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So in the last clip, I introduced you to the basic idea of hypothesis testing. And we established that if your estimate lies within a particular interval, then you do not reject the hypothesis. But if it lies outside that interval, then you reject the hypothesis H_0 in favor of H_a . So in this clip, let me do some examples to actually put that into, into to show you how those work. So, let's go back to our online learning example, with 100 students. And remember, what we have is data on their scores of learning online. And before that when we were doing it in person. And we came up, and let's say that the mean that we calculated is minus three, right, and the standard deviation is S is 2.5. So we have the data for the 100 students, and suppose we computed the mean and standard deviation, this was minus three and 2.5. So right off the bat, you see that this is negative, right? So the mean, is negative three. So that means on average, the score online is less than the score in person, but isn't big enough to, to suggest the hypothesis that online learning does have an impact on student learning. So that's what we want to test. So what we want to test is whether μ is equal to zero, versus the alternate hypothesis where the μ is different from zero, right. So this is no change, no impact versus that there is some. And again, going back to our basics, right, basic idea of hypothesis testing, what we know is that the probability that if H_0 is true, that is if μ is equal to zero, right, then there's 95% chance that $\bar{X}$ should lie between zero minus two times S , which is 2.5 divided by square root of n . So that's square root of 100. And knew that zero plus two times 2.5 over square root of n and if you calculate this out, this comes out to be negative point five, and this comes out to be positive point five. So if H_0 is true, then 95% probability that your $\bar{X}$ should lie within within the center. But if you check, in this case, your $\bar{X}$ here is minus three. So it lies outside the center. Right? So it lies outside this 95% region. So that means therefore we reject the hypothesis μ equal to zero, right? So based on this data, it's very unlikely that that μ is equal to zero. That's it. This is how we do hypothesis test. So look. So going from that example. So let me go a little bit more general and talk about the mechanics of this process of hypothesis testing exactly what are the steps that you need to do? Right, and you'll see, it's essentially what I just talked about, but just done using a slightly different formula. Now, so the basic idea is that we're testing whether μ is equal to μ_0 versus whether μ is not equal to μ_0 , right? So this is our more general version. So in the previous example, this μ_0 is 0 Right, so but you could test for other values,

whether μ is equal to point five or whether it's 10 right versus not. So the basic idea again, going back to the central limit theorem, is that if this hypothesis H_0 is true, then $\bar{x}$ would be normally distributed with mean μ_0 and variance σ^2/n . Now, here's an important property of the normal distribution, which we did in probability module four. This was called standardization. So if the normal distribution is distributed, if a variable is normally distributed with this mean and this variance, right? If you take out the mean, if you do $\bar{x} - \mu_0$, right, and divided by the square root of that variance. So if you divided by the square root of σ^2/n , right, then then this variable, it follows the normal distribution, right. So this is called the standard norm. So this is something which we covered in probability module four. And this is how we are connecting statistics back with probability. So so this process where we subtract out the mean, right, and divided by the square root of the variance, this is called standardization. And there I talked to you talked about how you standardized scores in a test, like the GRE or the LSAT, where different people take the test at different times are different versions of the test, how do you standardize so that you can compare between people who took one test versus the students who took another test? But let me come back here, right? So this standardized variable, this follows the standard normal, right? If you look at the standard normal distribution, so this is the normal distribution is bell shaped right? Now, 95% of the observations, read for a standard normal, lie between negative 1.96 and positive 1.96. Remember, this is this is the variance and the standard deviation of the norm. Right. So what we had already learned before is that for a normal distribution, 95% of the observations lie between minus two standard deviation and plus two standard deviation. So this here, the negative 1.96, this is what I had rounded off earlier to negative two, and this I had rounded off to positive two. Right. But I'm mentioning this number here, because in a lot of places, you will see the number 1.96 crop up in a lot of statistics courses, and a lot of statistics books, right? So I want to stress home, the actual number for that 95% region is negative 1.96 up to positive 1.96. Right? I rounded it up to minus negative two plus positive two, just for ease of memory, right? But the actual number is negative 1.96. But coming back, right, so what it means is that if your statistic right, if whatever the variable which follows the normal distribution, if it lies between negative 1.96 and positive 1.96, in this case, you do not reject the H_0 , rather you accept the H_0 . But if your variable lies below this, or above 1.96, then those are the rejection regions. So that's where you reject the null hypothesis. So coming back, right, so what we are really interested in is this variable now, right? So this follows the normal distribution, right? So what we do is that once we have our data, we construct what's called the Z statistic. Right? So this is $\bar{x}$, right? So this $\bar{x} - \mu_0$ and divided by the square root of the variance, right? So we don't have data on σ^2 , so we replace it by the sample standard deviation s , right? And this is the square root of it because if you if you take the square root, right, so then this is $\sigma/\sqrt{n}$, we don't have data on σ , so we replace that σ by s . So you construct the Z statistic, right. So this is your Z statistic. And then what you have to check is whether it lies within this region minus 1.96 to 1.96. If it's outside, then you reject it. And if it's inside, then you do not reject our null hypothesis. So that's your mechanics. for hypothesis testing, you construct this statistic, right, and then you check whether it lies in this region. If it does, write, sorry, you check if it lies outside this region. If it does, then you reject the null hypothesis in favor of H_a . If it does not, then you cannot reject. So let's do an example. Again, going back to our online learning, I'm sure you're getting sick and tired of this particular example. But I just wanted to show you because we have already become so familiar with this, right? So we're testing the hypothesis $\mu = 0$ versus $\mu \neq 0$ in this case. So what we want to construct here is the Z statistic, which is $\bar{x} - \mu_0$, right? So here, $\bar{x}$ is negative three, so this is negative three minus μ_0 , not that zero, divided by $s/\sqrt{n}$. So s here is 2.5, divided by square root of n , that's 100. Right? So this is negative three divided by this will be point two, five. And this will

turn out to be negative 12. And we're going to check this with this interval, negative 1.96 up to positive 1.96. So this is way outside this interval, right? So in this case, we reject the null hypothesis of μ equal to zero. Right? So in favor of the, of the hypothesis that μ is not equal to zero, or in other words, that online learning does have an impact on student learning. Right. So what I've given you in this clip is the process that the mechanical process that you need to follow for hypothesis testing. You look at the test statistic, see if it lies within negative 1.96 to 1.96. If it lies outside, then you reject the null hypothesis. If it lies within, then you cannot reject the null hypothesis. So that's the mechanics of hypothesis testing.