
**Module 1 Practice Problems: Introduction to Algebraic Expressions**

**1.** Every month, Sarah spends \$800 on rent, \$300 on food and drinks, and \$200 on various subscriptions. She is considering buying $x$ BABA stocks, each priced at \$200, $y$ APPL stocks, each priced at \$125 and $z$ TD stocks at \$75 each.

(a) What is her total expenditure for the month?
(b) If Sarah earns \$2000 in a month, can she afford to buy 2 each of BABA, AAPL and TD stocks?
(c) If Sarah only buys TD stocks, what is the maximum number that she can buy in a month?

**2.** In a course, the overall mark depends on one's mark in an assignment, a midterm exam and a final exam. The weight on the three are 20%, 30% and 50% respectively. Josh's score on the assignment was 90%, on the midterm was 70% and he has the final exam coming up.

(a) If Josh's score on the final exam is $x$%, what will be his overall score for the course?
(b) Josh needs to get 85% or more overall to get an A in the course. If he scores 90% in the final, will it be enough for Josh to get an A?
(c) What is the minimum that Josh needs to score in the final to secure an A for the course?

**3.** Sam's current weight is 200 lbs. He wants to lose some weight and so signs up with a nutrition company who promise that if he follows their diet, he will lose 4 lbs every month.

(a) What will be Sam's weight after 6 months if what the company says is true?
(b) What will be Sam's weight after $t$ months if what the company says is true?
(c) How many months does Sam need to follow this diet plan in order to bring his weight down to 150 lbs?

**4.** Sam's sister, Pam, signs up with a different nutrition company who promise that if she follows their diet, she will lose 5% of her weight every month. Pam's current weight is 150 lbs.

(a) What will be Pam's weight after 1 month if what the company says is true?
(b) What will be Pam's weight after 2 months if what the company says is true?
(c) What will be Pam's weight after 6 months if what the company says is true?
(d) What will be Pam's weight after $t$ months if what the company says is true?

*Activity*: Use Excel/Google sheets to figure out Pam's weight after 1 year and after 2 years of following this diet.

**5**. The distance between two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

A player on a basketball court is located at the point $(2, 2)$. He has the ball and is considering passing to one of two other teammates, one located at the position $(10, 12)$ and another at $(6, 16)$.

(a) Which of the two is closest to this player?
(b) Then he notices another player come up, who is located at the position $(8, 14)$. Now, who is closest among the three?

**6.** The government is trying to estimate how many people in Canada smoke. Canada has about 32 million individuals aged 15 years and older, with about equal numbers of males and females. A telephone survey was conducted for this purpose. 1000 males and 1200 females responded to this survey. Of them, 150 males and $y$ females answered that they were smokers.

(a) What fraction of males in the survey were smokers? What about females?
(b) Can we use this data to estimate the total number of smokers in Canada?

*Fact*: The actual number of smokers in 2017 was estimated to be 2.5 m males and 2.1 m females.
*Source*: Canadian Tobacco, Alcohol and Drugs Survey 2017: Smoking prevalence by sex | Tobacco Use in Canada | University of Waterloo (uwaterloo.ca)

**7.** Chez Iggy Fine Foods is considering procuring its vegetables from one of three possible farms. Farm 1 offers to sell vegetables at \$4000 a month and is located 30 km away; Farm 2 offers a lower monthly price of \$3000, but is located 60 km away, while Farm 3 offers the lowest price of \$1000 a month but is located $y$ km away. The cost of transportation, which Chez Iggy will have to bear, is given by $0.4 * (distance)^2$. In deciding on its choice, Chez Iggy will have to take into account both the price and the transportation costs.

(a) Between Farm 1 and Farm 2, which one will Chez Iggy choose?
(b) Between Farms 1, 2 and 3, which one will Chez Iggy choose if $y = 100$?
(c) Between Farms 1, 2 and 3, which one will Chez Iggy choose for any given $y$? For which values of $y$ will Farm 3 be the optimal choice?

8. Kim is considering two possible options in her commute from home to work. Option one involves driving a distance of 40 km; her speed on this route depends on the time of day and is given by $x$ km/h. Option two involves driving 5 km to her nearest subway station using neighborhood streets, then a 50 minute subway ride and finally a 10 minute walk. Suppose her speed on the neighborhood streets is always 50 km/h.

(a) How much time will it take Kim take if she drove to work?
(b) How much time will it take Kim if she used option two?
(c) Suppose $x = 30$. Which option will Kim choose?
(d) For what values of $x$ is option one better? When is option two better?

**9.** A firm XYZ Inc. uses three inputs $X$, $Y$ and $Z$ to produce an output $I$ according to the formula: $I = \sqrt{X} + \sqrt{Y} + \sqrt{Z}$. This is an example of a production function, a concept that is commonly used in economics and business.

(a) If it uses 4 units of $X$, 16 units of $Y$ and 9 units of $Z$, how much $I$ will it produce?
(b) Suppose it doubles its inputs to 8 units of $X$, 32 units of $Y$ and 18 units of $Z$, how much $I$ will it produce?
(c) If it already has 4 units of $X$ and 16 units of $Y$, how many units of $Z$ will it need to produce 12 units of $I$?

**10.** Consider a model of infection spread. Suppose each person interacts with $N$ people each day. If a person is infected, the chance of him/her spreading it to a person that he/she meets is $p$. Thus on average, an infected person will spread the infection to $pN$ persons. This number is called $R_0$ and has been much talked about in the context of the Covid-19 pandemic.

Assume for simplicity that an infected person is contagious for only 1 day i.e. stops spreading it after 1 day.

(a) If 1 infected person enters a population today without any previous infection, how many people are likely to be infected in 1 day, in 5 days, in $t$ days?

(b) The $R_0$ numbers for Ontario varied from a high of 2.8 early on in the pandemic to a low of 0.7 during the summer of 2020. Starting with 10 infected persons, compute how many infections there are likely to be in two weeks when $R_0 = 3$ versus when $R_0 = 2$ or $R_0 = 1$ or $R_0 = 0.7$.
(c) *Activity*: Suppose $p = 0.2$ and $N = 20$ so that initially $R_0 = 4$. Start with 5 infected persons. After 10 days of spread of the infection, the government imposes lockdowns so that $N$ now drops to $N = 4$. Use Excel/Google sheets to figure out the number of likely infections after 10 days (i.e. without any intervention), and then what happens after 20 days (i.e. 10 days without any intervention and then 10 days of lockdowns) and after 30 days.
(d) *Activity*: Vaccinations reduce the probability of the spread of infections i.e. they reduce $p$. Suppose instead in (c) $p$ was 0.06 (due to widespread vaccination). Use Excel/Google sheets to figure out the number of likely infections after 10 days and after 20 days, both in the absence of any lockdowns and with lockdowns after 10 days of spread.
(e) *Activity*: Try other values of $p$ and $N$ and different lockdown policies in (c) and (d) to see how the number of infections changes. Although this is a much simplified version of actual models used, this sort of modelling exercises are done by policy experts to determine the impact of different policies.

**11.** Oak wilt is a fungal infection that affects oak trees in many parts of North America. It spreads via insects and underground via tree-roots. Consider the following model of its spread. If a tree in a forest is infected with oak wilt, it will spread the infection to $R$ trees in a year and then die.

(a) If 1 tree gets infected in a forest, how many trees are likely to be infected in 1 year, in 2 years, and in $t$ years?
(b) Suppose a firm has developed a technology that can identify 50% of the trees infected with oak wilt every year and destroy them before they can infect other trees. Now, how many infected trees are likely to be there after 1 year, 2 years, and after $t$ years?
(c) *Activity*: The firm is trying to convince the government to buy and use its technology by demonstrating the consequences otherwise. Currently there are 4 infected trees in a forest. Suppose $R = 3$. Use Excel/Google sheets to figure out the number of likely infected trees after 10 years if nothing is done. What would the number of infected trees be after 10 years if this firm's technology was used instead?


 This project is made possible with funding by the Government of Ontario and through eCampusOntario's support of the Virtual Learning Strategy. To learn more about the Virtual Learning Strategy visit: https://vls.ecampusontario.ca/.