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SPEAKERS

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So so far, I've introduced you to the idea of continuous random variables, and the distributions, the probability distributions in terms of the PDF, the probability density function, and the distribution in terms of the cumulative distribution function of the CDF. So now, let me talk about couple of the most common continuous distributions that you receive courses. So the first one is what's called the uniform distribution. So we've already done some examples of the uniform distribution earlier, you saw the uniform distribution over zero to 10. So I'm going to introduce a slightly more general version of that. So when we say that random variable X is uniformly distributed over an interval a to b , but it means is that let's take an interval, E to B is the bottom point B is the topmost point, right? So this random variable is distributed over this interval. So we'll call the range, remember is the difference between the maximum value that the variable can take and the lowest value the variable can take. So in this case, the range is going to be B minus A . Now, so the important thing about any distribution, of course, is its PDF, or the density. So the density of a uniform distribution is given by f of x is equal to one over the range one over b minus a , for all values of x between A and B . And it's zero elsewhere. So what it means is that the density will look like this. So so this is the, the minimum point, right? A , right, so below it, it will be zero, this is the maximum point B , right above it, it will be zero. But in between A and B , it takes the same value, one over b minus a . Right, so the CDF, sorry, the PDF, the density is the same for all values of x in between. So that's why this is called uniform. All of these values between A and B , these are all equally likely, in the sense that they all have the same density. So that's what defines the uniform distribution. So once you have the PDF, now, of course, you can also derive the CDF or the cumulative distribution function of it. So f of b , so that's the CDF at the point B is given by Remember, it's the integral of the density up to the point B . Another way to look at it is that it's the area under the under the PDF up to the point. And it is given by by this thing here. And this is easy to derive because it's the area up to this point. So it's basically this is the area of this rectangle here. And what's the base of this rectangle? That's B minus A ? Right? What is the height of the rectangle? It's one over b minus a , right? So this area is going to be the product of the the small b minus a times one over b minus a . Which is exactly this. So that's the cdf of the uniform distribution. So what's the mean, and what's the variance? So the mean of this is A plus B over two. So remember, this is this random variable is distributed over the interval a to b . So the great thing about the

uniform distribution is that its mean is exactly the middle point of it. That's a plus b over two. What is its variance? You could derive this using the formulas that we had before, but let me just present it here. It's $b - a$ whole thing squared divided by two. So that's the variance. have of the uniform distribution? Right? So now where do we use uniform distribution? So you use the uniform distribution anywhere where you think that all values are equally likely. So for example, if you're introducing a new product, and you're trying to estimate what might be the sales of it, and you are, you don't have a very good idea, you think like all the sales level between, let's say, zero and 100, are equally likely to occur, then you would use a uniform distribution. Similarly, suppose you're introducing a new stock IPO, and you don't have very good idea of where the stock value may end up, right? And you think that all these values are equally likely, that's when you take a uniform distribution. So the characteristic of the uniform distribution is that all values within that interval are equally likely. So let's use it in a particular case. Right? So let's say a new stock is launched. And let's say its price at the end of the day is given by x tilde. And let's think that this is uniformly distributed over 10, and 50. So that means it's uniformly distributed over the interval 10 to 50. So now, if you want to try to figure out what is its mean price, remember, the mean price is just the midpoint of it, right? So that'll be $50 + 10$, divided by two, so that 60 divided by two. What is its variance. So again, we can use the formula, it's going to be $B - A$, it's $50 - 10$, whole thing squared divided by 12. So that will be 40 squared, divided by 12. And you can calculate that out. So that'll be 1600 divided by and whatever that is, is going to be the variance of this of this. Now you can calculate other probabilities, right? So you can calculate, suppose you want to know what's the probability that the stock is value is going to be less than or equal to 20. So that means it's the CDF at 20, F of 20. And remember, the formula for for the uniform distribution is it's $b - a$ divided by $B - A$. So it's the value there. So 20, minus the bottom most point, which is 10, over the range, which is 50, minus 10. So that's 10 over 40. So that's one four, or point two, five. So that's the probability that the stock's value will be less than or equal to 20. Similarly, suppose you wanted to calculate, what's the probability that the stock will take a value greater than four? So remember, you can figure this out by looking at the complementary probability, what's the probability that one minus the probability that it will be less than or equal to so it'll be one minus. So remember, to figure out again, F of 40, it'll be 40 minus the bottommost point 10 over the range. So that'll be one minus 30 of 40. So one minus three fourth. So that's one four. And this is also point two, five. So this is how you can calculate the probabilities for a uniform distribution. So here, I have a clicker question for you based on the uniform distribution. So please stop the video at this point. So hope you are able to solve it without too much trouble. So what it says is that suppose the water level in a lake is uniformly distributed over 500 to 600. What's the mean water level? Remember, again, the mean of the uniform is just the midpoint, right? So the midpoint of this interval is going to be 550. So the correct answer here is this. So that's it for the uniform distribution. It's the easiest continuous distribution, it has the easiest density, it has the easiest CDF. And therefore it's sometimes used. If you want to compute something very easily, right? Use it to its expectation can be derived very easily, its variance can be derived very easily. So let me stop this clip here. And in the next clip, I'll introduce you to what's probably the most common of the continuous distributions. And that's going to be the normal distribution.