GAMES Practice Problems – Applied Differential Calculus GAMES Practice Problem Solutions – Applied Differential Calculus

1. Find $\frac{\mathrm{d}w}{\mathrm{d}v}$ using implicit differentiation. Make sure you have the correct answer by rearranging the expression to isolate w and taking the derivative w.r.t. v

$$3w + 6v^2 = 12$$

2. Find the first and second derivative $(\frac{\mathrm{d}w}{\mathrm{d}v}, \frac{\mathrm{d}^2w}{\mathrm{d}v^2})$ using implicit differentiation. Make sure you have the correct answer by rearranging the expression to isolate w and taking the derivative w.r.t. v

$$w^2v = 1$$

3. Find y' using implicit differentiation where b is a constant for the following

   (a) $x^2 + y^2 = b^2$

   (b) $\sqrt{x} - \sqrt{y} = \sqrt{b}$

   (c) $x^4 - y^4 = x^2y^3$

   (d) $2xy - 3y^2 = 16$

4. The differentiable function h is given. y is an implicit function of x. Find y' for each of the following:

   (a) $g(x) - y^3 = -xy$

   (b) $g(x + y) = -x^2 - y^2$

   (c) $(xy - 1)^2 = g(xy^2)$

5. Excel: find the linear approximation of $\sqrt{1+x}$ when x is close to (about) 0. Use Excel to create a graph with both $y = 1+0.5x$ and $y = \sqrt{1+x}$ on the domain $x \in [-1, 3]$ and compare the two. $f'(x) = 0.5, f(x) = 1, soy \approx 1 + 0.5x$ See "GAMESAppliedCalcPracticeQuestionSolutions.xlsx for the solution in Excel"

6. Find linear approximations for the following about $x = 0$

   (a) $(1 + x)^{-1}$

   (b) $(1 + x)^5$

   (c) $(1 - x)^{1/6}$

   (d) $(5x + 3)^{-2}$

7. Compute the actual change in y: $\Delta y = f(x + \mathrm{d}x) - f(x)$ and the approximate change in y: $\mathrm{d}y = f'(x)\mathrm{d}x$ for each of the following:

   (a) $f(x) = x^2 + 2x - 3$ when $x = 2$ and $\mathrm{d}x = 0.1$

   (b) $f(x) = \dfrac{1}{x}$ when $x = 3$ and $\mathrm{d}x = -0.01$

   (c) $f(x) = f(x) = \sqrt{4}$ when $x = 4$ and $\mathrm{d}x = 0.05$

8. Find the linear approximation of $h(k) = B\,(1 + k)^{\frac{c}{c+d}} - 2$ where c, d, B are positive constants.

9. An Economy with Labour and Capital – Consider an economy that produces output, Q, using both capital and labour K, L. All other influential factors are represented by the constant, A.

$$AK^{2/3}L^{1/3} = Q$$

When taking Q as a constant value, recognize that the value of K depends on the value of L.

(a) Use implicit differentiation to find the marginal rate of substitution, $\frac{\mathrm{d}K}{\mathrm{d}L}$. $\frac{\mathrm{d}K}{\mathrm{d}L} = -\frac{K}{2L}$

(b) When K=100 and L=400 and the economy gives up one unit of capital, approximately how much labour would the economy need to add to maintain the same amount of output? $-\frac{K}{2L} = -\frac{100}{2(400)} = -\frac{1}{8}$. 1/8 is the slope of the tangent line when k=100, L=400. 1/8 is also the amount of capital needed to replace one unit of labor. Since the tangent line is a straight line, when the economy gives up one unit of capital, it must add 8 units of labour to maintain the same amount of output, Q.

10. Simple Macroeconomy – Consider a general version of the macroeconomy where output (Y) is equal to consumption (C) plus investment ($\bar{I}$). Note that consumption depends on output:

$$\text{(i)}Y = C + \bar{I}$$
$$\text{(ii)}C = f(Y)$$

a) Combine these equations and find $\dfrac{\mathrm{d}Y}{\mathrm{d}\bar{I}}$ assuming Y is an implicit function of $\bar{I}$. $\dfrac{\mathrm{d}Y}{\mathrm{d}\bar{I}} = \dfrac{1}{1+f'(Y)}$

b) If $f'(Y) = 2$, what is $\dfrac{\mathrm{d}Y}{\mathrm{d}\bar{I}}$? $\dfrac{\mathrm{d}Y}{\mathrm{d}\bar{I}} = \dfrac{1}{1+2} = \dfrac{1}{3}$

11. Supply-Demand Model with a Tax – Consider a linear supply and demand model where consumers pay a tax of τ per unit.

$$D = a - 500\,(P + \tau) \qquad\qquad S = \alpha + 100P$$

$aand\alpha$ are positive constants. The equilibrium price is found when supply equals demand:

$$a - 500\,(P + \tau) = \alpha + 100P$$

(a) Solve the above for Price, P, then find $\dfrac{\mathrm{d}P}{\mathrm{d}\tau}$ using the combined equation. ys

(b) When the tax rate increases, does the price, P, increase or decrease? Use implicit differentiation to find $\dfrac{\mathrm{d}P}{\mathrm{d}\tau}$ and interpret your results. ys

12. Consider the general forms of the supply and demand functions:

$$D = f(P + \tau) \qquad S = g(P)$$

where $f' < 0$ and $g' > 0$

Determine if the Price paid by the consumer, $P + \tau$, increases or decreases when the tax rate, τ, is increased? ys