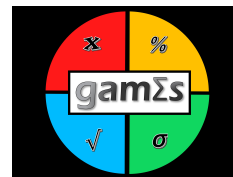


GAMES Practice Problems – Applied Differential Calculus

GAMES Practice Problem Solutions – Applied Differential Calculus



- Find $\frac{dw}{dv}$ using implicit differentiation. Make sure you have the correct answer by rearranging the expression to isolate w and taking the derivative w.r.t. v

$$3w + 6v^2 = 12$$

- Find the first and second derivative ($\frac{dw}{dv}$, $\frac{d^2w}{dv^2}$) using implicit differentiation. Make sure you have the correct answer by rearranging the expression to isolate w and taking the derivative w.r.t. v

$$w^2v = 1$$

- Find y' using implicit differentiation where b is a constant for the following

- $x^2 + y^2 = b^2$

- $\sqrt{x} - \sqrt{y} = \sqrt{b}$

- $x^4 - y^4 = x^2y^3$

- $2xy - 3y^2 = 16$

- The differentiable function h is given. y is an implicit function of x . Find y' for each of the following:

- $g(x) - y^3 = -xy$

- $g(x + y) = -x^2 - y^2$

- $(xy - 1)^2 = g(xy^2)$

- Excel: find the linear approximation of $\sqrt{1+x}$ when x is close to (about) 0. Use Excel to create a graph with both $y = 1 + 0.5x$ and $y = \sqrt{1+x}$ on the domain $x \in [-1, 3]$ and compare the two. $f'(x) = 0.5$, $f(x) = 1$, so $y \approx 1 + 0.5x$ See "GAMESAppliedCalcPracticeQuestionSolutions.xlsx for the solution in Excel"

- Find linear approximations for the following about $x = 0$

- $(1 + x)^{-1}$

- $(1 + x)^5$

- $(1 - x)^{1/6}$

- $(5x + 3)^{-2}$

- Compute the actual change in y : $\Delta y = f(x + dx) - f(x)$ and the approximate change in y : $dy = f'(x)dx$ for each of the following:

- $f(x) = x^2 + 2x - 3$ when $x = 2$ and $dx = 0.1$

- $f(x) = \frac{1}{x}$ when $x = 3$ and $dx = -0.01$

- $f(x) = f(x) = \sqrt{4}$ when $x = 4$ and $dx = 0.05$

- Find the linear approximation of $h(k) = B(1 + k)^{\frac{c}{c+d}} - 2$ where c, d, B are positive constants.

9. An Economy with Labour and Capital – Consider an economy that produces output, Q , using both capital and labour K, L . All other influential factors are represented by the constant, A .

$$AK^{2/3}L^{1/3} = Q$$

When taking Q as a constant value, recognize that the value of K depends on the value of L .

- (a) Use implicit differentiation to find the marginal rate of substitution, $\frac{dK}{dL}$. $\frac{dK}{dL} = -\frac{K}{2L}$
 (b) When $K=100$ and $L=400$ and the economy gives up one unit of capital, approximately how much labour would the economy need to add to maintain the same amount of output? $-\frac{K}{2L} = -\frac{100}{2(400)} = -\frac{1}{8}$. $1/8$ is the slope of the tangent line when $k=100$, $L=400$. $1/8$ is also the amount of capital needed to replace one unit of labor. Since the tangent line is a straight line, when the economy gives up one unit of capital, it must add 8 units of labour to maintain the same amount of output, Q .

10. Simple Macroeconomy – Consider a general version of the macroeconomy where output (Y) is equal to consumption (C) plus investment ($\bar{I}$). Note that consumption depends on output:

$$(i) Y = C + \bar{I}$$

$$(ii) C = f(Y)$$

- a) Combine these equations and find $\frac{dY}{d\bar{I}}$ assuming Y is an implicit function of $\bar{I}$. $\frac{dY}{d\bar{I}} = \frac{1}{1 + f'(Y)}$

- b) If $f'(Y) = 2$, what is $\frac{dY}{d\bar{I}}$? $\frac{dY}{d\bar{I}} = \frac{1}{1 + 2} = \frac{1}{3}$

11. Supply-Demand Model with a Tax – Consider a linear supply and demand model where consumers pay a tax of τ per unit.

$$D = a - 500(P + \tau) \quad S = \alpha + 100P$$

a and α are positive constants. The equilibrium price is found when supply equals demand:

$$a - 500(P + \tau) = \alpha + 100P$$

- (a) Solve the above for Price, P , then find $\frac{dP}{d\tau}$ using the combined equation. $\frac{dP}{d\tau} = -\frac{1}{6}$
 (b) When the tax rate increases, does the price, P , increase or decrease? Use implicit differentiation to find $\frac{dP}{d\tau}$ and interpret your results. $\frac{dP}{d\tau} = -\frac{1}{6}$

12. Consider the general forms of the supply and demand functions:

$$D = f(P + \tau) \quad S = g(P)$$

where $f' < 0$ and $g' > 0$

Determine if the Price paid by the consumer, $P + \tau$, increases or decreases when the tax rate, τ , is increased? $\frac{d(P + \tau)}{d\tau} = \frac{1}{1 - \frac{f'(P + \tau)}{g'(P)}}$



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