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So we know studying the concept of covariance. And as I said in the last clip, it measures the relationship between two random variables. And it's often used in a lot, lot of contexts. For example, you may be interested in the relationship between two stocks, right? Maybe the stock of apple and the stock of tests, or maybe the stock of Apple, or maybe the stocks of Tesla and Ford and GM. Or you could be interested in what's the relationship between unemployment and inflation. Or you could be interested in the exchange the what's the relationship between the exchange rate of the US versus Canadian dollar and the price of oil. And in the last group, I also developed for you the formula for measuring covariance. So let's now do an example of how you would actually compute covariance context. So here is an example of two random variables X and Y. And this table here gives the joint probabilities of X and Y. So this is the sort of table which we have done before, in in the previous module when we are talking about conditional probabilities. So here is the random variable X, which takes two possible values 10 and 20. And Y takes values 1, 2, and 3. And this here is the joint probability of X taking the value 10 and Y taking the value 2. This is the joint probability that X takes the value 20 And Y three. And what we want to calculate here is the covariance between X and Y, which is given by this formula, which is expectation of X times Y minus expectation of X times expectation. First thing if we were to calculate the expectation of XY. So what we have to look at is all the possible values of x and y. So let's take x 10, y equal to one. So that's 10. And what's the probability of that occurring, that's, that's, that's zero. What else, so 10 and two, so that's 10 times two is 20, times its probability, which is point two. So remember, we're taking the expectation of X times Y. So we're looking at all the possible products of X and Y, that's 10 times three will be 30 times its probability to corresponding probabilities, point four. Similarly, this is 20 times Y, that's 20, the probability of that is 2, then 20 times 2, that's 40 times its probability point two here again. And the last 120 times three, that's 60 times its probability. And if you add all of these up, that's going to be 20. So this first term here is equal to 28. And minus now we need to figure out the expectation of X and the expectation of y. Now see, x takes two values, 10, and 20. So what's the probability of 10? So we have to look at what's called the marginal probability. And we find that by adding up these three probabilities, right, this is all of these are X equal to 10, that Y takes the value 1, 2, or 3. So if we add these three up, we get the marginal probability of X taking the value 10. Next one, if we add these three up, that gives the marginal probability of

X taking the value of 20. So those two so if you want to calculate expectation of X, that's going to be 10 times this 10 times point six plus 20 times that's corresponding probability of point four. And if you do that, that comes out similarly, we need the expectation of Y. So again, we need to calculate Y takes the values one, two and three, we need to calculate the marginal probabilities, this is just achieved by adding these two. So that's point 242. That's for the students. So expectation of Y is one times its probability point two, plus two times its probability point four. Three times its probability point four. These three add up to two point. And now if you plug it back into the formula for covariance, right, so this is going to be minus expectation of X, that's 14 times the expectation of y, that's 2.2. And this comes out to be negative two. So that here is the covariance between X and Y. So what this suggests is that the relationship between X and Y is negative in the sense, on average, when X is higher, Y tends to be lower, and vice versa. So this is how you use this particular formula to calculate covariance between two random variables. In an example, like here, where I've given you the joint probabilities. Now, here's a quick clicker question. A very simple one, you may want to stop the video here. And so what the clicker question asks is, which one of the following is measured by covariance? The average of a random variable? That's not it, right? That's the expectation. The dispersion of a random variable. That's not it either. That's the variance. The maximum value for random variable? No, that's not covariance, the relationship between two random variables, that's what covariance measures, right, so D is the correct answer space. So let me end this clip by talking about some particular properties of covariance. Now, the covariance between X and Y is the same as the covariance between y and x, right? So the relationship doesn't matter if you like, if you change the order. The relationship between height and weight is the same as the relationship between weight and height. Secondly, if X and Y are independent, remember, independent is the case where one does not influence the occurrence of so there is no relationship between them. So in that case, the covariance between X and Y is zero. Thirdly, if you add a constant to X, and if we add a constant Y, that that does not change the covariance. The covariance of $X + C$ or $Y + d$ is the same as the covariance between X and Y. So in particular, what if I go back to our original graph, which showed you the intuition for deriving covariance, what it means is that if I shift these points this side, right, or if I shift this point, this side, or if I shift these points up, or if I shift these points down, the relationship still remains positive or negative, so the covariance doesn't change. So let me go back to my properties. So if you add a constant or subtract a constant, right, the covariance doesn't change. Now, if you multiply the random variables by constants by C and D, then the covariance then those two come out. So it becomes C times D times the covariance between X and Y, for example, right, suppose you're meant, suppose you're looking at the core covariance between height and weight. And let's say you're measured height by meters and weighed by kgs. And now you want to change those measurements and make this into yards and pounds. So in this case, you multiply by 1.1 for yards and 2.2 for pounds. And if you want to change your meter into yard, you divided by point nine. So then what happens is that the then the covariance in this case, is just going to be the product of point nine times 2.2, times the original covariance that you had with height in meters and the weight in kgs. And one last thing if you just look at the covariance between X and X in because it's the variance of x, and you can also see this by the formula, see, the formula for covariance is $x \times y$ minus expectation of X times expectation of y. So now if you replace y by x, then this becomes expectation of X squared minus expectation of X times expectation of x. And that is precisely the formula for variance that we derived. So these are the properties of covariance. And so we're going to stop this clip here. And in the next clip, I'm going to use this to develop a related measure called correlation.