Pre-emptive and Reactive Resource Allocation in Emergency Response

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A Thesis Submitted to

the Faculty of Graduate Studies

In Partial Fulfillment of the Requirements

for the Degree of

Master of Applied Science

Graduate Program in Civil Engineering

York University

Toronto, Ontario

May 2025

Abstract

Emergency response departments face significant challenges due to resource scarcity and fluctuating incident demands. Traditional static resource allocation, assigning resources to fixed stations, often results in inefficiencies, leaving resources underutilized or insufficient in high-demand areas. To address these shortcomings, this research develops a dynamic resource allocation framework tailored specifically for fire departments. The framework integrates preemptive strategies based on historical data and predictive modeling with reactive strategies responding to real-time emergencies. Validated through a case study in Fredericton, Canada, using data from 2017 to 2021, results demonstrate that dynamic allocation significantly reduces response times and enhances resource efficiency compared to static methods. This study provides a data-driven, practical approach to optimizing resource management in fire departments, effectively addressing operational constraints such as geographic challenges, varying demands, and limited resource availability, thereby improving efficiency, readiness, and effectiveness in emergency response operations.

Keywords: Dynamic resource allocation; Emergency response optimization; Fire department management; Spatiotemporal demand modeling; Preemptive and reactive strategies; Operational efficiency; Resource utilization; Mathematical modeling; Response time reduction; Incident management.

Dedication

I am genuinely thankful for the amazing support from so many special people in my life. First, a big thank you to my parents. Their love, support, and belief in me have been so important. They've always been there for me, helping me keep going. To my wonderful wife, who has always been by my side, her support has been extraordinary. She is not just been there during the easy times but has stood strong with me through all the challenges, offering encouragement, love, and understanding that goes beyond words. Her faith in me never wavers, even when I doubt myself, and her presence is a constant reminder of the beauty and strength of our journey together. She is my partner in every sense, making every step of this path brighter and filled with love. And to my friends in Canada, who are like my family now. Their friendship, support, and kindness have meant the world to me. They've made me feel at home, even when I'm far from where I started.

To all of you, thank you so much. Your support and belief in me have made this journey possible and so much more meaningful. For all of us, this thesis is a sign of what we've been through together. Thank you for everything.

Acknowledgements

I wish to extend my sincerest gratitude to my supervisors, Dr. Mehdi Nourinejad and Dr. Peter Park, for their exceptional guidance, trust, and sharing of their vast expertise throughout the course of my research. The enthusiasm and intellectual curiosity they inspired in me made the journey of completing this thesis an enlightening and engaging experience. Their invaluable advice and insights were crucial in shaping this work, and for that, I am deeply thankful. I am also profoundly grateful to York University, the Natural Sciences and Engineering Research Council of Canada (NSERC), the Regional Municipality of Peel, and the Smart Freight Centre. Their generous financial support and resources were indispensable to my research efforts, providing me with the necessary foundation to pursue my studies and achieve my goals.

This acknowledgment would be incomplete without expressing my appreciation for the wider academic and supportive community that surrounded me. The environment fostered by my advisors, colleagues, and the broader academic community has been nurturing and stimulating, contributing significantly to my personal growth and the success of this project.

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1 Chapter One: Introduction

This chapter provides the motivation and goal for the research followed by research objectives. Following this, the thesis structure is briefly described to give an overview of the topics covered.

1.1 Motivation

Emergency response departments are among the organizations that play an important role in saving lives. These departments face growing limitations due to the scarcity of resources, including personnel, fleets, and equipment. One contributing factor to these limitations is the static allocation of personnel to stations, requiring them to be at the same location and ready to respond to high-risk situations during their shifts. Consequently, fixed allocation leads to the mismanagement of human and financial resources, particularly because of the uneven spatiotemporal distribution of emergency events. For example, fire response data shows that fire incidents are more likely to occur during the day in downtown areas and at night in residential zones, simply due to higher crowds in business centers during working hours and vice versa in residential areas during the evening. An alternative to static allocation is dynamic allocation, where resources are relocated between stations within a day or across multiple days of operation. Dynamic allocation facilitates faster response operations by positioning resources closer to the potential areas of need, and resources will be relocated based on spatioteporal features of demand. To implement this, however, decision-making tools are required to generate strategies that effectively allocate resources over time. Without such tools, planners may be unaware of the long-term costs and benefits of their decisions, resulting in wasted resources that could be more effectively distributed elsewhere.

Resource management in the public sector differs significantly from that in the private sector, which aims to maximize profit or minimize costs. In contrast, public sector goals are centred around minimizing social costs, ensuring efficiency, and maintaining equity in service delivery [1]. The complexity of resource management is further compounded by political decision-making, data limitations, and conflicting goals [2]. The multifaceted nature of emergency responses highlights the need for dynamic resource allocation. Different types of incidents—such as structural fires, vehicle accidents, and hazardous material incidents—require specialized response protocols and strategies. Effective resource management requires a thorough understanding of incident types and a strategic approach to resource deployment, whether preemptive or reactive. From the moment an emergency call is received to the completion of incident handling, resource allocation plays a crucial role in determining response effectiveness.

Preemptive resource allocation involves positioning resources in anticipation of events based on historical data and predictive models, offering the advantage of proactive preparedness. By forecasting the likelihood and potential locations of incidents, emergency response departments can optimize resource utilization, reduce response times, and enhance overall effectiveness. In contrast to preemptive resource allocation, reactive resource allocation adapts to unforeseen incidents, addressing the unpredictable nature of emergency events. Both strategies play an essential role in the decision-making framework for effective emergency management, which highlights the importance of advanced models that integrate both preemptive and reactive elements, allowing for dynamic optimization of resource allocation.

Integrating dynamic optimization of resource allocation strategies into resource management systems can significantly enhance the capacity of emergency response departments to respond efficiently and promptly, ensuring maximum readiness and operational effectiveness. The findings of this study highlight the need to reassess current resource allocation models and invest in the development of advanced decision-making tools that can accommodate the dynamic nature of emergency incidents. Such advancements could improve resource allocation, ensuring optimal utilization at the times and locations of greatest need.

This study focuses on the logistics of dynamic resource allocation, specifically examining its application within fire departments. The proposed model captures two types of resource movements: preemptive and reactive. In preemptive relocation, resources are strategically scheduled to move between stations based on anticipated demand patterns. In reactive relocation, additional resources, such as fleets and personnel from other regions, are deployed to incident sites when the scale of the emergency exceeds expectations and the initially allocated resources are insufficient to manage the situation effectively.

1.2 Research Objectives

This research aims to advance the understanding and application of dynamic resource allocation strategies in emergency response departments, specifically focusing on fire departments. The study sets the following objectives:

1. **Evaluate the limitations of current static resource allocation models**: Analyze the inefficiencies, imbalances, and constraints of static methods, particularly in addressing the spatiotemporal variations of emergency demands.

2. **Develop and optimize a dynamic resource allocation framework tailored to emergency response operations**: Design and refine a mathematical model integrating pre-emptive strategies based on historical demand patterns and reactive strategies for real-time emergencies. Incorporate key operational factors such as spatiotemporal demand variations, maximum allowable distances for resource reallocation, response time targets, resource availability, geographic challenges, and cost efficiency. Systematically analyze their impact to ensure the framework is practical, adaptable, and effective across various operational contexts.

3. **Validate the framework through real-world application**: Apply the proposed model to the City of Fredericton, Canada, using historical incident data to evaluate its performance. Demonstrate its effectiveness in optimizing resource utilization, minimizing response times, and addressing operational inefficiencies in emergency response.

In summary, this research seeks to bridge critical gaps in emergency resource management by advancing both theoretical understanding and practical application of dynamic allocation strategies. The outcomes aim to provide emergency response departments with actionable tools to enhance their efficiency, adaptability, and preparedness in addressing evolving challenges.

1.3 Scope & Thesis Organization

1.3.1 Scope

The scope of this research is defined by the modeling assumptions and constraints under which the dynamic resource allocation framework is developed. In constructing the model, all emergency response units are assumed to be homogeneous in capability and performance, meaning each resource is treated as identical and interchangeable. Travel times and costs between stations and incident locations are estimated through a consistent, simplified method that provides a baseline for movement decisions without capturing the full complexity of real-time traffic or road network conditions. The demand for emergency services is modeled as a Poisson process, reflecting an assumption that incidents occur randomly over time at a steady average rate derived from historical data. Furthermore, the model operates under an assumption of perfect information, implying that it has immediate and accurate knowledge of incident occurrences and the status of all resources at any given moment. These assumptions create a controlled environment in which pre-emptive relocations (based on anticipated demand patterns) and reactive deployments (in response to unfolding incidents) can be optimally coordinated.

Within these assumptions lie the limitations that delineate the scope of the study. Treating all resources as uniform in capability means the model does not account for differences in vehicle type, crew expertise, or specialized equipment – a simplification that may overlook important heterogeneity in real emergency operations. The use of a fixed, predetermined travel cost estimation disregards dynamic traffic congestion and other routing uncertainties, which could lead to deviations between the model's predicted response times and actual field conditions. Likewise, assuming Poisson-distributed incident arrivals provides analytical tractability but may not fully capture peak demand periods or spatial-temporal clustering of emergencies that occur in practice. The perfect information assumption is an idealization, as real-world decision-makers often face incomplete or delayed data during emergencies. The evaluation framework also employs a static cost structure for optimization (e.g., treating all relocations or responses with uniform cost parameters), which simplifies analysis but fails to reflect potential variations in cost or priority for different incident types or severity levels.

By clearly stating these assumptions and acknowledging the associated limitations, the scope of the research is established, focusing on a tractable subset of the emergency response allocation problem and providing a basis for interpreting the model's results within an idealized context.

1.3.2 Thesis Organization

Chapter Two: Literature Review provides a comprehensive review of existing literature on emergency response resource management. It evaluates different methodologies and identifies gaps in current approaches, laying the groundwork for the development of the proposed preemptive and reactive resource optimization model.

Chapter Three: Methodology, the research methodology is elaborated, detailing the conceptual framework and mathematical models used in designing the preemptive and reactive resource optimization strategy. The theoretical foundations of the proposed approach and its practical applicability in real-world scenarios are discussed.

Chapter Four: Simulation Results and Discussion analyzes and discusses simulation results obtained from implementing the proposed model. The findings are interpreted in the context of their implications for resource allocation and operational efficiency in emergency response scenarios.

Chapter Five: Case Study validates the effectiveness of the preemptive and reactive resource optimization model in a real-world emergency response scenario. Practical insights into the implementation and performance of the developed model are provided.

Chapter Six: Conclusion summarizes the key findings, highlights the contributions to the field of emergency response resource management, and suggests directions for future research.

2 Chapter Two: Literature Review

This literature review is structured as follows: We begin by examining the critical aspects of staffing in fire departments, focusing on both qualitative and quantitative dimensions. Next, we investigate fleet sizing, by considering factors including the strategic locations of fleets and their deployment strategies. Concluding the review, we explore the operational intricacies and procedural dynamics within fire departments, aiming to provide a comprehensive overview of their functioning and challenges.

2.1 Staff Allocation Strategies

Ensuring that a fire company[1] is adequately staffed is of paramount importance. Proper staffing is essential for ensuring operational efficiency and significantly enhancing the overall effectiveness of fire incident management by ensuring that enough personnel are available to perform a variety of critical tasks on the fire ground. The direct and indirect impacts of staffing on incident outcomes and the safety of both firefighters and civilians are significant and highly impactful [3]. Determining the appropriate staffing levels for a fire station requires a comprehensive analysis that considers the financial implications of allocating additional resources, such as firefighters and fire apparatus, alongside the potential risks and consequences of inadequate response capabilities. Staff sizing is a complex process that requires careful consideration of several critical factors, including the imperative to save lives, ensure public safety, and minimize the economic impacts of fire incidents. Given the diverse nature of losses incurred during fire in-

[1]The term 'fire company' refers to the group of firefighters dispatched to an incident. It can also denote a single firefighter attending to an emergency.

cidents—ranging from loss of life and property to significant economic detriment—it is essential to evaluate the justification and feasibility of investing in additional protective measures in light of potential incident-related losses.

Although it is widely recognized that increasing staffing levels improves fire protection capabilities, the exact relationship between the size of a crew and overall fire safety remains unclear. To address this, the National Fire Protection Association (NFPA) in the United States has issued general recommendations for appropriate staffing levels at fire stations. Despite these guidelines, there is still no consensus on a universally accepted method to determine the optimal staffing level for each individual fire station [4]. Lack of a standardized approach highlights the complexity and variability inherent in fire protection and emergency response, posing a significant challenge for fire department management and policy development.

There is ongoing discussion in the firefighting profession about establishing an international standard for the number of teams assigned to specialized fire equipment, such as aerial ladder trucks or heavy rescue units. This discussion is pivotal as it influences various aspects of fire management and public safety, including fire safety, the economic impacts of fires, employment opportunities for firefighters, and the overall costs associated with fire protection. At the heart of this discussion is the relationship between fire company size and the extent of fire losses. Proponents of standardized larger crews argue that such teams are not only safer but also more effective, supporting their claims with various research studies. However, there is disagreement over the use of these studies to set national standards. Critics contend that these studies often fail to sufficiently consider or counter alternative hypotheses that could explain the causal relationship between specific local fire challenges and the frequency of fire outbreaks. This critique implies that the discussion may stem more from philosophical differences than from technological or empirical evidence [3, 5]. Standards that demand a higher-than-necessary number of firefighters per company could inflate firefighting costs, affecting other essential public services. Small, volunteer, or underfunded career departments may face closure or risk being perceived as substandard if they fail to meet these strict requirements. Alternatively, some regions might be compelled to reduce the number of fire stations to maintain minimum staffing levels per company, which could unintentionally increase station coverage areas and prolong response times [3].

Two main factors must be considered when determining the appropriate staffing level for a given fire station, commonly known as the effective response force. These two factors are the number of firefighters per fire truck (i.e., the size of the crew), and the number of fire trucks that will be required to respond to the fire [4]. The staffing levels and number of firefighters per fire truck are commonly discussed according to two standards in the fire protection community [6]. Among the most notable works on fire station staffing that have been published by the National Institute of Standards and Technology (NIST) are the field experiments conducted by the NIST on fire ground staffing [6]. To determine the effect of varying the size of crews per fire truck on the overall scene time while maintaining a fixed number of fire trucks, NIST conducted 60 field experiments to investigate the impact of varying the number of crews per fire truck. Regarding the choice of crew size, NIST does not provide any specific conclusions, but it provides various performance measures that can be applied to select the appropriate size of the crew based on the performance measures provided. The National Fire Protection Association (NFPA) established the NFPA 1710 standard [7], a significant reference regarding fire station staffing, in which the number of personnel required for a single station is not specified. According to NFPA 1710, every fire department should be prepared to respond to fires with a minimum of 15 personnel, including firefighters and emergency medical services (EMS) personnel qualified to handle various types of fires. However, the number of firefighters may need to be increased depending on the type or location of the incident.

Although fire departments use staffing models, career departments, which consist of fully paid and professionally trained firefighters, typically staff fire engines with crews of three or four firefighters. Unlike volunteer departments, where personnel may have other primary jobs and respond to emergencies on a part-time basis, career departments maintain a full-time workforce dedicated to fire and emergency services. Industry standards recommend a minimum of four firefighters on each type of apparatus to ensure adequate manpower for effective and safe operations. Nevertheless, local departments have the flexibility to adjust their staffing based on specific requirements, be it financial constraints or service demands [8]. NFPA-1710 suggests that both fire engines and ladder trucks should be staffed with four firefighters each. However, it is commonplace for many fire departments in the United States and Canada to operate with a minimum crew of three firefighters. As highlighted by Lawrence [3], the number of personnel assigned to a fire apparatus typically ranges from two to five. This staffing varies based on the community's requirements and the specific financial capacity of the fire department.

Alongside determining the required number of staff in each station and truck, a thorough understanding of how fire departments schedule resources during different time periods is crucial. Effective scheduling ensures that resources are optimally allocated to meet varying demand levels, enhancing response times and overall operational efficiency. The scheduling within any fire department is tailored to local requirements and preferences, reflecting the unique needs and challenges of the community it serves. The National Fire Fighter Near-Miss[2] Reporting System provides an optional, non-penalizing approach to gather and learn from incidents and close calls. Its summary report serves as an informative sample of the diverse shift arrangements employed by firefighters. From the reports contributed to this system, it was found that 12% of the departments implemented a two-shift schedule (day and night), each lasting between 10 to 14 hours. However, the bulk of the report originated from departments using three rotating shifts or platoons. A 24-hour shift followed by a 48-hour rest period was reported by 30%, while an alternative 24-hour rotation was indicated by 23%. The latter typically follows an on-off-on-off-on schedule, followed by a four-day rest, referred to variably as 3/4, modified Detroit, or modified Berkeley, depending on specifics. More than 19 variations of the fundamental three-shift rotation patterns are used. Most departments provide an off day, known as a "Kelly" day, every eighth shift to prevent hours worked from being classified as overtime. Certain departments operate a smaller fourth platoon to cover for Kelly's days and leaves. The result is that the average work week for firefighters generally ranges from 48 to 56 hours, excluding any overtime. In recent years, a 48-hour work followed by a 96-hour rest schedule has gained popularity [9]. According to Near-Miss reports, this newer schedule is used by 3% of departments. Originating in Southern California, this format was adopted because local housing was unaffordable for many firefighters leading to long commutes. This schedule effectively halved their travel time. The 24-hour shift pattern is a long-established tradition among firefighters, receiving few complaints. However, those challenging this schedule for safety or economic reasons have faced strong backlash from

[2]http://www.firefighternearmiss.com

comment-submitting firefighters [10].

To optimize fire station staffing, various operations research approaches have been developed. A comprehensive literature review by Simpson and Hancock [11] examines operations research methodologies applied to emergency response facilities, including fire stations, police stations, and ambulance services, over the past 50 years. To determine appropriate staffing levels for a fire station, Lawrence [3] proposed an analytical approach that moves beyond a one-size-fits-all solution. Their methodology uses community-specific demands to calculate the number of employees required, providing a tailored approach to resource allocation. The relationship between crew size and fire losses was tested using four simulated fire and rescue experiments, and different staffing levels were compared using data collected from these experiments. The analysis recommends staffing fire squads with three firefighters, as performance improvements beyond this number are minimal. Tests showed a significant 20-second improvement between two and three firefighters, but negligible gains with additional personnel. As a result of a similar approach, Geason [12] compared four different crew sizes at several fire stations by conducting seven practical experiments simulating common fire station tasks. Geason showed a significant difference in efficiency and safety between alternative staffing levels and recommended that the number of firefighters at those fire stations be increased. Fry et al. [13] developed a model aimed at minimizing the total expected costs for a fire department while meeting minimum staffing requirements. The model accounts for employee unavailability and hiring constraints within firefighter schedules, drawing inspiration from the newsvendor inventory problem to address these challenges effectively.

An examination of Management Science and Operations Management applications in emergency and fire operations was conducted by Green and Kolesar [14]. A decision analysis was used by Dalby [15] to calculate the expected risk associated with certain staffing decisions. Scenarios were analyzed based on various staffing levels to provide the decision-maker with quantifiable estimates of the benefits, risks, and costs associated with the different staffing levels. An analysis of the total daily cost of fire suppression resources was conducted by Wei et al. [16] using a two-stage probabilistic model. This model considers several assumptions to estimate the appropriate daily cost of fire response operations and determine which resources should be allocated to these activities. Lolli et al. [17] developed a bi-objective model aimed at minimizing travel distance and response time when managing multiple micro-calamities simultaneously. Their work emphasizes a focus on humanitarian and risk-based objectives, distinguishing it from models primarily driven by cost considerations. The required level of firefighters for special events was determined using a linear programming model by Stevanović et al. [18].

To reduce the total cost of the daily allowances for multiple shifts, the objective of study will be to minimize the total cost of those allowances based on meeting the special event requirements of firefighters in each shift at the same time. There have been several optimization methods applied to optimizing fire station staffing, but none combines the number of fire trucks and the crew size per truck at the same time.

2.2 Fleet Sizing Strategies

Emergency fleets, comprising vehicles dispatched to incidents, are a cornerstone of emergency response departments because they are essential for delivering staff and equipment to the scene promptly, ensuring effective and timely responses to emergencies. The efficacy of this resource is intrinsically tied to the number of vehicles available at each station, as insufficient fleet capacity can delay response times and limit the ability to address multiple incidents simultaneously, thereby compromising the overall effectiveness of emergency operations. Fire department planners and operators face the complex task of ensuring an optimal fleet size, which involves balancing the need to meet expected demand with the costs of owning and maintaining these vehicles, while also accounting for the potential service shortfalls that could arise from an inadequate fleet. Given that the demand for emergency response services fluctuates considerably over the course of a day, maintaining 24/7 readiness often means having a surplus of idle vehicles, which necessitates a strategic redistribution of vehicles to areas where they are most needed. Consequently, the composition and positioning of the fleet at any given moment depend on the department's vehicle redistribution strategy [19]. Effective management of emergency response vehicle fleets is paramount, as it goes beyond ensuring vehicle availability and focuses on optimizing deployment strategies to guarantee swift response times, which are essential for efficiently and promptly meeting the community's needs and enhancing the overall effectiveness of emergency response services.

Optimizing fleet size is a complex and crucial process in transportation management, involving the determination of the most efficient number and types of vehicles needed to meet specific operational demands. This optimization requires a thorough analysis of various data sets. In the context of emergency services, such data includes historical records of incidents, routes taken to incident locations, and comprehensive response times — including dispatch, arrival, response, and return phases. By carefully evaluating this data, fire department planners and operators can make informed decisions about the size and composition of their fleet, often guided by insights from mathematical models developed by researchers and experts in the field.

The concept of fleet sizing transcends numerous transportation sectors, each contributing unique insights to the field. In passenger transport, Zuckerman and Tapiero [20] focused on identifying the optimal fleet size. Mole [21] introduced a dynamic model for vehicle fleet sizing, adaptable to changing demands and conditions. Turnquist and Jordan [22] specialized in the container transportation sector, formulating a deterministic model for fleet sizing. The logistics sector saw contributions from Beaujon and Turnquist [19], who aimed to ascertain the optimal number and strategic locations for freight carpools. In the broader context of the automobile and railroad industries, Sherali and Tuncbilek [23] provided a comparative analysis of static versus dynamic models for fleet sizing. Emphasizing contemporary concerns, Tomasz et al. [24] developed a model that integrates safety and ecological factors in fleet management decisions. Furthering this line of research, Pozueco et al. [25] conducted an in-depth study on the impact of fleet management on professional drivers' performance throughout their workday.

In the area of emergency services, fleet sizing has been a topic of significant interest and research. Notably, Yang et al. [26] introduced a mathematical programming model for effective vehicle routing and fleet management in fire departments, leveraging real-time traffic data to optimize fleet allocation across different stations. This model primarily focused on dynamic reallocation of resources in response to emergent needs. Another innovative approach was proposed by Huang and Fan [27], who used an integer programming model for the strategic assignment of vehicles to emergencies, factoring in budgetary and resource constraints. This model conducted an uncertainty analysis, considering variables like network congestion and emergency incident variability. These studies collectively enrich the understanding of fleet management in emergency services, providing critical frameworks and tools for optimizing response capabilities and resource allocation in times of crisis.

Some approaches, such as those by Kolesar and Walker [28], have focused on the dynamic management of emergency fleets to maintain service coverage and minimize response times during high-demand scenarios by reallocating resources based on real-time data, addressing the limitations of static allocation models in complex urban emergency conditions, while Swersey [29] suggested a model based on decision rules for determining how many resources to allocate to a given event. Andersson and Värbrand [30] developed a decision support tool for dynamic ambulance planning, which Gustafsson and Granberg [31] extended to fire and rescue services, creating a model to estimate resource requirements for different types of events. Pérez et al. [1] conducted an empirical study aimed at proposing a fleet assignment model for the Santiago Fire Department. The primary goal of the model is to maximize the number of incidents that can be successfully attended. Another relevant integer programming model was proposed by Schmid and Doerner [32], where the dynamics of coverage areas were analyzed to enable the evolution of fleet allocation over time to achieve a desired level of emergency coverage. In another work, Dzator and Dzator [33] explored the classical p-median problem for ambulance fleet redeployment, seeking to minimize the average distance between demand nodes and the nearest facility.

Considering all the mentioned factors, optimizing fleet size offers several benefits, such as increased efficiency, improved resource utilization, enhanced planning and decision-making capabilities, and greater environmental sustainability. This holistic approach not only streamlines operations but also contributes to more sustainable practices within the fleet management sector.

One of the primary advantages of optimizing fleet size is increased efficiency. By leveraging data to thoroughly understand transportation needs and demand patterns, companies can allocate their resources more efficiently, minimizing waste and inefficiencies. Such an approach enhances the quality of transportation services and results in cost savings [19]. Additionally, optimizing fleet size improves fleet utilization, reducing vehicle downtime and boosting productivity. Such improvements enhance public satisfaction by ensuring timely responses to incidents and minimizing the risk of delays. Furthermore, the need for fewer vehicles reduces costs associated with purchasing, maintenance, and fuel [19].

Data-driven decision-making is another significant benefit of fleet optimization. Fire departments can use historical data on demand and response times to make informed decisions about fleet size and composition, ensuring an optimal mix of fleet types to meet operational needs. This adaptability allows fire departments to respond swiftly and effectively to changing demands, enhancing overall efficiency [34, 35]. Optimizing fleet size can also reduce maintenance and repair costs by minimizing wear and tear on vehicles. With a streamlined fleet, optimized in size and composition, each vehicle is utilized more effectively, leading to reduced downtime, lower maintenance costs, and enhanced operational longevity, ultimately improving the efficiency and reliability of the fleet. Furthermore, fleet optimization offers significant environmental benefits by carefully managing the number of vehicles and their travel distances. This approach reduces fuel consumption, associated costs, and carbon dioxide emissions, thereby decreasing the overall carbon footprint [36].

2.3 Incident Response Breakdown

Fire incidents include a variety of incidents such as structural and vehicle fires, vehicle accidents, hazardous materials and natural gas incidents, and trench and water rescues. Each of these incidents has unique elements, and responding to them requires specialized procedures. Although there is a general outline of various stages of an emergency response to a fire incident, from receiving the initial emergency call to the eventual return to the fire station post-incident, specific protocols must be followed depending on the type of incident.

The process begins with an emergency call reporting a fire incident, usually placed to a designated number (e.g., 911 in the U.S. and Canada). The call is received by a dispatcher who collects critical information such as the location and nature of the fire. The dispatcher then alerts the nearest fire department and relays the necessary details, initiating the response. The alerted fire department prepares for action, with firefighters putting on their protective clothing and equipment before boarding the fire engine and heading to the location. While en route, the crew receives additional information about the incident, allowing them to formulate an initial action plan.

Upon arrival, the crew leader or Incident Commander (IC) conducts an immediate size-up to assess the scene and gather precise information. Based on this assessment, the IC communicates the Incident Action Plan to the team. Firefighting operations then proceed according to the plan, involving simultaneous tasks such as establishing a water supply, setting up equipment, performing rescues, and suppressing the fire. Active firefighting includes applying water or foam to the fire, either by entering the building or attacking from the exterior, depending on the fire's severity and spread. Once the fire is controlled and extinguished, the overhaul phase begins.

Firefighters check for hidden fires using thermal imaging cameras, remove burned debris, and take measures to minimize water and smoke damage.

After the fire is extinguished, a fire investigator may examine the scene to determine the cause of the incident. This process involves inspecting the site, interviewing witnesses, and collecting evidence. Once the scene is deemed safe and no further services are required, firefighters pack up their equipment and return to the station, ready to respond to the next call [37].

This sequence of events provides a general framework for fire response, but variations often arise depending on the specific nature of the incident. For example, scenarios involving hazardous materials, large structure fires, or wildfires may require specialized response elements and tactics, significantly influencing the resources needed and the strategies employed. Further studies into these specific incident types could enhance the development of more efficient and effective response strategies, ultimately improving outcomes and resource utilization.

The complexity and diversity of factors influencing fire department staffing result in regulations that vary significantly between countries. As highlighted in the ”Staff Allocation Strategies” section, the required number of crew members per truck may differ depending on the type of truck and whether the region is urban or rural, sometimes exceeding or falling below four. Furthermore, given the operational procedures and response protocols of fire departments, this research focuses on fire trucks and fleet reallocation, which directly impacts staffing dynamics. Fleet reallocation inherently requires the reassignment of staff to accommodate the newly organized units, affecting both personnel availability and operational efficiency.

2.4 Pre-emptive and Reactive Resource Allocation

Resource allocation in various domains can be broadly categorized into preemptive and reactive strategies. Preemptive resource allocation involves allocating resources before they are actually needed, based on predictions or proactive measures [38]. In contrast, reactive resource allocation involves responding to resource needs as they arise, often in real-time or based on immediate requirements [39]. The distinction between these two approaches lies in the timing and decision-making process involved in assigning resources.

Preemptive resource allocation strategies are characterized by their proactive nature, where resources are allocated in anticipation of future demands or events [40]. This approach is often based on forecasts, historical data, or predictive models to optimize resource utilization and performance [41].

Reactive resource allocation involves allocating resources in response to immediate needs or changing conditions [42]. This approach is more adaptive and flexible, allowing for adjustments based on real-time requirements or unexpected events [41]. Reactive resource allocation is often used in dynamic environments where demand fluctuates or where precise predictions are challenging [39].

The choice between preemptive and reactive resource allocation strategies depends on various factors such as the predictability of resource demands, the cost-effectiveness of proactive measures, and the flexibility required to adapt to changing conditions [43]. Preemptive strategies are beneficial when future resource needs can be accurately forecasted, leading to optimized resource utilization and potential cost savings [44]. On the other hand, reactive strategies are more suitable in dynamic environments where resource demands are uncertain or when real-time adjustments are necessary to meet immediate requirements [45].

In many contexts, preemptive resource allocation helps optimize energy consumption, performance prediction, and load sharing by allocating resources in advance based on workload forecasts [46]. In contrast, reactive resource allocation focuses on dynamically adjusting resources in response to real-time changes in workloads, user demands, or performance metrics [47]. This approach is particularly relevant in scenarios where resource demands are unpredictable or where rapid adjustments are critical to maintaining service levels.

Reactive resource allocation plays a vital role in environments like emergency response, where resource needs can shift dramatically due to the nature of incidents and their locations. By enabling dynamic reassignment of resources, such as personnel or equipment, reactive strategies ensure a more agile response to fluctuating demands, reducing response times and optimizing resource utilization. Understanding the interplay between preemptive and reactive approaches is essential for developing systems that balance the cost-effectiveness of proactive measures with the adaptability required for real-time challenges, particularly in dynamic and high-stakes scenarios like fire and emergency services.

3 Chapter Three: Methodology

3.1 Model Description

Consider a set of facilities defined by the set I. Without loss of generality, we assume there is no limit to sharing resources between two stations. Let x_{ij} be a binary decision variable, which is one if facility $i \in I$ and $j \in I$ are permitted to share resources. Let $t \in T$ be the index of the set of time periods T.

Let r_i^t be a continuous decision variable representing the level of resources allocated to facility i at time t. For example, r_1^2 represents the level of resources allocated to facility 1 during the second time period. As previously discussed, we consider resources as a combination of staff and fleet, collectively referred to as a company. Based on the literature review and the definition of r_i^t, we infer that r_i^t initially represents preemptively allocated resources in the model, denoted as r_i^0. Let v_{ij}^t represent the number of resources relocated from facility i to be available at facility j at time $t+1$. Similar to r_i^0, v_{ij}^t reflects reactive resource allocation. Thus, we establish:

$$r_i^{t+1} = r_i^t + \sum_j (v_{ji}^t - v_{ij}^t) \quad \forall i, \forall t \in T, \tag{1}$$

which ensures resource balance during reallocations, maintaining that the resources at the next time step (r_i^{t+1}) equal those at the previous step (r_i^t), minus resources transferred to other facilities ($\sum_j v_{ij}^t$), plus those received from other facilities ($\sum_j v_{ji}^t$).

Let q_{ik}^t represent the resources deployed from facility i to incidents in the region $k \in K$ at time t. The distinction between q_{ik}^t and v_{ij}^t lies in their timing and purpose: v_{ij}^t occurs between time steps as resource reallocation between facilities, making it a preemptive measure to optimize future availability. In contrast, q_{ik}^t occurs during time step t as the deployment of resources to an incident, representing a reactive response to immediate demands.

Fire departments strategically locate fire stations based on factors like response time, population density, geographic barriers, risk assessments, and existing infrastructure. Using Geographic Information Systems (GIS), planners may decide not to place fire stations in certain zones due to low population density, lower risk levels, or budget constraints. In these cases, resources are strategically allocated from neighbouring zones to ensure coverage. Since not all regions may contain a facility, it follows that $I \leq K$. Taking these variables into account, there are specific restrictions on the values of both q_{ik}^t and v_{ij}^t as follows:

$$\sum_k q_{ik}^t \leq r_i^t \quad \forall i, \forall t \in T \tag{2}$$

$$\sum_j v_{ij}^t \leq r_i^t \quad \forall i, \forall t \in T \tag{3}$$

In equation 2, we ensure that the total allocation of deployed resources to either the same region or neighbouring regions does not exceed the total available resources. Similarly, in equation 3, we ensure that the total reallocated resources to other regions do not exceed the total available resources. As previously mentioned, equation 1 balances the available resources at every station for the subsequent consecutive time steps.

In certain scenarios, practical factors such as geographic remoteness, strategic priorities, or budgetary limitations may lead planners to impose restrictions on resource reallocations between specific regions. These constraints are designed to enhance system efficiency while addressing logistical challenges. Long-distance reallocations can introduce inefficiencies, such as increased travel time, resource unavailability at the origin facility, and delays in response. Additionally, frequent or excessive reallocations may strain personnel and equipment, ultimately reducing overall effectiveness. To prevent undesirable reallocations, we ensure that v_{ij}^t equals zero whenever $x_{ij} = 0$. To enforce this constraint, we introduce a large constant N, resulting in the following condition:

$$v_{ij}^t \leq x_{ij} \times N \quad \forall i, j, \forall t \in T \tag{4}$$

This constraint effectively enforces v_{ij}^t to be zero whenever $x_{ij} = 0$, contributing to the precision of the resource allocation model.

Let d_{ij} be a continuous parameter representing the distance between facility i and j. Without loss of generality, we assume a threshold ε as the maximum acceptable distance for the reallocation of resources between facility i and other permitted facilities. Therefore, we have:

$$\sum_j x_{ij} d_{ij} \leq \varepsilon \quad \forall i \tag{5}$$

Moreover, let D be a continuous random variable representing the total demand observed at a facility, typically measured in terms of the number of incidents or resource requests within a given time period. Therefore, the probability distribution function of demand at facility i at time t is $F_i^t(D)$. Similar to d_{ij}, d_{ik} is a continuous parameter that represents the distance, which in this case is not between two facilities, but between facility i and the center of incidents in region k. We use the Poisson distribution because it models discrete events that occur randomly and independently of one another.

A key component of the model is the maximum satisfied demand, expressed as:

$$\tau(\sum_{i} q_{ik}^{t} \times \exp(-\alpha d_{ik})), \tag{6}$$

here, τ is a scaling factor that adjusts the satisfaction level, q_{ik}^{t} represents the resources deployed from facility i to incidents in region k at time t, and $\exp(-\alpha d_{ik})$ accounts for the effect of distance on resource effectiveness. The impedance factor, α, determines the extent to which distance influences demand satisfaction. A higher α value indicates greater sensitivity to distance, meaning that resources farther from an incident are significantly less effective. Conversely, a lower α implies that distance has a smaller impact, allowing resources from farther locations to still be useful.

To ensure a reliable level of service, we define μ as the threshold for unsatisfied demand. Essentially, the maximum allowable probability that demand exceeds the allocated resources. This constraint ensures that the likelihood of unmet demand remains within acceptable limits. Formally, the cumulative distribution function (CDF) must satisfy:

$$\int_{\tau(\sum_{k} q_{ki}^{t} \times \exp(-\alpha d_{ik}))}^{\infty} F_{i}^{t}(D) \cdot \mathrm{d}D \leq \mu \quad \forall i, \forall t \in T \tag{7}$$

This equation guarantees that the probability of unmet demand at facility i during time t does not exceed the predefined threshold μ. The integral of the CDF, $F_{i}^{t}(D)$, ensures that the system maintains a specified level of reliability in meeting demand. By incorporating distance-based efficiency, resource availability, and probabilistic constraints, this formulation strikes a balance between effective resource allocation and operational limitations, ensuring that service levels remain consistent across different scenarios.

3.2 Model Schematic

Figure 1 visually depicts the distribution of resources between two stations over consecutive time steps, illustrating the quantities of resources transferred, denoted by v_{ij}^t and q_{ji}^t, for intervals where $I = K$. It is not logical for a station to act as both a sender and receiver of resources simultaneously.

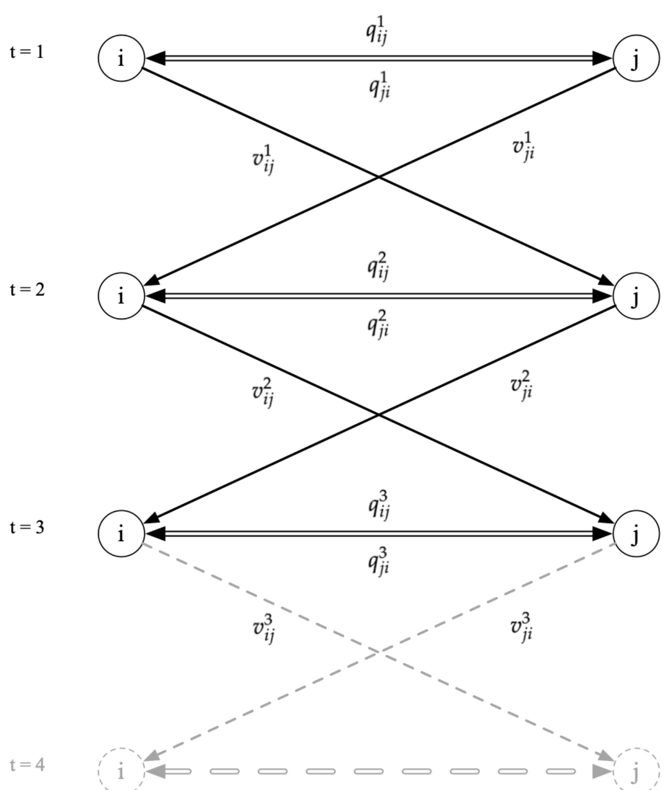


Figure 1: Resource distribution depiction during reallocation and emergency situations

As shown in the figure, as long as the resource balance is maintained according to equations 1 and 2, the number of time steps can be increased, allowing resources to be deployed or relocated to other facilities. To illustrate this continuity, $t = 4$ is blurred in the figure, indicating that the process can extend beyond the depicted time steps.

3.3 Objective Function: Minimizing the Cost

Fixed resources in each station incur costs, and reallocating resources adds additional expenses. The model aims to minimize the combined cost of these two factors while identifying the peak and optimum values for each. Consequently, the objective function accounts for both the acquisition cost of resources at each station and the transportation cost incurred when reallocating resources between stations. Specifically, the first term, $\sum_t \sum_{i,j} v_{ij}^t d_{ij}$, represents the total cost of reallocation, where v_{ij}^t denotes the quantity of resources moved between stations i and j at time t, and d_{ij} represents the distance-dependent cost of transportation. The second term, $\sum_i r_i^0 \beta_i$, captures the initial acquisition cost of resources at each station, where r_i^0 is the fixed number of resources at station i, and β_i is the per-unit acquisition cost. Together, these components define the total cost function, which the model seeks to minimize:

$$\min\left(\sum_t \sum_{i,j} v_{ij}^t d_{ij} + \sum_i r_i^0 \beta_i\right), \tag{8}$$

where β_i is the cost of maintaining resources at station i, calculated as a coefficient of the initial resources r_i^0. To clarify, β specifically refers to the cost associated with the initial allocation or provisioning of resources at each station, reflecting the inherent setup costs. Additionally, the model's objective function seeks to minimize another type of cost, which accumulates across all stations. To ensure clarity and avoid confusion, we distinguish this cumulative cost by referring to it as an *expense*. This distinction allows for unambiguous discussion of costs and outcomes throughout the analysis.

4 Chapter Four: Simulation Results & Discussion

4.1 Static Model

In the status quo static model of resource allocation, resources are permanently assigned to specific stations and dispatched from and returned to those locations. In this approach, resource reallocation during a given time period is not permitted. While this approach has limitations, it is particularly useful for analyzing maximum potential demand and determining the resources needed to address that demand effectively.

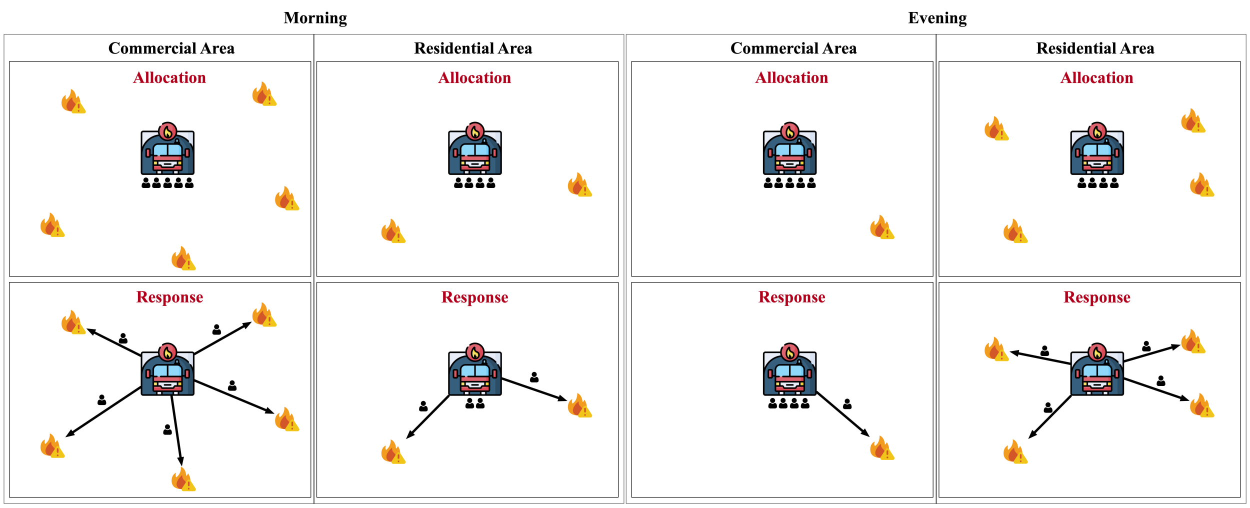


Figure 2: Schematic of static model

Figure 2 illustrates the schematic of a static model, showing resource allocation and response scenarios during two distinct time periods: morning and evening. In the morning, the commercial area experiences higher activity and a greater likelihood of incidents, while in the evening, the residential area becomes busier with increased incident probabilities. The static allocation model assigns resources to meet the maximum anticipated demand, preparing for the worst-case scenario within a day. For example, in the morning, the commercial area requires five groups of resources, whereas in the evening, only one group is needed. However, to ensure preparedness for peak demand, the allocation remains at five groups even during periods of reduced activity.

In static allocation, it is also possible to request resources from other stations in case of emergencies. Fire departments often use this approach to respond to fires, where resources such as firefighters, fire engines, and equipment can be quickly mobilized from neighbouring stations to respond to a fire.

With these assumptions, we present a simple static model involving two regions, each with one station ($I = J = \{1,2\}$). The stations are permitted to share resources, meaning $x_{12} = 1$. In this static scenario, $r_1^t = r_1^0$ and $r_2^t = r_2^0$, as resources remain fixed over time. The objective is to explore the optimal resource allocation based on varying costs and demands. For simplicity, the model assumes that d_{ij} and d_{ji} are identical, reflecting symmetric travel between stations.

According to equation (1), since there are no relocated resources $v_{12}^t = v_{21}^t = 0$, it follows that $r_1^t = r_1^{t-1} = r_1^0$ and $r_2^t = r_2^{t-1} = r_2^0$. Consequently, equations (2), (3), and (4) are always satisfied. Additionally, with only two stations and the assumption of resource exchange $x_{ij} = 1$, the d_{ij} value adheres to the imposed restrictions and does not exceed the established upper bound, ensuring that equation (5) is satisfied.

We define the maximum satisfied demand for each station as $\tau\left(\sum_k q_{ki}^t \times \exp(-\alpha d_{ik})\right)$. From equation (7), it follows that the cumulative distribution function (CDF) of the maximum satisfied demand must satisfy the condition $\tau\left(\sum_k q_{ki}^t \times \exp(-\alpha d_{ik})\right) \geq CDF^{-1}(\mu)$, where $CDF^{-1}(\mu)$ represents the inverse cumulative distribution function (also known as the quantile function). This implies that the maximum satisfied demand must meet or exceed the threshold value determined by the given probability level μ.

The use of $CDF^{-1}(\mu)$ is critical for establishing a threshold value for the maximum satisfied demand based on a predefined confidence level. The quantile function provides the value below which a specific percentage of the data falls. By ensuring that the maximum satisfied demand satisfies the inequality $\tau\left(\sum_k q_{ki}^t \times \exp(-\alpha d_{ik})\right) \geq CDF^{-1}(\mu)$, we guarantee that the demand meets a minimum probability requirement, making this approach particularly valuable in probabilistic and statistical models where setting bounds based on likelihoods is essential.

For the specific example at hand, the maximum satisfied demand for Station 1 is given by $\tau\left(q_{21}^0 \exp(-\alpha d_{12}) + q_{12}^0 \exp(-\alpha d_{21})\right)$. Furthermore, the objective function, as given by equation (8), is to minimize $r_1^0\beta_1 + r_2^0\beta_2$.

Table 1 displays the results yielded by the static model under an array of scenarios. In conducting these experiments, we assigned a value of 0.1 to α, and 0.05 to μ for the purpose of calculating CDF^{-1}, given the context of an exponential distribution. The calculated distances amount to 1.065. we have chosen very high and low values for costs and demands. This is to make sure our model works well, even under extreme conditions.

$\alpha = 0.1, \mu = 0.05$		Scenario 1		Scenario 2		Scenario 3	
		Region 1	Region 2	Region 1	Region 2	Region 1	Region 2
Inputs	Demand	100	0	100	0	100	100
	Cost	10	1	1	10	10	1
Outputs	r_i	0	333	300	0	0	633
	q_{ii}	0	0	300	0	0	300
	q_{ij}	0	333	0	0	0	333

Table 1: Results of static modeling in different scenarios

In Scenario 1, we observe that if there is high demand in one region, but the cost of fixed resources in that same region is higher, the model places resources in the other region and transports them to the first one. Conversely, in Scenario 2, Region 2 has a higher cost, so it is logical to place resources in Region 1 and use them within the same region. Finally, in the last scenario, both regions have the same demand, but the cost is lower in Region 1. As a result, the resources are allocated in Region 1 and then distributed to both regions. In the context of the second scenario, it is essential to note that the spatial distribution of resources remains confined within the same region. As a result, alterations in distance have no impact on the ultimate outcomes or the value of these resources. However, in the first scenario, changes in the distance parameter significantly affect the resultant values. For instance, when the distance is adjusted to 0.5, the variables r_2 and q_{21} change to 315. This shift demonstrates how variations in distance directly influence resource allocation and operational outcomes within the model.

4.2 Dynamic Model

Contrasting the static model's fixed approach, the dynamic model facilitates the redistribution of resources across multiple time intervals. This methodology accounts for the resources available at the beginning of each interval and incorporates their expected availability in subsequent periods.

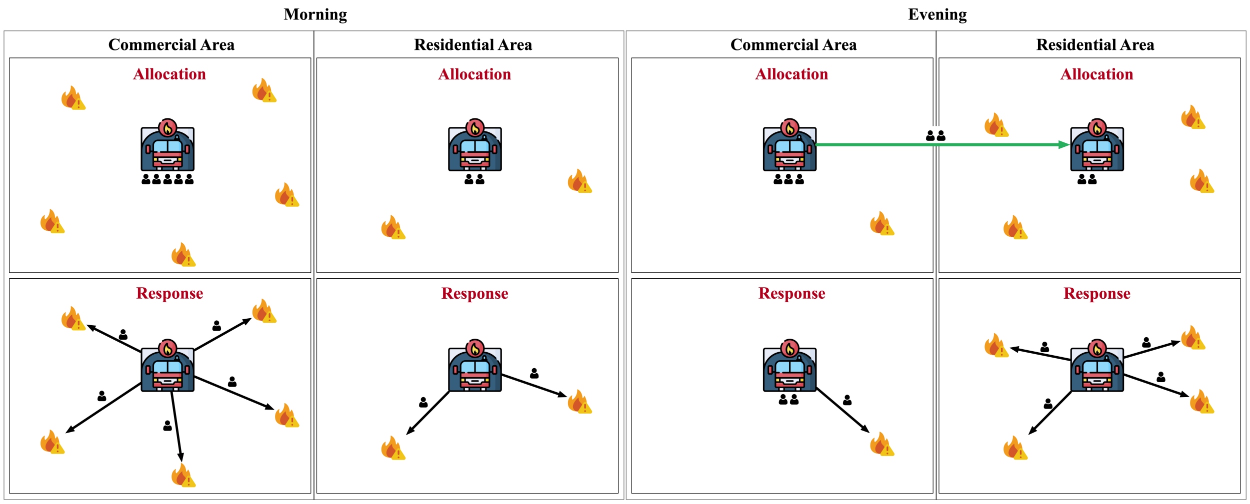


Figure 3: Schematic of dynamic model

Figure 3 shows the schematic of a dynamic model under the same incident scenarios. Here, the green arrow indicates the reallocation of resources between morning and evening to meet the demand specific to those time periods. A comparison of the two figures reveals a reduction in the number of resource groups in the dynamic model.

As shown earlier in Figure 1, a dynamic allocation manifests connections denoted by q_{ij}^t and v_{ij}^t between stations. With this in mind, the objective function works as a whole, with its initial segment playing a crucial role in minimizing costs. For illustrative purposes, we incorporated all constraints in the modeling phase and resolved the scenario over three sequential periods for two stations. Analogous to the preceding example, we allocated values of 0.1 to α and 0.05 to μ for computing CDF^{-1} in equation (7) as we explained in the last section within the framework of an exponential distribution.

The demand and cost values for each station were generated randomly, as presented in Table 2, along with the results of the example.

$\alpha = 0.1, \mu = 0.05$		$t = 1$		$t = 2$		$t = 3$	
		Region 1	Region 2	Region 1	Region 2	Region 1	Region 2
Inputs	Demand	0.618	0.281	0.925	0.056	0.941	0.863
	Cost	0.064	0.773	0.064	0.773	0.064	0.773
Outputs	r_i^t	5.404	0.000	2.819	2.585	2.819	2.585
	v_{ij}^t	2.585	0.000	0.000	0.000	-	-
	q_{ii}^t	1.851	0.000	2.819	0.168	2.819	2.585
	q_{ik}^t	3.553	0.000	2.418	0.000	0.000	0.000

Table 2: Results of dynamic modeling in three time steps

The results suggest that, between time periods and with alterations in the cost and demands of each region during these periods, we observe a variation, v_{ij}^t, modifying the resources for the subsequent time step; nonetheless, the total resource remains unchanged. In this instance, it equates to the sum of r_1^1, since $r_2^1 = 0$.

Further analysis reveals that Region 1's demand steadily increases from 0.618 at $t = 1$ to 0.941 at $t = 3$, with a stable cost of 0.064. In contrast, Region 2's demand fluctuates, dropping to 0.056 at $t = 2$ before rising to 0.863 at $t = 3$, with a constant cost of 0.773. This shows that Region 1 has a growing demand, while Region 2's demand is more variable.

Resource allocation changes over time: Region 1 starts with 5.404 units at $t = 1$, decreases to 2.819 units by $t = 2$, and stays constant, while Region 2, initially without resources, receives 2.585 units at $t = 2$ and maintains that level.

Internal output, denoted as q_{ii}^t, for Region 1 increases from 1.851 at $t = 1$ to 2.819 at $t = 2$, staying stable, while Region 2's internal output rises from 0 at $t = 1$ to 2.585 at $t = 3$. Region 1's external output, denoted as q_{ik}^{t}, drops from 3.553 at $t = 1$ to 0 by $t = 3$, while Region 2 has no external output throughout.

In the following section, we will explore the differences between the models described and examine the impact of various parameters on total expenses.

4.3 Comparing Static and Dynamic Allocation

In this section, we examine the nuanced differences between static and dynamic models. While both models address similar challenges, their behavior varies across scenarios. We compared the two systems by calculating the associated expenses for each station, highlighting their comparative efficiencies and potential advantages.

The mathematical model was implemented using Python and solved using Gurobi Optimizer [48]. The complete code, including data preprocessing and model constraints, is provided in Appendices A, B and C for reference and reproducibility.

One of the key factors examined in relation to expenses was the physical distance between neighbouring stations. Analyzing the impact of distance on expenses provided valuable insights for improving resource allocation and logistical planning.

We assess the impact of the impedance factor associated with distance, denoted by α, on several key variables: total expenses, total preemptive resources, total reallocated resources, and total deployed resources to incidents over all regions. To isolate the effects of α, we maintained constant values for the other parameters, setting μ at a fixed value of 0.05, without imposing an upper bound on distance. This control allowed for a systematic variation of α from 0 to 2, thereby enabling us to meticulously track and quantify the influence of distance impedance on the system's financial outlay and resource allocation. Through this process, we aimed to draw correlations between the impedance factor and the operational efficiency of the system under study. Given that the demand and costs were randomly generated, we subjected the model to a range of scenarios and iteratively ran the solver to observe the variability in outcomes. This approach enabled us to assess the robustness of the model and to understand the range of possible results under stochastic conditions. To summarize the scenarios and avoid confusion, Table 3 presents the scenarios used in this section:

Scenario	Number of Regions	Number of Stations per Region	Number of Time Steps
Scenario 1	3	1	2
Scenario 2	8	1	2
Scenario 3	8	1	4

Table 3: Assessed scenarios

The subsequent results pertain to scenarios involving three regions with one station in each region in two time steps.

In certain scenarios featuring randomly generated demand and expenses, we have observed a consistent outcome: the total reallocated resources remain at zero, as illustrated in Figure 4d.

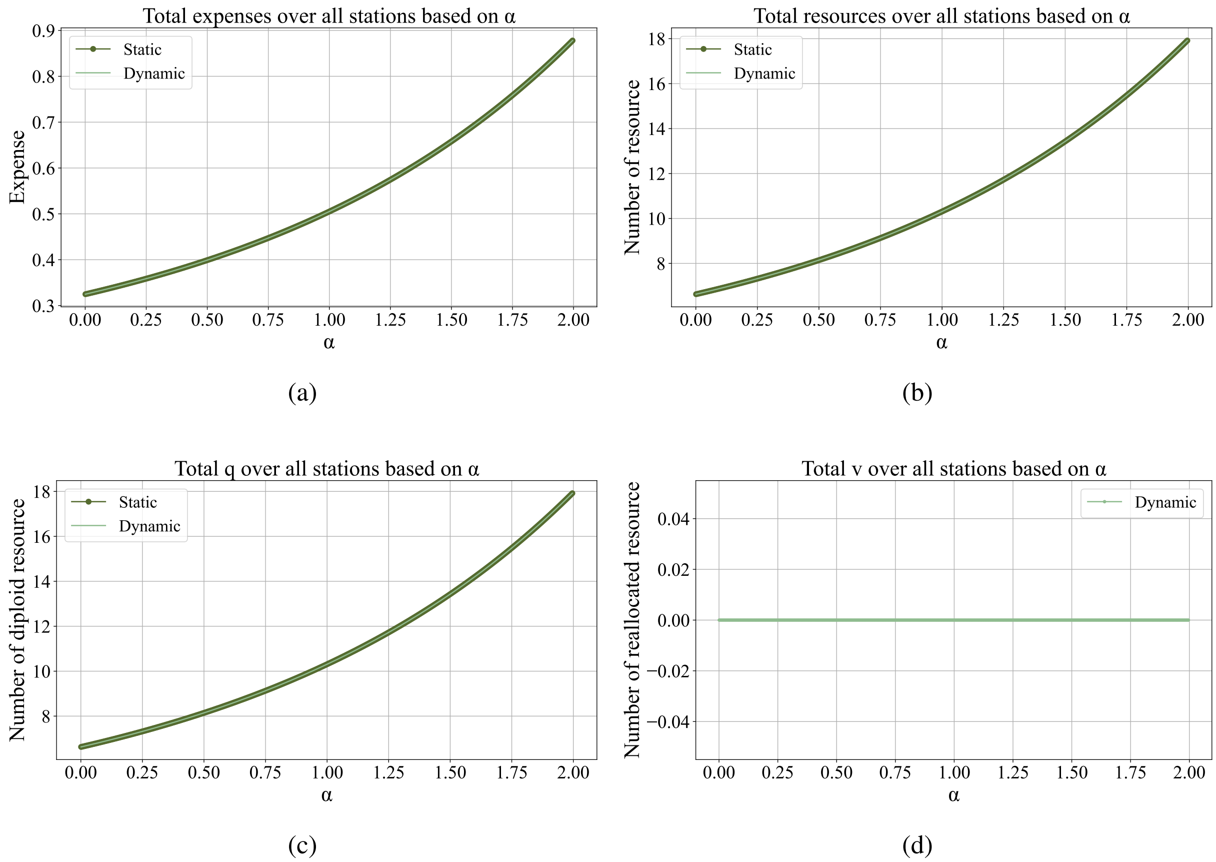
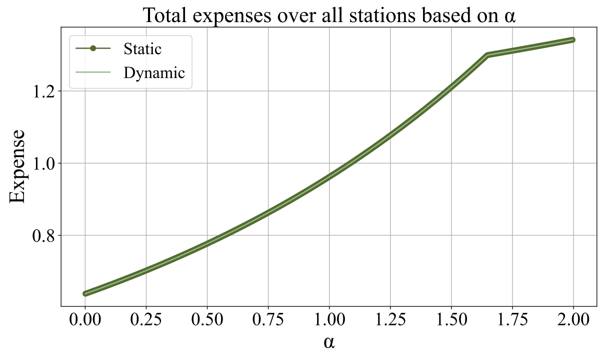


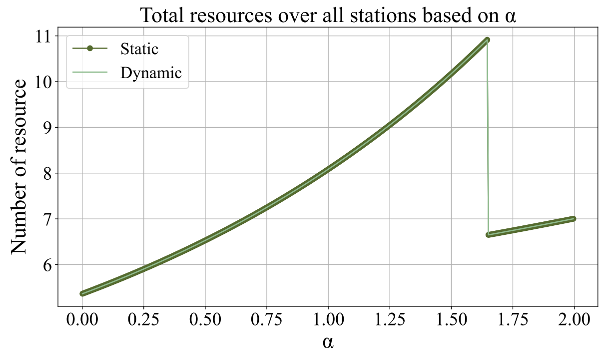
Figure 4: Impact of α in scenario 1; part 1
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

Upon closer examination of Equation 8, it becomes clear that when no reallocated resources are required, the total expenses for both models are identical, as the dynamic term reduces to zero. This hypothesis is validated by Figure 4a, where both expense lines closely overlap in the graph, affirming the accuracy of the solver's solutions. This pattern repeats in Figure 4b and Figure 4c, with the lines consistently overlapping.

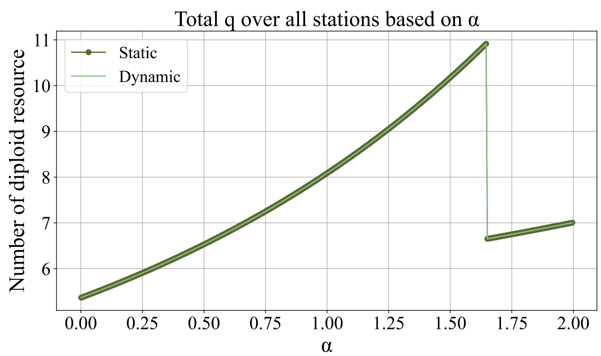
Additionally, these graphs demonstrate that alterations in the α value result in corresponding changes in the total resources and, consequently, the total deployed resources for incidents. These changes exhibit the same pattern and range as anticipated, confirming our expectations. This relationship consistently applies within the context of the specific demand and cost scenario being analyzed.



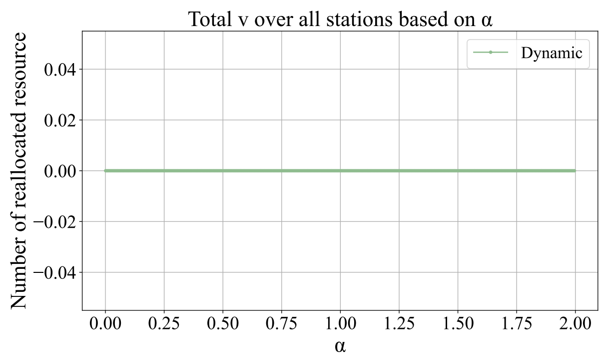
(a)



(b)



(c)



(d)

Figure 5: Impact of α in scenario 1; part 2
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In the subsequent phase of the scenario, although the reallocated value remains consistently at zero, as shown in Figure 5d, the recurring overlay pattern observed in the previous section persists. However, a significant trend emerges as the α value increases: both the total resources and total deployed resources exhibit a clear downward shift, as illustrated in Figures 5b and 5c. This shift implies a reduction in the available resources and, consequently, the allocation of deployed resources for incidents. While this shift moderates the slope of increase, it does not completely stop the overall upward trend in expenses, as shown in Figure 5a. What is particularly intriguing is that, even with the observed shift and reduction in resources, the trend remains upward as the α values increase in the aforementioned figures.

The rationale behind this trend is that an increase in the α value diminishes the financial benefit of deploying resources from a lower-cost region to a higher-cost one. Consequently, allocating a smaller number of resources in a more expensive region beyond a certain α threshold results in a reduced total resource count. This, in turn, slows the rate of increase in expenses, as fewer resources are required in higher-cost areas.

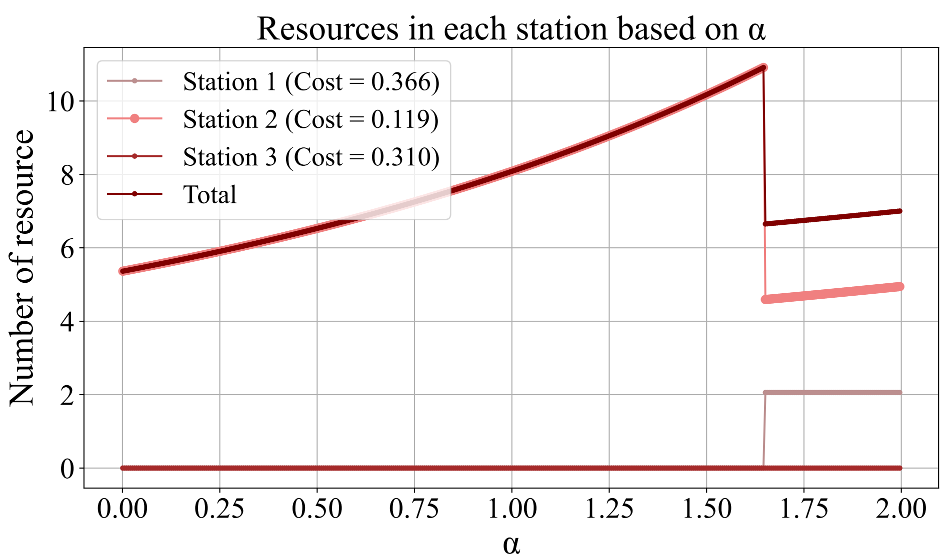


Figure 6: Resource allocation in each region

This trend is depicted in Figure 6, which details the changes that occur with an increase in the α value and the corresponding cost at each station. Furthermore, this suggests a complex relationship between the α values and the allocation of resources, indicating that while higher α values may lead to resource reduction, they do not result in expense reduction. This phenomenon underscores the importance of the impedance factor in both models.

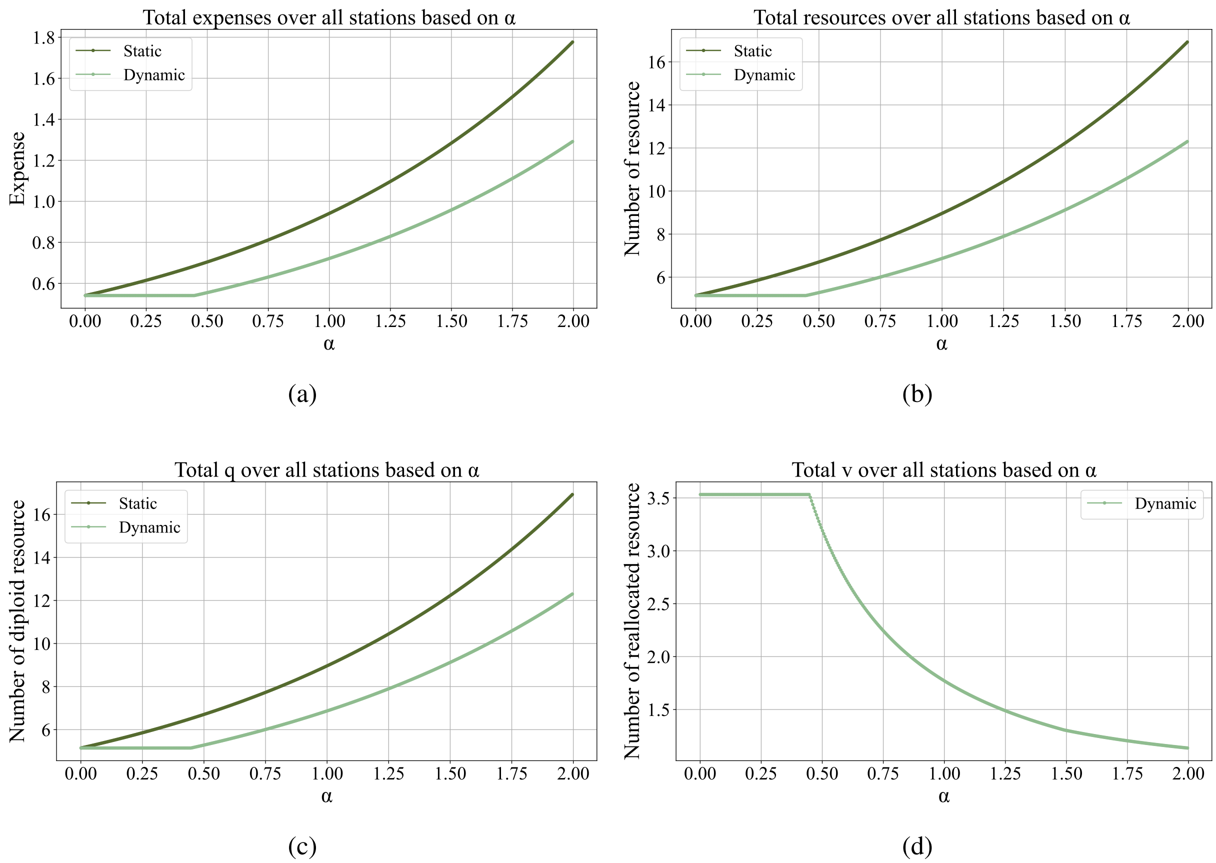


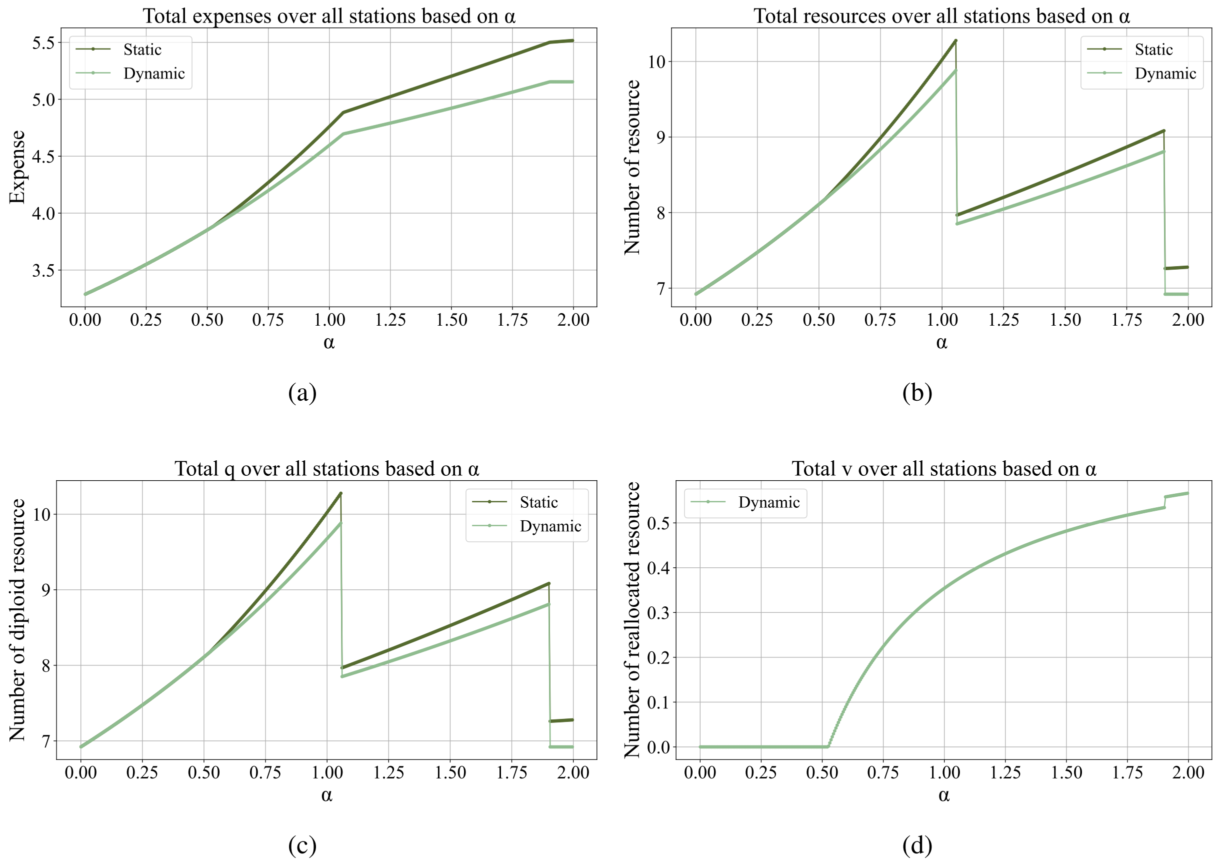
Figure 7: Impact of α in scenario 1; part 3
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In another facet of our scenario analysis, a noteworthy pattern emerges. Initially, at lower α values, the number of reallocated resources (v) exhibits a non-zero value, as illustrated in Figure 8d. However, as we progressively increase the α value, an interesting trend emerges: v steadily decreases. This phenomenon suggests that the α parameter plays a crucial role in determining the magnitude of reallocation, influencing the value of v in the process.

Examining Figures 8b and 8c, we observe that when the α value does not cause a decrease in v, the total resources and total deployed resources in the dynamic model remain relatively constant.

This stability is reflected in the consistency of expenses, as demonstrated in Figure 8a. However, as v decreases, we observe an increase in the number of resources (r) and the number of deployed resources (q), subsequently resulting in higher expenses for the dynamic model. Notably, the slopes of these changes in r, q, and expenses in the dynamic model closely align with those of the static model.

These findings shed light on the nuanced dynamics between the impedance factor (α), reallocation, and resource allocation within the context of our optimization model. Remarkably, a lower impedance factor can sometimes trigger reallocation, rendering the dynamic model more efficient, particularly at lower α values.



(a) (b) (c) (d)

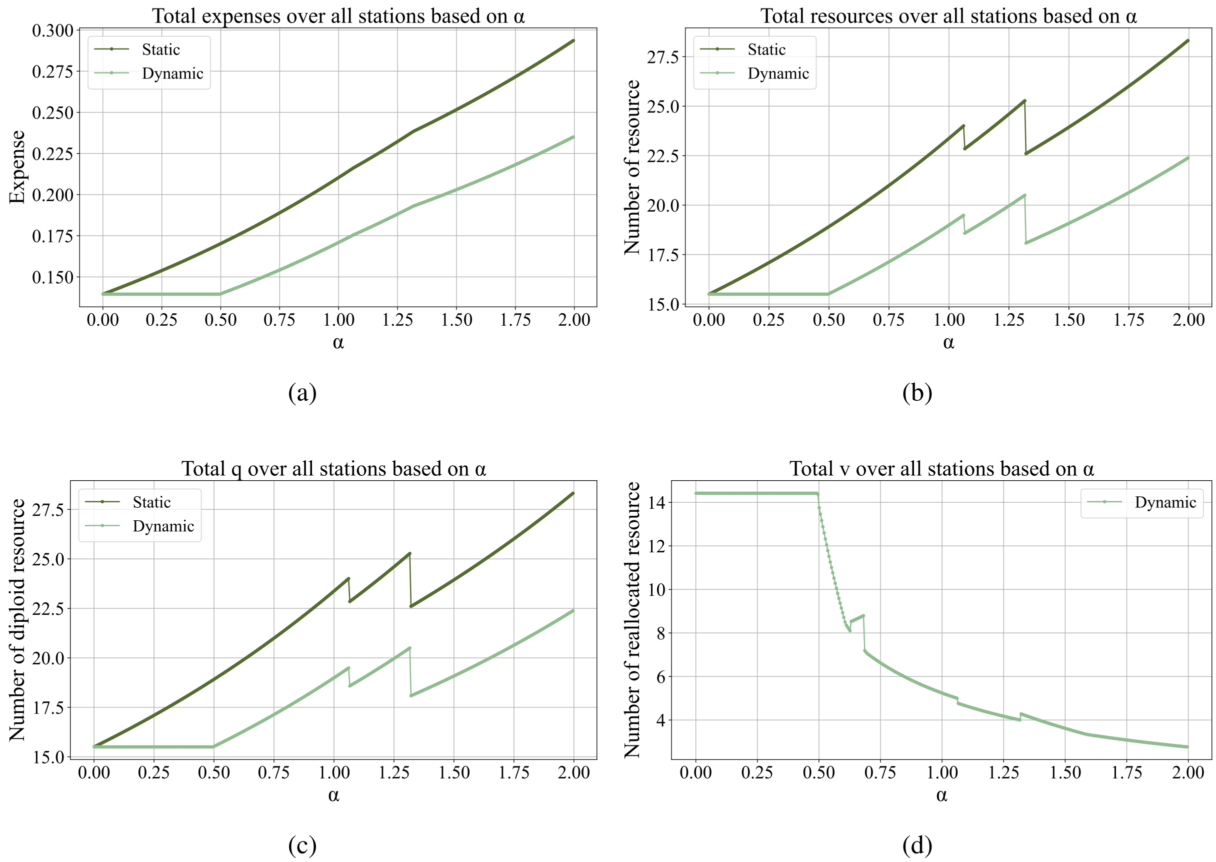
Figure 8: Impact of α in scenario 1; part 4
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In the last part of the scenario, we continue to observe a consistent trend when the reallocated value remains at zero, mirroring the patterns seen in the previous parts. However, a significant shift occurs as we increase the α value, leading to the emergence of a distinctive pattern. At the point when v obtains a non-zero value, as depicted in Figure 8d, a noticeable separation in the lines becomes evident. Specifically, the dynamic model exhibits a lower slope compared to the static model, as illustrated in Figure 8a.

These changes persist when further variations in the values of the reallocated resources are introduced. In Figures 8b and 8c, we continue to observe a similar pattern. In addition to the fractions discussed in the second part, it becomes evident that the presence of a non-zero v value results in discrepancies between the total resources and total deployed resources in the two models. Moreover, the reallocation of resources leads to a decrease in the overall number of resources available.

This pattern highlights the nuanced relationship between the α value, reallocated resources, and the allocation of resources in both the static and dynamic models. It emphasizes that certain critical thresholds or conditions can trigger variations in resource allocation dynamics, ultimately impacting expenses and resource availability in the optimization process.

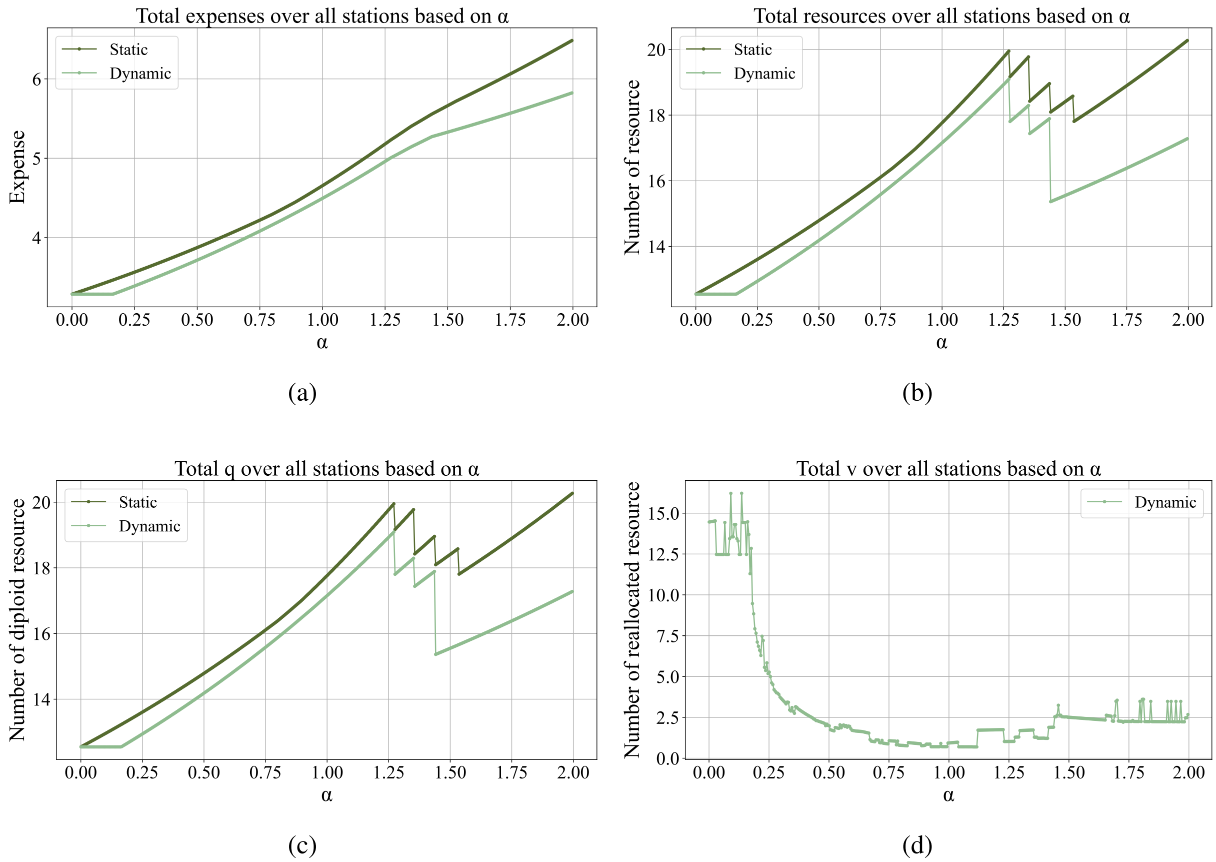
The implementation of traffic signal preemption can substantially improve the response times of emergency services. Allowing emergency vehicles to navigate through traffic with fewer delays, reduces the time it takes for them to reach the scene of an incident. This system grants emergency vehicles the right of way at intersections, momentarily interrupting the regular traffic flow.



(a) (b) (c) (d)

Figure 9: Impact of α in scenario 2
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In the subsequent scenarios, the number of regions with a station is increased to eight, while the number of time steps remains unchanged. Although similar trends to those observed in previous scenarios are evident, there is a noticeable increase in the number of fractions and deviations from a consistent pattern. Furthermore, the expense slope as shown in figure 9a remains positively correlated with an increase in α, thereby affirming the validity of our solving method even in more complex situations.



(a) (b) (c) (d)

Figure 10: Impact of α in scenario 3
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In the latter part of this scenario, while the number of regions is maintained at eight, we increased the time steps to four. This adjustment resulted in a significantly higher number of fractions, particularly in the number of reallocated resources, as depicted in Figure 10d. Notably, it can be observed that for α values below 0.75, the rate of change is more rapid compared to the rate after 0.75. Additionally, Figures 10b and 10c exhibit more pronounced fluctuations than in any other test scenario we have conducted.

4.4 Variables Impact

As outlined earlier in our discussions, our analytical model is predicated on three primary variables: α, μ, and ε. Each of these parameters plays a distinctive role in influencing the outcomes of our investigations. We investigated the impact of α in the previous section to explore the effect of changing the impedance factor on the expenses and resources. In the following sections, we will investigate the effects of the remaining parameters using the same scenarios presented in Table 3 and analyze how variations in their values may influence the results.

4.4.1 μ (Unsatisfied Demand Threshold)

In Equation 7, μ is defined as the threshold for unsatisfied demand, leading to the expectation that an increase in this parameter will result in lower expenses and fewer resources utilized, as the model is conditioned to respond to a reduced number of incidents under identical scenarios. For the analyses in this section, we maintain $\alpha = 1$, without imposing an upper bound on distance.

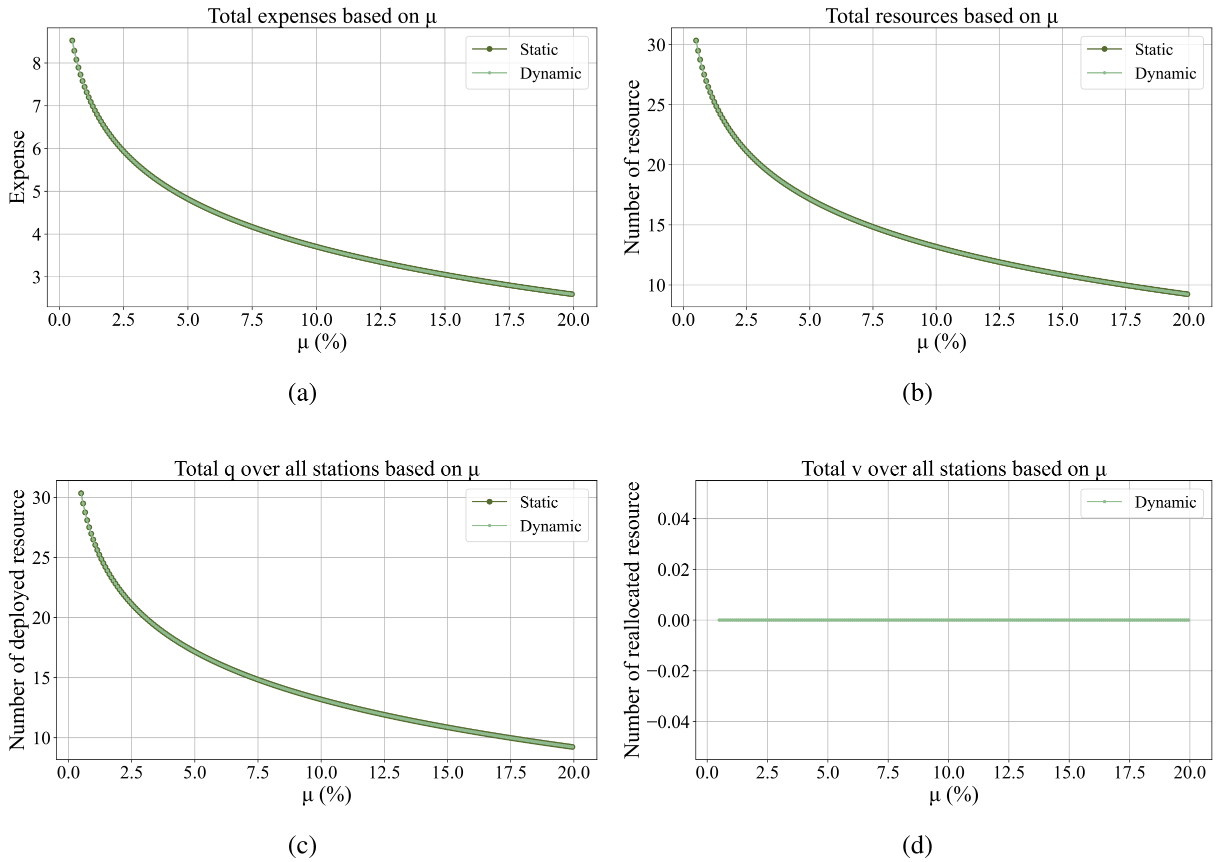


Figure 11: Impact of μ in scenario 1; part 1
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In scenarios characterized by the demands and costs outlined in Figure 4 and Figure 5, which previously demonstrated the effect of α, we observe trends as depicted in Figure 11. Consistent with expectations, an increase in μ leads to a reduction in both the number of resources required and the associated expenses. Furthermore, as μ increases, the rate of this decrease becomes more gradual, indicating that the impact of μ is more pronounced at lower percentages.

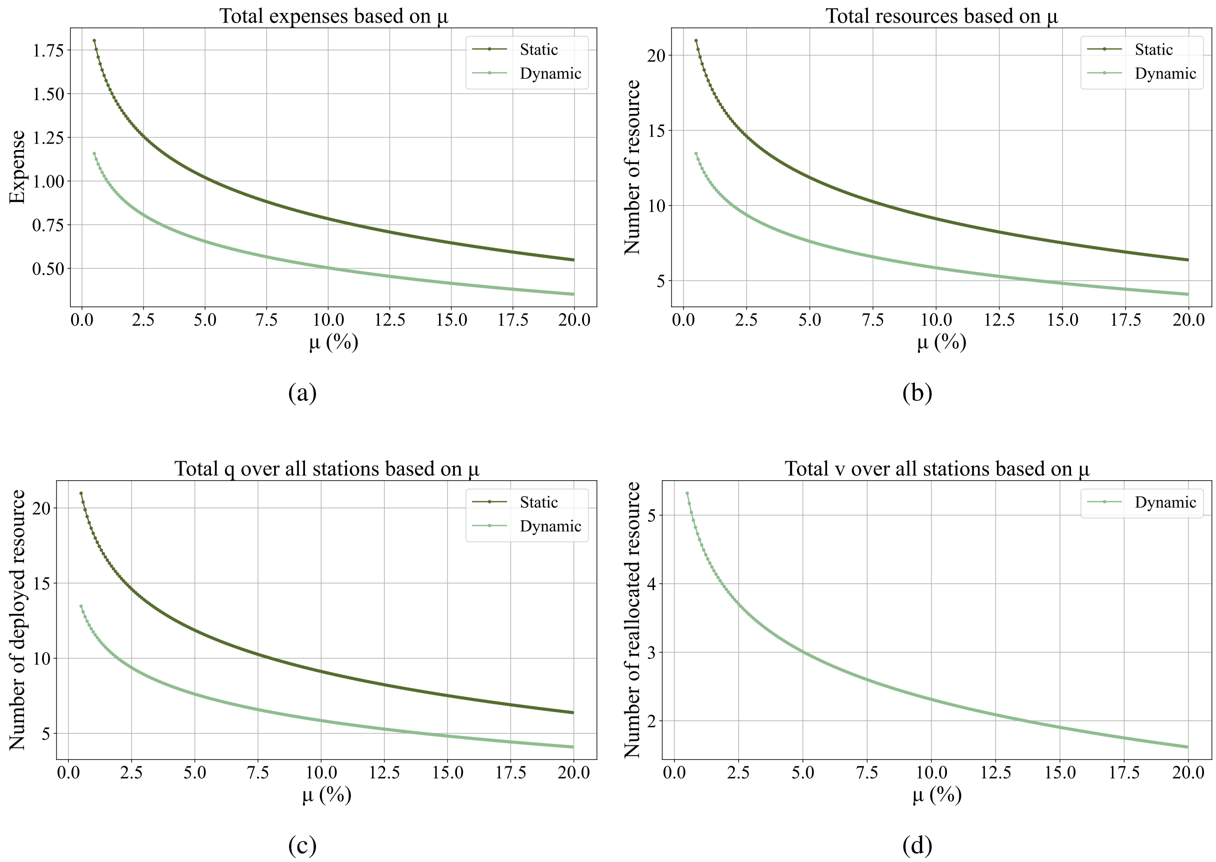
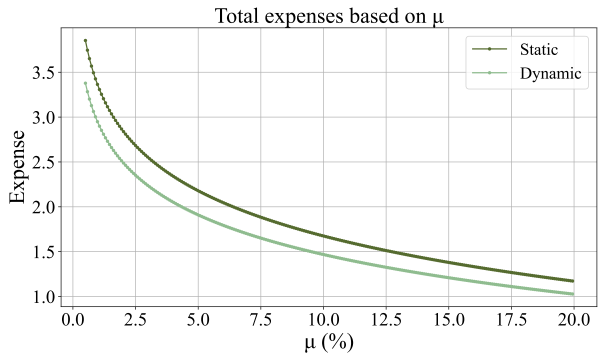
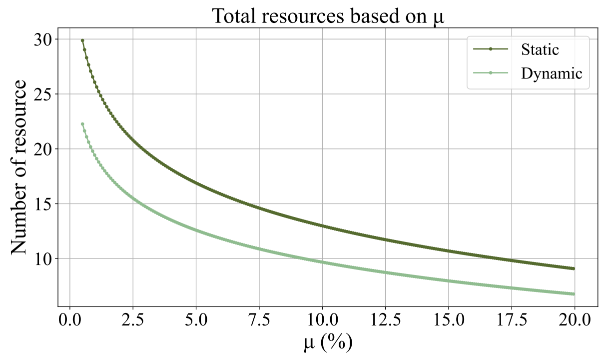


Figure 12: Impact of μ in scenario 1; part 2
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

In the next scenario, as depicted in Figure 12, there is a noticeable performance gap between the dynamic and static models, with the dynamic model outperforming the static one. This pattern is consistent across Figures 12a, 12b, and 12c. As we increase the value of μ, the gap diminishes, indicating that the dynamic model offers significant advantages when there is a need to satisfy more demands. Additionally, the rate at which this gap decreases slows as μ increases.



(a)



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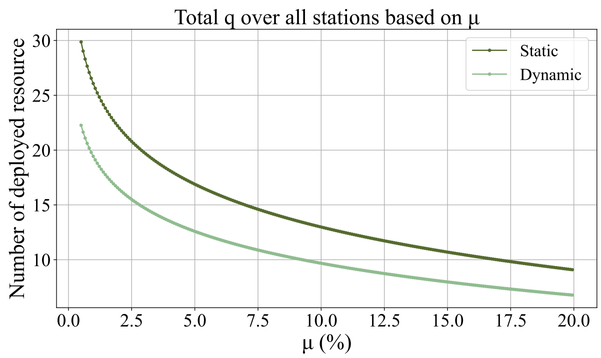

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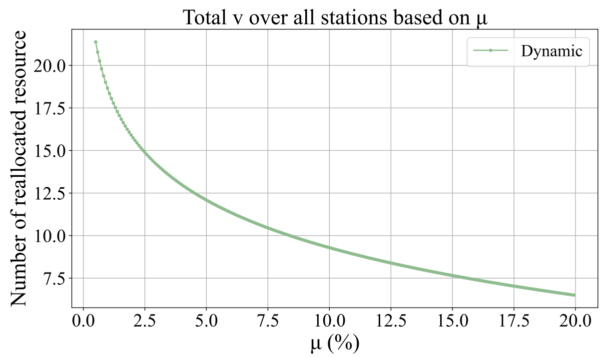


(d)

Figure 13: Impact of μ in scenario 2
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

Similar to Figure 9 in the model comparison, for the following scenario the number of regions containing a station has been increased to eight while maintaining the same number of time steps as before. As shown in Figure 13, the trend is similar to previous sections, with a decrease observed in all areas as we increase μ. However, increasing the number of regions impacts the values. Compared to the expenses in the previous section, there is an increase as depicted in Figure 13a. These changes are also evident in Figures 13b, 13c, and 13d.

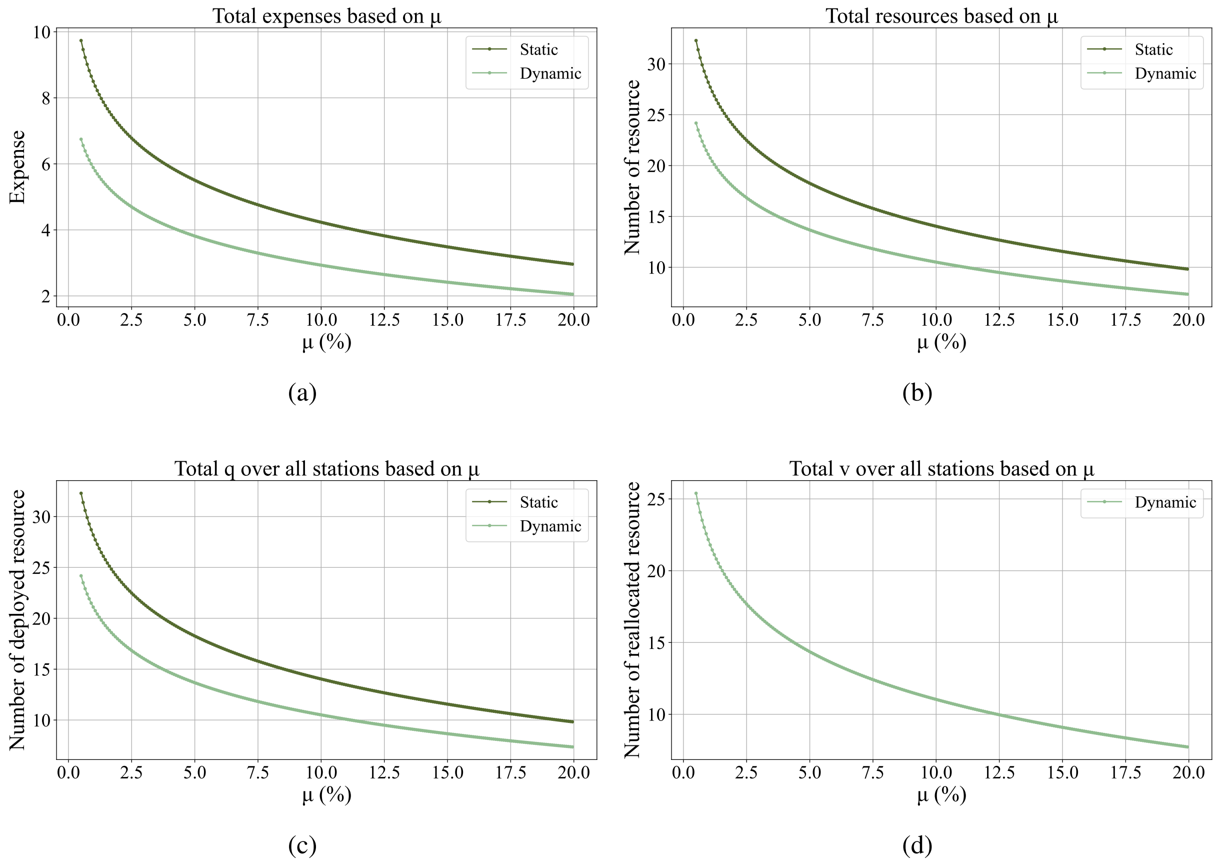


Figure 14: Impact of μ in scenario 3
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

Lastly, similar to Figure 10 in the model comparison section, while maintaining eight regions, we increased the time steps to four. Consistent with other sections, the trend remains similar, but the values change. As anticipated, having more regions and time steps necessitates more resources, resulting in higher overall expenses for the entire system. These results are illustrated in Figure 14.

Although this outcome might be expected, it not only confirms that the modeling yields predictable results in clear-cut situations but also demonstrates that it can be extended to more complex and large-scale models.

4.4.2 ε (Maximum Acceptable Distance Threshold)

In Equation 5, ε is defined as the threshold for the maximum acceptable distance, which ensures that resources are not allocated to regions significantly distant from their current locations. This is particularly important since our model does not consider the time required for resource reallocation, thereby placing a critical emphasis on the impact of distance in our simulations. Evaluating how distance influences resource distribution and response effectiveness is therefore vital.

As previously discussed in Section 4.1, to execute the scenarios, we generated random points within a one-by-one dimensional surface, ensuring that the distance between every pair of points was meticulously computed. Figure 13 provides an example of this random generation, showing the location of each station and the distance between each pair of stations. The figure shows three stations, with the values in parentheses representing their coordinates and the blue numbers indicating the distances between them.

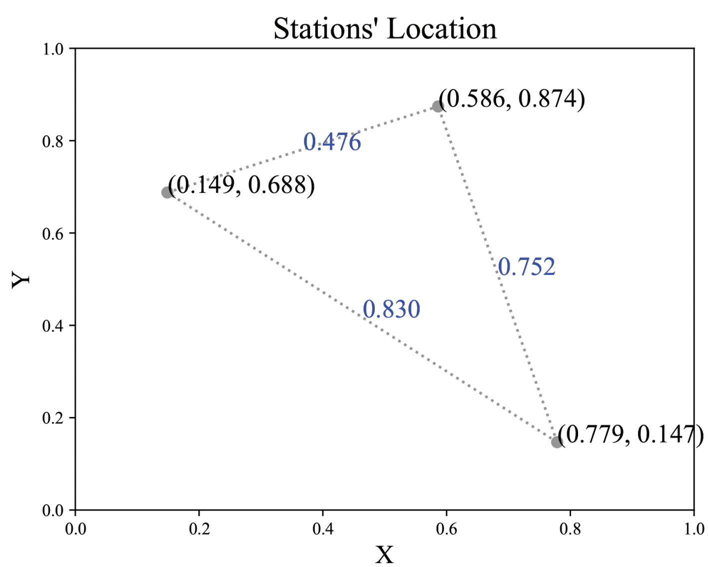


Figure 15: Example of stations generation in the model

For the analyses in this section, we maintain the parameter settings with α at 1 and μ at 0.1, allowing us to consistently observe the effects of these settings across various scenarios.

Similar to what is depicted in Figures 4, 5, and 11, there are scenarios where both models perform similarly, showing no significant differences in performance or expenses. This outcome also occurs when the models are compared by altering the distance threshold. The primary reason for this similarity is that the total reallocated resources remain at zero within the defined range.

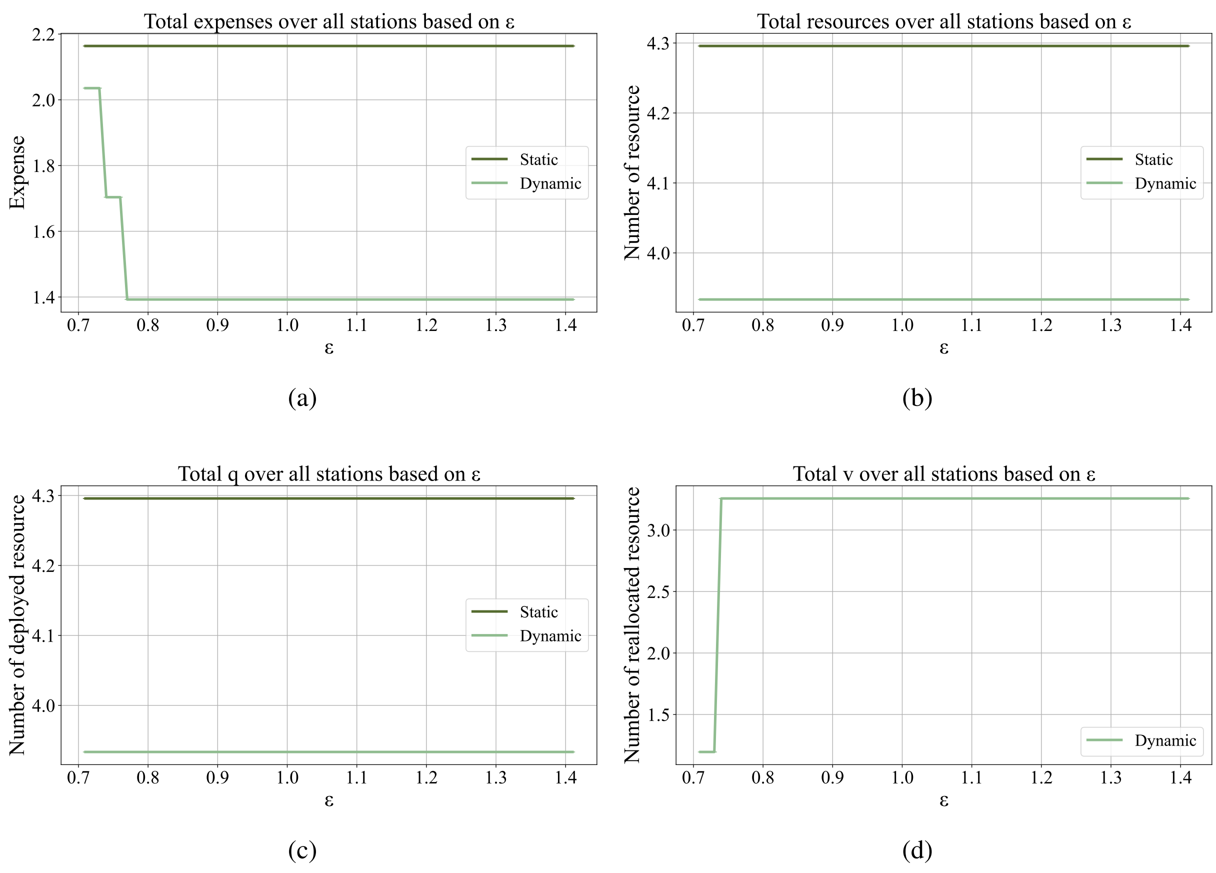


Figure 16: Impact of ε in scenario 1
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

Aside from the scenarios mentioned, differences between the two models are observable in other cases. As depicted in Figure 16, the dynamic model exhibits better performance within the defined range. As the threshold increases, so does the difference in expenses, as shown in Figure 16a, which corresponds with an increase in the number of reallocated resources (Figure 16d). However, it is important to note that this expense gap is solely driven by the variable v, while other parameters (q and r) do not affect it.

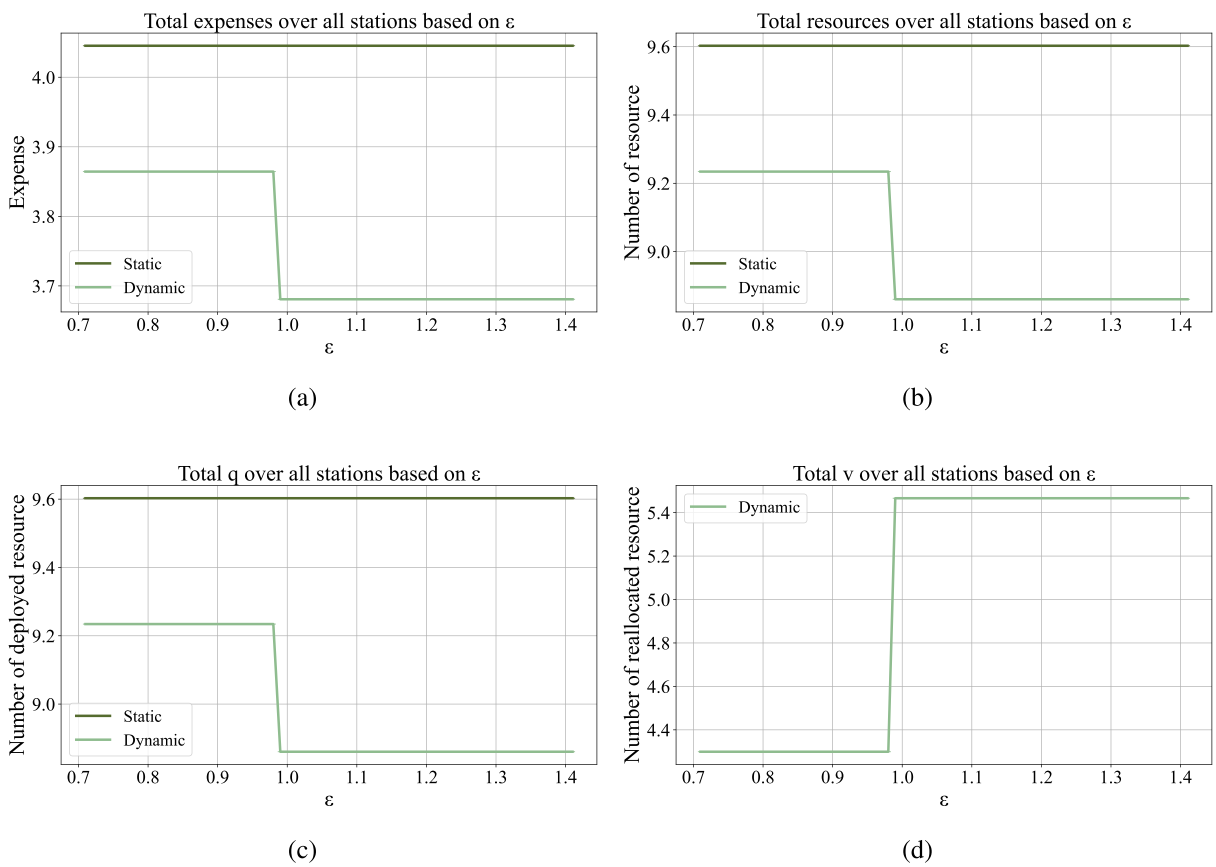


Figure 17: Impact of ε in scenario 2
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

Similar to previous parameter analyses, in another scenario where we increased the number of stations to eight while maintaining the same number of time steps, we observe in Figure 17 that, consistent with earlier scenario findings, there is a noticeable difference in expenses between the two models. This time, however, the variation in expenses stems not only from the variable v but also from changes in the total and deployed resources.

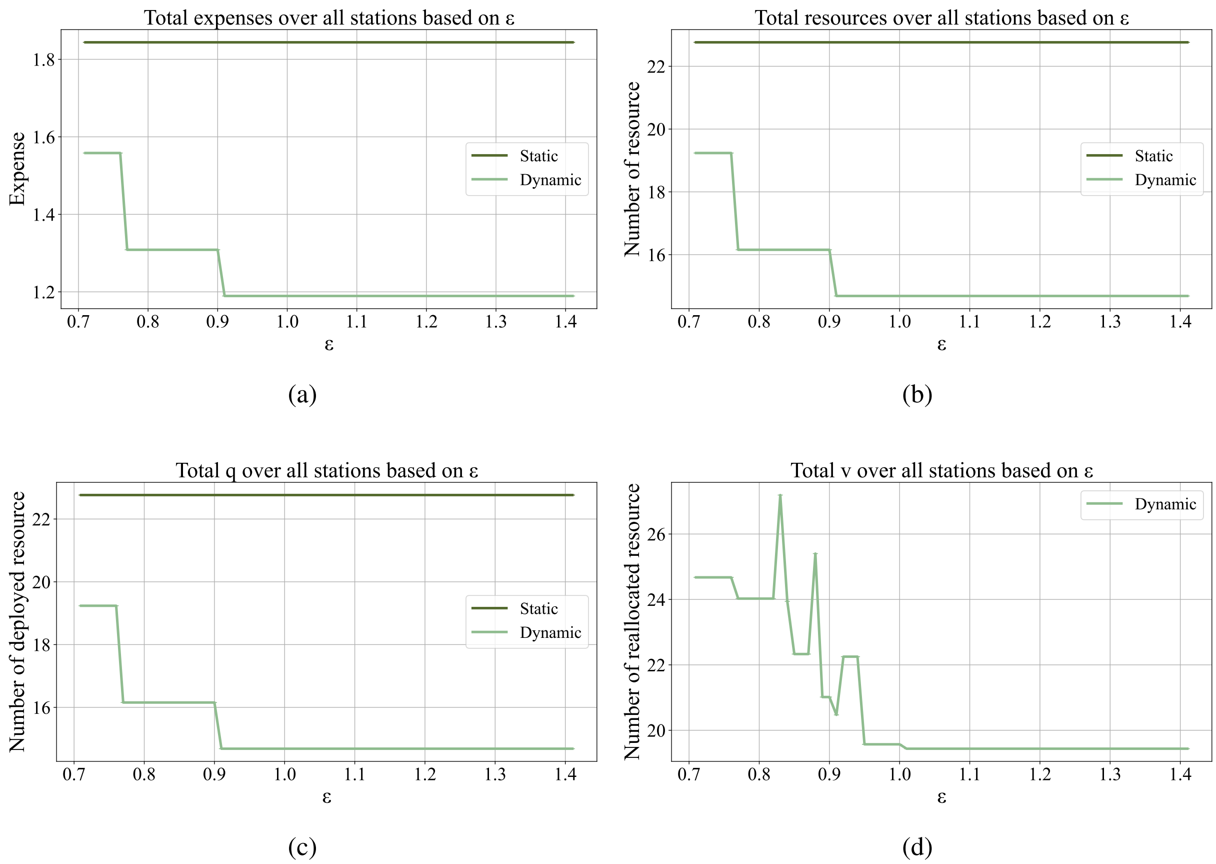


Figure 18: Impact of ε in scenario 3
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

Lastly, when we increase the time steps to four while keeping the station count at eight, we observe a consistent pattern in Figures 18. As the threshold increases, there is a noticeable decrease in expenses, total resources, and deployed resources. It is important to highlight that while the parameters r and q consistently decrease, the variations in v fluctuate with changes in the ε, showing both increases and decreases. This trend is expected as the model becomes more complex.

These outcomes demonstrate that certain results, such as changes to the ε, do not affect the static model since it controls the reallocation of resources in the dynamic model. Additionally, we observe that restricting the stations from reallocating resources among themselves makes the dynamic model more similar to the static model and increases the expenses of the dynamic model. It is important to note that the impedance factor and the threshold have distinct impacts, and both should be carefully adjusted for each scenario to ensure optimal performance.

5 Chapter Five: The Case Study

5.1 Introduction: City of Fredericton, NB, Canada

Fredericton, the capital city of the Canadian province of New Brunswick, is situated alongside the Saint John River. Its geographical positioning and growth trends present unique challenges and opportunities from an engineering standpoint.

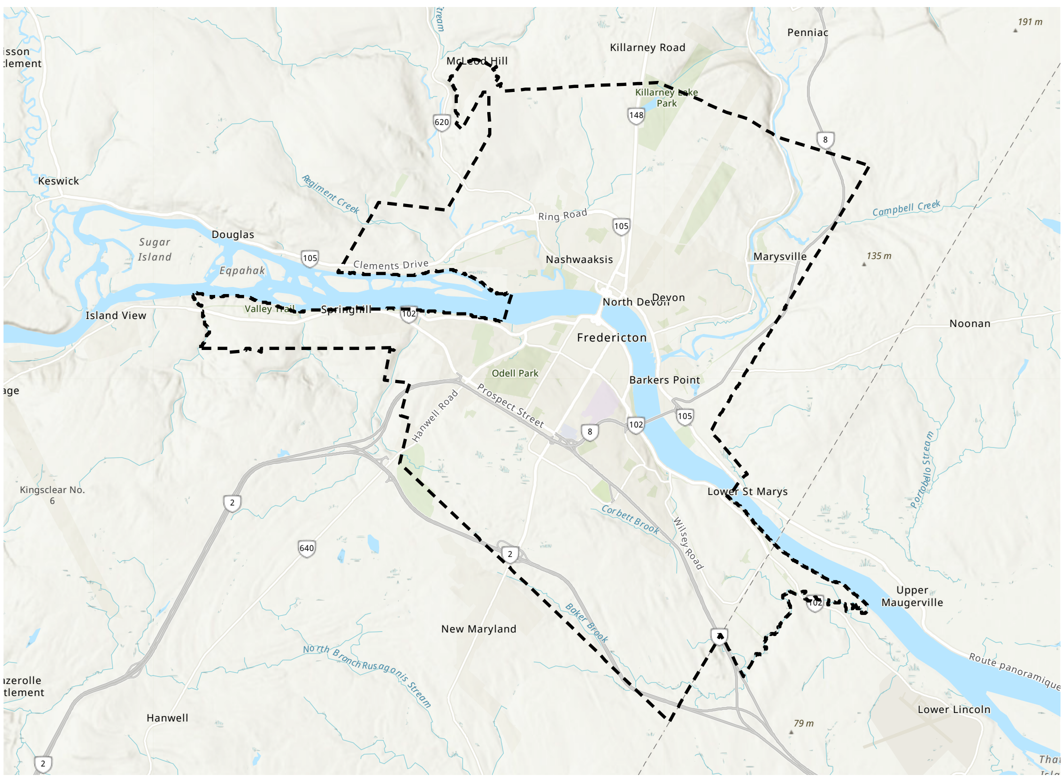


Figure 19: City of Fredericton

Located in the west-central part of New Brunswick, Fredericton's development has been significantly influenced by its proximity to the Saint John River. Originally inhabited by the Wolastoqiyik (Maliseet) peoples, the region became a European settlement stronghold in the 18th century. By 1785, Fredericton was designated as the capital of New Brunswick due to its strategic central location and defensible topography. Over the years, the city's infrastructural development has evolved to accommodate a blend of historic structures and modern urban architecture. The presence of institutions like the University of New Brunswick has also catalyzed infrastructural expansion and technological advancement in the area.

As Fredericton's population has grown and diversified, its infrastructure has been subjected to continuous upgrades to meet the demands of its residents. The city's road networks, public transportation systems, water and wastewater management, and energy supply have all seen evolutionary changes. The economic drivers of Fredericton, including education, government services, and information technology, have further necessitated the creation of robust infrastructural systems that can sustain the city's growth trajectory and its positioning as a significant urban center in New Brunswick.

A critical component of Fredericton's infrastructural framework is its emergency response system. The Fredericton Fire Department plays a vital role in ensuring the safety and security of the city's residents and their properties. Fire stations strategically located throughout the city ensure minimal response times and extensive coverage. These facilities are equipped with advanced engineering solutions to improve the efficiency and effectiveness of emergency responses. Design considerations for these stations include equipment storage, personnel readiness, energy efficiency, and resilience to potential natural disasters.

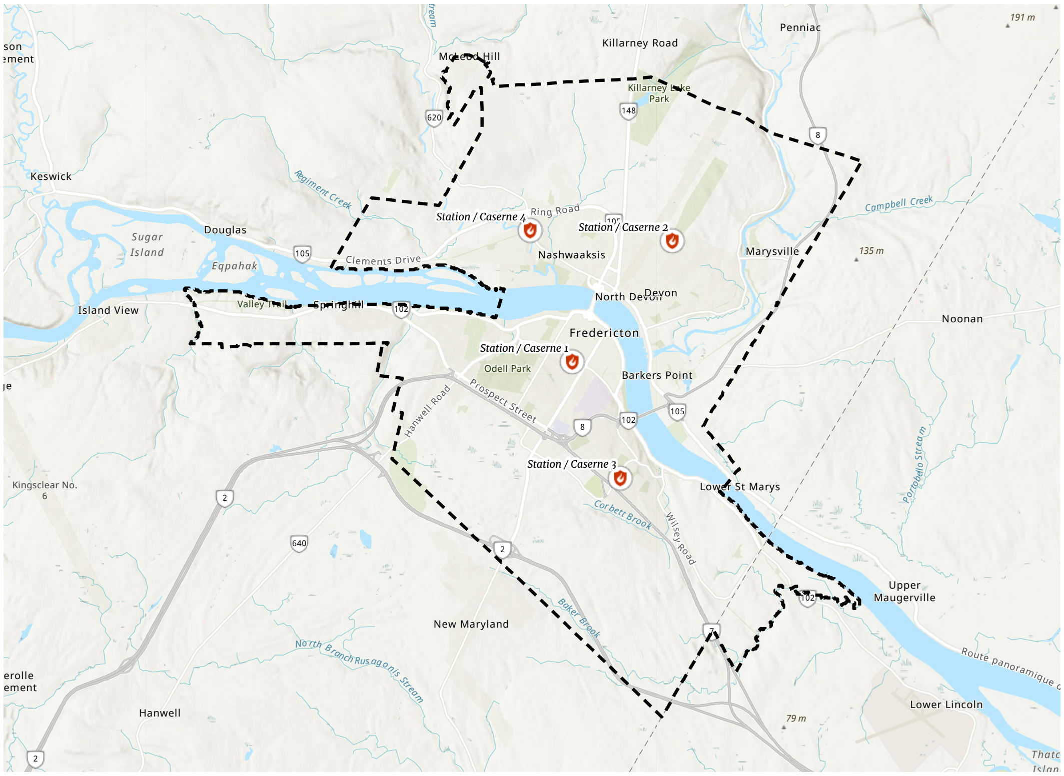


Figure 20: Fredericton's fire stations

Fredericton's growth and development, viewed through an engineering lens, offer valuable insights into urban planning, infrastructure development, and sustainability in modern cities. This case study of Fredericton highlights the critical importance of ongoing infrastructural adaptations to meet the changing needs of the city. It emphasizes the essential role that engineering plays in defining and shaping the contours of contemporary urban environments.

To gain a more comprehensive understanding of the city's circumstances, we conducted an analysis of historical fire incident data spanning from 2017 to 2021. This investigation encompassed temporal and spatial aspects, incident type categorization, response urgency evaluation, and service efficiency analysis. These facets are detailed in the following section.

5.2 Historical Data Analysis

In the temporal analysis section, we delve into the patterns and trends of fire incidents by scrutinizing data across various time frames: annually, monthly, and by day of the week. This approach allows us to identify any significant temporal fluctuations or consistencies in incident occurrence. The accompanying figures offer a detailed visualization of this analysis, illustrating the distribution and frequency of fire incidents over the specified time periods, thereby providing insight into potential cyclical or emerging trends.

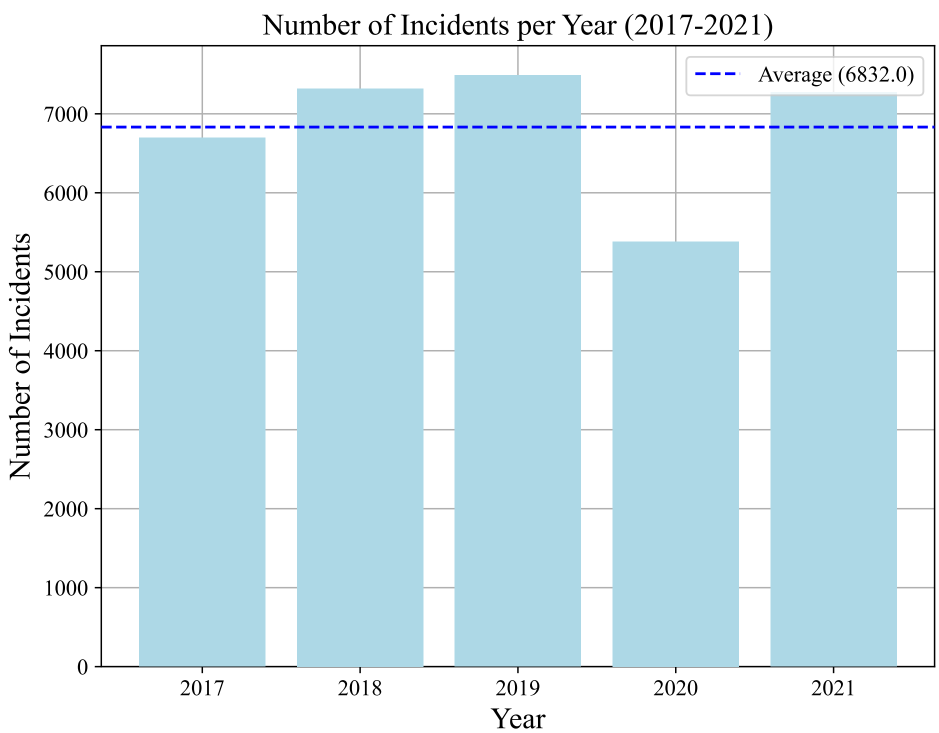


Figure 21: Yearly incident analysis

Figure 21, shows that the city of Fredericton averages 7,250 incidents per year. Excluding 2020, which marked the onset of the COVID-19 pandemic, the incidents recorded in the other four years analyzed typically range from 7,000 to 8,000. An examination of the 2020 statistics reveals a noticeable decrease in calls to the fire department during the first year of the pandemic. This reduction can be attributed to the widespread implementation of lockdowns and social distancing measures, which significantly curtailed commercial and social activities, thus reducing the potential for incidents typically requiring fire department intervention. However, in 2021, as the first wave of the pandemic subsided, the number of incidents returned to the usual range. This resurgence indicates that the temporary reduction in incidents was directly linked to the unique circumstances of the pandemic and suggests that once routine activities resumed, they reinstated the typical frequency of incidents. Excluding 2020, the data show an upward trend in the number of incidents as we move forward, underscoring the need for future planning and the importance of considering that incident rates may not remain at these averages.

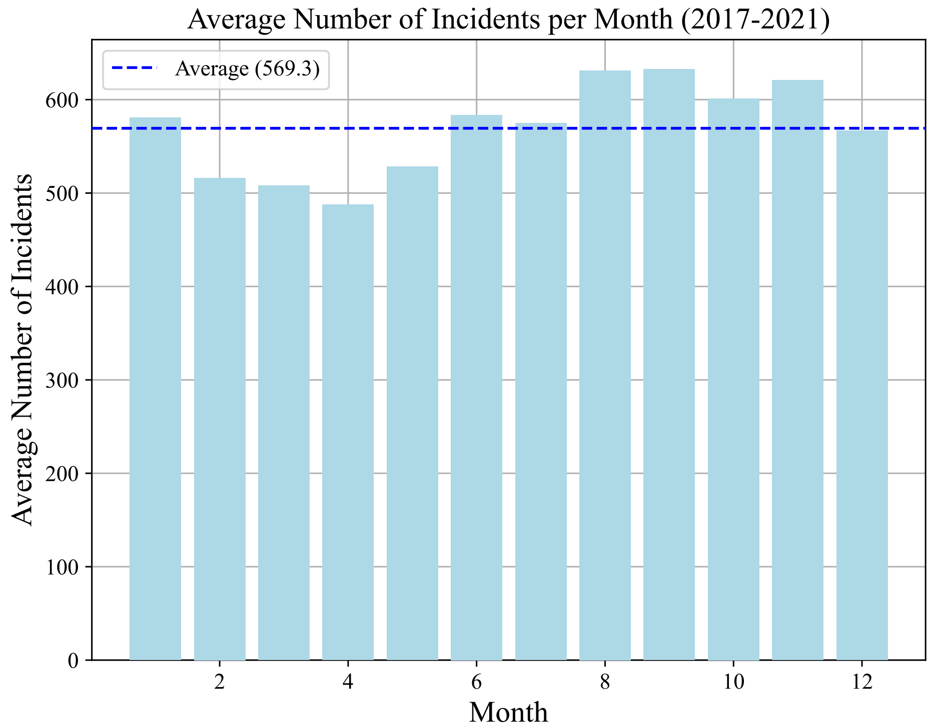


Figure 22: Monthly incident analysis

By examining the monthly averages presented in Figure 22, it is observed that the average is around 600 incidents per month, but this figure is not consistent throughout the year. During the winter and in cold weather conditions, the number of incidents decreases and falls below the average, while in summer, it increases and exceeds the average. This pattern suggests that seasonal variations significantly influence the frequency of incidents. Factors such as weather conditions, changes in social activities, and variations in industrial and commercial operations according to the season may contribute to these fluctuations. Therefore, it may be beneficial for emergency response planning and resource allocation to consider these seasonal trends in incident occurrence.

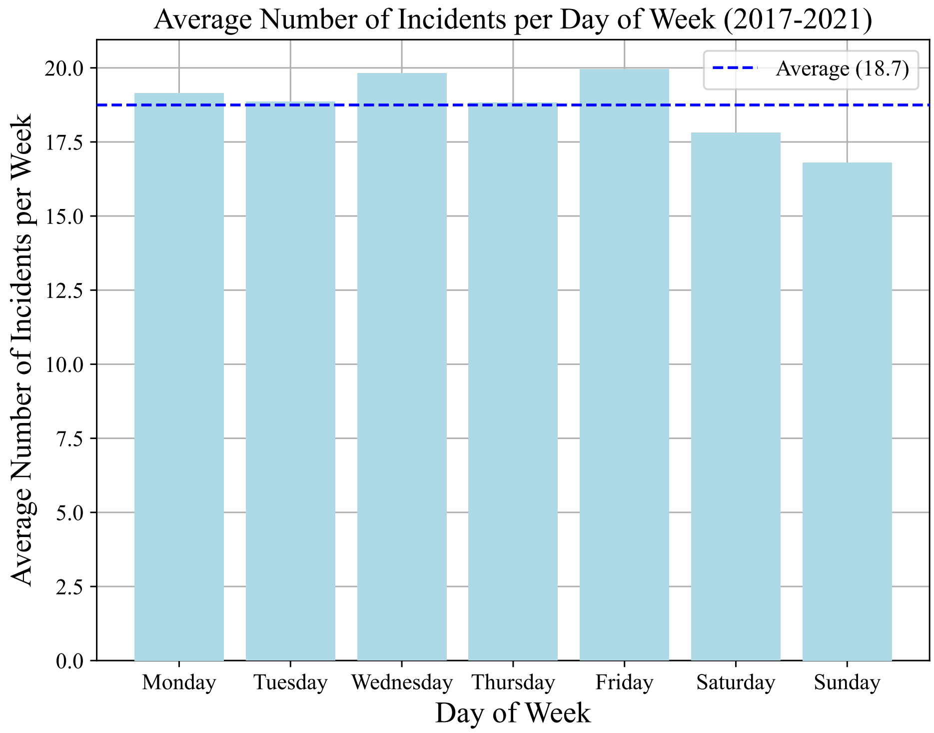


Figure 23: Daily incident analysis

By analysis of Figure 23 the data reveals the number of incidents by day of the week. As indicated in the graph, the number of incidents decreases during the weekends, falling below the average, while for weekdays, the incidence rate is relatively consistent except on Wednesdays and Fridays, which are above average. The higher incident rates on these particular weekdays could be related to increased mid-week commercial activities and social events, especially prevalent on Wednesdays and Fridays. Conversely, the lower weekend rates may be attributed to a reduction in industrial operations and possibly less traffic and related activities.

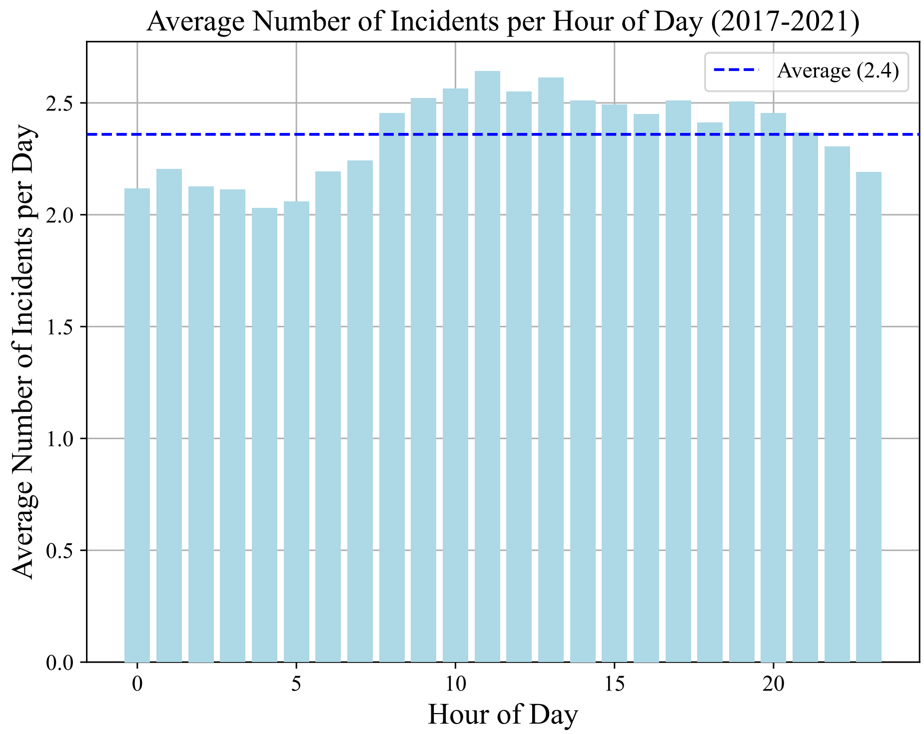


Figure 24: Hourly incident analysis

This examination of Figure 24 represents the final part of our temporal analysis. The data clearly shows, as one might expect, a variation in the number of incidents throughout the day. The frequency of incidents is below average between midnight and the early working hours, gradually increasing as the day progresses. Notably, there are peaks in incidents at 11:00 and 13:00, aligning with the middle of the day. This pattern may reflect the increased activity and movement of people during daytime hours, particularly around midday, which could contribute to a higher likelihood of incidents occurring. Understanding these temporal patterns is crucial for the strategic planning and allocation of emergency services and resources, ensuring an effective response to the varying incident rates throughout the time.

For the next phase of our study, we conducted a spatial analysis to investigate the distribution of incidents across the city. This examination aimed to identify patterns and trends in the geographical occurrence of incidents, providing insights into areas with higher frequencies and potential underlying factors contributing to these distributions. Understanding spatial patterns is crucial for optimizing emergency response strategies and resource allocation across different regions of the city.

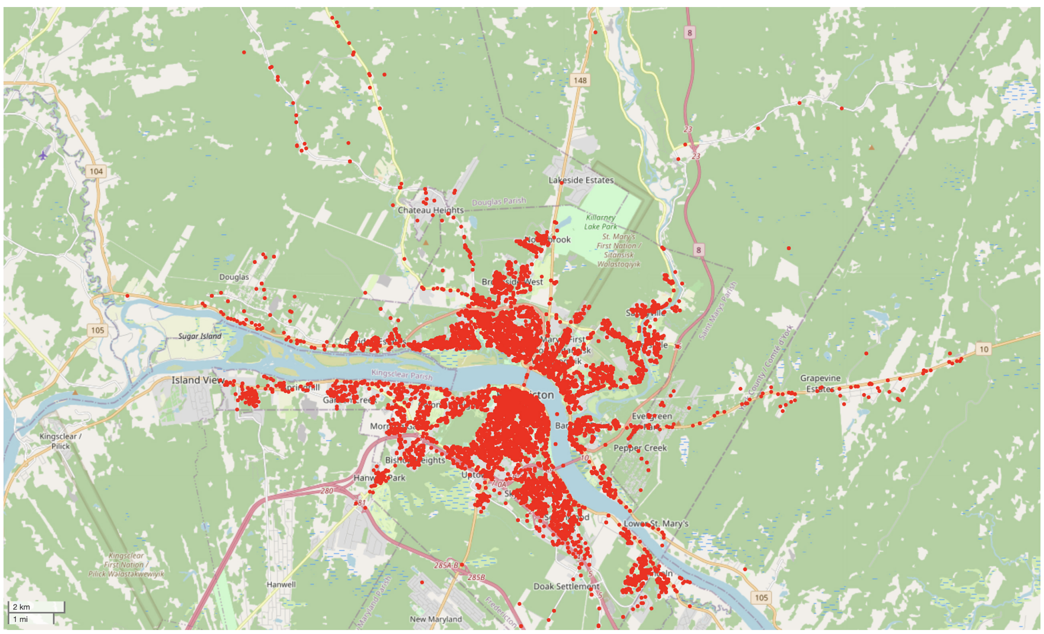


Figure 25: Incidents' location in Fredericton between 2017 and 2021

Figure 25 presents the locations of incidents within the city, illustrating that they occur in most areas. To maintain privacy and enhance understanding of incident concentration, we have clustered the incidents in Figure 26. This clustering reveals that the majority of incidents are concentrated in downtown Fredericton. Intriguingly, as shown in Figure 20, this area is in close proximity to Station 1. This spatial proximity could have implications for response times and resource allocation strategies for emergency services.

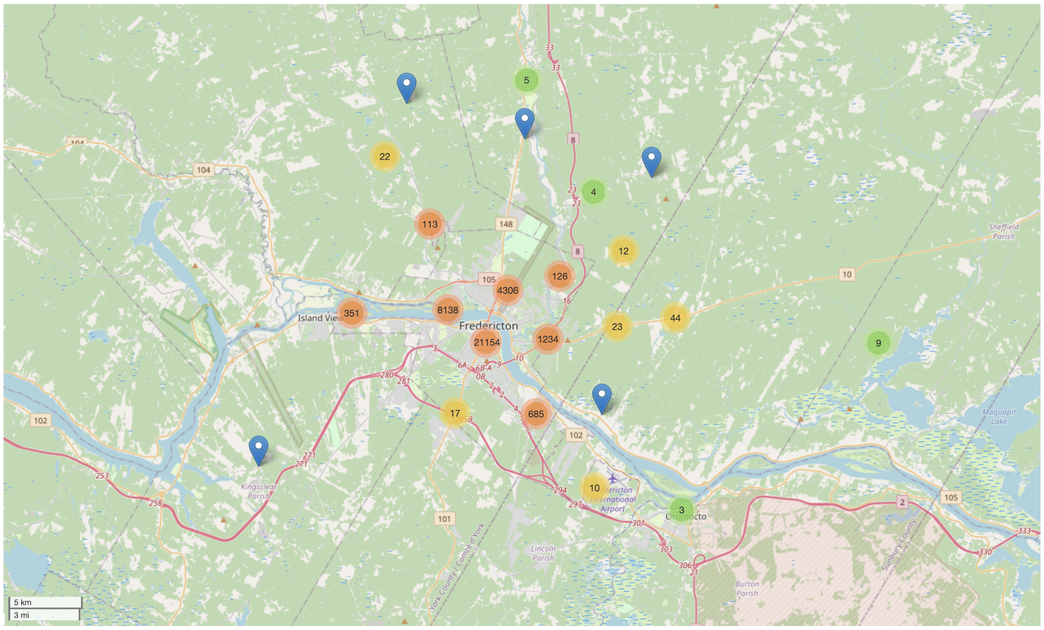


Figure 26: Incidents' clusters in Fredericton between 2017 and 2021

Furthermore, to better understand the areas with higher incident concentrations, a detailed, zoomed-in view is presented in Figure 27. This detailed perspective allows us to examine the specific neighbourhoods that exhibit a higher frequency of incidents. Such granular analysis is crucial for tailoring emergency response strategies to the unique needs and characteristics of these particular areas.

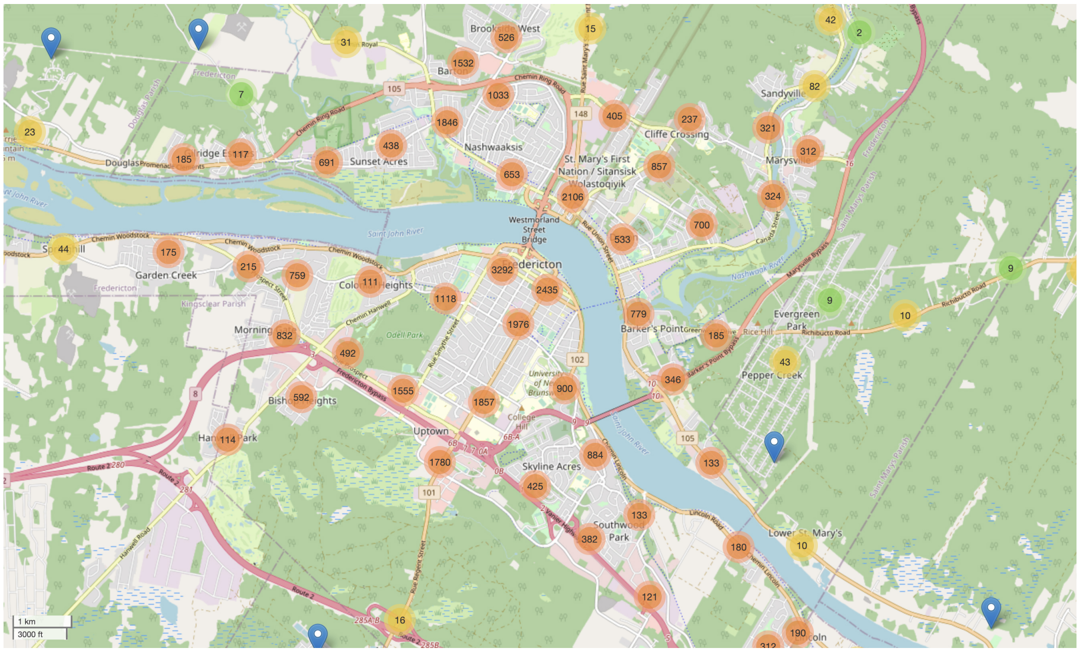


Figure 27: Incidents' cluster in downtown Fredericton between 2017 and 2021

Having identified the areas with the highest incident rates and understanding where incidents are most likely to occur, we conducted additional analyses using data that included responder station locations and response times. We examined the number of incidents each station responded to over five years, as well as the response times for each station. This comprehensive analysis helps to assess the efficiency and coverage of the stations across the city.

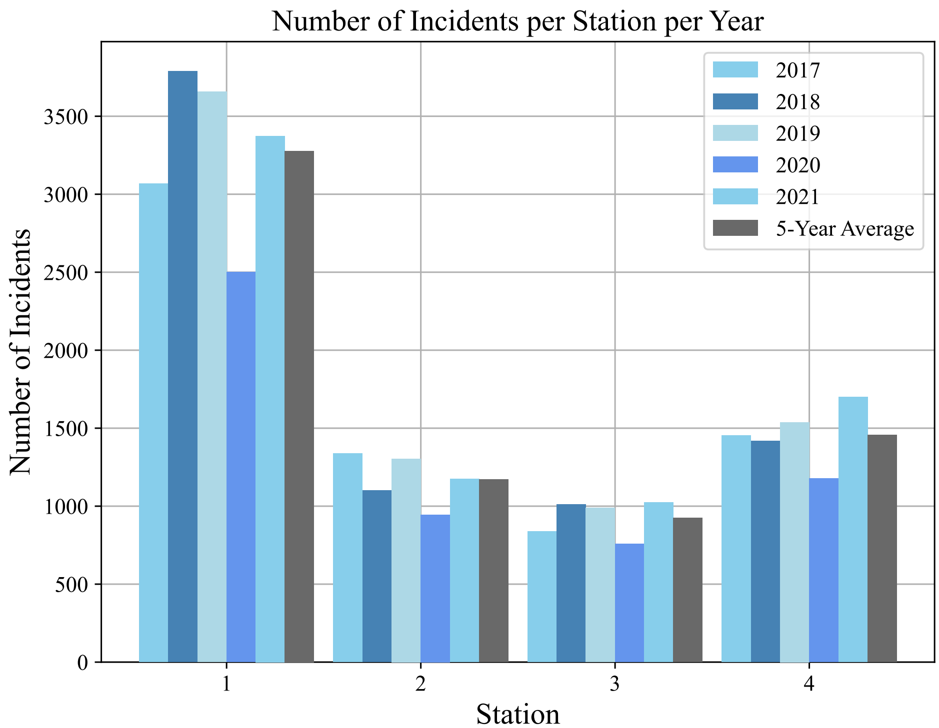


Figure 28: Number of incidents per station on average and per year

As previously discussed and illustrated in Figure 28, the number of incidents and their average at Station 1 is more than double that of the other stations, which exhibit similar incident counts. This significant disparity highlights the central role of Station 1 in the overall safety operations of the city. Its higher incident rate suggests that it is a key area for resource allocation and response efforts, reinforcing its importance in maintaining citywide safety and addressing critical needs more efficiently than other stations.

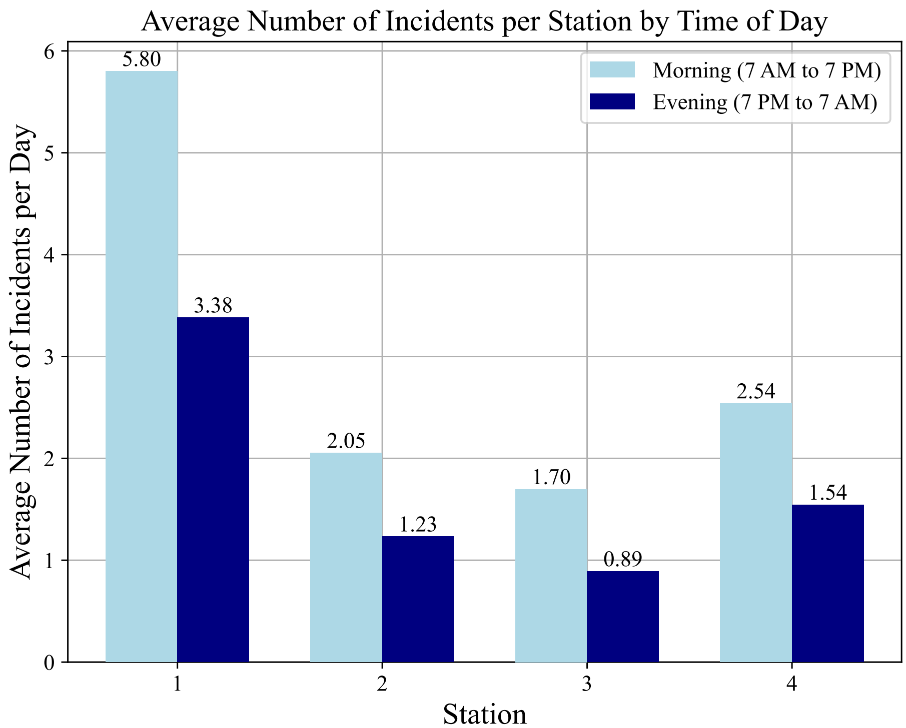


Figure 29: Number of incidents per station on average in two time periods

	Station 1	Station 2	Station 3	Station 4
Morning Demand	5.80	2.05	1.70	2.54
Evening Demand 2	3.38	1.23	0.89	1.54

Table 4: Demand values for each station in two time periods

As our model focuses on the reallocation of resources across two time periods, we analyzed the data to determine the average number of incidents for each station during two specific intervals, as illustrated in Figure 29. The first interval spans from morning to evening (7 AM to 7 PM), while the second corresponds to the night shift. The values from these graphs are also presented in Table 4 for use in the modeling section. The results from the mentioned figure do not differ significantly from previous ones and continue to highlight the crucial role of Station 1. However, it is important to remember that the number of incidents alone is not an accurate measure of a fire station's performance, prompting further analysis.

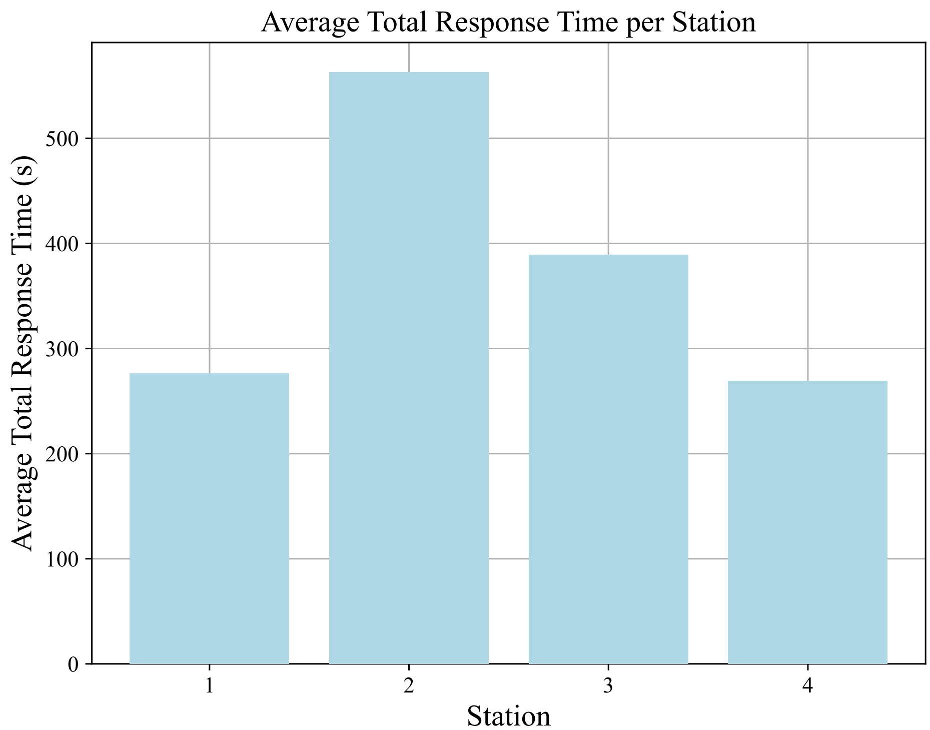


Figure 30: Average response time for each station

Although Station 1 handles the highest number of incident responses, Figure 30 reveals that both Station 1 and Station 4 have the best (lowest) response times, averaging about 4 minutes. In contrast, Station 2 shows the poorest performance, with response times that are double those of Stations 1 and 4, while Station 3's performance falls in between.

Based on these analyses, we now have the necessary data to run the model preemptively and determine the variables needed to evaluate the performance of the city should it opt for a reactive resource allocation approach.

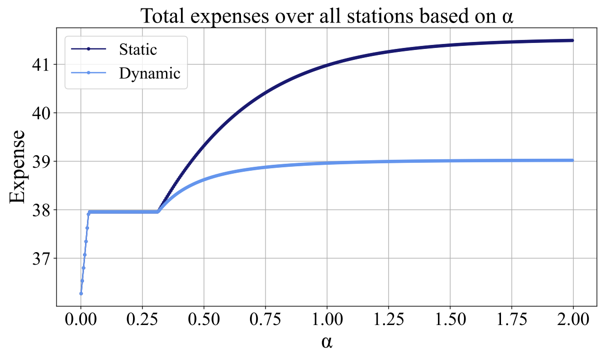
5.3 Data Integration into the Optimization Model

The essential inputs for our model include costs, which we have assumed to be equal across all stations, and demand, represented by the average number of incidents, as previously detailed and summarized in Table 4. Additionally, for our modeling, we require the distances between stations, with these values presented in Table 5.

	Station 1	Station 2	Station 3	Station 4
Station 1	0.0	4.169	2.992	3.625
Station 2	4.1690	0.0	6.186	3.807
Station 3	2.992	6.186	0.0	6.614
Station 4	3.625	3.807	6.614	0.0

Table 5: Distances between stations (in KM)

Now that we have gathered all the necessary data from the city of Fredericton, NB, we can use the Gurobi optimizer to solve our model. This will allow us to assess the differences between reactive and preemptive resource allocation. It is important to note that, for each of the following analyses, although we vary one parameter, all other parameters are kept at the values specified in Section 4.



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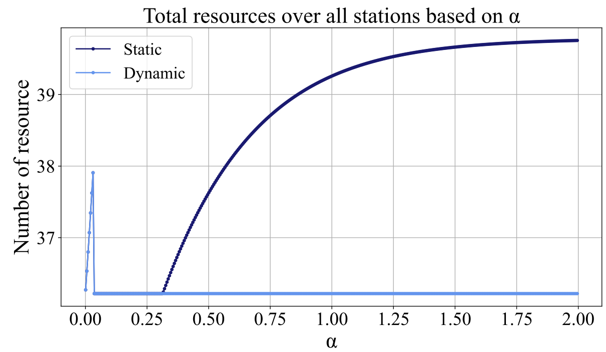

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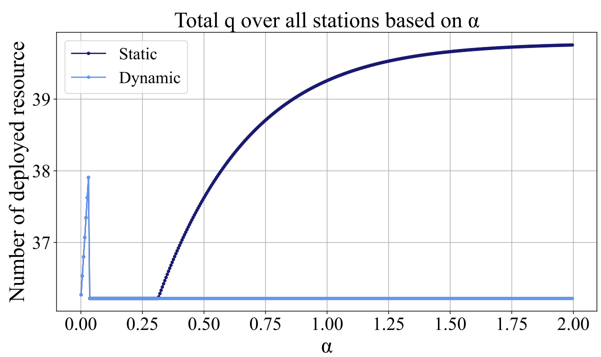

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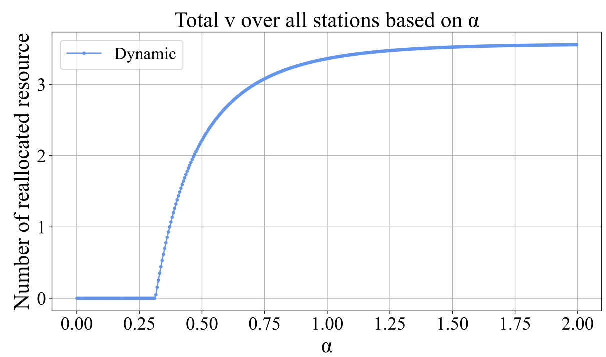

(d)

Figure 31: Impact of α in the Fredericton, NB case study
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

The scenario presented in our case study, illustrated in Figure 31, aligns with the findings from part 4 of scenario 1 in the α analysis (section 4, Figure 8). As the impedance factor α increases, the dynamic model outperforms the static model, demonstrating lower expenses at higher α values (Figure 31a). This cost reduction is accompanied by a lower total number of resources and fewer reallocated resources (Figures 31b and 31c), despite an overall increase in reallocations (Figure 31d). Notably, at very low α values, no significant difference is observed between the models, as resource reallocation remains unnecessary.

Additionally, Figures 31b and 31c reveal a brief fluctuation, resembling the pattern previously observed in Figure 5 and further analyzed in Figure 6. This shift indicates a temporary reduction in available resources, subsequently affecting the allocation of deployed resources for incident response.

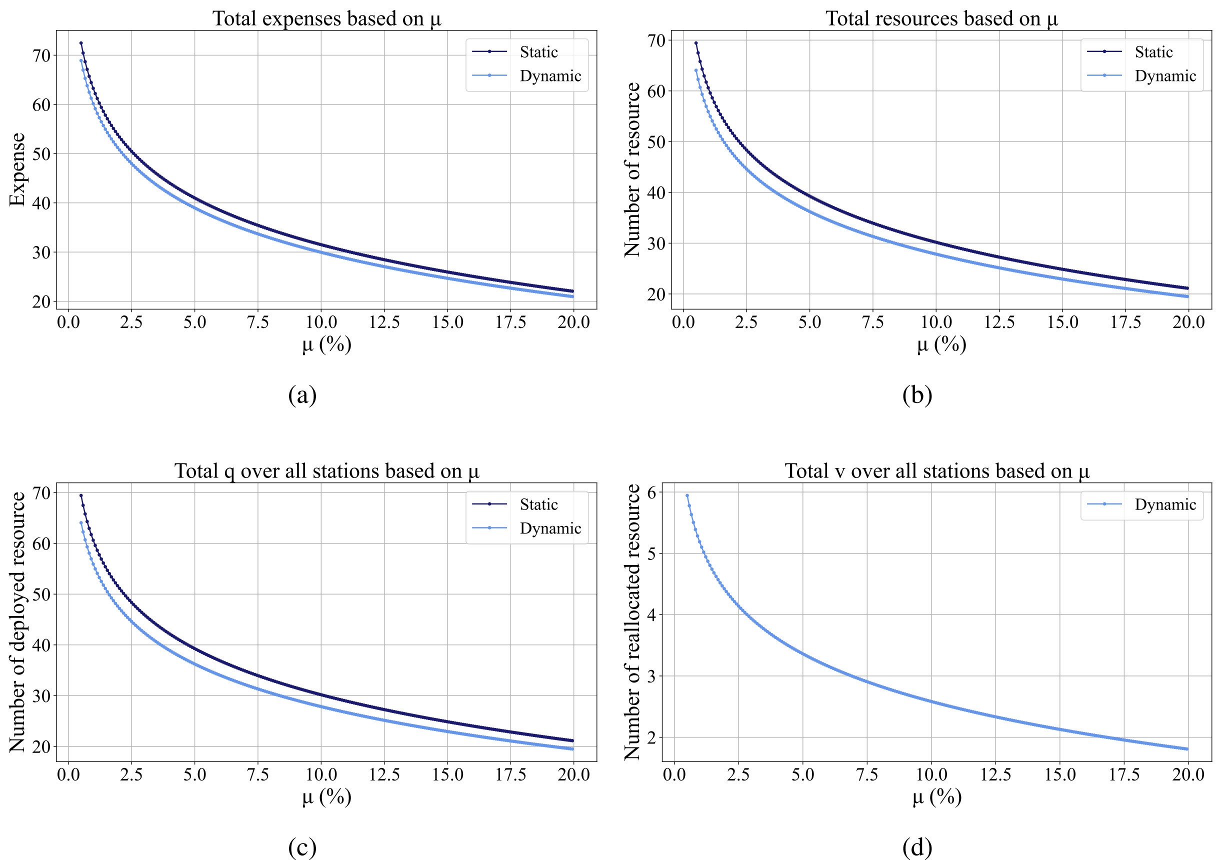

Figure 32: Impact of μ in the Fredericton, NB case study
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

The sensitivity analysis of μ in our case study aligns with our previous findings in the section 4. Although the differences across comparison figures in Figure 32 are not substantial, the dynamic model consistently outperforms the static model across the entire range of μ values shown. Additionally, in this specific dataset, while higher μ values reduce expenses, this benefit applies to both models similarly, as both follow a comparable pattern of expense reduction with increasing

μ.

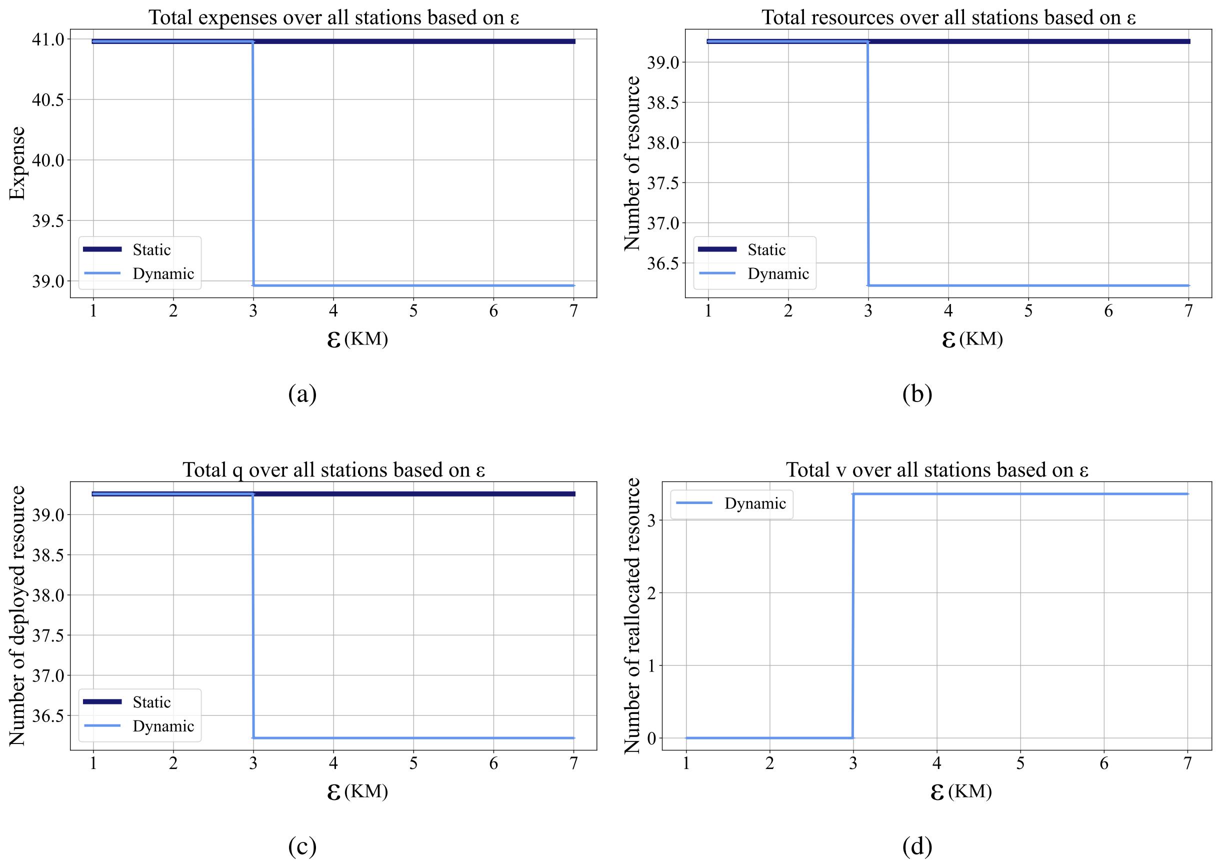

Figure 33: Impact of ε in the Fredericton, NB case study
(a) Total expenses (b) Total resources (c) Total deployed resources (d) Total reallocated resources all in over all stations

The maximum value in Table 5 is 6.614, so the limit for ε should exceed this distance, which represents the maximum range within which the dynamic model can reallocate resources. Therefore, the upper bound in Figure 33 is set to seven kilometers. The results indicate that if resources are allowed to be reallocated within a range of three kilometers, the dynamic model gains an advantage. However, for ε values below three kilometers, both models perform similarly.

6 Chapter Six: Conclusion

6.1 Summery

This research aims to evaluate the performance of dynamic resource optimization and compare it to traditional static allocation. By breaking down the problem into smaller components, we replicated the actual performance of fire stations using a static allocation model as a benchmark. This objective was achieved by first mathematically modeling the relationship between resource quantity and expenses associated with demand and its costs, followed by computationally solving the model. A case study analysis for the city of Fredericton, New Brunswick, was then conducted, where results from data analysis were applied to assess the impact of using a dynamic model compared to a static one. To support reproducibility and practical application, the full implementation code is included in Appendices A, B and C.

The literature review encompasses various aspects, beginning with an examination of individual resource types and how they are assigned and function within the system. This is followed by an exploration of the operational logistics of fire station response, focusing on incident response procedures and routing strategies. Additionally, the review delves into the concepts of preemptive and reactive resource allocation, analyzing their definitions, differences, and implications for effective resource management.

Our study's results provide qualitative support for the benefits of dynamic resource allocation in fire station operations. Through data analysis, we identified several key themes that emphasize the advantages of adopting a dynamic approach to resource management.

A primary finding was the importance of flexibility in addressing fluctuating needs and demands.

Our analysis showed that the current static allocation system often results in resource imbalances, with some stations overburdened while others are underutilized. In contrast, a dynamic allocation system would allow resources to be directed to areas with the highest need, leading to faster response times and more effective outcomes.

Another key theme was the potential for cost savings and enhanced resource utilization through a dynamic approach. Our findings suggest that a dynamic system could enable a more efficient use of personnel and equipment, reducing the need for redundant resources and maximizing the impact of available assets.

Overall, our results indicate that a dynamic resource allocation system could enhance operational efficiency, improve response times, and reduce costs for fire stations. By underscoring the benefits of flexibility and optimized resource use, this study provides valuable insights for fire station managers and policymakers aiming to optimize their operations.

6.2 Research Limitation

In our study, we operated under the assumption that each fire station is responsible for a predefined geographic region, generating specific demand within those boundaries. When an incident occurs within a station's designated region, that station is assigned to manage the incident, regardless of its actual distance from the event or whether another station might be closer. This framework allowed us to calculate the necessary resources for each station or region to meet the expected demand.

However, an alternative approach could involve redefining station boundaries to assign incidents based on proximity rather than fixed regional assignments. Such an adjustment might improve response times and resource utilization by dynamically allocating incidents to the nearest available station, thereby optimizing operational efficiency and potentially altering emergency response outcomes across the city. Implementing this approach would require a different analytical framework and additional resources to monitor and adapt to demand fluctuations in real time.

It is also worth noting that, due to certain limitations, we did not include the most recent data. The intervening period of the COVID-19 pandemic may have affected our case study results and data analysis, given the pandemic's impact on patterns of demand and resource use. Incorporating more recent data could reveal behavioral changes in incident trends and resource needs post-COVID-19. This insight would offer policymakers and managers a more comprehensive understanding of the city's evolving resource allocation requirements and help them anticipate future needs more effectively.

Finally, a detailed cost analysis for both demand and resources was beyond the scope of this project. However, determining an actual price for each resource and examining cost trends over time could yield valuable insights. Such an analysis would enhance the model's accuracy by capturing real-world financial impacts, thereby supporting more informed decisions on budget allocation and resource management. Including cost considerations in future work could further refine the model's practicality and make it a more robust tool for strategic planning.

6.3 Future Outlook

Given the diverse staffing configurations that exist across different regions and even among individual fire stations, future research could benefit from adopting a more localized focus. This would enable a granular exploration into the impact of dynamic resource allocation strategies.

Such research should aim to evaluate not only the distribution and utilization of the various fleet types, but also the staffing levels associated with each fire truck category. It is expected that this more detailed analysis would yield more specific insights into optimal resource management strategies within unique operational contexts.

In the current study, we made several simplifying assumptions within our model, such as employing linear modeling and positing equal distances for departure and return trips. Future research should strive to adjust these assumptions to more accurately reflect real-world conditions. Enhancing the realism of our modeling parameters will likely improve the precision of our findings and enhance their transferability to actual fire station operations.

Further development of our model should also consider the inclusion of facility resources, representing a third dimension to our resource considerations. The implications of taking into account all resource types simultaneously—fleet, staff, and facility—would provide a more holistic view of resource allocation within fire services. This integrated approach could pave the way to more sophisticated optimization models, contributing to more effective and efficient resource utilization in fire service operations.

Moreover, future studies should aim to establish a scalable framework for emergency resource management that extends beyond fire services. By generalizing the methodology, it can be adapted to other emergency response operations such as ambulance services and disaster relief. This broader applicability would enhance the relevance and utility of the model across various sectors of emergency management.

Lastly, we recommend that future work should involve rigorous testing and refinement of the model against real-world datasets. By iteratively comparing model predictions with actual data, researchers will be able to progressively refine the model, enhancing its reliability and applicability. This process would increase the model's value as a decision-making tool for fire service resource allocation, fostering its practical implementation within targeted localities and potentially leading to significant operational improvements.

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Appendices

A Appendix A: Static Model

```
static = gp.Model("Static␣Allocation")
static.setParam('OutputFlag', 0)

results_df = pd.DataFrame(columns=['Alpha', 'Objective␣Value'])

r = static.addVars(ii, tt, lb = 0, ub = gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="r")
v = static.addVars(ii, ii, tt, lb = 0, ub = gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="v")
q = static.addVars(ii, ii, tt, lb = 0, ub = gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="q")

static.setObjective(gp.quicksum(cost[i] * r[i, 0] for i in range(ii))
    + gp.quicksum(v[i,j,t] * distances[i][j] for i in range(ii)
        for j in range(ii) for t in range(tt))*co, gp.GRB.MINIMIZE)

for t in range(tt-1):
    for i in range(ii):
        static.addConstr(r[i,t+1] == r[i,t] + gp.quicksum((v[j,i,t]
            - v[i,j,t]) for j in range (ii)))

for t in range(tt):
    for i in range(ii):
```

```
        static.addConstr(gp.quicksum((q[i,j,t]) for j in range (ii))
        <= r[i,t])

for t in range(tt):
    for i in range(ii):
        static.addConstr(gp.quicksum((v[i,j,t]) for j in range (ii))
        <= r[i,t])

for t in range(tt):
    for i in range(ii):
        static.addConstr(gp.quicksum(q[j,i,t]
        * exponential_distance(j,i,alpha) for j in range(ii))
        >= demand[i,t]* exponential_cdf_inv(0.95, 1))

for i in range(ii):
    static.addConstr(gp.quicksum(v[j,i,tt-1] for j in range(ii)) == 0)

for i in range(ii):
    static.addConstr(gp.quicksum(v[j,i,t] for j in range(ii)
    for t in range(tt)) == 0)

static.optimize()

if static.status == gp.GRB.Status.OPTIMAL:

    results_df = results_df.append({'Alpha': alpha,
```

```
        'Objective␣Value': static.objVal}, ignore_index=True)

    print("\nVariable␣values␣for␣r:")
    for t in range(tt):
        for i in range(ii):
           print("r[{}{}]:{:.3f}".format(i,t, r[i,t].x))

    print("\nVariable␣values␣for␣v:")
    for t in range(tt-1):
        for j in range(ii):
            for i in range(ii):
                print("v[{}{}{}]:{:.3f}".format(i, j,t, v[i,j,t].x))

    print("\nVariable␣values␣for␣q:")
    for t in range(tt):
        for j in range(ii):
            for i in range(ii):
                print("q[{}{}{}]:{:.3f}".format(i, j,t, q[i,j,t].x))

else:
    print("Optimal␣solution␣not␣found")

sum_r1 = 0
sum_q1 = 0

for i in range(ii):
```

```
    sum_r1 = sum_r1 + r[i,0].x

print(sum_r1)

for i in range(ii):
    for j in range(ii):
        if j!=i:
            for t in range(tt):
                sum_q1 = sum_q1 + q[i,j,t].x

print(sum_q1)

sum_q1 = 0
for i in range(ii):
    for j in range(ii):
        if j==i:
            for t in range(tt):
                sum_q1 = sum_q1 + q[i,j,t].x


print("\n",results_df)
print(sum_q1)
```

B Appendix B: Dynamic Model

```
dynamic = gp.Model("Dynamic␣Allocation")
dynamic.setParam('OutputFlag', 0)

results_df = pd.DataFrame(columns=['Alpha', 'Objective␣Value'])

r = dynamic.addVars(ii, tt, lb = 0, ub = gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="r")
v = dynamic.addVars(ii, ii, tt, lb = 0, ub = gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="v")
q = dynamic.addVars(ii, ii, tt, lb = 0, ub = gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="q")
x = dynamic.addVars(ii, ii, lb=0, ub=1, vtype=gp.GRB.BINARY,
    name="x")

dynamic.setObjective(gp.quicksum(cost[i] * r[i, 0] for i in range(ii))
    + gp.quicksum(v[i,j,t] * distances[i][j] for i in range(ii)
        for j in range(ii) for t in range(tt))*co, gp.GRB.MINIMIZE)

for t in range(tt-1):
    for i in range(ii):
        dynamic.addConstr(r[i,t+1] == r[i,t] + gp.quicksum((v[j,i,t]
            - v[i,j,t]) for j in range (ii)))

for t in range(tt):
    for i in range(ii):
```

```
        dynamic.addConstr(gp.quicksum((q[i,j,t]) for j in range (ii))
        <= r[i,t])

for t in range(tt):
    for i in range(ii):
        dynamic.addConstr(gp.quicksum((v[i,j,t]) for j in range (ii))
        <= r[i,t])

for i in range(ii):
    for j in range(ii):
        dynamic.addConstr((distances[i, j]) * (x[i, j]) <= eps)

for t in range(tt):
    for i in range(ii):
        for j in range(ii):
            dynamic.addConstr(v[i, j, t] <= x[i, j] * pow(10,6))

for t in range(tt):
    for i in range(ii):
        dynamic.addConstr(gp.quicksum(q[j,i,t]
        * exponential_distance(j,i,alpha) for j in range(ii))
        >= demand[i,t]* exponential_cdf_inv(0.95, 1))

for i in range(ii):
    dynamic.addConstr(gp.quicksum(v[j,i,tt-1] for j in range(ii)) == 0)
```

```
dynamic.optimize()

if dynamic.status == gp.GRB.Status.OPTIMAL:

    results_df = results_df.append({’Alpha’: alpha,
        ’Objective␣Value’: static.objVal}, ignore_index=True)

    print("\nVariable␣values␣for␣r:")
    for t in range(tt):
        for i in range(ii):
           print("r[{}{}]:{:.3f}".format(i,t, r[i,t].x))

    print("\nVariable␣values␣for␣v:")
    for t in range(tt-1):
        for j in range(ii):
            for i in range(ii):
                print("v[{}{}{}]:{:.3f}".format(i, j,t, v[i,j,t].x))

    print("\nVariable␣values␣for␣q:")
    for t in range(tt):
        for j in range(ii):
            for i in range(ii):
                print("q[{}{}{}]:{:.3f}".format(i, j,t, q[i,j,t].x))

else:
    print("Optimal␣solution␣not␣found")
```

```
sum_r2 = 0
sum_q2 = 0

for i in range(ii):
    sum_r2 = sum_r2 + r[i,0].x

print(sum_r2)

for i in range(ii):
    for j in range(ii):
        if j!=i:
            for t in range(tt):
                sum_q2 = sum_q2 + q[i,j,t].x
print(sum_q2)

sum_q2 = 0
for i in range(ii):
    for j in range(ii):
        if j==i:
            for t in range(tt):
                sum_q2 = sum_q2 + q[i,j,t].x

print("\n",results_df)
print(sum_q2)
```

C Appendix C: Model Solving

```
Results = pd.DataFrame(columns=['Alpha', 'Obj_Static', 'Sum_r_Static',
    'Sum_q_Static', 'Sum_v_Static', 'Obj_Dynamic', 'Sum_r_Dynamic',
    'Sum_q_Dynamic', 'Sum_v_Dynamic'])

for alpha in [i/1000 for i in range(1, 2000, 5)]:

    # Static Model
    static = gp.Model("Static␣Allocation")
    static.setParam('OutputFlag', 0)

    r_static = static.addVars(ii, tt, lb = 0, ub = gp.GRB.INFINITY,
        vtype=gp.GRB.CONTINUOUS, name="r")
    v_static = static.addVars(ii, ii, tt, lb = 0, ub = gp.GRB.INFINITY,
        vtype=gp.GRB.CONTINUOUS, name="v")
    q_static = static.addVars(ii, ii, tt, lb = 0, ub = gp.GRB.INFINITY,
        vtype=gp.GRB.CONTINUOUS, name="q")

    static.setObjective(gp.quicksum(cost[i] * r_static[i, 0]
        for i in range(ii)) + gp.quicksum(v_static[i,j,t]
        * distances[i][j] for i in range(ii) for j in range(ii)
        for t in range(tt)) * co , gp.GRB.MINIMIZE)

    for t in range(tt-1):
        for i in range(ii):
            static.addConstr(r_static[i,t+1] == r_static[i,t]
```

```
        + gp.quicksum((v_static[j,i,t] - v_static[i,j,t])
        for j in range (ii)))

for t in range(tt):
    for i in range(ii):
        static.addConstr(gp.quicksum((q_static[i,j,t])
        for j in range (ii)) <= r_static[i,t])

for t in range(tt):
    for i in range(ii):
        static.addConstr(gp.quicksum((v_static[i,j,t])
        for j in range (ii)) <= r_static[i,t])

for t in range(tt):
    for i in range(ii):
        static.addConstr(gp.quicksum(q_static[j,i,t]
        * exponential_distance(j,i,alpha) for j in range(ii))
        >= demand[i,t]* exponential_cdf_inv(1-mu, 1))

for i in range(ii):
    static.addConstr(gp.quicksum(v_static[j,i,tt-1]
    for j in range(ii)) == 0)

for i in range(ii):
    static.addConstr(gp.quicksum(v_static[j,i,t]
    for j in range(ii) for t in range(tt)) == 0)
```

```
static.optimize()
sum_r_static = sum(r_static[i, 0].x for i in range(ii))
sum_q_static = sum(q_static[i, j, t].x for i in range(ii)
        for j in range(ii) for t in range(tt))/tt
sum_v_static = sum(v_static[i, j, t].x for i in range(ii)
        for j in range(ii) for t in range(tt))

# Dynamic Model
dynamic = gp.Model("Dynamic␣Allocation")
dynamic.setParam('OutputFlag', 0)

r_dynamic = dynamic.addVars(ii, tt, lb=0, ub=gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="r")
v_dynamic = dynamic.addVars(ii, ii, tt, lb=0, ub=gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="v")
q_dynamic = dynamic.addVars(ii, ii, tt, lb=0, ub=gp.GRB.INFINITY,
    vtype=gp.GRB.CONTINUOUS, name="q")
x_dynamic = dynamic.addVars(ii, ii, lb=0, ub=1, vtype=gp.GRB.BINARY,
    name="x")

dynamic.setObjective(gp.quicksum(cost[i] * r_dynamic[i, 0]
    for i in range(ii)) + gp.quicksum(v_dynamic[i, j, t]
    * distances[i, j] for i in range(ii) for j in range(ii)
    for t in range(tt)) * co, gp.GRB.MINIMIZE)
```

```
for t in range(tt - 1):
    for i in range(ii):
        dynamic.addConstr(r_dynamic[i, t + 1] == r_dynamic[i, t]
        + gp.quicksum((v_dynamic[j, i, t] - v_dynamic[i, j, t])
        for j in range(ii)))

for t in range(tt):
    for i in range(ii):
        dynamic.addConstr(gp.quicksum((q_dynamic[i, j, t])
        for j in range(ii)) <= r_dynamic[i, t])

for t in range(tt):
    for i in range(ii):
        dynamic.addConstr(gp.quicksum((v_dynamic[i, j, t])
        for j in range(ii)) <= r_dynamic[i, t])

for i in range(ii):
    for j in range(ii):
        dynamic.addConstr((distances[i, j]) * (x_dynamic[i, j]) <= eps)

for t in range(tt):
    for i in range(ii):
        for j in range(ii):
            dynamic.addConstr(v_dynamic[i, j, t] <= x_dynamic[i, j]
            * pow(10,6))
```

```
for t in range(tt):
    for i in range(ii):
        dynamic.addConstr(gp.quicksum(q_dynamic[j, i, t]
        * exponential_distance(j, i, alpha) for j in range(ii))
        >= demand[i, t] * exponential_cdf_inv(1-mu, 1))

for i in range(ii):
    dynamic.addConstr(gp.quicksum(v_dynamic[j, i, tt - 1]
    for j in range(ii)) == 0)

dynamic.optimize()

sum_r_dynamic = sum(r_dynamic[i, 0].x for i in range(ii))
sum_q_dynamic = sum(q_dynamic[i, j, t].x for i in range(ii)
        for j in range(ii) for t in range(tt))/tt
sum_v_dynamic = sum(v_dynamic[i, j, t].x for i in range(ii)
        for j in range(ii) for t in range(tt))

Results = Results.append({'Alpha': alpha,
                        'Obj_Static': static.objVal,
                        'Sum_r_Static': sum_r_static,
                        'Sum_q_Static': sum_q_static,
                        'Sum_v_Static': sum_v_static,
                        'Obj_Dynamic': dynamic.objVal,
                        'Sum_r_Dynamic': sum_r_dynamic,
                        'Sum_q_Dynamic': sum_q_dynamic,
```

```
                                   'Sum_v_Dynamic': sum_v_dynamic},
                                   ignore_index=True)

print(Results)
Results.to_csv('optimization_results.csv', index=False)
```