}{169n^{10}}\right)\\
&=\frac{65n^6}{896m}
\end{aligned}$$

Students were asked to search for the errors, explain why they were incorrect, and make corrections. Another reason this form of questioning was used was students' familiarity with worked examples (e.g., textbook examples; presentation of worked examples in math lectures in students' past school experiences). Each step required the student to work out the operation in which a decision emerged: agree with the displayed operation or disagree because it conflicts with what students previously understood. Sharing why they agreed or disagreed allows students to discuss their understanding of the underlying properties. For this reason, contrasting what students knew with the incorrect worked examples may lead to opportunities for students to discuss their understanding of mathematical ideas.

Open-ended questions elicit uncertainty as outcomes are non-readily verifiable (Zaslavsky, 2005). These questions were assigned to students to address understanding rather than procedure. For example, students could not start the question by deducing the next step. The following example was assigned: Some numbers can be expressed as the sum of a string of consecutive positive numbers. For example, 9=2+3+4, 11=5+6. List different properties associated with adding consecutive numbers. Open-ended questions prompted students to consider questions differently than repeating procedures. From my experience, developmental mathematics students frequently encounter questions that ask them to perform the correct procedure to get the right answer. Thus, uncertainty may arise from facing a question they were unfamiliar with. Not knowing how to start is a form of struggle (Warshauer, 2015a), which can be discussed with their partner or myself, the instructor. Thus, questions that students could not solve by applying a set of procedures may give rise to uncertainty to be discussed.

Some questions focused on procedure. For example, evaluate $10-2\times\left(4^2\div4\right)\div\frac{1}{2}+9$. The questions allowed students to refresh what they knew about the topic. The question may lead to several incorrect answers. As students discuss their solutions, uncertainty may arise if they lack the necessary knowledge or conviction that their solution is correct (Zaslavsky, 2005). Questions with verifiable outcomes may provide students with a starting point instead of diving into uncertainty or being overwhelmed by the questions described above. Four to five questions focused on procedure during the first two weeks. I noticed that students could quickly find the answer on an app or on the internet. The solutions were shared instead of discussed, which diverted away from the intention of the design. Thus, no more than three questions that focused on procedure were included after the third week.

**Students' Experiences: Phenomenographic Data Collection and Analysis**

Phenomenography aims to see the world as it appears to participants (Marton, 1986). The interview, popularly used in phenomenography (Bruce, 1994; Kinnunen & Simon, 2012), was adopted in this study. Artifacts can trigger memories of experiences during semi-structured interviews (Abildgaard, 2018; de Leon & Cohen, 2005). Thus, students' journals and summaries of class discussions were collected during the course to support the phenomenographic interview.

Journals and summaries can be used as artifacts to support the interview. Wildemuth (2017) stated that artifacts serve as an unobtrusive method of data collection, compared to intrusive methods such as a researcher observing a participant. Other studies have found that artifacts trigger memories (de Leon & Cohen, 2005) and elicit elaborate conversation (Danby, Ewing, & Thorpe, 2011) during interviews. No studies were found in this review that applied artifacts specifically to a phenomenographic interview, but there were examples in phenomenology (e.g., Abildgaard, 2018) and ethnography (e.g., de Leon & Cohen, 2005). Class discussions at the end of class were prompted by questions such as, 'How did it feel to struggle?', 'What did you struggle with?', or 'Describe productive struggle in your own words.' Due to a lack of participation after the first few weeks, an app called Mentimeter was used for class discussions. Students could enter their responses through their phones without identifying themselves, and the class could see the collection of responses on the screen in front of the class. I summarized the experiences students shared after the class and posted the document on our course webpage. The following is an excerpt from the document summarizing a class discussion on Oct. 29.

Describe productive struggle in your own words.

What you shared:

- Improving on your mistake or failing
- Practice until you feel comfortable with the concepts
- Finding a more productive way to come up with answers, using a more difficult, challengeable way instead of going the easy way
- Looking for simpler solutions
- Productive struggle is the ability to overcome a struggle in a productive/positive way.

Students were encouraged to document moments of struggle and productive struggle during each class in their journals. There was an option for students to submit these notes for bonus marks at the end of the course. An excerpt from one of the assignments can be found in Appendix A (Figure A1). The artifacts were only used to trigger memory and conversation during the interview. Thus, artifacts were not included in the data analysis. The participant's own journal and the document summarizing class discussions were laid out on the table during the interviews. Some participants reread the artifacts to answer the interview questions. For example, during the interview question, *what did you think productive struggle was in the first week of the course?*, a few participants read their early journal entries before answering the question.

Many phenomenographers (e.g., Marton et al., 1993; Marton & Pong, 2005; Säljö, 1979; Trigwell et al., 1994) use interviews to bring forth the participants experience, as it is descriptive, specific, focused on certain themes, open to change, and takes place as an interpersonal interaction (Bruce, 1994; Kvale, 1983). For these reasons, audio-recorded semi-structured one-on-one interviews were conducted for data collection between January and March 2020. Marton (1994) explained that the phenomenographic interview should facilitate the thematization of aspects of the person's experience and conceptualizations that had not been previously thematized. This applies to developmental mathematics students as the theme of productive struggle has never been explicitly identified in their past experiences. Bruce (1994) states that the focus of the phenomenographic interview is specific to the relation between the participant and the phenomenon. The following semi-structured interview questions focused on the participants' experiences of productive struggle.

1. What did you know about productive struggle before the course?
2. What did you think productive struggle was in the first week of the course?
3. What do you think productive struggle is now?
4. How would you explain productive struggle to a friend/classmate?
5. How did your understanding of productive struggle change throughout the course? Please use concrete examples and descriptions.
6. Can you describe a few experiences of productive struggle in this course?
7. How did the experience impact your learning? Good or bad. Please tell me how. Concrete examples are helpful.
8. Did you find yourself struggling during parts of the lessons? Please describe the feeling and describe which part of the lecture you struggled with.
9. Was there a lesson you found more helpful for your learning? Please tell me why.
10. Was there a lesson you found less helpful? Please tell me why.
11. How were the lessons in this course different from what you have experienced in other courses?
12. Is there anything else you want to share?

The role of the interviewer is “to try to see the phenomenon as it is seen by the interviewee” (Bruce, 1994, p.50). Marton and Booth (1997) argued that the interviewer needs to adopt specific strategies to break down or bypass the interviewee’s resistance to sharing. Thus, semi-structured questions were used to encourage reflection, probe for analogies (e.g., you said it was like), and confront and pursue areas of confusion (Bruce, 1994). During such moments, I probed the participant with inquiries such as: *Please explain. Tell me more about… Can you explain that in a different way? Give me an example. Describe that a little more, please*. The aim of the semi-structured interviews is for students to elaborate their own descriptions in more detail. Avoided was asking a question based on the researcher’s preconceptions, for example, “Would you describe that as a struggle getting started?” This question emerges from the researcher’s preconception of struggle from the literature (e.g., Warshauer, 2015a) and not the participants’ description.

### *Data Analysis*

Phenomenographic analysis aims to uncover qualitatively different categories of experiences and conceptions within participants’ descriptions (Marton, 1994). The claim is not that this is an objective analysis, nor is it a description of all the different experiences and conceptions of struggle and productive struggle. The similarities and differences emerge from the participants’ words and not the interpretation of the researcher (Marton, 1994). After reading through the transcripts a couple of times, participants’ quotes that provide insight into the research questions were highlighted. The selected quotes that address the research questions contained the word struggle or a synonymous statement. The quotes formed the “pool of meanings” (Marton, 1994, p.4428). Similar quotes in the pool of meanings were placed into categories, and the categories were redefined until the final categories, known as the outcome space, emerged (Marton, 1994).

The sorting of statements into categories follows the rule of horizontalization. Horizontalization treats every statement in equal value (Eddles-Hirsch, 2015). I was required to consider the categories that emerge from the words of each statement. What was avoided was trying to place a statement into an established category. The statements were sorted by placing similar quotes into categories. I added the reason the quotes were placed into a category as justification for review after each round of analysis. Initially, there were two overarching categories: (1) experiences of productive struggle while working on math problems and (2) impacts on learning. Category 1 had three subcategories: problem solving, communicating, and not being told the answer. Category 2 had ten subcategories: developing mindset, time and effort, responsibility, active learning, reflection, retention, importance of math, knowing and understanding, a different way to learn, and transferability to other classes. Feedback from my supervisor on the common features and differences pointed out the need to redefine the categories again. For instance, there were several quotes that could fall into multiple categories. The review required a “step backward and [redefine] meta-level differences between the categories” (Kinnunen & Simon, 2012, p.205). The redefinition of the categories emerged into a final set on how students experienced and conceptualized productive struggle presented in the findings.

**My Noticing and Data Analysis**

My experience supporting productive struggle addresses the research questions on what accounts of struggle and productive struggle are noticed and how these accounts evolved my practice. Accounts-of are useful when others can recognize the incident being described (Mason, 2002, 2011). Making data identifiable begins with moments that caught my attention in the classroom. I marked the moments of noticing with a brief note (Figures A2 and A3), for example, “looking up conversions” or “struggle understanding question.” After each class, I recorded the accounts-of incidents of struggle and productive struggle as objectively as possible in my journal. Here is an example of an entry from my journal:

> Account – Ratios and Proportions. There seems to be a struggle in connecting ratios with proportions. I did not notice this immediately. A student asked me for help when the ratio questions were completed, and the student began working on proportion questions. The student stated, “I am not sure how to do this.” I started noticing the struggle when more students asked for help when they reached the same question.
>
> This prompted me to ask myself when and if I should stop the class to summarize the difference between ratios, rates, and proportions. I chose to wait until every student had completed the ratio questions and got a chance to try a proportion question. (November 19)

The account aims to be recognizable by describing what was said at the moment. The subsequent actions describe my particular sensitivity to that moment. For example, one can say I was attending to classroom management in deciding to ask when I should stop to explain. I could reenter the account during the analysis to consider what I missed, as my attention was drawn to managing the class. Others can criticize my actions or consider what they would do at that moment. Such action by the reader is evidence that the account is recognized. There were also reminders for subsequent lessons and reflections on previous lessons recorded. For example, “take more time to catch these instances” (October 15) reminded me to be sensitive to specific moments of struggle noticed in future lessons. The following text in the journal was an example of a reflection recorded on how students engaged and worked independently.

> As I am writing in this journal, I am reflecting on the different struggles and ways to mitigate the struggles. It has been a class where engagement is not natural. Students rather work independently. Struggle has been communicated, but we will see what it means to them during interviews. (November 19)

Phenomenography was used as a method to analyze my accounts as a robust analysis is required for professional learning to become research (Mason, 2002). It also provides a method to "locate what is the same, and what is different about different accounts" (Mason, 2002). Stressing sameness and difference provides "a matter of context and point of view" (Bateson, 1994, p.89) and a way to abstract (Gattegno, 1987). Hence, fresh possibilities were sought from the different themes after the analysis. Themes are issues or tensions that emerged from my experience (Mason, 2002). These tensions are critical because they mark turning points in one's awareness, which initiates a rethinking of beliefs or practices of teaching (Mason, 2002). This may lead to some principle that is open to inquiry and modification, and can be tested in future practice to locate incidents in which the principle might be appropriate (Mason, 2002).

The collection of classroom instances, reminders, and reflections from my journal composed the data. A phenomenographic analysis of the data began with the determination of whether and how each account related to the research questions. Mason (2002, 2011) states that we notice many things, but some of our attention is directed and occupied elsewhere. Thus, accounts were omitted when they did not relate to the research questions. Accounts were omitted because they did not relate to struggle. For example, moments when students walked in late or were absent, student reactions to reminders for an upcoming assessment, and students returning from Reading Week. The following account of students being late to the first class is an example that was omitted:

> Several students were thirty minutes late to class. I had to take time to repeat the demonstration on sharing mathematical ideas with their partner. (September 5)

The remaining accounts formed the "pool of meanings" (Marton, 1994, p.4428). These accounts remained because they answered the research questions by capturing my noticing of struggle. For example, the following account made the pool because it states students' confusion with a topic (synonymous with struggle).

> The topic of scientific and engineering notation was confusing for students. This was noticed as hands were raised shortly after reading over the questions, asking me to explain what to do. Students reported that this topic was new to them. (September 17)

Accounts from the pool of meanings were reread to define the key features. Each account was tagged with a few words that embodied its key features. For example, the account above was coded with the words *students' action, confusion,* and *new topic*. Two distinct themes emerged in the first coding round: students' actions and overarching ideas on struggle and productive struggle. The first theme included accounts tagged with *students' action* because they described students' actions during struggle. A new theme was formed for the remaining accounts that did not fit into the first theme because they did not focus on students but on overarching ideas about struggle and productive struggle. The following account is an example in the second theme about the overarching idea of assessment in relation to productive struggle: "The assessment is a fixed one, will it conflict/support/deter productive struggle?" (October 1). This process of identifying subthemes based on each account's features continued within each of the two overarching themes. Presented in the findings are the qualitatively different themes that I noticed struggle and productive struggle in the classroom.

The phenomenographic analysis addressed what I noticed and provided a means for me to stand back to seek fresh possibilities. The themes emerging from the analysis respond to the research question: What accounts of struggle and productive struggle did I notice as the instructor of the course? The reported accounts represent what was noticed through experiencing what it is like to support productive struggle in developmental mathematics. The phenomenographic analysis adds to the ways I can stand back and make sense of what I experienced. For example, I can reflect on individual moments, common themes, and distinctions between themes. Mason and Spence (1999) state that keeping the reflective process fresh is vital as one loses sensitivity from repeating the same action. As the goal of seeking fresh possibilities is to be more aware of what I did not notice in those moments, different ways to reflect on moments enhance the possibility of something new emerging.

Seeking fresh possibilities after the phenomenographic analysis addresses the research question, 'What did I learn from my accounts of noticing struggle and productive struggle?' Fresh possibilities can be sought by considering why I felt at odds with an account, from the point of view of someone else, and what I wished I had done differently (Chapman, 1999; Mason, 2002). My learning is evidenced by the new actions emerging from rewriting the story of the initial account (Chapman, 1999; Mason, 2002). This process of new potential actions emerging from investigating possibilities is called acting-as-if (Jaworski & Watson, 1994). Acting-as-if is vital to one's change as "we cannot be sure until we have tried something, but we cannot have really tried it until we have acted-as-if we believed it will work" (Mason, 2002, p.124). Thus, the findings address research questions by reporting the different ways I noticed struggle and the fresh possibilities of actions sought from those accounts.

# Chapter Five – Findings

This chapter is organized into two sections. The first section presents the outcome space from the phenomenographic analysis of students' experiences and conceptions. The second section includes the themes emerging from my journal and how I sought fresh possibilities from my experience.

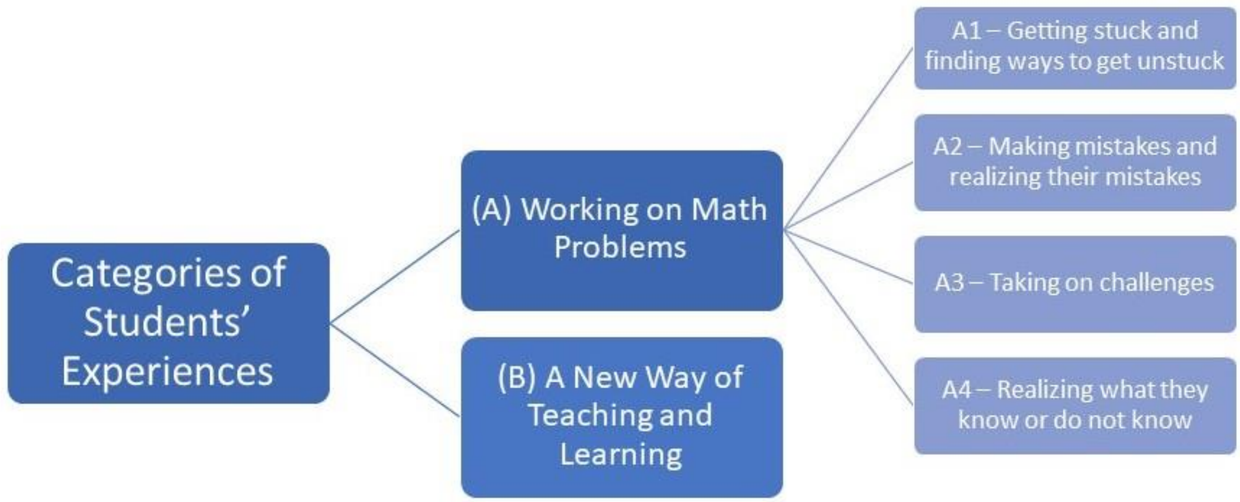
## Part 1: Students' Experiences and Conceptions

As described in the phenomenography data analysis section, the outcome space described the qualitatively different ways students experienced and conceptualized productive struggle. The final categories and subcategories were presented to my supervisor with an emphasis on how each category was qualitatively different, the interview question which the quotes originated from, and why they were placed in the selected category. This section presents how the following criteria of the outcome space were met: a) there must be something distinctive about the conception in each category; b) the categories are optimal and made with the fewest assumptions; and c) the relation between the categories are clearly stated (Marton & Booth, 1997).

Students experienced and conceptualized productive struggle through two overarching categories: (A) working on math problems and (B) a new way of teaching and learning. Figure 2 lays out the categories and subcategories from the analysis. The first category centers around how students synonymize productive struggle with experiences of working on math problems in class. The second category focuses on students' conception that productive struggle is a new way of teaching and learning math. The categories emerged from the analysis of participants' responses to interview questions. Quotes are provided to outline participants' experiences and conceptions in each category. Phenomenography aims to describe the variety of ways people conceptualize a phenomenon and not the most common experience (Marton, 1986). Thus, the categories may have emerged from comments from one or all the participants. The pronoun 'they' is used to represent students in these findings since genders were not identified.

**Figure 2**

*Categories of Students' Experiences and Conceptions of Productive Struggle*




***Category A – Working on Math Problems***

Students tended to describe productive struggle through their experiences in class, a determining factor of comments placed in this category. Experiences emerged from working on problems designed to elicit uncertainty. Students referred to these sets of problems as worksheets. Statements under this category were prompted by interview questions about what productive struggle meant to them or how they would describe it to somebody else. Participants conceptualized productive struggle as: (1) getting stuck and finding a way to get unstuck; (2) making mistakes and realizing their mistakes; (3) taking on challenges; and (4) realizing what they know or do not know.

**Category A1: Getting Stuck and Finding a Way to Get Unstuck.** When Randy was prompted to explain what productive struggle means, they connected learning to getting stuck on questions: "In order to learn, you have to kind of get stuck and figure out why you got stuck." Stuck was perceived as not knowing how to proceed with the next step or not being able to complete the question. Aaron explained that it was helpful to work with someone on questions because "you could get stuck, … [and] don't know the next step." On the follow-up question, what does stuck mean? Aaron said, "Stuck would be not being able to complete a certain question."

After responding to the question on how they would explain productive struggle to a friend, Jackie explains that getting told exactly what to do can lead them to being stuck again. "I actually worked on it by myself, instead of somebody just telling me, 'Okay, just use this formula. Do this, do that.' Then, after that person is gone and you do it on your own, you get stuck again."

When participants were asked how being stuck felt, they used words that indicated stress and frustration. The following statement expresses their stress and frustration: "I should know this; it's a math problem; it's not impossible to solve; something is possible" (Felipe). Randy's response to how stuck felt highlights the uncertainty and doubt they grappled with: "Is this even a foot in the right direction? Or am I taking a step back? Am I just not progressing at all?"

When prompted to explain what they did when they grappled with uncertainty, Randy stated, "Getting stuck makes me step back and look at the problem in a different way." These different ways of becoming unstuck included understanding when to apply specific techniques. Students noticed that there were certain techniques associated with specific topics. For example, Randy reported that they got stuck dividing polynomials using long division. When prompted about what they did to get unstuck, they explained:

> Well, certain questions have certain techniques to approach it, right? So just knowing those techniques on how to approach it … helped me kind of identify the

question and move on from there as opposed to just looking at it and "I don't know what I am looking at!" So [the question] identified what I can do here. (Randy)

Students related productive struggle to getting stuck and unstuck. The state of being stuck was frustrating and filled with uncertainty. However, it prompted them to step back, look at the problem in a different way, and identify the techniques appropriate to the question.

**Category A2: Making Mistakes and Realizing Their Mistakes.** The experiences of this category characterize how students perceived productive struggle through making mistakes. This category is different from the previous one, as it pertains to students' experiences when an error is detected, compared to a state of not knowing what to do. When Jackie was asked how they would explain productive struggle to a friend, they commented that they could learn from making mistakes in their statement. "I would say you work on the problem, try to figure it out yourself, like even from the mistakes. Cause you learn from the mistakes." In a follow-up question in the interview, Randy explained that making mistakes is part of the journey of understanding. "It's kind of like trial and error, and the mistakes you make along the way will help you understand it. It's kind of a journey of learning" (Randy).

Students explained that they discovered the benefits of making mistakes in their journey of learning through productive struggle. When Randy was prompted on how they would describe productive struggle to a friend, they reported that making mistakes improved their ability to recall strategies to solve problems. Randy said, "You have to make the mistakes in order to learn from it. Shortcuts are nice, but unless you practice and make those mistakes, you won't memorize it as well." Randy continued to explain that shortcuts included someone else telling them what to do. In contrast, students described how they preferred to take time to work out the problem on their own, and if they needed help, they only wanted hints. The following statement from Jackie, as they continued to describe how they would explain productive struggle to a friend, captures this preference.

> Try to work it out, even if you make mistakes, and take more time to work on the problem on your own. And then, if you absolutely can't figure it out, just ask somebody, just to nudge you in the right direction, not to tell you exactly how to do it. Like, say, bring this down, divide this. Just nudge, just give you a little hint to nudge you in the right direction.

Students described how they realized their mistakes when the answer was incorrect or sensed something did not look right. The comments were brought up when students responded to the question of how productive struggle impacted their learning. Jackie alluded to mistakes and explained how they went back through their steps to realize what they did wrong. "Once I started doing it, and then made a mistake and not come up with the right answer. Like, okay, what did I do wrong? And then [I] went back, and then I could figure it out." As Jackie was prompted on how they knew something was wrong, they added, "It doesn't look right … it's just maybe the value didn't look right, maybe the way or the steps I took didn't give me the answer that I predicted." When asked what they realized from looking back through their steps, Jackie responded that they were able to resolve the mistake by recalling prior knowledge. "Okay, what can I remember? Oh, I remember that rule!" (Jackie).

Participants in their discussion about productive struggle also realized their mistakes through interactions with the instructor. When asked how others helped Felipe through their struggle, Felipe recalled a discussion they had with the instructor after a class. This interaction highlighted how the teacher's redirection helped Felipe uncover their mistake.

> One time at the end of class, we were free to go, and I came to you [the instructor] and was like, "Oh, is it like this?" and you asked me, "How did you do this?" And I was, "I did this, this [pointing at their steps on the page]. Oh! I have a mistake here!" (Felipe)

Students reported that they thought of productive struggle as making mistakes and then realizing the mistakes they made. They believed they could learn from their mistakes, and mistakes were part of their journey of understanding. Participants were able to use prior knowledge to resolve their mistakes. These realizations occurred when they noticed the answer was incorrect or something did not look right. Mistakes were also realized through interactions with the instructor. This category describes the struggles experienced by students through making mistakes; realizing their mistakes was conceptualized as a productive way to work through their struggles.

**Category A3: Taking on Challenges.** Statements in this category describe how students perceived productive struggle as taking on challenges or working on harder problems. The category differs from previous categories, as these experiences focus on how students perceived the challenge of tackling harder problems, compared to previous categories that reported their experience of working through questions. When Felipe was asked what they believed productive struggle was, they connected it to a challenge. Felipe said, "It was like, you are given something difficult, [and you have] to try to figure how to solve the problem or something." Later in the interview, Felipe was prompted whether they found some questions too hard. Felipe described that word problems were difficult, but working on more challenging problems prepared them for the test.

> Especially the word problems were very hard. But when you work on something that is hard, and then it is going to be on the test, it's great! Because I feel more confident when you see the questions on the test. (Felipe)

When asked to elaborate on how that challenge helped, Felipe compared working on easy questions to hard problems. Felipe believed that easy questions would not prepare them for tests while working on more challenging problems allowed them to push themselves.

> If you give us something easy and then on the test, something harder appears. Then I will say, 'Oh, I didn't train for this!' So, on the worksheets, we had a chance to push ourselves more than you would ask on the test. (Felipe)

Students expressed ideas about challenge in relation to productive struggle. Randy agreed when they were asked if the challenge was important to productive struggle. A follow-up question in the interview prompted Randy to describe what they would do if the challenges were too hard. Randy responded that they would try to figure it out, but if they were in a prolonged state of not knowing, they would skip the question. "I mean, I probably try to figure it out. But if it took too long and, like, I have no idea what I am looking at? Then I probably skip the question." Randy's reference to "I have no idea what I am looking at" was not placed into the previous category of getting stuck because it stemmed from a question asking them to describe if challenge was important. Randy compared their response when facing challenge to working on easier questions, where they reported that they would "zoom right through it." However, participants perceived the benefits of working on easier questions. Responding to whether easy questions helped with productive struggle, Felipe expressed how starting with easier ones gave them confidence.

> At the beginning, when you got something easy, and then you solve it. It helps on the second question. 'Cause you feel something's changed in your mind. Like, 'I got it!' I want the second one because 'math is cool!' Otherwise, if you are struggling with the first question, you didn't do anything. You look beside you and see your partner on the fifth question. You feel terrible. 'What is wrong with me?' (Felipe)

The variation of challenges gave rise to creativity. In Aarya's response to the question of how challenge played a part in productive struggle, they revealed that the variability of questions demanded their thinking to be creative.

> The questions you gave me was not the same. Each and every one was not the same; each one has some different thing. Like two, three are the same, then the fourth, I have to implement my own thing. It is creative, so you learn how to use our minds and solve problems in a different way. (Aarya)

In summary, productive struggle was articulated in terms of taking on a challenge by students. The challenge helped students prepare for tests. However, the challenge could be too hard, and completing easier questions fostered confidence. In addition, having a variety of challenges promoted creative approaches to solving problems.

**Category A4: Realizing What They Do Not Know.** Comments in this category spoke to students' realizations of what they do not know, or identifying and working on their weaknesses through their experiences of productive struggle. Specifically, students used the words "do/did not know" and "weakness," as they conceptualized productive struggle through what they realized, how they felt about the realization, and what they did as a result. When asked how they would describe productive struggle to a friend, Aarya disclosed that the struggle is to figure out what they do not know, and productive struggle would be to learn and get help on what they do not know. Aarya states, "When they failed a quiz or do not know, they just give up like, 'I don't know.' But if they keep productive struggle in mind and face the struggle, then it would be easier for them to study." Prompted to elaborate on what struggle meant to them, Aarya explained two struggles that led them to realize what they did not know. The first struggle is not retaining information due to a lack of practice. "When we are learning something, we just look at the board, but we don't practice it. If you don't practice, you won't be able to learn" (Aarya). The other struggle is not retaining the lesson after class. "We just go to the class, and we don't take anything back" (Aarya). In a follow-up interview question, Aarya recalled that the struggle helped them realize what they did not know so they could get help from the instructor in class. "So we solve the problems on our own, we were able to know this, and some of the stuff we don't know. If we face any struggles, you [the instructor] were there to help" (Aarya).

Participants thought of productive struggle as facing their weaknesses. Similar to how they described realizing what they did not know, identifying their weakness revealed what they needed to work on. In Felipe's response to what they thought of productive struggle now, they reported that they faced their weakness by writing their struggles in a journal. "It's something like a journal. When you think about it, you describe your weakness to yourself. And when you see that, it's like talking with yourself. Oh, I need to improve on that part. It makes it clear" (Felipe). Using journals to support what they did not know was referred to when Aaron was prompted to elaborate on what helped during struggle.

> Writing down steps was really helpful, especially on the algebraic stuff. I wrote in my journal after every class because after I realized that some of the steps are going to be the exact same. I would write in words a description of the step. Writing them down definitely helped me remember and progress to other stuff. (Aaron)

Working with fractions was identified by Felipe as their weakness, and Felipe reported that they have struggled with fractions since grade eight. However, Felipe felt responsible for overcoming this topic, as it is fundamental to their field of study.

> Fractions could be in anything, like sharing a pizza with someone. And especially when you are an engineer. It [fractions] was something I missed before, and [I] have to work on it again. Thank god fractions was first. Otherwise, it would be my struggle again. It would be my weakness again. (Felipe)

When asked what it felt like to overcome their weaknesses, the following statement from Felipe depicts relief and achievement. “When you realize you have improved. … You feel better. You feel free because you know you achieved your goal.” When prompted on how they knew that they improved, Felipe responded, “It became easy to solve.”

Students perceived productive struggle as realizing their weaknesses or what they did not know. Identifying what they did not know allowed them to focus their effort on improving their weaknesses. For example, participants asked for help in class and practiced what they did not know to prepare for the test. Weaknesses were identified by describing the struggle in their journals. Facing and overcoming their weaknesses led to a sense of relief, and topics became more comfortable to solve.

***Category B – A New Way of Teaching and Learning***

This category included students' experiences and conceptions that productive struggle was a new way of teaching and learning. These statements could not be classified under 'working on math problems' because students described the productive struggle classroom compared to other math classrooms they have experienced. Comments often responded to the question of how the experience impacted their learning. An idea within the experiences was that productive struggle was a new way of teaching and learning, and it was beneficial in comparison to the traditional lecture. For example, Felipe said:

> This type of new classes that you teach right now. In the beginning, it was weird. I felt very bad. Like I pay a lot of money for a teacher who doesn't work [Felipe laughs]. The teacher doesn't work and leaving the class without any lectures. It was kind of weird for me. But now I understand the point. Yeah, it was very useful. It worked very good on my struggles. This is new, right?

Students conveyed that the productive struggle learning experience was different in terms of not being a passive learner, having time to practice in class, and having the ability to communicate with classmates. In comparison to previous traditional math experiences, participants reported that it was better to be able to work on math. In response to how the course impacted their learning, Jackie described their preference for getting to work on math in class compared to just sitting there and taking notes.

> In the math class I have now. She just did the question, and we just watched. She did a few examples, and she moved on to the next topic. We are copying down what she said, what she is doing, but we are not actually doing it ourselves. I prefer doing

> it the other way, in our class… Working through some of the problems is way more beneficial. Even a classmate who was in my previous class says the same thing; he prefers actually working on it, struggling a little bit, instead of just watching somebody just doing the question and we are just copying. (Jackie)

Prompted to elaborate on what they preferred, Jackie reported that it was harder to recall learning. "It's hard to remember what she taught in the class before. We see it once, and she might do a review before the test, and we don't see it again. Unless you already know the stuff, it's not good for remembering." Jackie continues to add that they still recall learning from productive struggles they experienced in the course. The following statement from Felipe indicates the difficulties students may experience in a traditional lecture.

> I will tell you why. 'Cause if you were to explain something on the board. And you call my attention. I cannot do both. I cannot write down and, at the same time, pay attention to the professor. It is very, very difficult. That's why sometimes, like, I give up to write something and start to pay attention to the professor. I cannot do both. It is impossible for me. Someone, they have the skills to pay attention and write at the same time. But it is not for me. It doesn't work for me. (Felipe)

Participants explained that they felt responsible for their learning in the course, which supported productive struggle. For example, Felipe states, "You became the first person. In a traditional class, the teacher is the first person, and the students are the second person. When you change this, yeah, you come to class, you have to do something, there is no escape, you have to do something, which is really, really great." Felipe continues to express that to understand math, they had to work on it.

> If you want to know how to run a marathon, you don't go to class to learn how to run a marathon. You have to wear a pair of shoes, go to the street and start running. In your class, it is something like this. To understand math, sometimes you have to grab a pencil and start to try solving the problem. (Felipe)

Students added that the worksheets provided opportunities to practice through productive struggle. For example, Aarya's response highlights their onus to practice.

> Back home, our professor just writes, we copy in our notebooks, and he just explains, we never practiced. He just gives us homework, but we don't do it, never do it. But here we just have to do it, because we have a sheet which has to be completed. (Aarya)

Using class time for practice was essential for students' time management. When prompted on how the experience of productive struggle helped, Aarya revealed that it lessened the burden of study time at home. Aarya said, "If we have to do it at home, we don't have time; we have job and such things." Participants reported that it takes a certain mindset to motivate themselves to practice at home. For example, Felipe said, "When you get home, and you didn't practice anything in class. If you have a good mindset, you are going to sit down and start practice. Otherwise, you are going to skip this, and the test will come, and you are going to fail."

Students also reported that compared to taking notes in a traditional class, being able to explain math to someone near them when they struggled impacted their learning. For example, when responding to the question on how this course was different from previous math experiences, Aaron said,

> Yeah, it impacted my learning. Maybe I got this question wrong, talk to whoever is sitting beside me, and maybe I didn't notice it was wrong, so I was explaining it, and oh wait, you did this part wrong. And I look at it again, and I will be, like, oh yeah. It is sometimes good to hear it from another person.

Participants were able to validate their own thinking by talking to other people. Randy also responded that working with someone else was different in this course compared to past experiences. When asked to elaborate, Randy described what they talked about when they had struggles; they responded, "If we both got the same answer, it kind of validates my own thinking. Like, 'Oh, this is correct.' Or if I got it wrong and they got it right. Then I can see where I went wrong."

In summary, students conceptualized productive struggle as a new way of teaching and learning mathematics. They explained that this was different from traditional lectures they had experienced because they got to practice problems in class instead of sitting and writing notes. They believed that understanding math requires practice. Instead of being passive learners, they preferred the responsibility of learning math by practicing in class. The class practice also allowed them to discuss math with their peers, which was beneficial as they could compare and share explanations.

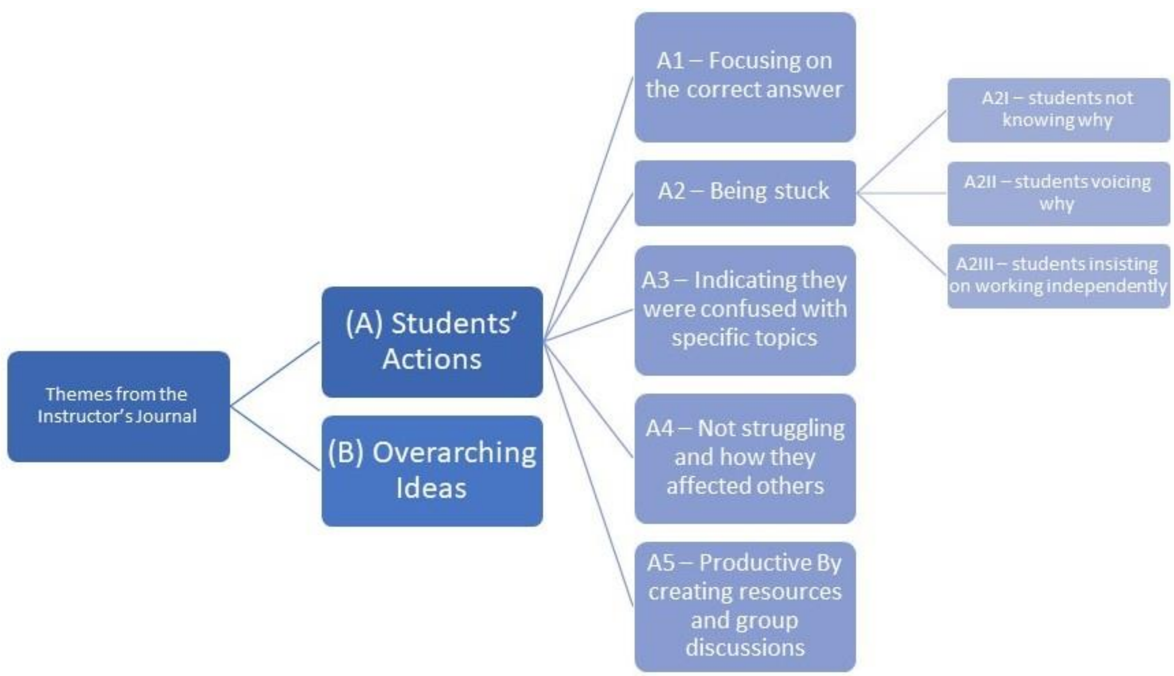
## Part 2: Instructor's Experiences

My experience supporting productive struggle is presented in two ways. It begins with a phenomenographic analysis of my journal, identifying qualitatively different themes of struggle and productive struggle that I noticed. These themes encapsulate surprise, concerns, issues, and tensions that caught my attention. Such tensions are important for my learning because incidents are deemed critical when they fit or contrast with my experience. The phenomenographical analysis resulted in two overarching themes: I noticed struggle and productive struggle through (A) students' actions and (B) overarching ideas. Figure 3 provides a visualization of the themes and subthemes from the analysis. The analysis of my journal addresses the question: What accounts of struggle and productive struggle did I notice as the instructor of the course?

The next section, 'seeking fresh possibilities,' encapsulates what I learned from supporting productive struggle. This section addresses the research question: What was learned from the accounts of noticing struggle and productive struggle? Specifically, the themes were revisited to articulate alternative courses of action that I could have undertaken. The manner in which I became aware of these new possibilities is described in this section.

**Figure 3**

*Themes From My Experience Supporting Productive Struggle*



***Theme A: Students' Actions***

The first theme describes students' actions reported in the journal that relate to struggle. These accounts were sorted into five subthemes based on their distinct features. These features speak to my noticing of struggle as: (1) students focusing on the correct answer, (2) students being stuck, (3) students indicating they were confused with specific topics, (4) students not struggling and how they affected others; and (5) students being productive by creating resources and group discussions.

**Subtheme A1: Focusing on the Correct Answer.** Accounts placed in this subtheme feature students responding to struggle by focusing on the correct answer. The following journal excerpt documenting a student's comment exemplified moments classified under this subtheme, "If I don't have the answer, how do I know that the struggle was productive?" (September 24). Reported in the journal were students looking for the correct answer by asking their peers, searching for solutions on their phones, and asking me for verification. After the answer was found, no further discussion of mathematical ideas emerged. When I attempted to support productive struggle, students got frustrated or could not explain how they got the answer.

Reported in the journal were accounts of students comparing answers with their partners. If both students had the same answer, students deemed the answer correct. No discussion of mathematical ideas occurred when answers matched. The following is an excerpt from the journal under this subtheme: "Students asked each other, 'What did you get for number 1?'" (November 26). If their partner replied with an answer that matched their result, the next question was verified (e.g., "What did you get for number 2?"). If the answer did not match, one student proceeded to tell their partner the correct steps of the question. The journal reported that accounts sorted under students comparing answers with their partners did not lead to further discussion about their struggles.

Another action of students focusing on the correct answer was through their phones. The journal documented accounts of students searching for solutions on their phones when they did not know how to answer the question. The question was inputted into an app that displayed the complete solution to the question. These accounts also did not lead to further discussions about their struggles. The following excerpt from the journal exemplified accounts classified under this subtheme: “Students skipped the sharing with their partner and pulled up their phone to reveal a worked example” (September 10). The account described students copying the solution from their phones into their notebooks and moving on to the next question.

Another method was to ask me to verify the correct answer. The following are questions students asked to verify answers: “Is this the answer? What is the answer? Can you check if I am right?” (September 24). Students got frustrated, and some could not provide explanations when I redirected their requests and asked them to explain their thinking. For example, reported in the journal were my attempts not to verify answers during class in hopes that students would discuss them with each other. However, I noticed instances when students got frustrated when the answer was denied. The following text evidenced this in the journal, “What instances do I notice struggle bordering in frustration?” (October 15). A detailed account of the interaction between Michael (pseudonym) and I depicted the student’s frustrations when their request for the answer was denied. The following sequence of events was written in the journal. The entry started as,

> Students were working on worksheets with multiple-choice and open-ended questions on exponents. While working on a specific question, the interaction

> between the student and I led me to redirect [Michael's] attention away from the answer (September 24).

The redirection of Michael's attention referred to how correct answers and verification were denied. Instead, Michael was asked to explain their process, search for patterns, and hypothesize what the answer could be. This was documented in the account with the following text: "leading [Michael] to explain [their] process, looking for a pattern to solve the specific problem, and then conjecturing" (September 24). Michael explained their steps briefly, and Michael's explanation was followed by another request to confirm if the steps were correct. When I asked for a more detailed explanation, Michael stated in frustration, "If I don't have the answer, how do I know that the struggle was productive?" (September 24). Also placed into the ways students asked me to verify the correct answer were accounts when students could not provide an explanation on how they got the answer. The following journal text exemplified these accounts, "I asked them why they chose that answer. The student said they did not know, it just looks right" (November 21).

All accounts sorted into this subtheme emphasized students' struggles by seeking the correct answer. The journal reports that students compared correct answers with their partners, searched for solutions on their phones, and asked me to confirm their answers. Comparing answers and searching for the solution through an app resulted in no further sharing of their struggles. When verification of correct answers was denied, students were asked to explain their work. Some students got frustrated, and others could not explain how they got their answers.

**Subtheme A2: Being Stuck.** Journal accounts of students being stuck formed another subtheme of noticing students' actions relating to struggle. The following text exemplified a defining feature of all the accounts that were sorted under this subtheme: "When I walked around, I saw nothing written in their [students'] notebooks" (September 5). All the journal accounts classified under “stuck” shared the feature of students not writing anything for an extended period and me checking in with students. I typically supported struggle by asking questions such as, “How are you doing?” (November 21) and “Can you explain your thinking?” (September 24). In this subtheme are the three different ways students responded to the check-in questions: I) students not knowing why they were stuck; II) students voicing why they were stuck; III) students insisting on working independently while stuck.

***Subtheme A2I.*** Students not knowing why they were stuck was exemplified by responses such as “I don’t know…” or a synonymous statement during the check-ins. For instance, “I don’t know what to do” (November 21) in the journal referred to students voicing that they did not know how to start the question. Students also used the term in the form of a guess, “I don’t know, maybe add them [fractions] together?” (September 12). The guesses marked students’ uncertainties about how to proceed. The following text exemplified this: “Sstudents voiced they were stuck on the question when they made guesses at what the next step was” (September 12). Students’ “I don’t know…” statements led to me writing the question, “How do I get them to realize … what they don’t know?” (November 14) in the journal. This subtheme reported students voicing “I don’t know…” and inability to start or proceed with the question after I checked in on students not writing anything down for some time.

***Subtheme A2II.*** Students also responded to the check-in questions by voicing why they were stuck. The stated reason separated these accounts from subtheme A2I of students not knowing why they were stuck. Students identified reasons such as the topic being new to them and not understanding the words within the question. For example, “students were unfamiliar with the topic” (October 1) reported students voicing that they were stuck because the topic was new to them. The account referred to a lesson on scientific and engineering notation. The journal spoke about how students said they had not seen the topic before and asked for some examples. Another reason voiced by students was the difficulty in understanding the words within the question. The following text in the journal exemplified this difficulty, “words confusing sometimes” and “hard to understand” (November 19). The text referred to a student who voiced that they struggled to start the question because they had difficulty defining some words. For example, “a student asked what the word diameter meant” (November 21). Students’ reasons for being stuck led to me questioning the difficulty of the questions. For instance, “Can struggle be set up instead of throwing them into the deep end?” (October 1) referred to the consideration of gradually increasing the difficulty of questions.

***Subtheme A2III.*** The remaining accounts could not be placed in the previous two subthemes because they speak to students who responded to the check-in questions by insisting they continue to work alone. The journal text, “Students preferred to do it themselves” (October 31), was evidence of students preferring to continue on their own after I saw them not writing anything down for some time. One account reported a student insisting on working through the state of being stuck by themselves, stating, “I’ll figure it out myself” (October 31) when I invited them to share their thinking with their partner. The account also documented how the student continued to be stuck. The account led to me asking about the role working with others played in students’ struggles, “Is collaborating a key to productive struggle?” (October 29). This subtheme differed from the previous (A2I and A2II), as students voiced that they wanted to continue to work independently compared to stating they did not know or the reason they were struggling.

In summary, featured in this subtheme were accounts of students not writing anything down and me checking in on them. These accounts were further sorted into subthemes that report the different ways they responded during the check-in. Three subthemes emerged, including students not knowing why they were stuck, voicing why they were stuck, and insisting on continuing independently.

**Subtheme A3: Indicating They Were Confused with Specific Topics.** During my support of students' struggles, I noticed confusion with mathematical topics. This new subtheme was formed because the journal reported students' confusion with a mathematical property/topic they previously believed was correct. The subthemes mentioned previously did report students working on mathematical topics. Still, the distinguishing feature of accounts classified under this subtheme was that students were not stuck. As they explained their thinking, confusion was reported because of incorrectly held conceptions of the mathematical topic. The accounts differed from the first subtheme because students were not focused on the correct answer. The following text exemplified the confusion based on a mathematical property students thought was true, "students were confused to why $(a-b)^2$ does not equal to $a^2-b^2$" (December 5). Reported are the struggles students faced on three specific topics: i) order of operations, ii) exponents, and iii) factoring and expanding algebraic expressions.

Confusion, a synonymous statement to struggle, arose during students' explanations due to incorrect conceptions of the order of operations and exponents. The journal text, "Confusion on the implied rules on the order of operations" (September 12), reported a moment in which a student thought that addition came before subtraction because they were following the order of the acronym BEDMAS. The account described Pavli (pseudonym) working on the question $-10\div(4-2+1)\times8-2$ and arriving at an error evidenced by Pavli's description that adding 2 + 1 came before subtracting 4 – 2 inside the bracket. Another example occurred while students were working with exponents. The journal text evidenced this, "students were confused as to why

$\left(a-b\right)^2$ does not equal to $a^2-b^2$" (December 5). This account reported Jun (pseudonym) stating to their partner that they thought they could distribute the exponent into a binomial when expanding $\left(3a-2b\right)^2$. The journal described that Jun was confused when their partner replied that they could not do that because the rule was $\left(a-b\right)^2=a^2-2ab+b^2$. The struggles reported in this subtheme were revealed when students explained an incorrect procedure to their partners.

Students' struggles compounded when factoring and expanding higher-order expressions because of difficulties with the specific topics mentioned in this subtheme. The following text from the journal exemplified the compounding struggle: "Students were confused with [factors and expansions of] cubic expressions … which brought up past difficulties such as exponents and order of operations" (December 5). Reported in the journal was an interaction between Alex (pseudonym) and me while Alex was seeking help on the following question:

Factor the expression: $729e^6-64i^6$

a. $=\left(27e^3+8i^3\right)\left(27e^3-8i^3\right)$
b. $=\left(9e^2-4i^2\right)\left(81e^4+36e^2i^2+16i^4\right)$
c. $=\left(3e+2i\right)\left(3e-2i\right)\left(9e^2+6ei+4i^2\right)\left(9e^2-6ei+4i^2\right)$
d. $=\left(3e-2i\right)\left(3e+2i\right)\left(81e^4+36e^2i^2+16i^4\right)$

The journal text "There were struggles implementing the rules even when recognized" (December 5) described Alex's difficulties distinguishing between cubic factor and expansion rules. The account described Alex pointing at the factoring and expanding formulas (see below) in their notebook and voicing their confusion to the professor because they thought that $a^3-b^3$ and $\left(a-b\right)^3$ were the same:

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Alex thought the power of 3 could be distributed into the bracket, making $a^3 - b^3$ and $(a-b)^3$ equivalent statements. Alex's confusion was written in the journal as "confused on what to cube" (December 5). In the same account, Alex's struggle with the question was compounded when they expressed uncertainty in applying the order of operations. Alex was reported to have reached $(9e^2 - 4i^2)$ as one of the factors. However, the journal text, "Confusion with the coefficients" (December 5), described Alex's uncertainty as they asked me if they could subtract the coefficient 4 from 9 before factoring the difference of squares.

Students struggled with specific topics because they incorrectly understood mathematical properties. These struggles were reported on the order of operations, exponents, and factoring and expanding higher-order operations. While factoring and expanding, students' difficulties compounded when they faced struggles from past topics.

**Subtheme A4: Not Struggling and How They Affected Others.** The instances placed under this subtheme focus on how I noticed students who completed their questions affecting the struggle of other students. The feature of students who "completed the worksheets quickly" (November 19) distinguishes these accounts from previous subthemes of students working through the worksheet. The journal described how these students "played on their phones," "left the class for a long break," and "did not participate in class discussion" (November 19) after they completed the questions. One account reported a student, Sasha (pseudonym), asking: "Is this all we are doing today?" (November 19). Sasha eventually left the class early, shortly after a conversation with me.

Evidence showed that I perceived that students who finished early hindered others' participation during tasks. The following excerpt exemplified this, "students were scared to answer publicly" (November 19). The account focused on a class discussion about students' struggles when students still working on their questions were hesitant to share. Also described was my unsuccessful attempt to include students who completed the questions in the same discussion on productive struggle, "As I brought them back to productive struggle in a classroom discussion, again, there was disinterest" (November 19). The effect students who finished their questions had on other students heightened my emotions. The following text evidenced this: "My anxiety rose as I noticed there was not much interest in the feedback" (November 19). I described the anxious thoughts in the journal, "What do I do, what do I say, constantly ran through my head" (November 19). The heightened emotions led to an early dismissal of the entire class.

In summary, this subtheme reported the behavior of students who finished early, how other students were scared to answer questions, and my anxieties attempting to manage the class when students completed their questions.

**Subtheme A5: Productive By Creating Resources and Group Discussions.** The accounts in this subtheme involved students creating resources and discussing as a group to struggle through questions without my assistance. The students were not stuck nor focused on the correct answer, and confusion did not stem from previous conceptions of mathematical rules or words within the questions. Students affected each other's work, which can draw similarities to the previous subtheme. Still, these accounts differed because students acted by working together on questions and creating resources, and not instances based on students who had completed the questions.

The first way students made progress through questions was to create a resource from what they had learned earlier in the semester. I considered the account "productive struggle" (November 26) because creating the resource was evidence that they had a plan for the questions they were working on. This is evidenced in the journal entry "knowing what to do" (November 26). In this account, the journal reported Seren (pseudonym) creating a sheet of rates to work on the current lesson featuring ratios and proportions. The account described Seren searching for rates discussed earlier in the semester and writing them out on sticky notes, "student wrote down conversions on separate sticky notes for reference" (November 26). The notes were then stuck onto a piece of paper to form a conversion sheet to help answer the questions from the ratio and proportions lesson. Seren's actions were described in the journal as a "productive search for rates to apply to the topic" (November 26).

Another way students progressed through questions was to collaborate. This account was placed in this subtheme because I stated in the journal, "They [Students] got a taste of the emotions felt during productive struggle" (September 12). This instance described pairs of students working together as a group and discussing possible ways to solve the question. Reported was a pair of students conferring with another pair nearby on how to solve the problem. Soon, others joined in, and there was a group discussion. The account had the following description in the journal: "resourceful by asking each other" and "realiz[ing] something was missing" (September 12). The group was reported discussing different ideas, and the journal text "Didn't give up" (September 12) referred to how the group spent a good portion of the class trying to solve the question.

In summary, this subtheme reported examples of what I considered productive struggle. These examples included ways students created resources and discussed as a group to work through questions. These actions were described in the journal as students being productive, resourceful, and knowing what to do.

***Theme B: Overarching Ideas***

This theme encompassed the accounts that did not fit into Theme A because these accounts spoke about struggle and productive struggle on a broader level compared to moments of students' actions reported in the first theme. The accounts are reflections and comments of previous classes written in my journal as I planned for subsequent classes. Hence, these accounts were placed in a separate theme from the previous one because they do not refer to students' actions I noticed in the classroom. Within the overarching ideas were the common threads of how to engage students to question their work, standardized assessments, the passive culture of learning, and the dense curriculum in developmental mathematics.

Recorded in the journal was a reflection on the first half of this course, during the Reading Week. The account inquired into ways to educate students to question their work. The following text evidenced this, "Is self-questioning a key to PS [productive struggle]?" and "What kind of questions should students ask of the mathematical content?" (October 29). These questions influenced how I supported struggle during the next class. The journal documented me asking students, "Which questions did you find difficult? And why?" (October 31).

A different overarching idea written in the journal was the conflict between a standardized assessment and supporting struggle and productive struggle. The following text exemplified this idea, "The assessment is a fixed one, will it conflict/support/deter productive struggle?" (October

1). The comment focused on the standardized tests created for all sections of the course. I believed that the test questions focused on procedure and not deeper understanding: “Other constraints include the need for deeper understanding rather than a surface (know the formula) approach as implied by the test and systems” (October 15).

I questioned whether the passive learning culture in developmental mathematics hindered students’ learning from my experience of supporting productive struggle. This is reported in the following journal excerpt: “Are students in a culture that needs to change before fully accepting struggle? How do I challenge this mindset or culture to support change?” (October 15). The demands of the curriculum added to the challenge, “There is a challenge balancing curriculum” (October 15). The comment was focused on how a new topic was introduced in every class to meet the curriculum demands in the developmental mathematics course.

In summary, there were remaining accounts that could not be sorted into Theme A. These accounts formed a qualitatively different theme, discussing overarching ideas on the experience of productive struggle in developmental mathematics. The ideas include how to engage students to question their work, standardized tests, a passive learning culture, and a dense curriculum in developmental mathematics.

### ***Seeking Fresh Possibilities***

This section presents my learning in terms of seeking fresh possibilities for supporting productive struggle. The examples describe how new possibilities emerged from the data. After the phenomenographic analysis of my journal, I revisited themes or account(s) within themes that I was at odds with. I reentered these moments to consider possibilities from the perspective of the student and what I would do differently in my future practice. Awareness of new possible actions is described through the following five examples:

1. Mismatched conceptions of struggle
2. Overhelping
3. Misconceptions brought into the classroom
4. Reacting Emotionally
5. Struggles beyond the mathematics

**Mismatched Conceptions of Struggle.** There is evidence of a mismatch between students' conceptions of struggle and mine. Some students related productive struggle to finding the correct answer. For instance, one student, Michael, remarked, "If I don't have the answer, how do I know that the struggle was productive?" (September 24). In contrast, I held the belief that students should learn to verify their own solutions. As a result, I pursued explanations rather than confirm Michael's answer. Unaware of this mismatch at the time, this discrepancy in perspective resulted in mutual frustration. Michael grew frustrated that I refrained from providing a direct answer, while my frustration stemmed from Michael's reluctance to verify their solution. Ultimately, I proposed that Michael convince classmates nearby regarding the correctness of the answer and walked away. My heightened emotions made the moment vivid, and when I recounted this incident in my journal, I thought Michael harbored doubts about my knowledge of productive struggle. Thus, I concluded at the time that Michael relied on someone else to confirm their answer. It was not until my supervisor suggested the possibility of a mismatch of conceptions that I began to view the account differently.

New possibilities can emerge from considering the student's perspective. It is evident that the answer held significant importance. If Michael had grown accustomed to having immediate access to answers (such as the answer key in the back of the textbooks or past teachers confirming correctness), the notion of engaging in mathematics without readily available answers could have presented a challenge. It may have developed into habitual practice, which explains why the situation led to frustration when I refrained from confirming the answer.

Seeking fresh possibilities brought forth new action that I was previously unaware of. I am now aware that these moments are opportunities for negotiation, and not simply an acceptance or denial of confirming an answer. A request for confirmation is a chance to dive into students' experiences and conceptions with the answer during problem solving. A shared meaning can be negotiated on how and why verifying one's answers is a productive way to struggle. The shared meaning can lead to a shared goal in subsequent lessons, in which I may initially confirm answers, but the student and I understand that this support will slowly fade over time.

**Overhelping.** An important insight arose from my account of Avery staring at their notebook, not writing anything down. Initially, I made nothing of it, but I noticed Avery's actions persisted for over five minutes. This naturally prompted me to intervene and inquire about Avery's thoughts. Avery's response, "I don't know what to do," confirmed my suspicion that they were struggling. In response, I guided Avery through the question. However, upon revisiting accounts after the phenomenographic analysis, I noticed a theme when student moments of impasse and "I don't know" responses caught my attention. A number of these accounts involved Avery. In a conversation with my supervisor, the idea of overhelping was brought up, prompting me to reflect on whether I was overhelping Avery throughout the semester.

Viewed from Avery's perspective, a conception may have developed that productive struggle entails investing effort in thinking about questions, and help will eventually arrive. Perhaps Avery thought that even though they were at an impasse, just thinking about the question was an example of struggle. It is possible that Avery's only conception of productive struggle was waiting for someone's assistance. The possibility came to light as a theme emerged from similar accounts of me coming to the aid when I noticed Avery at an impasse.

This experience taught me that the undesired outcome of Avery's struggle might be linked to overhelping. By assuming and determining when and what assistance Avery required, I inadvertently limited Avery's struggle. A more productive action would involve Avery realizing their need for help, seeking assistance, and expanding their strategies to tackle moments of impasse. Hence, a different action I will adopt in my future practice is to notice student moments of an impasse as opportunities to discuss possible strategies. For example, students could identify where to seek and ask for help (e.g., from me or their peers). The task in itself increases awareness of possible supports, motivates students to seek help, and uncovers reasons students avoid help. A discussion could have drawn students' attention to the fact that seeking help is more productive than spending time at an impasse.

**Misconceptions brought into the Classroom.** Students' misconceptions caught my attention because they resonated with my past experiences teaching developmental mathematics. For example, Jun believed the exponent could be distributed into a binomial. As a result, they concluded that $(3a-2b)^2=(3a)^2-(2b)^2$. When Jun's partner tried to explain that $(a-b)^2=a^2-2ab+b^2$ and not $a^2-b^2$, Jun was reluctant to accept the claim. Based on my experience teaching developmental mathematics, this was a typical misconception. Targeting misconceptions motivated the design of certain questions students worked on in each class. It is likely that my inclination to notice instances of misconceptions was influenced by my prior experience, explaining why I considered these examples significant to record in my journal.

More could be sought from moments that are familiar. The process of seeking fresh possibilities made me consider the complexity of misconceptions. Jun's previous experiences might have included the distributive property holding true when multiplying a constant and an exponent over a single term. For example, $3(a-b)=3a-3b$ or $(ab)^3=a^3b^3$. Jun may have thought that the distributive property can apply to binomials, resulting in the misbelief that $(3a-2b)^2=(3a)^2-(2b)^2$. Over time, the belief may have solidified. Hence, it was difficult for Jun to accept that it does not abstract into the context involving multiple terms. The erroneous application may extend into other topics (e.g., $\sin(a-b)=\sin a-\sin b$). Based on my experience with Jun, I learned that even in accounts that seem familiar, more possibilities can be sought, such as different roots of misconceptions.

Expanding what is familiar informs mindful action. It was evident Jun misunderstood aspects of exponents, but less evident was the root of the misconception. Especially in a developmental mathematics course, where an array of topics are covered, students have less time to connect topics. A different action that I will apply in my future practice is to explore the roots of misconceptions with students. Awareness of one's own misconception may uncover another. My future classroom tasks can elicit struggle from a topic like exponents but not limit students to search for misconceptions within one topic. Bringing back previous topics for students to test their beliefs and address misconceptions is a fruitful exercise. The tasks can even preview upcoming topics for students to consider abstracting what they have learned.

**Reacting Emotionally.** Notable in my accounts was my reaction to students who were not struggling. One account reported the distraction caused by students playing on their phones after completing the questions. This distraction seemed to deter students from sharing their thinking with others. My reaction was to initiate a class discussion to re-engage these students. However, students continued to play on their phones. As my attempts failed, my anxiety escalated, ultimately leading me to dismiss the class early. This specific account held weight due to my heightened anxiety and the discrepancy between my expectations and students' classroom behavior. How my attention was diverted to managing the classroom and away from students' struggles was an action worth investigating.

A fresh possibility emerged, diving into the students' perspective. Perhaps students needed a break from more struggle. This perspective was not realized until after the course and the analysis of my journal, when my emotions had time to subside. I was able to consider that the students had completed the task, evidence that they were productive with struggling through the task. Effort is inherent to productive struggle; sustained effort over an extended period is difficult. Taking a break so students have the capacity to re-engage more meaningfully can be considered a productive way to struggle. Instead of drawing my attention to how they were distracting others, it could have been an opportunity to discuss the productive action of taking breaks.

These possibilities equip me to act more flexibly in the future. I may be less anxious because I am not limited to trying to stop students from distracting others. In my future classes, I can better understand how students want to spend time after completing the questions and draw attention to the productivity of taking breaks.

**Struggles Beyond the Mathematics.** There were accounts of students' struggles extending beyond understanding mathematical concepts. For example, a student insisted on working alone despite being stuck. This caught my attention because collaboration was a vital part of my task design. The analysis drew more attention to this issue as accounts, such as students being afraid to share, emerged. In my journal, I even questioned the role of collaboration: “Is collaborating a key to productive struggle?”

It may be a daunting task to share one’s struggle. Developmental mathematics students may have experienced mathematics as something done individually, quietly, and while sitting at a desk. Collaborating to struggle through a task for these students might be difficult and risky (Murdoch et al., 2020). Students risk getting judged by their peers, especially if they are still forming their thoughts and ideas. The fear of stating a wrong answer or having their ideas judged as inadequate may be overwhelming. Under these circumstances, it is easier not to share at all.

I learned from these moments that supporting productive struggle is not limited to supporting students to think about how they can solve a problem. The process of embracing collaboration is complex. For instance, students need time and support to adapt to an environment that embraces collaboration, especially in developmental mathematics, where students may have long histories of being passive learners in mathematics classrooms. The risk some students take to share their struggles should be valued. In my future classrooms, I look to discuss how difficult it may be to share and negotiate ways students can collaboratively work, as it is an important aspect of productive struggle.

# Chapter Six – Discussion

This chapter is organized into four sections. The first section describes how the findings address the research questions relating to students' experiences and conceptions of struggle and productive struggle. The second section provides insight into the research questions on my learning supporting productive struggle in developmental mathematics. The significant contributions of this research to professional development and the blurred boundaries between teaching and researching are discussed in the final two sections.

## Addressing the First Two Research Questions

The categories and subcategories provide insights into how students experience and conceptualize struggle and productive struggle. While tackling mathematics problems, participants articulated distinct perspectives and contrasted their encounters in this course with those in other mathematics classrooms. The alignment of the findings with documented experiences in existing K-12 literature suggests that supporting productive struggle in developmental mathematics deserves more attention. For example, Randy's response statement, "Getting stuck makes me step back and look at the problem in a different way," is evidence that when a student's prior knowledge is insufficient to address the given problem, a struggle is initiated (Hiebert & Grouws, 2007). This struggle, in turn, gives rise to various experiences and conceptions as evidenced by the subcategories: getting stuck and finding ways to get unstuck, making mistakes and realizing their mistakes, taking on challenges, and realizing what they know or do not know. The subcategory focusing on tackling challenges underscores that students can persevere in the face of difficulties – an often-discussed characteristic of productive struggle (e.g., C. Townsend et al., 2018; Warshauer, 2015b). The NCTM (2014) reports that students who struggle productively realize that confusion and errors are a natural part of learning. Randy's statement, "You learn from the mistakes," exemplifies the NCTM's perspective on the value of productive struggle. The consistency of the findings with current K-12 literature is evidence that productive struggle can be supported in the context of developmental mathematics. This viewpoint is shared with students' descriptions in Category B, where students contrasted their learning experience in this study with passive learning prevalent in traditional settings (e.g., "just watching somebody just doing the question, and we are just copying" – Jackie).

*How do students' experiences and conceptions of struggle and productive struggle impact their learning?* The second research question can be addressed through the connection of the findings of this study with deep approaches to learning, perseverance, affect, and habits of mind.

### ***Deep Approaches to Learning***

Students experienced productive struggle as deep approaches to learning. This is significant because the concern with and limitations of students in developmental mathematics learning through rote memorization is well documented (e.g., Stigler, Givvin, & Thompson, 2010; Wheland, Konet, & Butler, 2003). Students of this study reiterated this issue of rote memorization by saying: "I actually worked on it by myself, instead of somebody just telling me, 'Okay, just use this formula. Do this, do that.' Then after that person is gone and you do it on your own, you get stuck again" (Jackie). Framing the findings through Marton, Dall'Alba, and Beaty's (1993) deep approaches to learning reveals students finding meaning in their learning rather than knowledge to be picked up, taken in, and regurgitated when called upon.

Students displayed three conceptions related to deep approaches to learning as they (1) specifically mentioned understanding mathematics, (2) seeing something in a different way, and (3) changing as a person (Marton et al., 1993). Randy used the word "understanding" when explaining the importance of mistakes: "It's kind of like trial and error, and the mistakes you make along the way will help you understand it. It's kind of a journey of learning." Marton, Dall'Alba, and Beaty (1993) describe the conception of seeing something in a different way as a change in the learner's conception of something. Evidenced was how students changed how they saw the mathematics classroom. "In the beginning, it was weird. I felt very bad. Like I pay a lot of money for a teacher who doesn't work. The teacher doesn't work and leaving the class without any lectures" (Felipe). Felipe ends the quote by describing how new this type of classroom was: "But now I understand the point. Yeah, it was very useful. It worked very good on my struggles. This is new, right?" Jackie even prefers the class that supports productive struggle when comparing it to their current course: "We are copying down what she said, what she is doing, but we are not actually doing it ourselves. I prefer doing it the other way, in our class."

The final category that demonstrates students in this study finding deeper meaning in learning mathematics is changing as a person. One way Marton, Dall'Alba, and Beaty (1993) evidence this conception is when the learner "sees oneself as a more capable person" (p. 293). This is evident when Felipe refers to how the "first person" switched from teacher to student in their experience in this study:

> You became the first person. In a traditional class, the teacher is the first person, and the students are the second person. When you change this, yeah, you come to class, you have to do something, there is no escape, you have to do something, which is really, really great. (Felipe)

This is certainly a change from being a "second person," which Felipe described as in charge of taking notes and listening. The evidence of deep approaches to learning fostered by productive struggle in this study is a significant deviation from students being knowledge receivers, a well-documented issue in developmental mathematics research (Bickerstaff & Edgecombe, 2019; Bryson, 2013; Grubb & Gabriner, 2013; Stigler et al., 2010b).

It should be noted that students did refer to memorizing in the findings. However, the meaning expressed by students in this study differs from the surface approach that learning is memorizing and reproducing knowledge without meaning (Marton et al., 1993). Even though students referred to memorizing, they articulated the meaning of the importance of making mistakes in learning. For instance, Randy said, "You have to make the mistakes in order to learn from it, … unless you practice and make those mistakes, you won't memorize it as well." This connection to making mistakes when recall can be enhanced separates Randy from the surface approach of memorizing facts that can go away after the facts are reproduced (Marton et al., 1993).

***Perseverance***

Students conceptualized productive struggle as perseverance. Perseverance is influential in students' ability to complete developmental mathematics programs (Bonham & Boylan, 2011; Edwards & Beattie, 2016). Felipe's quote, "We had a chance to push ourselves more," embodies perseverance. The quote exemplifies Middleton et al.'s (2015) definition of perseverance as the continuance of effort to overcome difficulty. The authors define perseverance as learners amending their attack in difficult situations. Amending one's attack is described in Jackie's following description of productive struggle:

> Try to work it out, even if you make mistakes, and take more time to work on the problem on your own. And then, if you absolutely can't figure it out, just ask somebody, just to nudge you in the right direction, not to tell you exactly how to do it. Like, say, bring this down, divide this. Just nudge, just give you a little hint to nudge you in the right direction.

Students' descriptions of how they persevered through moments they were stuck, made errors, tackled challenging questions, and overcame weaknesses are significant deviations from Merseth's (2011) report on how developmental mathematics students lack the motivation to persevere in classes.

### *Affect*

Students' experiences speak to how the affective domain cannot be ignored when speaking to achievement and retention in developmental mathematics (Bonham & Boylan, 2011). Empowering affective structures are evident in these findings if viewed through DeBellis and Goldin's (2006) conceptualizations of meta-affect, mathematical intimacy, and mathematical integrity. Affect functions in relation to cognition, and meta-affect is the individual's "thinking about the direction of one's feelings" (DeBellis & Goldin, 2006, p.136). Meta-affect is evident when Randy gets stuck, eliciting feelings of discomfort: "Is this even a foot in the right direction? Or am I taking a step back? Am I just not progressing at all?" Randy's meta-affect is positive and safe because it productively informs their cognition: "Getting stuck makes me step back and look at the problem in a different way." Mathematical intimacy is captured through the bond Felipe has constructed through deep emotions (e.g., "thank God," or "you feel free"), as Felipe constructs meaning and purpose for overcoming fractions (e.g., "achieved your goal" or "it became easy to solve"), suggesting the intimate engagement Felipe has with the content (DeBellis & Goldin, 2006). Finally, mathematical integrity is "honesty and a degree of openness" (DeBellis & Goldin, 2006, p.138) about one's ability. Felipe displays integrity when they identify fractions as their "weakness." Felipe acknowledges their insufficiency but makes a change: "Otherwise, it would be my struggle again. It would be my weakness again." Action to make the change is done by identifying their weakness and ways to improve within Felipe's journal accounts: "It's something like a journal. When you think about it, you describe your weakness to yourself. And when you see that, it's like talking with yourself. Oh, I need to improve on that part. It makes it clear." Students descriptions of meta-affect, mathematical intimacy, and mathematical integrity are evidence of the importance of the affective domain in productive struggle for developmental mathematics students.

### *Habits of Mind*

Students' conceptions contained many habits suggesting they think the way mathematicians do. Cuoco, Goldenberg, and Mark (1996) state that students should be pattern sniffers. Randy displayed such habits when they became aware that "certain questions have certain techniques to approach it, right? So just knowing those techniques on how to approach it … helped me kind of identify the question and move on from there." Randy's articulation of the pattern and why it is important is at the heart of doing mathematics (Cuoco et al., 1996). Seeing mistakes as a source of learning is a behavior of professional mathematicians (Watson, 2008). In order to do so, students need the habit of describing when precise descriptions of the process are expressed and argued (Cuoco et al., 1996). Jackie's quote: "It doesn't look right … it's just maybe the value didn't look right, maybe the way or the steps I took didn't give me the answer that I predicted" is a description of how Jackie detects a mistake. Jackie's ability to say what they mean and describe a process to correct their mistake is evidence of what Cuoco, Goldenberg, and Mark (1996) call "mathematical sophistication." Jackie articulates the specifics of the process: "Once I started doing it, and then made a mistake and not come up with the right answer. Like, okay, what did I do wrong? And then [I] went back, and then I could figure it out." Although none of the participants of this study aimed to be mathematicians, the habits they displayed were closely tied to the workplace needs in their fields. Evidence can be found comparing mathematical habits of mind to the program learning outcomes. For example, “analyze and solve routine technical problems related to electrical systems by applying mathematics and science principles,” “assemble, test, modify …,” and “use, verify, and maintain…” (Centennial College, 2021) echo the importance that students need to display mathematical habits of mind to describe and sniff out patterns. The connection between habits of mind is also evident across various program learning outcomes (e.g., nursing, aviation, business) that developmental mathematics serves.

**Addressing the Research Questions on My Experience Supporting Productive Struggle**

The themes and subthemes provide an insider’s perspective on what can be noticed in supporting productive struggle in developmental mathematics. Through this perspective, the findings illustrate interpretations and lived experiences of supporting productive struggle. This perspective is significant in understanding how new curricula, like supporting productive struggle, can be implemented (Chapman, 2012). For example, the different subthemes exemplify the diverse ways struggle manifests. These themes underscore the notion that there are various aspects to the concept of struggle. Such insight is significant, as literature typically focuses on productive struggle, which is the positive outcome of struggle (e.g., C. Townsend et al., 2018; Warshauer, 2015b). Instead, the findings dive into the diversity of struggles one can notice in their learning to support productive struggle.

The themes provided a basis for me to learn more about supporting productive struggle. For instance, the accounts reporting students focusing on correct answers contrast with what I read about productive struggle (e.g., NCTM, 2014). The disconnect between my classroom experiences and documented literature serves as a catalyst for deeper exploration. This process is enriched as I record and subsequently analyze accounts into themes distinguishing my experience from ordinary-noticing, which can be easily forgotten (Mason, 2002). Connecting accounts into themes of similarities and differences forms part of an inquiry into learning more about my own practice of supporting productive struggle. Mason states, “It is through becoming explicit about what is beheld that the beholder learns about their beholding, their own practice” (2002, p.249). Yet, the journey of becoming an expert instructor involves knowing more deeply about one’s practice, to expand one’s range of possibilities (Mason & Davis, 2013). The section where I seek fresh possibilities represents my learning from supporting productive struggle and addresses the research question: *What did I learn from my accounts of noticing struggle and productive struggle?* In seeking different interpretations and perspectives from my accounts-of noticing, I adopt a stance “to expand and enrich what is noticed, becoming ever more aware of both what one is attending to and in what manner” (Mason & Davis, 2013, p.195). Emerging from the possibilities are new actions for others and myself to test in future practice, providing fresh avenues for further learning in supporting productive struggle.

**The Discipline of Noticing as Professional Development**

The previous sections summarized the findings as related to the research questions. This section considers seeking fresh possibilities as a form of professional development. My professional learning from engaging with the Discipline of Noticing 1) aligns with literature calling for effective professional development to expertise one’s awareness, 2) is relevant to one’s practice, and 3) provides examples of ongoing change to habitual practices.

Seeking fresh possibilities in this dissertation expertizes one’s awareness, responding to calls for effective professional development (Mesa, 2011; Schoenfeld, 2020; Sweeney, 2003). Expertizing one’s awareness is effective professional development because it fosters change relevant to one’s practice over an extended period. The learning emerging from this study’s systematic inquiry acts as an alternative to the prevalent ineffective mode of professional development that consists of one-stop workshops promoting the acquisition of best practices (Bonham & Boylan, 2011; Edwards et al., 2015). What I learned from this form of professional development is that I was able to notice what is happening inside my teaching (Mason, 2011). For example, learning I was overhelping, as I decided Avery “required help and chose the type of support” when I sought fresh possibilities from my practice. According to Simon and Tzur (1999), this perspective of learning is important for research in mathematics teacher development, where researchers attempt to articulate how instructors make sense of their experiences to inform professional development interventions.

There is a case to be made that this dissertation embodies the notion of becoming more expert (Mason, 2002). Specifically, expertizing awareness-in-discipline is observed by gaining access to one’s choices and the facts of the choices (Mason, 1998, 2011). This expertise is vital for effective professional development of instructors, as “through such critical scrutiny, by teachers, teaching develops” (Jaworski, 1998, p.7). There are many examples of awareness-in-discipline in the findings, and Michael’s account exemplifies how I expertized my awareness through a finer discernment of the facts of the choices. The two choices I was aware of at that moment were to deny or grant Michael’s request for the answer. My choice to “redirect [Michael’s] attention away from the answer” (September 24) was justified by my belief that I was attempting to educate Michael to work without the answer. Through the analysis process, a finer discernment was uncovered, such as Michael’s conception that the correct answer is an essential element of productive struggle: “If I don’t have the answer, how do I know that the struggle was productive?” (September 24). A mismatch between my and Michael’s conceptions of productive struggle came to the forefront through analyzing and writing this dissertation. Mason (2002) describes that becoming more expert is to have a finer discernment of the facts of the choices. For example, considering my beliefs from the point of view of Michael and what I would do differently. Expertizing my awareness broadens the choices I can call upon the next time the answer is requested, such as negotiating a shared meaning of productive struggle with the student on how to decrease their dependence on the answers.

Becoming more aware of possibilities aligns with calls for effective professional development to be relevant to one's practice. Overhelping is an often discussed teaching problem, as it may negatively affect the learner's autonomy and competence (e.g., Errico, Leone, & Poggi, 2010; Glover & Brown, 2006; Towers & Proulx, 2013; Warshauer, 2015; Xu, Du, Cunha, & Rosário, 2021). Overhelping is particularly important in supporting productive struggle as teachers need to learn how to strike a balance on "keep[ing] struggling students engaged while focused on attending to their struggles" (Warshauer, 2015, p.393). However, there is a distinction between what teachers need to do and how they come to know. Seeking fresh possibilities describes how I came to know that I was overhelping. The recording of overhelping in my journal is evidence that the problem is relevant to my teaching, and once noticed and interrogated, I felt there was a need to change. This relevance contrasts with listening to someone else's recommendation (on a topic like overhelping) during a workshop on what should be done that can be ignored, because it is perceived as far away from one's practice (Attard, 2007; Bonham & Boylan, 2011; Edwards et al., 2015). Revisiting, analyzing, and reporting the findings to seek fresh possibilities made the experience more meaningful. Abstracting meaning was effective in my professional development because it moved learning beyond simply recalling the event (Harrison, 2008). The incident continues to build meaning, as it triggers an ongoing process to improve instruction. As new choices are sought, choices become conjectures, leading to a new cycle of inquiry (Mason, 2002). Through seeking fresh possibilities, "the problem develops, shifts, and changes in response to the continual shifts in teaching" (Loughran, 2002, p.243), embodying a never-ending quest to improve one's practice.

Seeking fresh possibilities outlines pathways that change habitual approaches to teaching. Tackling the overhelping problem brought attention to my habitual routine, that students who were stuck needed my guidance to get through questions. Attention to the issue allowed my habitual routine to emerge from the subconscious (Mason, 2020), permitting access to perceive deficiencies in my teaching and bringing forth my need to change (e.g., realizing I was overhelping Avery habitually during the course). My determination to change is an alternative to the easier course of sticking to my routine (e.g., guiding Avery through the question when they responded, "I don't know what to do"), which reinforces the status quo and denies access to new experiences (Attard, 2007). Seeking fresh possibilities enhances my will to change further, as I gain a deeper understanding of why I need to change: my sensitivities were broadened as I considered when helping becomes overhelping (I was responding to Avery in a similar manner throughout the course, guiding Avery through questions). This depth was made possible as standing back allows my reactions, emotions, and immediate cognition to quiet down, giving me access to more appropriate action during Avery's incident (Mason, 2020). In considering more appropriate action, I learned that my habitual action of guiding Avery through questions might deny them access to identify their own struggles. Now, I plan to provide affordance for students to self-identify and seek help for their struggles. These new insights will be tested in my future practice, as I continue to consciously seek fresh possibilities in new experiences and opportunities, reinforcing ongoing change (Mason, 2002).

The ongoing change was made possible over an extended period of time, a characteristic of effective professional development (Attard, 2017; Bates & Morgan, 2018; Hunzicker, 2011).

For instance, the account with ‘students playing on their phones’ was noticed on November 19, 2019. Focus on the incident was sustained over months and turned into years, as the account was discussed with my supervisor, included in my initial analysis, the reanalysis of my journal, and the writing of the findings during 2020 and 2021. The sustained focus over months and years contrasts with professional learning restricted to minutes and hours in a one-stop workshop (Edwards et al., 2015; Wei, Darling-Hammond, & Adamson, 2010). A discussion with my supervisor in 2021 provoked the idea that students could have just been taking a break, leading to the fresh possibility of teaching students to use breaks so they can reengage more productively at a later time. The focus on this incident is still ongoing, as I use this example to write my claims. In fact, each fresh possibility in the findings came to mind over sustained periods during the course of this dissertation as different sensitivities emerged over time (Mason, 2002). This is why researchers describe sustained focus over time as a key feature of effective professional development (Attard, 2017; Bates & Morgan, 2018), and challenge the depth of learning that occurs during a pre-planned moment such as a one-stop workshop (Cochran-Smith & Lytle, 2001; Sweeney, 2003).

In summary, seeking fresh possibilities characterizes effective professional development as it expertizes awareness. This form of effective professional development tackles teaching problems relevant to one’s practice over time. This form of professional learning has much to offer faculty of developmental mathematics, as it reframes how they see students (Severs, 2017) by learning to be responsive rather than depending on habits (Mason & Davis, 2013). Mason claimed that the Discipline of Noticing is a groundbreaking approach to professional development (Mason, 2002). Still, very few studies (e.g., Breen et al., 2014; J. Brown, Brown, Coles, & Helliwell, 2020; Helliwell, 2017) apply the Discipline of Noticing to learn from one’s practice. This study exemplifies the worth of engaging with the Discipline of Noticing as it contributes an insider perspective on how an instructor learns professionally.

**Blurred Boundaries Between Teaching and Researching**

This dissertation embodies how professional learning becomes research, blurring the boundaries of research and practice (Cochran-Smith & Lytle, 1993; Schoenfeld, 2014). The methodology enacted in this dissertation contributes to under-utilized self-based practitioner research for researching mathematics teacher professional development (Chapman et al., 2020; Matos et al., 2009). The shortage of self-based methodologies published in mathematics education journals (Helliwell, 2019), coupled with the infrequent engagement of college instructors in research endeavors (Maciejewski & Matthews, 2010; Tinberg et al., 2007), creates an opportunity to contribute towards the blurring of teaching and researching within developmental mathematics. By intentionally blurring the boundaries, knowledge from the inside of teaching practice is made accessible and usable in other contexts, thus transforming it into knowledge for others (Cochran-Smith & Lytle, 2012). Generating over time and across others' practices on how one supports productive struggle contributes to a deeper understanding of how instructors learn in mathematics teacher professional development (Simon & Tzur, 1999). The blurring created a space for me to reconsider teaching in relation to the research, which in turn reshaped my instruction in developmental mathematics. Unpacking this claim requires a description of this dissertation's process. The motivation for this study was to improve my teaching in the developmental mathematics classroom; my act as an instructor learning to enhance my practice. I drew upon research on productive struggle to design a course, theorizing what it would take to improve the course experience, an act of a researcher. The recommendations from the research were taken into this course to determine what would work and how it worked as I enacted the theory. My roles as a researcher and instructor are blurred here, but I mainly acted as an instructor attending to students while recording brief accounts to be analyzed later. Analyzing the students' interviews and reflecting on my experience refined my understanding of supporting productive struggle, which I could then better theorize as a researcher. However, within this analysis space, traditionally attended to by a researcher, important ideas to improve my practice came to mind, blurring the boundaries of researching and teaching. Schoenfeld also claims that within his theoretical work, he becomes aware of "suggested avenues for the improvement of practice" (2014, p.408). This is why educators researching their own practice will likely learn much more about their context than an outside researcher (Chapman, 2009; Loughran, 2007). The learnings from the findings are also likely to be valuable and informative for other teachers (Loughran, 2007). For example, it can evoke similar experiences in readers, encouraging them to seek fresh possibilities in their experiences. Whether the reader rejects or accepts my interpretations, it has the potential to prompt further stories and interpretations from the reader (Conle, 2000). Thus, blurring the boundaries enhances my learning and, potentially, the learning of others.

# Conclusion

This study illustrates how productive struggle may be supported in developmental mathematics. It can serve as a reference for instructors looking for a research-based design to support their students' learning, emphasizing the various ways developmental mathematics students experience and conceptualize struggle and productive struggle. The conceptual framework, methodology, and findings may also inspire educators to adopt the Discipline of Noticing to learn from one's experience. The remainder of this chapter discusses the limitations of this study, suggestions for developmental mathematics, and avenues for future research, including how the framework of this study can be applied to learn from one's experience of becoming a mathematics education researcher. The chapter ends with my concluding remarks.

## *Limitations*

The findings from this study were not meant to be generalized. Instead, findings, such as hesitancy to share and mismatched conceptions of productive struggle, are meant to be tested to see if they fit future experiences. If it does not fit, the challenge produces new inquiries for examination. In this sense, "noticing begets noticing" (Mason, 2002, p.194). This also applies to readers of this study, as noticing something in reading makes you think of ways to adapt it to your research and practice or becomes a choice to challenge the findings, both leading to more sensitivity for a time (Mason, 2002, 2020).

The voluntary nature of participant interviews and the small number of participants does not guard against social-desirability bias, tendency for participants to over-report positive experiences. It is crucial to note that the findings do not claim to capture all experiences and discrepancies. What holds significance is that there are different experiences and conceptions of struggle and productive struggle. Anticipated are more differences to be identified in future studies in the context of developmental mathematics, including potential differences in my future investigations of struggle and productive struggle within my classes.

### *Suggestions for Developmental Mathematics*

Some of the most interesting and provocative insights by blurring the boundaries occur when researchers' own practice brushes up against what has traditionally been valued (Cochran-Smith & Lytle, 2007). For developmental mathematics faculty in Canada, educational research and the role of instructors researching their own practice are incongruent with what has been considered professional learning. Systematic inquiries into one's practice disrupt the traditional notion of providing 'best practices' in one-stop workshops often delivered by someone considered an expert (Edwards & Beattie, 2016). This dissertation made a case for developmental mathematics to embrace the stance of instructors researching their own practice, as it acts as an example of exactly this. Embracing such a stance requires genuinely inviting inquirers to challenge the hegemony of what is considered knowledge in the college and encourage opposing viewpoints (Cochran-Smith & Lytle, 2007). For example, the knowledge from the inside emerging from this study supports the claim that instruction can simultaneously support mathematics learning and skills, such as productive struggle, in helping students transition to higher education mathematics (Lake et al., 2017).

### *Future Research*

Mason (2002) claims that others can use the research from the Discipline of Noticing. By reporting professional learning in the form of research, the fresh possibilities offered can be recognized by others (Mason, 2002). My next step is to continue my research as fresher possibilities enhance my practice. The first step is publishing my professional learning to share it with the public. This contribution is important as it is rare to find self-based methodologies published in mathematics education journals (Helliwell, 2019). Others may resonate with the publication, triggering possibilities in their practice. Mason describes this process as offering others task-exercises based on what one noticed, “task-exercises are offered to colleagues to see if they also notice what I am noticing; to see if they also find that they can mark incidents and have access to choices not previously available” (p.92). What others do or do not notice continues my professional learning, as it triggers analysis to seek fresher possibilities for my future practice.

The term *understanding* featured prominently throughout this dissertation. Pirie and Kieran’s (1994) model portrayed the dynamic movement between layers of mathematical understanding. Subsequent research endeavors could investigate how supporting productive struggle *triggers* (Towers & Proulx, 2013) movement forward and backward through the layers.

In this study, the term *struggles* in the plural was employed to capture diverse conceptions, experiences, and perspectives from both the students and myself. However, the singular form productive struggle was consistently used in alignment with the literature (e.g., C. Townsend et al., 2018; Warshauer, 2015a). The complexities of productive struggle, as revealed in this research, are intricately described through various conceptions, experiences, perspectives, and interpretations of noticing. Future research should embrace the diversity inherent in *productive struggles*, continually seeking out different possibilities of students’ productive struggles and instructor’s support of productive struggles.

### *Becoming a Researcher*

There is an underlying narrative of what it is like to become a researcher. Boaler, Ball, and Even (2003) discussed the range of problems novice researchers face in learning how to be a mathematics education researcher. Of note in the authors' paper was the challenges novice researchers face in writing research. I can seek fresh possibilities from my struggles to write this dissertation. For example, past drafts, which include feedback from others, serve as data. The feedback and changes can be analyzed. Common and different themes can be detected. The themes become triggers to stand back and consider what I was unaware of at the time and how I can act differently when I write in the future.

### *Concluding Remarks*

The enduring impact of my journey through this dissertation is one of its most gratifying aspects. Vivid to this day was how hesitant students were at the beginning of the course. It felt as though a profound chasm separated us, and the students were waiting for someone to prove that the bridge to cross was indeed stable. Supporting students to take risks and embrace struggle has left an indelible mark on my understanding of teaching and learning within developmental mathematics. What continues to resonate is the shared experience of traversing this metaphorical bridge. As students tested strategies in moments of being stuck, making mistakes, and realizing their limitations, a similar process unfolded for me. I, too, got stuck, made errors, and realized gaps in supporting students' struggles. The ongoing emergence of possibilities is forging a novel purpose in my conceptualization of teaching and learning developmental mathematics. At the core of this new perspective is that fostering skills such as productive struggle aligns harmoniously with teaching mathematics content. This alignment is ever more evident in the current landscape, where students enter college with their learning experiences significantly disrupted by the pandemic's stoppage. I perceive this as a new challenge to expand my practice, as previously inaccessible possibilities of struggle and productive struggle in developmental mathematics may come to the forefront.

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# Appendix

## Appendix A: Figures

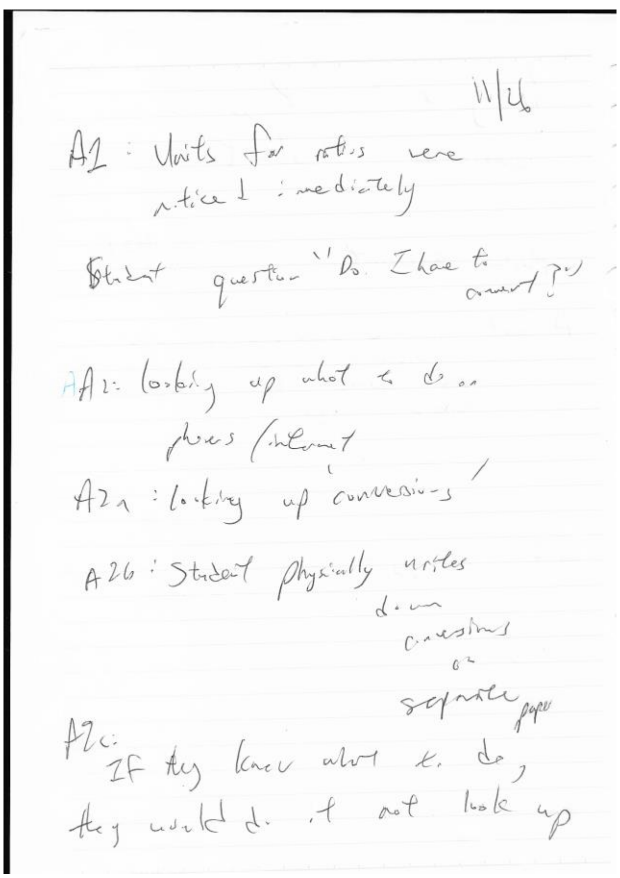
**Figure A1**

*Excerpt from an assignment submitted by a student*

**Class 7:** somethings that I found a struggle with is fractions I have always struggled with fractions I've never been the strongest in that unit, I also didn't think that rounding, precision and accuracy was going to be on the test so I didn't really study that too much. I need to be more prepared for incoming tests and quizzes and spend more time on being fully prepared.

**Figure A2**

*Sample of Instructor's Notes 1*



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A1: Units for ratios were noticed immediately

Student question "Do I have to convert?"

A2: looking up what to do on phones/internet

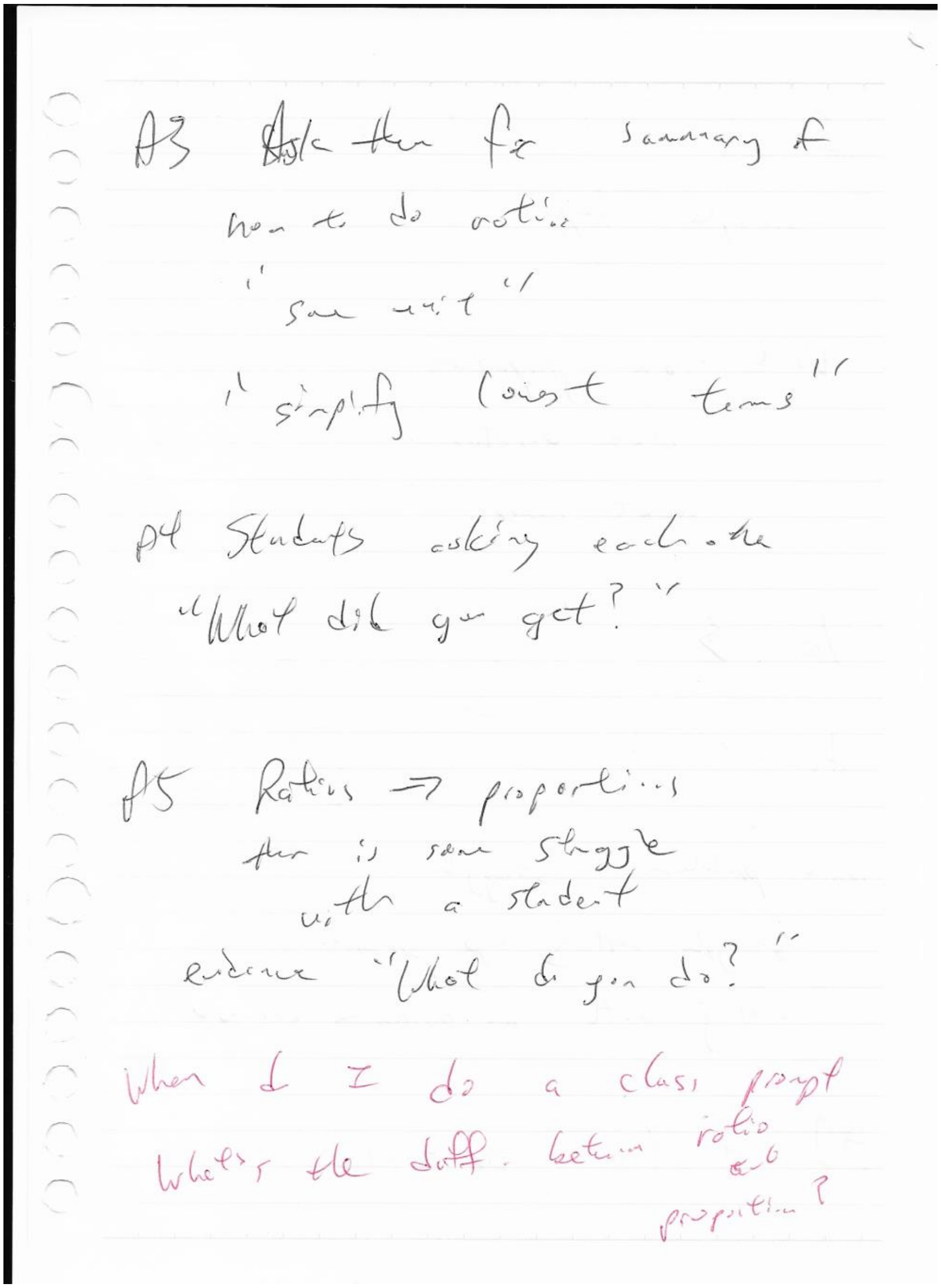
A2a: looking up conversions

A2b: Student physically writes down conversions on separate paper

A2c: If they knew what to do, they would do it not look up

**Figure A3**

*Sample of Instructor's Notes 2*



A3 Ask them for summary of
how to do ratios
"same unit"
"simplify lowest terms"

A4 Students asking each other
"What did you get?"

A5 Ratios → proportions
there is some struggle
with a student
evidence "What do you do?"

When I do a class prompt
What's the diff. between ratio and proportion?