Towards an Ecological Macroeconomics: Linking Energy and Climate in a Stock-Flow Consistent Input-Output Framework

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A Dissertation
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy

Graduate Program in Environmental Studies

June 2021

Abstract

This dissertation examines the development of a stock-flow consistent input-output (SFCIO) model in continuous time and its applications to energy analysis and climate change at the macroeconomic level. The approach used in the dissertation is to explore the SFCIO model via the development of a sequence of progressively more detailed models. The first portion of the dissertation is concerned with the analysis of several key problems: first, the integration of physical quantities (energy and power) with macroeconomic variables and the issue of dimensional analysis; second, the introduction of physically determined investment equations; third, the modelling of endogenous prices as "physical market clearing" mechanisms; and fourth, the presentation of an overall model calibration scheme based on a life-cycle net energy approach.

The second portion of the dissertation presents a sequence of energy transition models designed to explore the dynamics of economies attempting both a transition in how energy is generated (from fossil-fuels to renewables), and how energy is used (the electrification of end use). Using scenario and sensitivity analysis, the conditions under which economies succeed or fail at generating emissions trajectories consistent with a 1.5-degree target are explored. A three sector (renewable, fossil-fuel, and manufacturing) energy transition integrated assessment model is developed that includes endogenous depletion-based fossil-fuel EROI, an endogenous energy storage based renewable EROI, and climate interactions via coupling with the BEAM carbon cycle model and a two-component energy balance model. This is further extended to incorporate trade dynamics under assumptions of capital mobility and immobility.

A principal finding of this work is that, assuming no large-scale deployment of negative emissions technologies, the only conditions under which an SFCIO modelled economy can achieve emissions trajectories consistent with a 1.5 degree target is immense up-front investment in renewables and a rapid rate of decommissioning and replacing non-electrified capital with electrified capital and that degrowth alleviates the magnitude of these requirements.

Acknowledgements

There are many people to whom I owe a great debt of gratitude. First and foremost, my supervisor Peter Victor. The writing of a dissertation, and graduate school as a whole, is often said to be a lonely and difficult affair. Contrary to this, I might generally summarize large parts of the process as fun and fascinating. No small part of this is due to the exceptional supervision, mentoring, and friendship that was offered by Peter. A major thanks is also owed to Peter Brown an Dina Spigelski for their roles in the Economics for the Anthropocene (E4A) program which was so fundamental to my doctoral experience.

I would also like to thank Martin Bunch for inviting me, on a whim, to be part of the SPEAR lab at York. Many good discussions were had both in lab at various campus pubs. I am grateful to both Martin and Atif Kubursi for being part of my academic committee these past years and for their valuable inputs to the process.

I want to extend a major thank you to the CANSEE and ecological economics community in Canada (and Vermont) as well. I was fortunate during this process to encounter fascinating, kind, and just plain friendly people with whom I have made many lasting friendships.

Finally, I must say thank you to family for providing love and encouragement during the whole process.

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Chapter 1

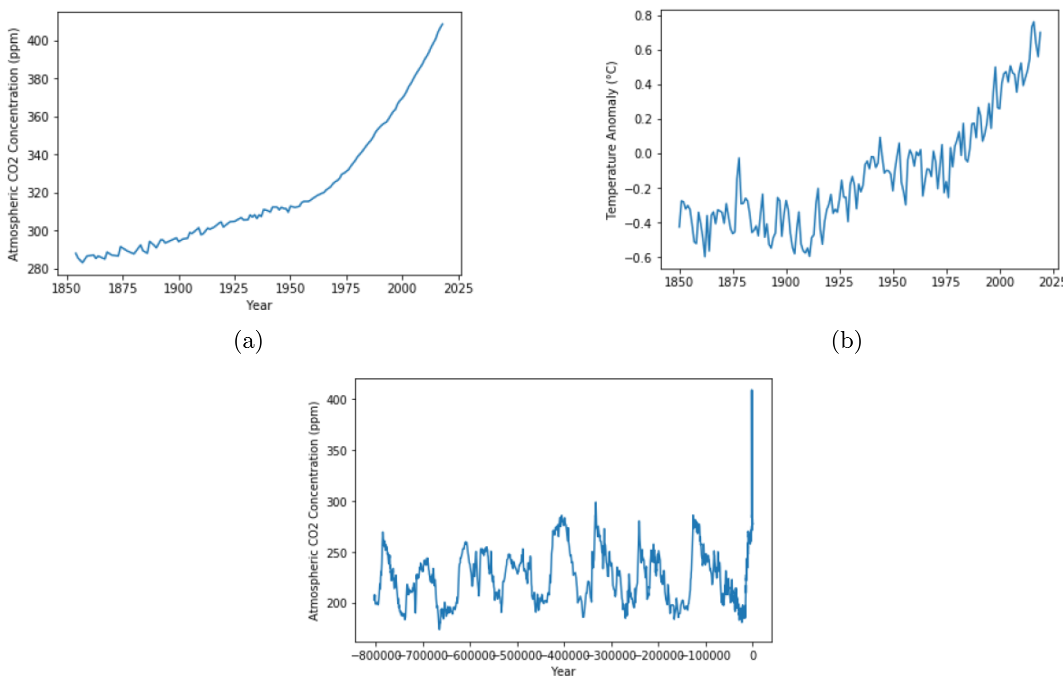
Chapter One: Introduction

In their recent landmark paper, (Sherwood et al, 2020) have narrowed the range of likely values for a key climate parameter (the equilibrium climate sensitivity) to between 2.6 and 4.1 degrees Celsius (66% confidence). In a simplified sense, this parameter represents the expected long term (on the order of centuries) change in global-average temperatures that would arise from a doubling of atmospheric carbon dioxide (CO_2) over pre-industrial levels. Since pre-industrial times, atmospheric concentrations have increased by nearly 50% from a baseline of approximately 280 parts per million (ppm). Emissions over the next decades of this century may make the modelling experiment of doubling atmospheric concentrations a real one with civilization-scale existential consequences. For example, (Xu et al, 2020) note that under business as usual warming assumptions, up to one third of the global population may experience mean annual temperatures well in excess of the historical conditions in which most of humanity has lived. This grand geophysical experiment being undertaken today, and the possibly near term catastrophic issues it presages, provides the complicated background context in which any long-term macroeconomic inquiry must now be situated.

In May 1958, the atmospheric carbon dioxide (CO_2) concentration was recorded at 317.51 parts per million (ppm), at the Mauna Loa observatory in Hawaii. A symbolic milestone of sorts was crossed as of May 2020 as concentrations increased past 417 ppm to hit a record high of 417.16 ppm (Keeling et al, 2001).[1] In slightly over 60 years atmospheric concentrations increased by approximately 100 ppm representing a dramatic and near instantaneous (in geological time) shift. Atmospheric CO_2 concentrations, reconstructed from ice-core data, indicate that this is the highest level in the past 800 thousand years (Luthi et al, 2008). More recent,

[1] Atmospheric Concentrations display a natural seasonality with annual peaks occurring in May.

though less certain, evidence suggests current concentrations may be the highest in the past 23 million years (Cui et al, 2020). The following figure presents historical time series data for atmospheric CO_2 concentrations and the global average temperature anomaly.



(a) (b) (c)

Figure 1.1: Panels (a) and (b): Evolution of atmospheric CO_2 and global average temperature anomaly since 1850. Panel (c): Reconstructed CO_2 concentrations over the past 800,000 years.

It is very well understood that this remarkable increase in concentration and the immensely rapid rate at which it has occurred is driven by anthropogenic emissions and it is also well understood that this increase has led, and will lead to, the additional warming of the planet (IPCC, 2018). What is of course far less well understood is how human civilization will manage this change. It is worth noting for perspective that modern anatomical humans emerged in the last 800 thousand years, and civilization only in the last ten thousand years of relative Holocene stability.

Modifying the concentrations of atmospheric CO_2 and influencing the trajectory of planetary warming is only one facet of the large-scale changes being wrought on the earth-system by human actions. As documented in (Rockstrom et al, 2009) and (Steffen et al, 2015), anthropogenic activities are perturbing a whole suite of earth-system processes. These perturbations include, for example, the destruction of biodiversity, the acidification of oceans, and the change in biogeochemical cycling. Taken together, human activity has in a very short span of time altered the physical system in which it is embedded and is currently, short of fairly dramatic alterations in function, likely to induce changes of even larger magnitude. In (Steffen et al, 2018), the authors note the possibility of a self-reinforcing feedbacks in the earth-system leading to a shift from the current inter-glacial state to a hot-house state much different from the current climate we are collectively accustomed to, which they suggest, can be averted by "collective human action".

If environmental and climatic changes present one set of difficult to analyse problems, then what comprises "collective human action" presents another. The third chapter of the IPCC special report on the global Warming of 1.5 °C (SR1.5) indicates that fundamental changes are necessary across many basic aspects of societies such as energy generation, agriculture, and transportation; and furthermore, that we may be reliant on yet unproven negative emissions technologies even taking into account other actions (de Coninck et al, 2018).

Environmental degradation, the threat of catastrophic and irreversible climate change, and the possibility that successful mitigation and adaption to climate change may require attempting immensely complex changes under increasingly narrow time constraints define, arguably, the key underlying features any macroeconomics must be able to address going forward. However, such analysis is complicated by more than just the inherent physical uncertainties in modelling the climate system or capturing energy system transitions. At the core of any attempt to understand the future of economies in a world dominated by considerations of climate and energy is the requirement to understand how these factors may impact us collectively, and how to incorporate them into the tools used to study the problem.[2] The magnitude of the challenge to accomplish this was put rather succinctly by economist Martin Weiztman:

> I believe that the most striking feature of the economics of climate change is that its extreme downside is non-negligible. Deep structural uncertainty about the unknown unknowns of what might go very wrong is coupled with essentially unlimited downside liability on possible planetary damages (Weitzman, 2011).(p.275)

Understanding how economies might realize the deep emissions reductions necessary to generate 1.5°C or 2 °C consistent emissions pathways is a problem composed of, in the most basic

[2]Aspects of this problem are addressed in some detail in Chapter four.

sense, three parts; the energy problem, the climate problem, and the macroeconomic problem. In order to model the system under study, the modeller must be able to say something about energy and energy systems; be able to relate this to the greater earth-system; and ultimately link both to some representation of a macroeconomy. Ultimately, reaching net zero emissions by some target date will necessitate both rapid and fundamental changes to the basic physical systems that modern societies are built upon (especially energy, transportation, and agriculture).

The challenge for the macroeconomist aiming to study this problem is simple to state, and rather more difficult to answer. How does one meaningfully include concepts from the physical and engineering sciences into mathematical models meant to describe a whole economy? A large and growing body of work on models built to capture aspects of energy, environment, and economy are surveyed in chapter three. These integrated assessment models (IAMs) generally (but not always) combine various elements of neoclassical theory (growth theory, production functions, and utility functions for example) with a few notable exceptions working from heterodox traditions generally built on a synthesis of post-Keynesian and ecological economics principles. The degree to which these models incorporate physical systems varies with both the complexity of the model and their stated purpose.

Unlike, more conventional neoclassical approaches to macroeconomics, one will not find anything resembling utility functions, infinite horizon optimization, representative agents, or microfoundations in the modelling work undertaken here. The energy-transition models to be developed in this dissertation are dynamic systems whose evolutions through time are determined, not by equilibrium considerations, but by interaction of key physical and monetary mechanics. Importantly, we eschew entirely the notion of discount rates and the cost-benefit analysis of future climate damages as mechanics that drive model outcomes. Rather, the inspiration for the models is derived from several heterodox schools of economic thought; principally the stock-flow consistent approach to macroeconomics, biophysical economics, and ecological economics.

The basis of the modelling in this dissertation rests on the syncretic union of two different macroeconomic mathematical formalisms; the transactions flow and balance sheet matrices from the stock-flow consistent approach and the input-output matrix arising from input-output analysis. The former of these formalisms (in the most basic sense) provides the rules and accounting logic governing monetary flows and financial stocks in an economy[3] while the latter captures both the sectoral interdependencies of economic sectors and the relationship between final demand and total outputs. The purpose of such an approach is to construct a coherent modelling framework that moves seamlessly from financial considerations to physical ones. Put simply, we need to be able to relate things measured in dollars to other things measured in kilowatts or gigatonnes and doing so consistently is a key feature of the approach taken here.

There are therefore two principle aims to this dissertation. The first is to present the development of the stock-flow consistent input-output (SFCIO) framework in continuous-time and show how energy and climate may be integrated with the framework in a physically meaningful manner. At the risk of employing some pompous language, this aim is one of basic scientific interest. For example, we know that the operation of an economy like Canada's is heavily dependent on a stable electric power system operating at a frequency of 60 Hz. The operation of such a system in the context of macroeconomics is not obvious, yet the future is dominated by considerations of electrification and the mass-deployment of variable renewable generation. We know these systems are interrelated in reality which spurs a curiosity to understand how they might be related in our formal economic modelling. In this dissertation we will work out the basics on how some of these considerations may be addressed.

The second aim is to use the resulting framework to explore an understudied aspect of transition studies; the possibility of transition pathways that do not rely on negative emissions technologies (NETs) and do not necessarily assume long term economic growth. While more will be said about these technologies (examples include afforestation, bio-energy carbon capture and storage, and direct air capture) in chapter three, we note here that in integrated assessment macroeconomic modelling, NETs play an outsize role in reducing cumulative emissions sufficiently by the end of century to keep warming within given targets (generally 1.5°C and 2°C). Given the importance of these technologies in modelling there is little evidence for their use at the scales required (Anderson and Peters, 2016). Generally, IAMs are characterized by what we might here term a double-blind spot; they rely on an unproven category of technologies and generally fail to explore scenarios not characterized by long term economic growth. In the

[3] We must be careful here. The basic rules of a transactions flow table (rows and columns sum to zero) do not themselves tell us the precise nature of any given flow. Rather, these overarching accounting necessities tell us how any possible set of flows must behave when viewed as a whole. The issue of what we can learn solely from these rules is addressed more fully in chapter four.

latter chapters of this dissertation, we will explore scenarios that are neither reliant on NETs or predicated on long-term growth.

A second feature of IAM modelling is the inadequate or complete absence of a representation of the financial dimension of a macroeconomy. A telling statement appears in (McCollum et al, 2013) noting that in the IAMs examined in their multi-model study, how and through what mechanisms the investment necessary to facilitate the transition was not accounted for. Ultimately, any successful transition will require immense investments in areas like new renewable generation capacity, energy efficiency measures, and electrification. Failing to account for finance in macroeconomics is already, we hold, indefensible but doing so in the climate context is even more egregious given the possibility that climate damages and economic contraction may wreck havoc on the stability of the system in the first place (see (Dafermos et al, 2018) for an exploration of this particular topic).

In this dissertation we will therefore ask a "how" question followed by a "what" question. The how question concerns **how** we might put together a model that links, in a sensible fashion, various issues ranging from electrification of end use, the energetics of storage, financing, emissions, global average temperatures, and impacts on the economy. The first four modelling chapters (four through seven) present the fundamentals of the SFCIO model in continuous time, while chapters eight and ten introduce key physical processes. The what question asks under **what** conditions (economic and physical) is it possible for an IAM model to generate a 1.5°C consistent emissions pathway without assuming the use of negative emissions technologies, this is primarily dealt with in chapters twelve and thirteen.

The following presents the structure of the dissertation.

- **Chapter 2** introduces the decoupling problem. It first presents a brief overview of historical climate, energy, and macroeconomic trends over the past half century, and then presents two simple mathematical models of growth and decoupling to generate carbon budget consistent emissions pathways.

- **Chapter 3** reviews the literature on global integrated assessment models (IAMs) used to study the relationship between future macroeconomic, emissions, and temperature increases.

- **Chapter 4** presents four modelling principles concerning the construction of macroeconomic theory on physically consistent grounds. The chapter also presents an abstract analysis of the nature of the stock-flow consistent transactions flow matrix and several of its mathematical properties.

- **Chapter 5** presents the first SFCIO model to be developed focusing on issues of the integration of energy and power into a macroeconomic framework, and the union of the SFC and input-output formalizations.

- **Chapter 6** introduces energy capital to the SFCIO framework and explores more thoroughly the problem of dimensional analysis. The chapter also presents an example of how a macroeconomic model can be shown to violate the law of conservation of energy.

- **Chapter 7** relaxes the static pricing assumption allowing the energy price to become endogenously determined. This modification moves the model beyond the framework of linear-differential equations of the previous models.

- **Chapter 8** introduces a more complete economic model by adding in a simple private banking sector, manufacturing capital, profits, and employment. The chapter concludes by deriving the model's underlying conservation of mass equations.

- **Chapter 9** presents a simple method to calibrate model energy parameters on the basis of life-cycle energy return on investment (EROI) for renewable and fossil-fuel energy providers.

- **Chapter 10** presents a three sector "linear" energy transition SFCIO model designed to explore emissions pathways generated by varying assumptions about the rate of renewable capacity construction and future economic growth. Three economic scenarios (growth, steady-state, and degrowth) are developed in order to study the relationship between growth and emissions pathways, for a given rate of renewable construction.

- **Chapter 11** presents the core physical feedback mechanisms to be implemented in the dissertation's IAM model. Fossil-fuel and renewable EROI's are made endogenous via the introduction of depletion and energy storage losses, respectively. Furthermore, the chapter introduces the BEAM carbon-cycle model as well as a two-component energy balance model whose union will form the basis of the IAM model's climate component. Finally, the chapter ends on a discussion of the problem of relating temperature feedbacks to the economy.

- **Chapter 12** presents the non-linear IAM model linking the SFCIO macroeconomic approach with climate and energy system feedbacks. The model is designed to answer the question of whether it is possible to, given current conditions, generate 1.5 degree consistent pathways assuming no negative emissions; and if so, what are the assumptions necessary to do so.

- **Chapter 13** extends the IAM model of chapter 12 to a two-region trade model in order to explore the interactions between trade and the stringent emissions reductions policies introduced in the closed economy framework. key questions concern the nature and feasibility of the degrowth mechanism for emissions reductions assuming a trading partner pursuing minimal climate policies and focusing on economic growth.

- **Chapter 14** concludes the dissertation by discussing the implications of the model pathways generated in the previous chapters.

Chapter 2

Chapter Two: Historical Trends and the Decoupling Problem

Close to three fourths of total anthropogenic emissions are related to energy use and as of the latest 2019 data, 84% of primary energy is sourced from fossil-fuels (BP, 2020). Changing how energy is obtained and how it is used presents arguably the greatest challenge to overcome in order to reduce emissions. This is laid out unequivocally in the SR1.5 that meeting emissions targets requires "pathways with no or limited overshoot include a rapid decline in the carbon intensity of electricity and an increase in electrification of energy end use (high confidence)" (Rogelj et al, 2018). This statement captures a key problem; to reduce emissions and mitigate climate change requires a transition both in how energy is generated and how it is used. It is this dual transition that we explore in the latter chapters of this dissertation.

To gain some sense of the magnitude of the challenge it is instructive to examine a breakdown of global primary energy consumption by type and by year in order to see the evolution of the total over time as well as the change in the relative importance of certain primary energy sources.[1]

[1] The United States Energy Information Administration defines primary energy as "Energy in the form that it is first accounted for in a statistical energy balance, before any transformation to secondary or tertiary forms of energy. For example, coal can be converted to synthetic gas, which can be converted to electricity; in this example, coal is primary energy, synthetic gas is secondary energy, and electricity is tertiary energy. " See https://www.eia.gov/tools/glossary/index.php?id=Primary%20energy

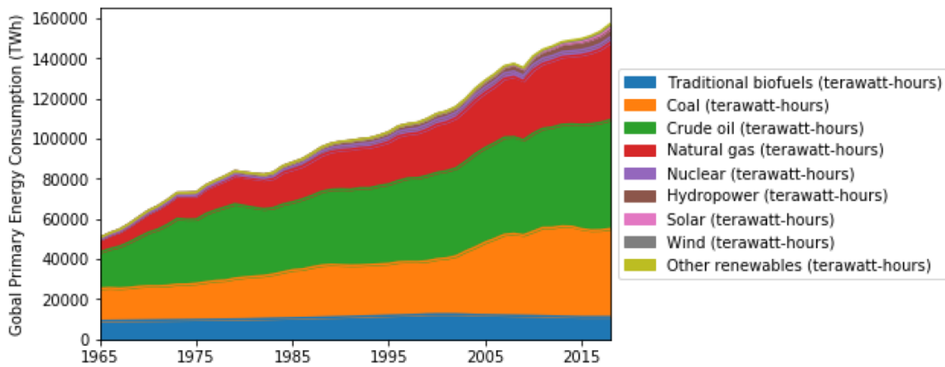


Figure 2.1: Global Primary Energy Use by Year and Type (BP, 2020)

Two key features emerge from a cursory exploration of figure 2.1: first, the share of renewable primary energy (including hydroelectric generation) in the primary energy supply forms only a small, though growing fraction, of the total.[2] Second, primary energy consumption has roughly tripled over the 53 years plotted. Given the relative dominance of fossil-fuels in the primary energy supply, this increase naturally correlates with the rapid increase in atmospheric concentrations of CO_2 over the same time period as displayed in Figure 1.1.

This immense growth in primary energy consumption over this time reflects an approximate doubling of consumption by regions such as North America and Europe, and a near 14-fold increase by the Asia pacific region. Figure 2.2a shows the regional breakdown of primary energy consumption from 1965 for six global regions. As coal, crude oil, and natural gas form the largest share of primary energy consumption in the above data their growth overtime is reflected in a commensurate growth in emissions. This is displayed in Figure 2.2b which plots global as well as regional emissions (for Europe, North America, and the Asia-Pacific) per year since 1750.

Over the same time period the global economy has grown by a factor of more than 8 times which is shown in Figure 2.2c. While at this point we can only note correlations, it is clear that the immense growth in economic activity is strongly correlated with the growth in primary energy consumption as well as emissions.[3] In examining these series, a relatively obvious and familiar pattern begins to become apparent. Emissions, primary energy use, economic output,

[2]One Terawatt Hour is equivalent to 0.0036 Exajoule, or 10^{18} Joules.

[3]We can only say correlation here as we do not yet have a specific mechanism to link energy use and GDP. In later models every unit of GDP produced will be explicitly tied to energy use but for the time being we can only speak in intensities and correlations. An obvious example of a physical mechanism that ties two series together is the combustion process linking the conversion of primary energy from fossil-fuels to emissions. In such a case we can clearly make stronger statements than correlations.

and atmospheric concentrations at the global level undergo dramatic growth in the second half of the twentieth century. In fact, this pattern is well documented across a much larger number of physical and social phenomena and has been termed the great acceleration (see Figures 1,2, and 3 in (Seffan et al, 2015)) for a more detailed examination.

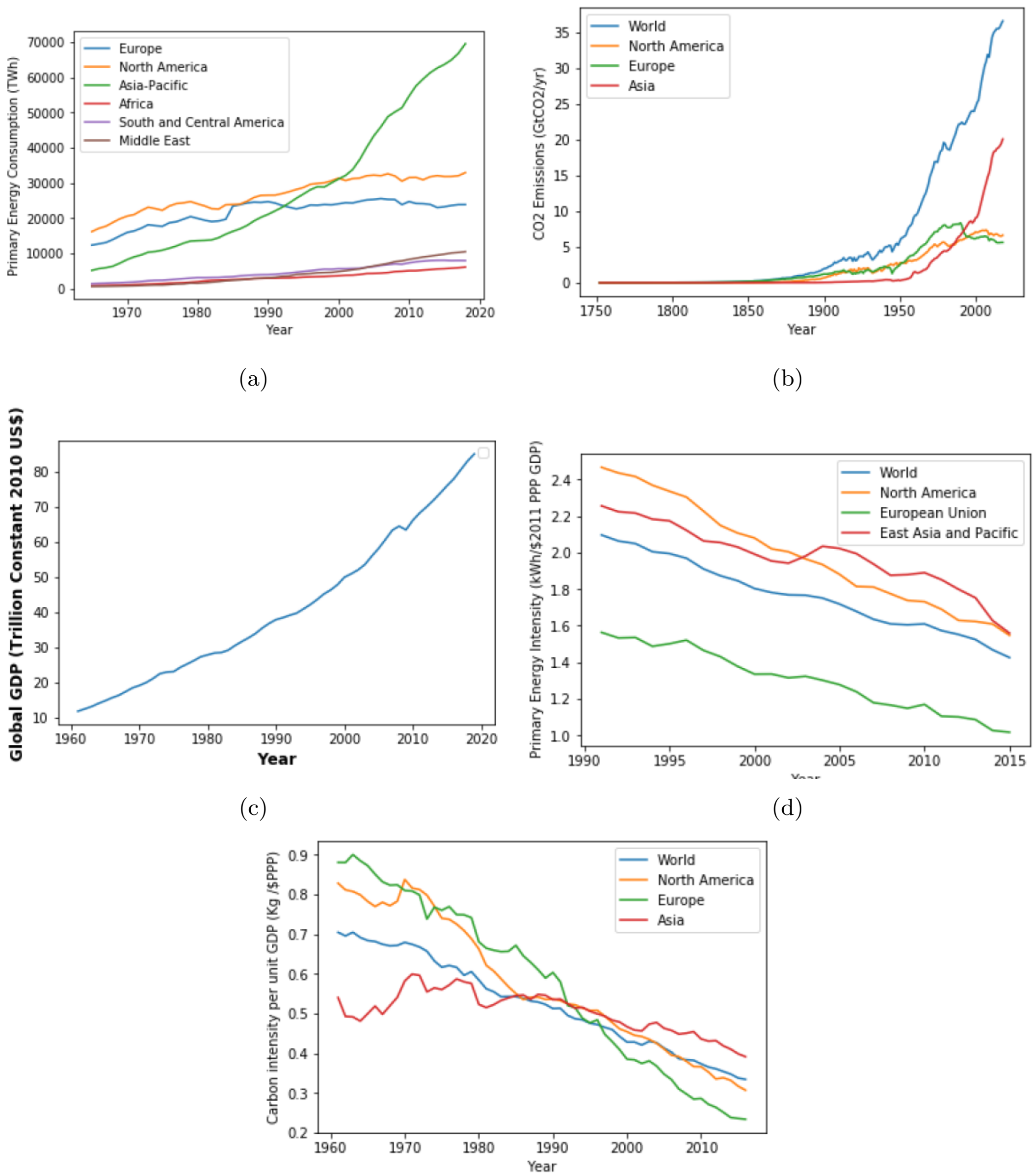
The correlation between economic growth and the material inputs and waste outputs is an immensely important one that will be explored in greater detail in the following subsection. At present however we may define the basic concept of a physical GDP intensity which gives the magnitude of some physical phenomenon per unit of GDP. The global (and regional) level trend for primary energy intensity is displayed in Figure 2.2d. Primary energy intensities all decline over the thirty years of data availability from the initial levels indicating, broadly, both an increasingly efficient use of primary energy (inputs) in the production of gross domestic product as well as a change in the composition of gross domestic product itself. However there is another key source of efficiency gains that is occurring; which is the efficiency of converting primary energy to final use.[4] Estimates, for example, for aggregate primary to end stage energy use conversion efficiencies are estimated as approximately 20% in 1900, 35% in 1950, and 50% in 2015 (Smil, 2017).

The same long-term intensity decline is evident from the data for carbon intensities of GDP as shown in Figure 2.2d again indicating that GDP is becoming more emissions "efficient". Now, without the context of the previous graphs concerning the levels of primary energy, emissions, and GDP growth, these figures might appear fairly optimistic as, in a loose sense, every new unit of GDP is being produced with less consumption of primary energy and less output of emissions. However, in the context of the above graphs, it is strikingly clear that the increases in efficiencies are outweighed by the overall increase in magnitude of GDP over the same time horizon.

Figures 2.2d and 2.2e are examples of relative decoupling (the decline in intensity) of some physical metric from GDP. As evidenced by the data, it is manifestly possible to witness consistent relative decoupling and increasing total magnitudes of the same physical quantities over time.[5] What there is little evidence for here at the global level is of absolute decoupling of

[4]For example, coal represents a source of primary energy which can be used to produce heat and drive a turbine to produce electricity. This electricity itself may be converted to heat and light in lightbulbs. The final use for the original primary energy here is illumination. Increases in the conversion efficiencies at each step reduce the total primary energy input required.

[5]For a very readable introduction to relative and absolute decoupling, see chapter five in Jackson, (2010).



(a) (b) (c) (d) (e)

Figure 2.2: Global and Regional Data for primary energy consumption, CO_2 emissions, economic growth, and primary energy and emissions intensities. (BP, 2019) and (Ritchie and Roser, 2017).

emissions and economic growth. Absolute decoupling of emissions, for example, may be simply understood as a decline in emissions in an absolute sense (not just the intensity). Ultimately ameliorating or mitigating climate change (and a host of other environmental issues) will require absolute decoupling to occur. In the following section we will work out mathematically the exact rates at which such decoupling must occur to meet any given Carbon budget.

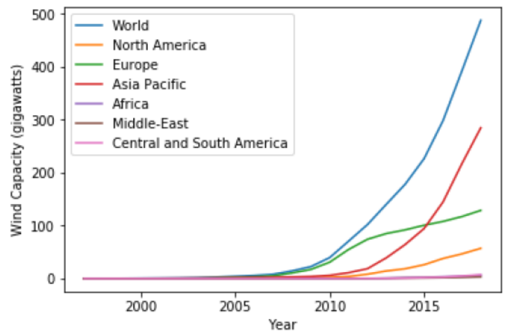
Concerning renewables we see that solar and wind capacity has been growing in an exponential fashion since the last decade of the twentieth century; this is shown in Figures 2.3a and 2.3b. Given this large increase in wind and solar capacity is it puzzling that the renewable share of electricity generation (Figure 2.3f) has increased more substantially? Not necessarily. Electricity consumption has also increased over the same time period for the same regions, with the largest growth occurring in the Asia-Pacific region which is displayed in Figures 2.3c and 2.3d. While it is certainly the case that renewable capacity has increased, those gains are offset by the underlying growth in electricity demand itself. Another way to note this is to examine the modern renewable share of energy final demand.[6][7] The global modern renewable share of final energy consumption is plotted in Figure 2.3e. While the share of renewable energy in in consumption is growing it is not at a sufficient rate to offset the underlying growth in energy consumption itself.

It is evident over the period of data availability that the renewable share is indeed increasing; it is also clear that at the very aggregated global level, that there is a relatively large gap between the role renewables are currently playing in meeting final energy demand and the role they must play in order to achieve substantial energy emissions reductions. A more detailed look at this issue may be found by examining the most recent Lawrence Livermore National Laboratory energy Sankey diagram for the United States displayed in 2.4.

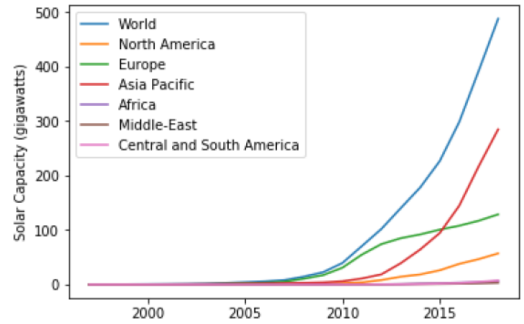
One major necessary condition for emissions reductions involves the simultaneous reduction in the carbon intensity of electricity generation coupled with the large-scale electrification of end use. In 2018, the last year of full data availability, combustible fuels accounted for approximately 66.3% of electricity generation, nuclear approximately 10.1%, and renewables (including hydro), the remaining fraction (IEA, 2020); this share has not changed substantially since 1990 (World

[6]Final Energy is defined by EUROSTAT as “Final energy consumption is the total energy consumed by end users, such as households, industry and agriculture. It is the energy which reaches the final consumer’s door and excludes that which is used by the energy sector itself”.

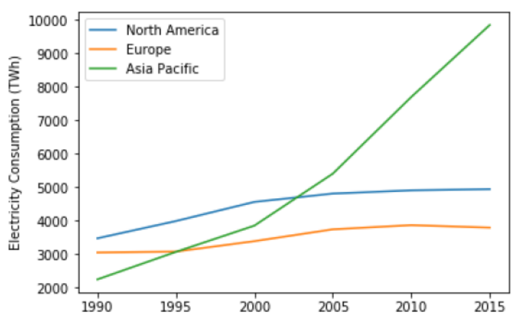
[7]Modern renewables are renewables that exclude a category of fuels termed traditional biomass. Traditional biomass is comprised of wood fuels, agricultural by-products, and dung.



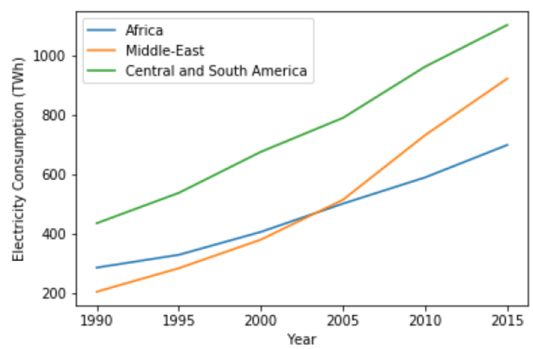
(a)



(b)



(c)



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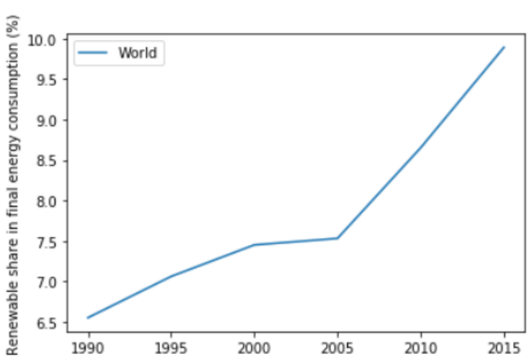
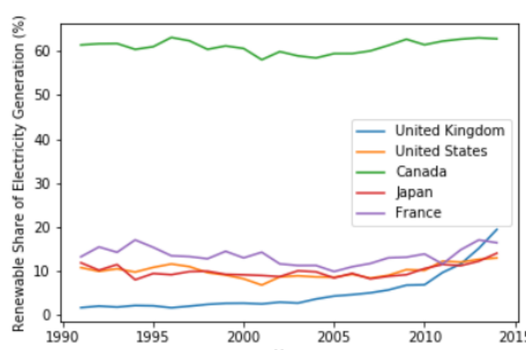


(e)



(f)

Figure 2.3: Global and Regional Data for renewable capacity, electricity consumption, and renewable shares of final energy and electricity generation.

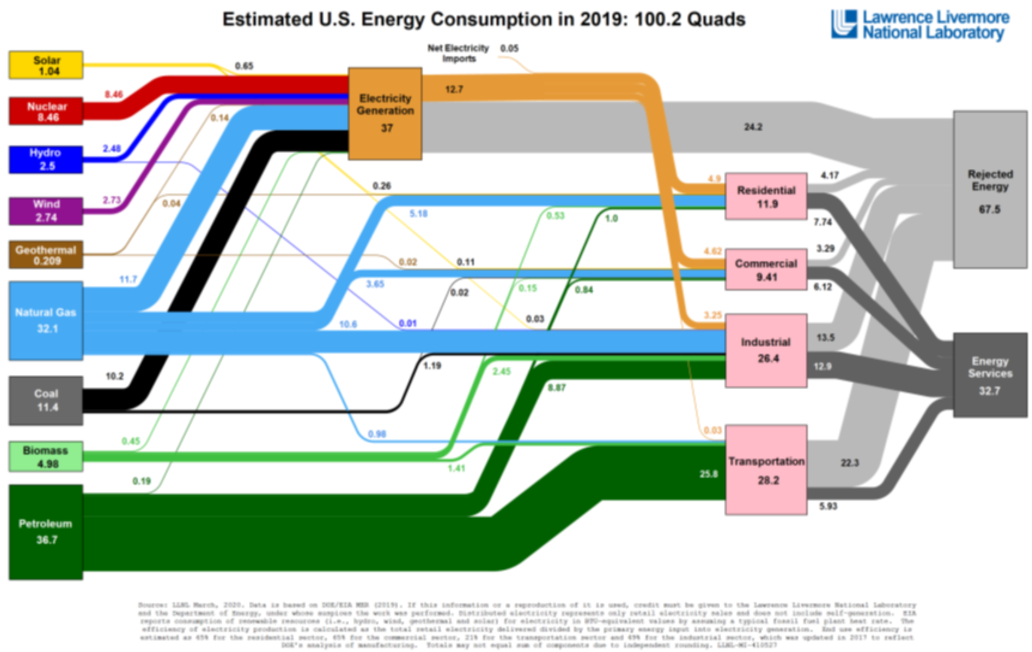


Figure 2.4: Sankey Energy Diagram. Retrieved from https://flowcharts.llnl.gov/ (LLNL, 2020)

Bank, 2014). The aggregate global figure hides several interesting country level differences; for example, Figure 2.3f plots the renewable share of electricity generation for five G7 countries and indicates large differences between several select countries. First, the large difference between Canada and the other countries reflects the fact that approximately 61% of Canadian electricity is generated by hydroelectric sources (CER, 2019). Secondly, while each of the renewable shares for the five countries plotted do increase for the five countries plotted, it is not, except in the case of the United Kingdom, a large improvement over the initial period value. However, some caution is necessary in interpreting these results as, in the case of France, a very large fraction (on the order of 70%) of electricity is generated by Nuclear.

Figure 2.4 indicates, for an advanced economy like the United States, the complexity of a dual transition in generation and end use. First, in 2019, the shares of coal and natural gas in electricity generation are still relatively large; second, the share of end use (residential, commercial, industrial, and transportation) that is non-electrified (e.g., uses natural gas, petroleum, and biomass) is relatively large. In particular the share of transportation energy consumption that is electrified is very small in comparison to the sector's total consumption.

Taken together the above data indicate that, at least at a global level, there is a very substantial gap between the necessary energy system changes required to achieve large-scale emissions reductions, and the current state the global system. While such a broad analysis necessarily misses the nuance at the country and local levels it does indicate the nature of the challenge. Put simply, achieving emissions reductions in line with a 1.5 °C budget requires both rapid and fundamental changes to many facets of modern economies. In the following section we will begin exploring the magnitudes underlying these changes by examining the relation between economic growth, intensity declines, and remaining carbon budgets.

2.1 The Simple Mathematics of Growth and Decoupling With a Carbon Budget Constraint

While the previous section reviewed evidence for and against relative and absolute decoupling it did not indicate the magnitude of the necessary rates of intensity declines for emissions that are now required to meet emissions targets associated with, say, the 1.5 $^\circ C$ warming carbon budget outlined in (Rogelj et al, 2018). In this section we construct a simple mathematical model linking emissions, economic growth, emissions intensity, and a carbon budget and derive analytical expressions for both the time to budget exhaustion and the rate of intensity decline associated with emissions pathways consistent with a carbon budget of a given magnitude. [8]

Let $Y(t)$ [\$ yr^{-1}], $E(t)$ [GtCO$_2$ yr^{-1}], and $I(t)$ [GtCO$_2$ \$$^{-1}$] denote the GDP, emissions, and emissions intensity at time t respectively and note that $E(t) = Y(t)I(t)$ by definition.[9] Denoting the rate of economic growth as g and the rate of intensity decline as r then we may write the following expressions for the evolution of $Y(t)$ and $I(t)$ as:

$$Y(t) = Y(0)e^{gt} \tag{2.1}$$

$$I(t) = I(0)e^{-rt} \tag{2.2}$$

[8]Similar calculations have, for example, been undertaken in (Victor, 2010), (Victor, 2012), and (Sers and Victor, 2020).

[9]While it is conventional to write the units of the relevant quantities in square brackets as done above we shall not always do so in this dissertation. Units will be discussed explicitly when necessary and otherwise left unstated when obvious.

where $Y(0)$ and $I(0)$ are initial year values for GDP and intensity. Emissions per year $E(t)$ is simply the product of $E(t) = Y(t)I(t)$ which after some simplification results:

$$E(t) = E(0) \cdot e^{(g-r)t} \tag{2.3}$$

Since the $B(t)$ is the remaining carbon budget the emissions $E(t)$ act to reduce the remaining budgets magnitude. The differential equation governing the evolution of the remaining carbon budget can be given as:

$$\frac{dB}{dt} = -E(0) \cdot e^{(g-r)t} \tag{2.4}$$

Letting $B(0)$ be the initial period remaining carbon budget (2.4) has the solution:

$$B(t) = -\frac{E(0)}{g-r} \cdot e^{(g-r)t} + B(0) + \frac{E(0)}{g-r} \tag{2.5}$$

From (2.5) two useful quantities can be determined; the time of budget exhaustion t_e (the time for the remaining carbon budget to reach 0) and the rate of emissions intensity r_e such that the budget will just be exhausted over an infinite horizon.[10] The year of budget exhaustion is given by (2.6) and the rate of intensity decline r_e is given as by (2.7). The year of budget exhaustion t_e is found by setting $B(t) = 0$ in (2.5) and solving for t which leads to:

$$t_e = \frac{\ln\left[\left(\frac{g-r}{E(0)}\right) \cdot \left(B(0) + \frac{E(0)}{g-r}\right)\right]}{g-r} \tag{2.6}$$

Here (2.6) states that year from present that the carbon budget B will be exhausted given g, r, $B(0)$, and $E(0)$.

The second quantity of interest is the rate of intensity reductions that, for given values of the other variables, leads to the carbon budget being just exhausted over an infinite horizon. Defining the rate of intensity in this manner may seem somewhat abstract but its connotation is simple. Any value of intensity declines greater than r_e indicates that the carbon budget B will never be exhausted while any value less implies the budget will be exhausted in some finite time. To obtain this value, we make two assumptions; first that as t goes to infinity $B(t)$ goes to 0; and second that $g - r < 0$. The second condition simply states the obvious requirement

[10] An emissions pathway that exhausts a budget over an infinite horizon is one whose cumulative emissions, summed from the initial period to time infinity, equals the budget.

that the rate of intensity declines is greater than the rate of growth. If this condition were not true, then decoupling would not occur at all. Therefore, we have:

$$\lim_{t\to\infty} B(t) = \lim_{t\to\infty}\left[-\frac{E(0)}{g-r}\cdot e^{(g-r)t}\right] + \lim_{t\to\infty}\left[B(0)+\frac{E(0)}{g-r}\right]$$

By assumption the left-hand side (LHS) is 0 and since $g - r < 0$ the first limit on the right hand side (RHS) also goes to 0. Therefore, we have:

$$0 = B(0) + \frac{E(0)}{g-r}$$

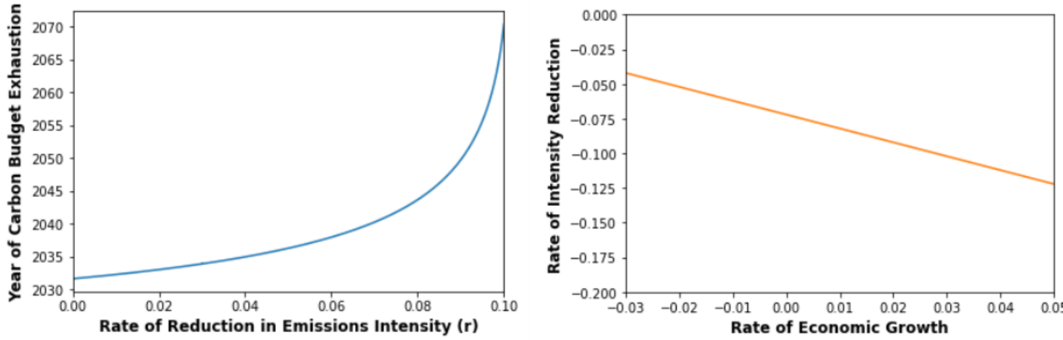
which we may rearrange as follows to obtain the value for r which we denote as r_e:

$$r_e = g + \frac{E(0)}{B(0)} \tag{2.7}$$

Equation (2.7) provides a simple relationship between the rate of intensity reductions and the rate of economic growth. For given initial year emissions and remaining carbon budget, the rate of intensity decline increases linearly with the rate of economic growth.

Figure 2.5a plots t_e as a function of the rate of reduction in emissions intensity r and 2.5b plots r_e as a function of the rate of economic growth g for a range of values.[11] For example, reading from panel (a) of Figure 2.5, we see that for an assumed rate of economic growth of 3%, a remaining carbon budget of 520 $GtCO_2$, and yearly rate of intensity decline of 8%, the carbon budget is exhausted by approximately year 2042.

[11]Note well, (2.6) has no solution if the carbon budget is not exhausted (this occurs for sufficiently high values of r).



(a) (b)

Figure 2.5: **Panel (a)**:For an assumed initial budget of of $B(0) = 520$ $GtCO_2$, a rate of economic growth of $g = 0.02$, and initial year emissions of $E(0) = 37.5$, the plot displays the year of budget exhaustion te (beginning at 2020) for a range of intensity declines r. **Panel (b)**: For an assumed initial budget of 520 $GtCO_2$ the plot displays rates of intensity reduction (r_e) that correspond to the budget just being exhausted over an infinite time-horizon as a function of rate of economic growth (g).

From (2.7) we know that $g - r_e$ is simply equal to $-\frac{E(0)}{B(0)}$. Substituting this into the emissions trajectory equation 2.3, we have:

$$E(t) = E(0) \cdot e^{-\left(\frac{E(0)}{B(0)}\right)t}$$

which defines the emissions trajectory with constant decay rate that will lead to the carbon budget being exhausted over an infinite horizon. The emissions pathway and cumulative emissions pathway are plotted in the following figure.

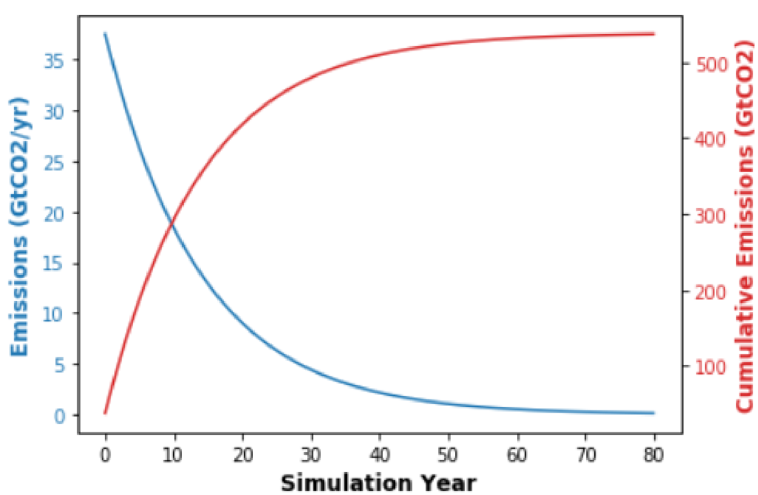


Figure 2.6: Emissions and cumulative emissions trajectories assuming a remaining carbon budget of $B(0) = 520$ $GtCO_2$, a rate of economic growth $g = 0.02$, and initial year emissions $E(0) = 37.5$ $GtCO_2$ yr^{-1}.

The salient feature of Figure 2.6 is that emissions must decline initially very rapidly. As this trajectory is constructed from approximate global level data it indicates an example of a global emissions pathway that assumes no overshoot and no carbon dioxide removal (CDR).[12][13] The shape of the trajectory is dependent on the magnitude of the remaining carbon budget assumed. Trajectories for several different carbon budgets are shown in the following figure.

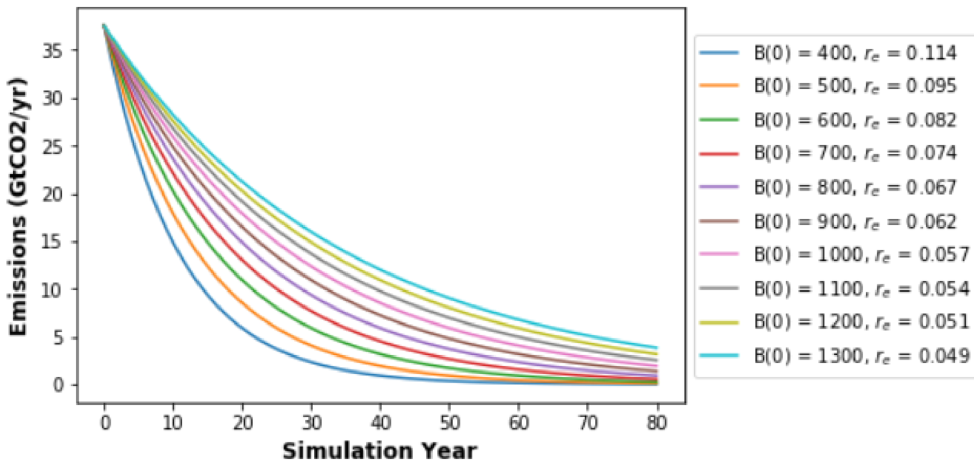


Figure 2.7: Emissions trajectories assuming a range (400 - 1300 $GtCO_2$) of values for the remaining carbon budget, a rate of economic growth $g = 0.02$, and initial year emissions $E(0) = 37.5$ $GtCO_2$ yr^{-1}. Legend values display the value of the assumed budget and the magnitude of the intensity declines r_e assumed for each pathway.

All else being equal, Figure 2.7 indicates (the obvious) that emissions pathways associated with larger remaining carbon budgets are characterized by slower reductions and longer times to reach zero emissions. Notably, even for relatively larger budgets the intensity reductions are still on the order of 5% per annum.[14,15] The rates of intensity decline necessary to just exhaust carbon budgets over this range varies from approximately 11% to 8%; that is, every year emissions intensity must decline by these amounts.

In order to contextualize these numbers it is necessary to examine global and country level data. Figure 2.8 presents the historical trajectory of CO_2 intensities as well as the annual percentage change in these intensities. Figure 2.9 presents the same data at the country level for several example countries. At the global level intensity declines fluctuate around approximately

[12]CDR will be discussed in greater detail in the following chapter.

[13]Similar pathways could just as easily be constructed at the individual country level by using country level emissions and an appropriate share of the remaining budget as the target.

[14]In 2018 the carbon budget for a 50% chance of staying within a 1.5 °C temperature rise was 580 GtCO2 and 1500 GtCO2 for a 50% chance of staying within a 2 °C temperature rise. (Rogelj et al, 2018)

[15]Naturally these budgets have since declined given the emissions produced since the publication of the report.

−2% per annum over the period of data availability (1960-2016) which is substantially below the lower end of the intensity declines derived above. Put simply, the intensity declines at the global level are, on average, not even close to sufficient to lead to emissions reductions in line with any of the above budgets.

The complexity of the decoupling story is obviously masked at the global level. In Figure 2.9 we see steady declines in CO_2 intensities for Canada, the United Kingdom, France, the United States, and Japan after 1970. This pattern does not hold for Brazil, China, or India however. While substantial reductions in intensities are observable in some countries there are none in which we observe rates of decoupling approaching those derived in the theoretical model above. This agrees with the earlier data indicating that global emissions have been rising over the period and have at best remained only level in some regions.[16]

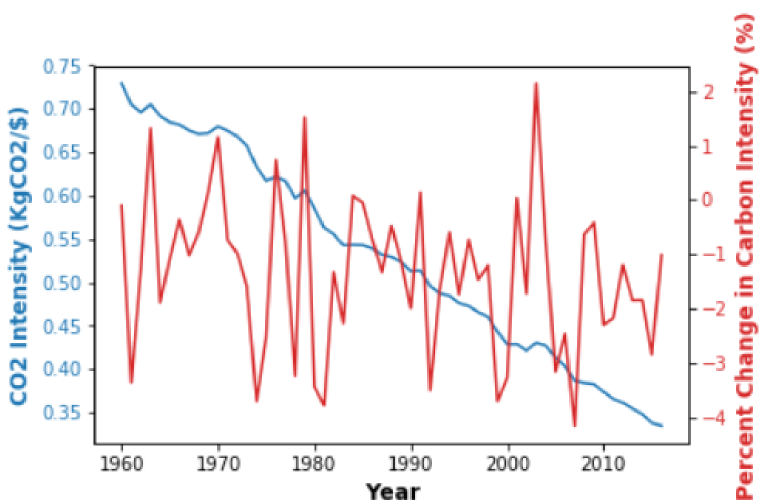
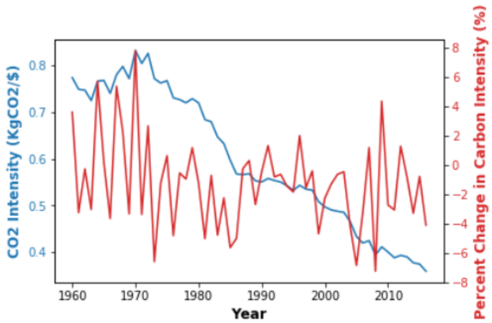
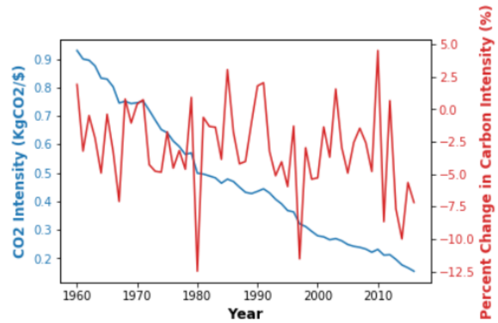


Figure 2.8: Global $CO2_2$ intensity ($kgCO_2$/2011 ppp \$) and annual percentage change in intensity (%).

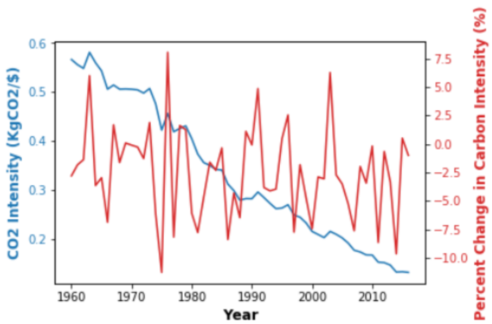
[16]Note well that the emissions for each country is calculated on a consumption basis. For an analysis of the impact of territorial assumptions on emissions accounting see (Dolter and Victor, 2016).



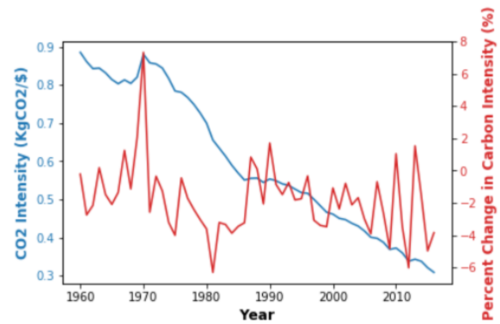
(a) Canada



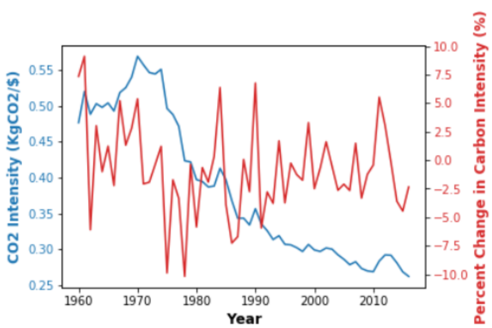
(b) United Kingdom



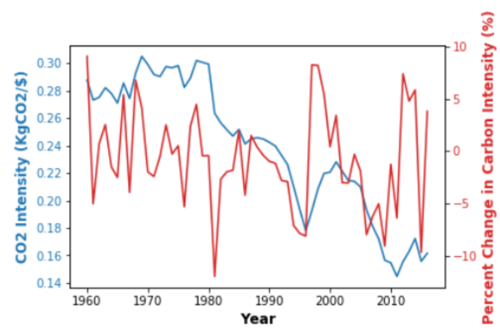
(c) France



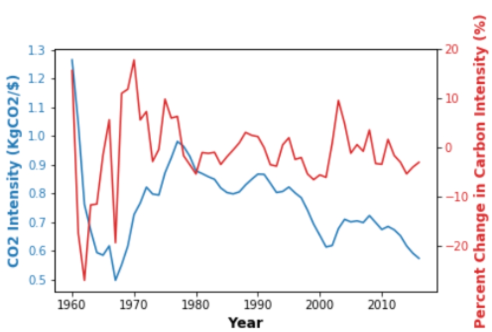
(d) United States



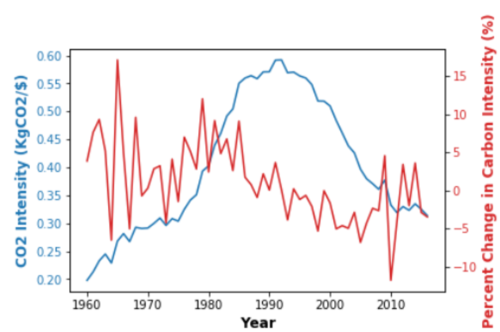
(e) Japan



(f) Brazil



(g) China



(h) India

Figure 2.9: Historical CO_2 intensities and annual percent changes CO_2 intensities for select countries.

2.2 A More Complex Decoupling Model

In the above simple model, we calculated rates of intensity declines that, if realized immediately, would result in emissions pathways consistent with remaining carbon budgets of a given magnitude. A reasonable question is to consider what occurs if there is a delay in the decline in emissions? In this section we adapt the linear growth-exponential decline emissions function in (Raupach et al, 2014) to relate it to economic growth in order to calculate intensity declines.[17]

As before let us assume that output $Y(t)$ is growing at some constant rate g:

$$Y(t) = Y(0)e^{gt}$$

and that emissions per year are given as:

$$E(t) = E(0)[1 + (rg + m)t] \cdot e^{-mt} \tag{2.8}$$

where g is the rate of economic growth, m is the rate of mitigation of emissions, and r a coefficient that relates economic growth rates with growth in emissions.[18] If the rate of mitigation $m = 0$ then emissions simply grow linearly with further economic growth, positive values of m imply that the linear growth in emissions is off-set by an exponential decay factor (this eventually dominates the linear growth leading to emissions declines). Finally, emissions intensity is defined in the usual manner as the ratio of emissions $E(t)$ to output $Y(t)$.

As before we may express the evolution of the remaining carbon budget as the following linear differential equation:

$$\frac{dB}{dt} = -E(0)[1 + (rg + m)t] \cdot e^{-mt} \tag{2.9}$$

[17] Several very similar calculations to those carried out below are undertaken in the supplementary material of (Raupach et al, 2014). We wish to make it very clear that though we have undertaken the derivations below, similar calculations have already been done and the result are in the supplementary results section of the (Raupach et al, 2014) paper.

[18] Unlike the simple model of the previous section, the formulation invoked here captures a slightly more complex version of emissions reductions. Initially emissions increase linearly in mitigation rate m capturing the near-term emissions involved in activities such as the build out of new renewable capacity and energy efficiency measures; emissions "eventually" decline exponentially in the same rate. A higher value of m therefore corresponds to a faster near-term growth in emissions followed by a faster exponential decline in emissions afterwards. The purpose of this formulation is to capture the persistence in emissions during the initial periods of emissions reductions activities (Raupach et al, 2014).

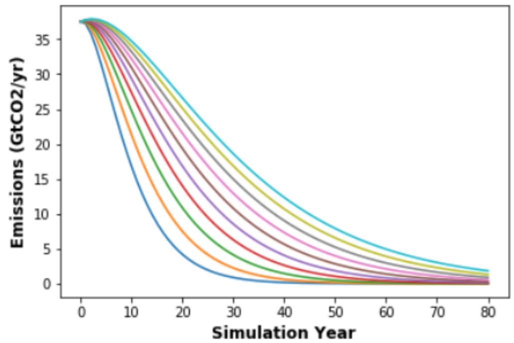
which has a solution of:

$$B(t) = e^{-mt}\left(\frac{E(0)}{m}\right)\left[1+\frac{rg+m}{m}+(rg+m)t\right]+B(0)-E(0)\left(\frac{rg+2m}{m^2}\right) \tag{2.10}$$

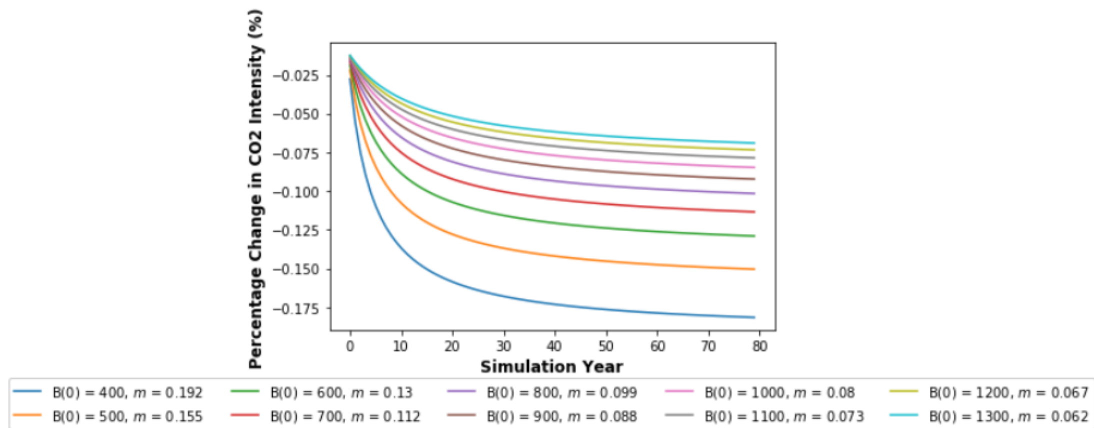
assuming an initial condition of time $t = 0$ remaining carbon budget being given by $B(0)$. As in the previous section we may inquire as to what the necessary rates of mitigation m are given assumptions about the rate of economic growth and remaining carbon budgets. Assuming as before that the carbon budget is just exhausted over an infinite horizon (that is $\lim_{t\to\infty} B(t) = 0$) we can find the following positive quadratic solution for the rate of mitigation m:

$$m = \frac{2E(0)+\sqrt{4E(0)^2+4B(0)E(0)rg}}{2B(0)} \tag{2.11}$$

For an initial remaining budget of $B(0) = 520GtCO_2$, initial year emissions $E(0) = 37.5$ $\mathrm{GtCO_2}$ per year, $g = 0.02$, and $r = 0.5$, we obtain a value of $m \approx 0.149$. This rate is noteworthy as it is substantially larger than the similar rate in the previous model which reflects the added challenges associated with emissions decline being delayed.



(a)



(b)

Figure 2.10: **Panel (a)**:For an assumed initial budget of of $B(0) = 520$ GtCO$_2$, a rate of economic growth of $g = 0.02$, and initial year emissions of $E(0) = 37.5$, the plot displays emissions pathways consistent with each of the initial budgets specified in the legend. **Panel (b)**: Trajectories for the change in emissions intensity for budget consistent pathways and continual economic growth of 2%. The Figure legend displays the assumed remaining carbon budget $B(0)$ and the magnitude of emissions mitigations m necessary per annum to obtain budget consistent emissions pathways.

The simple models elaborated in this section are designed to highlight that the rates of emissions intensity, for a given rate of economic growth, must be substantially larger than observed. The above calculations, however, do not specify any physical mechanism that might account for the declining intensities, only the necessary year over year magnitudes required to realize budget consistent pathways. One set of mechanisms of critical importance are relatively obvious given the above discussion; these are the transition of energy systems from primarily fossil-fuel based to renewable energy along with the necessary electrification of end stage energy use. Incorporating these mechanisms into an integrated assessment model will be a key task of this dissertation.

In the following chapter we introduce the literature on 1.5 °C pathways and survey the many types of integrated assessment models (IAMS) that have been deployed to study the problem of energy and climate from a macroeconomic perspective.

Chapter 3

Chapter Three: 1.5 °C Consistent Pathways: Background and Modelling Approaches

The purpose of this chapter is to survey the relevant literature concerning the physical and economic characteristics of achieving global emissions reductions in line with keeping warming within 1.5 °C as well as introduce the new strand of macroeconomic modelling called ecological macroeconomics. To this end the chapter is split into three sections. The first surveys the literature underlying the 1.5 °C consistent pathways and the various major multi-model studies that underpin the IPCC analyses. The second surveys the integrated assessment models (and their limitations) and the third explores a promising alternative development in macroeconomic modelling (ecological macroeconomics) that may be deployed to analyse pathways to 1.5 °C.

3.1 Modelling of 1.5 °C Pathways

The landmark Paris agreement, signed in 2015 codifies a global agreement to attempt to limit temperatures "well below" 2 °C and strive to pursue efforts to limit warming to within 1.5 °C (United Nations, 2015). The basic nature of the problem may be summarized as follows. Keeping global mean temperatures limited to below some given threshold will necessitate that CO_2 emissions reach net zero (Rogelj et al, 2018) while achieving such emissions reductions will require profound changes across the entire "global energy–agriculture–land-economy system" (Clarke et al, 2014). Emissions pathways consistent with no or very limited overshoot of the 1.5 °C target requires declines in emissions (over 2010 levels) by approximately 45% by 2030 and reaching net zero emissions by approximately 2050 (Rogelj et al, 2018).

Achieving emissions reductions consistent with obtaining a 1.5 °C pathway is, however, a rather complex affair requiring transitions in many heterogeneous and interrelated aspects of society. These have been broadly collected under the following categories of energy, land and ecosystem, urban and infrastructure, and industrial systems in the latest IPCC special report (de Coninck et al, 2018). Under these headings appear such diverse issues as the deployment of increased electrical energy storage, the management of wetlands, the electrification of transport, and agroforestry.[1]

A principal feature of the literature concerning 1.5 °C pathways (and emissions pathways in general) is that there is no singular set of assumptions that are necessary to meet the required emissions reductions (Clarke et al, 2014). Of the 90 1.5 °C consistent pathways obtained from the set of scenarios examined in the special report, 9 kept warming to below 1.5 °C over the 21st century, 44 exhibited low transitory overshoot of the 1.5 °C target before return to 1.5 or below (overshoot of less than 0.1 °C), and the remaining 37 exhibited higher overshoot (in the range of 0.1-0.4 °C) (Rogelj et al, 2018).

The high proportion of scenarios defined by low and high overshoot indicate the varying assumptions concerning the use of carbon dioxide removal (CDR) in the models.[2]. The variance in model assumptions concerning the inherently uncertain aspects of the future (population growth, the nature and magnitude of energy demands, the rate of economic growth, etc.) contribute significantly to the future emissions trajectories and consequently the requirements for CDR deployment. Five broad classifications for scenarios, covering a variety of assumptions about the future have been introduced called the shared socio-economic pathways (SSPs) (Kriegler et al, 2012) (O'Neill et al, 2014) and (O'Neill et al, 2017). For example, SSP1 broadly defines a sustainable development scenario with key features including low population growth, high economic growth, high technological progress, and low energy and food demand per capita (see (Rogelj et al, 2018) or (O'Neill et al, 2017) for full SSP descriptions). This is contrasted with SSP5 (fossil-fueled development), for example, which maintains the low population, high economic growth, and high technological change assumptions but assumes high energy and food demands. SSP2 (Middle of the Road) broadly characterizes less stringent assumption than the sustainable development SSP1. It was found in a recent study of six global IAMs (see (Rogelj

[1] The following very clear statement in the report is worth considering; "no single solution or option can enable a global transition to 1.5°C-consistent pathways" (de Coninck et al, 2018). Given the sheer range of issues (some more amenable to mathematical modelling than others) any model attempting to generate "transitions" consistent with 1.5 °C will exclude numerous possibly critical features.

[2] More on the role of CDR shortly

et al, 2018b))that 1.5 °C consistent pathways could be found for each model assuming SSP1 characteristics and 4 of the six participating models could generate 1.5 °C trajectories for the SSP2 scenario (Rogelj et al, 2018). Similar quantitative modelling was also undertaken in the context of a 2 °C target in (Riahi et al, 2017).

Four representative 1.5 °C pathways are presented in (IPCC, 2018) representing the low energy demand scenario, (see (Grubler et al, 2018), SSP1, SSP2 and SSP5 respectively demonstrating the existence of pathways under a broad range of socio-economic assumptions concerning the future. There are two caveats that need to be made here. First, all four scenarios are reliant to some extent on negative emissions arising from changes in agricultural, forestry, and other land use (AFOLU). Second, except for the low energy demand scenario, increasing quantities of bioenergy with carbon capture and storage are assumed to make the pathways possible.

Clearly negative emissions technology play an important role in the modelling of 1.5 and 2 °C scenarios as they are present in most 1.5 °C pathways and are also included in 104 of the 114 2 °C scenarios examined in (van Vuuren et al, 2018). The magnitudes of bioenergy with carbon capture and storage (BECCS) deployed in no or limited over shoot pathways require 0-1, 0-8, and 0-16 $GtCO_2$ yr^{-1} to be removed by BECCS in 2030, 2050, and 2100 (de Coninck and Benson, 2014). The survey of potentials for BECCS and other forms of CDR in (Fuss et al, 2018) indicate magnitudes in the range of 0.5–3.6 $GtCO_2$ yr^{-1} for afforestation and reforestation, 0.5–5 $GtCO_2$ yr^{-1} for BECCS, and 0.5-2, 2-4 $GtCO_2$ yr^{-1} for biochar, 2-4 $GtCO_2$ yr^{-1} for enhanced weathering, 0.5-5 $GtCO_2$ yr^{-1} for direct air capture with carbon capture and storage, and up to 5 $GtCO_2$ yr^{-1} for soil carbon sequestration. (Fuss et al, 2018) The deployment of CDR on large-scale has a number of possible biophysical impacts ranging from possibly very large land surface requirements, water usage, and modifications of albedo. For a more complete discussion of the various impacts of CDR technologies see (Smith et al, 2016).[3]

The assumptions of the magnitude of CDR assumed in 1.5 °C is large ranging from 100-1000 $GtCO_2$ depending on the nature of the pathway and the degree of overshoot that occurs (Rogelj et al, 2018). Major concerns have been raised about the potential for negative emissions technologies to scale sufficiently (de Coninck and Benson, 2014), (Smith et al, 2016) (Anderson and Peters, 2016). That significant issues may prevent the actual realization of CDR on the

[3]Albedo is the fraction of incoming solar radiation that is directly reflected back into space by earth's atmosphere and surface. See Goosse (2015) chapter two for a further discussion of albedo and its use in a simple energy balance model.

scale assumed in IAM analysis has led Anderson and Peters (2016) to note "Negative-emission technologies are not an insurance policy, but rather an unjust and high-stakes gamble" (p.183). This use of an unproven suite of technologies as model assumptions making possible the 1.5 °C and 2 °C represents an arguably key weakness in the endeavour. Should bioenergy with carbon capture and storage (BECCS) not be possible to scale to required magnitudes then the "success of many of the modelled pathways" disappears.

The second caveat concerns the assumption of long term economic growth. For example, a relatively recent four IAM multi-model study (on the impacts of different assumptions concerning economic growth and fossil availability) concluded:

> Based on an integrated assessment model comparison exercise, we show that economic growth and fossil resource assumptions substantially affect baseline developments, but in no case do they lead to the significant greenhouse gas emission reduction that would be needed to achieve long-term climate targets without dedicated climate policy. The influence of economic growth and fossil resource assumptions on climate mitigation pathways is relatively small due to overriding requirements imposed by long-term climate targets. (Kriegler et al, 2016) (p.7-8)

Indeed, each of the SSPs and all of the scenarios presented in the Special report on 1.5 °C share the feature of assuming at least some level of long term economic growth at the global level.[4] While more will be said on this in further chapters, the omission of no growth (steady-state) and degrowth economic trajectories in the featured IAM literature indicates a modelling blindspot in the community. A question of major import is whether no growth and even degrowth trajectories may reduce or even negate the need for unproven negative emissions technologies.[5][6]

[4] A key contribution of this dissertation is to explore the relationships of steady-state and degrowth economic trajectories with 1.5 °C consistent carbon budgets.

[5] There is a certain irony in the more mainstream economic community excluding or dismissing no and degrowth trajectories on the basis of realism but very happily adopting a belief in, and generally relying on, unproven negative emissions technologies to reach emissions targets.

[6] Note well, when we discuss steady-state and degrowth trajectories in the context of the SSPs we are being insufficiently granular. Steady-state and degrowth trajectories are likely, at this point in time, not viable or beneficial for many developing countries that might benefit greatly from additional growth; rather, it might be noted that degrowth and steady-state trajectories in rich developed regions may make "space" for necessary growth in others.

3.1.1 1.5 °C Pathways and Investment

A general (if obvious) feature of the above literature is that 1.5 °C pathways exists across a variety of socio-economic, physical, and technological assumptions. However, the large-scale transformations implied by any successful 1.5 °C pathway, necessarily involve immense new investments in low carbon energy technologies and other critical areas such as energy efficiency improvements. There are two seemingly simple questions that arise from these observations: (1) what are the rough magnitudes of investment expenditures associated with the transitions underlying 1.5 °C pathways; and (2) how will such investment be paid for?

Estimates of the magnitude of investment necessary for 1.5 °C pathways (global level) are "relatively sparse" with the majority of such studies focused on 2°C pathways (Rogelj et al, 2018). Rough analysis suggests that investments in low carbon energy consistent with 1.5 °C pathways must increase by a factor of 4-10 by 2050 over 2015 (Rogelj et al, 2018). A 2013 multi-model IAM ensemble analysis was used to determine rough ranges and magnitudes concerning global investments in low carbon energy and efficiency improvements in the context of 2 °C warming pathways finding that approximately 1.1 $trillion USD annually would be required which represented a gap of about $800 billion over then present day emissions-reductions policies (McCollum et al, 2013). A more recent analysis by the International Renewable Energy Agency (IRENA) notes that total energy sector investment would need to be on the order of 3.5 trillion (as compared with the $1.8 trillion in 2015) out to 2050 to meet the 66% 2°C carbon budget and that "Total demand-side investment into low-carbon technologies would need to surge by a factor of ten over the same period" (OECD/IEA and IRENA, 2017).[7]

Another recent multi-model IAM ensemble study finds:

> As a share of global GDP, the total energy investments projected by the models do not rise significantly from today in any of the scenarios, hovering just over 2% (model range: 1.5–2.6%) in 'CPol' and 'NDC' and growing to 2.5% (1.6–3.4%) and 2.8% (1.8–3.9%) in the '2C' and '1.5C' pathways, respectively (McCollum et al, 2018) (p.6).[8]

For their 1.5 °C consistent scenario this implies total energy sector investments on the order

[7]Very simply energy supply side denotes generation and distribution while demand side denotes the uses of energy.

[8]The population and economic development assumptions used in this study are in line with those in the middle of the road narrative captured by SSP2.

of 3.3 trillion US dollars per annum with a significant fraction corresponding to non-biomass renewables (0.73 trillion), electricity transmission, distribution, and storage (0.75 trillion), and demand side energy efficiency (0.82 trillion) which substantially outweighs the combined extraction, conversion, and electricity (without CCS) investments in fossil-fuels of 0.522 trillion.[9] (McCollum et al, 2018).

There are relatively few large multi-model comparisons of IAMs (Duan et al, 2019), and as of the writing of the IPCC special report in 2018, only a single multi-model IAM study of investment patterns necessary to achieve systemic energy system transformations in line with a 1.5 °C warming target (Rogelj et al, 2018). What is relatively clear however from the above multi-model study is that both the total magnitude of energy investments must increase over the current baseline established by the nationally determined contributions (NDS) and much of that increased investment must be directed away from the fossil-fuel sector to renewable generation, electricity infrastructure, and demand side energy efficiencies (see Table 1 in (McCollum et al, 2018)).

An additional complexity arises when considering the problem of infrastructure lock-in. Contemporary and near-term investment in new fossil-fuel infrastructure as well as currently installed infrastructure may be incompatible with emissions reductions should these displace low carbon energy infrastructure. This may itself necessitate the premature retirement of said infrastructure (Bertram et al, 2015) (Johnson et al, 2015). More broadly, 1.5 $^{\circ}C$ consistent pathways very likely necessitate also leaving significant quantities of fossil-fuels "in the ground". This suite of issues is given the broad name of "stranded assets". It is beyond the purpose or scope of this dissertation to explore this issue in any detail so we instead point the reader to a recent survey of the literature and stock-flow consistent input-output modelling undertaken by in the 2019 doctoral dissertation of Andrew Jackson.[10]

[9]The other types of energy investment calculated in the model are fossil and non-fossil hydrogen, extraction and conversion of bioenergy, extraction and conversion for nuclear energy, and carbon capture and storage. Taken together, these investment types cover energy both investment in generation capacity itself and the costs associated with changes to energy end use. As noted in the paper "the six models have broad coverage of different types of energy technologies across the entirety of the global energy system, including resource extraction, power generation, fuel conversion, pipelines/transmission, energy storage, and end-use/demand device" (McCollum et al, 2018) and therefore capture many key energy system features.

[10]This dissertation may be found at http://epubs.surrey.ac.uk/850106/.

3.1.2 Integrated Assessment Models: Classifications and Issues

Integrated assessment models continue to play a major role in the analysis of climate policy and pathways as evidenced by their significant role in the previous section in relation to 1.5 °C pathways. However, it is critical to note here that a significant strand of literature exists arguing that IAM modelling is, at best, often misleading and, as one author's recounts the views of some economists, "worse than useless" (Weyant, 2017).[11] Indeed, economist Robert Pindyck notes in his 2017 paper on the use and misuse of models for climate policy that:

> In a recent article (Pindyck 2013a), I argued that integrated assessment models (IAMs) "have crucial flaws that make them close to useless as tools for policy analysis" (page 860). In fact, I would argue that the problem goes beyond their "crucial flaws": IAM-based analyses of climate policy create a perception of knowledge and precision that is illusory and can fool policymakers into thinking that the forecasts the models generate have some kind of scientific legitimacy (Pindyck, 2017).(p.1)

Before addressing the literature is it worth thinking carefully about the nature of IAMs and their purpose.[12] For an IAM to be useful it must somehow be able to couple global temperature changes (an immensely complex process characterized by deep uncertainties) with a macroeconomic model (hardly an exact science), and must either make dramatic simplifications or attempt to capture the byzantine complexities of (at bare minimum) the energy system and its various emissions.[13] Supposing the hypothetical IAM modeller accomplishes the above, then they are faced with arguably the most difficult and uncertain aspect of the enterprise; capturing how the climate system (however modelled) feeds back into the functioning of the macroeconomic system that is the cause of its perturbation. It need hardly be stated that the sheer range of possible earth-system changes makes this a daunting task. For example, a possible (though heavily debated) collapse in insect populations presents very real issues that are hardly obvious

[11] See (Gambhir et al, 2019) for another recent overview of the various criticisms levied against IAM modelling.

[12] Here we are explicitly only considering IAMs that are used for global climate analysis. The models discussed in the previous section are examples of this type of IAM.

[13] The following definition, taken from the Dynamic Integrated Climate-Economy (DICE) model is a good example, of a dramatic simplification. The so called back-stop technology is defined as "... one that removes carbon from the atmosphere or an all-purpose environmentally benign zero-carbon energy technology. It might be solar power, or carbon-eating trees or windmills, or some as-yet undiscovered source." (Nordhaus and Sztorc, 2013). It is easy to be critical about such an assumption but as will become apparent in some of the modelling work in this dissertation, linking macroeconomic phenomena with physical phenomena in a coherent manner is non-trivial.

how to meaningfully integrate within a macroeconomic model.[14]

The conceptual difficulties in incorporating complex, possibly irreversible, and potentially catastrophic earth system feedbacks into any sort of macroeconomic framework represent perhaps the greatest unknowns in any IAM modelling. A notable recent attempt to incorporate one such geophysical phenomenon (the disintegration of the Greenland ice sheet) is undertaken in (Nordhaus, 2019), with a further commentary on the possibility of extending social-cost of carbon analysis to other geophysical processes such as the melting of the Antarctic ice-sheet (Pizer, 2019).[15] However, it must be stated here that the changes associated with the disintegration of the Greenland ice sheet, let alone the Antarctic ice-sheet, imply such a dramatically different set of earth-system conditions that any economic analysis of their impacts are highly suspect.[16]

Arguably, given the awarding of the 2018 economics Nobel prize to William Nordhaus, the most well known IAM model is The Dynamic Integrated Climate-Economy (DICE) model. However, the relatively simple DICE model is not representative of the suite of IAM models currently in use. In his overview of IAMs, Weyant makes several key classifications. First, he defines an IAM of global climate change as "any model that covers the whole world and, at a minimum, includes some key elements of the climate change mitigation and climate impacts systems at some level of aggregation" Second he further classifies IAMs as either detailed process (DP) or benefit-cost (BC) IAMs (Weyant, 2017). Detailed process IAMs generally are far more disaggregated, including much more detailed physical modelling, and possess the functionality to explore detailed sectoral impacts. Benefit-Cost IAMs are generally much simpler and more highly aggregated models used to attempt to model optimal climate policy.[17]

This classification scheme is similar to the slightly earlier overview of IAMs undertaken by Farmer, Hepburn, Mealy, and Teytelboym. Here IAMs are classified as either policy optimization models (POMs) which are characterized by their "focus on complete cost-benefit analysis of climate change mitigation and optimal policy', while policy evaluation models (PEMs) are

[14]See (Sanchez-Bayo and Wyckhuys, 2019)) for an analysis of extinction evidence for insects and (Simmons et al, 2019)) for a cautionary review of the claims.

[15]It is worth noting here that while we very much disagree with the methodology employed in the Nordhaus analysis, the attempt to extend the analysis to include important but tremendously complex phenomena such as ice-sheet dynamics is commendable.

[16]For a recent examination of the problem of applying cost-benefit style analysis to climate change see (Victor, 2020).

[17]A further point noted in (Weyant, 2017) concerns the immense difference in model complexity that is masked by lumping IAMs together. This issue is well illustrated by comparing the model in (Nordhaus, 2017) with the vastly more complex REMIND model whose latest documentation is found at Aboumahboub et al (2020).

used to "look at the cost-effectiveness of achieving a particular mitigation target with a given policy" (Farmer et al, 2015). Very simply, the POM and PEM categories roughly map onto the BC and DP classifications mentioned above. In the review the authors cite four main issues with POM IAMS covering uncertainty, aggregation problems, technological change, and damage functions. However a further criticism is levied in that "... most IAMs do not explicitly model money, finance or banking". It is worth restating a similar acknowledgement concerning the IAMs studied in (McCollum et al, 2018): "From where exactly these investment dollars are summoned is outside the scope of our study, and for the most part beyond the capability of the models employed".

This lack of sound analysis of money, finance, and banking in the context of IAM models has been relatively recently raised by Pollitt and Mercure who contend that many of the commonly computable general equilibrium (CGE) models are characterized by assumptions that are essentially at odds with the observed nature of how financial systems actually function (Pollitt and Mercure, 2018). They note the relative paucity of CGE models and E3 (energy-environment-economy) models in general that address money and finance in a more realistic fashion. The authors introduce a further classification system for IAMs that separate the models into the neoclassical microeconomics based CGE models and large-scale post-Keynesian based macroeconometric models (E3ME and GINFORS). [18]

A notable characteristic of many IAMs as observed in both the Politt and Mercure work and the survey by (Nikas et al, 2019) is that many of the existing IAMs, though often incorporating detailed physical modelling, have at their core basic neoclassical constructs (e.g., utility maximization, representative agents, production functions) and generally absent or minimally specified financial sectors. It is argued that modelling aspects of the low carbon transformation can lead to opposite results depending on the treatments of finance and innovation (Mercure et al, 2019). The modeller, working within the broader neoclassical tradition, may accept without much question the use of optimization and utility functions as found, for example, in the DICE model. However, these tools are by no means universally accepted and are indeed outright re-

[18]Yet another recent survey of 61 IAM models proposes 6 different classifications of optimal growth models, general equilibrium models, partial equilibrium models, energy-system models, macroeconometric models, and a residual category of "other models" (Nikas et al, 2019). This survey is particularly useful in that important characteristics of the many IAMs are made explicit.

jected in favour of other approaches in the broader macroeconomic modelling community.[19],[20]

A major strand of criticism levied against IAMs concerns what might be termed the gross misrepresentation and likely immense understatement of the nature of climate damages (feedbacks from the climate-system on economies) (Pindyck, 2013), (Pindyck, 2017), (Keen, 2020). Robert Pindyck makes the argument repeatedly that the functional forms of damage functions are arbitrary and the immense variance in their predictions provides little in the way of useful information. To see this, consider Figure 3.1 which plots the well known Nordhaus and Weitzman damage function formulations. Here the damage parameter measures the percentage by which output is reduced (compared to a baseline of no warming) as a function of the change in global average temperatures. [21]

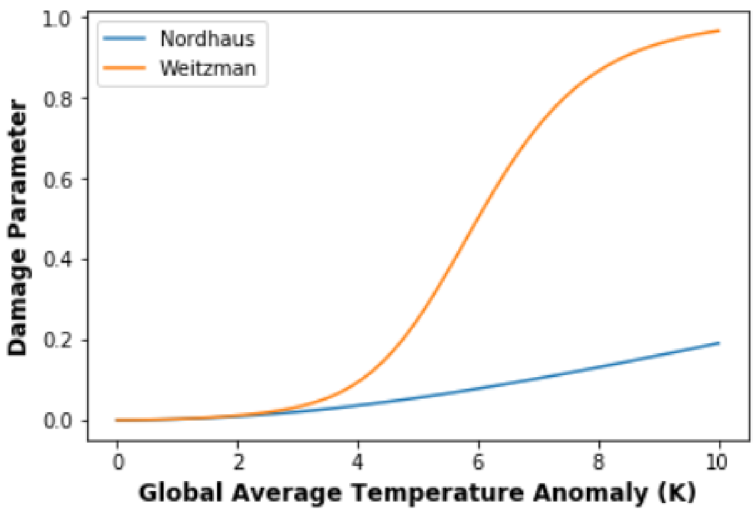


Figure 3.1: The damage parameter as a function of global average temperature changes for the Nordhaus and Weitzman damage formulations. Data for the functional form parameters were obtained from (Dietz and Stern, 2015).

There are two salient features of the plots in Figure 3.1 worth noting. First, such functional forms are largely arbitrary. For "low" temperature increases the two formulations produce

[19]For example, the following somewhat defiant statement appears on page 11 of the opening chapter of Monetary Economics: An Integrated Approach to Money, Credit, Income, Production, and Wealth. "We have no compunction whatever about aspiring to describe the behaviour of whole sectors, in defiance of the putative maximization of utility by individual agents " (Godley and Lavoie, 2007)

[20]Most advanced undergraduate and graduate students of economics will be familiar with the mathematical neoclassical construction of the consumer preference and firms. A different "world view" is described in enumeration of principles describing the post-Keynesian consumer theory is found in (Lavoie, 2005). Fascinatingly, a key figure in this work is Georgescu-Roegen who is also a seminal figure in the history of ecological economics.

[21]A more detailed discussion of damage functions is undertaken in chapter 10 which addresses some of the ambiguities of the existence of a singular damage parameter acting to unilaterally reduce output.

roughly similar results, but for "high" temperature increases the Weitzman formulation leads to substantially higher climate damages. It is noteworthy that the damages in both formulations are relatively small (below 2 °C) which is arguably somewhat dubious given the sheer variety of impacts that might occur under warming of just 1.5 °C (Hoegh-Guldberg et al, 2018). Second, the Nordhaus formulation displays results that seem at odds with reality. That damages are relatively small above 2, 4, and even 6 °C warming with even 10 °C leading to less than 20% reduction in output is indicative of deep flaws in the methodology.[22]

The Weitzman specification is parametrized so that output losses are 50% at 6 °C warming while the more recent work by Dietz and Stern introduce a "high-damage" function that obtains the same result but for 4 °C of warming (Dietz and Stern, 2015). On the one hand it may be argued that these forms better capture the possible existential consequences of warming above 1.5 and 2 °C it is critical to note that they are still essentially arbitrary mathematical calibrations that at best reflect different guesses about the future. The IAM model results underpinning the IPCC 1.5 scenario analysis, for example, simply excluded the temperature feedback altogether (Rogelj et al, 2018).

3.1.3 Ecological Macroeconomics

In the previous section we noted several standard problems with traditional IAMs used for climate policy analysis: (1) their general lack of financial system modelling; (2) their reliance on unproven negative emissions technologies; and (3) the limiting of future scenarios to ones assuming long term economic growth. In this section we survey the emerging strand of modelling termed ecological macroeconomics which, in its various forms, address the above concerns. Ecological macroeconomics, though derived on premises from the significantly older and broader field of ecological economics, is a relatively new undertaking. In their survey of ecological macroeconomic models, (Hardt and O'Neill, 2017) note that the 2007 study (Victor and Rosenbluth, 2007) is a seminal contribution to the emerging field. [23]

The work in (Victor, 2012) is particularly interesting in the context of this dissertations anal-

[22]A very recent and bluntly critical evaluation of IAMs is found in (Keen, 2020) The reader can likely guess the nature of this paper from its rather direct title: 'The appallingly bad neoclassical economics of climate change". The criticisms within however highlight the deeply problematic nature of attempts to integrate climate (and physical phenomenon in general) to macroeconomics.

[23]The LOWGROW model employed in the paper and its further developments underpinned further analysis in several other key journal publications and books (Victor, 2012) (Victor, 2008), and (Victor, 2019).

ysis as it represents both one the earliest and one of the few examples of formalized degrowth modelling.[24],[25]The degrowth scenario, predicated on very large declines in GDP per capita and designed to simulate the reduction in Canada's ecological footprint to one in line with the global average via reductions in GDP per capita, corresponds with very large (on the order of 80%) greenhouse gas emissions.

In their survey Hardt and O'Neill categorize ecological macroeconomic models based on the three following criteria "(1) describe the total monetary economy in mathematical terms; (2) include different groups of agents or sectors, typically households, firms, and the government; and (3) aggregate the economy at the level of a nation-state or region.". They also specifically exclude models predicated on optimizing assumptions as these are not typical of heterodox approaches (Hardt and O'Neill, 2017). The models are categorized first as either analytical or numerical with the numerical models being broken into a further four categories of stock-flow consistent, monetary input-output, physical input-output, and systems dynamics models.[26]

As noted in the commentary on the Farmer, McCollum, and Pollitt reviews in the previous section, the lack of financial sector representation is a key weakness in many conventional IAM models. On the other hand, a key feature of the models surveyed is their incorporation of various aspects of financial systems. The authors note:

> The representation of the monetary system is one of the eight themes identified where considerable progress has been achieved to date. This progress is almost entirely due to the use of stock-flow consistent models, which are the only class of models reviewed here that include an explicit representation of the banking and financial system (Hardt and O'Neill, 2017).(p.206)

A source of disagreement in the broader ecological economics community directly concerned the financial sector implications arising from the use of stock-flow consistent and post-Keynesian type of modelling, in particular the compatibility (or lack thereof) of money created as interest bearing debt. This disagreement is summarized in (Cahen-Fourot and Lavoie, 2016) with the

[24]As noted in a survey of 91 degrowth articles, formalized modelling efforts to study degrowth only largely began in 2012 (Weiss and Cattaneo, 2017).

[25]A particularly unusual example of degrowth modelling is found in (Germain, 2016) who studies voluntary degrowth utilizing the Ramsey growth model framework.

[26]These are not exclusive categories. Virtually all the models in this dissertation implicitly combine monetary and physical input-output approaches within a stock-flow consistent model. It would be entirely possible to simulate them using a standard systems dynamics tool such as Stella. Finally, some of the models will have analytical solutions.

authors providing mathematical modelling indicating the compatibility. A more complex model developed a year prior achieves the same result ((Jackson and Victor, 2015)). In the remainder of this section, we will briefly examine key ecological macroeconomics models and their most salient features.

In their 2010 paper, (D'Alessandro et al, 2010) introduce a relatively simple dynamic model of fossil-fuel depletion and renewable investment. The model is designed to explore the trade-offs between the additional investment capacity in a fossil-fuel growth scenario and depletion. A key feature of the model is a use of a Leontief production function with capital and energy as inputs. This production function approach was developed and expanded in the Energy-Emissions trap model which more explicitly examined the links between the EROI of renewables, remaining carbon budgets, and the investment rates necessary to facilitate the transition (Sers and Victor, 2018). A key result of this work was to indicate that successful energy-transitions under a variety of assumptions required immense upfront investment in new renewable capacity. Furthermore, in agreement with the results of (King and van den Bergh, 2018), this work noted that the rapid and large scale transition to renewables would lead to a decline in net energy available to society. However, both of these works themselves have been criticized for relying on now suspect data concerning the EROI of various renewable technologies (overly low estimates) (Diesendorf and Wiedmann, 2020)).[27]

In a more recent paper (Bernardo and D'Alessandro, 2016) construct a model assuming that energy is the only intermediate sector. The model uses a more sophisticated variant of the Leontief style production found in the previous two studies with the explicit inclusion of labour.[28] An important finding of the paper is that, to achieve 80% reductions in carbon emissions, investment in renewable energy must be on the order of 5.6% of GDP. The authors note that this is considerably higher than the numbers assumed in the Stern Review.[29] This general result is not however found in the dynamic modelling of (Taylor et al, 2016) where they suggest (from their model) that approximately 1% of world GDP would suffice.

A criticism of the modelling mentioned thus far lies in their relatively loose treatment of the

[27] A further criticism of dynamic EROI modelling studies along the lines of the Energy-Emissions trap model is found in (Brockway et al, 2019) which raises yet another set of concerns around the EROI values used.

[28] The production function is a nested Leontief function with one component being energy and the other a CES function of capital and labour.

[29] We will encounter even larger investment fractions still in the integrated assessment model developed in the latter chapters. Intuitively however, the high numbers are not surprising as the sheer magnitude of the transition in terms of the new capital that must be built in conjunction with the now minimal timeframe to accomplish it naturally leads to simulations producing such results.

concept of energy; for example, by aggregating the energy obtained from renewables and fossil-fuels in a production function. Aggregation of different energy types is not necessarily physically meaningful (Giampietro et al, 2013). A second issue concerns dimensionality associated with the use of production functions and the inclusion of the physical phenomenon (energy) with macroeconomic phenomenon. We leave discussion of dimensionality problems to the modelling chapters where considerable effort is put into making the mixed physical/economic equations dimensionally sensible.

The nested Leontief production function with energy as a non-substitutable component is used again in the stock-flow consistent agent based simulation model of (Ponta et al, 2018). This work introduces two assumptions that will be replicated in the work of this dissertation. That the electrical output is proportional to the magnitude of built renewable capacity, and that the renewable generation has "negligible operational costs" in comparison to fossil-fuel derived electricity.

Compared to the relatively simpler energy analysis models discussed already, there has been a marked increase in ecological macroeconomics models explicitly built on stock-flow consistent foundations. The SFCIO model of (Berg et al, 2015) is particularly relevant to the work of this dissertation as it explicitly combines the SFC framework with the mathematical logic of input-output analysis. In the relatively simple model, the authors assume two types of goods, energy and a composite all-purpose manufacturing good. The input-output coefficients are assumed static in the two sector model.[30] Intriguingly the paper explores the problem of anthropogenic heat emissions arising necessarily from the thermodynamic properties of energy conversions processes. The authors note that this contribution at current magnitude is relatively negligible but assuming the maintenance of current trends, this anthropogenic heat flux becomes a large component of the earth's energy balance.[31]

In addition to energy, a geophysical component is explored in the stock-flow consistent model built by (Dafermos et al, 2017). This work combines the stock-flow consistent approach as developed in (Godley and Lavoie, 2007) with the flow-fund model developed in Georgescu-Roegen's magnus opus *The Entropy Law and the Economic Process* (Georgescu-Roegen, 1971). This

[30] We relax this assumption over this dissertation by deploying a form of dynamic input-output model based on the relative magnitude of sectoral capital stocks.

[31] A very readable introduction to the concept of energy balance models and the various incoming and outgoing radiation terms may be found in Goosse (2015) chapter two.

model, as with the previous models, makes use of an aggregate energy term that is the summation of renewable and non-renewable energies, with the overall assumption that the energy required for production is simply proportional to total output (modified by an efficiency parameter). Interestingly, the model also includes the discrete time climate and carbon-cycle models used by Nordhaus (see (Nordhaus, 2017)). The model discusses two ways in which climate damages are modelled including direct damages to the fund-service resources (capital and labour) and their productivities. This dual approach to climate damages (direct damages and indirect productivity effects) is the general form for damages utilized in this dissertation (see chapter 10).

A more recent model due to (Bovari et al, 2018) utilizes a similar climate and carbon-cycle model (adapted for continuous time) in their stock-flow consistent macroeconomic model. This work addresses energy only indirectly via mitigation parameters and the notion of a "back-stop" technology. The assumption of this generalized back-stop technology is problematic as it essentially corresponds to, in practice, the assumption of some idealized and readily available non-emitting energy technology. A more general criticism of such model technologies is noted in (D'Alessandro et al, 2010). The authors note that their modelling approach leads to the 2 °C warming target being largely out of reach in the absence of large scale deployment of negative emissions technology and a realized climate sensitivity lower than 3 °C.

A criticism of the carbon-cycle model used in (Dafermos et al, 2017) and (Bovari et al, 2018) is that it is problematic when used in simulations over the long run, due to its linearized nature. As discussed in (Glotter et al, 2013), such models over-predict the CO_2 uptake in the ocean leading to quite substantially lower atmospheric concentrations of CO_2, and consequently global temperate increases, over century and millennia time-scales. This over-prediction of CO_2 uptake may act to unrealistically suppress atmospheric concentrations in IAM models and consequently lead to lower temperature increases.

Three new stock-flow consistent based ecological macroeconomic studies appear in 2020. The first of two papers appearing in the journal Ecological Economics is also the latest explicitly SFCIO model found in (King, 2020). The paper presents a model linking the Goodwin-Keen model (Keen, 2013), with the human and nature dynamics model of (Motesharrei et al, 2014). A notable feature of this model is that two of the models four underlying input-output technical coefficients are endogenous. Particularly interesting is King's discussion concerning power return ratios (PRR). In continuous-time when discussing “energy flows” we are actually discussing instantaneous rates (e.g., a flow of some magnitude per unit time) and hence power rather than energy.[32] The King paper also raises one critically important criticism of neoclassical integrated assessment models (IAMS); chiefly that many of the neoclassical IAMs are predicated on a neoclassical growth model involving a term called "total factor productivity". As King notes in the following two passages:

> “Many IAMs assume TFP grows between one and two percent per year. This assumption dominates economic growth such that no matter the magnitude and rate of a transition to low-carbon energy, economic growth is not only reported as positive, but robust.”.(p.2)
>
> “A major reason and problem is that by its construction, TFP and economic growth are independent of changes in the energy system itself (Figure A.II.1 of (Krey et al., 2014)). Instead of assuming changes to the energy system and calculating effects on economic factors, most IAMs first assume economic growth and fill in options for the energy system. This is not the type of modeling we need."(p.2)

This approach to IAM modelling is indefensible given there is growing literature (Exergy Economics) that shows that TFP is largely connected to energy system characteristics in the first place (see (Ayres et al, 2002), (Ayres and Warr, 2010), and (Brockway et al, 2018)).

The second SFC based paper to appear in the journal *Ecological Economics* presents the LOWGROW SFC model (Jackson and Victor, 2020). In comparison with the models previously discussed in this section, LOWGROW SFC is substantially more complex and is able to address in some detail key energy transition features including electrification, decarbonization of electric power production, and decarbonization of non-electricity related sectors making it particularly useful for simulating energy transition dynamics. Specifically, via a combination of policies including increasing renewables in the energy mixture and the electrification of road, rail, and transport, the modelling indicates that deep (80%) reductions are obtainable with even faster reductions possible with the model reaching zero emissions by 2040 in the sustainable prosperity scenario. The authors find that in attempting a large-scale transition that the model's endogenous economic growth rate declines as a result of the diversion of investment

[32] Energy is simply power integrated over some time period. We might measure energy in Joules and power in Watts (Joules per second).

towards “non-productive” green investment. [33]

The other major SFC ecological macroeconomics study published in 2020 utilizes the EUROGREEN model to explore joint issues of emissions reductions and changing socio-economic indicators (e.g., income distribution). Like LOWGROW SFC, the EUROGREEN model is a relatively large systems dynamics simulation model calibrated for a single country (France) and shares the feature of having a more detailed energy sector than the simpler SFC models previously discussed. The authors present on of the few degrowth simulations and find emissions reductions of up to 80% in their degrowth scenario.

Importantly, neither LOWGROW SFC or the EUROGREEN model (two large scale SFC ecological macroeconomics models) assume the deployment of negative emissions technologies while both find (to different extents) the greatest emissions reductions occur in scenarios with the least economic growth (quasi steady-state and degrowth). Furthermore, these models both address investment in models containing rather more sophisticated treatments of financial sectors than found in the typical IAM models discussed in the previous sections.

A second review of ecological macroeconomic models was published in 2020 and finds 11 non-optimization based ecological macroeconomic models that have specific relevance to the energy transition (Hafner et al, 2020).[34] While the authors note that the majority of the models surveyed are not IAM models (e.g., including geophysical components) they do point to four examples of models that do have some aspects of this consideration.

The first of four models is the Dystopian Schumpeter meeting Keynes (DSK) model in (Lamperti et al, 2018). The model is, as the authors note, the first attempt to provide an integrated assessment framework based on the agent-based modelling approach. The macroeconomic portion of the model is built on the so-called Keynes plus Schumpeter (K + S) model (see (Dosi et al, 2013) for example). The energy sector modelling assumes a monopolist that generates energy via power plants embodying “green and dirty technologies” both of which supply electricity to firms producing capital and consumption goods. The authors assume (as do we in our own models) that green energy production is always used at full capacity. A key result of the modelling is that the climate damages arising from uncontrolled emissions are much higher than

[33] The authors note that a full description of the LOWGROW SFC model may be found in (Jackson and Victor, 2019).

[34] Some of the models surveyed overlap with the above. For a short description of the nature and purpose of each of the eleven models see Table 2 (Hafner et al, 2020).

typically produced in cost-benefit IAMs which is a feature of the heterogeneity of the agents.

The extended EIRIN model in (Dunz et al, 2018) is notable for its approach to modelling climate damages as the authors eschew a full climate model (carbon-cycle and temperature model) and make direct arguments on the probabilistic nature of damages arising from both the increased severity and frequency of extreme weather events. This approach differs from the more standard damage function approach discussed in the previous sections and presents an alternative probability-based formulation.

The MEDEAS model (see (Capellán-Pérez et al, 2017)) is a global scale single region Post-Keynesian style energy transition model covering energy, climate, materials, and land-use. MEDEAS is notable for its rather detailed energy systems considerations and perhaps even more so for centering the energy return on energy invested (EROI) metric in its model. MEDEAS utilizes the standard Nordhaus DICE climate modelling framework but also makes the unusual decision to include only afforestation as a negative emissions technology.[35] Critically, the MEDEAS model introduces a discussion of the energy consequences of storage and utilizes the grid storage framework elaborated in (Barnhart et al, 2013). This model is also used in the energy transition models in this dissertation as an approach calculate the net energy impacts on renewable technologies. The authors use the MEDEAS model to explore the SSP2 scenario (middle of the road). [36]

The final model E3ME-FTT-GENIE model couples the global scale macroeconomtric model E3ME with the technology diffusion models (FTT), and an earth-system model of intermediate complexity (Genie). E3ME-FTT-GENIE is rather different than the more traditional IAMs in that it is a technology diffusion model and does not operate under the more standard assumptions about cost/utility optimization. Secondly, the authors note that BECCS is not a major feature of their modelling. The model is used to explore a very broad range of policy options and is used to provide detailed pathway to achieve the 66% 2 °C carbon budget.

[35] Nothing critical is meant by stating that this is unusual. The choice of excluding BECCS as it is unproven at scale is unusual only in that the vast majority of IAMs do not do so.

[36] One possible source of confusion arising from the work concerns the reported temperature changes (Figure 70, page 213) which indicate the transgressing of 1.5 °C in 2015 and two °C warming before 2040. These reported simulation results are at odds with contemporary temperature measurements.

3.2 Modelling Motivation and Research Questions

A very large number of models have been constructed to simulate the impact of various policies on emissions. A more specialized class of such models (global scale IAMs) include key geophysical features to evaluate the success of policies against the metric of global mean surface temperature increases. However, the major IAM studies underlying pathways to 1.5 °C are characterized by two principal modelling frailties (lack of proper financial sector modelling and the reliance on unproven negative emissions technologies) and a narrow focus on scenarios assuming long term economic growth.

Unlike these IAMs, ecological macroeconomic modelling generally has at its core a detailed and robust set of methods to include financial sector analysis and (using the two major models mentioned above), do not assume negative emissions technologies to obtain emissions trajectories. Perhaps more importantly, these models expanded the conventional universe of future economic states beyond the limitations of the SSPs to include steady-state and degrowth analysis.

We argue that a critical area of further exploration lies in the gaps of the above literature. While there exist models that have studied degrowth pathways, these have not been IAMS, and those ecological macroeconomic models that do have IAM features were either not designed to explore the energy transition or did so only in the case of growth scenarios.

The overarching purpose of this dissertation is therefore to construct a set of stock-flow consistent input-output models (SFCIO) that capture high-level features of the energy transition in order to study the dynamics of reaching 1.5 °C warming targets without the assumption of negative emissions technologies and without assuming economic growth as a feature of the future. Specifically, we will evaluate the possible role that degrowth economic trajectories play in leading to emissions pathways consistent with 1.5 °C.

Before turning to the construction of the SFCIO models, we must introduce the principle underlying techniques (stock-flow consistent modelling and input-output analysis) as well as contextualize their usage in the context of ecological economics. The following chapter therefore begins with a discussion of the modelling philosophy underpinning the work in this dissertation; discusses the notion of the scientific foundation of macroeconomic models and introduces the key features of the primary modelling methods to be utilized throughout the remainder of the dissertation. Finally, the full elaboration of the key research questions underpinning this dissertation are introduced at the conclusion of the chapter.

Chapter 4

Chapter Four: Ecological Macroeconomics: Foundational Issues

Having introduced the stock-flow consistent input-output model in chapter one and surveyed a variety of other approaches to integrated assessment model building in chapter three, we are left facing a large array of possible modelling approaches to address nominally the same set of phenomena.

Perhaps the most basic question we might ask of such models is the following: How do we, the modeller, know if the model actually corresponds to some degree with reality? If we construct a macroeconomic model in order to study the linked issues of energy, climate, and economy, by what criteria can we judge the model to state whether it successfully captures the phenomena it is meant to study?

While this question is simple to pose, it is not at all obvious how to answer. Macroeconomic models (neoclassical and heterodox alike) are ultimately just sets of equations whose quantities and dynamics we hope capture some aspect of the evolution of a much more complicated reality. Unlike in the physical sciences, macroeconomic modellers, cannot truly perform a set of controlled experiments to judge the validity of their theories. This, unfortunately, deprives the modeller of the simplest and most effective way of ascertaining whether their mathematical model actually models what it is supposed to. However, this problem is more complex than simply lacking an ability to test models in a laboratory.

First, the very system economists study undergoes fundamental changes over time. Quite obviously the present-day economy is different than that studied by Adam Smith. That economies evolve and undergo qualitative change leads to a problem not necessarily shared in the physical sciences where change occurs at a much slower pace; we shall address this issue in brief when we introduce the so-called ergodicity problem. Second, economics suffers from a type of observer effect problem in which the attempt to understand and model the system under observation (the economy) can change the behaviour of the system itself.[1][2][3] Ultimately, the economist must contend with the fact that not only does the system under study (the economy) evolve relatively rapidly but also may well evolve as a function of the study of economics itself. That "the economy" is both a phenomenon that changes fundamentally overtime and can react to its own study makes it a particularly problematic system to pin down formally with mathematics.

Answering the question of whether a macroeconomic model is "correct" is certainly beyond the scope of this chapter; rather, we will ask a slightly different question. If knowing when a macroeconomic model is "correct" is not achievable, can we at least know when a macroeconomic model is "wrong"? We will examine four criteria that may be useful in answering such a question.

It is useful to dispense with one argument at the outset, however. A common bit of modelling wisdom, attributed to George Box, is that "all models are wrong, but some are useful". This is often taken to mean that our abstractions of reality are never going to capture the underlying reality perfectly, however, these simplifications can be useful in, for example, generating testable predictions. We will argue a different meaning. A model is not wrong because it fails to fully capture the complexities of the underlying reality; any model capable of such a feat would be too complex to be of any use. The simplification and abstraction allowed to us by mathematical modelling is not, we argue, the grounds by which models are wrong but precisely the reason they can be useful in the first place. As we will see in the section on observability and consistency, there are very concrete ways that mathematical models can be wrong. The ethos of this section and those that follow is instead: all models are simplifications, some of which are wrong, and some of which are useful.

[1] This issue is of course well known to economists in the form of the Lucas critique.

[2] A famous literary example of the intersection of social science modelling and the observer effect is found in Asimov's Foundation series. A principle axiom of the fictional science of psychohistory is "that the population should remain in ignorance of the results of the application of psychohistorical analyses because if it is aware, the group changes its behaviour".

[3] To be clear, the observer effect problem is not unique to the social sciences. The reader can find a fascinating example in the realm of quantum mechanics arising from the famous double slit experiment.

4.1 Modelling: Some Foundational Issues

In Einstein's address to the Prussian academy of sciences he made the following statement: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality". While the statement originates in the context of physics, its meaning looms over any physical or social science that employs mathematical formalism. Mathematics allows for a level of precision that is unwarranted in most other domains of knowledge; for example, in mathematics it is possible to speak of proofs and truthfulness in the most exact sense.[4] Indeed, in physics, mathematics has been remarkably and profoundly useful, or borrowing the language of Eugene Wigner, it has displayed "unreasonable effectiveness" (Wigner, 1960).

What of its use in the social sciences (especially economics)? In the introduction to their text Mathematics for economists, authors Simon and Blume note that "for better or worse, mathematics has become the language of modern analytical economics " (Simon and Blume, 1994). While this is clearly in reference to the dominant neoclassical approach it applies equally well to some other approaches to economics and certainly to the heterodox approach taken in this dissertation. The deployment of mathematical abstractions allows the economist to study problems in a precise way but carries several considerable risks, for example the risk of committing the fallacy of misplaced concreteness. This fallacy, defined by Whitehead in his 1929 work "Process and Reality" (Whitehead, 1929), is given a simple English translation by Herman Daly as:

> In plainer English, the fallacy of misplaced concreteness is to confuse the model with the real world, to mistake the map for the territory. The verb "to abstract" means "to draw away from," and the fallacy of misplaced concreteness is to lose track of how far we have drawn away from the actual entity in defining our analytical image

[4] For example, take the seemingly trite statement that $1 > 0$. This assertion is of course intuitively obvious, but to prove it is "true" is more subtle. As an undergraduate student this author was assigned just this problem in a course in Real Analysis. It is possible to prove the statement absolutely by logical deduction from the set of complete ordered field axioms. Once done in this manner one can be content that the statement $1 > 0$ is indeed absolutely true in the firmest of senses. To reach this conclusion however requires the assumption of a set of axioms which are statements or premises that are simply taken to be true by definition. In pure mathematics the assumption of such axioms is usually uncontroversial; but what of the social sciences? A set of inarguable premises from which things may be proven true or false based on systematic deduction seems a tall order. However, the advanced undergraduate or graduate student in microeconomics might encounter just such a set of assumptions in the form of the axioms of consumer choice (see (Jehle and Reny, 2001) chapter one). Insofar as these axioms are precisely written they most certainly do not accurately model actual human psychology; and, in reverse, it is likely impossible to reduce the immense complexities of human behaviour down to a set of certain statements.

of it (Daly, 1980).

Why might it be easier to commit the fallacy of misplaced concreteness in economics than in some other heavily mathematical branch of knowledge such as physics? Unlike physics, economics suffers from two linked problems. First, as discussed above, it is rather more difficult to carry out replicable and properly controlled experiments in many domains of economic theory. While the econometric toolbox contains a variety of statistical techniques to aid in theory testing this is only a rough approximation of proper controlled experimentation. Second, economics is plagued by constructs that, though mathematically tractable, do not correspond well with entities in observable reality; utility and production functions being prime examples. Put more simply, it is easy in economics to mistake the map (mathematical formalism) for the land itself (real economic phenomena) as testing the validity of the map is relatively difficult and the map often contains intuitively useful features that do not correspond to any known "geographies".

In the beginning of this section we argued that it is simpler to know when a macroeconomic model is "wrong" than when it is "right". In the following section we will explore the two foundational notions of observability and consistency and provide examples of when we can definitively say a macroeconomic construct is "wrong".

4.1.1 Observability and Consistency

Suppose we have some mathematical model whose purpose is to describe a macroeconomy; we might ask of it the following two simple questions. Do the quantities in the model correspond to something that can be measured (observability), and are the quantities in the model linked together in a meaningful way (consistency)? To understand these two concepts let us first examine two simple examples from physics. First, consider the following equation which gives the force due to gravity on earth.

$$F = mg \tag{4.1}$$

where F is the force due to gravity measured in Newtons [N], m is mass [kg], and g is the acceleration due to gravity [m s^{-2}]. Here, m (the mass) and g (the acceleration) are measurable quantities that correspond to SI base unit (kilograms for mass) and some derived unit formed of a composite of base units (meters per second squared for acceleration). Importantly, if we know how the RHS quantities are measured we know something about the nature of the LHS term

F; namely, it is something measured in units of mass multiplied by meters per second squared. Of course, this follows immediately from the statement of Newton's second law which famously defines force as $F = ma$ or force equals mass times acceleration.

Now let us consider a second equation which expresses the conservation of energy for an object falling vertically due to gravity. The equation states that the sum of kinetic and potential energies is some constant c,

$$\frac{1}{2}mv^2 + mgh = c \tag{4.2}$$

Since c is an energy term, we know that it is measurable in the unit of Joules [kg m^2 s^{-2}]. For the RHS to be dimensionally consistent with the LHS it must be the case that the quantities in the sum on the LHS are also measured in the same units. Since velocity v is measured in units of meters per second, and the mass term m in kg, it follows that the first of the RHS terms is measured in the same units as c. Similarly, since the g in the second LHS term is the acceleration due to gravity it is measured in units of meters per second squared, and the height h is measured in meters, it follows that its units are again the same as those of c. In sum, the equation is dimensionally consistent (as it must be) in that the units on both sides of (4.2) match; that is, the RHS of the equality is measuring the same thing as the LHS.

In the following example we will examine a common construct in microeconomic theory, familiar to any economics student, that manages to violate both the logic of observability and consistency.

A Utility Function Example

Consider the following example Cobb-Douglas utility function:

$$U(x, y) = x^{\alpha}y^{\beta} \tag{4.3}$$

where $U(x, y)$ is the utility arising from the consumption of goods x and y.[5] Now, let us ignore the LHS temporarily and naively inquire as to the nature of the two RHS quantities. Suppose, for example, that we take goods x and y to be two fairly innocuous examples of coffee and apples respectively. How might such goods be represented in the utility function? To operationalize the goods in the utility function it is necessary to be able to attach a magnitude to each. For exam-

[5] Chapter one in (Jehle and Reny, 2001) presents a formal derivation of utility functions and their properties.

ple, we might intuitively think of one consumption bundle containing four apples and another containing five. This however leads to a problem that we shall address momentarily. However, carrying on with this scheme we face an immediate issue when giving the consumption good coffee a magnitude in the same fashion. It is not sensible to have four coffee or five coffee. Why? Put simply, there is no unique instance of a thing called "coffee" that can be counted in the same way that that we can count discrete instances of the good called apples.[6]

Therefore, while we might have apples as a good in the function, we must modify our logic for coffee and give it some identifying metric which we will suppose is "cups". Therefore, we might have good x represent the number of apples and good y represent the number of cups. Unfortunately, this approach hides two rather critical problems. First, the metric for the consumption good apple is only sensible if every instance of the good apple is identical to every other instance. This, however, we know is false in reality, and leads to a metric that does not actually capture what it intends to. The measure "unit of apple" fails as it has no unique unit. One instance of apple larger than another would surely give additional utility and since there is no guaranteed unique measure it cannot in practice be used as the metric for the consumption good. Therefore, to make the consumption good apple accounting rigorous, we must find some other way to count it. Let us now suppose the obvious that we count apples in the unique unit of kilograms (kg). If the consumption good x is now kilograms of apples we have a workable metric as one kilogram of apples, as compared by the metric of mass only, is the same as some other kilogram of apples.[7]

Finally, how are cups of coffee measured? To operationalize this notion, we must define more precisely the notion of a cup. The most sensible definition of such a metric will be as one that denotes volume. Therefore, we now have consumption good x which is measured in kg and consumption good y which is measured in litres (L) and in doing so have ensured that the consumption goods are properly representative of the phenomena they are designed to represent.

Let us return to equation (4.3) and consider the dimensions of the RHS. If good x is measured in kg and good y in L then the dimensions are [$\text{kg}^{\alpha}\ \text{L}^{\beta}$]. Let us take the simplest case to make an example and suppose that $\alpha,\beta = 1$, then the RHS has units of $\text{kg} \cdot \text{L}$. The dimensions

[6] Note well, the production function example here is done without respect to time for the purposes of simplicity. In chapter 8, we examine the dimensionality problem for a production function that, in theory, is designed to relate output (a flow variable) with stock variables such as labour and capital.

[7] Note well, any two instance of a kilogram of apples may differ by any number of other metrics such as age, colour, etc. This is immaterial however, as the consumption good is defined only as kilograms of apples for which now we can now compare instances in the chosen metric.

of the RHS are even less sensible should the exponents be any other number. For example, take the common case of $\alpha, \beta = 0.5$, under this set of assumptions the dimensions of the RHS is the square root of kilograms multiplied by the square root of litres which is essentially meaningless.

Now, one may argue that we purposefully picked incoherent units; suppose instead we had decided to measure both RHS goods in a common unit (kilograms of apples and kilograms of coffee), does this ameliorate the problem? Certainly not. Now, the RHS has dimensions of $[\mathrm{kg}^{(\alpha+\beta)}]$ which is arguably no more meaningful than the previous example. Quite obviously, if we attempted to make one of the "goods" in the utility function a service such as haircuts the dimensionality of the RHS quantity (any its interpretation) becomes even less obvious.

What is the LHS of (4.3) measured in? The function $U(x, y)$ gives the utility associated with any given consumption bundle but utility, as an abstract formalization of a immensely complex psychic phenomena (happiness), represents an unobservable in the truest sense of the word.[8] One approach is to measure utility in the abstract units of utils [u] which we shall use here. But if equation (4.3) is to be consistent the LHS and RHS dimensions must be the same which means that utility [u] has the utterly meaningless dimensions (assuming we return to measuring coffee in litres) of $\mathrm{kg}^{\alpha}\mathrm{L}^{\beta}$. Therefore, we are faced with a dual problem. If the equation is dimensionally consistent then utils [u] must have measurable if absurd units that have no linkage to any psychological phenomena; if the equation is inconsistent (a rather major issue) then utility is simply measured by some unobservable and unmeasurable unit.[9]

The purpose in exploring the above example is to note the relative ease with which one can construct mathematical models that violate any semblance of either the consistency of equations, or the notion of observability of the modelled quantities. Are models that violate one or both principles ruled out as simply incorrect? We argue that a model failing a check of consistency is, ultimately, simply incorrect as equations with different dimensions on RHS and LHS represent a fundamental model construction error.[10] The problem of observability of the underlying model

[8]There are other ways of attempting to measure happiness. For example, the Cantril ladder method used in the World Happiness Reports (See (Helliwell et al, 2020)) .

[9]The reader may here object and state that the Cobb-Douglas function is often modified by some additional parameter A so that:

$$U(x, y) = Ax^{\alpha}y^{\beta}$$

In this case for both sides to have dimensions of just [u], the unobserved utility unit, the parameter A must have the dimensions of $\frac{u}{kg^{\alpha}L^{\beta}}$. However, this parameter now has an unobservable numerator combined with a physically meaningless denominator. Worse still, A is hardly an exogenous parameter but is dependent on the magnitude of the exponents α and β and on how the consumption goods are measured.

[10]A cautionary note on the interpretation of this statement is necessary. If the RHS and LHS dimensions do not match than the statement is incorrect in so far as it states an erroneous equality between two entities that

quantities is however a more nuanced issue.

Successful mathematical modelling relies on simplifications and abstractions of the underlying system of study to produce a tractable model. Consider the following very simple example. In macroeconomics we might include the household consumption portion of GDP as some term $C(t)$ knowing in reality that such a term corresponds to the aggregation of many actual purchases by households in some time period. Furthermore, these purchases themselves are given as the product of the price and quantity of the various consumption goods; two "observable" measures.[11] While we do not directly observe and measure the aggregate entity $C(t)$, it is a useful simplifying abstraction with strong correspondence to real phenomena. Compare this with the notion of the Cobb-Douglas utility discussed function above which does not correspond to any real process.[12]

In discussing model abstractions and the notion observability we argue the following: mathematical modelling, of necessity, relies on abstractions often with no exact first order correspondence to a measurable phenomenon and such a strict criterion to evaluate mathematical models (especially in macroeconomics) would be too restrictive. Rather, abstract model quantities should be connected to the actual phenomenon by a plausible chain of assumptions and abstractions. It is possible to arrive at the term $C(t)$ by summing the actual individual purchase data; it is not possible to arrive at a Cobb-Douglas utility function for coffee and apples from some psychological first principles.

Let us now return to the motivating question for this section, how does the modeller know if their economic model has good (or any) correspondence with the phenomena the model is meant to study? Are consistency and a reasonable claim to observability sufficient conditions? No, as we will see in the following purposefully absurd example. Suppose we are measuring the energy consumption expenditure of an economy that consumes some constant quantity g_n of Joules per year with a constant energy price P_e which has dimensions of dollars per Joule. Now,

do not share a common unit of measure. However, can we necessarily jump from this issue to the invalidation of the model as a whole? While we have argued in this section that consistency can play a role in aiding one determine the validity of model it is a rather large stretch to work from this principal alone in order to state that all economic models with problems of dimensional consistency (and what we can learn from them) are simply "wrong".

[11] Prices are of course a problematic measuring stick as they are not constants.

[12] The reader may justifiably object at this comparison between an accounting term $C(t)$ and a behavioural equation. The point to be made here is that the components of $C(t)$ are at least comprised of "observables" which, as per the above discussion, is not necessarily true for the utility function.

consider the following equation:

$$C(t) = P_e g_n$$

where g_n is Graham's number, a quantity we will discuss shortly. The equation is certainly consistent as the LHS is measured in dollars per year while the RHS is measured in dollars per Joule multiplied by Joules per Year. The equation also certainly conforms to notions of observability as the various quantities (joules, years, dollars) are each sensible units of measure. Why then is the above equation absurd? Put simply, the quantity g_n (Graham's number) is so immensely vast that its use to measure a quantity of Joules does not apply to any energetic phenomena on any possible physical scale, let alone energy use by an economy.[13] Here observability and consistency are insufficient criteria to reject the equation as one that may sensibly model some economic phenomena; we must instead reject it by calling upon knowledge from the physical sciences.

While we can use the concepts of consistency and observability to help identify economic models that do not correspond sufficiently closely to reality to be useful, they are not sufficient criteria on their own to state whether an economic model corresponds meaningfully with the phenomena it is meant to capture. Put another way, we cannot ascertain an economic model's validity based only on the information captured in its mathematical presentation, we require some exogenous and "objective" standard against which to assess the model. As will become apparent in the discussion on the ergodicity problem, it is likely that such external, time invariant laws used for assessing economic theory will not be obtainable from just the historical study of economies and econometric analysis.

The ergodic-hypothesis, due originally to Boltzmann, concerns the subject of statistical mechanics may be stated simply as follows: The time spent by a system in some region of phase space is proportional to the volume of that region in the phase space. This hypothesis has a close counterpart in the field of econometric time-series analysis. Roughly stated a covariance stationary stochastic process is said to be ergodic if the time average of the process converges to

[13] Graham's number was, for a time, the largest integer used in a mathematical proof. While it is a finite number, its size is literally indescribably large and requires a special mathematical notation to represent. To provide an evocative comparison, if one were to inscribe the digit 9 on every plank volume ($\approx 4.2217 \cdot 10^{-105}$ m^3) in the observable universe, and produce a single long number by ordering these digits into a line, the resulting number would be meaninglessly small in comparison to Graham's number. If one were to take said number and raise it to the power of itself, itself many times (e.g., $3^{3^{3^3}}$), the result would still be meaninglessly small in comparison.

the ensemble average (see Hamilton, (1994) chapter 3 for a rigorous definition of these terms). That is, the time average will eventually converge to the average of a large number of such processes at a given time. Ergodicity in the time-series sense also implies stationarity; here stationarity roughly implies that the probability distribution of the process does not change as a function of time (See again Hamilton, (1994) chapter 3 for technical definitions). Or as stated in (Kirstein, 2015), removes time as a creative force in the evolution of systems.

Why is this discussion of ergodicity important? The ergodic hypothesis, according to the seminal neoclassical economist Paul Samuelson is necessary in order to treat economics scientifically and not have economists simply be viewers to unfolding history (Samuelson, 1968). As noted by Kirstein, this assumption removes the creative aspect of time which fundamentally allows the economic system to be modelled as if it were a natural system with set rules independent of the time of study; that is, if we only look at sufficiently long time series, we should be able to derive time-invariant principles. Armed with these fundamental and eternal economic principles we would possess the tool, along with consistency and observability, to validate or reject our economic models.

However, even a basic knowledge of the history of social systems and economies renders this assumption rather hard to accept. Clearly the nature and structure of economies have evolved dramatically and unpredictability over time and very clearly the system is not ergodic in the statistical sense. Put a different way, there is no reason why rules, relations, and empirical relationships that underpin the study of the economy in some window of time should remain unchanged as time passes. An example is useful to illustrate the point. Historically, the inverse relationship between inflation and unemployment, as presented in the Phillips curve, serves as a good example of an unstable empirical relationship across time. The phenomenon of stagflation (high unemployment and inflation) is a particularly famous example of the relation breaking down; the relationship is not necessarily stable across any large enough time period.

The problem for economics is that the condition of ergodicity and the philosophy it embodies for economic modelling are very unlikely to function as underpinnings for economics. There is no necessary reason why empirical relationships between economic variables, and consequently theories explaining these relations, need remain valid across time in systems that undergo qualitative change. In examining only economic phenomena it seems that economists are, in Samuelson's words, perhaps more akin to viewers of unfolding history than scientists discovering immutable laws.

Supposing the above argument is correct and there are no purely economic time-invariant laws, then need we discard the idea of testing economic models against some exogenously valid principles? The answer to this question is of course no, as economists do have access to a different set of laws and principles; namely those obtained from the physical sciences. While there may be no timeless laws of economics, by using physical laws it may be possible to gain an understanding of economic systems that is independent of the qualitative changes amongst purely economic variables that occurs.

4.1.2 Economics and Physical Law

In the previous section it was noted that the mathematization of economics rests on relatively shakier grounds than physics as the former generally lacks the ability to carry out experimental work that partly defines the latter. Rather than taking this difference as some unimpeachable separation of the two branches of knowledge is the contention here that it should represent a starting point for how to construct "testable" economic models.

The fundamental starting point for ecological economic analysis is the realization that any economic system is ultimately embedded in the greater earth-system. Herman Daly, evoking thermodynamic terminology, has defined economies as "open" subsystems of the greater "closed" earth-system (Daly, 1993). Here open systems are understood as systems in which matter and energy may be exchanged across system boundaries while closed systems allow only the exchange of energy.[14] This understanding of economies as physically embedded systems is of fundamental theoretical importance; the assertion that economies are characterizable partly as physical sub-systems is equivalent to stating that they must play by the same rules (physical laws) as the greater system in which they are embedded.

This vision of the economy as a sub-system has been expressed in several different manners by notable ecological economists. For example, Herman Daly states that:

> The necessary change in vision is to picture the macroeconomy as an open subsystem of the finite natural ecosystem (environment), and not as an isolated circular flow

[14] The earth-system is only a closed system only in approximation as, for example, matter in the form of meteorites enters frequently, hydrogen gas is lost to space from the upper atmosphere, and humans leave and return on occasion sometimes bringing rocks with them. There is also a third type of system classification called an isolated system. It is defined as a system that allows no exchange of either matter or energy.

> of abstract exchange value, unconstrained by mass balance, entropy and finitude. (Daly, 1996) (p.48)

This sentiment has been expressed much earlier in the writing of Kenneth Boulding:

> The closed earth of the future requires economic principles which are somewhat different from those of the open earth of the past. For the sake of picturesqueness, I am tempted to call the open economy the "cowboy economy," the cowboy being symbolic of the illimitable plains and also associated with reckless, exploitative, romantic, and violent behaviour, which is characteristic of open societies. The closed economy of the future might similarly be called the "spacema" economy, in which the earth has become a single spaceship, without unlimited reservoirs of anything, either for extraction or for pollution, and in which, therefore, man must find his place in a 8 cyclical ecological system which is capable of continuous reproduction of material form even though it cannot escape having inputs of energy. (Boulding, 1966). (p.8-9)

Finally, another early example is found in:

> Taking the view, then, that economic activity is a part of human society, and that in turn, society itself is only a subset of the phenomena that constitute the universe, the focus of this study will be the connection between human society and the rest of the universe that are attributable to economic activity. (Victor, 1972). (p.17-18)

A key element of the above statements is the realization that economies ultimately take place in the physical universe (and more narrowly almost entirely within the confines of the earth-system), and the (utterly revolutionary) implications it implies for the study of economics.

This realization leads to a partial resolution of the problems outlined in the previous section. If real economies must cohere to the set of physical rules that seemingly govern other aspects of reality then it should be the case that the mathematical models of economies do likewise. Therefore, one source of external validity of economic theory is its compliance, or lack thereof, with physical laws and theories. To illustrate this point, consider the following. The law of conservation of energy states that energy is neither created or destroyed, only transformed.[15] If

[15] See (Feynman et al, 1963) chapter four for an informative and accessible introduction to the concept.

an economic model were to produce behaviour that contradicts this conservation law, then it could be invalidated on the basis of a known physical law. Hypothetically, invoking physical laws and principles as tests for the realism of economic models presents a strategy for establishing their validity or lack thereof. This leads to the following question raised in another work that the present author has contributed to:

> If new modelling or theory is proposed can it (a) be compared in any fashion to physical principles or quantities, and (b) does it violate these principles? (Sers and Victor, 2020). (p.90)

In practice it is far more difficult to undertake the comparison than the statement above may imply. To undertake such a comparison requires that a model be formulated in such a way that a comparison is possible in the first place. Take for example the standard presentation of the Solow growth model (Solow, 1956). The model is defined without reference to the physical concept of energy and it is therefore not possible to state whether it violates the law of conservation of energy or generally any other law one might be interested in validating it with.

A further problem arises concerning which laws and physical principles are actually relevant. Should macroeconomic models be so constructed so that it can be verified that their behaviour does not violate the boundary set by the speed of light or that angular momentum is conserved? Likely not. Quite obviously not all physical principles are relevant to macroeconomic modelling though it is not obvious which are, and which are not. For example, much effort was expended on debating the relevance of the second law of thermodynamics and the illusive concept of entropy to ecological economics (see for example (Lozada, 1991), (Khalil, 1990), (Khalil, 2004), and (Townsend, 1992)). On the one hand the direct relevance to economics of the broad statement of the second law (entropy never decreases in an isolated system), is difficult to determine. On the other hand, the second law can be used to determine limit of energy conversion efficiencies of heat engines which are ubiquitous in society and therefore provides a fundamental upper bound for models with energy production technologies modelled on heat engines.

4.1.3 A Lurking Macroeconomic Conservation Law

While we might not be able to observe, courtesy of the ergodicity problem, timeless principles of economics by which we can judge our macroeconomic models, there is one set of very simple "axioms" we can invoke that we argue should apply to all coherent macroeconomic models. These axioms are themselves simple accounting principles that have, as we shall see, several profound implications. The first such axiom concerns the nature of financial transactions and is simply that the outflows of funds by one party (or sector) is necessarily matched by the inflow of funds by some other sector. The obvious example here is wages. Firms, over some time period, pay out wages to households and record an outflow of funds. Households receiving these funds must naturally record an inflow of the same magnitude. This logic can be captured in an ingenious construct called a transactions flow matrix which we shall discuss below.

A key element of all the models to be developed in this dissertation is the transactions flow matrix. This matrix, which contains information on both the financial transactions between the sectors of an economy and the logic governing the change in each sector's holdings of financial stock variables, has several particularly interesting features of which we will discuss in abstract terms in this section. Principally, we will show that any macroeconomic model whose flows are represented by a transactions flow table observes a type of conservation law.[16]

The following figure displays an example of a transactions flow matrix for an economy with four sectors (households, Energy, Manufacturing, and Government). Note that unlike the more conventional presentation of such tables, we are operating in a continuous-time environment so the changes in financial stocks (in this case just money H) is represented by the expression $\frac{-dH}{dt}$ rather than the more usual discrete-time ΔH.[17]

[16] Here we use conservation law to mean an equation which shows that some model quantity does not change as the system evolves through time.

[17] Despite this change the logic is ultimately the same. The first column of the transactions flow matrix gives an expression for the evolution of the stock of money held by households regardless of whether its specified in continuous time or discrete time. We can read the differential equation governing the time evolution of H from the first column as:

$$\frac{dH}{dt} = WB(t) - C(t) - T(t)$$

In discrete time this would be:

$$\Delta H = H_t - H_{t-1} = WB_t - C_t - T_t$$

$$\begin{array}{l} \\ \text{Consumption} \\ \text{Wages} \\ \text{Government} \\ \text{Taxes} \\ \text{Inter-Industry} \\ \Delta\text{Money} \\ \Sigma \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Households} & \text{Energy Sector} & \text{Manufacturing Sector} & \text{Government} & \Sigma \end{array} \\ \begin{pmatrix} -C(t) & C^e(t) & C^m(t) & 0 & 0 \\ WB(t) & -WB^e(t) & -WB^m(t) & 0 & 0 \\ 0 & G^e(t) & G^m(t) & -G(t) & 0 \\ -T(t) & 0 & 0 & T(t) & 0 \\ 0 & z_{12}(t) - z_{21}(t) & z_{21}(t) - z_{12}(t) & 0 & 0 \\ \frac{-dH}{dt} & 0 & 0 & \frac{dH}{dt} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{4.4}$$

Figure 4.1: Transactions Flow Matrix

Examining Figure 4.1 we note several important features. First, the row sums are all necessarily equal to zero. In a properly specified macroeconomic model this is an unsurprising feature as any transaction must necessarily involve two parties one of which expends funds and the other which necessarily receives the funds. The simplest example of this logic in the above table is that household tax payments at time t, denoted $-T(t)$ (and given a negative sign to indicate an outflow of funds) is necessarily exactly matched by the inflow of funds by the government sector $T(t)$. The logic governing the zero sum of the columns, as noted in (Godley and Lavoie, 2007) is simply that the columns represent the budget constraint of each sector. For example, examining the household column, we see that if the wages earned by the household exceeds the consumption and taxation expenditures then the remaining unspent funds are captured as an increase in the household's holdings of money.

To see this more formally we write out the equation defined by column one as:

$$\frac{dH}{dt} = WB(t) - C(t) - T(t)$$

which is the differential equation governing the evolution of the household's stock of money. First, household savings will only remain constant when wages received are exactly matched by the consumption and tax outflows, otherwise its magnitude will change depending on the sign of the LHS. Secondly, given our discussion of dimensions in the previous section, we need to be precise in how the entries of the transactions flow matrix are measured. Clearly, the LHS of the above equation ($\frac{dH}{dt}$) has dimensions of dollars per unit time.[18] For the equation to be

[18]Why? The term $\frac{dH}{dt}$ is the ratio of the infinitesimal change in the stock of money (measured in \$) to the infinitesimal change in time dt measured in some time unit. Hence the full terms have dimensions of dollars per unit time.

dimensionally consistent it must be the same for the RHS quantities. However, the RHS has terms such as wages, consumption, and taxes which are all flow variables; that is, they denote some magnitude of expenditures in some prescribed time period. Therefore, the dimensions match as necessary.

Now, suppose for argument that one had constructed a transactions flow table and found that a row did not sum to zero. What would this imply? A non-zero row sum would imply a breakdown in the logic that all transactions involve a symmetric exchange of funds; that is, funds would be leaving or entering from some sector (or multiple sectors) that are not accounted for in the model. Now, this non-zero row sum necessarily implies that the model is incomplete in that either the magnitude of funds leaving or entering are incorrectly specified, or that one or more sectors are simply missing from the model. Put another way, such a non-zero sum would imply that funds were either coming from nowhere or disappearing to nowhere.[19]

As discussed in (Godley and Lavoie, 2007) chapter two, the requirement of zero row and column sums implies that a change in any one element of the transactions flow matrix must necessarily lead to a change in at least three other elements. This is readily understood in the context of the above transactions flow matrix. Suppose the tax outflow in the household sector column were modified to be $-T(t) + A$ where A is some number. If nothing else were modified, then the taxes row would no longer sum to zero. Therefore, by the logic of the table (and common sense) the tax entry in the government sector must also be modified to $T(t) - A$. Now, this modification is still insufficient to maintain the logic of the tables as, unmodified, both the household sector and government sector columns would not sum to zero and so an element in each of those columns must also change (and hence four elements are modified). In this manner transactions flow tables follow the principle of quadruple entry book-keeping

Now, suppose we know nothing at all about an economy except that it follows the rules governing a properly specified transactions matrix. We can represent this by the below abstract or generalized transactions flow matrix which models an economy with n sectors, r types of financial transactions and m types of financial stocks. Since we know nothing about the economy at hand we denote the entries with arbitrary f_{ij} elements (denoting financial transactions) and $\frac{dS_{u,v}}{dt}$ denoting changes in the financial stocks. Some of these entries are positive (representing

[19]A properly specified model therefore would be one in which, to quote, a phrase from the preface of (Godley and Lavoie, 2007), "everything comes from somewhere and everything goes somewhere".

inflows of funds), some of which are negative (representing outflows), and some of which may be zero.[20] The arbitrary transactions flow matrix $\mathbf{T}$ is shown in the following Figure 4.2.

$$\mathbf{T} \quad = \quad \begin{array}{l} \\ \text{transaction 1} \\ \vdots \\ \vdots \\ \text{transaction r} \\ \Delta\text{Stock 1} \\ \vdots \\ \Delta\text{Stock m} \\ \Sigma \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Sector 1} & \ldots & \ldots & \text{Sector n} & \Sigma \end{array} \\ \left(\begin{array}{ccccc} f_{11} & \ldots & \ldots & f_{1n} & 0 \\ \vdots & \ddots & & & 0 \\ & & \ddots & & \\ f_{r1} & \ldots & \ldots & f_{rn} & 0 \\ \frac{dS_{1,1}}{dt} & & & \frac{dS_{1,n}}{dt} & 0 \\ \vdots & \ddots & & \vdots & 0 \\ \frac{dS_{m,1}}{dt} & & & \frac{dS_{m,n}}{dt} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \end{array} \tag{4.5}$$

Figure 4.2: Balance Sheet Matrix

While we distinguish between the transactions and changes in flows in this matrix it is convenient also to discuss the matrix elements as a whole. Since there are n sectors, r transaction types, and m financial stock variables, $\mathbf{T}$ is an $(r + m) \times n$ matrix. Let us also denote each of the entries in this matrix as t_{ij} for $i \in [1, r + m]$ and $j \in [1, n]$.[21]

Now, the rule mandating zero row and column sums necessarily implies the following:[22]

$$\sum_{i=1}^{r+m} \sum_{j=1}^{n} t_{ij} = 0 \tag{4.6}$$

which states that the sum of all the elements in the transactions flow matrix sum to 0. Put another way, the sum of all outflows of funds is matched exactly by the sum of all inflows of funds. If this equality does not hold then it must be the case that something is incorrectly specified in the model; that is, funds are entering or disappearing unaccountably. Recalling from above that the t_{ij} have dimensions of dollars per unit time and represents flows of funds, we arrive at the conclusion that a properly specified macroeconomic model must be one where the sum of all financial flows, as recorded by all sectors, must be zero.

[20] A zero entry here represents when a sector does not engage with the particular type of financial flow. For example, considering 4.1, there are 0s in the productive sectors taxes row as they are assumed not to pay taxes.

[21] For example, $t_{11} = f_{11}$, and $t_{r+m,1} = \frac{dS_{m,1}}{dt}$.

[22] Zero row and column sums necessarily require the sum of all the entries to sum to zero. The opposite is however not true, a matrix with the sum of all elements equalling zero does not imply the zero row and column sum condition as may be seen with the simple example:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Each of the elements of matrix $\mathbf{T}$ denote either an inflow or outflow of funds from the perspective of the sector in whose column they appear. However, as the measurements of all of the entries are the same it is possible to aggregate these flows to express the condition displayed in Equation (4.6). Taken as a whole the equation states that the sum of the inflows is balanced by the sum of the outflows, or put differently, the net flow is zero. The relevant question here is what is the entity for which we describe the flows into or out of?

Let us proceed with a simple analogy. Suppose we have a bath with a number of taps producing inflows of water and some number of drains providing an outflow. If the only thing we know is that the sum of these inflows is matched exactly by the sum of the outflows at all points in time, then we know that the quantity of water in the hypothetical bath is constant across time. From an abstract view of a transactions flow matrix we obtain a similar result in the sum of inflows into "something" is exactly balanced by the sum of outflows from "something".

Carrying on with this abstraction, we note that the double sum of flows in (4.6) represents the sum of the changes in some quantity per unit time. Let us denote the aggregation of these changes as $\frac{dF}{dt}$. Therefore, we have:

$$\frac{dF}{dt} = 0$$

The above equation simply states that the time-rate of change in the underlying model quantity F is always 0, and hence, that its magnitude is unchanging over time. We may solve this equation as:

$$F(t) = c$$

where c is some constant of integration, and both sides (after integration with respect to time) are measured in dimensions of just dollars. Starting from a properly specified transactions flow matrix, we find that there is some quantity, denoted here by F(t) and measured in dollars, that is conserved. No matter how the hypothetical SFC model embodied by the arbitrary transactions table evolves through time, F(t), by construction, is invariant.

Now, the question at hand is what is this conserved quantity $F(t)$? Clearly, it is a stock-variable as it has dimensions of dollars. To understand this, let us return to the arbitrary transactions flow matrix T and note the following. For an arbitrary column j we have that:

$$f_{1j} + f_{2j} + \cdots + f_{rj} = \frac{dS_{1,j}}{dt} + \cdots + \frac{dS_{m,j}}{dt}$$

which leads to:

$$\sum_{i=1}^{r} f_{ij} = \frac{d(S_{1,j} + \dots S_{m,j})}{dt}$$

which states that the net flow of funds into sector j (the RHS term) is balanced by the change in the quantity $(S_{1,j} + \dots S_{m,j})$. What is this bracketed quantity? This is simply the sum of the sector's financial stock variables which we might term their financial net worth. Therefore, the above equation states that the change in the sector's financial net worth as would be found on a balance sheet is equal to the net flow of funds into that sector per unit time. Now, if we sum over all rows we have:

$$\sum_{i=1}^{r}\sum_{j=1}^{n} f_{ij} = \sum_{j=1}^{n} \frac{d(S_{1,j} + \dots S_{m,j})}{dt}$$

The LHS of the above equation is the net inflow of funds across all sectors and the RHS is the net change in the total value of all of the economies financial assets.[23] Now, let us rewrite the above as:

$$\sum_{i=1}^{r}\sum_{j=1}^{n} f_{ij} - \sum_{j=1}^{n} \frac{d(S_{1,j} + \dots S_{m,j})}{dt} = 0$$

which is of course simply a restatement of (4.6) as the above equation is simply a different way of noting that all the elements in the transactions flow matrix are equal to zero. However, we now have that:

$$\sum_{i=1}^{r}\sum_{j=1}^{n} f_{ij} - \sum_{j=1}^{n} \frac{d(S_{1,j} + \dots S_{m,j})}{dt} = \frac{dF}{dt} = 0$$

Now, since the double sum of the flows on the LHS of the above equation is zero by the assumption of zero row sums we obtain the following:

$$-\sum_{j=1}^{n} \frac{d(S_{1,j} + \dots S_{m,j})}{dt} = \frac{dF}{dt} = 0 \tag{4.7}$$

The RHS of (4.7) is simply the negative of the change (the negative sign here does not matter) in the sum of the financial net worth's of each of the sectors, which is ultimately 0 by the above equation. As before, if the time-rate of change of a quantity (here the aggregate financial net worth) is zero, then the underlying quantity is constant. Therefore, we derive from the

[23] since the transactions flow matrix has zero row sums, this implies that both of these terms are actually equal to 0. Why? If all row sums are equal to zero, then the sum of the rows of all the transactions must also be equal to zero so the terms are zero.

transactions flow table that the overall financial net worth of an economy (the sum of all financial assets and liabilities) is constant. Put differently, the financial net worth of the economy is a conserved quantity in a properly specified macroeconomic model. This can be stated formally as:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij} = F(t) = c$$

It is worth pausing here to reflect on the above; beneath the formalism lies a powerful statement of macroeconomic modelling coherence. With only an arbitrary transactions flow table and assumptions concerning row sums and column sums we are able to obtain a wholly non-trivial result that the sum of financial assets and liabilities in a properly specified model economy is a conserved quantity. This is obtained with the utterly uncontroversial assumptions that transactions between sectors must be of the same magnitude but opposite sign and that the net inflow of funds into a sector simply leads to changes in the financial net worth of those sectors.

We argue that these simple logical accounting rules and the consequences derived from them are contenders for criteria against which we ascertain the validity of macroeconomic models. In addition to consistency, observability, coherence with exogenous natural laws, we might add a fourth criteria for macroeconomic model validity which is that the sum of all sector financial stocks is invariant because financial assets and liabilities in an economy are always equal.

Now, the above only tells us that, in aggregate, financial assets and liabilities balance. From this alone we cannot necessarily say anything about any given stock. However, the transactions flow table contains the information necessary to do this. Let us consider the row sum for the first financial stock S_1. We have:

$$\frac{dS_{1,1}}{dt} + \cdots + \frac{dS_{1,n}}{dt} = 0$$

which we can again write as:

$$\frac{d(S_{1,1} + \cdots + S_{1,n})}{dt} = 0$$

The bracketed quantity is the sum of the holdings of financial stock S_1 by each of the sectors. As before, since the time rate of change of the sum of the "holdings" of S_1 by all sectors is zero, the amount of S_1 held by all the sectors is some constant. Finally, from the above we obtain:

$$S_{1,1} + \cdots + S_{1,n} = c_1$$

We can of course carry out this exercise for the remaining stocks resulting in a similar equation for each type of financial stock variable. Let us organize this information by putting it into a new matrix $\mathbf{B}$ whose columns are the model's sectors and whose rows correspond to the different financial stocks in the economy.

$$\mathbf{B} \quad = \quad \begin{array}{l} \\ \text{Stock 1} \\ \vdots \\ \text{Stock m} \\ \text{Financial Net Worth} \\ \Sigma \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Sector 1} & \dots & \dots & \text{Sector n} & \Sigma \end{array} \\ \begin{pmatrix} S_{1,1}(t) & & & S_{1,n}(t) & c_1 \\ \vdots & \ddots & & \vdots & \vdots \\ S_{m,1}(t) & & & S_{m,n} & c_m(t) \\ -fnw_1(t) & \dots & \dots & -fwn_n(t) & -c \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \tag{4.8}$$

Figure 4.3: Balance Sheet Matrix

The first thing to note in Figure 4.3 is that the column sums are zero by construction as we include a financial net worth term which is simply equal to the sum of stocks on each individual sector's balance sheet. The rows, in each case, sum to the above obtained, and presently unknown conserved quantities, the c_i. What is the nature of the entries in this matrix? Without any further assumptions we can state only the following. The S_{ij} are quantities with dimensions of dollars and that the sum of each individual sectors "holding" of one of these quantities is a constant, or conserved, quantity over time.

Now, the reader may recognize the above matrix as simply the financial portion of an economy's balance sheet matrix (it excludes non-financial assets such as built capital) but it is worth noting again that this matrix is entirely obtainable from the information provided in the transactions flow matrix; that is, the basic accounting structure of the financial portion of the balance sheet matrix is already contained in the transactions-flow matrix. However, this is as far as we can progress with our abstract deduction, to continue we need one further assumption concerning the nature of these stock variables.

First, since the only thing we know about the above quantities is that they are measured in units of dollars then they are free to be positive, negative, or simply zero. Let us now define two elementary concepts; a positive quantity on the balance sheet matrix above is considered an asset, and a negative quantity is considered a liability. To complete the table above we need invoke only one additional rule.

Let us consider the row sum for stock one. We know from our derivation that:

$$S_{1,1} + \cdots + S_{1,n} = c_1$$

that is, the sum of the entries of stock on for all sectors is constant. Now, we know nothing about the magnitude of these entries, only that some may be positive, some may be negative, and some may be zero, and as such c_1 may itself be negative, positive, or zero. If it is positive then sum of stock one held as an asset across the sectors is greater than the sum of stock one held as a liability; the reverse is true if c_1 is negative. When c_1 is zero, the sum of the stock held as assets and the sum held as liabilities across the sectors are equivalent.

The one additional logical accounting rule we require is that the magnitude of a financial stock held as an asset must always be matched by the magnitude of same the stock held as a liability. To simplify, one sector's financial asset is by definition some other sector's financial liability. Invoking this additional assumption means that the c_i are all zero and that the total financial net worth of the economy c is also 0. From this we obtain the following more conventional balance sheet matrix:

$$\mathbf{B} \quad = \quad \begin{array}{l} \\ \text{Stock 1} \\ \vdots \\ \text{Stock m} \\ \text{Financial Net Worth} \\ \Sigma \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Sector 1} & \dots & \dots & \text{Sector n} & \Sigma \end{array} \\ \begin{pmatrix} S_{1,1}(t) & & & S_{1,n}(t) & 0 \\ \vdots & \ddots & & \vdots & \vdots \\ S_{m,1}(t) & & & S_{m,n} & 0 \\ -fnw_1(t) & \dots & \dots & -fwn_n(t) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \qquad (4.9)$$

Figure 4.4: Balance Sheet Matrix

The only information excluded from Figure 4.4 is the non-financial stocks that appear on balance sheets, e.g., built capital in the form of machinery owned by the productive sector. Including such information in the table would lead to the following result; since the financial net worth of the economy (the sum of financial assets and liabilities) is a conserved quantity always equal to zero, then the net worth of the economy is simply the value of the non-financial assets held by each sector. Ultimately, the financial assets and liabilities cancel out in the accounting leaving the "real" economy as the object by which the value of the economy is measured.

In summary, if one accepts the basic accounting principles that define the transactions flow table then we arrive at purely macroeconomic criteria with which we can test a macroeconomic model. Put simply, any macroeconomic model for which the sum of financial assets and liabilities is not conserved is one that possesses a transactions flow table that has either non-zero row or column sums. This implies that funds come from, or go to, nowhere and indicates that the model is incomplete in that either the magnitude of the flow is misspecified or that the model is missing one or more sectors or types of financial transactions. In conclusion, a macroeconomic model with a complete set of flows is one in which the financial net-worth is always conserved.

4.2 An Outstanding Issue

In order to study the coupled phenomena of climate change, energy transitions, and macroeconomics we will need to construct models that seamlessly integrate physical concepts such as power with economic phenomena such as wages paid or interest on loans. Such models by their very nature run the risk of incoherently combining physical concepts with economic ones. As we will see in the following chapters, a great deal of attention is paid to ensuring the dimensional consistency of the underlying model equations.

As discussed in this section consistency alone is not sufficient as a criteria for the analysis of model "realism". In fact, the four criteria discussed in this chapter (observability, consistency, coherence with knowledge from physical sciences, and coherence with basic accounting principles) are not sufficient when taken as a whole. The criteria as elaborated in this chapter all apply to ensuring that the model quantities are sensible, that the equations are consistent, and that basic macroeconomic accounting is followed. What we cannot provide are equivalent criteria for the behavioural components of the models that follow.

This is perhaps a subtle point but it worth noting before committing to the analysis. The above criteria say nothing at all about the validity of the behavioural assumptions at play in a model. If the modeller specifies an equation for a firm's desire to invest in new built capital, is the underlying behaviour correct? Different schools of economic thought will propose different behavioural assumptions on their models on the argument that these perhaps best reflect the behaviour of the underlying entities in question.

In the following models we will often impose behavioural conditions that are rather different from most conventional analysis (e.g., energy firms construct capital to match power demand and supply exactly, or households and governments voluntarily reduce consumption to simulate degrowth). Ultimately we are less concerned with whether the modelled behaviours are good approximations of historical behaviour and more concerned with the question of what outcomes a model characterized by the four criteria of this chapter produce when subjected to novel behavioural assumptions.

In the following chapters we will examine a sequence of SFCIO models that meet the four criteria laid out in this chapter. Particular emphasis will be placed on ensuring that the equations in the models are dimensionally consistent. Beginning in the following chapter we will examine how energy, power, and emissions (physical quantities) are integrated into a relatively simple continuous-time two sector SFCIO model.

Chapter 5

Chapter Five: A Simple Stock-Flow Consistent Input-Output Model

Let us begin by considering arguably the simplest interesting SFCIO model (hereafter SFCIO-IOSIM) which takes the form of a closed economy comprised of four main sectors: households, energy firms, manufacturing firms, and a government sector. In this simple economy, the firms comprising the energy and manufacturing sectors do not require any capital equipment to operate and distribute all profits implicitly as part of wages to the household sector; furthermore, households purchase a fixed quantity of energy goods from the energy sector and the remainder of their consumption on goods from the manufacturing sector.[1] Households pay taxes as a fixed percentage of their incomes to the government which makes purchases from both the energy and manufacturing sectors.

In order to produce the goods demanded by the household and government sectors the energy and manufacturing sectors must purchase intermediate goods from each-other as well as use some of their own outputs as inputs to their production. Firms receive income in the form of payments for the sales of household and government consumption goods and have as expenditures the payments for intermediate goods and wages. Wages in this simple model are simply taken to be the difference between sales and intermediate costs.

5.1 Balance Sheet, Transactions Flow, and Input-Output Matrices

We adopt the practice undertaken by Godley and Lavoie (Godley and Lavoie, 2007) in presenting the balance sheet and transactions flow matrices at the outset of the analysis. As the models are

[1] The nature of "energy goods" will become be discussed in far greater detail in the following chapter.

fundamentally input-output models we also present the models extended input-output matrix which details the sectoral dependencies of SFCIO-IOSIM.

$$\begin{array}{c c} & \begin{array}{cccc} \text{Households} & \text{Energy Sector} & \text{Manufacturing Sector} & \text{Government} \end{array} \\ \begin{array}{l} \text{Money} \\ \text{Net Worth} \end{array} & \begin{pmatrix} H(t) & 0 & 0 & -H(t) \\ -V(t) & 0 & 0 & V(t) \end{pmatrix} \end{array} \tag{5.1}$$

Figure 5.1: Balance sheet matrix for model SFCIO-IOSIM

Figure 5.1 presents the balance sheet matrix for the economy; note well here that there is only one financial asset and liability present which is the stock of money. Net worth is added in as a balancing item.[2] For households the stock of money (entered with a plus sign) is an asset and correspondingly is a liability (entered with a negative sign) of the government. It is assumed in this model that the energy and manufacturing sectors hold no money and do not possesses assets such as a tangible capital stock.

An asset such as money is an example of a stock variable, a quantity that can be measured in some given units. For example, the quantity of water in a container at some point in time (measured in litres) is a stock. The inflow or outflow which changes the stock as measured in, say, Litres per second is a flow variable which acts to modify the magnitude of the stock variable.[3]

Much like the quantity of water in a container changes according to the physical inflow and outflows, the stock variables in the SFCIO model change according to the transactions in the economy. Wages paid by firms and consumption purchases made by households in a given time period are examples of these flows. The complete set of flows is summarized in the following transactions flow table where each entry possesses dimensions of dollars per unit time.

[2]What is this net worth term and why is it on the balance sheet? A sector's net worth is the difference between a sector's assets and its liabilities, or to use the term put forward in (Godley and Lavoie, 2007), a residual. As those author's note, balance sheets, ultimately, must balance; however, there is no reason whatsoever that the magnitude of a sector's assets must be exactly equivalent to its liabilities. Hence, it is necessary to add the balancing accounting entity of net worth in to complete the accounting. Now, in the household column of 5.1 we see that money $H(t)$ is in as a positive which means it's an asset of the households. The equation in column one $(H(t) - V(t) = 0)$ is simple a restatement of the net worth equation $V(t) = H(t)$. Now since $H(t)$, the money issued by the government is a liability of the government and an asset to the households, it is entered as a negative. By the same logic as for households this implies that the government has a negative net worth, which by balancing logic, implies a positive entry of $V(t)$ which is simply government debt.

[3]We will often also use the term state-variable to denote the stock variables.

$$
\begin{array}{l} \\ \text{Consumption} \\ \text{Wages} \\ \text{Government} \\ \text{Taxes} \\ \text{Inter-Industry} \\ \Delta\text{Money} \\ \Sigma \end{array}
\begin{array}{c}
\begin{array}{ccccc} \text{Households} & \text{Energy Sector} & \text{Manufacturing Sector} & \text{Government} & \Sigma \end{array} \\
\begin{pmatrix}
-C(t) & C^e(t) & C^m(t) & 0 & 0 \\
WB(t) & -WB^e(t) & -WB^m(t) & 0 & 0 \\
0 & G^e(t) & G^m(t) & -G(t) & 0 \\
-T(t) & 0 & 0 & T(t) & 0 \\
0 & z_{12}(t) - z_{21}(t) & z_{21}(t) - z_{12}(t) & 0 & 0 \\
\frac{-dH}{dt} & 0 & 0 & \frac{dH}{dt} & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
\tag{5.2}
$$

Figure 5.2: Transactions flow matrix for model SFCIO-IOSIM

Figure 5.2 presents the full set of transactions taking place in the simple model. The defining feature of such tables, as discussed in the previous chapter, is that the row sums (transactions between sectors) and the column sums (sectoral budget constraints) all sum to zero. For example, in the first row households expend $C(t)$ (entered as a negative) and energy and manufacturing firms record payments received of $C^e(t)$ and $C^m(t)$ respectively.

The first column displays the budget constraint of the household sector. Households receive inflows in the form of wages $WB(t)$ which is entered as a positive and have outflows in the form of consumption expenditures $C(t)$ and taxes $T(t)$ which are entered with negative signs. Finally, the change in the stock of money held by households $\frac{dH}{dt}$ is then given simply as the difference between wages earned and expenditures for consumption and taxes paid. Since the household column sums to zero, we can simply read the differential equation governing the evolution of the stock of money $H(t)$ directly from the table as:

$$
\frac{dH}{dt} = WB(t) - C(t) - T(t) \tag{5.3}
$$

Now, the left-hand side (LHS) of (5.3), has dimensions of dollars [\$] (H is measured in dollars) per unit time [s] which we write more compactly as $[\frac{\$}{s}]$. This of course must be the case as it is a flow variable; however, this implies that the right hand side (RHS) variables (e.g., $C(t)$) are flow variables also measured in units of $[\frac{\$}{s}]$. Of course, consumption $C(t)$ is a flow variable as it is measured as the magnitude of some expenditures for a given time period so that the LHS and RHS units are equivalent is not surprising. Put another way (5.3) states the LHS flow is simply the sum of the three RHS flows.[4]

[4]In later chapters the dimensional analysis will become more involved. However, as this dissertation invokes both physical and economic quantities, such dimensional analysis is necessary to ensure the coherence of the

It is worth comparing this briefly with the discrete time case and to make a point about the precision of terminology. The discrete-time formulation of (5.3) is:

$$\Delta H_t = H_t - H_{t-1} = WB_t - C_t - T_t$$

Here the LHS (the change in the stock of money H) is simply the difference between two stocks (the current and previous stock of money) and hence is still a stock variable. Dimensional homogeneity requires that the RHS terms also be stock variables. This however is counter intuitive as its common to think that flows (recall the bathwater example) modify the stock which is clearly not the case here. Reconciling this requires noting that the discrete time consumption C_t is really the consumption per unit time multiplied by the length of the discrete time interval.

Returning to continuous time, (5.3) states the instantaneous change in the stock of money (LHS) is equal to the difference between inflows and outflows of funds to the household sector. Multiplying through the above equation by the infinitesimal time interval dt returns

$$dH = (WB(t) - C(t) - T(t))dt \tag{5.4}$$

which is the essentially the same as the discrete time formulation except the time intervals are of length dt. Here dH denotes the magnitude of the change in the household stock of money that occurs over an infinitesimally small time period. Since the RHS flow variables are now all multiplied by dt their dimensions cancel to [$] as required. As noted, this preoccupation with dimensional analysis may seem unnecessary in this simple model but as the models become more complex it is increasingly necessary to carefully check and ensure that the equations are physically meaningful.

Where the SFCIO transactions flow table begins to differ from the framework presented in the text by Godley and Lavoie (aside from the continuous time presentation) is that the transactions between firms are explicitly represented via the inclusion of inter-industry transactions.[5] Energy firms here record sales of $z_{12}(t)$ to the manufacturing sector and also make purchases of $z_{21}(t)$

undertaking.

[5]The reader may wonder where the $z_{11}(t)$ and $z_{22}(t)$ terms are which denote the purchase (or sale) of goods from an industrial sector to itself. Put simply, a sector's expenditures on its own goods would be included by a negative outflow and the receipt of funds for the expenditure would be received by an inflow of the same magnitude but opposite sign. This of course will simply always cancel to zero and hence it is not included.

from the manufacturing sector. By symmetry, this entry but with opposite sign is found in the manufacturing sector column and as such, the sum of inter-industry transactions necessarily sums to zero as it must to be included in the transactions flow table.

The final matrix necessary to complete the accounting description of the simple SFCIO model is the input-output table. This table is shown in the following figure.

$$\begin{array}{l} \\ \text{Energy} \\ \text{Manufacturing} \\ \text{Payments Sector} \\ \hline \text{Total Outlays} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Energy} & \text{Manufacturing} & \text{Final Demand} & & | \;\text{Total Output} \end{array} \\ \left(\begin{array}{cccc|c} z_{11}(t) & z_{12}(t) & C^e(t) & G^e(t) & X^e(t) \\ z_{21}(t) & z_{22}(t) & C^m(t) & G^m(t) & X^m(t) \\ WB^e(t) & WB^m(t) & 0 & 0 & WB(t) \\ \hline X^e(t) & X^m(t) & C(t) & G(t) & X(t) \end{array} \right) \end{array} \tag{5.5}$$

Figure 5.3: Extended input-output matrix for model SFCIO-IOSIM

It is important here to note that the input-output and transactions flow matrices, while very different constructs, contain much of the same information. Many of the entries in Figure 5.3 are also found in the transactions flow table above and the question now is whether these two representations are consistent. By the logic of the input-output tables the row sum for the energy sector and the column sum for the energy sector are equivalent and equal to $X^e(t)$. This however implies the following must hold:

$$WB(t) + z_{11}(t) + z_{21}(t) = C^e(t) + G^e(t) + z_{11}(t) + z_{12}(t) \tag{5.6}$$

Is this equation consistent with those in the transaction flow table? For example, from Figure 5.2 another equation with $WB^e(t)$ on the left hand side can be obtained which is:

$$WB^e(t) = C^e(t) + G^e(t) + z_{12}(t) - z_{21}(t) \tag{5.7}$$

For the two different accounts of the various economic phenomena to be consistent it must be the case that the equality in (5.6) should hold if the definition for $WB^e(t)$ in (5.7) is substituted in. Indeed, as shown, the substitution reduces (5.8) to an identity and so there is consistency. By similar logic this holds for the manufacturing sector.

$$C^e(t) + G^e(t) + z_{12}(t) - z_{21}(t) + z_{11}(t) + z_{21}(t) = C^e(t) + G^e(t) + z_{11}(t) + z_{12}(t) \tag{5.8}$$

5.2 Model Equations

Armed with the accounting structure presented in the previous section it is now necessary to present the behavioural equations that define the model itself. It is important to note that the balance sheet, transactions flow, and input-output matrices provide only the model identities that must be satisfied, not the "behavioural" equations that drive the model itself which must also be defined to complete the model.

To provide a point of comparison the simple SFCIO model is built using many of the same equations that are used to construct the first and simplest model in (Godley and Lavoie, 2007) (Model-SIM in chapter 3).[6] From the first row of the transaction flow table we may simply read off the identity that consumption must satisfy as:

$$C(t) = C^e(t) + C^m(t) \tag{5.9}$$

Equation (5.9) is simply an accounting device or identity which ensures that household consumption is equivalent to the consumption expenditures received by the industrial sectors. We must also define consumption from a behavioural perspective. Households are paid wages some of which they must pay to the government sector. As such the disposable income of households is given as:

$$YD(t) = WB(t) - T(t) \tag{5.10}$$

which is the difference between wages and taxes. Let us further assume that the magnitude of taxes paid is some fraction θ of the wage bill:

$$T(t) = \theta WB(t) \tag{5.11}$$

In the model, household consumption is comprised of consumption out of disposable income and consumption from savings which is governed by the following consumption function.

$$C(t) = \alpha_1 YD(t) + \alpha_2 H(t) \tag{5.12}$$

[6]In fact the model in this chapter is constructed to produce identical results to Godley and Lavoie's model SIM as a manner of checking the sensibility of the SFCIO-IOSIM model. That it produces identical results with the added IO structure will become clear from the model derivation.

where $0 < \alpha_2 < \alpha_1 < 0$. In this very simplified model a constant population is tacitly assumed who collectively demand a constant quantity of energy services over time.[7] Let this quantity of energy services be denoted as J_H and assume it is measured in some unit of energy such as Joules or Kilowatt Hours.[8] Letting P_e denote the price (more accurately the conversion factor with units of dollars per Joule) of energy then we may write:

$$C^e(t) = P_e \cdot J \tag{5.13}$$

From equation (5.9) $C^m(t)$ is given as simply the residual of consumption expenditure after the energy goods are purchased.[9] Similarly, assume that the government also requires a constant quantity of energy services to function and denote this J_G; furthermore, assume that government purchases are constant over time so that we have:

$$G(t) = G \tag{5.14}$$

and therefore,

$$G = P_e \cdot J_g + G^m \tag{5.15}$$

where G^m is the purchases by the government of manufacturing sector output.

Having determined expressions for both consumption and government expenditures we may define output in the economy as calculated by either expenditures or income; this is shown in the following equation.

$$WB(t) = Y(t) = C(t) + G(t) \tag{5.16}$$

[7] With an assumed constant population and no technological change, a constant energy demand might be understood simply as the routine energy requirements necessary to run a household (heating) or fuel costs for transportation.

[8] In the simple SFCIO model being presented here the energy sector is purposefully left vague and the actual characteristics of the energy being provided is unimportant. For now, it suffices to think of the output as energy but, due to the continuous time nature of the model, it is actually power which will be made clear in the following chapter.

[9] Therefore we must have that $\alpha_1 YD(t) + \alpha_2 H(t) = C^e(t) + C^m(t)$. This derived condition is an important limiter of what sensible values the model can take. Since we assume a constant energy consumption $C^e(t)$, it must be the case that in a sensible model run, the household has at least sufficient disposable income and wealth to finance the purchase of this constant portion of the consumption.

Finally, the wages paid to the household sector are simply the sum of wages paid from the energy and manufacturing sectors:

$$WB(t) = WB^e(t) + WB^m(t) \tag{5.17}$$

Reading from the transactions flow matrix we see that the wages paid to households by either sector is taken to be simply the difference between sector incomes (the sum of consumption, government expenditure, and purchases by the other sector) and outlays in the forms of payments for intermediate goods from the other sector. Therefore we have the following equations for the wage bill faced by each sector:[10]

$$WB^e(t) = C^e(t) + G^e(t) + z_{12}(t) - z_{21}(t) \tag{5.18}$$

$$WB^m(t) = C^m(t) + G^m(t) + z_{21}(t) - z_{12}(t) \tag{5.19}$$

In (5.18) and (5.19) wages are determined partly by the interindustry flows $z_{12}(t)$ and $z_{21}(t)$. Obtaining expressions for these inter-industry flows is slightly more involved and it is to this we turn in the following section.

The final equations to present are the equations governing the evolution of the model stock variables which are taken directly from the first or the fourth columns of the transaction flow matrix. These are the differential equations that determine how the money stock evolves through time; it is worth noting that either equation may be used.

$$\frac{dH}{dt} = YD(t) - C(t) \tag{5.20}$$

$$\frac{dH}{dt} = G(t) - T(t) \tag{5.21}$$

Equations (5.20) and (5.21) are not in a form that can be solved (either by hand or by computer) currently. The two differential equations above are in what we will term flow variable form; that is, their LHS components are written in terms of the model flow variables. In order to solve (or put in simulable form) it will be necessary to obtain the LHS terms as functions of the

[10] Specifying wages in this manner implies that there are no profits. It is useful to imagine that the firm's profit is also entirely distributed to households and are simply bundled in with the wage bill. Note well that this is a strong simplification as profits and wages are not necessarily treated in the same manner in more realistic models.

underlying stock or state variables. When we have done this the resulting set of differential equations will be termed as being in state-variable form. [11]

5.3 Interindustry Flows, Technical Coefficients, and Sectoral Total Outputs

We turn now to deriving expressions for the interindustry flow terms appearing (5.18) and (5.19) which will allow us to complete the model. First, we require the definition of an input-output technical coefficient, denoted a_{ij}, which is given as:

$$a_{ij} = \frac{z_{ij}}{X_j} \tag{5.22}$$

In order to determine the interindustry flows and the magnitude of total outputs for both energy and manufacturing sectors it is necessary to invoke a set of these technical coefficients which we assume for now are exogenous and constant; these are presented in the following technical coefficients matrix: [12]

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{5.23}$$

Where the first and second rows (and columns) are the energy sector and manufacturing technical coefficients respectively. From a units perspective the first element of A is of special interest. Here a_{11} is the dimensionless ratio of the purchases from (or sales to) the energy sector from itself. Because we assume that the energy sector price P_e is constant in the model this ratio may also be understood as the ratio of the energy required by the energy sector (the intermediate input) to produce its total output of energy that is used by the entire economy. This however,

[11] Some computer software such as Stella can be presented a discrete time model in flow-variable form and numerically perform the translation to state-variable form to perform simulations. The advantage of Stella in this case is that the user avoids the often immensely complicated model derivations altogether; on the other hand, solving the models by hand ensures that one understands very clearly how the model works and ensures it is doing what it is theoretically supposed to be doing.

[12] In more standard input-output analysis these technical coefficients are found empirically using data. One may think of the "exogenous parameters" invoked in this section as the hypothetical result of those calculations undertaken at the initial period of analysis. In later chapters these coefficients become time-dependent and the rules under which they follow become substantially more complex. For now it is sufficient to consider these coefficients as simply exogenously determined model parameters.

is just energy invested on energy return or the reciprocal of EROI at the economy wide level.[13]

$$a_{11} = \frac{\text{Energy Invested}}{\text{Energy Return to Economy}} = \frac{1}{\text{EROI}_{\text{Society}}} \left[\frac{\text{Joules}}{\text{Joules}}\right] \tag{5.24}$$

The above ratio is the ratio of the direct energy requirements by the energy sector to produce the total of energy demanded to the total energy produced. This ratio does not include the indirect energy costs associated with the other intermediate purchases the energy sectors makes to produce energy. In a similar fashion, a_{22} is the ratio of some measure of the physical quantity of manufacturing goods used by the manufacturing sector to meet total manufacturing demand. The off diagonal elements are not, in a physical units, dimensionless but involve the ratio of Joules to Manufacturing goods and the inverse.

In order to calculate explicitly the z_{ij} terms in equations (5.18) and (5.19) we need to calculate the total outputs of the energy and manufacturing sectors which are defined as follows:

$$X^e(t) = L_{11}(C^e(t) + G^e(t)) + L_{12}(C^m(t) + G^m(t)) \tag{5.25}$$

$$X^m(t) = L_{21}(C^e(t) + G^e(t)) + L_{22}(C^m(t) + G^m(t)) \tag{5.26}$$

Where the Leontief coefficients (the L_{ij}) in the above equations are the Leontief or total requirements coefficients and are derived from the following:

$$L = (I - A)^{-1} \tag{5.27}$$

Using some basic linear algebra, the specific elements of the Leontief matrix may be calculated:

$$L_{11} = \frac{(1 - a_{22})}{|A|} \tag{5.28}$$

$$L_{12} = \frac{a_{12}}{|A|} \tag{5.29}$$

$$L_{21} = \frac{a_{21}}{|A|} \tag{5.30}$$

$$L_{22} = \frac{(1 - a_{11})}{|A|} \tag{5.31}$$

[13]Technically we will see that this is actually the ratio of power invested on power return but for now it is sufficient to think of this as energy; more on this in the following chapter.

where $|A|$ is the determinant of A and is calculated as:

$$|A|= a_{11} * a_{22} - a_{21} * a_{12} \tag{5.32}$$

Since the technical coefficients are assumed to be exogenous parameters the total industry outputs can be determined by substituting in equations (5.28), (5.29), (5.30), (5.31), into equations (5.25) and (5.26). Finally, having both the a_{ij} and X_j terms, it is possible to solve the for interindustry flows z_{ij} by using equation (5.22) specifically. For example, $z_{12}(t)$ in (5.18) is therefore:

$$z_{12}(t) = a_{12} \cdot [L_{21}(C^e(t) + G^e(t)) + L_{22}(C^m(t) + G^m(t))]$$

Having solved for the interindustry flows we can return to the sectoral wage equations (5.18) and (5.19) to determine the wages paid to households by each sector.

5.4 Solving The Model

In the previous section the list of model equations was elaborated; in this section we turn to obtaining an analytical solution to the model itself.

It is worth briefly considering what it means to obtain a model solution in the context of the above equations. Many equations, such as that for the taxes or disposable income, are simply identity relations; for example, disposable income at time t is simply equal to wages minus taxes at time t. These identity relations are static in so far as they relate variables in some given time period. The element of the model that is dynamic and drives the model forward in time is the stock equation. In this simple model there is a single stock variable; money, and, hence solving the associated differential equation. Either of (5.20) and (5.21) can be used as they both describe the evolution of the same quantity; however, neither are written in a solvable form. We turn to this now; note well that for ease of exposition the (t) is dropped temporarily.[14]

substituting (5.12) into (5.16) and rearranging we have that:

$$Y = C + G$$

[14]That is, we will write C instead of C(t) during the algebraic manipulations.

and therefore

$$Y = \alpha_1 YD + \alpha_2 H + G$$

substituting in (5.10), (5.11) leads to:

$$Y = \alpha_1(1-\theta)Y + \alpha_2 H + G$$

Rearranging the above and solving for Y we obtain,

$$Y = \frac{(\alpha_2 H + G)}{1 - \alpha_1(1-\theta)} \tag{5.33}$$

Now, substituting (5.9) into (5.20) and using again that $Y = WB$ and hence that $YD = (1-\theta)Y$ we have:

$$\frac{dH}{dt} = (1-\alpha_1)(1-\theta)Y - \alpha_2 H$$

and substituting in the expression for Y in (5.32)

$$\frac{dH}{dt} = \underbrace{\frac{(1-\alpha_1)(1-\theta)}{(1-\alpha_1(1-\theta))}}_{\psi_1} * (\alpha_2 H + G) - \alpha_2 H$$

and therefore,

$$\frac{dH}{dt} = \underbrace{\alpha_2(\psi_1 - 1)}_{\omega_1} H + \psi_1 G$$

and finally,

$$\frac{dH}{dt} - \omega_1 H(t) = \psi_1 G(t) \tag{5.34}$$

(5.33) is now written in the form of a non-homogenous first order linear differential equation which can be solved using the method of integrating factors. Here the integrating factor is simply:

$$\mu(t) = e^{\int \omega_1 dt}$$

as such, the expression for $H(t)$ is:

$$H(t) = \frac{1}{e^{\omega_1(t)}} \left(\int (e^{\omega_1 t} \omega_2 G(t)) dt + c \right)$$

which simplifies to:

$$H(t) = \frac{\omega_2}{\omega_1} G(t) + \frac{c}{e^{\omega_1(t)}} \tag{5.35}$$

where the lowercase c is the constant of integration. If we specify an initial condition that, say $H(0) = 0$, then we can solve fully and get rid of the arbitrary constant of integration:

$$H(t) = \left(\frac{\omega_2}{\omega_1} - \frac{\omega_2}{\omega_1 e^{\omega_1 t}} \right) G(t) \tag{5.36}$$

Since it is assumed in (5.14) that total government spending is constant across time then the $G(t)$ in the above solution is simply a constant G. Therefore, we have solved explicitly for the time-path of the money stock in the model.

With this it is possible to solve for the remaining variables. For example, substituting (5.36) back into (5.12) we have the following time-path for GDP:

$$Y(t) = \left(\frac{1}{1 - \alpha_1(1-\theta)} \right) \left(\alpha_2 * \frac{\omega_2}{\omega_1} G + \frac{c}{e^{\omega_1(t)}} + G \right) \tag{5.37}$$

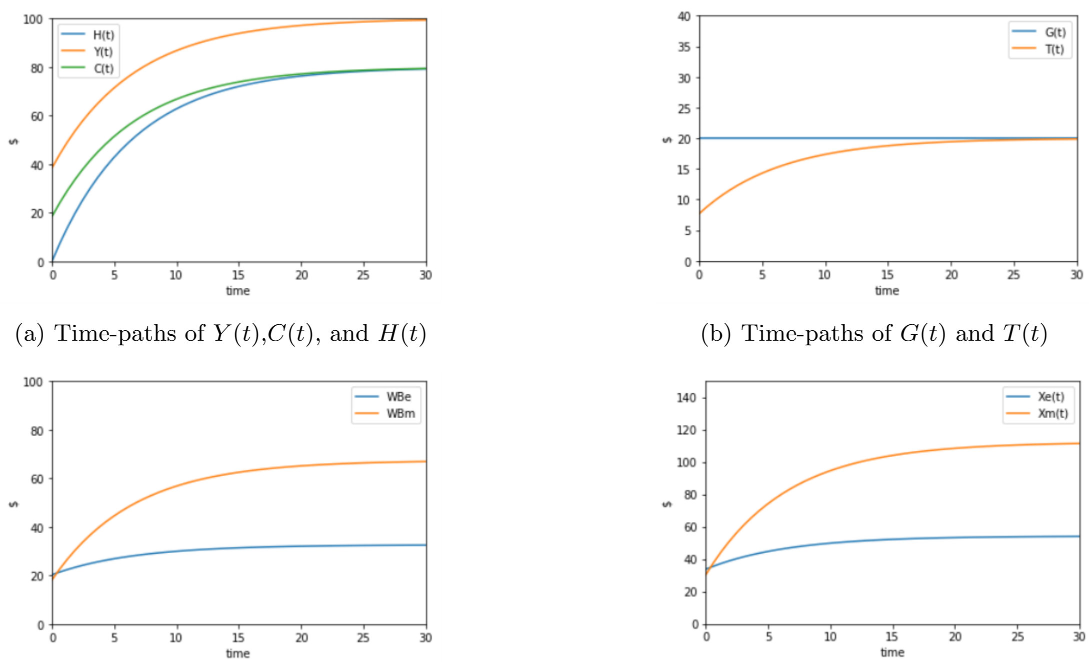
A point to note is that knowing the value of $H(t)$ at any point in time is sufficient to completely determine all aspects of the model. Once we know the value of the stock at any point in time we can always work backswords to re-express a flow equation using the function for $H(t)$.

An examination of (5.37) reveals that the time-path of GDP is independent of any input-output elements and depends solely on the exogenous parameters and the level of government spending. This result is in fact an artefact of the simplicity of the model, but it usefully illustrates an important notion. Changes to the technical coefficients in this simple setup impact only the magnitude of inter-industry flows and not the final demands. From (5.6) any changes to inter-industry flows impacts the economy at the sectoral level but cancel out at the macroeconomic level. Furthermore, GDP is calculated only as the sum of final demand elements and therefore changes to inter-industry flows by definition have no influence on the time-path of GDP.

That changes to the activity levels at the sectoral level do not influence the level of GDP also showcases the model as properly stock-flow consistent. As the model is a simple closed economy, changes in one sector are exactly balanced by changes in the other sector leaving total wages paid unchanged. In fact, had changing the input-output technical coefficients changed GDP in this simple setup it would indicate that the model was not properly specified in the first place.[15]

5.5 Simulation and Analysis

Having solved the model analytically in the previous section we may use (5.35) to simulate the model. Letting $H(0) = 0$ be the initial condition, simulations for the main model variables are shown in the following figure 5.4c.



(a) Time-paths of $Y(t)$,$C(t)$, and $H(t)$

(b) Time-paths of $G(t)$ and $T(t)$

(c) Time-paths of $WB^e(t)$ and $WB^m(t)$

(d) Time-paths of $X^e(t)$ and $X^m(t)$

Figure 5.4: Evolution of main model variables from an initial condition of $H(0) = 0$.

In 5.4a we see that, $Y(t)$ and $C(t)$ grow initially and appear to approach some equilibrium values. That the model appears to trend towards some steady-state is also shown clearly in 5.4b where taxes increase steadily until there is no yearly government deficit and hence no additional growth in the stock of money (recall (5.21)). In similar fashions the wages paid by the respective sectors and the total output of these sectors increases initially and levels off towards some steady-state values as shown in 5.4c and 5.4d.

The steady-state value of the stock of money and the other variables is readily calculable. From 5.4b we note that the steady state occurs when $T^* = G$. Since $T^* = \theta Y^*$ this implies

[15]This independence of GDP from the activity of the sectors disappears when any number of more realistic assumptions are made, as will be seen in later chapters.

that:

$$G = \theta Y^*$$

and finally that:

$$Y^* = \frac{G}{\theta} \tag{5.38}$$

which leads to a numerical value of $20/0.2 = 100$. With the model in hand we may explore questions along the following lines. Suppose the economy is in the steady state and there is, at some time t, a permanent increase in government expenditures, a permanent increase in the taxation rate θ, or an increase in the propensity to consume, how will the model evolve? To explore these questions in the context of the simple model we conduct three separate "experiments" assuming that the model is at its steady state at time $t = 0$ and that these changes occur at time $t = 5$.

5.5.1 A Increase in Government Expenditures

The first experiment we will conduct with SFCIO-SIM is to examine the impact of a permanent increase in government expenditures from 20 to 25. The evolution of key model variables is displayed in the follow figure. From the calculation above we know that the new steady-state value of output will be $25/0.2 = 125$. The evolution of output this level is shown in panel (1) of 5.5. As a result of this additional government expenditures we see in panels (2), (3), and (4) that the stock of money, disposable income, and consumption adjust slowly over time to new steady-state values. However, as the increase in expenditure to its new level is instantaneous while the increase in tax revenue is not, the government sees a sharp transient increase in its deficits which declines back down to a balanced budget over the simulation run. As taxes are simply assumed to be some constant fraction θ of the wage bill earned, the reason for this prolonged deficit is made clear in panel (8) which shows that the wage bill takes time to adjust to its new higher steady-state level. Finally, panels (6) and (7) show that the total outputs of the energy and manufacturing jump suddenly to meet the new higher demand and rise over the simulation run as the increased expenditure raises household consumption to a new permanently higher level.

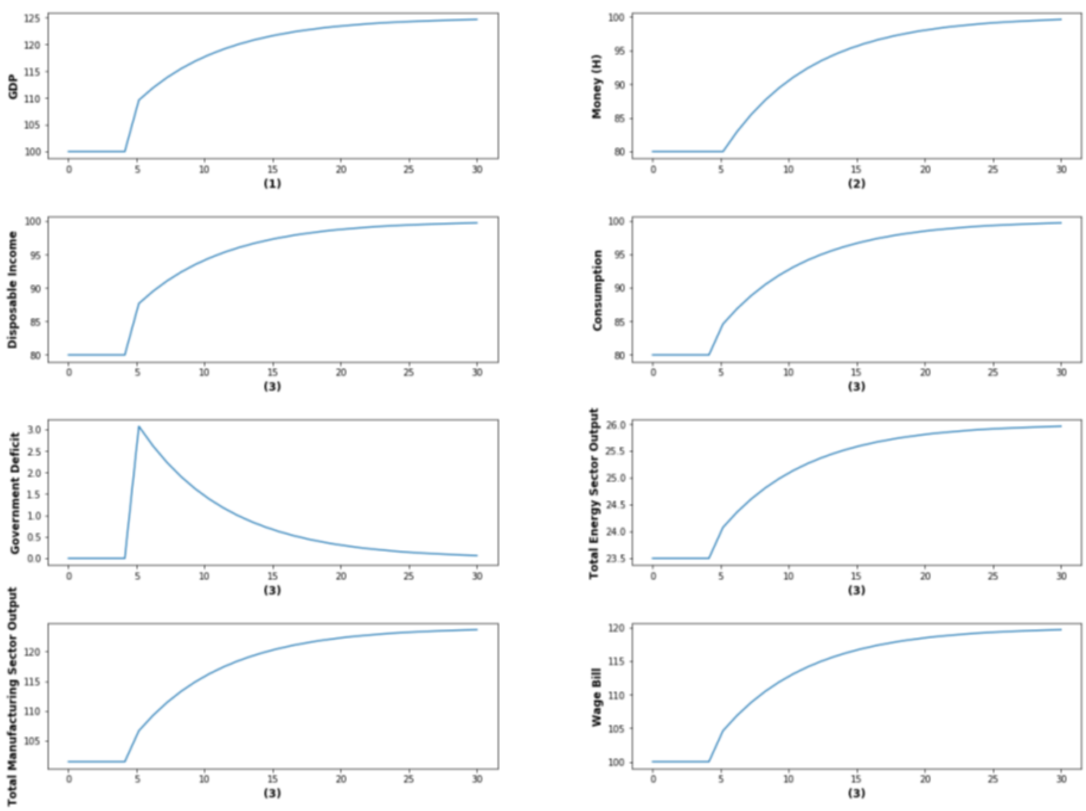


Figure 5.5: Evolution of select SFCIO-IOSIM model variables following a permanent increase in government expenditure from 20 to 25 at time $t = 5$.

5.5.2 A Increase in the Tax Rate

The second experiment to conduct with SFCIO-IOSIM is a sudden increase in the rate of taxation from 0.2 to 0.3. The results of this are shown below.

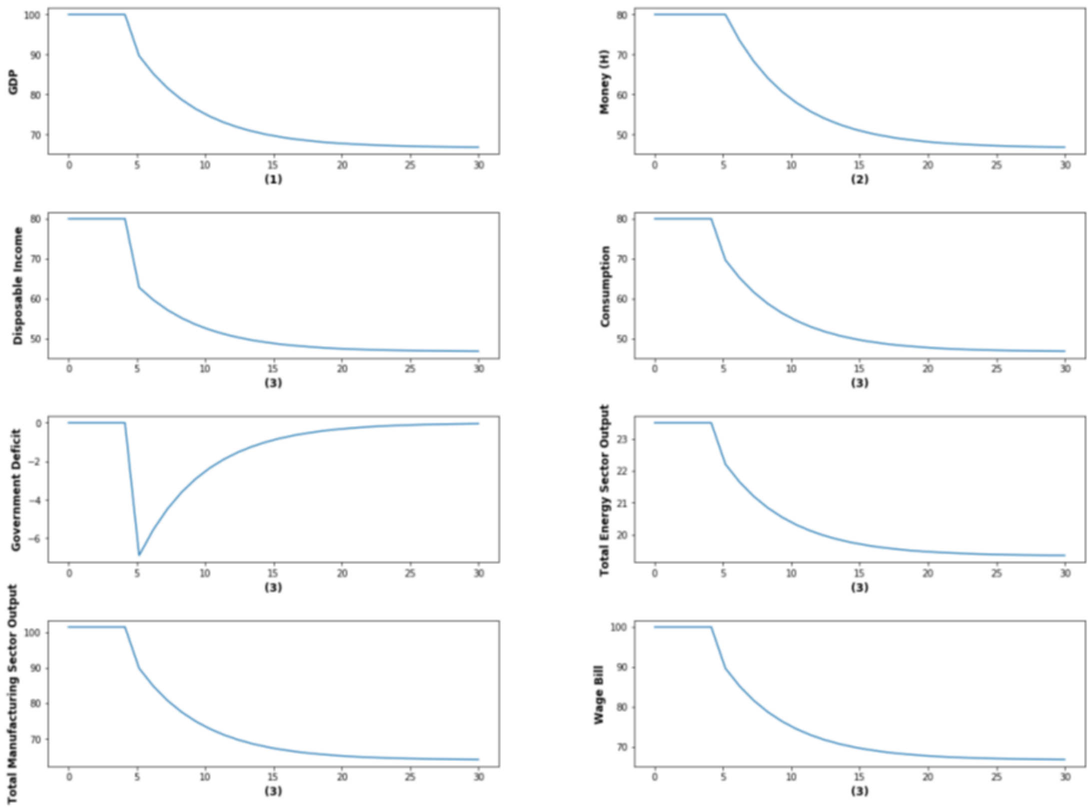


Figure 5.6: Evolution of select SFCIO-IOSIM model variables following a permanent increase in the tax rate θ from 0.2 to 0.3.

The increased taxation experiment works essentially in reverse to the increase in government experiment scenario above with the variable trajectories essentially tracing out mirror images of those in 5.5. While the SFCIO-IOSIM model is a particularly simple model it does have implicit feature that makes it relevant to the recent public discourse concerning modern monetary theory. [16] In the increased government spending scenario, the government is the currency issuer and therefore spends into existence the new government expenditure. This new high-powered money circulates throughout the economy and overtime wages, consumption, taxes, etc., adjust. The transitory period of deficits implies a period of time when net new money is added to the

[16] The publication of Stephanie Kelton's *The Deficit Myth* has brought modern monetary theory (and more broadly a non-neoclassical form of economic thinking) decidedly to the forefront of public consciousness if its sales are any indication (Kelton, 2020).

economy. It is certainly not the case that this additional expenditure had to be financed via taxes or borrowing.

In the taxation scenario, the government runs a transitory period of surpluses implying the net "destruction" of money circulating in the economy as it is taxed back by the government.[17] The higher taxation leads to lower disposable income, which via the consumption function, leads to lower expenditures by households to the manufacturing sector (recall that energy consumption is assumed constant). This direct decline in manufacturing demand, via the sectoral interdependences, leads to an indirect decline in manufacturing sector purchases of energy sector output and hence a decline in the energy sector output over the model run time.

5.5.3 A Increase in the Propensity to Consume

Finally, let us examine the impact of a permanent increase in the propensity to consume

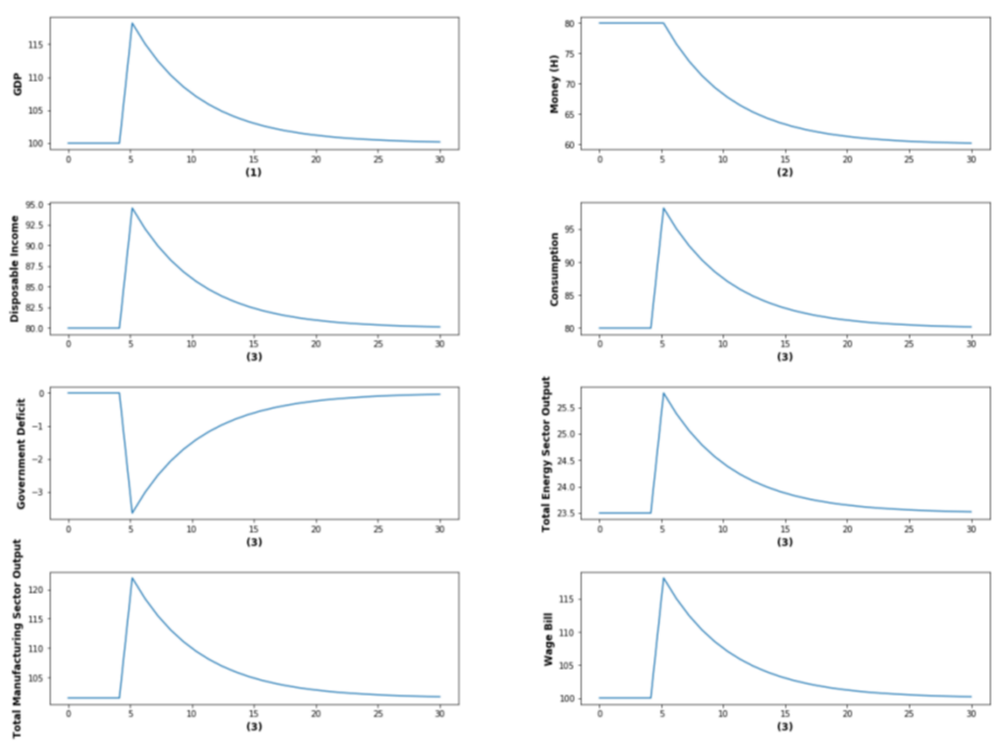


Figure 5.7: Evolution of select SFCIO-IOSIM model variables following an increase in the propensity to consume parameter α_1 from 0.6 to 0.7.

[17]Conventional political rhetoric often demonizes the deficit and government debt while holding surpluses to be intrinsically good based on the incorrect view of the government as a household. While model SFCIO-SIM is extremely simple, it displays the opposite logic in that, all else equal, the government deficit is the private sectors surplus and the government debt is the public's assets.

Figure 5.7 indicates that the impact of the increased propensity to consume is transitory for the main model variables except that the money holdings H by households are permanently lower at the end of the simulation period. This effect, as noted by in Godley and Lavoie's model-SIM (the comparator model for this chapter) arises from the upward change in the propensity to consume while holding the propensity to save constant.

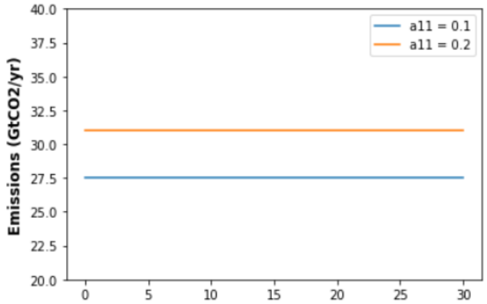
5.6 Bringing in Emissions

Having explored the basics of the model we can turn to adding the simplest version of a key component of later models, emissions. Let us now assume that the energy sector is a fossil fuel sector which produces emissions in proportion to the total energy it produces (energy for intermediate use and energy to meet final demand). Denoting emissions as $E(t)$ we have:

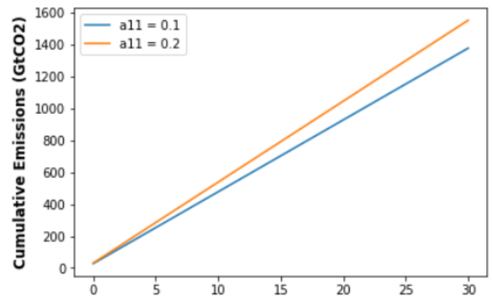
$$E(t) = \eta X^e(t) \tag{5.39}$$

where η is some constant of proportionality.[18] In this simplest of models, emissions increase proportionally with increases or decreases in output and do not display any particularly interesting dynamics. However one model result does emerge; modifications to the a_{11} parameter do have important impacts on emissions and cumulative emissions. Recalling that a_{11} can be interpreted as the reciprocal of $\text{EROI}_{\text{Society}}$ what happens to the trajectory of emissions given changes to this EROI? Let us assume for simplicity that every Joule of energy produced produces a unit of emissions (that is, $\eta = 1$). The following figure displays the emissions trajectories from the steady state for decreasing EROI values for society (and hence increasing a_{11} term).

[18] Here η converts the total output of the energy sector (which is in dollar monetary terms) to emissions. To be precise, since $X^e(t)$ is a monetary flow variable it is measured in dollars per unit time. Emissions, a physical flow variable, are measured, generally, in units such as megatonnes (Mt) or gigatonnes (Gt) per year. Therefore, assuming $X^e(t)$ is measured in dollars per year, it is the case that η must have dimensions of megatonnes or gigatonnes per dollar.



(a) Emissions



(b) Cumulative Emissions

Figure 5.8: Panel (a) displays emissions trajectories corresponding to $a_{11} = 0.2$ and $a_{11} = 0.3$; the increase in the technical coefficient implies a decline in EROI. Panel (b) displays the trajectories for cumulative emissions under the same assumptions.

Here we begin to see the utility of combining the SFC and IO approaches. While changing the technical coefficients does not impact GDP in the simple model it very much has an impact on total energy usage and therefore emissions. In 5.8 it is clear that, from the steady-state, a decline in energy sector EROI and therefore an increase in the a_{11} technical coefficient corresponds directly to larger total energy requirements and emissions. In both scenarios, emissions remain constant over time with the lower EROI economy producing greater emissions than the higher per unit of energy output. As a result, the lower EROI economy has an increasingly greater quantity of cumulative emissions over time than does the higher EROI economy.

This relation between emissions and EROI is fundamental and some effort is put toward capturing the EROI-depletion dynamics in chapter 11. It is worth noting here however that this dynamic is particularly amenable to being captured using the IO framework given the built-in notion of sectors purchasing their own outputs as inputs to produce their goods to meet final and intermediate demands.

5.7 The Way Forward

The model introduced in this chapter was built to introduce the SFCIO framework in as simple a manner as possible while introducing certain key concepts such as the EROI-emissions relation. In the following three chapters we will steadily relax the simplifying assumptions made here and explore in turn the addition of physical capital, dynamic pricing, and a simple private bank framework. The next step in the trajectory of model development is to introduce the concept of physical capital that underlies the generation of energy in physical terms.

Chapter 6

Chapter Six: Introducing Capital to the Model

In this chapter the simple SFCIO model of the previous chapter is advanced considerably by having the energy sector construct and maintain a capital stock in order to produce its output. The task in this chapter is to consider how energy capital fits into the SFCIO framework and how to deal with the issues that arise in doing so.

What will become readily apparent in this chapter is that simply adding an energy capital stock to the formulation in the previous chapter is a less obvious task than it might appear; issues of both the accounting structure of the model and dimensional analysis become of critical importance. As these issues will appear in all subsequent models it is useful to explore them in depth in the context of a still very simple model.

6.1 Capital and Production: Some Theoretical Issues

Capital, and how it is deployed in models, is subject with a somewhat controversial history in economics. Issues ranging from capital's circular definition in the Cambridge Capital Controversies (see (Cohen and Harcourt, 2003)) to the problem arising from the assumption of its substitutability with other factors production (See (Ayres, 2007) for example) indicate deep underlying issues with how we understand and model the often nebulous concepts of capital and production. The inclusion of energy in standard production functions is even more troubling (see (Keen et al, 2019)) as it is often included as just another factor of production that can, via the deployment of, say, additional capital or labour, be reduced while still leaving aggregate output unchanged. Ultimately, such a view of the substitutability of inputs leads to the absurd scenario when energy (and material) inputs can be reduced to virtually zero if sufficient capital and labour are provided. This view of production and energy, while convenient mathematically, violates basic physical sense.

Given that the SFCIO framework is explicitly multi sectoral with production based on an input-output structure it is likely clear that the standard neoclassical production function will play no role in the modelling undertaken in this dissertation. However, it is worth looking briefly at an example of the more conventional approach in order to highlight some of the issues that will be addressed in this chapter.

Consider the following standard Cobb-Douglas production function:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{6.1}$$

where A is the so-called total factor productivity and $K(t)$ and $L(t)$ are some measures of capital and labour inputs (we shall return to the measurement problem momentarily). A classical problem in a microeconomic theory class is, given the existence of a wage rate w and a rental rate on capital r, to determine the profit-maximizing or, equivalently, the cost minimizing set of inputs to produce a given quantity of output.

In response to changing costs of inputs, the producer needs simply substitute to some new combination that minimizes costs. In a physical sense however the above production function and its elegant curved isoquants are mostly meaningless constructs. The obvious problem in the specification of (6.1) is that "output" (however measured) is produced only by a relation of the factors of production with no reference to the material basis that must form the output itself (See (Daly and Farley, 2004) chapter four for a discussion on this).[1] If this problem is ignored, a second is introduced by noting that for many production function forms (the Leontief formulation is an example of an exception), some quantity of output may be produced with any of the infinite combinations making up a given isoquant. This latter assumption allows for output to be produced with vanishingly small quantities of one input assuming sufficiently large quantities of the other.[2]

In this chapter we will spend some effort ensuring that each of the equations encountered

[1]Either that or the output of the economy defined by (6.1) is somewhat alarming in nature

[2]The basic Leontief production function in capital and labour is:

$$Y = \min(K, L)$$

. See (Miller and Blair, 2009) chapter two for a readable discussion relating the Leontief production function with the input-output model structure.

are physically meaningful to avoid a problems along the lines of the following example. In his seminal 1956 paper, Solow introduces the now standard Solow model (Solow, 1956). The first sentence of section two states that "There is only one commodity, output as a whole, whose rate of production is designated Y(t)". We assume that this means that $Y(t)$ is defined in the standard way as a flow variable with units of commodity per unit time given the explicit use of the term rate. Capital is then defined as "The community's stock of capital K(t) takes the form of an accumulation of the composite commodity"; which in this language is unambiguously a stock variable. Confusingly Solow originally defines the variable $L(t)$ as "...labor, whose rate of input is L(t)" only to later define it as total employment which in the model is equal to the total labour force which is governed by the equation $L(t) = L_0 e^{nt}$. The latter definitions and the equation of growth clearly imply labour is a stock variable and not a rate as originally denoted. Now to see why this leads to an absurdity let us examine the Cobb-Douglas functional form deployed in the paper:[3]

$$Y = K^{\alpha} L^{1-\alpha} \tag{6.2}$$

where α is a dimensionless constant. By the above we have that $Y(t)$ is measured in commodity per unit time $[c/s]$, capital $K(t)$ is the accumulation of the commodity and therefore measured simple in units of the commodity $[c]$, and labour is measured as the labour force (number of people) $[n]$. The LHS dimensions are therefore commodity $[c/s]$ while the RHS dimensions are $[c \cdot n]$ or commodity times the unit that measures labour. The multiplication of capital times labour produces a quantity (machine times labour) that (a) is not meaningful; and (b), does not match $Y(t)$ on the RHS. A key equation of the famous Solow model is, at its core, dimensionally inconsistent and relies on evoking quantities with no real meaning. The total factor productivity term A can rescue the above Cobb-Douglas function only if it has the units of $[1/(\mathrm{s} \cdot n)]$ or "per unit time labour". This term is equally inscrutable as to what it means or measures. Applying dimensional analysis to the Cobb-Douglas function, interestingly, was an activity of brief interest in an Austrian economics journal with the publication of an analysis of the units problem in (Barnett, 2004) and the response in (Folsom and Gonzalez, 2005).

The purpose of this discussion is not to point out flaws in Solow's work per se but to note that the common neoclassical approach to representing capital and its use in the production

[3]We drop the time subscripts to match the style in the paper.

process is often dimensionally inconsistent which raises questions as to the meaningfulness of any results obtained using such an approach. In this chapter we will introduce a capital stock and very carefully work out how it fits into the SFCIO framework, ensuring at all times that equations are dimensionally homogenous.

The concerns raised here are of course not novel. The economist Joan Robinson had the following to say concerning production functions and capital:

> . . the production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $Q = f(L, K)$ where L is a quantity of labor, K a quantity of capital and Q a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labor; he is told something about the index-number problem in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units K is measured. Before he ever does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next. (Robinson, 1953)

6.2 Balance Sheet, Transactions Flow, and Input-Output Matrices

We begin, as with the previous chapter, by displaying the three principal matrices describing what we will call model SFCIO-CAP. Unlike the previous model, the rows of the balance sheet matrix do not sum to zero. The energy sector's built capital is an asset to itself but a liability to no other party. Therefore, the sum of the capital row is simply $K(t)$ which in turn is equivalent to the net worth of the economy. By definition, the value of financial assets and liabilities cancel and therefore ultimately the net worth of the economy is simply the value of the economy's tangible assets.

$$
\begin{array}{lc}
 & \begin{array}{ccccc} \text{Households} & \text{Energy Sector} & \text{Manufacturing Sector} & \text{Government} & \Sigma \end{array} \\
\begin{array}{l} \text{Money} \\ \text{Capital} \\ \text{Government Loans} \\ \text{Net Worth} \end{array} &
\left(\begin{array}{ccccc}
H(t) & 0 & 0 & -H(t) & 0 \\
0 & K(t) & 0 & 0 & K(t) \\
0 & -L(t) & 0 & L(t) & 0 \\
-V_h(t) & -V_e(t) & 0 & -V_g(t) & -V(t)
\end{array}\right)
\end{array}
\tag{6.3}
$$

Figure 6.1: Balance Sheet Matrix for model SFCIO-CAP

It is worth noting that the balance sheet in Figure 8.1 a is now considerably more complex than that of the model SFCIO-IOSIM. Why should this be the case when the only stated addition to the model is the energy capital stock? The answer, as will become apparent, is due to the rigorous accounting enforced by the SFC approach. Examining the balance sheet we see that energy firms now have an asset called $K(t)$ representing their built capital stock, a liability $L(t)$ denoting loans, and a net worth $V_e(t)$. It is readily apparent that the net worth of the energy sector is the value of its capital stock less the liabilities it holds in the form of loans.

As SFCIO-CAP does not contain a private banking sector (which would normally supply the business loans) the role of loan provisioning falls to the government sector (which implicitly includes a central bank). Therefore, the model is a very basic example of pure public money economy where the central bank assumes some of the roles of the financial sector where loans are assumed here to be interest free. In this case the government extends loans to the energy sector which uses these funds to finance the construction of its physical capital stock. The transactions flow matrix is presented in the following figure.

$$
\begin{array}{lc}
 & \begin{array}{cccccc} \text{Households} & \text{Energy Sector Current} & \text{Energy Sector Capital} & \text{Manufacturing Sector} & \text{Government} & \Sigma \end{array} \\
\begin{array}{l} \text{Consumption} \\ \text{Wages} \\ \text{Government} \\ \text{Taxes} \\ \text{Inter-Industry} \\ \text{Investment} \\ \text{Loan Repayment} \\ \Delta\text{Money} \\ \Delta\text{Loans0} \\ \Sigma \end{array} &
\left(\begin{array}{cccccc}
-C(t) & C^e(t) & 0 & C^m(t) & 0 & 0 \\
WB(t) & -WB^e(t) & 0 & -WB^m(t) & 0 & 0 \\
0 & G^e(t) & 0 & G^m(t) & -G(t) & 0 \\
-T(t) & 0 & 0 & 0 & T(t) & 0 \\
0 & z_{12}(t) - z_{21}(t) & 0 & z_{21}(t) - z_{12}(t) & 0 & 0 \\
0 & 0 & -I(t) & I(t) & 0 & 0 \\
0 & -\epsilon L(t) & \epsilon L(t) & 0 & 0 & 0 \\
\frac{-dH}{dt} & 0 & 0 & 0 & \frac{dH}{dt} & 0 \\
0 & 0 & \frac{dL}{dt} & 0 & -\frac{dL}{dt} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{array}
\tag{6.4}
$$

Figure 6.2: Transactions Flow Matrix for model SFCIO-CAP

There are several additional components in model SIM-CAP compared to the model from the previous chapter that must be discussed. First, the transactions flow table now includes both a current and capital account for the energy sector. In the capital account we see an entry for investment $-I(t)$ with a negative sign representing an outflow of funds. The manufacturing sector is assumed to be the producer of these capital goods and therefore records an inflow of funds $I(t)$ as payment for the capital goods it sells to the energy sector.

How the loan repayments are represented in the model requires some care. The current account of the energy sector shows an outflow $-\epsilon L$ where ϵ is the fraction of total outstanding loans $L(t)$ paid back per unit time. By this logic one would expect these repayments to flow from the energy sector to the government sector column which is not the case in 6.2. To understand this, consider what occurs when the energy sector pays back some fraction of its loan. The energy sector's liability declines by the amount paid and commensurately the government sector sees its loan asset decline by that amount. Because we explicitly assumed, for the time being, that the loans are interest free, the repayment of loan principal in this manner simply acts to reduce the magnitude of the outstanding loans. This is exactly represented in the capital account column of the energy sector where the loans are assumed to grow as money is borrowed to finance investment and decline with repayment. From the third last row, the change in loans as captured in the capital account is of the same magnitude and opposite sign of the government sector as it must be.

Now, let us make a check to see that the models accounting is correct and see if GDP calculated from the expenditures is equivalent to GDP calculated via income. GDP by expenditures at time t is:

$$Y(t)_{\text{Expenditure}} = C(t) + I(t) + G(t)$$

what is GDP from the income side? In the previous chapter income was simply the wage bill $WB(t)$. By definition, GDP by the income approach is the sum of incomes (wages) and profits. At first pass we might assume the same in model SFCIO-IOSIM and would therefore calculate that:

$$Y(t)_{\text{Income}} = WB(t) = C(t) + I(t) + G(t) - \epsilon L(t)$$

which clearly differs from the expenditure approach by a factor of $-\epsilon L(t)$. Now, to understand how to remedy this problem let us consider the case where none of the loan principal is repaid. Then after the payment of wages and inter-industry expenditures, the energy firm has a remaining $\epsilon L(t)$ in funds. These funds are the difference between revenue and costs or, put simply, profit. This profit is transferred to the capital account and plays a role in financing investment via the repayment of loans. However, since $\epsilon L(t)$ is ultimately a form of profit it must be included in the income calculation of GDP Therefore, we find that:

$$Y(t)_{\text{Income}} = WB(t) + \epsilon L(t) = C(t) + I(t) + G(t) - \epsilon L(t) + \epsilon L(t)$$

which simplifies to:

$$Y(t)_{\text{Income}} = C(t) + I(t) + G(t)$$

and so is therefore identical to GDP as calculated by the expenditure approach. Finally, the accounting for model SIM-CAP is completed by presenting the input-output table in the following figure.

$$\begin{array}{l} \\ \text{Energy} \\ \text{Manufacturing} \\ \text{Payments Sector} \\ \hline \text{Total Outlays} \end{array} \begin{array}{ccccc} \text{Energy} & \text{Manufacturing} & \text{Final Demand} & | & \text{Total Output} \end{array} \\ \begin{pmatrix} z_{11}(t) & z_{12}(t) & C^e(t) + G^e(t) & | & X^e(t) \\ z_{21}(t) & z_{22}(t) & C^m(t) + G^m(t) + I(t) & | & X^m(t) \\ WB^e(t) + \epsilon L(t) & WB^m(t) & 0 & | & WB(t) + \epsilon L(t) \\ \hline X^e(t) & X^m(t) & C(t) + G(t) + I(t) & | & X(t) \end{pmatrix} \tag{6.5}$$

Figure 6.3: Input-Output Matrix for model SFCIO-CAP

Having dealt with the additional accounting complexities associated with the introduction of capital to the SFCIO model we are faced with a more subtle issue; how to represent the energy capital stock and the output it is used to produce. Clearly energy capital must produce something related to the physical concept of energy but doing this properly in a continuous time SFC setting requires very careful attention to detail regarding the dimensions of the quantities being considered. Before turning to the model equations, we must first spend some effort in investigating how to relate investment (a monetary flow variable) to the output of the energy capital stock.

6.3 A Brief Foray into Dimensional Analysis

For the purposes of developing this still relatively simple model it is not necessary to specify the nature of the energy capital in any detail. We assume simply that the energy sector operates a stock of machines $k(t)$ that is used in the production process of energy goods which might represent technologies ranging from solar panels to coal-fired power plants. The lower case $k(t)$ denotes the number of physical machines measured in units of what we will simply call machines [m].[4] while the upper case $K(t)$ denotes the value of the machines at current prices. The relation between the two is given simply as:

$$K(t) = P_m \cdot k(t)$$

There are some subtleties concerning the representation of energy capital in the SFCIO framework

[4]It is only fair to Solow to note that manufacturing sector output in this model takes the form of a composite good that acts as both capital goods (the aforementioned machines [m]) and output for consumption. While this is itself certainly a very strong simplifying assumption, we will work to make it at least physically consistent.

that will crop up repeatedly in the following chapters and therefore will be addressed in some detail in this chapter. First, how should the energy output of energy capital be represented? Energy, unlike capital, is a precisely defined physical concept and integrating it into this framework requires care. Consider the following equation relating what we will temporarily (and incorrectly) term energy supply $E_S(t)$:

$$E_s(t) = \phi k_t \tag{6.6}$$

Equation (6.8) states that the energy supply is proportional to the magnitude of the energy capital by the constant of proportionality ϕ. To understand the units of $E_s(t)$ and ϕ it is instructive to first look at the energy demand equation and work our way back.

Total "energy demand" in the model is given as follows:

$$E_d(t) = \frac{X^e(t)}{P_e} \tag{6.7}$$

where total "energy" demand in physical units at time t is the dollar value of the total output of the energy sector necessary to meet intermediate and final demands for energy divided by P_e, the price of energy. First, what are the units of $X^e(t)$? Since this term represents the sum of intermediate and final demands the dimensions of $X^e(t)$ must be dollars per unit time or [$/s] because it is the sum of energy consumption expenditures $C^e(t)$ and the interindustry transaction terms $z_{11}(t) + z_{12}(t)$ which are flow variables.

What dimensions does the price of energy have? In the model we assume the energy price has the intuitive dimensions dollars per unit of energy; for example, dollars (or cents) per kilowatt hours. For simplicity here we will reduce this to dollars per Joule [$/J]. Therefore, the RHS of (6.7) has dimensions:

$$\left[\frac{[\$/s]}{[\$/J]}\right] = [J/s] = W$$

That is, the dimensions cancel to Joules per second, which is the unit of power, Watts. Therefore, the energy-demand function $E_d(t)$ is actually no such thing; rather, it is a power demand function. This function denotes the total power demanded at each point in time.[5]

To relate this power demand back to energy one must simply integrate the power demand function over some definite time period t_0 to t_1 as follows:

$$E = \int_{t_0}^{t_1} E_d(t)dt$$

[5] Recall that energy is measured in joules, while power is measured in joules per second. Power measures the work done per second by some process where work is the energy transferred to or from some "object" due to the application of a force.

as by definition power multiplied by time returns energy. Now, we may return to the original energy supply function and complete the determination of its units. Knowing that energy demand is in fact power demand, it should follow that the energy supply function is itself a power supply function. Ultimately, the two functions should be comparable and their equality (physical supply equalling physical demand) will be used as a key element to determine the behavioural component of the energy sector investment equation. As such we need $E_s(t)$ to be measured in units of Watts and, fortunately, this requirement lends the coefficient ϕ a very natural interpretation. Recalling that $k(t)$ is just measured in units of machines it must be the case that ϕ has units of Watts per machine. Put more simply, ϕ is the power output per unit of energy capital which is a standard way to describe generation; for example, one might have a two-megawatt wind turbine or a 250 watt solar panel.

If we were careless and simply assumed that $E_s(t)$ and $E_d(t)$ were energy, rather than power demand and supply functions then integrating such energy supply and demand functions over some time interval would return the physical quantity of Joule-seconds which is not meaningful in this context. It is important to note that it is the assumption of continuous time which leads to power being the prime unit of analysis rather than energy. In a discrete time formulation, we would have energy supply and demand functions rather than power.

6.4 Model Equations

The equations of model SFCIO-CAP are largely identical to those of the model elaborated in the previous chapter with the necessary additions to facilitate the inclusion of the energy capital stock and investment. The behavioural component of the investment equation used here is somewhat idiosyncratic to this work. For example, in two recent ecological macroeconomics SFC models, (Bovari et al, 2018) and (King, 2020), profit shares and profits are used to drive investment behaviour. In the simple model with private bank money in (Godley and Lavoie, 2007), a target capital stock is used. In the SFCIO-CAP model it is assumed that the energy firm sets a target capital stock that equates power supply with power demand.

From the previous section two new equations were introduced for power supply and demand; $E_s(t)$ and $E_d(t)$ respectively. In model SIM-CAP the energy sector invests in new energy capital in order to bring power supply in line with power demand. This is represented in the following behavioural equation:

$$I(t) = \Delta P_m(\phi^{-1}(E_d(t) - E_d(t))) \tag{6.8}$$

Broadly, (6.8) states that investment is undertaken to reduce the gap between power demand and power supply. The LHS term $I(t)$ is measured in dollars per unit time [$/s] and this ultimately must be the case for the RHS. Therefore it is necessary to move from the difference in power as measured in Watts of the bracketed term $(E_d(t) - E_d(t))$ to dollars per unit time.

To accomplish this first note that since ϕ has units of Watts per machine its inverse has units of machines per Watt. Therefore $\phi^{-1}(E_d(t) - E_d(t))$ has, after cancelling, units of machines [m] and indicates the total number of new machines necessary to close the power gap. Since P_m has units [$/m], then $P_m(\phi^{-1}(E_d(t) - E_d(t))$ has units of dollars. What then is the parameter Δ and how is it measured?

The parameter Δ in the model is a partial adjustment parameter which represents the rate at which the gap between, in this case, the power demand and power supply is closed. A Δ value of 1 for example implies the gap is closed instantaneously while a value less than one implies that such a closure happens only over time. While this parameter makes good economic sense, it is also precisely what is required to obtain the correct units. By virtue of Δ being a rate parameter its units are by definition “per unit time” or s^{-1}. Therefore the term $\Delta P_m(\phi^{-1}(E_d(t) - E_d(t)))$ has units of dollars per unit time as necessary.[6]

The following equation describes the time-evolution of the stock of energy capital in monetary terms and is represented by the following differential equation.

$$\frac{dK}{dt} = I(t) - P_m \sigma k(t) \tag{6.9}$$

which may also be represented in physical terms by dividing through by the manufacturing sector price P_m to obtain:

$$\frac{dk}{dt} = i(t) - \sigma k(t)$$

where $i(t)$ is investment in real terms.

The final additional equation for the model concerns the evolution of the magnitude of the stock of loans and is given, from the capital account column of the transactions flow table as:

$$\frac{dL}{dt} = I(t) - \epsilon L(t) \tag{6.10}$$

[6]Note well, the Δ should not be confused with the symbol for the first difference that arises in discrete time modelling.

6.5 Solving the Model

While the model elaborated in this chapter is still very simple, we will see that already acquiring analytical solutions is much more difficult than for model SFCIO-SIM. As before the time subscript is dropped for convenience.

Beginning with the definition of GDP we have that:

$$Y = C + I + G$$

substituting in for C and I we have:[7]

$$Y = \alpha_1 YD + \alpha_2 H + \underbrace{\Delta P_m \phi^{-1} p_e^{-1} \psi}_{\omega_1}(L_{11}(C^e + G^e) + L_{12}(C^m + G^m + I)) - \Delta P_m k + G$$

Now, the above expressions may be considerably simplified by noting that; (1) $YD = (1-\theta)WB = (1-\theta)(Y - \epsilon L)$; and (2) that $C^m + G^m + I = Y - C^e - G^e$. Making these substitutions and rearranging terms we have:

$$Y = \underbrace{(\alpha_1(1-\theta) + \omega_1 L_{12})}_{\omega_2} Y + \underbrace{\omega_1(L_{11} - L_{12})(Ce + G^e)}_{\omega_3} - \Delta P_m k + G - \alpha_1(1-\theta)\epsilon L + \alpha_2 H$$

Now, bringing the Y term over to the LHS and solving we have:

$$Y = \underbrace{\left(\frac{1}{1-\omega_2}\right)}_{\omega_4} \cdot (\alpha_2 H - \Delta P_m k - \underbrace{\alpha_1(1-\theta)}_{\omega_5} \epsilon L + \underbrace{\omega_3 + G}_{\omega_6})$$

Which in more compact form is:

$$Y = \omega_4(\alpha_2 H - \Delta P_m k - \omega_5 \epsilon L + \omega_6)$$

Now, we have Y purely in terms of the state variables H, k, and L, and a constant term which we can use to write the differential equations governing the model stocks in a solvable form. First the evolution of the stock of money is given as:

$$\frac{dH}{dt} = YD - C$$

[7] To keep the solutions as tidy as possible we will collect and rename parameters using braces under sets of terms.

which, substituting in for the definition of disposable income and consumption and rearranging leads to:

$$\frac{dH}{dt} = \underbrace{(1-\alpha_1)(1-\theta)}_{\omega_7}(Y - \epsilon L) - \alpha_2 H$$

Now, substituting in the expressions for Y determined above and collecting terms we have that:

$$\frac{dH}{dt} = \underbrace{(\omega_7\omega_4\alpha_2 - \alpha_2)}_{\psi_1} H - \underbrace{\omega_7\omega_4\Delta P_m}_{\psi_2} k - \underbrace{\omega_7(1+\omega_4\omega_5)\epsilon}_{\psi_3} L + \underbrace{\omega_7\omega_4\omega_6}_{\psi_0}$$

which leaves us with the following linear differential equation for H:

$$\frac{dH}{dt} = \psi_1 H - \psi_2 k - \psi_3 L + \psi_0$$

Given there are three state variables we clearly require an additional two equations. First, we find the differential equation for the evolution of the capital stock as:

$$\frac{dK}{dt} = I - P_m\sigma k$$

which, using some of the substitutions already made in finding the expression for Y is:

$$\frac{dK}{dt} = \omega_3 + \omega_1 L_{12} Y - \Delta P_m k - P_m\sigma k$$

Again, substituting in the expression for Y and collecting terms we have:

$$\frac{dK}{dt} = \underbrace{\omega_1 L_{12}\omega_4}_{\delta_1} H - \underbrace{(\omega_1 L_{12}\omega_4\Delta P_m + \Delta P_m + P_m\sigma)}_{\delta_2} k - \underbrace{\omega_1 L_{12}\omega_4\omega_5\epsilon}_{\delta_3} L + \underbrace{\omega_3 + \omega_1 L_{12}\omega_4\omega_6}_{\delta_0}$$

and finally, expressing in compact form:

$$\frac{dK}{dt} = \delta_1 H - \delta_2 k - \delta_3 L + \delta_4$$

Finally, we may easily solve for governing equation for L making use of the delta coefficients just derived:

$$\frac{dL}{dt} + I - \epsilon L$$

which equals:

$$\frac{dL}{dt} = \delta_1 H - (\delta_2 + \sigma)k - (\delta_3 + \epsilon)L + \delta_0$$

The model may ultimately be written in more compact form as:

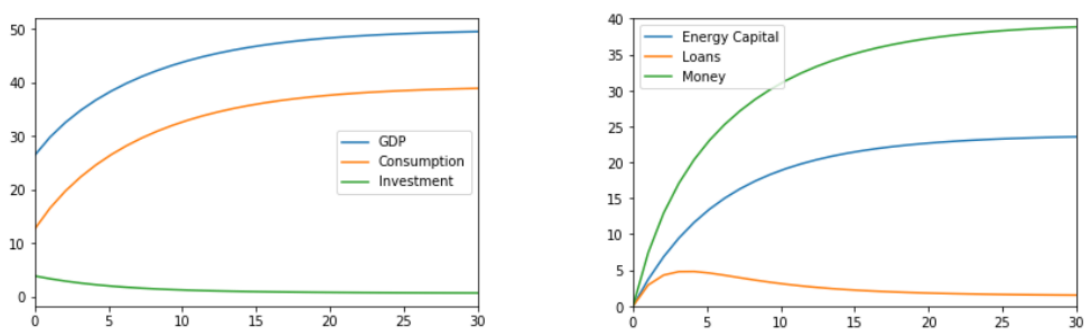
$$\begin{bmatrix} \dot{H} \\ \dot{K} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \psi_1 & -\psi_2 & \psi_3 \\ \delta_1 & -\delta_2 & -\delta_3 \\ -\delta_1 & -(\delta_2+\sigma) & -(\delta_3+\epsilon) \end{bmatrix} \cdot \begin{bmatrix} H \\ K \\ L \end{bmatrix} + \begin{bmatrix} \psi_0 \\ \delta_0 \\ \delta_0 \end{bmatrix}$$

and ultimately be expressed as the following linear matrix differential equation:

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{A}\mathbf{X}(t) + \mathbf{f} \tag{6.11}$$

6.6 Simulation and Analysis

Having solved for the model into its state-variable form we turn to simulations. To illustrate an important point we will, as in the previous chapter, explore some simulations assuming all flows and stocks are initially at 0. The following figures shows the evolution of several key flow variables and the three stock variables over-time. Note well, that much like the model elaborated in the previous chapter government spending G and energy demand C^e are exogenous and constant.



(a) Time-paths of $Y(t)$,$C(t)$, and $H(t)$ (b) Time-paths of $K(t)$, $L(t)$, and $H(t)$

Figure 6.4: Evolution of main model variables assuming $H(0), K(0)$, and $L(0)$ are all zero. Panel **(a)** displays the increase in GDP and consumption to their steady-state values. Investment is initially large and declines until its level is that required to replace depreciated capital in the steady-state. Panel (b) displays the evolution of the model state-variables to their steady-state levels. The rate at which the model reaches its steady-state is dependent on the rapidity with which the capital stock adjusts.

Careful consideration of the above simulations reveals an interesting problem with the nature of the model. If at time zero, there is no stock of energy capital then how is it possible that the model is able to not only construct new energy capital but supply energy to meet government and household final demand? In model terms we have a physically dubious scenario where actual power use (given by the power demand function) is greater than the actual power supplied (given by the power supply function).

This mismatch is displayed in the following figure.

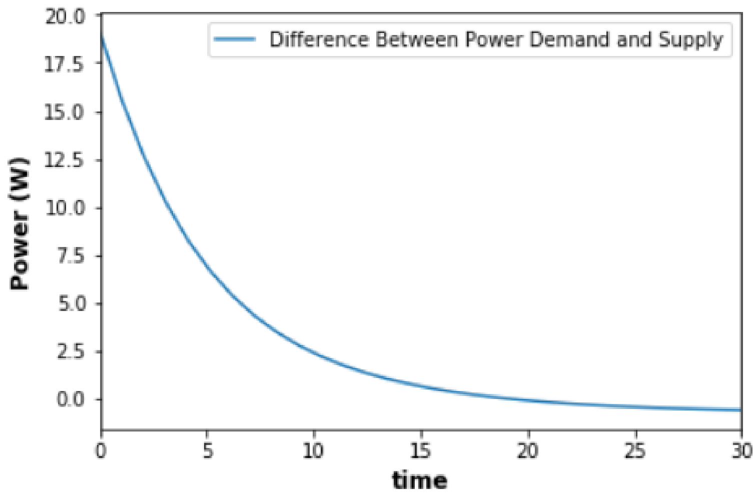


Figure 6.5: Evolution of the gap between power demand and power supply

The problem in figure 6.5 is easily stated. The model is using more energy than it is generating; a clearly impossible scenario that largely arises from assuming a model with zero initial conditions. This however exposes a fatal flaw in the model, in that it can generate behaviour that violates, in essence, the law of conservation of energy.

The total quantity of energy produced over the model run is simply the sum (integral) of the power output of the energy capital stock or:

$$E_{produced} = \int_0^{30} \phi k(t) dt$$

From Figure 6.5 we know that the power demand over most of the model run exceeds the power supply, so the total energy consumed, given by:

$$E_{consumed} = \int_0^{30} \phi E_d(t) dt$$

is such that $E_{consumed} > E_{produced}$. Now, let us assume, for the sake of argument, that the energy technology is something like solar photovoltaic panels and consider model SFCIO-CAP in terms a system and surroundings thermodynamic diagram.

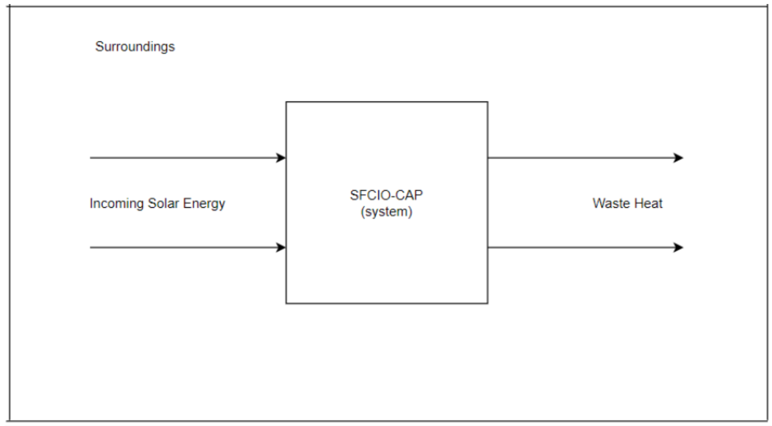


Figure 6.6: Systems and surroundings

The model and its equations represent the system and the surroundings are "everything else" which in this case we assume contains the source of the solar energy. Continuing with the thermodynamic terminology the union of the system and the surroundings is called the universe which is an isolated system. The first law of thermodynamics states that the energy of an isolated system is constant so let us denote the energy of this isolated system as U. From inspection of Figure 6.6 we see that energy enters the system (SFCIO-CAP) in the form of solar energy and leaves the system as waste heat. This waste heat arises as the useful energy (exergy) is destroyed in the process of performing work in the system and therefore the waste heat is proportional to the energy consumed. Let us denote the energy of the surroundings as SU and the energy of the system as SY. The law of conservation of energy always requires that $U = SU + SY$. But since $E_{consumed} > E_{produced}$ the energy gained by the system is less than that gained by the surroundings (as waste heat is eventually equivalent to $E_{consumed}$). But this implies that the total energy of surroundings is $SU - E_{produced} + E_{Consumed}$ which, when summed with the energy of the system SY, gives

$$SU - E_{produced} + E_{Consumed} + SY > SU + SY = U$$

which violates the law of conservation of energy as new unaccounted for energy was produced.

While the above argument is not particularly rigorous it illustrates a key point; without a mechanism to equate physical supply and demand, a model with energy production and consumption can produce behaviour that violates the most basic of physical laws.

It is useful to return briefly to the discussion in chapter four. Unlike the Solow model discussed at the outset of this chapter or the SFCIO-IOSIM Model of the previous chapter, model SFCIO-CAP, despite its physical failings, is a model that allows for comparison with some physical law. That we can find problems in the model by invoking an exogenous law from the physical sciences is interesting in its own right and indicates the utility in attempting to specify macroeconomic models in such a fashion. In the following two chapters we will consider the problem of ensuring that the models generated are consistent with key physical notions such as the conservation of mass and energy.[8]

[8] Note well, when we introduce dynamic prices, they will take the role of matching physical supply and demand. As pointed out by a reviewer of this work, such dynamic prices do not, however, solve the problem of starting the model from a zero starting condition. In the models to be developed further in this dissertation it must always be assumed that there is some positive quantity of energy capital. Starting the model from zero as done in the above example inevitably leads to a problem where the energy to construct the initial energy capital must come from somewhere which is not accounted for in such a scenario.

Chapter 7

Chapter Seven: Dynamic Prices

The modelling of the previous chapter led inexorably to the problem of more energy being consumed than was physically being produced which is a violation, in model terms, of the conservation of energy. That more energy is being used in the system and eventually dissipated as waste heat (this dissipation is implied but not explicitly modelled) than enters the system via the generation mechanism is an error that renders the model results incorrect on physical grounds. In this short chapter we make the energy price P_e endogenous in order to ensure parity of power supply and demand.

Let us first return to the formalism of the previous chapter to more clearly express the problem. The total output of the energy sector (sum of intermediate and final demands) is given as $X^e(t)$ which, as a monetary flow variable, necessarily has dimensions of dollars per unit time. The energy sector also operates a stock of energy capital $k(t)$ (measured in machines) that when multiplied by some factor ϕ (power output per machine) indicates the total power supply of the economy. Now, given some static energy price P_e (measured in dollars per Joule) there is no logic in the model that ensures that the resulting power demand $\frac{X^e(t)}{P_e}$ need equal the power supply $\phi k(t)$ when both demand and supply in physical terms are variable. This leads to the possibility of power consumption exceeding generation which is erroneous on physical grounds.[1]

While making the energy price serve the role of a physical market clearing mechanism is seemingly simple theoretical extension, expanding the model in this manner entails both a host of new technical complications and strays into debates concerning the nature of pricing itself. The purpose of this chapter is simply to see how dynamic prices fit into the SFCIO framework under construction; a fuller discussion of pricing theory and investment is left to the following more substantive chapter which will complete the sequence of simple models by introducing private banks, profits, labour, and manufacturing sector capital.

[1]Excess generation capacity over demand, however, is a common feature of energy generation and is termed curtailment. See (NREL, 2013(@) for an overview of curtailment in electric power generation.

7.1 Physical and Monetary Technical Coefficients

To keep the analysis simple, we will continue with a modified version of the model elaborated in the previous chapter. However, unlike model SIM-CAP we cannot at the outset assume a set of constant and exogenous technical coefficients (the a_{ij}). To understand why this is the case let us begin with the assumption that that there are a set of fixed physical technical coefficients that relate the quantity of some physical input to the total physical output of a given sector.

Let us first define the physical interindustry matrix $\mathbf{E}$ as follows:

$$\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \tag{7.1}$$

Here, the e_{ij} terms are the physical analogue of the monetary interindustry transactions represented by the z_{ij}. Just as the z_{ij} represent the dollar value of some interindustry transaction over some time period, the e_{ij} represent physical magnitude of some transaction over a given time period. For example, the physical interindustry term e_{12} represents the sale of energy (in Joules) over some time period and hence has units of Joules per unit time, or Watts. We can obtain the transaction term z_{12} by multiplying the physical interindustry transaction e_{12} by the price of energy P_e which returns a quantity with units of dollars per unit time as necessary.

The physical technical coefficients a m_{ij} are constructed on the same logic as the monetary a_{ij} coefficients with the caveat that the underlying units are not necessarily dimensionless in the matrix.

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{7.2}$$

For example, m_{11} is the ratio of e_{11} to the total power output of the energy sector x_e and is therefore a dimensionless ratio.[2] The physical technical coefficient m_{12} is given as the ratio of the energy sold per unit time to the manufacturing sector e_{12} to total physical manufacturing sector output per unit time x_m; this is a mixed unit coefficient and is not dimensionless. For simplicity it is sufficient to think of the physical manufacturing sector output as abstract all-purpose machines as there is no need in the current model to elaborate more carefully the nature of this output. We denote this output with units of m for machines; as such, x_m has units of machines per unit time. Therefore the technical coefficient m_{12} has units of [J/s]/[m/s] or Joules per machine [J/m]. Analogously, the coefficient m_{21} has units of machines per Joule, and m_{22} is dimensionless.

[2]Note well: the uppercase X_e denotes the total output of the energy sector in monetary terms with units [\$/s], while the lowercase x_e denotes the total output of the energy sector in physical terms with units of [J/s].

The m_{ij} technical coefficients are assumed to be constant in the model which implies that the physical requirement of production are assumed to be constant; however, in introducing dynamic prices it will become clear that the monetary technical coefficients do not remain constant but adjust with changes in the relative prices of energy and manufacturing goods.

Having defined the physical interindustry and technical coefficients it is possible now to obtain the monetary technical coefficients by multiplying the physical quantities in the numerators and denominators by their respective prices. This converts the physical flows to the dollar flows as necessary.

$$a_{11} = \frac{z_{11}}{X_e} = \frac{P_e \cdot e_{11}}{P_e \cdot x_e} = m_{11} \tag{7.3}$$

$$a_{12} = \frac{z_{12}}{X_m} = \frac{P_e \cdot e_{12}}{P_m \cdot x_m} = \left(\frac{P_e}{P_m}\right) m_{12} \tag{7.4}$$

$$a_{21} = \frac{z_{21}}{X_e} = \frac{P_m \cdot e_{21}}{P_e \cdot x_e} = \left(\frac{P_m}{P_e}\right) m_{21} \tag{7.5}$$

$$a_{22} = \frac{z_{22}}{X_m} = \frac{P_m \cdot e_{22}}{P_m \cdot x_m} = m_{22} \tag{7.6}$$

Therefore, the monetary technical coefficients matrix $\mathbf{A}(\mathrm{t})$ is given as:

$$\mathbf{A}(t) = \begin{bmatrix} m_{11} & \left(\frac{P_e}{P_m}\right) m_{12} \\ \left(\frac{P_m}{P_e}\right) m_{21} & m_{22} \end{bmatrix} \tag{7.7}$$

In order for $\mathbf{A}(\mathrm{t})$ to be sensible as a monetary technical coefficients matrix it must be the case that each of its entries are dimensionless. As previously noted the m_{11} and m_{22} elements are dimensionless, so it is necessary only to check the off-diagonal entries. First, the price ratio P_e/P_m has units of [\$/J]/[\$/m] or simply machines per Joule [m/J]. Recalling that m_{12} has units of Joules per machine [J/m] we have that $\left(\frac{P_e}{P_m}\right) m_{12}$ is dimensionless as required; furthermore, by symmetry the same holds for $\left(\frac{P_m}{P_e}\right) m_{21}$.

Beginning with physical input-output coefficients we have shown how to construct a sensible monetary technical coefficients matrix. More interestingly, it is evident that changes in the relative prices act to change the monetary technical coefficients and so, unlike previous analysis, the standard assumption of constant technical coefficients no longer holds which, as will become apparent, will complicate the analysis somewhat.

Finally, the Leontief matrix is given in the standard way as:

$$\mathbf{L}(t) = (\mathbf{I} - \mathbf{A}(t))^{-1}$$

which, after employing the formula for the 2x2 natrix inverse, is:

$$\mathbf{L}(t) = \underbrace{\left(\frac{1}{(1-m_{11})(1-m_{22}) - m_{12}\cdot m_{21}}\right)}_{D} \begin{bmatrix} 1-m_{22} & \left(\frac{P_e}{P_m}\right) m_{12} \\ \left(\frac{P_m}{P_e}\right) m_{21} & 1-m_{11} \end{bmatrix} \tag{7.8}$$

The individual Leontief coefficients are given as:

$$L_{11} = D(1-m_{22}) \tag{7.9}$$

$$L_{12} = D \cdot \left(\frac{P_e}{P_m}\right) m_{12} \tag{7.10}$$

$$L_{21} = D \cdot \left(\frac{P_m}{P_e}\right) m_{21} \tag{7.11}$$

$$L_{22} = D(1-m_{11}) \tag{7.12}$$

7.2 Prices and Investment

The key issue raised in the previous chapter concerned the mismatch between the power demanded $X_e(t)P_e^{-1}$ and the power supply $\phi K(t)$. The purpose of this chapter is to employ endogenous prices such that the physical power demanded is always equivalent to that produced. That is, when the energy sector's capital stock is insufficient to generate power to meet demand for some given price, it will raise prices until supply and demand are equivalent. The same dynamics apply in reverse when there is an overabundance of generation at a given price.

Only one additional equation is required in order to specify dynamic prices. To determine the functional form that the price of energy $P_e(t)$ will take is simply to choose prices so that the following equality always holds:

$$\frac{X_e(t)}{P_e(t)} = \phi K(t)$$

It follows immediately that the price $P_e(t)$ which maintains equality is:[3]

$$P_e(t) = \frac{X_e(t)}{\phi K(t)} \tag{7.13}$$

where the RHS has units, after cancelling, of dollars per Joule as required.[4] Now, an additional problem

[3]This price formulation implies that the total output of the energy sector is always used; that is, there is no case in which spare capacity will exist as the price will always adjust to absorb any slack. This is a strong assumption especially if we consider the model's energy sector as simply representing electricity generation as such generation is often characterized by spare capacity. Quite obviously, the existence of generation capacity such as peaker plants (electrical generating capacity that is used during periods of high demand) imply that not all capacity is always fully utilized.

[4]In chapter 5 we noted that the energy price can be understood in an abstract manner as a conversion factor.

arises; the substitution of the above price equation into the standard investment equation (7.14) below would lead to the bracketed term being equivalent to zero in all cases. This would imply that no investment would occur at any price and is clearly a perverse emergent feature from the modification of the investment equation.

$$I(t) = \Delta P_m (X_e P_e^{-1} - \phi K(t)) \tag{7.14}$$

To solve this problem a modified investment equation is used where the energy sector makes investment decisions on the basis of an assumed normal price. This equation, modified with this normal price P_0 is shown as follows:

$$I(t) = \Delta P_m \phi^{-1} (X_e P_0^{-1} - \phi K(t)))$$

What is this normal price? As noted a fuller discussion of pricing is left to the following chapter but for now we might understand the normal price P_0 as a target price held by the energy sector as whole.[5] Equation (7.15) states simply that the energy sector will undertake investment activities so that its capital stock is sufficient to meet energy demand at the normal price P_0. This investment equation will not however, without an additional modification, lead a to capital stock that ensures that $P_e(t) = P_0$. In the absence of depreciation, the above formulation is sufficient, but given any positive depreciation rate σ this investment function will result in chronically less actual energy capital than is necessary to bring about the parity of the realized price and its target. The following investment function includes an additional term $\rho(\sigma)$ to offset this:

$$I(t) = \Delta P_m \phi^{-1} (\rho(\sigma) X_e P_0^{-1} - \phi K(t)) \tag{7.15}$$

where the term $\rho(\sigma)$ may simply be thought of as the amount of extra capital the energy sector must

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It is worth pursuing this thought. Suppose, for argument, that we do not have any conception of the concept of price, but we know that the total demand for energy sector output in monetary terms must somehow be related to the energy sectors power output. The total monetary demand for energy sector output is $X_e(t)$ and that the total physical output of the energy sector is $\phi k(t)$. If we argue that there is some, not necessarily constant, relation between these variables we might write the following:

$$X_e(t) = \psi(t) \phi k(t)$$

The expression above simply states that, at any given time the total energy sector demand $X_e(t)$ is related to the power output of the energy sector by some unspecified variable $\psi(t)$. Given we know what $X_e(t)$ and $\phi(t)$ are we can divide both sides by $\phi k(t)$ to isolate $\psi(t)$. This ultimately leads us to the conclusion that $\psi(t)$ is a variable with dimensions of dollars per joule which we intuitively understand as a price. In the early chapter we noted that the energy price intuitively has these units but by the above it becomes clear that anything that relates $X_e(t)$ and $\phi k(t)$ by equality must in fact have such units and that the energy price is really just a possibly varying conversion factor.

[5]There are several possible justifications for the existence of this price in abstract. Consider, for example, a sector made up of many firms each with free entry and exit. In the neoclassical world we might imagine that a "relatively" higher price acts as a signal for profits inducing new firms to enter, construct capital, and increase supply thereby driving down the price back to the "normal" price. Alternatively, and following the logic we will deploy in the following chapter, this normal price might represent the mark-up over production costs that occur at a "standard" level of output.

construct (due to the continuous depreciation) so that the realized energy price $P_e(t)$ will tend towards the normal price P_0.[6]

7.3 Simulation and Analysis

While the only change of substance between model SFCIO-Price and SFCIO-Cap is the addition of a dynamic energy price, this change is sufficient to make the model non-linear and substantially complicate the derivation of the model from its flow variable form to its state variable form. We leave the derivation of the model in state-variable form to the chapter appendix for the interested reader and simply note here its interesting features.[7]

From the appendix derivation we obtain the following expression for the energy price:

$$P_e(t) = \frac{-\Phi_2(t) + \sqrt{\Phi_2(t)^2 - 4\Phi_1(t)\Phi_3(t)}}{2\Phi_2(t)} \tag{7.16}$$

where the $\Phi_i(t)$ terms are functions of the underlying state variables.[8] Here we might understand the energy price as a reactive mechanism that varies around an assumed normal price in order to keep physical power supply and demand equivalent. To see this, consider the following figure which plots the principal model variables after a sudden one time decline in energy capital.[9]

First, panel (4) of Figure 7.1 shows the sudden decline in the stock of energy capital at time $t = 5$ which, via the transitory spike in investment expenditures in panel (5), returns to its original magnitude. The power generated by the energy sector capital (panel (6)) declines and increases in lock-step with the pathway of the energy sector capital (recall they are related by the constant of proportionality ϕ).

Now two different mechanisms conspire to lead to the energy sector price increase witnessed in panel (3). A first consequence in the decline in the energy capital stock is that the energy sector will generate

[6]The additional term $\rho(\sigma)$ is a function of the depreciation rate σ and is defined as follows:

$$\rho(\sigma) = 1 + \frac{\sigma}{\Delta}$$

If $\Delta = 0.1$ and $\sigma = 0.03$ then $\rho(\sigma) = 1 + 0.03/0.1 = 1.3$. To see why this term exactly offsets depreciation losses we expand out the RHS of (7.15) to obtain:

$$\Delta P_m \phi^{-1} X_e P_0^{-1} + \Delta P_m \phi^{-1} \frac{\sigma}{\Delta} X_e P_0^{-1} - \Delta P_m K(t)$$

In the steady-state $X_e P_0^{-1} = \phi k(t)$, therefore the middle term of the above reduces to (after cancelling the Δ terms) to $P_m \sigma k(t)$ which is the same magnitude but opposite in sign to the depreciation in monetary terms.

[7]Note that non-linear here implies that the underlying model differential equations are non-linear. While analytical solutions can be obtained for sets of first order linear differential equations generally no such solutions will exist for sets of non-linear differential equations.

[8]In the following chapter when endogenous manufacturing prices and capital are introduced the price variables will no longer be expressible analytically.

[9]Such a sudden decline might occur from a large power station being rendered suddenly inoperable and taken offline due to damage from some natural disaster event.

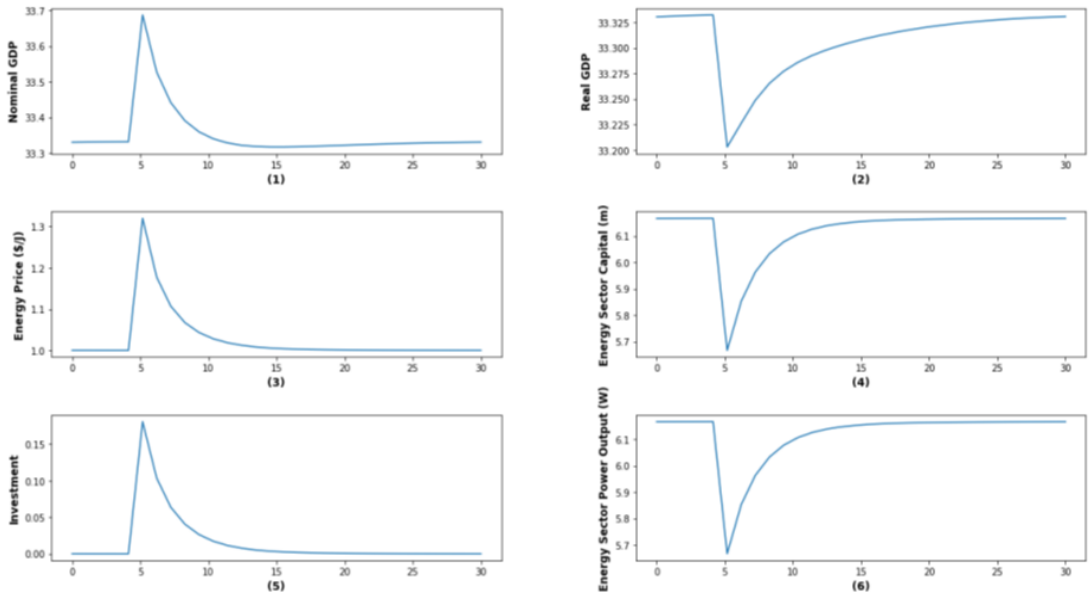


Figure 7.1: Evolution of select SFCIO-Price model variables following a one time decline in the energy capital stock $k(t)$ from 6.2 to 5.7. for an assumed normal energy price of $P_0 = 1$.

less power and the quantity of energy demanded at the normal price P_0 would suddenly exceed what is possible to generate assuming an unchanging energy demand. However, energy demand itself increases as a result of the loss of the capital stock as the energy sector increases its investment expenditures to rebuild its capital stock. This increase in investment is a direct transitory increase in demand for manufacturing sector output and via the logic of the input-output structure an indirect increase in energy sector demand. Put simply, at time $t = 5$ the demand for energy increases while the actual generation capacity decreases; both effects act to drive up the energy price while the energy sector capital returns to its normal level.

Panels (1) and (2) plot the trajectories of nominal and real GDP respectively. The spike in nominal GDP in panel (1) reflects the spike in investment expenditures from panel (5) as well as the sudden sharp increase in the energy price displayed in panel (3). The trajectory in real GDP reflects that, during the period of increased energy prices, energy consumption by households and governments (in real terms) is lower than would be the case at normal prices which outweighs the real effect of the transient increase in investment activities.

The second experiment to be considered concerns a sudden unexpected and permanent decline in energy sector EROI as modelled via an increase in the physical technical coefficient m_{11} from 0.1 to 0.2.[10] From Figure 7.2 the decrease in EROI leads to the build out of a permanently larger energy

[10] That the change is unexpected is important. If the firms in the energy sector knew ahead of time that (for whatever reason) that the decline in EROI would occur, then it stands to reason that additional energy capital

sector capital stock.[11] This occurs as, all else being equal, the energy sector must generate more power to run its operations as a result of the EROI decline. Because the decline is unexpected, there is a temporary power deficit which leads to a transient spike in the energy price and a temporary increase in investment activities. However, unlike the case in experiment one where the economy returns to its original steady-state values, the permanent decline in EROI leads to permanently higher steady-state prices and investment activities.

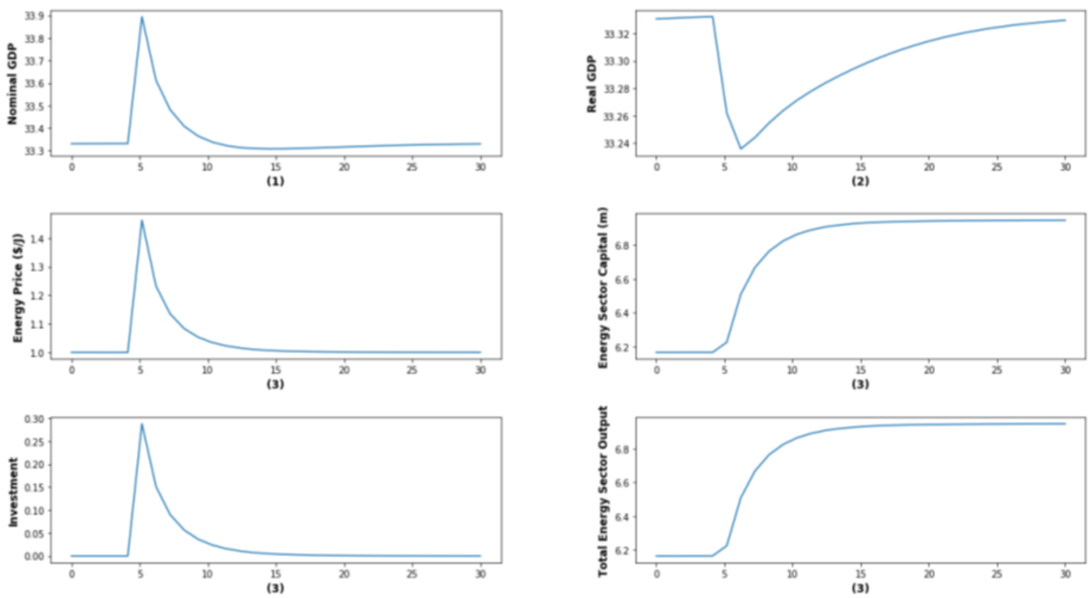


Figure 7.2: Evolution of select SFCIO-Price model variables following a permanent increase in the m_{11} coefficient from 0.1 to 0.2.

The purpose of this short chapter was to introduce a mechanism (prices) that would rectify the problem of the previous chapter in which more power was consumed by the model than generated.[12] To accomplish this, we specified a dynamic price which "reacts" to imbalances between power supply and demand in order to ensure that physical demand always matches physical supply. To incorporate this into the model in a sensible manner we also modified the investment equation around the concept of a normal price.

A simple question to conclude this chapter is whether these model mechanisms are reasonable approximations of how firms set prices and make investment decisions or are simply contrived formulae

would have been constructed in preparation.

[11] The relationship between oil prices and EROI is more complex than the simple dynamics produced by the SFCIO-Price model. This is explored in (Heun and de Wit, 2012).

[12] We do not show that power demand is equivalent to power supply graphically because it is trivially true by equation (7.13). A similar plot to 6.5 would simply return a graph of $y(t) = 0 \; \forall \; t$ which is not very interesting.

designed to ensure a specific outcome? There is rather good reason to assume that the price mechanism proposed here (perfectly reactive to supply and demand imbalances) is not necessarily accurate to how prices are actually formed (see (Nordhaus and Godley, 1972) for an empirical examination of price setting behaviour). It is important to note that the price mechanism invoked here is designed specifically to ensure the parity of power supply and demand and is therefore some idiosyncratic in nature. We will, in the first section of the following chapter, contextualize this pricing scheme by examining more broadly how prices are modelled in a variety of other macroeconomic approaches.

7.4 Chapter 6 Appendix

We begin with prices by expanding the numerator of the price equation using the Leontief coefficients as follows. Note well that household and government energy expenditures are still assumed constant and therefore we denote $C^e + G^e$ as J. As usual, the time argument (t) is dropped for simplicity.

$$P_e = X^e \cdot \phi^{-1} K^{-1}$$

Expanding the expression for X^e we have:

$$P_e = [L_{11} J + L_{12}(Y - J)]\phi^{-1} K^{-1}$$

and therefore,

$$P_e = \left(L_{11} J - D\frac{P_e}{P_m} m_{12} J + D\frac{P_e}{P_m} m_{12} Y \right) \phi^{-1} K^{-1}$$

Now, bringing the terms with P_e to the LHS and solving we have:

$$P_e \left(1 + \frac{D m_{12} J}{P_m \phi K} - \frac{D m_{12} Y}{P_m \phi K} \right) = \frac{L_{11} J}{\phi K}$$

and finally rearranging and simplifying,

$$P_e = \frac{L_{11} J}{\left(\phi K + \frac{D m_{12} J}{P_m} - \frac{D m_{12} Y}{P_m} \right)} \tag{7.17}$$

Letting $\omega_0 = D m_{12} J P_m^{-1}$ and $\nabla = D m_{12} P_m^{-1}$ the above simplifies to:

$$P_e = \frac{L_{11} J}{(\phi K + \omega_0 - \nabla Y)} \tag{7.18}$$

Having an expressions for price dependent on the state variable K and Y it is necessary to obtain an expression for Y.

$$Y = C + I + G$$

which solving out YD for Y and ϵL gives:

$$Y = \underbrace{\left(\frac{1}{1 - w11}\right)}_{\omega_2}(\alpha_2 H - \omega_1 \epsilon L + I + G)$$

where $\omega_1 = \alpha_1(1-\theta)$. Now, expanding the investment term using the modified investment equation and noting that X_e may be written as $\phi K P_e$ using the price equation we have:

$$Y = \underbrace{\omega_2\alpha_2}_{\omega_3} H - \underbrace{\omega_2\omega_1\epsilon}_{\omega_4} L + \underbrace{\omega_2 \Delta P_m P_0^{-1}}_{\omega_5} K P_e - \underbrace{\omega_2 \Delta P_m}_{\omega_6} K + \underbrace{\omega_2 G}_{\omega_7}$$

and finally,

$$Y = \omega_3 H - \omega_4 L + \omega_5 K P_e - \omega_6 K + \omega_7$$

Now, having an expression for Y in terms of P_e and the underlying state variables we may substitute it back into the energy price equation and obtain:

$$P_e = \frac{L_{11} J}{\phi K + \omega_0 - \nabla(\omega_3 H - \omega_4 L + \omega_5 K P_e - \omega_6 K + \omega_7)}$$

Multiplying both sides by the denominator of the RHS and factoring for P_e we obtain:

$$\underbrace{-\nabla\omega_5 K}_{\Phi_1} P_e^2 + (\underbrace{\phi K + \omega_0 - \nabla(\omega_3 H - \omega_4 L - \omega_6 K + \omega_7)}_{\Phi_2}) P_e + (-\underbrace{L_{11} J}_{\Phi_3}) = 0$$

and finally,

$$\Phi_1 P_e^2 + \Phi_2 P_e + \Phi_3 = 0 \tag{7.19}$$

It is possible to solve for P_e from the above equation by employing the quadratic formula as follows:

$$P_e = \frac{-\Phi_2 \pm \sqrt{\Phi_2^2 - 4\Phi_1\Phi_3}}{2\Phi_2} \tag{7.20}$$

However, since we are only interested in positive prices we exclude the solution with the negative square root term and obtain:

$$Pe = \frac{-\Phi_2 + \sqrt{\Phi_2^2 - 4\Phi_1\Phi_3}}{2\Phi_2} \tag{7.21}$$

As each of the Φ terms are written in terms of model state-variables the expression for P_e is a rather complicated non-linear combination of state-variables. The underlying differential equations of the model are finally found as:

$$\frac{dK}{dt} = I - \sigma P_m K$$

which leads to after some rearranging of terms,

$$\frac{dK}{dt} = (\omega_5 P_e - \omega_6 - \sigma P_m)K \tag{7.22}$$

The differential equation governing the evolution of loans is:

$$\frac{dL}{dt} = (\omega_5 P_e - \omega_6)K - \epsilon L \tag{7.23}$$

and finally, the differential equation for the evolution of the stock of money is given as:

$$\frac{dH}{dt} = YD - C$$

which after rearranging and undertaking several substitutions leads to:

$$\frac{dH}{dt} = (\omega_8\omega_3 - \alpha_2)H - \omega_8(\omega_4 - \epsilon)L + \omega_8(\omega_5 P_e - \omega_6)K + \omega_8\omega_7 \tag{7.24}$$

Due to the presence of the non-linear term P_e in each of the differential equations, the system is a set of coupled non-linear first order differential equations for which is extremely likely no analytical solution exists.

Chapter 8

Chapter Eight: A Simple Ecological Macroeconomic Model

In this chapter we present the last of the sequence of simple models and add in many of the important factors omitted in the previous three chapters. Specifically, this chapter adds in manufacturing capital, an endogenous manufacturing price, a simple private banking system, employment, and profits. In addition to presenting a more complete generic SFCIO model, we begin the process of extending the model to a proper ecological macroeconomic model by introducing materials and energy flows and the materials balance principle in the latter portion of the chapter. We begin with the definition of manufacturing capital and its output.

In the previous chapters energy capital was defined as the stock of abstract machines that, when multiplied by the power output per unit machine parameter ϕ, defined the models power supply. Manufacturing capital will be defined in an analogous manner and as before careful attention needs to be paid to the units. Unlike energy capital which produces power, the manufacturing sector capital in this model represents, in an abstract sense, the economies stock of machines that are used both to produce non-energy consumption goods and also to produce new energy and manufacturing capital. [1]

Denote the stock of manufacturing capital, as measured in units of physical machines [m] as $k_m(t)$. This stock of machines represents the productive capacity of the economy, and importantly for the energy transition models in the latter portion of this dissertation, a fundamental constraint on the rate at an economy can produce additional capital goods. To make this clear it is useful to consider the operation of capital in a more qualitative manner before returning to mathematical formalism. Suppose we have one unit of manufacturing capital that is tasked with producing new manufacturing capital. A feature of any real manufacturing process is that new capital cannot be constructed instantaneously but requires some production period to elapse. Therefore, we might imagine that the unit of manufacturing

[1] To be absolutely clear, the manufacturing sector output is taken to be a type of pseudo-universal good that be used for consumption purposes or as an investment good that can be purchased to construct new energy and manufacturing capital. This is rather obviously a simplification, but it is necessary to constrain the number of industrial sectors.

capital, operating continuously for some set period of time, should produce some new quantity of capital at the end of this time.

The above logic may be formalized using a rate parameter, similar to that used for energy capital, denoted ϕ_m. Now, the units of this parameter become clear by considering the following. Supposing that one full production cycle takes time T to complete, and that over such a time period, the manufacturing capital runs continuously, and produces one new unit of capital. Then we have that:

$$1 = \int_0^T \phi_m \cdot 1 \cdot dt$$

where the ones on the LHS and RHS are measured in units of machines [m]. Now, solving the RHS integral we have that:

$$1 = \phi_m \cdot 1 \cdot T$$

Now, the LHS of the above is measured in units of machines, and therefore so must the RHS. T is simply a length of time and therefore is measured in some time units and the one unit of capital (represented by the 1) is measured in units of machines [m]. To ensure dimensional homogeneity, the parameter ϕ_m must have dimensions of [m/s]/[m] or machines per unit time per machine which cancels to simply per unit time.[2]

Therefore, in physical terms, manufacturing capital $k_m(t)$ is defined as the stock of machines [m] that, when multiplied by the production rate parameter ϕ_m, produce new capital at some given rate. Defining manufacturing capital and its associated parameter ϕ_m in this manner imposes a fundamental physical constraint on the model economy. New capital of all types (energy and manufacturing) as well as non-energy consumption goods can only be produced in proportion to the magnitude of the existing physical stock of manufacturing capital which limits the rapidity of the build out of new capital.

In the following chapter we integrate this notion of capital into a relatively more complete closed economy (simple) SFCIO model that includes government and central bank activities, private banks, energy and manufacturing firms, and households. The following section presents the model's structure via balance sheet, transactions flow, and input-output matrices.

[2]Recall that the analogous parameter for energy capital was measured in Watts per unit machine or Joules per second per unit machine. Here machines per unit time per machine is a similar rate parameter to Watts in that capital produces new capital at some rate (e.g., machines per second) and this rate per unit machine determines how fast a single unit of capital will produce new capital. That the units of ϕ_m reduce to per unit time [1/s] is because the output and the capital producing the output are, for simplicity, measured in the same units.

8.1 Balance Sheet, Transactions Flow, and Input-Output Matrices

Figure 8.1 displays the balance sheet matrix for the SFCIO-Fin model. The inclusion of the simplified private banking sector is indicated by the addition of the new Banks column. The assets on the banking sector balance sheet are the loans $L(t)$ to the industrial sectors; the liabilities include the interest earning deposits by households $M(t)$, and a new term $A(t)$. To understand this column, we must note two salient points. First, from the previous chapters, loans increase in magnitude with borrowing by the production sectors and decrease in magnitude as the principal is paid back. Second, in the SFCIO-FIN model, the household sector will hold its savings as interest earning deposits at private banks.

Now since we are assuming at the outset that private banks play the role of passive lenders with zero net-worth we must reconcile this with the problem that there is no a priori reason that the magnitude of deposits from households (the banks liabilities) should be equal to the magnitude of the loans to the industrial sectors (the bank's assets). Let us examine both cases. Suppose first that the magnitude of deposits $M(t)$ is greater than the magnitude of loans $L(t)$, then from the bank's column of 8.1 we have that:

$$A(t) = M(t) - L(t) > 0$$

Now, since an entry on a balance sheet that is positive denotes an asset and a negative entry denotes a liability we see that when $M(t) > L(t)$ that the entry $A(t)$ is some asset to the banking sector and a liability of the government sector. What is the nature of this asset? In a simple sense the deposits the banking sector holds (a liability) are ultimately just government backed money which is itself a liability of the government sector and an asset to the holder.

To understand this let us consider a hypothetical scenario by temporarily forgetting loans altogether and assuming an economy at a zero starting state. Suppose at some time $t(0)$ the government spends into existence some amount of high-powered money by purchasing outputs from the industrial sectors. Wage payments and profits will naturally arise from these sales; if some portion of wages and profits are saved as deposits at private banks then we suddenly have new private bank liabilities (interest earning deposits) being created. However, the banks also have a new asset as a result of households depositing their savings; this asset is simply the high-powered money that makes up the deposits itself. In this simple world the bank liabilities (deposits) exactly match the banks new assets (government high powered money) and the net worth is zero as desired. Now, these reserves of government money held by the banks which are counted as assets from the banks perspective are necessarily the liability of the government sector. Should households suddenly desire to store their deposits as physical cash under mattresses (for whatever reason) then the banks decline in reserves matches the decline in its liability exactly.

$$\begin{array}{l} \\ \text{Money} \\ \text{Deposits} \\ \text{Capital} \\ \text{Loans} \\ \text{Advances} \\ \text{Net Worth} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{Households} & \text{Energy Sector} & \text{Manufacturing Sector} & \text{Banks} & \text{Government} & \Sigma \end{array} \\ \begin{pmatrix} H(t) & 0 & 0 & 0 & -H(t) & 0 \\ M(t) & 0 & 0 & -M(t) & 0 & 0 \\ 0 & K^e(t) & K^m(t) & 0 & 0 & K(t) \\ 0 & -L^e(t) & -L^m(t) & L(t) & 0 & 0 \\ 0 & 0 & 0 & A(t) & -A(t) & 0 \\ -V_h(t) & -V_e(t) & -V_m(t) & 0 & -V_g(t) & -K(t) \end{pmatrix} \end{array} \tag{8.1}$$

Figure 8.1: SFCIO-FIN Balance Sheet Matrix

The purpose of the above story is simply to make clear the logic that the bank deposits which are considered a liability from the banks perspective must necessarily also create some asset for the banking sector as it now also holds the money comprising the deposit which is ultimately a liability of the government sector. The term $A(t)$ introduced above is simply the difference in reserves held by the bank and that lent out in the form of loans. However, in model SFCIO-FIN we add slight additional complexity to this story in that we assume that the banking sector will use these reserves of government money to purchase interest earning government treasuries rather than simply holding inert reserves of money.

This interpretation of $A(t)$ as an interest earning asset/liability works in reverse as well. Suppose instead that we have that $L(t) > M(t)$ (loans are greater than deposits), then $A(t)$ takes the form of private bank borrowing from the central bank upon which it pays interests back to the government sector. In this formulation the funds extended by private banks to the manufacturing sectors in the form of loans is not dependent on the magnitude of bank deposits and loans may exceed deposits, while keeping net worth exactly zero, assuming that banks can borrow as necessary from central banks.

The other salient feature of the balance sheet matrix is that the manufacturing sector now has entries for capital $K^m(t)$ and loans $L^m(t)$. It is useful now to return to the prior discussion about the measurement of capital to note that the capital stocks of both sectors in model SFCIO-FIN are (in monetary terms) valued at replacement cost. What is the replacement cost? This is simply the price of manufacturing sector output. Unlike in previous chapters, the price of manufacturing sector output is no longer assumed to be constant. Therefore, the value of the capital stock of either sector can change due to changes in the magnitude of the underlying physical capital stock or due to changes in price.[3]

Figure 8.2 presents the transactions flow matrix for model SFCIO-FIN. It is worth noting again the

[3]A clever way to both visualize and compute the two effects is shown for the discrete time case in (Godley and Lavoie, 2007) chapter 5, page 137-138 deploying a diagrammatic representation termed an Ostergaard diagram.

slightl subtlety that arises with the how the repayments of loan principals are recorded. First, each sector will pay interest on its outstanding stock of loans which is represented by an outflow of funds from the current account columns of each sector and an inflow to the current account column of the banking sector. However, these interest payments do not actually reduce the size of the outstanding loans; another mechanism is necessary to achieve this in order for the magnitude of held loans to actually decrease.[4] Since new investment must constantly occur just to offset depreciation in the physical capital stock such a mechanism is necessary to avoid a sector's loans from increasing continuously.

$$
\begin{array}{l|ccccccccc}
 & \text{Households} & \text{Energy Current} & \text{Energy Capital} & \text{Manufacturing Current} & \text{Manufacturing Capital} & \text{Bank Current} & \text{Bank Capital} & \text{Government} & \Sigma \\
\hline
\text{Consumption} & -C(t) & C^e(t) & 0 & C^M(t) & 0 & 0 & 0 & 0 & 0 \\
\text{Government} & 0 & G^e(t) & 0 & G^M(t) & 0 & 0 & 0 & -G(t) & 0 \\
\text{Wages} & WB(t) & -WB^e(t) & 0 & -WB^M(t) & 0 & 0 & 0 & 0 & 0 \\
\text{Profits} & F(t) & -F^e(t) & 0 & -F^m(t) & 0 & 0 & 0 & 0 & 0 \\
\text{Taxes} & -T(t) & 0 & 0 & 0 & 0 & 0 & 0 & T(t) & 0 \\
\text{Investment} & 0 & 0 & -I^e(t) & I(t) & -I^m(t) & 0 & 0 & 0 & 0 \\
\text{Inter-Industry} & 0 & z_{12}(t) - z_{21}(t) & 0 & z_{21}(t) - z_{12}(t) & 0 & 0 & 0 & 0 & 0 \\
\text{Interest Deposits} & rM(t) & 0 & 0 & 0 & 0 & -rM(t) & 0 & 0 & 0 \\
\text{Interest Loans} & 0 & -rL^e(t) & 0 & -rL^e(t) & 0 & rL(t) & 0 & 0 & 0 \\
\text{Interest Advances} & 0 & 0 & 0 & 0 & 0 & rA(t) & 0 & -rA(t) & 0 \\
\text{Loan Repayment} & 0 & -Z_eL^e(t) & Z_eL^e(t) & -Z_mL^m(t) & Z_mL^m(t) & 0 & 0 & 0 & 0 \\
\text{dMoney} & -\frac{dH^h}{dt} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dH}{dt} & 0 \\
\text{dDeposits} & -\frac{M}{dt} & 0 & 0 & 0 & 0 & 0 & \frac{dM}{dt} & 0 & 0 \\
\text{dLoans} & 0 & 0 & \frac{dL^{FF}}{dt} & 0 & \frac{dL^M}{dt} & 0 & -\frac{dL}{dt} & 0 & 0 \\
\text{dAdvances} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dA}{dt} & -\frac{dA}{dt} & 0 \\
\Sigma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\tag{8.2}
$$

Figure 8.2: SFCIO-FIN Transactions Flow Matrix

The mechanism, using the energy sector as the example, works as follows. At time t the energy sector has taken out loans worth $L^e(t)$ of which is seeks to pay back some Z_e. To this end the energy sector collars a portion of its revenue from sales $Z_eL^e(t)$ which is used to pay down the loan principle. The repayment of the loan implies simply that the magnitude of the loan (the banking sector asset and the energy sector liability) declines. The change in loans, like that of any other state variable, is given as a differential equation. Now, in the capital account column of the energy sector the differential expression for energy sector loans $\frac{dL^e}{dt}$ appears and from this we may obtain the following expression for the evolution of energy sector loans:

$$\frac{dL^e}{dt} = I^e(t) - Z_eL^e(t)$$

that is, energy sector loans increase as the sector requires additional financing to construct new capital and decreases with the repayment of principal. The total change in banking sector loans is simply the sum of this differential expression and that of the manufacturing sector; this is represented in the dLoans

[4]The attentive reader may note that the same interest rate is applied to each of deposits, Loans, and Advances. Naturally these interest rates are likely to differ in order for the banking sector to generate profit which, in this simple model would simply be distributed back to households. As this is not an important part of the modelling in this chapter, we assume identical rates to avoid having to explicitly include bank profits. It is simple addition in the model codes to set different rates and redistribute profits but for the sake of this chapter presentation the interest rates are kept identical as a simplifying assumption.

row. While the accounting works, it requires that loan repayments go from a industrial sectors current account to its capital account rather than to the bank as might seem more intuitive; the accounting logic for this is discussed in more detail for chapter five's model SFCIO-CAP.

Finally, figure 8.3 presents the input-output matrix for the model.

$$
\begin{array}{l|ccc|c}
 & \text{Energy} & \text{Manufacturing} & \text{Final Demand} & \text{Total Output}\\
\text{Energy} & z_{11}(t) & z_{12}(t) & C^R(t)+G^R(t) & X^R(t)\\
\text{Manufacturing} & z_{21}(t) & z_{22(t)} & C^m(t)+G^M(t)+I(t) & X^M(t)\\
\text{Wages} & WB^e(t) & WB^m(t) & 0 & WB(t)\\
\text{Loans} & (r+Z_r)L^R(t) & (r+Z_r)L^R(t) & 0 & \sum_i (r+Z_i)L^i(t)\\
\text{Distributed Profits} & F^e(t) & F^m(t) & & \\
\hline
\text{Total Outlays} & X^e(t) & X^m(t) & Y(t) & X(t)
\end{array} \tag{8.3}
$$

Figure 8.3: SFCIO-FIN Input-Output Matrix

8.2 Model Equations

As with the previous models total consumption by households is governed by the standard consumption function but rather than money $H(t)$ appearing in the second term we have $V(t)$ which is a portfolio of the two types of financial asset held by the households (money $H(t)$ and deposits $M(t)$). The logic for how households make their two asset portfolio decision is given later in the section but it is worth noting now that $V(t) = M(t) + H(t)$. The modified consumption function is therefore given as:

$$C(t) = \alpha_1 YD(t) + \alpha_2 V(t) \tag{8.4}$$

This consumption is split into energy consumption $C^e(t)$ which is exogenously given and manufacturing consumption $C^m(t)$ which is given as the residual $C^m(t) = C(t) - C^e(t)$. Government expenditures are exogenously determined and split into energy and manufacturing sector expenditures:

$$G(t) = G^e(t) + G^m(t) \tag{8.5}$$

Letting pr_e and pr_m denote the labour productivities of energy sector and manufacturing sector labour, the total labour employed in each sector is given as:[5]

$$N^e(t) = \frac{X^e(t)}{P_e(t)} \cdot \frac{1}{pr_e}$$

[5]While we do not focus on unemployment in this work, we note that it might be introduced via the following employment equation. One might postulate the existence of some exogenously defined labour force, and define unemployment using this number and that arising via the model's labour productivity calculations.

Since $X^e(t)$ denotes the total output of the energy sector, its division by $P_e(t)$ gives output in real terms. This output in real terms, divided by the labour productivity (output per worker) consequently gives the total employment in the sector. Finally, the wages paid for this labour by the energy sector to the household sector are:

$$WB^e(t) = N^e(t) \cdot W^e$$

where W^e is the energy sector wage rate and is assumed to be exogenously determined. For the manufacturing sector we have similar equations with:

$$N^m(t) = \frac{X^m(t)}{P_m(t)} \cdot \frac{1}{pr_m}$$

and therefore,

$$WB^m(t) = N^m(t) \cdot W^m$$

The total wage bill paid to households from both industrial sectors is the sum of these and is given by:

$$WB(t) = WB^e(t) + WB^m(t) \tag{8.6}$$

Sectoral profits are given as the residual between the revenues arising from sales and the various costs arising from production. As shown in the model's transaction flow matrix, all profits are assumed to be distributed to households. Energy and manufacturing sector profits are given as:

$$F^e(t) = C^e(t) + G^e(t) + z_{12}(t) - z_{21}(t) - WB^e(t) - (r + Z_e) \cdot L^e(t) \tag{8.7}$$

$$F^m(t) = C^m(t) + G^m(t) + I(t) + z_{21}(t) - z_{12}(t) - WB^m(t) - (r + Z_m) \cdot L^m(t) \tag{8.8}$$

where total profits are $F(t) = F^e(t) + F^m(t)$. Household disposable income is given in the standard way as the sum of wages, profits, and interest on deposits minus taxes owed to the government.

$$YD(t) = WB(t) + F(t) + rM(t) - T(WB, F) \tag{8.9}$$

We assume that the same tax rate applies to wages as does profits (both accrue to households) and therefore that the tax function above can be given as:

$$T(WB, F) = \theta(WB(t) + F(t)) \tag{8.10}$$

From the balance sheet matrix households now hold two types of financial asset; money (which can be thought of as cash) and interesting earning deposits (which might be usefully imagined as a savings account). We follow the logic developed in (Godley and Lavoie, 2007) for their portfolio choice model and utilise the following equation to determine the proportion of wealth held as money and that held as deposits at banks.

$$\frac{M(t)}{V(t)} = \lambda_0 + \lambda_1 r - \lambda_2 \frac{YD(t)}{V(t)} \tag{8.11}$$

Equation (8.11) states that households want to hold some proportion λ_0 of their wealth in interest earning deposits which increases as a function of the interest rate on deposits r and decreases with the ratio of disposable income $YD(t)$ to wealth; this latter term capturing the transactions demand for money. By logic, the proportion of wealth held as money is simply one minus the above proportion as household portfolio is comprised of only two assets.

8.2.1 Pricing, Investment, and Total Outputs

Pricing in the SFCIO-FIN model is relatively complicated affair in part due to the sectoral interdependencies and also as prices must fulfil the role of matching physical supply and demand. In addition to this, firms will make long run investment decisions on the basis of some desired or normal price based on a mark-up of unit costs. This normal price is loosely based on the notion of normal-cost pricing as explored in (Godley and Lavoie, 2007) chapter 8.

Let us begin by defining the following normal price equations for both sectors:

$$p_{e0} = (1 + \psi_e)UCN_e \tag{8.12}$$

$$p_{m0} = (1 + \psi_m)UCN_m \tag{8.13}$$

where ψ_e and ϕ_m are energy and manufacturing sector costing margins which form the mark-up component of the price. Here UCN_e and UCN_m denote the "normal" unit costs faced by each sector. These normal unit costs are not obvious to define in a framework constructed around interdependent production sectors with endogenous pricing. To understand this, let us examine the unit costs for the two sectors that prevail at any given time in greater detail.

For the energy sector the unit cost at some time t is simply the ratio of the total production costs to the total physical output of the energy sector given as follows:

$$UC_e(t) = \frac{z_{21}(t) + WB_e(t) + (r + Z_e)L_e(t)}{\frac{X_e(t)}{p_e(t)}}$$

where the first term in the numerator represents the costs of inputs purchased from the manufacturing sector, the second is the wage costs, and the last term captures the loan interest and principle repayments. The denominator is simply the total physical output of the energy sector. The same logic holds for the manufacturing sector:

$$UC_m(t) = \frac{z_{12}(t) + WB_m(t) + (r + Z_m)L_m(t)}{\frac{X_m(t)}{p_m(t)}}$$

Given pricing and production interdependencies the two unit cost terms are naturally interdependent. For example an increase in the price of manufacturing sector output implies that the interindustry purchases of that sector's output by the energy sector ($z_{21}(t)$) becomes more expensive which drives up the unit cost of production (assuming total physical production remains unchanged). Now, it should be clear from the two equations that the unit costs of one sector are partially determined by changes in the other sector and as such there is no uniquely determinable normal unit costs. Given this we will proceed by assuming some given normal unit costs and, as per usual, examine the model's behaviour arising from changes in the various underlying parameters when it is originally in a steady-state.

The investment equations for the energy and manufacturing sectors take the now familiar form developed in the last two chapters. These are given as:

$$I^e(t) = \Delta_e p_m(t)\left[\phi_e^{-1} X^e(t) p_{e0}^{-1} - k^e(t)\right] \tag{8.14}$$

$$I^m(t) = \Delta_m p_m(t)\left[\phi_m^{-1} X^m(t) p_{m0}^{-1} - k^m(t)\right] \tag{8.15}$$

Furthermore, the total sectoral outputs $X^e(t)t$ and $X^m(t)$ are given in the standard way using the Leontief coefficients derived in the previous chapter. These are given as:

$$X^e(t) = L_{11}[C^e(t) + G^e(t)] + L_{12}[C^m(t) + G^m(t) + I(t)] \tag{8.16}$$

$$X^m(t) = L_{21}[C^e(t) + G^e(t)] + L_{22}[C^m(t) + G^m(t) + I(t)] \tag{8.17}$$

Finally, the prices for each sector that equate physical supply and demand are given as:

$$p_e(t) = \frac{X^e(t)}{\phi_E k^e(t)} \tag{8.18}$$

$$p_m(t) = \frac{X^m(t)}{\phi_m k^m(t)} \tag{8.19}$$

There are two important features that are worth noting. First, as noted previously, the prices are interdependent with each price depending on the other price. This is simple to see by inspection of the above equations. This pricing interdependency is an inevitable outcome of having both multiple sectors and allowing prices to be endogenously determined. Second, it is no longer possible to obtain expressions for the prices in terms of the underlying state variables as we have done in previous chapters. Why? It is relatively simple to solve for $p_e(t)$ in terms of $p_m(t)$ but the resulting equation, when substituted back into the expression for $p_m(t)$ is a rather immensely complicated non-linear expression that does not readily admit analytical solutions. Since we cannot obtain expressions for prices written purely in terms of the underlying state-variables it follows that we cannot write the underlying model differential equations in such form either, where this was at least possible in the simpler models. A discussion on how the model is solved and the numerical methods utilized is found in the chapter appendix.

8.3 Simulation and Analysis

Of the several new features added in SFCIO-FIN, perhaps the most important is the manufacturing capital stock and its dynamics. Why? In the previous chapters we went to some length to ensure that the energy demanded by the model would, in physical terms, be consistent with the supply yet made no mention of the analogous issue faced by the manufacturing sector. This issue is in fact more complex when considering that the output of the manufacturing sector can itself take the form of new physical capital as well as consumption goods. Investing in new manufacturing capital in some time $t(0)$ takes a portion of the output of the sector in that time period which might have gone to the energy sector or have been consumed by households; however, this time $t(0)$ investment leads to a larger future stock of manufacturing capital and ultimately a larger productive base in the future. The notion that the output of the manufacturing sector (the productive base of the economy) might be a limiting factor will become apparent in the IAM model to be developed in chapter twelve, where the manufacturing sector becomes responsible for the simultaneous provisioning of new renewable capacity, electrified capital, and (in growth scenarios) ever increasing consumption demands. Keeping these issues in mind we turn to explore several simulations.

8.3.1 An Increase in Government Spending

The first scenario we will examine is a sudden increase in government expenditures occurring in 2025. We assume that total government expenditures increase from 10 to 15 with 80% of the new expenditures directed towards the manufacturing sector and the remaining 20% towards the energy sector. Prior to this change the economy is assumed to be in a steady-state with prices and unit costs at their normal levels and capital stocks being operated at full capacity.

The following set of figures display the trajectories for key SFCIO-FIN model variables. Examining the first four panels of Figure 8 we see that the sudden increase in government expenditures leads

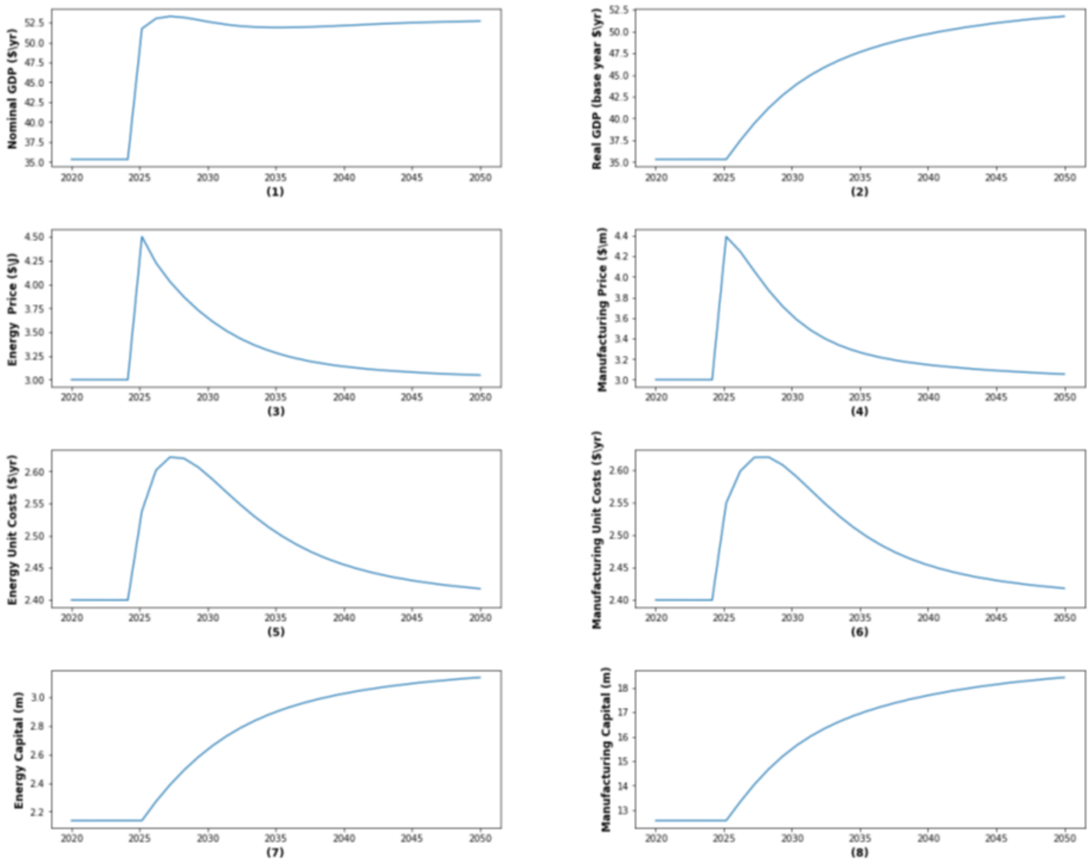


Figure 8.4: Evolution of select SFCIO-FIN model variables following an increase in government spending from 10 to 15.

initially to spikes in the energy and manufacturing sector prices reflecting the instantaneous increase in demand; both prices return to their normal levels over time as new capital comes online. The spike in prices and higher government spending leads to a jump in nominal GDP (panel (1) and a slower increase of real GDP to the same level (panel 2). Since both sectors are reliant on the other sector's inputs, the increase in prices naturally raises the costs associated with production; this is shown in panels 5 and 6. Finally, given the permanent higher demand, panels 7 and 8 show that both capital stocks adjust upwards.

Figure 8.5 presents the trajectories for the sectoral profit and wage trajectories. Why do profits increase immediately and wages at a less rapid rate over the model run? To see this, we recall that the wage bill is defined as the product of the labour employed by the sector multiplied by the exogenous wage rate and that the number of employed is given as (using the energy sector as an example):

$$N^e(t) = \frac{X^e(t)}{P_e(t)} \cdot \frac{1}{pr_e}$$

Here $N^e(t)$ is determined only by the magnitude of real physical output; this along with an exogenous wage rate means that in an economy with no spare capacity the increase in sectoral prices does not lead to a sudden jump in wages but rather a steady increase matching the adjustment of the capital stock. Consequently, the wage cost portion of profits does not "jump" to a new higher level immediately while the revenues do (given the sudden increase in prices) which is indicated by the transient spike in sectoral profits. However, over the long run as the model adjust to its new steady-state, both wages and profits are higher than before the increased government expenditures.

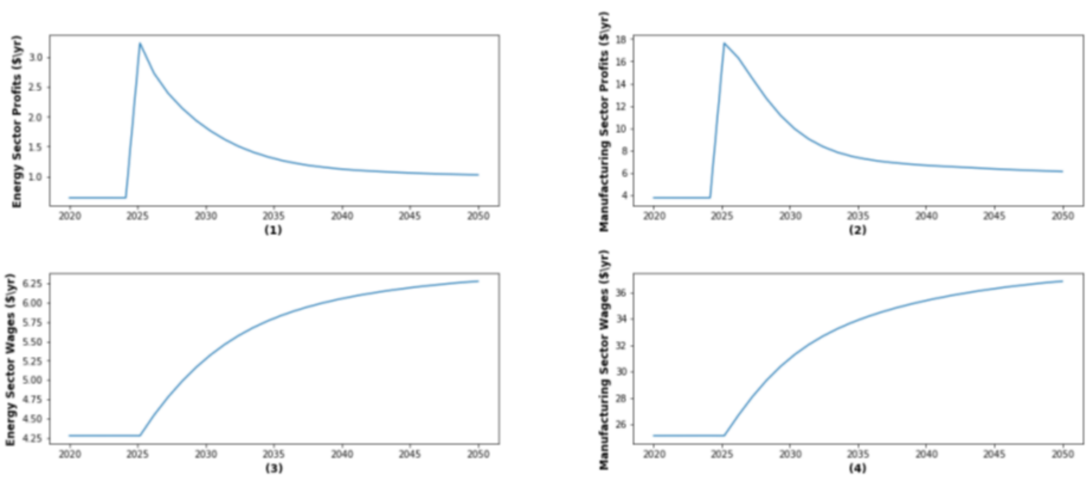


Figure 8.5: Evolution of select SFCIO-FIN model variables following an increase in government spending from 10 to 15.

Given that we assumed that both sectors were operating at capacity in the steady-state with both capital stocks being operated at capacity, the additional demands put forward by the new government expenditure might seem to raise a physical problem. The increase in the demand for both sector's output by the government leads to several indirect increases in sectoral demands due to the input-output and investment structure of the model. For example, the increase in demand for the manufacturing sector leads to increased manufacturing sector demand for energy sector output, which in turn leads to energy sector demand for new capital (investment demand) from the manufacturing sector. Neither of these direct or indirect demands can be met initially and ultimately require increased investment in new physical capacity. In order to see this, Figure 8.6 plots real GDP shares consumption, investment, and government expenditures. Notably, the real consumption share decreases by virtually the same magnitude as the increase in the real GDP share.

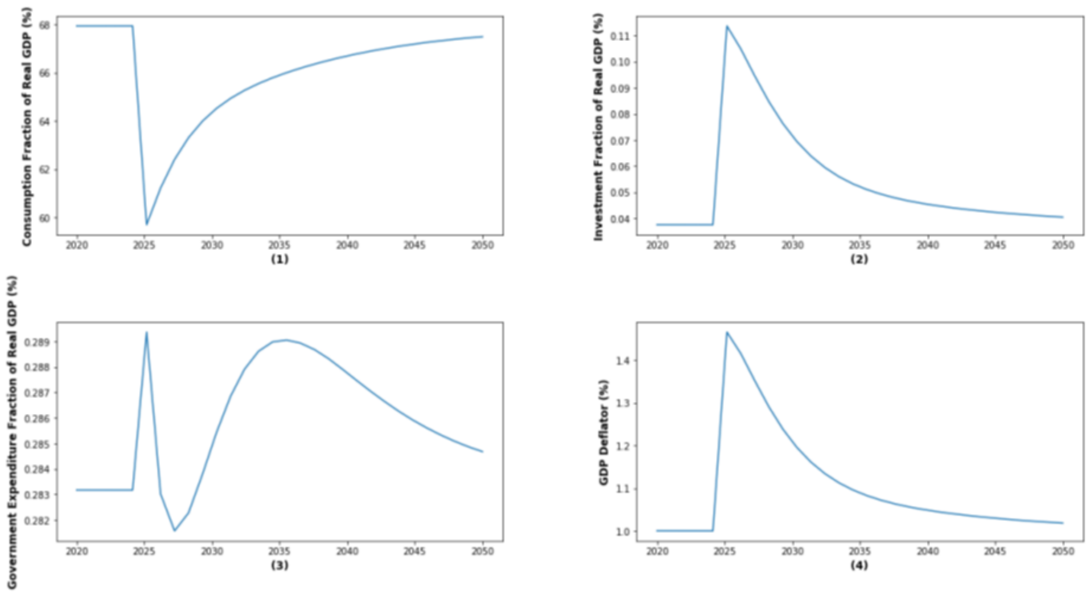


Figure 8.6: E
volution of select SFCIO-FIN GDP Shares Evolution of select SFCIO-FIN model variables following an increase in government spending from 10 to 15.

Ultimately, this re-prioritizing of how the economy deploys its resources is inevitable given both the assumption of full capacity and the assumption of how capital generates output. However, it does indicate an interesting conceptual problem that will become important in later chapters. In the opening section of this chapter we introduced physical capital and noted that one unit of physical capital could only "replicate" itself at a given rate. This provides a physical constraint on what the economy can achieve at any given time and disallows any instantaneous increases in output.

Finally, Figure 8.8 presents the evolution of the key banking sector metrics. Panels (1) and (2) indicate the fairly obvious result that the energy and manufacturing sector outstanding loans both increase initially as the sectors invest to meet additional demand and decline over the model run as the principal is paid back. Panel (3) indicates that household savings decrease initially (implying a greater temporary increase of consumption from savings) before increasing to a new higher steady-state level. Panel (4) shows a transient decline in the balancing term $A(t)$ commensurate with the increase in funds flowing from banks to the industrial sectors before a return to steady-state levels.

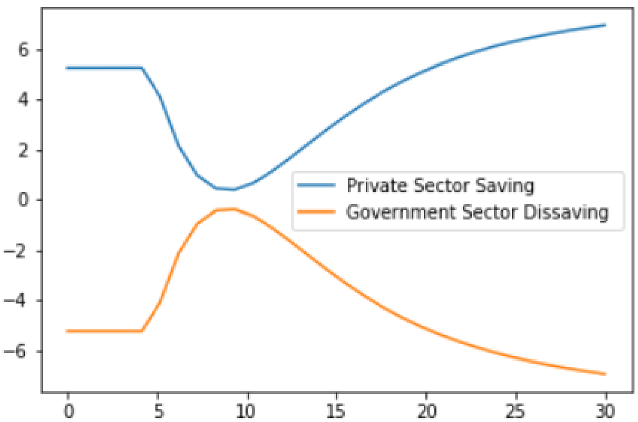


Figure 8.7: Evolution of net private sector savings and government sector dissaving.

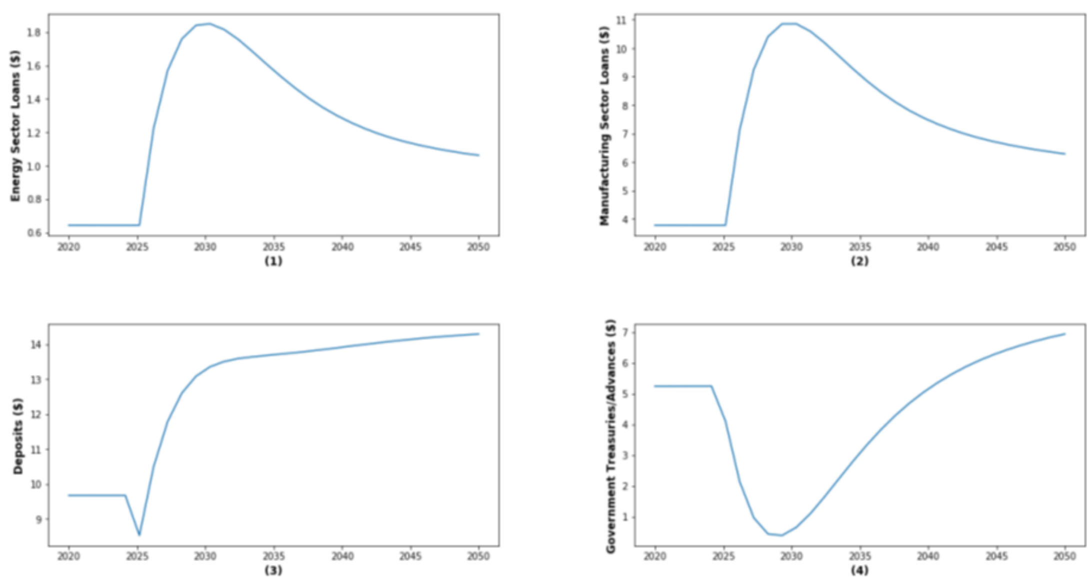


Figure 8.8: Evolution of select SFCIO-FIN model variables following an increase in government spending from 10 to 15.

Finally, let us note the following which arises from the SFC nature of the model. In the simple financial world of the model, the net savings by the "private" sector amounts to the difference between the magnitude of household interest earning deposits $M(t)$ and loans taken out by the production firms $L(t)$. As noted in the opening of this chapter, there is no reason that this net savings term should be zero. However, if the model's private sector has non-zero net savings it must, by logic (and by construction), be the case that some other sector of the model must be lending or accruing dissavings. In this case we know (from the balance sheet matrix) that this takes the form of the increase in "borrowing" by the government sector. The net savings of the private sector should be equivalent in magnitude (and opposite in sign to reflect the difference in assets and liabilities) to the interest bearing treasury term $-A(t)$ which appears on the government column of the balance sheet. We see this in the following Figure 8.7. This result is, of course, not surprising as it is written explicitly in the balance sheet matrix.

8.4 A Decrease in Energy Capital

The second scenario we will examine is a sudden decline in the energy capital stock in year 2025. Figure 8.9 displays the trajectories for the same variables as in Figure 8.4. While the decline in the energy capital stock leads to higher prices in both sectors and higher unit costs as in the previous scenario, real GDP undergoes a transient negative shock before recovering back to steady-state levels mirroring the decline in real output due to the decline in the energy capital stock in panel (7). Panel (8) indicates a temporary increasing in manufacturing sector capacity arising to meet the increased energy sector investment needs to rebuild the capital stock.

As in the previous scenario, the decline in energy sector capital leads to a sudden increase in investment which cannot be met instantaneously. The evolution of the real GDP shares are shown in the Figure 8.10.[6]

[6] It is important to address a point of some possible contention arising in the above scenarios. Be it a sudden increase in government spending or a sudden decline in energy capital, both changes were initially inflationary as demand temporarily outstripped supply. This of course arises from the assumption that prices in the model act to match physical supply with demand which in turn implies full capacity utilization. Under such assumptions, any change that leads to adjustments in the demands for one or both sectors faster than the sectors can adjust their capital stocks will lead to temporarily higher prices. This however is not an inevitable feature of this modelling approach but a specific outcome of the manner in which prices and the investment behaviour of the sectors is specified. Two alternative approaches are mentioned here for completeness

First, one alternative specification might be to specify that firms will aim to construct enough capital to meet demand with an additional margin for error. The inflationary behaviour in such a “spare capacity” formula would then only occur if the change in demand exceeded the additional threshold. Second, one might also specify that firms will only increases prices in the face of increased demand but not lower their prices below their unit costs. In such a formulation spare capacity would arise when demand for outputs declined sufficiently to push prices down to this lower threshold.

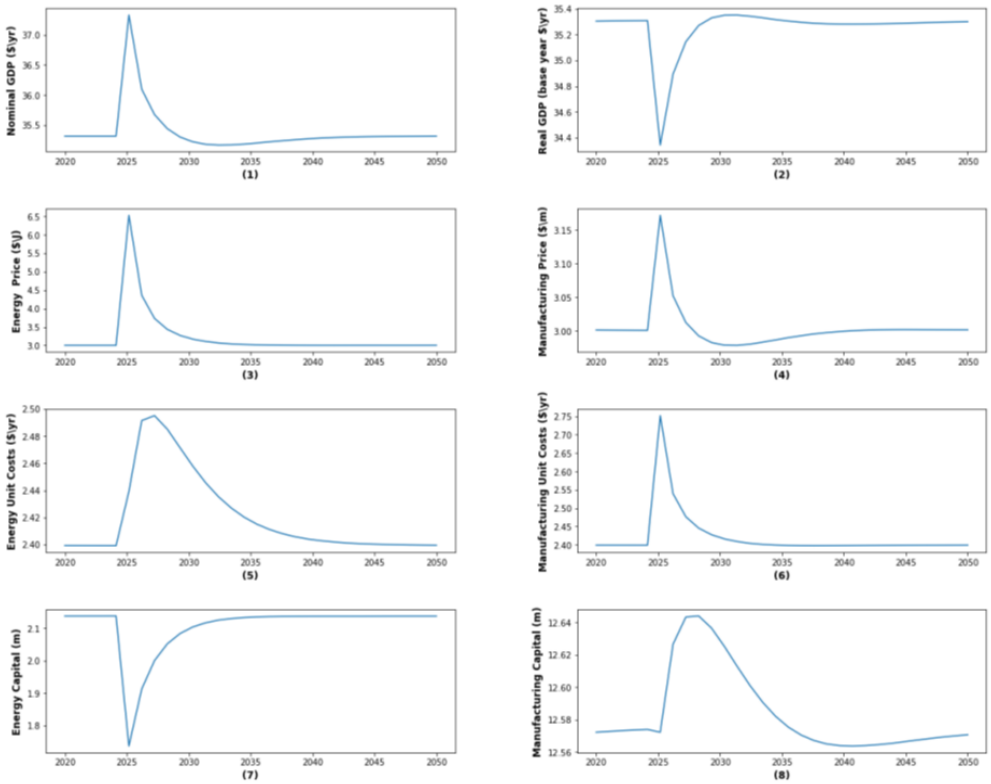


Figure 8.9: Evolution of select SFCIO-FIN model variables following a sudden and unexpected decrease in the magnitude of the energy capital stock.

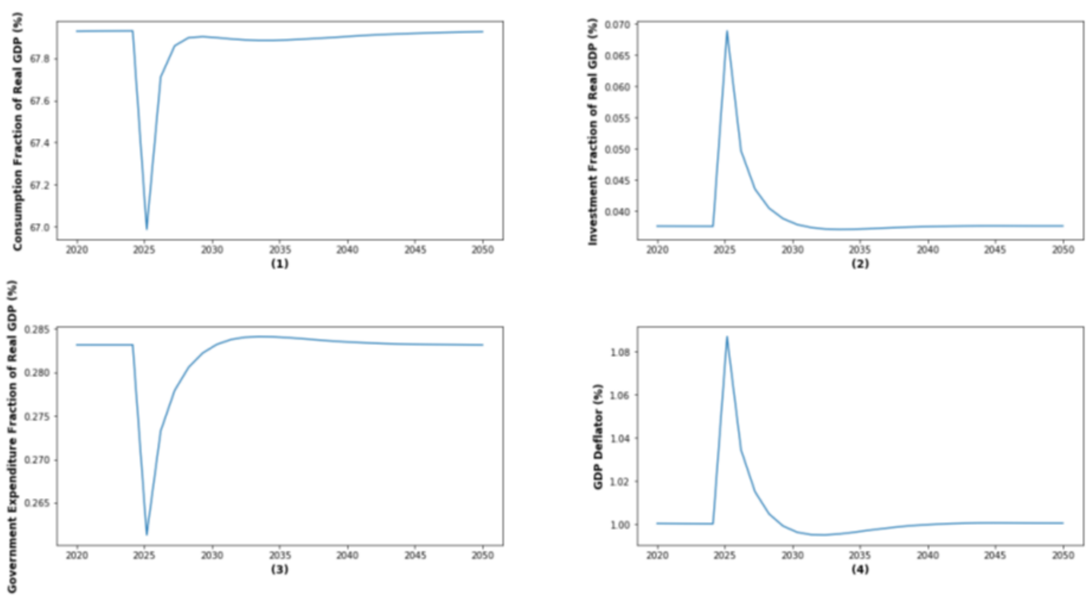


Figure 8.10: Evolution of select SFCIO-FIN model variables following a sudden and unexpected decrease in the magnitude of the energy capital stock.

8.4.1 Growth and Degrowth

In the previous subsections we noted that the sudden shocks imposed on the model lead to a transient period of inflationary behaviour followed by a longer-term process of capital stock adjustments which eventually return to the economy to a steady state. In this final experiment we examine what occurs when the sudden “shock” is the imposition of long-term economic growth or degrowth where there is no return to a steady state. In order for SFCIO-FIN to be sensible it is necessary that the price levels for the energy and manufacturing sectors do not rise indefinitely (or decrease indefinitely in the degrowth scenario). The following Figures 8.11 and 8.12 plot the GDP trajectories, sectoral prices, and capital stocks for scenarios assuming that government expenditures begin growing (or de-growing) by 2% per annum beginning in 2025.

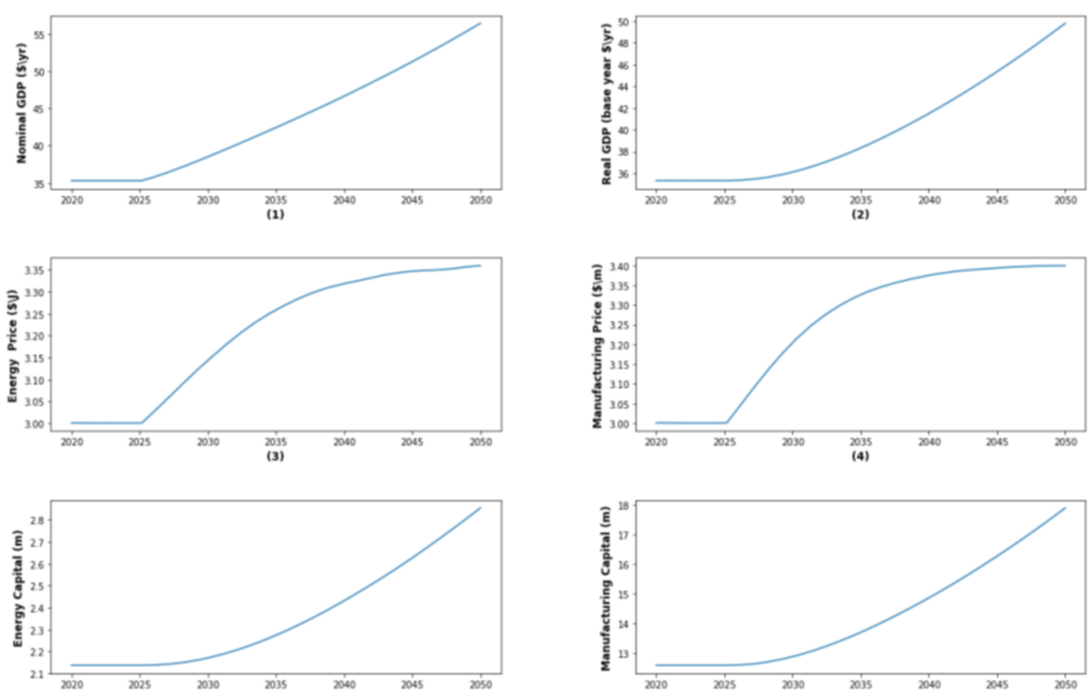


Figure 8.11: Evolution of select SFCIO-FIN model variables following the the imposition of 2% growth in government expenditures per annum beginning in 2025.

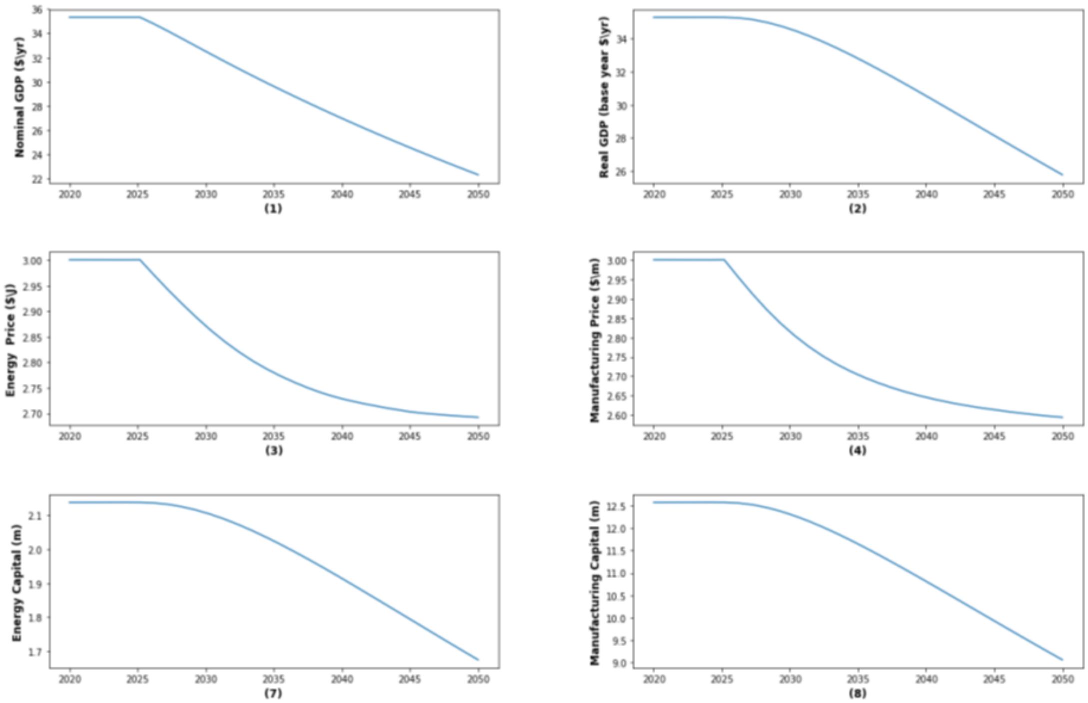


Figure 8.12: Evolution of select SFCIO-FIN model variables following the the imposition of -2% growth in government expenditures per annum beginning in 2025.

8.5 Materials, Energy, Scale Invariance, and Ecological Macroeconomics

From an ecological economics view point the model in this, and the preceding three, chapters all share a similar flaw in that they are untethered from any notion of a larger environment. While some effort was put forth to ensure that the energy calculations were internally consistent it is still largely the case that the materials and energy used by the economy come from nowhere and that the wastes produced by their usage go nowhere. More formally, in failing to specify the sources and sinks for the inputs and waste outputs, and not modelling the magnitude of the throughput itself, we are left with models that can generate behaviour that is absurd.

To understand this let us pursue the following thought experiment. First, let us assume that model SFCIO-SIM is in a steady-state. A feature of such a steady-state is that if we multiply the magnitude of the state variables by the same factor, say 2, and do the same for the exogenous components of the model (government expenditures and household energy consumption) then the resulting model will still be in a steady state with all of the model outputs increased by the multiplicative factor of 2. Now instead of a factor of two let us use a slightly larger number, say 10^{80}, which is the approximate number of atoms in the observable universe. Scaling up the model, even by such a preposterous factor, is entirely feasible as the new steady-state values produced will simply be 10^{80} times larger than the original trajectories. The point here is, without specifying any notion of sources, sinks, and throughputs, it is mathematically perfectly acceptable to have the model generate power, for example, on literally cosmic scales.

In reality the above is obviously nonsense given the very real energetic and materials constraints placed on economic activities, but the thought experiment is useful in illustrating a key problem. There is nothing whatsoever in the mathematical model that disallows the model from being scaled to any size; or put a different way, the model's behaviour is scale invariant.[7] We argue here that any mathematical model of an economy, whose behaviour is scale invariant is fundamentally at odds with the observation that economies are, using the term loosely, a subset of the environment that they are embedded within. Why? Economies are dependent on flows of material and energy inputs and must ultimately convert these to waste outputs. As the source of inputs and the sinks for outputs are necessarily finite, it cannot be the case that an economy scaled to the point of exhausting the capacity of one or both of these sources or sinks cannot undergo a change in behaviour.

In the following sections we will lay the ground work to make the SFCIO-FIN model a more complete ecological macroeconomics model by introducing the materials and energy flows that are required for the model economy to function, and in doing so we will derive the underlying conservation of mass equations that the model must respect. Before turning to this however, we must clarify an issue concerning model time.

8.5.1 Model Time and Parameter Magnitudes

In order to deal more rigorously with the concepts of materials and energy in the model we must address the issue of model time scales that were, for lack of necessity, largely ignored in the previous several chapters. First, in the above simulations we are assuming that 1 time step (e.g., from 2020 to 2021) represents a year's passage of time. While this time-scale is sensible for macroeconomic analysis it does introduce some conceptual difficulties when we begin to think about the denominations of the units of energy at play in the model and the meaning of the magnitude of the parameter ϕ_e, as it is these quantities that will be used to determine the material flows in the energy sector it is important to make them precise.

In the above simulations we set the magnitude of the energy parameter $\phi_e = 1$ and, from the previous chapter, noted that this term is measured in energy per unit time. A single unit of energy

[7] In fact, we can even scale the model by a negative quantity and still observe scale invariance in its behaviour.

capital (assuming away depreciation for the moment) operating over the 30-year time horizon in the above simulations would produce:

$$\int_0^{30} \phi_e \cdot 1 \cdot dt = 30$$

Now, the reasonable question is 30 what? Clearly, this quantity 30 cannot be measured in just Joules as that would indicate a truly miniscule quantity of energy generation; to be sensible this 30 must be measured in some far larger denomination. To obtain a more realistic figure let us make the following simple assumption that each unit of energy capital generates power at the rate of one kilowatt; that is, in one second of operation, the capital will have provided 1000 Joules of energy. However, the natural time-scale of the model is in years (not seconds) so it is necessary to multiply this figure by a factor of 31536000 (the seconds per year) to obtain a quantity of 31.536 gigajoules or in more numerically tractable (though less conventional terms) one kilowatt-year.[8]

In the following section we will derive the conservation of mass equations for the energy sector of the SFCIO model for a particularly simple type of fossil-fuel energy system. Quite obviously energy systems are more complex, but the following section is designed to illustrate the concept of the materials balance principle rather than explore the complexity of energy systems.

8.5.2 Energy

Let us, for the purpose of illustration, assume that the energy capital K_e takes the form of a technology such as a steam or gas turbine that generates electricity and is powered by the combustion of natural gas. Furthermore, let us assume that the natural gas is entirely comprised of methane gas.[9] Let us also assume that the power output per unit of energy capital is defined as 1 kW so that it produces one kilowatt-hour (kWh) of energy over one hour of operation. The combustion reaction equation for methane is given as follows:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \tag{8.20}$$

with the heat of combustion being 50.1 $\frac{kJ}{g}$ of methane gas (Engineering Toolbox, 2017). The material inputs to the economy here is the mass of the CH_4 and O_2 while the wastes outputs are the mass of the resulting CO_2 and H_2O produced. The law of conservation of mass requires the mass of the LHS reactants equal the mass of the RHS products.

To move from this chemical equation to energy in the model let us work out the mass of methane required to generate 1 kWh of electrical energy. To do this we first require a conversion factor relating

[8] One kilowatt-year is simply the energy produced by a process that produces power at a rate of one kilowatt operating continuously over one year. For reference, a single solar panel on average produces power at a rate in the range of 220-350 Watts.

[9] Natural gas is composed primarily of methane but not entirely. We make this assumption as to facilitate the simplest possible calculations in this section.

heat to electrical energy produced which is readily available from the United States Energy Information Administration (EIA) (see (Eia, 2018)). For the purposes of this calculation we will use the data noting that heat rate for combined cycle turbine energy generation was estimated (for 2015) by the EIA to be about 7340 Btu/kWh of electrical energy generation. Given that 1 Btu = 1.05506 kJ, and the heat of methane combustion is 50.1 $\frac{kJ}{g}$, the total mass of CH_4 (in kg) necessary to generate 1 kWh is:

$$1 \cdot kWh \times \frac{7340}{1} \cdot \frac{Btu}{kWh} \times 1.05506 \cdot \frac{kJ}{Btu} \times \frac{1}{50.1} \cdot \frac{g}{kJ} \times \frac{1}{1000} \cdot \frac{kg}{g} \approx 0.155 \cdot kg$$

In the previous subsection we assumed that the energy capital generated energy at the rate of one kilowatt which leads to one kilowatt hour generated per hour (by definition) and 3600 kilowatt-hours generated per year (assuming continuous operation). Therefore, it directly follows that one unit of energy capital operating continuously over one year will require $3600 * 0.155\ kg = 0.558\ t$ of CH_4.

Finally, we may solve for the masses of the other quantities in the combustion equation. First, the molar mass of CH_4 is 16.02 g/mol so the number of mols of CH_4 in 0.56 tonnes is:

$$\frac{1}{16.04} \cdot \frac{mol}{g} \times 1000000 \cdot \frac{g}{t} \times 0.56 \cdot t \approx 34788\ mol$$

From the RHS of the combustion equation each mole of CH_4 reacts with two moles of O_2 and therefore, the total mass of the O_2 corresponding to the combustion of the 0.558 tonnes of CH_4 is (noting that the molar mass of O_2 is 32 g/mol) $2 \cdot 34788 * 32 \approx 2.22t$. Now, to obtain the total mass of material inputs directly required to generate energy per year in model SFCIO-FIN we must multiply these masses by the actual rate of energy generation per year which is simply $\phi_e K_e(t)$. Therefore, the mass of the material inputs per year used by the energy sector per year $M_e^i(t)$ is given as:

$$M_e^i(t) = \phi_e K_e(t)[0.56 + 2.22] \cdot t\ yr^{-1}$$

Now, the conservation of mass requires that the mass of the LHS reactants in the combustion equation equal the mass of the RHS products. We see this if we undertake the same molar mass calculations as above to obtain, per each kilowatt-year of energy generated, approximately 1.53 t of CO_2 and 1.25 t of H_2O is the same as the inputs. Therefore, the mass of material wastes or outputs generated per year by the energy sector $M_e^w(t)$ is given as:

$$M_e^w(t) = \phi_e K_e(t)[1.53 + 1.25] \cdot t\ yr^{-1}$$

Finally, as we possess the mass of inputs and mass of outputs in per year rates, we may state the materials balance or conservation of mass conditions for the model as follows:

$$\int_0^{t_r} M_e^i(t)dt = \int_0^{t_r} M_e^w(t)dt \tag{8.21}$$

Here (8.21) states simply that the total mass of inputs to the model SFCIO-FIN energy system over the definite time interval $[0, t_r]$ is equal to the total mass of outputs (wastes) generated over the same period. This result is guaranteed by the conservation of mass argument obtained from a simple first principles chemical equation analysis.

It is important to note several aspects of the above work. First, while assuming that all energy is produced from methane combustion is a fairly dramatic assumption it is only the logic of the process that is important to show as the same logic would hold for more complex energy systems based on the combustion of other types of fossil-fuels as well. However, what we have accomplished is the linking of the mostly economic SFCIO-FIN model to basic first principles physical analysis concerning the materials balance principle. The model, extended in this manner, now has an irrefutable component that states that the economy must always take in methane and oxygen from the environment and will, in the process of obtaining useful energy, produce CO_2 and H_2O as "wastes" as an inevitable feature.

This however leads to a further result of some conceptual importance in that the conservation of mass necessarily implies that model SFCIO-FIN is a throughput model. Because neither CO_2 or H_2O are themselves inputs to any other model components the model as a whole acts as a mechanism for converting CH_4 and O_2 into CO_2 and H_2O. If we assume the existence of some exogenous stocks of methane and oxygen supplying the inputs and stocks for the accumulation of the waste outputs then we see that model SFCIO-FIN (regardless of its internal economic behavioural components) viewed from a physical perspective will simply be a black-box process that converts material inputs into different material outputs.

We conclude this section by noting that, in a simple fashion, we can relate the model to first and second law considerations by considering again the heat rate of the energy generating technology. First, we assumed a heat rate of 7340 Btu/kWh which implies that for every (approximately) 2.18 units of heat energy put into the energy generation capital, 1 unit of electrical energy is obtained after the conversion. Now the conservation of energy requires that the energy input is ultimately equal to the energy output of the process. Therefore, we must have that 2.18 - 1 units of energy are produced as waste heat (we assume there is no useful application of this waste heat in the model) as a by-product of the energy conversion process. That some of the heat energy input is lost as waste heat in the above is a consequence of the second law of thermodynamics.[10]

8.5.3 Manufacturing

Extending the materials balance equation to the manufacturing sector involves one additional complexity not present in that of the energy sector, namely that some material inputs to the economy do not exit "immediately" but remain embodied in the economy's stock of built energy and manufacturing capital. As such, a unit of materials entering the economy at some time is not necessarily balanced by an outflow of wastes of equal magnitude at the same time. What we will show in this section is that each unit of materials entering the economy balanced by the sum of the materials embodied in built capital and that leaving the economy as wastes.

Let us first begin by postulating the existence of a stock of raw materials $R(t)$ from which the manufacturing sector draws from to create its output. From the beginning of this chapter we noted that each unit of manufacturing capital produces output at a rate of ϕ_m per year which for tractability we will assume is equal to one.[11] Let us further suppose that r_m units of the raw material are required to produce one unit of manufacturing output.[12] We know that the rate of production of manufacturing sector output is the rate of production per unit manufacturing capital ϕ_m multiplied by the magnitude of the manufacturing capital stock $k_m(t)$, for a total output per unit time of $\phi_m k_m(t)$. A portion of this production $i(t)$ is used to construct new capital goods, while the remaining output $\phi_m k_m(t) - i(t)$ is used to meet interindustry demands and final demands by governments and households.[13]

Let us assume that the manufacturing sector outputs not used for the construction of new capital are disposed as wastes as they are utilized. Therefore, of the incoming raw materials into the economy per unit time $r_m[\phi_m k_m(t) - i(t)]$ worth of mass is brought in and discarded back to the environment as wastes. The remaining $r_m i(t)$ of raw material input is embodied in new energy sector and manufacturing capital and not immediately disposed of as wastes. Thus far, it is clear from the materials balance principle that inputs to, and outputs from, from the economy should balance in mass terms does hold as the materials embodied in the built capital are not accounted for. The mechanism that ensures the accounting is correct in the long run is capital depreciation.

In the models thus far we have assumed that capital depreciates at some given percentage per annum (e.g., 3%). Let us note that the physical depreciation of the energy and manufacturing capital stocks must also be counted as a source of waste outflow from the economy. As r_m units of raw

[10] One can use both the first and second laws to prove the existence of the Carnot limit which gives the maximum theoretical conversion efficiency for heat to work done between two different temperatures.
[11] As we will see in the next chapter, this number is essentially arbitrary.
[12] r_m has units of mass of raw materials per unit of manufacturing sector output.
[13] Note well, $i(t)$ is the physical quantity of investment goods and is given as $i(t) = I(t)/P_m(t)$.

material are required per unit of capital output we require that the sum of all the depreciation waste outflows arising from the depreciation of a single unit of capital equal the original raw material input magnitude of r_m. Consider that for one unit of energy capital K_e constructed at time $t = 0$, the waste outflow of depreciated materials per unit time is $\sigma_e r_m e^{-\sigma_e t}$. Therefore, over the infinite time horizon for depreciation we have that:

$$\int_0^\infty \sigma_e r_m e^{-\sigma_e t} dt = r_m \tag{8.22}$$

as required.[14] This however means that the materials embodied in the built capital eventually are entirely discarded as wastes. Now, having worked out the magnitude of the material wastes per unit time arising from the depreciation of a single unit of the energy capital stock it is simple to note that the material wastes per unit time arising from the depreciation of the total stock of capital $k_e + k_m$ is simply:

$$w_\sigma(t) = r_m[\sigma_e k_e e^{-\sigma_e t} + \sigma_m k_m e^{-\sigma_m t}]$$

Therefore the material inputs to the economy per unit time is given as $r_m \phi_m k_m$ and the material outputs leaving the economy are the sum of the consumed and discarded consumption goods $r_m[\phi_m k_m(t) - i(t)]$ plus the depreciation induced stream of material wastes $w_\sigma(t)$.

For an economy that is continuously investing in new capital ($i(t) \neq 0$) the materials balance principle requires that the sum of materials brought into the economy over some definite time interval $[0, t_r]$ is equal to the materials embedded in new capital constructed plus the materials discarded from the economy which can be written formally as:

$$\int_0^{t_r} r_m \phi_m k_m dt = \int_0^{t_r} r_m[i(t) - w_\sigma(t)]dt + \int_0^{t_r} r_m[\phi_m k_m(t) - i(t) + w_\sigma(t)]dt \tag{8.23}$$

All that equation (8.23) really states is that the total mass of raw materials brought in over some time period (the RHS) is equal to the mass of raw materials embodied in new capital construction less that lost to depreciation (the first term on the RHS) plus the materials discarded as wastes (the second term on the RHS).

These equations, in conjunction with the mass calculations of the previous subsection, define the materials balance principle at work in model SFCIO-FIN model. All incoming mass must be equal to the net gain in the embodied mass of the economies built capital plus the mass embodied in the wastes discarded back to the environment. Furthermore, it follows from (8.22) that should there be a cessation in investment ($i(t) = 0$) then over an infinite time horizon all the mass that entered the economy will be equivalent to the mass discarded back to the environment.

[14]Note well the r_m on the RHS of the depreciation equation is multiplied by a 1 with dimensions of machines (m) ensuring that the RHS is measured in the mass of raw material inputs as required.

8.5.4 Ecological Macroeconomic Models

While the previous two subsections may seem a somewhat abstract departure from the work in the previous chapter they showcase, in some detail, both a method for how to link a SFCIO model to concepts such as the conservation of mass and conservation of energy, and also that economies, when viewed from a certain physical perspective, are processes that convert material inputs and useful energy into material wastes and waste heat. Borrowing the language of thermodynamics again, we have demonstrated in the above subsections that the SFCIO-FIN model can be viewed also as what is termed a dissipative system; an open thermodynamics system exchanging materials and energy with its environment.

In extending the mostly monetary based SFCIO-FIN model to include the throughput of materials we are essentially pursuing the goal of embedding the macroeconomics in some representation of a greater containing physical environment. Materials and energy must come from somewhere and ultimately they must be disposed back to somewhere; this somewhere is of course the environment or biosphere in which economies are situated. In this simple model we have just assumed the existence of some nebulous stocks of methane, oxygen, and raw materials and assumed the material wastes arising from the economic processes were disposed back into some equally nebulous waste sinks. However, as a primary concern of this dissertation lies in the impacts arising from the production of excessive CO_2 emissions a more accurate accounting of flows of carbon through the climate system will become a key element of later chapters.

8.6 Chapter Appendix: Model Solution and Numerical Method to Obtain Prices

As per usual we drop the time arguments to keep the solution neat and begin with the definition of national output:

$$Y = C + I + G$$

Substituting in first equation for C and re-arranging terms we have

$$Y = \alpha_1((1-\theta)(WB + F) + rM) + \alpha_2 V + I + G$$

Now by inspection of current account columns for the industrial sectors in the transaction flow matrix we see that we can rewrite the bracketed expression $(WB+F)$ as $Y-(r+Z_e)L^e-(r+Z_m)L^m$. Defining $\omega_1 = \alpha_1(1-\theta)$, $\omega_2 = (r + Z_e)$, and $\omega_3 = (r + Z_m)$ we may rewrite the above, after bringing the LHS Y term over to the RHS as:

$$Y = \underbrace{\left(\frac{1}{1-\omega_1}\right)}_{\omega_4}(-\omega_1\omega_2 L^e - \omega_1\omega_3 L^m + \alpha_1 r M + \alpha_2 V + I + G)$$

from which we simplify further as:

$$Y = \underbrace{\omega_4\alpha_1 r}_{\omega_5} M - \underbrace{\omega_4\omega_1\omega_2}_{\omega_6} L^e - \underbrace{\omega_4\omega_1\omega_3}_{\omega_7} L^m + \omega_4 I + \omega_4 G + \omega_4\alpha_2 V$$

Now in order to progress we need expressions for M and I. We begin first with an expression for M that follows from a rearrangement and substitution as follows:

$$M = \underbrace{\lambda_0 + \lambda_1 r}_{\omega_8} V - \underbrace{\lambda_2((1-\theta)}_{\omega_9}(WB + F) + rM)$$

Collecting M terms on the LHS and defining $\omega_{10} = \frac{1}{1+\lambda_2 r}$ we have that:

$$M = \underbrace{\omega_{10}\omega_8}_{\omega_{11}} V - \underbrace{\omega_{10}\omega_9}_{\omega_{12}} Y + \underbrace{\omega_{10}\omega_9\omega_2}_{\omega_{13}} L^e + \underbrace{\omega_{10}\omega_9\omega_3}_{\omega_{14}} L^m$$

Substituting this expression back into the earlier equation for Y and rearranging we have:

$$Y = \underbrace{\left(\frac{1}{1-\omega_5\omega_{12}}\right)}_{\omega_{15}}[\underbrace{(\omega_4\alpha_2 + \omega_5\omega_{11})}_{\omega_{16}} V + \underbrace{(\omega_5\omega_{13} - \omega_6)}_{\omega_{17}} L^e + \underbrace{(\omega_5\omega_{14} - \omega_7)}_{\omega_{18}} L^m + \omega_4 I + \omega_4 G]$$

which we may rewrite as

$$Y = \underbrace{\omega_{15}\omega_{16}V + \omega_{15}\omega_{17}L^e + \omega_{15}\omega_{18}L^m + \omega_{15}\omega_4 G}_{g_2} + \underbrace{\omega_{15}\omega_4}_{\omega_{19}} I$$

or more simply:

$$Y = g_2 + \omega_{19} I$$

It is worth noting again here that the approach to solving the model across all models in this dissertation is to rewrite the flows variables in terms of the underlying state variables with the intention of putting the model in state-variable form to be simulated. However, unlike the previous models, it is not possible to actually achieve this step fully as the algebra becomes rapidly far to complicated to make it sensible to look for analytical expressions. THis will be shown shortly.

We may write I, following some rearrangement of terms, as :

$$I = \underbrace{\Delta_e \phi_e^{-1} p_{e0}^{-1}}_{g_3} P_m X^e - \underbrace{\Delta_e k^e}_{g_4} P_m + \underbrace{\Delta_m \phi_m^{-1} p_{m0}^{-1}}_{g_5} P_m X^m - \underbrace{\Delta_m k^m}_{g_6} P_m$$

Now, using the definition of the input-output coefficients, and making use of an accounting trick we can rewrite the above equation as:[15]

$$\begin{aligned} Y = g_2 + \omega_{19} P_m [g_3 L_{11}(C^e + G^e) + g_3 Dm_{12} P_e P_m^{-1}(Y - C^e - G^e) - g_4 \\ + g_5 Dm_{21} P_m P_e^{-1}(C^e + G^e) + g_5 L_{22}(Y - C^e - G^e) - g_6] \end{aligned} \tag{8.24}$$

Bringing the RHS Y terms to the LHS and solving out we have:

$$\begin{aligned} Y = \underbrace{\left(\frac{1}{1 - \omega_{19} g_3 Dm_{12} P_e - \omega_{19} P_m g_5 L_{22}}\right)}_{z_1} [g_2 + w_{19} P_m \\ \left(\underbrace{g_3 L_{11}(C^e + G^e)}_{z_2} - \underbrace{g_3 Dm_{12} P_e P_m^{-1}(C^e + G^e)}_{z_3} - g_4\right) + \underbrace{g_5 Dm_{21} p_m p_e^{-1}(C^e + G^e)}_{z_4} - \underbrace{g_5 L_{22}(C^e + G^e)}_{z_5} - g_6] \end{aligned} \tag{8.25}$$

Now the above expression for Y is a somewhat complex affair in both prices p_e and p_m. Technically, we could substitute Y here into the expression for either p_e or p_m and solve for one price in terms of this other. However, if such a step were possible the resulting substitution back into the expression for the remaining price would certainly be too complex to solve analytically. Therefore, to solve for prices (and essentially all other model elements) we must turn to numerical methods.

This however is relatively simple to accomplish now that we have derived the z_i parameters which can be used to write X^e and X^m in terms of state-variables and the prices. First, let us rewrite the expressions for the prices as follows:

$$p_e - \frac{X^e}{\phi_e k^e} = 0$$

$$p_m - \frac{X^m}{\phi_m k^m} = 0$$

Now, by the above formulation, the value (or possibly values) of p_e that satisfies the first equation (e.g., its roots) is the energy sector price at time t. Similarly, the value of p_m that satisfies that second equation (again the equations root) is the manufacturing sector price at time t. However, since the prices are codependent these roots must in essence be determined simultaneously. This is accomplished via

[15]The final demand vector for the manufacturing sector is $C^m + G^m + I$ which is readily shown to be equal to the more useful expression $Y - C^e - G^e$.

the use of the scipy optimization package in python which given some initial guesses jointly determines the values of the prices.

Once numerical values for the prices are determined it is relatively simple to compute the values for all the other model components. For example, the numerical value of X^e at time t follows immediately given the knowledge of the state variable k^e at time t. Similarly, we can solve for the value of deposits, and loans. From these, using the accounting logic set out in the balance sheet matrix, it is simple to determine the value of advances A and so on.

Chapter 9

Chapter Nine: A Life-Cycle Energy Calibration Scheme

Up until this point we have been relatively cavalier in the discussion of the role of energy in the model and have left aside, until now, any discussion of how to properly assign magnitudes to the rate parameters ϕ_e and ϕ_m or how, necessarily, they must relate to each other. Recall from the previous chapter that these two parameters denote the power output per unit machine and machine output per unit machine per unit time respectively. No new model will be presented in this short chapter but rather we will explore a method to calibrate SFCIO models using a life-cycle energy approach such that they are energetically "internally consistent".[1]

In the following sections we introduce two new power output parameters that are use to model simplified and idealized energy generation via renewable energy capacity and fossil-fuel capacity and show how these parameters can be related back to life-cycle energy measurements as determined in the physical analysis of these technologies.

9.1 Power Requirements

Consider the parameter ϕ_M from the previous section. This parameter denotes the output per unit time per unit capital and hence when multiplied by some physical quantity of capital equipment, gives the output of manufacturing goods per unit time. Due to the highly aggregated nature of the model attempting to estimate some empirical value for this parameter is not especially useful; and fortunately, not required. Rather, ϕ_M will take the form of an arbitrarily chosen parameter from which the other model parameters will be obtained.

For simplicity let us set $\phi_M = 1[\frac{m/yr}{m}]$ which states that new manufacturing sector output (which, we recall, can take the form of either capital goods or consumption goods) is produced at a rate of one unit per year. Now, using the concept of energy return on investment, the overall logic for proceeding

[1] What is meant by internally consistent here will become clear in the following sections.

is to calibrate the remaining production parameters. Since one unit of capital is produced per year then it is simplest to assume that the process consumes power at the rate of one unit per year as well. Therefore, let us set the power consumption parameter as follows $\tau_E = 1[\frac{kW}{m}]$; that is manufacturing capital consumes power at a rate of one kilowatt.[2]. To be clear the previous assumptions imply that one unit of manufacturing capital operating over one year will produce one unit of output and consume one kilowatt-Year of energy. To see this formally consider the following. A single unit of manufacturing capital will, over some very small time interval of operation dt, produce $\phi_M \cdot 1 \cdot dt$ units of output [3] and likewise would consume $\tau_E \cdot 1 \cdot dt$ energy. Now if we "sum" up the output and energy consumption over an entire year we have:

$$\int_0^1 \phi_M \cdot 1 \cdot dt = 1[m]$$

and

$$\int_0^1 \tau_E \cdot 1 \cdot dt = 1[kWy]$$

These expressions simply formalize what was noted above. The output produced and the energy consumed by one unit of capital operating continuously over one year are both normalized to one. In the following section we will describe a method to link the other capital stock parameters to these using a life-cycle energy approach.

9.2 Power Output of Renewable Capacity

The one kilowatt-year of energy (one unit for short) represents the life-cycle or embodied energy cost of producing one unit of capital output. Let us think of this in terms of producing a unit of renewable generation capacity. This unit of capacity will generate power over its entire life cycle and hence over its lifetime the unit of capacity will produce some life-cycle energy return. From the previous section we know that the energy invested necessary to produce this unit of capital has been usefully normalized to 1 so the ratio of this life-cycle energy return to the 1 unit of energy invested gives a life-cycle energy return on investment (EROI) of the same magnitude as the energy return itself. We now need to relate this life-cycle energy return to a power output per unit renewable capital parameter ϕ_R. To determine this we first note that, due to the nature of how capital depreciation is specified, a unit of capital will

[2]To be clear this number is ultimately arbitrary; obviously different types of real world capital consumer power at different rates but it must be recalled that the number chosen here acts only as the anchor to determine the other parameters. One kilowatt of continuous power consumption over a year is equivalent to 31,447,600 kJ of energy

[3]The 1 in the equation is the unit of capital and therefore has units of machines [m].

depreciate to 0 only over an infinite time horizon.[4] Therefore power output ϕ_R of one unit of capacity must be chosen so that its life-cycle energy output is equal to the numerator of the EROI ratio. By construction we have chosen the energy invested (denominator) of the EROI term to have the magnitude of 1 so the energy return (numerator) has the same magnitude as the EROI value; that is, the unit of renewable capacity will produce EROI energy over its life-cycle.

As noted, the unit of renewable capacity is also constantly depreciating over time. So ϕ_R must be chosen such that the energy produced over the lifetime of a progressively depreciated unit of capacity is equivalent to the EROI. The value of ϕ_R that accomplishes this is determined by the following equation.

$$\int_0^\infty \phi_R \cdot e^{-\sigma_R \cdot t} \cdot dt = EROI$$

where $e^{-\sigma_R \cdot t}$ represents the time-path a single unit of capital depreciating constantly, and σ_R is the renewable capital depreciation rate.[5] If EROI is measured in kilowatt-years then ϕ_R has the units of kilowatts per unit of capacity [kW/m]. To determine the value of ϕ_R above we solve the above improper integral:

$$\lim_{T \to \infty} \int_0^T \phi_R \cdot e^{-\sigma_R \cdot t} \cdot dt = EROI$$

which leads to

$$\lim_{T \to \infty} \left[\left(\frac{-\phi_R}{\sigma_R} \right) e^{-\sigma_R \cdot T} - \left(\frac{-\phi_R}{\sigma_R} \right) e^{-\sigma_R \cdot 0} \right] = EROI$$

simplifying and rearranging leads to:

$$\phi_R = EROI \cdot \sigma_R \tag{9.1}$$

From equation (9.1) we can relate the model parameter ϕ_R to two exogenous physical parameters; the renewable technology's EROI and its physical rate of depreciation. For example, assuming an $EROI$ of 20 and a depreciation rate of $\sigma_R = 0.03$ would imply a power output of $\phi_R = 20 * 0.03 = 0.6$ $[\frac{kW}{m}]$ and would therefore produce a total of 0.6 kWy when operated continuously over a full year.

The process above derived a power output from known EROI and depreciation values assuming that manufacturing time and power consumption were so normalized as to ensure the energy invested term was equivalent to one. For completeness it should be noted that we can work in reverse to calculate a

[4] Recall that the differential equation governing the evolution of an arbitrary capital stock (k) is usually written as:

$$\frac{dk}{dt} = I(t) - \sigma k(t)$$

where σ is the depreciation term. In the absence of investment this equation simply models a process of exponential decay. In such a process the capital stock will only technically reach zero at infinite time.

[5] Since at time 0, $e^{-\sigma \cdot 0} = 1$ this represents an undepreciated unit of renewable capacity and is measured in units of [m]. Therefore, the units of the RHS reduce to kWy as required.

technology's EROI. Assuming again time values are in terms of years we can write:

$$EROI = \frac{\int_0^{\infty} \phi_R \cdot e^{-\sigma_R \cdot t} \cdot dt}{\tau_E \cdot \phi_M} \tag{9.2}$$

which determines the life-cycle EROI of an energy technology given information concerning its power-output, depreciation rate, and the embodied energy in its production.

9.3 Power Output of Fossil-Fuel Capacity

Determining the power output of renewable capacity in the previous section was made relatively simple as the only energy cost to consider was the embodied energy cost necessary to produce the renewable capital. The underlying simplifying assumption being made is that once installed the renewable capital passively generates energy from freely available wind and solar radiation. Fossil-fuel derived energy incurs a similar embodied energy cost associated with the construction of the capital stock necessary to produce and refine the raw material inputs, but it also incurs a more direct energy cost due to the requirement to power extraction itself.[6]

First, we will define the direct or extraction energy costs. Using the terminology of (Hall et al, 2014) we can define $EROI_m^F$ as the EROI at the mine-mouth. This EROI corresponds to the direct energy costs associated with extraction. A more comprehensive EROI that also takes into account the embodied energy costs of producing the fossil-fuel capital (again following the terminology of Hall et al) is the extended $EfROI_{ex}^F$ which includes both the direct and indirect energy requirements.

As noted in the previous chapter the technical coefficient a_{22} represent the ratio of the purchases of fossil-sector input by the fossil-fuel sector to its total output, or in physical terms, the physical quantity of fossil-fuel sector output used by the fossil-fuel sector to produce its output. This ratio is usefully the reciprocal of $EROI_m^F$ as it does not take into account the embodied energy used to produce its capital stock, only the direct requirements. Therefore, we define the following:

$$EROI_m^F = \frac{1}{a_{22}} \tag{9.3}$$

From equation (9.3) we see that an investment of a_{22} units of energy provides an energy return of 1 unit of energy. It follows from this that an investment of one unit of energy returns $\frac{1}{a_{22}}$ units of energy.

From the previous chapter we know that the parameter ϕ_F has units of power produced per unit capital (or machine). The fossil-fuel capital in the model can therefore be interpreted as a composite

[6]The boundaries of the following EROI calculation are defined at end use. The numerator of the term we will construct is the energy returned for all uses in the model economy while the denominator is the sum of direct and indirect energy expenditures to provide it.

capital stock that extracts, refines, and distributes energy. Like all other capital discussed so far in the model, a unit of fossil-fuel capital can only provide its service at some rate and therefore, over its life-cycle, will produce some energy return which we denote here as ER_F. Now, ignoring temporarily the direct extraction energy costs, what must the magnitude of ϕ_{FF} be such that over the life cycle of one unit of capital, ER_F energy is produced? This calculation is in fact identical to the calculation for renewable capacity above and is expressed as:

$$\int_0^{\infty} \phi_F \cdot e^{-\sigma_F \cdot t} \cdot dt = ER_F$$

which leads to:

$$\phi_F = ER_F \cdot \sigma_F \tag{9.4}$$

The direct costs to produce the life-cycle energy-return of ER_F is simply $ER_F * a_{22}$. Now summing the direct and indirect costs we have that[7]:

$$EROI_{EX} = \frac{ER_F}{ER_F * a_{22}(t) + 1} \tag{9.5}$$

For $a_{22} = 0.05$, $\sigma_F = 0.03$, and $ER_F = 80$, we have an extended fossil-fuel EROI of:

$$EROI_{EX} = \frac{80}{80 * 0.05 + 1} = \frac{80}{4 + 1} = 16$$

That is, the 80 units of energy returned on a direct expenditure of four energy to facilitate extraction and 1 unit of indirect energy expenditure embodied in the fossil-fuel capital. Much like ϕ_M it is not necessarily useful to try and estimate ϕ_F directly from data; rather, it can be calculated using known values for $EROI_m^F$ and $EROI_{EX}^F$. Rearranging equation (9.5) in terms of ER_F we have:

$$ER_F = \frac{1}{\frac{1}{EROI_{EX}} - a_{22}}$$

Now, from equation (9.3) we have that $a_{22} = \frac{1}{EROI_m^F}$. Substituting this and the expression for ϕ_F determined from equation (9.4) into the above, and rearranging we have an expression for ϕ_F purely in terms of exogenously determined physical properties:

$$\phi_F = \frac{\sigma_F}{\frac{1}{EROI_{EX}} - \frac{1}{EROI_m^f}} \tag{9.6}$$

[7] Remembering that the unit of fossil-fuel capital has an embodied energy cost of 1 by construction so the indirect cost in the denominator is simply 1.

Now some caution is necessary when considering (9.6). First, the two EROI components of the denominator are not independent of each other. For a given energy return we know from (9.5) that a lower $EROI^f_m$ implies a lower $EROI_{EX}$ as this extended measure takes into account the direct costs that are used to define $EROI^f_m$. It is therefore important not to think of (9.6) as a model mechanism where the various components may change endogenously but as an empirical relation used to calibrate the model. For example, assuming an increase in $EROI^f_m$ without modifying the extended EROI term would lead to a decline in the power output parameter; a counter-intuitive result since its expected that an increase in "EROI" would have the opposite effect.

In chapter eleven we will develop an approach to make the fossil-fuel EROI endogenous to the model framework based on the depletion of some stock of fossil-fuels. The calculation in this section is done purely to relate the magnitude of the ϕ_F parameter to certain physical quantities. [8]

9.4 Conclusion

In this chapter we have provided a formal method to link the abstract energy sector parameter ϕ_E of the previous chapter to life-cycle representations of either fossil-fuel or renewable energy generating capacity via a life-cycle energy return on investment formulation. This linkage will prove important in the following chapters as we introduce more complex energy transition dynamics into the SFCIO framework by explicitly considering both fossil-fuel and renewable energy "sectors".

[8] In chapter eleven we will invoke another EROI term which is a model variable rather than an external empirical value. It is important not to confuse the two.

Chapter 10

Chapter Ten: A Linear Energy Transition Model

10.1 Introduction

Total CO_2 emissions from energy generation and industrial activity has reached a record high of 33.2 $GtCO_2$ per year in 2019 (IEA, 2020).[1] Furthermore, projections indicate that total carbon emissions in 2019 for all activities will be even greater (Friedlingstein et al, 2019). In order for economies to decarbonize sufficiently quickly to meet emissions reductions targets in line with keeping warming below 1.5°C above pre-industrial levels, it will be necessary for rapid changes in the mixture of electricity generation technologies as well as a transition in how energy is used. This however implies the requirement for both a fast build out of renewable generation capacity and electrification of industry and end use.

However, the majority of emissions path-ways consistent with 1.5°C and 2°C carbon budgets also assume the large-scale deployment of negative emissions technology whose feasibility at scale is unknown (Anderson and Peters, 2016). This conclusion further noted in the 2018 IPCC special report that most scenarios consistent with the 1.5°C target assume carbon dioxide removal (CRD) to achieve negative emissions (Rogelj et al, 2018); and critically, that such technologies are unproven at scale. The reliance on such negative emissions technology arises partly from the observation that the necessary rates of decarbonization for economies to meet targets is higher than there is strong evidence to assume achievable (Holz et al, 2018).[2] However, as (Anderson and Peters, 2016) notes "Negative-emission

[1] While the reader is certainly aware, it is worth pointing out that the unusual circumstances caused by the current coronavirus pandemic and the decline in economic activities occurring with lock-down policies, has appeared in the emissions data. See (IEA, 2020) for a discussion of this.

[2] For example, (Hickel, 2019) notes that carbon emissions intensity must decline at a very large 10.5 percent per annum to remain within the 1.5°C carbon budget. This 10.5% decarbonization number is easily derived by solving the following expression for r; the rate of emissions intensity decline:

$$B = E_0 \int_0^{\infty} e^{(g+r)t} dt \tag{10.1}$$

which returns $r = \left(g + \frac{E(0)}{B}\right)$. For a GDP growth rate of 3 percent ($g = 0.03$), initial year emissions of $E(0) = 42$ $GtCO_2$, and a remaining 1.5°C carbon budget of, for example, 520 $GtCO_2$ a value of $r = 0.105$ is obtained.

technologies are not an insurance policy, but rather an unjust and high-stakes gamble". Betting on the success of negative emissions technology poses a considerable risk and alternatives without large-scale negative emissions technology have been proposed; for example, (Grubler et al, 2018) develop a low energy demand (LED) scenario devised to meet the 1.5°C target without assuming large-scale negative emissions. This scenario has the novel feature of assuming 40% decline in final global energy demand by 2050.

Like the LED scenario of (Grubler et al, 2018), and the noCDR scenario of (Holz et al, 2018), this chapter constructs an energy transition model that assumes no deployment of negative-emissions technology over the period examined (2020 - 2050). Assuming no negative emissions technology implies the necessity of a very rapid transition of energy systems in order to decarbonize sufficiently quickly to meet the 1.5°C target. One aspect of meeting such emissions reductions targets is an "aggressive electrification end use" (Dennis et al, 2016); this implies, for example, the electrification of transport and manufacturing activities. To explore this problem this chapter constructs a stock-flow consistent input-output energy transition model (hereafter SFCIO-ETM). A key feature of the model is to make use of a dynamic input-output framework to simulate both the changes in energy final demand and the change in the industrial structure of the economy via the endogenization of technical coefficients. This approach makes possible the examination of the simultaneous transition of how energy is generated as well as the transformation in its end-use in a stylized macroeconomic model.

The dynamics of the problem to be explored are as follows: economies whose productive sectors (e.g., those that create new capital equipment for example) are reliant to some degree on the usage of fossil-fuels implies an emissions build out cost per each additional unit of renewable capacity constructed. This emissions cost of additional renewable capacity declines as the manufacturing sectors electrify their operations and increasing proportions of the electricity necessary to power manufacturing is derived from renewables; that is, additional renewable capacity is increasingly constructed by existing renewable capacity[3]. Furthermore, as energy demand increases with economic growth the renewable energy system must not only scale up to meet some static demand but play catch-up with a possibly increasing demand over time.

The purpose of this chapter is two-fold. First, it serves as an addition to the small but growing literature on merging stock-flow consistent macroeconomic modelling with input-output analysis. This relatively new approach can be found in the works of (Berg et al, 2015), (Jackson, 2018) and, (King, 2020). More broadly, the incorporation of the stock-flow consistent approach, as elaborated most fully in (Godley and Lavoie, 2007), with ecological macroeconomics is found in a growing body of literature;

[3]The construction of new renewable capacity by existing renewable capacity is termed renewable energy breeding in (Howard et al, 2018).

see for example (Jackson and Victor, 2015), (Jackson and Victor, 2020), (Bovari et al, 2018), and (Barrett, 2018). The second purpose of the chapter is to simulate different economic scenarios, examine the general principles put forward in (Hickel, 2019) and (Grubler et al, 2018) that a transition based on declining energy demand is feasible, and that it may be the only possible route forward without assuming vast deployment of unproven negative emissions technology.

10.2 Balance Sheet, Transactions Flow, and Input-Output Matrices

The financial structure of the model in this chapter is based on that of the simple ecological macroeconomic model presented in more detail in chapter eight. Figure 12.11 presents the balance sheet (stock) matrix which contains the main model variables and presents the accounting structure of the model. The entries of Figure 12.11 display the value in current prices of the assets and liabilities of each sector. The model economy is divided into six sectors: households, three production sectors (renewable sector, fossil-fuel sector, and the manufacturing sector), private banks, and the government. Each of the production sectors operates capital equipment to produce its output and finances the construction and replacement of depreciated capital via loans obtained from the banking sector. The banking sector also holds and pays interest on household deposits.

$$\begin{array}{l} \\ \text{Money} \\ \text{Deposits} \\ \text{Fixed Capital} \\ \text{Loans} \\ \text{Advances} \\ \text{Net Worth} \end{array} \begin{array}{c} \begin{array}{ccccccc} \text{Households} & \text{Renewable Firms} & \text{Fossil-Fuel Firms} & \text{Manufacturing} & \text{Banks} & \text{Government} & \Sigma \end{array} \\ \begin{pmatrix} H^h(t) & 0 & 0 & 0 & 0 & -H(t) & 0 \\ M(t) & 0 & 0 & 0 & -M(t) & 0 & 0 \\ 0 & K_e^R(t)+K_f^R(t) & K^F(t) & K_e^m(t)+K_{ne}^M(t) & 0 & 0 & K(t) \\ 0 & -L^R(t) & -L_t^F & -L^M(t) & L(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & -A(t) & A(t) & 0 \\ -V(t)* & -V^R(t) & -V^{FF}(t) & -V^M(t) & 0 & -V^g(t) & K(t) \end{pmatrix} \end{array} \tag{10.2}$$

Figure 10.1: SFCIO-ETM Balance Sheet Matrix

Figure 12.12 displays the transactions (flow) matrix where the rows record the transactions between each sector and the columns and the columns display the budget constraint of each sector.[4][5] For example, the first row of Figure 12.12 states that consumption expenditure by households at time t, denoted $C(t)$, is matched by the sales recorded in the first row for the renewable, fossil fuel, and manufacturing sectors, denoted $C^R(t)$, $C^F(t)$, and $C^M(t)$ respectively. The negative sign on the consumption term

[4]See (Godley and Lavoie, 2007), Chapter 2, pages 38-40 for a more detailed discussion of the transactions flow matrix.

[5]Note well; the entries of the transaction (flow) table are flows variables and therefore have units of dollars per unit time. The entries in the balance sheet matrix are measured in dollars.

for households indicates that this is an outflow of funds; correspondingly, the positive sign of the sales terms for each industrial sector indicates inflows of funds. The first column states that the change in the stocks of money and deposits held by households is equal to the difference between income (wages and interest on deposits) and consumption and taxation expenditures. Considering interindustry transactions Figure 12.12 shows by symmetry that the sum of payments made by, and received from, each industry to each other industry is zero and therefore the inclusion of multiple industries into the SFC framework is consistent with the requirement of zero row sums.

$$
\begin{array}{lc}
 & \begin{array}{ccccccccccc} \text{Households} & \text{Renewable Current} & \text{Renewable Capital} & \text{Fossil Current} & \text{Fossil Capital} & \text{Manufacturing Current} & \text{Manufacturing Capital} & \text{Bank Current} & \text{Bank Capital} & \text{Government} & \Sigma \end{array} \\
\begin{array}{l} \text{Consumption} \\ \text{Government} \\ \text{Wages} \\ \text{Taxes} \\ \text{Investment} \\ \text{Inter-Industry} \\ \text{Interest Deposits} \\ \text{Interest Loans} \\ \text{Interest Advances} \\ \text{Loan Repayment} \\ \text{dMoney} \\ \text{dDeposits} \\ \text{dLoans} \\ \text{dAdvances} \\ \Sigma \end{array} &
\begin{pmatrix}
-C(t) & C^R(t) & 0 & C^{FF}(t) & 0 & C^M(t) & 0 & 0 & 0 & 0 & 0 \\
0 & G^R(t) & 0 & G^{FF}(t) & 0 & G^M(t) & 0 & 0 & 0 & -G(t) & 0 \\
WB(t) & -WB^R(t) & 0 & -WB^{FF}(t) & 0 & -WB^M(t) & 0 & 0 & 0 & 0 & 0 \\
-T(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T(t) & 0 \\
0 & 0 & -I^R(t) & 0 & -I^{FF}(t) & I(t) & -I^M(t) & 0 & 0 & 0 & 0 \\
0 & z_{13}(t) & 0 & z_{23}(t) & 0 & -z_{13}(t)-z_{23}(t) & 0 & 0 & 0 & 0 & 0 \\
rM(t) & 0 & 0 & 0 & 0 & 0 & 0 & -rM(t) & 0 & 0 & 0 \\
0 & -rL^R(t) & 0 & -rL^{FF}(t) & 0 & -rL^M(t) & 0 & rL(t) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -rA(t) & 0 & rA(t) & 0 \\
0 & -Z_R L^R(t) & Z_R L^R(t) & -Z_{FF} L^{FF}(t) & Z_{FF} L^{FF}(t) & -Z_M L^M(t) & Z_M L^M(t) & 0 & 0 & 0 & 0 \\
-\frac{dH^h}{dt} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dH}{dt} & \frac{dH}{dt} & 0 \\
-\frac{M}{dt} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dM}{dt} & 0 & 0 \\
0 & 0 & \frac{dL^R}{dt} & 0 & \frac{dL^{FF}}{dt} & 0 & \frac{dL^M}{dt} & 0 & -\frac{dL}{dt} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dA}{dt} & -\frac{dA}{dt} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
\tag{10.3}
$$

Figure 10.2: SFCIO-ETM Transactions Flow Matrix

The interdependencies between the production sectors are shown in the following input-output table in Figure 12.13. The manufacturing sector is assumed to purchase inputs in the form of electricity and fuels from both the renewable and fossil fuel sectors as well as use some of its output as an input in its production process. The fossil fuel sector is assumed to purchase as inputs only its own output (energy in the form of fuel) to power its operations (extraction, refinement, transportation, etc.). Finally, the renewable sector does not purchase inputs from any of the sectors; rather, the renewable sector purchases capital equipment from the manufacturing sector which is recorded as investment and hence appears in final demand. The purchase of capital equipment for the other two sectors is also considered investment and hence counted in final demand.[6]

[6] Figures 12.12 and 12.13 present two different economic accounting schemes and in order to successfully merge stock-flow consistent modelling with input-output analysis it must be the case that both systems of accounting are coherent. For example, the sum of the first four entries of row one of Figure 12.13 equals $X^R(t)$ which must also equal the column sum. This implies that, after rearrangement, $WB^R(t) = z_{13} + C^r(t) + G^R(t) - (r + Z_R)L(t)$; for the model to work this must be consistent with the accounting in the transactions matrix. Indeed, substituting the expressions for $WB^R(t)$ just found into the second column of the transactions flow matrix returns an identity as it must.

$$
\begin{array}{l}
\\
\text{Renewable} \\
\text{Fossil Fuel} \\
\text{Manufacturing} \\
\text{Wages} \\
\text{Loans} \\
\hline
\text{Total Outlays}
\end{array}
\begin{array}{c}
\begin{array}{cccc|c}
\text{Renewable} & \text{Fossil Fuel} & \text{Manufacturing} & \text{Final Demand} & \text{Total Output}
\end{array} \\
\left(
\begin{array}{cccc|c}
0 & 0 & z_{13}(t) & C^R(t) + G^R(t) & X^R(t) \\
0 & z_{22}(t) & z_{23}(t) & C^{FF}(t) + G^{FF}(t) & X^{FF}(t) \\
0 & 0 & z_{13}(t) & C^m(t) + G^M(t) + I(t) & X^M(t) \\
WB^R(t) & WB^F(t) & WB^M(t) & 0 & WB(t) \\
(r + Z_r)L^R(t) & (r + Z_r)L^R(t) & (r + Z_r)L^R(t) & 0 & \sum_i (r + Z_i)L^i(t) \\
\hline
X^R(t) & X^F(t) & X^M(t) & C(t) + I(t) + G(t) & X(t)
\end{array}
\right)
\end{array}
\tag{10.4}
$$

Figure 10.3: SFCIO-ETM Extended Input-Output Matrix

The previous three figures present the accounting structure of the model; the following section presents the identity and behavioural equations of the model and the derivation of the model as a system of differential equations

10.3 Model Equations

An important feature of this 'linear" energy transition model is that we assume, as in earlier chapters, that each sector's prices are held constant. This simplifying assumption, though inducing energetic complications as noted in chapter six, is necessary to keep the model linear in form and relatively tractable.[7] We will relax this, as well as a number of other assumptions in chapter twelve. Under this assumption it is possible to obtain the underlying model differential equations explicitly as we do in the chapter appendix and furthermore solve the model analytically.

10.3.1 Consumption and Wages

Household consumption $C(t)$ is given by the following consumption function:

$$C(t) = \alpha_1 YD(t) + \alpha_2 V(t) \tag{10.5}$$

where $YD(t)$ is disposable income and $V(t)$ is wealth where $V(t) = M(t) + H^h(t)$; furthermore, $\alpha_1, \alpha_2 \in (0, 1)$. Households are assumed to spend some fraction ψ on energy goods and the remaining fraction $1-\psi$ on manufactured goods. Denoting these as energy and manufacturing consumption, then $\psi C(t) = C^e(t)$ and $(1 - \psi) \cdot C(t) = C^m(t)$ respectively. Therefore,

$$C(t) = C^e(t) + C^m(t) \tag{10.6}$$

[7]That is, the model is a set of coupled linear differential equations.

Households purchase energy from both the fossil fuel sector and the renewable sector; as such energy consumption can be written as:

$$C^e(t) = C^R(t) + C^F(t) \tag{10.7}$$

which denote consumption expenditures on energy from the renewable and fossil fuel sectors respectively.

Government expenditures follow a similar logic with total government expenditure comprised of energy and manufacturing expenditures:

$$G(t) = G^e(t) + G^m(t) \tag{10.8}$$

Like consumption, government expenditure on energy can be separated into energy purchases from the renewable and fossil-fuel sectors.

$$G^e(t) = G^R(t) + G^F(t) \tag{10.9}$$

Household disposable income is made up of the difference between wages $WB(t)$ received from the industrial sectors, interest on deposits held at banks $r \cdot M(t)$ where r is the interest rate and $M(t)$ is the magnitude of the deposits, and taxes paid to the government $T(t)$.

$$YD(t) = WB(t) + r \cdot M(t) - T(t) \tag{10.10}$$

The total wages received by households is simply the sum of the wages paid to households from each sector so that $WB(t) = WB^R(t) + WB^{FF}(t) + WB^M(t)$. This sum can be found directly from the transactions flow matrix by summing the second, fourth, and sixth columns and solving for $WB(t)$; this results in:

$$WB(t) = C(t) + G(t) + I(t) - (r + Zr)L^R(t) - (r + Zf)L^F(t) - (r + Zm)L^M(t) \tag{10.11}$$

Here the Z_i represents the fraction of total outstanding loans $L^i(t)$ held by each sector that must be paid back at time t.[8] From the transactions flow table each of the individual sectors also engage in interindustry transactions which net to zero across all three sectors by necessity. As such, the wages earned by households are simply the difference between the sales of each sector and their costs.

Households in the model can hold two types of assets: cash $H(t)$ and interest earning deposits at banks $M(t)$. It is assumed that households desire to hold a certain proportion (λ_0) of their wealth $V(t)$ as deposits and the remainder as cash (Godley and Lavoie, 2007), (Brainard and Tobin, 1969). This proportion is however modulated by interest rates and disposable income. Households will desire to

[8] Note well that $i = R, F, M$.

hold relatively more of their wealth as interest earning deposits at banks given a higher rate of return r, and conversely, hold relatively less of their wealth as deposits at banks as disposable income increases leading to a greater demand for cash to undertake transactions. As such $M(t)$ can be determined as:

$$\frac{M(t)}{V(t)} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD(t)}{V(t)}\right) \tag{10.12}$$

Finally, the taxes levied by the government on households is simply a proportion θ of wages.

$$T(t) = \theta WB(t) \tag{10.13}$$

10.3.2 Investment

In the SFCIO-ETM model each sector will invest in capital equipment so that the total demand for the outputs of each sector is met by the productive capacity of the capital stock each sector operates. For example, meeting total demand for electricity from renewable sources requires a sufficient renewable capacity to generate that electricity. The logic is similar for the other two sectors, a stock of fossil-fuel capital can only produce so much output; the stock of manufacturing equipment can likewise only produce in proportion to its productive capacity.

The manufacturing sector operates two types of capital stock; electrified capital $k_e^m(t)$ and non-electrified capital $k_{ne}^m(t)$ (non-electrified capital may be thought of simply as capital powered by internal combustion engines). Note well that the lower case $k_e^m(t)$ denotes the number of machines while the uppercase $K_e^m(t)$ denotes the dollar value of the capital stock where $K_e^m(t) = P_m \cdot k_e^m(t)$; that is the dollar value of a capital stock is simply the price multiplied by the number of units of capital comprising the stock. Therefore, the manufacturing sector must purchase both electricity and fuels to operate its production equipment. Electricity to meet this manufacturing intermediate demand is sourced from both the renewable sectors stock of capital built to meet intermediate demand k_e^R and generated, via combustion, by the fossil-fuel sector using a portion of its capital stock k^F to do so.

It is assumed for simplicity that households and government's energy demand is only for electricity.[9] As with intermediate demand, some of this electricity is sourced from the renewable sector's capital built to meet final demand k_f^R and the rest is produced by the fossil-fuel sector. The demand for manufactured goods by households and government is met by the combined productive capacity of the electrified and non-electrified capital stocks of the manufacturing sector.

[9]This is a fairly strong assumption, though necessary to keep the model tractable. One notable consequence of this assumption is that this likely overestimates the decline in emissions due to households and government sectors as they increasingly source their energy from renewable sources. Why? This eliminates consideration of elements of the transition such as the electrification of private transport for example.

To reach zero emissions in the model it is necessary to both build up the capacity of renewable energy generation and also electrify the manufacturing sector. The investment necessary in each type of capital stock to achieve this outcome is captured in the following equations, first investment in renewable capacity to meet intermediate demand is examined.

$$I_e^R(t) = \Delta_1 P_m \left[\phi_R^{-1} \tau_E k_e^m(t) - k_e^R(t)\right] \tag{10.14}$$

Here ϕ_R represents the electric power output per unit of renewable capacity and has units of [Watts/unit of capital] or [W/k] for short.[10] The parameter τ_E denotes the electricity cost to run one unit of electrified manufacturing capital and also has units [W/k]. P_m, the price of manufactured output, has units of dollars per unit of capital equipment [$/k]. Finally, Δ_1 is a parameter that determines the rate at which the gap between the number of units of renewable capital required to power electrified capital and the number of units currently in operation is closed by and therefore has units of s^{-1}. Therefore, (10.14) states that the renewable sector will invest in renewable capacity (for intermediate demand) up until the energy output of renewables matches the energy requirement of electrified capital. [11]

The renewable sector also constructs capacity to meet energy final demand; the investment equation for this capacity is given as follows and states that the renewable sector will invest in:

$$I_f^R(t) = \Delta_1 P_m \left[\phi_R^{-1} P_R^{-1}(C^e(t) + G^e(t)) - k_f^R(t)\right] \tag{10.16}$$

Equation (10.17) states that the fossil-fuel sector will invest in its capital stock so that its capacity is equivalent to the capacity necessary to supply $X^{FF}(t)$, the sum of intermediate and final demand for fossil fuels. Here ϕ_F represents the quantity of fossil-fuel energy delivered per unit time per unit of fossil-fuel capital that can be produced per unit of fossil-fuel capital.

$$I^F(t) = \Delta_3 P_m \left[\phi_F^{-1} P_F^{-1} X^{FF}(t) - k^F(t)\right] \tag{10.17}$$

[10] The energy output of a unit of renewable capacity over the model time-horizon is the "sum" of its power output [Watts] (ϕ_R multiplied by one unit of capacity) multiplied by the run-time. The energy return (in Joules) is given therefore as:

$$\text{Energy Return Renewable} = \int_0^{t_{\text{final}}} \phi_R \cdot e^{-\sigma_R t} dt \tag{10.15}$$

Where σ_R is the assumed physical depreciation rate of a unit of renewable capacity.

[11] Since $I_e^R(t)$ (a flow variable) on the left hand side (LHS) is measured in dollars per time, then to be dimensionally consistent, the same must be true for the right hand side (RHS). The k_e^m and K_e^R terms represent the physical number of units of electrified manufacturing capital and renewable capital respectively; it is assumed for tractability that these share a common unit of physical capital k. Therefore, the dimension of the quantity within the brackets of (10.14) is units of physical capital. Since P_m has dimensions of [$/k] then P_m multiplied by the bracketed quantity has units of dollars which when multiplied by $Delta_1$, the rate of closure of the gap between the renewable capacity necessary to power electrified manufacturing capital and the currently constructed capacity, has the necessary dimensions of $ per unit time. A table of parameters and their units is provided in the appendices.

Finally, the manufacturing sector will invest in its stock of electrified capital to meet both increases in demand for manufactured output and to replace non-electrified capital as it depreciates. In the model it is assumed that there is no new investment in non-electrified capital in order to model the fastest possible transition away from fossil-fuels. The investment rule is as follows:

$$I_e^M(t) = \Delta_3 P_m \left[\phi_M^{-1} P_M^{-1} X^M(t) - k_e^m(t) - k_{ne}^m(t)\right] \tag{10.18}$$

Here ϕ_M denotes the quantity of manufacturing output per unit time per unit of manufacturing capital; since it is assumed that these are measured in the same units this is a dimensionless quantity. Equation (10.18) states that the manufacturing sector will invest in new electrified capital so that its total manufacturing capacity (electrified and non-electrified capital) is able to meet total demand for manufacturing output. Finally, total investment is simply the sum of the above:

$$I(t) = I_e^R(t) + I_f^R(t) + I^{FF}(t) + I_e^M(t) \tag{10.19}$$

The above investment rules require expressions for total fossil-fuel and manufacturing sector outputs which can be obtained from the input-output structure of the model which is represented by the following time-dependent matrix of technical coefficients $\mathbf{A(t)}$.[12]

$$\mathbf{A(t)} = \begin{pmatrix} 0 & 0 & \frac{P_r \phi_R k_e^R(t)}{X^M(t)} \\ 0 & a_{22} & \frac{P_f\left[CF[\tau_E k_e^M(t) - \phi_R k_e^R(t)] + \tau_{FF} k_{ne}^m(t)\right]}{X^m(t)} \\ 0 & 0 & a_{33} \end{pmatrix} \tag{10.20}$$

The model assumes that the technical coefficients relating the purchases of the fossil-fuel and manufacturing of their own goods is constant and exogenous; these are given by technical coefficients a_{22} and a_{33}. The a_{13} element is the ratio of the cost of electricity inputs purchased by the manufacturing sector from the renewable sector to the total value of manufacturing sector output. The a_{23} element is the ratio of the cost of fossil-fuel output purchased by the manufacturing sector to the total value of manufacturing output. As the renewable capacity grows, a larger portion of the electricity necessary to run the manufacturing sector's electrified capital can be obtained from the renewable sector. This implies that the a_{13} coefficient will become larger overtime pending sufficient renewable investment.

The manufacturing sector must purchase both electricity and fuel to power its electrified and non-electrified capital stocks respectively; that is it must purchase $\tau_E k_e^M$ electricity and $\tau_{FF} k_{ne}^M$ units of fossil-fuels. Here τ_{FF} is the fuel requirement per unit of non-electrified capital. The amount of electric-

[12]The a_{ij} element of a technical coefficient matrix is calculated as: $a_{ij} = \frac{z_{ij}}{X_j}$

ity purchased from the fossil fuel sector is therefore $\tau_E k_e^M(t)$ less that produced by the renewable sector $\phi_R k_e^R(t)$. It is necessary to obtain the fossil-fuel equivalent of this quantity of electricity so the term $\tau_E k_e^M(t) - \phi_R k_e^R(t)$ must be multiplied by the conversion factor CF which represents the conversion losses in electricity generation using fossil-fuels; this in addition to the quantity $\tau_F k^F(t)$ when multiplied by P_f gives the total dollar value per unit time of intermediate purchases of fossil-fuel output by the manufacturing sector.

The first condition necessary for the SFCIO-ETM model to completely shift away from fossil-fuel usage is for the a_{23} coefficient to go to zero. This, from examination of the numerator, will occur when renewable capacity is sufficient to meet the electric power demand of electrified capital and the manufacturing sector uses no non-electrified capital. The second condition is that the final demand for fossil-fuel derived electricity is reduced to zero.

Summing and rearranging equations(12.4) and (12.6) returns the following expression for the dollar value of final demand for fossil-fuels:

$$C^f(t) + G^f(t) = C^e(t) + G^e(t) - C^R(t) - G^R(t) \tag{10.21}$$

Here $C^R(t)$ and $G^R(t)$ represent the final demand for renewable electricity. The renewable sector constructs capacity k_f^R to meet this demand, and it is assumed that the final demand for renewables is simply equivalent to the output of this capacity; that is,

$$C^R(t) + G^R(t) = P_R \phi_R k_f^R(t) \tag{10.22}$$

and therefore:

$$C^f(t) + G^f(t) = C^e(t) + G^e(t) - P_R \phi_R k_f^R(t) \tag{10.23}$$

From the investment equation given in (10.16) the renewable sector will invest in capacity k_f^R until total energy final demand $C^e(t) + G^e(t)$ is met by renewable capacity which implies, via (10.23), that $C^f(t) + G^f(t) = 0$.

Finally, the final demand for manufacturing goods is the sum of consumption expenditure, government expenditure, and investment expenditure $C^m(t) + G^m(t) + I(t)$.

From (10.20) the Leontief coefficients can be calculated by solving for the individual matrix elements of:

$$\mathbf{L(t)} = (\mathbf{I} - \mathbf{A(t)})^{-1} \tag{10.24}$$

which results in:

$$L_{11} = 1, \quad L_{12} = 0, \quad L_{13} = \left(\frac{P_r \phi_R k_e^R(t)}{(1 - a_{33}) \cdot X^M(t)} \right), \tag{10.25}$$

$$L_{21} = 0, \quad L_{22} = \frac{1}{1 - a_{22}}, \quad L_{23} = \frac{P_f \left[CF[\tau_E k_e^M(t) - \phi_R k_e^R(t)] + \tau_{FF} k_{ne}^m(t) \right]}{(1 - a_{22})(1 - a_{33}) \cdot X^M(t)}, \tag{10.26}$$

$$L_{31} = 1, \quad L_{32} = 0, \quad L_{33} = \frac{1}{1 - a_{33}} \tag{10.27}$$

With these coefficients the $X^{FF}(t)$ and $X^M(t)$ terms from investment equations (10.17) and (10.18) can be specified noting that $\mathbf{X}(t) = \mathbf{L}(t)\mathbf{f}(t)$, where $\mathbf{X}(t)$ is the vector of total outputs for each sector and $\mathbf{f}(t)$ is the vector of final demands for each sector.

$$X^{FF}(t) = L_{22}[C^f(t) + G^f(t)] + L_{23}[C^m(t) + G^m(t) + I(t)] \tag{10.28}$$

and,

$$X^M(t) = L_{33}[C^m(t) + G^m(t) + I(t)] \tag{10.29}$$

To complete the model, it is necessary to specify equations for emissions and cumulative emissions. Denoting the total physical output of the fossil-fuel sector as $x^F(t)$ where $x^F(t) = \frac{X^t}{P_f}$ then total emissions per unit time is $\xi_F x^F(t)$ where ξ_F is a conversion factor giving emissions per unit of fossil-fuel energy used. Therefore, emissions $E(t)$ is given as:

$$E(t) = \xi_F P_F^{-1} L_{22}(C^e(t) + G^e(t) - P_R \phi_R k_f^R) + \frac{\xi_F \left[CF[\tau_E k_e^M(t) - \phi_R k_e^R(t)] + \tau_{FF} k_{ne}^m(t) \right]}{(1 - a_{22})(1 - a_{33}) L_{33}} \tag{10.30}$$

and cumulative emissions at time t is simply:

$$CE(t) = \int_0^t E(t)dt \tag{10.31}$$

10.3.3 Differential Equations and Analytical Model Solution

Having laid out the principle identities and behavioural equations of the model in the previous section, the underlying differential equations can now be stated. They are as follows:

Variable Name	Symbol	Differential Equation
Renewable Loans	$L^R(t)$	$\dot{L^R} = I_e^r(t) + I_f^R(t) - Z_R L^R(t)$
Fossil-Fuel Loans	$L^R(t)$	$\dot{L^F} = I^F(t) - Z_F L^F(t)$
Manufacturing Loans	$L^R(t)$	$\dot{L^M} = I_e^M(t) - Z_M L^M(t)$
Household Wealth	$V(t)$	$\dot{V} = WB(t) + rM(t) - C(t) - T(t)$
Renewable Intermediate Capital	$K_e^R(t)$	$\dot{K_e^R} = I_e^r(t) - \sigma_r K_e^R(t)$
Renewable Final Capital	$K_f^R(t)$	$\dot{K_f^R} = I_f^r(t) - \sigma_r K_f^R(t)$
Fossil-Fuel Capital	$K^F(t)$	$\dot{K^F} = I^F(t) - \sigma_F K^F(t)$
Electrified Manufacturing Capital	$K_e^M(t)$	$\dot{K_e^M} = I_e^M(t) - \sigma_M K_e^M(t)$
Non-Electrified Manufacturing Capital	$K_{ne}^M(t)$	$\dot{K_{ne}^M} = -\sigma_{ne} K_{ne}^M(t)$

Table 10.1: SFCIO-ETM State Variables and their Associated Differential Equations

Having specified a differential equation for each of the nine model state variables in Table (10.1) it is possible to solve the model analytically, using the equations from the previous section, as a set of nine coupled first order linear non-homogenous differential equations. The model model in this form is shown in the following matrix expression, [13]

$$\dot{\mathbf{p}} = \mathbf{B}\mathbf{p}(t) + \mathbf{f}(t) \tag{10.32}$$

where $\dot{\mathbf{p}}$ is the column vector of differentials, $\mathbf{B}$ is the matrix of coefficients, $\mathbf{p}(t)$ is the column vector of state-variables, and $\mathbf{f}(t)$ is a possibly time-dependent set of exogenous forcing functions. Equation (10.32) paired with a set of initial conditions $\mathbf{p}(0)$ is solvable analytically as:

$$\mathbf{p}(t) = \mathbf{\Psi}(t)\left(\int_0^{t_{\text{Final}}} \mathbf{\Psi}^{-1}(t)\mathbf{f}(t)\mathrm{d}t + \mathbf{\Psi}^{-1}(0)\mathbf{p}(0)\right) \tag{10.33}$$

where;

$$\mathbf{\Psi}(t) = e^{\mathbf{B}t}\mathbf{p}(0) \tag{10.34}$$

is the fundamental matrix for the homogenous case.[14]

[13] See Appendix B for the derivation of the model in this form.

[14] The interested reader may find an excellent and readable account of the matrix exponential and the solution form for sets of coupled linear non-homogenous differential equations in (Greenberg, 2012) Chapter Four.

10.4 Simulation and Analysis

Three different economic scenarios are considered which broadly correspond to growth, steady-state, and degrowth trajectories. Model runs of these three scenarios are explored in two different cases: "slow" renewable investment and "fast" renewable investment. In all scenarios the model economy attempts to transition fully away from the use of fossil-fuels. As government expenditure is left as an exogenous variable in the model it may be used to determine overall growth trajectories. Due to the interlinked nature of SFC models, increases (or decreases) in government expenditure will have knock-on effects for all of the remaining endogenous variables (consumption, investment, wages. etc.).[15] The growth and degrowth scenarios assume 3 percent growth (and degrowth) of the magnitude of government expenditure over time while the steady-state scenario assumes that government expenditures remain constant at their initial period level. Figures 10.4 and 10.5 display the trajectories for select model variables for each of the three growth scenarios under slow and fast renewable investment specifications respectively.

In figure 10.4 cumulative emissions transgress the 1.5°C carbon budget for the growth scenario in approximately year 2034; the steady-state scenario by 2036; and approximately 2041 in the degrowth scenario. Emissions peak and decline in the steady-state and degrowth scenarios but do not peak in the growth scenario. Renewable investment as a fraction of GDP for all three scenarios is approximately 2.8 percent in year 2020; this fraction increases continuously for at least 12 years in each scenario. The peak and decline in emissions for the steady-state and degrowth scenarios and the period of more rapidly increasing emissions in the growth scenario indicate partly what may be termed a transient spike in emissions associated with the build-out of renewable capacity.

[15] It is important to note that fist, degrowth is a far richer concept than is made simplistically here by associating it with declining GDP, and second, that the conflation of degrowth here with declining to government expenditure is an artefact of how the model was constructed. A similar result could have been obtained by assuming an autonomous portion of consumption that grows or degrows exogenously.

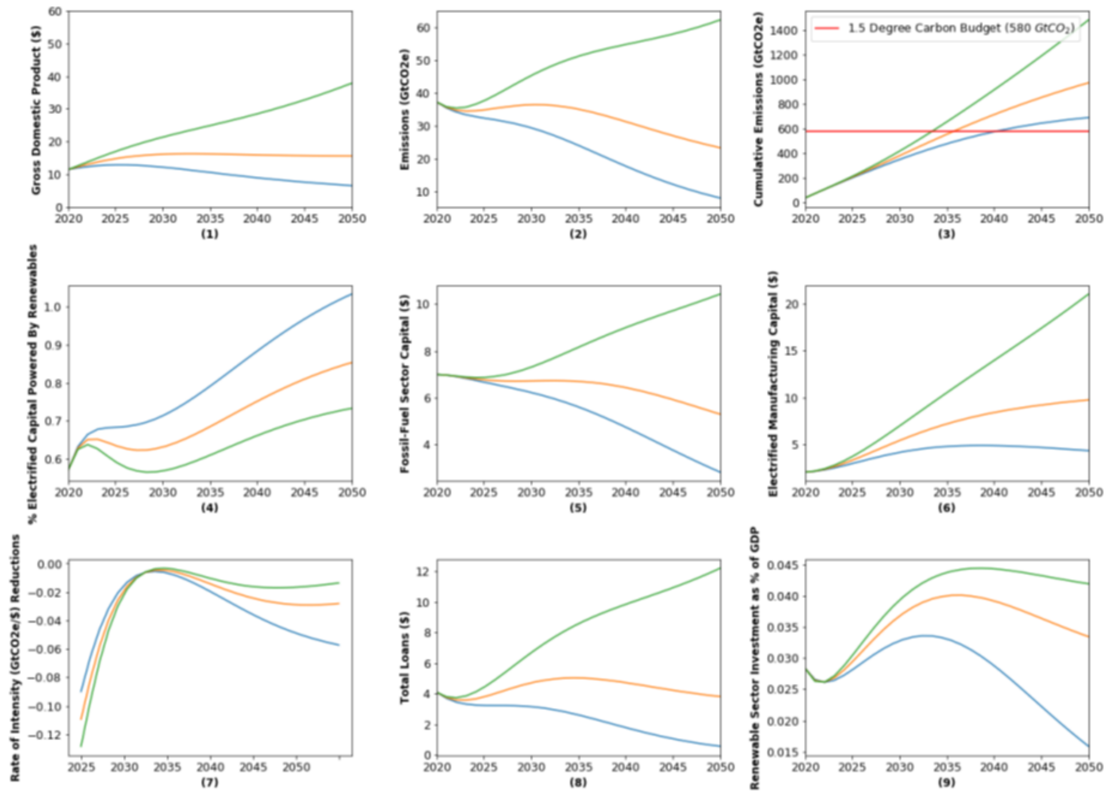


Figure 10.4: Trajectories for the main model variables in the degrowth scenario (blue line), the steady-state scenario (orange line), and the growth scenario (green line) for $\Delta_1 = 0.1$. By 2030 emissions have peaked and are declining in the degrowth scenario only. Emissions peak for the steady-state scenario in approximately 2032, and there is no peak in the growth scenario. Cumulative emissions surpass the 580 $GtCO_2$ carbon budget in all three scenarios.

Figure 10.5 displays the model trajectories under the "fast" investment assumption. Renewable investment as a percentage of GDP in all three scenarios begins at approximately 8.5 percent and declines over the remainder of the simulation. This corresponds to a very large initial burst of investment and renewable capacity construction. Emissions in all three scenarios decline immediately but only in the degrowth scenario do emissions decline sufficiently rapidly to keep cumulative emissions within the 1.5 degree budget. Cumulative emissions in the growth and steady-state scenarios, even with rapid investment, transgress the budget in approximately 2039 and 2048 respectively.

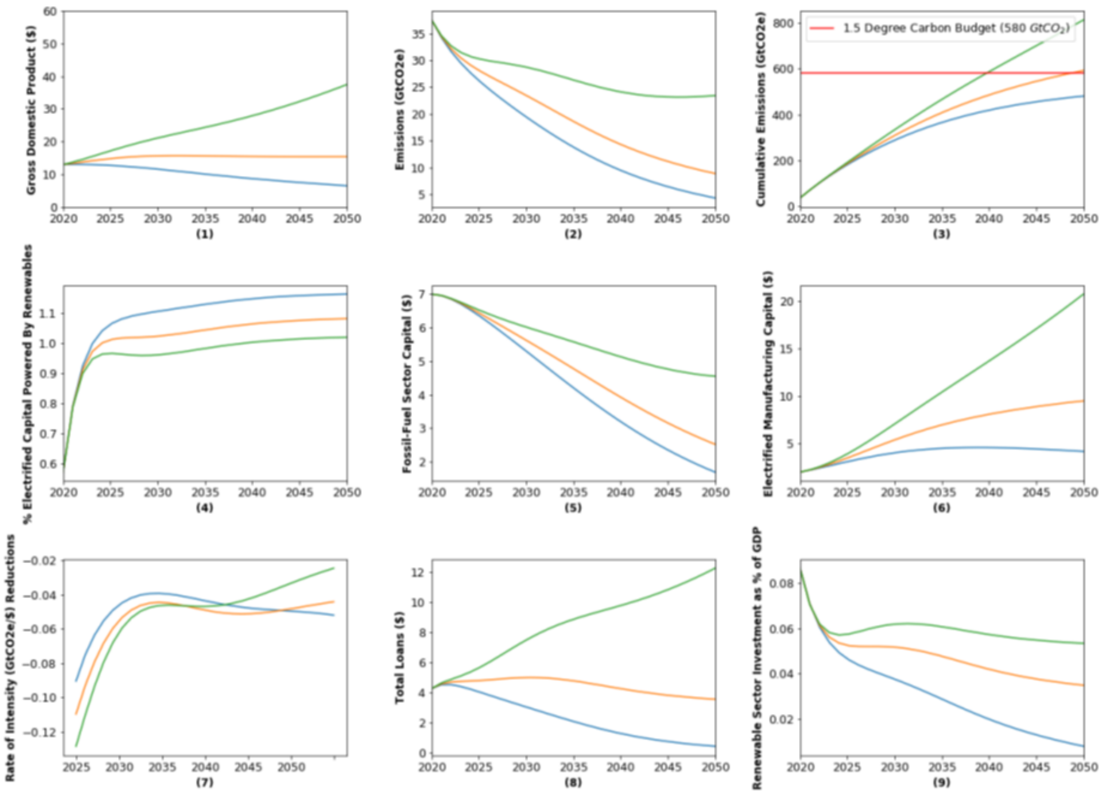
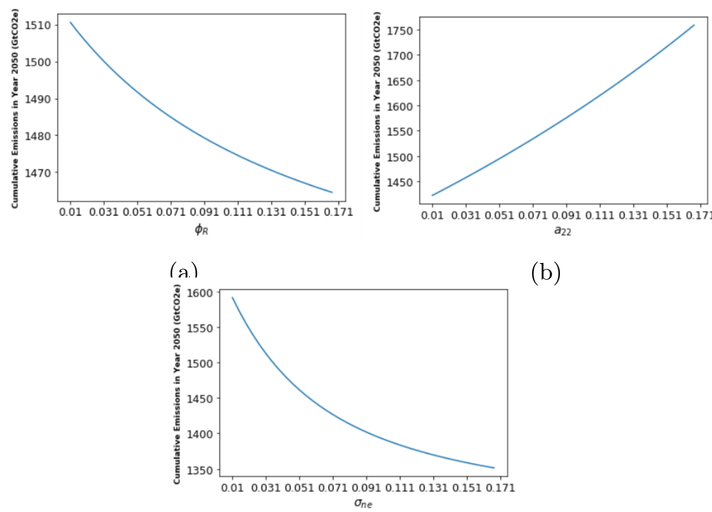


Figure 10.5: Trajectories for the main model variables in the degrowth scenario (blue line), the steady-state scenario (orange line), and the growth scenario (green line) for $\Delta_1 = 0.335$. Emissions in 2030 are 55 percent below the 2018 level in 2020 in the degrowth scenario only. Cumulative emissions by 2050 remain below the 580 $GtCO_2$ carbon budget in the degrowth scenario only.

Figure (11.7) presents a sensitivity analysis of cumulative emissions in year 2050 as impacted by changes in three key parameters for the "slow" renewable investment economic growth scenario. Panel (a) of Figure 11.7 shows that, all else being equal, increasing the power output per unit of renewable capacity leads to lower cumulative emissions in year 2050; however, the effect is not large for the range examined. Panel (c) shows a similar decrease in emissions but as a function of the increasing rate of the non-electrified depreciation and decommissioning parameter.

The technical coefficient a_{22} has a particularly interesting meaning in the model. It is a dimensionless ratio of the quantity of fossil-fuel energy purchased by the fossil-fuel sector as an input to power the production of its total output. In the model the fossil fuel sector is assumed to power its operations with its own output (fossil-fuel derived energy) and hence a_{22} is the reciprocal of a measure of energy return on investment (EROI). All else being equal, increasing the magnitude of the a_{22} parameter implies a declining fossil-fuel EROI which correspondingly leads to increased emissions. The impact of changes in this parameter on year 2050 cumulative emissions is shown in panel (b). The model assumes a value of $a_{22} = 0.05$ which roughly corresponds to an EROI of 20. This primary EROI declines to less than 10 when considering the energy costs of the construction of capital and the conversion losses in converting fossil-fuels to electricity. For an analysis comparing the differences between primary and final EROIs of fossil fuels and their comparison with renewable EROIs see (Brockway et al, 2019).



??

(a) (b)

(c)

Figure 10.6: Sensitivity of Cumulative Emissions in year 2050 of the Slow Investment Growth Scenario to changes in select model parameters. Panel (a) displays the variation in total emissions across a range of possible values of ϕ_R holding all else equal. Panels (b) and (c) display similar results for variations in the a_{22} and σ_{ne} parameters.

10.5 Conclusions

The SFCIO-ETM model examined in the preceding sections presents a simple model of the linkages between a monetary economy, an energy system in transition, and emissions. Under these assumptions and, excluding by design, several key complicating factors that would make the process of transition possibly considerably more complicated it is shown that achieving the 1.5°C target is virtually impossible without enormous upfront investment and simultaneous degrowth in final demand. This stark result is in large part due simply to the relatively small remaining carbon budget. So tight are the constraints on future allowable emissions that the only solution of the system (under ideal circumstances) is to transition while simultaneously witnessing declining consumption and government expenditures.

It is important to discuss qualitatively the possible impact of the omitted factors. First, from Figure 11.7 declining EROI leads to increased emissions. Modelling this endogenously would make achieving the target more difficult. Second, the model assumes a relatively high conversion efficiency for the heat from fossil-fuel combustion to electricity (50%). Such efficiencies are associated with combined-cycle power plants and are high in comparison to the heat rates for most plants (see (Eia, 2018)). Third, the complexities surrounding 100 percent renewable energy (for example the possible rapidly increasing energetic costs of storage with greater market share of generation) is excluded. Finally, the feedbacks of the climate system into the functioning of the economy are not modelled specifically. However, such feedbacks, which might range from increasing depreciation rates or damaging sectoral outputs, would result again in increased transition difficulty.

While the principal aim of this chapter was to derive and present a relatively tractable form of a continuous-time stock-flow consistent input-output model (with applications to the energy-transition) a secondary outcome of the analysis is a strong statement about the difficulty of reconciling growth with sufficient emissions reductions while simultaneously transitioning energy systems. Insofar as the simple dynamics of the SFCIO-ETM model reasonably capture aspects of a macroeconomy attempting a rapid transition from fossil-fuels to renewables, it is clear that economic growth and meeting the 1.5°C target are largely incompatible for any even remotely plausible magnitudes of renewable energy investment. Furthermore, the model does not consider the political and social complications that might arise under degrowth or even steady-state economic trajectories. Therefore, under generous assumptions and ideal conditions the model suggests that a transition without negative-emissions technology is just barely possible assuming both a very large renewable investment beginning immediately and conditions of long-term degrowth. Should negative emissions technology prove scalable then this result is less dire.

As noted earlier in the chapter, and in this concluding section, many important physical considerations were absent in the formulation of this model. In the following chapter we will not present any new model but introduce important climate and energy feedback processes that were absent from this chapter. We will then synthesize these with the core-structure of the

SFCIO-ETM model to produce a single country IAM model.

10.6 Appendix: Model Derivation

In this appendix we derive the matrix form of the model in (10.23); for convenience the time argument (t) is dropped to keep the derivation tidier. We begin with the definition of GDP.

$$Y = C + I + G \tag{10.1}$$

$$= \alpha_1 YD + \alpha_2 V + I + G$$

$$= \underbrace{\alpha_1(1-\theta)}_{w_1} WB + \alpha_1 rM + \alpha_2 V + I + G$$

$$= w1(Y - \underbrace{(r + z_R)}_{w_2} L^R - \underbrace{(r + z_F)}_{w_3} L^F - \underbrace{(r + z_M)}_{w_4} L^M) + \alpha_1 rM + \alpha_2 V + I + G$$

$$Y = w_1 Y - w_1 w_2 L^R - w_1 w_3 L^F - w_1 w_4 L^M + \alpha_1 rM + \alpha_2 V + I + G \tag{10.2}$$

Now, we need to find an expression for M in terms of Y, therefore From above we have (after rearranging and substituting in the definition of disposal income):

$$M = \underbrace{(\lambda_0 + \lambda_1 r)}_{w_5} V - \lambda_2((1-\theta)WB + rM) \tag{10.3}$$

$$M = \underbrace{\frac{1}{(1 + \lambda_2 r)}}_{w_6}(w_5 V - \underbrace{\lambda_2(1-\theta)}_{w_7} WB)$$

$$M = \underbrace{w_6 w_5}_{w_8} V - \underbrace{w_6 w_7}_{w_9} Y + \underbrace{w_6 w_7 w_2}_{w_{10}} L^R + \underbrace{w_6 w_7 w_3}_{w_{11}} L^F + \underbrace{w_6 w 7_7 w_4}_{w_{12}} L^M \tag{10.4}$$

Plugging this back into the expression for Y and collecting terms we have:

$$Y = \underbrace{(w_1 - \alpha_1 r w_9)}_{w13} Y + \underbrace{(\alpha_1 r w_{10} - w_1 w_2)}_{w_{14}} L^r + \underbrace{(\alpha_1 r w_{11} - w_1 w_3)}_{w_{15}} L^F + \underbrace{(\alpha_1 r w_{12} - w_1 w_4)}_{w_{16}} L^M + \underbrace{(\alpha_2 + \alpha_1 r w_8)}_{w_{17}} V + I + G$$

$$= \left(\frac{1}{1 - w_{13}}\right) \cdot (w_{14} L^R + w_{15} L^F + w_{16} L^M + w_{17} V + I + G) \tag{10.5}$$

Now having derived an expression for Y we turn to the investment functions. First, the equation for I_e^R, rearranged for convenience:

$$I_e^R = \underbrace{\Delta_1 P_m \phi_R^{-1} \tau_E}_{w_{19}} k_e^m - \underbrace{\Delta_1 P_m}_{w_{20}} k_e^R \tag{10.6}$$

Now, for I_f^R,

$$I_f^R = \underbrace{\Delta_1 P_m \phi_r^{-1} P_R^{-1}}_{w_{21}} C^e + \underbrace{\Delta_1 P_m \phi_r^{-1} P_R^{-1}}_{w_{21}} G^e - \underbrace{\Delta_1 P_m}_{w_{w22}} k_f^R \tag{10.7}$$

Now, since $C^e = \psi C$ we can obtain an expression for C^e from the above derivation for Y as:

$$C^e = \psi(w_{13} Y + w_{14} L^R + w_{15} L^F + w_{16} L^M + w_{17} V)$$

Plugging this into the equation for I_f^R and rearranging we have:

$$I_f^R = \underbrace{w_{21} \psi w_{13}}_{w_{23}} Y + \underbrace{w_{21} \psi w_{14}}_{w_{24}} L^R + \underbrace{w_{21} \psi w_{15}}_{w_{25}} L^F + \underbrace{w_{21} \psi w_{16}}_{w_{26}} L^M + \underbrace{w_{21} \psi w_{17}}_{w_{27}} V + w_{21} G^e - w_{22} k_f^R \tag{10.8}$$

Now for I^{FF},

$$I^{FF} = \underbrace{\Delta_2 P_m \phi^{-1} P_F^{-1}}_{w_{28}} X^{FF} - \underbrace{\Delta_2 P_m}_{w_{29}} k^{FF} \tag{10.9}$$

$$= w_{28} L_{22}(C^e + G^e - P_R \phi_R k_f^R) + \underbrace{\frac{w_{28} L_{33}^{-1} P_f}{(1-a_{22})(1-a_{33})}}_{w_{30}} \Big(CF(\tau_E k_e^M - \phi_R k_e^R) + \tau_{FF} k_{ne}^M)\Big) - w_{29} k^{FF}$$

$$\underbrace{w_{28} L_{22}}_{w_{31}} C^e + \underbrace{w_{28} L_{22}}_{w_{31}} G^e - \underbrace{w_{28} L_{22} P_R^{-1} \phi_R}_{w_{32}} k_f^R + \underbrace{w_{30} CF \tau_E}_{w_{33}} k_e^M - \underbrace{w_{30} CF \phi_R}_{w_{34}} k_e^R + \underbrace{w_{30} \tau_{FF}}_{w_{35}} k_{ne}^M - w_{29k} FF$$

$$\underbrace{w_{31} \psi w_{13}}_{w_{36}} Y + \underbrace{w_{31} \psi w_{13}}_{w_{37}} L^R + \underbrace{w_{31} \psi w_{13}}_{w_{38}} L^F + \underbrace{w_{31} \psi w_{13}}_{w_{39}} L^M + \underbrace{w_{31} \psi w_{13}}_{w_{40}} V + w_{31} G - w_{32} k_f^R + w_{33} k_e^M - w_{34} k_e^R + w_{35} k_{ne}^m - w_{29} k^{FF} \tag{10.10}$$

Finally, we bring in the last investment type:

$$I_e^M = \underbrace{\Delta_3 P_m \phi_m^{-1}}_{w_{41}} X^M - \underbrace{\Delta_3 P_m}_{w_{42}} k_e^M - \underbrace{\Delta_3 P_m}_{w_{42}} k_{ne}^M \tag{10.11}$$

$$w_{41} L_{33}(Y - C^e - G^e) - w_{42} k_e^M - w_{42} k_{ne}^M$$

$$\underbrace{(w_{41} - L_{33} w_{41} \psi w_{13})}_{w_{43}} Y - \underbrace{L_{33} w_{41} \psi w_{14}}_{w_{44}} L^R - \underbrace{L_{33} w_{41} \psi w_{14}}_{w_{45}} L^F - \underbrace{L_{33} w_{41} \psi w_{16}}_{w_{46}} L^M - \underbrace{L_{33} w_{41} \psi w_{17}}_{w_{47}} V - w_{41} G^e - w_{42} k_e^M - w_{42} k_{ne}^M \tag{10.12}$$

Finally, we find total investment I by summing the four investment categories previously derived and grouping terms.

$$\begin{aligned} &= \underbrace{(w_{23} + w_{36} + w_{43})}_{w_{48}} Y + \underbrace{(w_{24} + w_{37} - w_{44})}_{w_{49}} l^r + \underbrace{(w_{25} + w_{38} + w_{45})}_{w_{50}} L^F + \underbrace{(w_{26} + w_{39} + w_{46})}_{w_{51}} L^M + \underbrace{(w_{27} + w_{40} + w_{47})}_{w_{52}} V \\ &\quad - \underbrace{(w_{20} + w_{34})}_{w_{53}} k_e^R - \underbrace{(w_{22} + w_{32})}_{w_{54}} k_f^R - w_{29} k^F - \underbrace{(w_{42} - w_{19} - w_{33})}_{w_{55}} k_e^M - \underbrace{(w_{42} - w_{34})}_{w_{56}} k_{ne}^M + \underbrace{(w_{21} + w_{31} - w_{41})}_{w_{57}} \end{aligned} \tag{10.13}$$

Now, having an expression for I in terms of Y we may substitute it to obtain an expression for Y. This, after some rearranging and grouping of terms is shown as follows:

$$\begin{aligned} Y &= \underbrace{w_{18} w_{48}}_{w_{58}} Y + \underbrace{w_{18}(w_{49} + w_{14})}_{w_{59}} L^R + \underbrace{w_{18}(w_{50} + w_{15})}_{w_{60}} L^F + \underbrace{w_{18}(w_{51} + w_{16})}_{w_{61}} L^M + \underbrace{w_{18}(w_{52} + w_{17})}_{w_{62}} V \\ &\quad - \underbrace{w_{18} w_{53}}_{w_{63}} k_e^R - \underbrace{w_{18} w_{54}}_{w_{64}} k_f^R - \underbrace{w_{18} w_{29}}_{w_{65}} k^F - \underbrace{w_{18} w_{55}}_{w_{66}} k_e^M - \underbrace{w_{18} w_{56}}_{w_{67}} k_{ne}^M + \underbrace{w_{57} G^e + w_{18} G}_{w_{68}} \end{aligned} \tag{10.14}$$

and, finally we can solve for Y as a function of the nine state variables:

$$Y = \underbrace{\left(\frac{1}{1 - w_{58}}\right)}_{w_{69}} \Big(w_{59} L^R + w_{60} L^F + w_{61} L^M + w_{62} V - w_{63} k_e^R - w_{64} k_f^R - w_{65} k^F - w_{66} k_e^M - w_{67} k_{ne}^M + w_{68}\Big) \tag{10.15}$$

Having solved for Y we can now solve for each of of the investment types as functions of the underlying state variables:

$$\dot{k_e^R} = I_e^R - \sigma_R k_e^R = -(w_{20} + \sigma_R) + w_{19} k_e^M \tag{10.16}$$

$$
\begin{aligned}
I_f^R = &\underbrace{(w_{23}w_{69}w_{59} + w_{24})}_{\delta_1} L^R + \underbrace{(w_{23}w_{69}w_{60} + w_{25})}_{\delta_2} L^F + \underbrace{(w_{23}w_{69}w_{61} + w_{26})}_{\delta_3} L^M + \underbrace{(w_{23}w_{69}w_{62} + w_{27})}_{\delta_4} V \\
&- \underbrace{w_{23}w_{69}w_{63}}_{\delta_5} k_e^R - \underbrace{(w_{23}w_{69}w_{64} + w_{22})}_{\delta_6} k_f^R - \underbrace{w_{23}w_{69}w_{65}}_{\delta_7} k^F - \underbrace{w_{23}w_{69}w_{66}}_{\delta_8} k_e^M - \underbrace{w_{23}w_{69}w_{67}}_{\delta_9} k_{ne}^M + \underbrace{(w_{23}w_{69}w_{68} + w_{21}G^e)}_{\delta_0}
\end{aligned} \tag{10.17}
$$

$$
\begin{aligned}
I^F = &\underbrace{(w_{36}w_{69}w_{59} + w_{37})}_{\delta_{11}} L^R + \underbrace{(w_{36}w_{69}w_{60} + w_{38})}_{\delta_{12}} L^F + \underbrace{(w_{36}w_{69}w_{61} + w_{39})}_{\delta_{13}} L^M + \underbrace{(w_{36}w_{69}w_{62} + w_{40})}_{\delta_{14}} V \\
&- \underbrace{(w_{36}w_{69}w_{63} + w_{34})}_{\delta_{15}} k_e^R - \underbrace{(w_{36}w_{69}w_{64} + w_{32})}_{\delta_{16}} k_f^R - \underbrace{(w_{36}w_{69}w_{65} + w_{29})}_{\delta_{17}} k^F - \underbrace{(w_{36}w_{69}w_{66} - w_{33})}_{\delta_{18}} k_e^M \\
&- \underbrace{(w_{36}w_{69}w_{67} - w_{35})}_{\delta_{19}} k_{ne}^M + \underbrace{(w_{36}w_{69}w_{68} + w_{31}G^e)}_{\delta_{10}}
\end{aligned} \tag{10.18}
$$

$$
\begin{aligned}
I_e^M = &\underbrace{(w_{43}w_{69}w_{59} - w_{44})}_{\delta_{21}} L^R + \underbrace{(w_{43}w_{69}w_{60} - w_{45})}_{\delta_{22}} L^F + \underbrace{(w_{43}w_{69}w_{61} - w_{46})}_{\delta_{23}} L^M + \underbrace{(w_{43}w_{69}w_{62} - w_{47})}_{\delta_{24}} V - \underbrace{w_{43}w_{69}w_{63}}_{\delta_{25}} k_e^R \\
&- \underbrace{w_{43}w_{69}w_{64}}_{\delta_{26}} k_f^R - \underbrace{w_{43}w_{69}w_{65}}_{\delta_{27}} k^F - \underbrace{(w_{43}w_{69}w_{66}+]w_{42})}_{\delta_{28}} k_e^M - \underbrace{(w_{43}w_{69}w_{67} + w_{42})}_{\delta_{29}} k_{ne}^M + \underbrace{(w_{43}w_{69}w_{68} - w_{41}G^e)}_{\delta_{20}}
\end{aligned} \tag{10.19}
$$

$$
\begin{aligned}
\dot{V} = &WB + rM - T - C = \underbrace{(1-\theta)}_{w_{70}}(WB) + r(w_8V - w_9Y + w_{10}L^R + w_{11}L^F + w_{12}L^M) - w_{13}Y - w_{14}L^R - w_{15}L^F - w_{16}L^M - w_{17}V \\
&\underbrace{(w_{70} - rw_9 - w_{13})}_{w_{71}} Y + \underbrace{(rw_{10} - w_{70}w_2 - w_{14})}_{w_{72}} L^R + \underbrace{(rw_{11} - w_{70}w_3 - w_{15})}_{w_{73}} L^F + \underbrace{(rw_{12} - w_{70}w_4 - w_{16})}_{w_{74}} L^M + \underbrace{(rw_8 - w_{17})}_{w)75} V
\end{aligned} \tag{10.20}
$$

and finally, substituting in the expression for Y we have:

$$
\begin{aligned}
\dot{V} = &\underbrace{(w_{71}w_{69}w_{59} + w_{72})}_{\delta_{31}} L^R + \underbrace{(w_{71}w_{69}w_{60} + w_{73})}_{\delta_{32}} L^F + \underbrace{(w_{71}w_{69}w_{61} + w_{74})}_{\delta_{33}} L^M + \underbrace{(w_{71}w_{69}w_{62} - w_{75})}_{\delta_{34}} V \\
&- \underbrace{w_{71}w_{69}w_{63}}_{\delta_{35}} k_e^R - \underbrace{w_{71}w_{69}w_{64}}_{\delta_{36}} k_f^R - \underbrace{w_{71}w_{69}w_{65}}_{\delta_{37}} k^F - \underbrace{(w_{71}w_{69}w_{66} + w_{42})}_{\delta_{38}} k_e^M - \underbrace{(w_{71}w_{69}w_{67} + w_{42})}_{\delta_{39}} k_{ne}^M + \underbrace{w_{71}w_{69}w_{68}}_{\delta_{30}}
\end{aligned} \tag{10.21}
$$

a Finally, substituting the above into each of the differential equations, and putting in matrix form we have:

$$
\begin{bmatrix} \dot{L^R} \\ \dot{L^F} \\ \dot{L^M} \\ \dot{V} \\ \dot{K_e^R} \\ \dot{K_f^R} \\ \dot{K^F} \\ \dot{K_e^M} \\ \dot{K_{ne}^M} \end{bmatrix}
=
\begin{bmatrix}
(\delta_1 - z_R) & \delta_2 & \delta_3 & \delta_4 & -(\delta_5 + \omega_{20}) & -\delta_6 & -\delta_7 & -(\delta_8 - \omega_{19}) & -\delta_9 \\
\delta_{11} & (\delta_{12} - z_F) & \delta_{13} & \delta_{14} & -\delta_{15} & -\delta_{16} & -\delta_{17} & -\delta_{18} & -\delta_{19} \\
\delta_{21} & \delta_{22} & (\delta_{23} - z_M) & \delta_{24} & -\delta_{25} & -\delta_{26} & -\delta_{27} & -\delta_{28} & -\delta_{29} \\
\delta_{31} & \delta_{32} & \delta_{33} & \delta_{34} & -\delta_{35} & -\delta_{36} & -\delta_{37} & -\delta_{38} & -\delta_{39} \\
0 & 0 & 0 & 0 & -(\omega_{20} + \sigma_R) & 0 & 0 & \omega_{19} & 0 \\
\delta_{11} & \delta_{12} & \delta_{13} & \delta_{14} & -\delta_{15} & -(\delta_{16} + \sigma_R) & -\delta_{17} & -\delta_{18} & -\delta_{19} \\
\delta_{21} & \delta_{22} & \delta_{23} & \delta_{24} & -\delta_{25} & -\delta_{26} & -(\delta_{27} + \sigma_F) & -\delta_{28} & -\delta_{29} \\
\delta_{31} & \delta_{32} & \delta_{33} & \delta_{34} & -\delta_{35} & -\delta_{36} & -\delta_{37} & -(\delta_{38} + \sigma_M) & -\delta_{39} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma_{ne}
\end{bmatrix}
\cdot
\begin{bmatrix} L^R(t) \\ L^F(t) \\ L^M(t) \\ V(t) \\ K_e^R(t) \\ K_f^R(t) \\ K^F(t) \\ K_e^M(t) \\ K_{ne}^M(t) \end{bmatrix}
+
\begin{bmatrix} \delta_0 \\ \delta_{10} \\ \delta_{20} \\ \delta_{30} \\ 0 \\ \delta_0 \\ \delta_{10} \\ \delta_{20} \\ 0 \end{bmatrix}
\tag{10.22}
$$

Or, in more compact notation:

$$\dot{\mathbf{p}} = \mathbf{B}\mathbf{p}(t) + \mathbf{f}(t) \tag{10.23}$$

as was to be shown.

Chapter 11

Chapter Eleven: Earth System and Energy System Feedbacks

A principal feature of the ecological economics world view, as discussed in chapter four, is that the economic system is a component or sub-system of the greater (and containing) physical universe. In less abstract terms this generally boils down to noting that the economy can be thought of as a subset of the biosphere in which (almost) all of its activities take place.[1] Or, borrowing the words of Herman Daly: "The economy, in its physical dimensions, is a subsystem of the earth ecosystem" (Daly, 1993).[2] While this vision of the economy as a subset of the earth-system is a compelling conceptual starting point for the study of ecological economics it also introduces a number of issues outside the scope of conventional economic analysis. Put another way, this opens a veritable Pandora's box of issues for the ecological economist desiring to undertake analytical work.

First, while the notion of the economic system being a subset of the greater earth-system makes good intuitive sense, what does this mean in practice? Does it follow from this that macroeconomic models (the formal representations of the economic system) should also be subset of earth-system models (the formal representation of the earth-system) and what, if anything, does this mean in practical modelling terms? Second, the reader will recall in chapter three that a source of criticism for the neoclassical IAMs was due to their oversimplified nature and their sparse representations of the earth-system. Ecological economists, who recognize the impor-

[1]We use subset here in a colloquial fashion.

[2]Daly makes an intriguing point here in specifically stating the economy in its "physical dimensions" is a what constitutes the subset of the earth-system. One then might inquire if there is some other dimension of the thing called "the economy" that is not so contained? There are likely many answers to this question (ranging from spiritual to scientific) that one might provide depending on one's world view, but we shall note here one rather grand example. Cosmologists and astronomers quite literally work at the level of the entire cosmos both temporally and spatially. A telescope is certainly an object produced in an economy and yet it can be used to "see" radiation only now reaching the earth from the early universe. We must be careful as Daly was in his writing, when dealing with the notion of human activities as being constrained by the earth-system.

tance in understanding the economy as an embedded subsystem, face much the same problem. How should economy and environment be modelled together formally and which parts of the immensely complex earth-system should be included in formal models.

In this chapter we will not present any new SFCIO model but present several physical models that will be integrated into the SFCIO framework to produce the SFCIO-IAM model of the following chapter. We begin with a brief survey of the basic energy balance nature of the earth-system and introduce the notion of radiative forcings and climate sensitivities. We then will present the core models making up the earth-system component of the model. Finally, we will introduce the methods to make the renewable and fossil-fuel sectors endogenous to the model via accounting for energy storage costs and the increased energetic requirements of production arising from depletion.

11.1 Basic Notions

Climate models range enormously in complexity from the simplest model governed by a few equations (the one constructed in this section) to coupled ocean-atmospheric models that cover all relevant biogeophysical processes called coupled general circulation models Goosse (2015). While the more complex models capture a greater number of phenomena, they come at the cost of computational complexity with the largest models requiring supercomputer time to solve in anything approaching a reasonable time span. Evidently, such models are not reasonable to use in most IAM models. Historically, some IAMs have utilized much simpler "energy balance" models consisting of only a few equations (Calel and Stainforth, 2017) and simplified representations of the global carbon cycle to model the changes in atmospheric concentrations of CO_2 arising from anthropogenic emissions. We begin this chapter with an overview of arguably the most basic energy balance model in order to note both the utility of relatively simple models and to introduce several key concepts.[3] The "climate model" to be used in this dissertation is an example of an energy balance model which, in the hierarchy of climate models, ranks as the simplest. Before turning to this model however it is worth covering the basics of what is meant by an energy balance model in the first place.

[3]The following section is based on Goosse (2015) chapter two.

First, virtually all of the energy entering the climate system is in the form of solar (shortwave) radiation and this energy entering the climate system must logically be balanced by an equivalent loss of energy lest large shifts in temperature occur. Working from this premise alone, and invoking a few basic laws, it is possible to obtain equilibrium surface temperature estimates with and without the presence of the atmosphere. We shall do so by first considering the earth as a black body object and then considering the impact of the addition of an atmospheric layer.

Let us begin with the statement of the fundamentally important Stefan-Boltzmann law which relates the total energy radiated by a black body surface per unit time area per unit time to the fourth power of its temperature.

$$A = \sigma T^4 \tag{11.1}$$

where $\sigma = 5.67 \cdot 10^8 W \cdot m^2 \cdot K^{-4}$ is the Stefan-Boltzmann constant. This expression gives the flux density of a blackbody with units of Watts per square meter, which if solved for temperature, gives the so-called effective emission temperature.

Now, since the time of Kepler it has been known that the earth's orbit around the sun takes the form of an ellipse. Therefore, considering the mean earth-sun distance it can be shown that the solar irradiance on a surface perpendicular to the incoming rays at the top of the atmosphere receives approximately 1360 Wm^{-2} (this is termed the total solar irradiance and denoted by S_0). Due to the spherical geometry of the earth, the average irradiance on the earth is equivalent to the total solar irradiance multiplied by the cross-sectional area of the earth. However, some fraction of the incoming solar energy is reflected away from the earth; this fraction is given by the earth's planetary albedo, denoted α_p. Therefore, the incoming solar radiation per unit time on the earth's cross-sectional surface is given by:

$$S_{planet} = \pi R_E^2 (1 - \alpha_p) S_0 \tag{11.2}$$

where R_E is the earth's radius. Assuming, for now, that the earth is a blackbody surface then it will emit at a wavelength proportional to its temperature.[4] Furthermore, assuming earth is a blackbody (and therefore emits according to (11.1) and emits radiation in all directions then,

[4]See Wien's law.

using the formula for the surface area of a sphere, earth's radiation per unit time is given as:

$$F_{planet} = 4\pi R_E^2 \sigma T_E^4 \tag{11.3}$$

For the earth to have a steady temperature it must be the case that incoming shortwave solar radiation is balanced by outgoing longwave radiation. Therefore, setting the previous two equations to be equal, we can solve for the blackbody emissions temperature of the earth.

$$\pi R_E^2 (1 - \alpha_p) S_0 = 4\pi R_E^2 \sigma T_E^4$$

implies that

$$\frac{1}{4}(1 - \alpha_p) S_0 = \sigma T_e^4$$

and therefore:

$$T_e = \left[\frac{1}{4\sigma}(1 - \alpha_p) S_0\right]^{1/4} \tag{11.4}$$

For, $\alpha_p = 0.3$ we find that $T_e = 255K = -18°C$. This is, quite obviously, colder than the average earth temperature. Using Wien's law we can compare the peak emissions wavelengths for the earth and sun.

The above calculation is for an earth modelled as a blackbody with no atmosphere. To understand the impact of the atmosphere on surface temperature the calculation we add a very simplified single layer atmosphere. Following Goosse (2015) one can postulate a single layered atmosphere that is completely transparent to incoming shortwave radiation and completely opaque to outgoing longwave radiation. Denoting the temperature of the atmosphere as T_a, it must be the case that in equilibrium the incoming solar radiation is just balanced by the outgoing radiation from the top of the single layered atmosphere.[5] Therefore as before we can write:

$$\sigma T_a^4 = \frac{1}{4}(1 - \alpha_p) S_0 \tag{11.5}$$

for the irradiance of the atmosphere. Now, the atmosphere will radiate in both directions (up and down) so the total radiation incident on the earth's surface is the sum of incoming shortwave solar radiation and that energy re-radiated from the atmosphere. Therefore, the

[5] Why? If this were not true, then the system would not be in equilibrium as assumed.

radiative balance at the earth's surface is given by:

$$\sigma T_s = \frac{1}{4}(1-\alpha_p)S_0 + \sigma T_a^4$$

which implies that:

$$T_s = 2^{1/4}T_e = 1.19T_e \approx 303K = 30°C$$

This is now higher than the observed mean temperature of $15°C$ as the atmosphere is represented very simplistically. However, even the simple single layer atmospheric model indicates quite strongly the greenhouse effect.

11.1.1 The Concept of Radiative Forcing

In the previous subsection we computed a simple energy balance model predicated on the principle that incoming shortwave solar energy must be balanced by outgoing longwave radiation from the earth. In this section we examine how the surface temperatures change as a result of the equilibrium between incoming and outgoing radiation changing, or how temperature changes as a result of different types of radiative forcing. The definition for the concept of a radiative forcing is given in the IPCC as:[6]

> [Radiative forcing] is the net change in the energy balance of the Earth system due to some imposed perturbation. It is usually expressed in watts per square meter averaged over a particular period of time and quantifies the energy imbalance that occurs when the imposed change takes place. (Rogelj et al, 2018) (p.664)

There is some subtlety to the notion which is worth exploring with a simple example. Suppose that the sun instantaneously increased in brightness, and therefore the flux of radiation at the top of the atmosphere increased. The immediate impact of such a change would be an imbalance between the downward flux of solar radiation and the upward flux of thermal radiation; as a result the surface temperature would eventually increase until the outgoing longwave flux balanced the larger incoming solar flux and equilibrium is restored.

Now, if in the process of the surface temperature rising a substantial fraction of the cryosphere (the part of the earth-system where water is in its solid form) were to undergo melting this would

[6] This section is based on chapter 4 of Goosse (2015).

lessen the planetary albedo. This change in albedo is itself a forcing. As such, depending on the timescale, the original solar forcing would induce a follow on albedo forcing. If we were only interested in determining the direct impact of the initial change in the solar radiation this follow on effect would complicate the analysis.

Different definitions for radiative forcing are used to account for the many different timescales on which the various components of the earth-system operate. The **instantaneous radiative forcing** can be understood as the instantaneous change in the downward flux at the top of the atmosphere excluding all follow on responses of the system. **Radiative forcing** (RF) can be defined as the change in the net flux in the troposphere (bottom layer of the atmosphere) after allowing the stratospheric (atmosphere's second layer) temperatures to adjust, all the while holding surface temperatures and other state variables in the troposphere constant. A third concept is the **Effective Radiative Forcing** whereby surface temperatures are held constant whilst allowing more rapidly equilibrating atmospheric processes to readjust. For the scope of this chapter the precise use of the three terms is unimportant and only a generally term radiative forcing (RF) will be used from this point. It is worth recalling that a radiative forcing is measured in Wm^{-2} and represents the change in net downward flux.

The most important radiative forces and their estimated magnitudes are discussed by (Myhre et al, 2013). The radiative forcing due to anthropogenic emissions of CO_2 since pre-industrial times corresponds to a radiative forcing of approximately 1.8 Wm^{-2} and a greenhouse gas radiative forcing of approximately 2.8 $W\ m^{-2}$. The radiative forcing due to the presence of aerosols in the atmosphere from anthropogenic activity is approximately $-0.9Wm^{-2}$; this negative forcing partially offsets the positive forcing due to greenhouse gasses. An interesting problem lies in the fact that the burning of fossil-fuels and biomass produces aerosols which enter the atmosphere. As the lifetime of aerosols is on the order of only a few days, and the lifetime of CO_2 in the atmosphere is several orders of magnitude longer Archer et al, (2009), an immediate cessation of fossil-fuel and biomass combustion could potentially lead to the rapid decline in the negative aerosol forcing and lead to more rapid warming; there is enormous uncertainty concerning this outcome however.

Simple and useful approximations for the magnitude of the radiative forcing associated with the changing in atmospheric concentrations of greenhouse gasses are provided in (Myhre et al, 1998); the equation for CO_2 is discussed here. Letting $[CO_2]$ and and $[CO_2]_r$ denote current and reference period atmospheric concentrations of CO_2, one can express the radiative forcing due to a change in concentrations as:[7]

$$RF = 5.35 \ln \left(\frac{[CO_2]}{[CO_2]_r} \right) \tag{11.6}$$

A doubling of pre-industrial CO_2 concentration of approximately 280 ppm to 560 ppm would result in a radiative forcing of:

$$RF = 5.35 \ln(\frac{560}{280}) \approx 3.7 \ Wm^{-2}$$

This simple estimate is line with that of the IPCC (Myhre et al, 2017).

Using (11.6), an approximate value for the radiative forcing due to the change in the concentration of CO_2 can be determined. A critical question in the study of climate change is to what extent such a forcing will change the earth's surface temperature. In this section a relatively simple framework will be derived to study the relation between temperature change and radiative forcing.[8]

The sensitivity of the surface temperature to the radiative forcing is called the climate sensitivity, here denoted as λ with:

$$\lambda = \frac{dT_s}{dRF} \tag{11.7}$$

where T_s is the surface temperature and RF is the radiative forcing term. As noted before, a positive (or negative) radiative forcing will have both immediate and knock-on effects which would over time affect the value of the climate sensitivity. For simplicity, first consider a perfect blackbody earth. The climate sensitivity is given simply as:

$$\lambda_{bb} = \frac{dT_s}{dRF} = \frac{dT_e}{dRF}$$

where T_e is the black-body temperature from (11.4). By (11.1) we know that:

$$T_e = \left(\frac{F_{planet}}{\sigma} \right)^{1/4}$$

[7]A variation of equation (11.6) will play a key role in the dynamic energy balance model to be developed later in the chapter.

[8]This section is based primarily on chapter 10 of Wallace and Hobbs, (2006).

This expression can be used to show that:

$$\lambda_0 = \frac{T_e}{dF_{planet}} = 0.266\ \frac{K}{(Wm^{-2})}$$

From this formula the temperature of a black-body planet increases by one degree Kelvin for approximately each additional downward flux of 3.76 W m^{-2}. [9] As the earth is not a blackbody, the climate sensitivity parameter will depend on many other processes. A general expression for the parameter is given as:

$$\lambda = \lambda_0 + \sum_i \frac{\partial T_s}{\partial y_i} \cdot \frac{dy_i}{dRF} \tag{11.8}$$

where the second term (arising from use of the chain-rule) denotes the partial effects due to other processes (change in albedo, change in greenhouse gas concentrations, etc.). For a climate sensitivity parameter of 0.70 (Wallace and Hobbs, (2006) p. 446) we can estimate the change in temperature associated with the doubling of atmospheric CO_2 in the previous section using the following equation due to IPCC, (2001).

$$\Delta T_s = \lambda \cdot RF = 0.70 \cdot 3.7K \approx 2.6K$$

or an increase of approximately 2.6 degrees Kelvin.

11.2 A Two-Component Energy Balance Model

In the previous section we worked from what might be termed a static energy balance model to derive basic estimates of the earth's surface temperature with and without an atmosphere. While that model was useful to illustrate the concept of energy balance it lacks the dynamics necessary to incorporate into an integrated assessment model. What we require is a model that is able to translate increases in atmospheric concentrations of CO_2 from emissions into changes in global surface temperature. In this section we present just such a dynamic model which links the change in global mean-surface temperatures and deep ocean temperatures with changes in the radiative forcing. This model is the two component energy balance model of (Geoffroy et al, 2013).

[9] The exact calculation is shown on page 445 of Wallace and Hobbs, (2006).

This model, as discussed in (Calel and Stainforth, 2017), is the continuous-time version of the model used in Nordhaus' DICE framework. The other well known IAM models, FUND and PAGE employ a discrete time variation of a simpler single component energy-balance model found in Andrews and Allen (2008) In this section we will present the basics of this two component energy-balance (hereafter 2-EBM) model as originally presented in continuous time by the authors. A complete description of the model and an analysis of its properties and dynamics can be found in the original paper. The two principle model equations that govern the evolution of the global mean surface air temperature T and the temperature of the deep ocean T_0 are given as:

$$\mathcal{C}\frac{dT}{dt} = \mathcal{F}(t) - \lambda T - \gamma(T - T_0) \tag{11.9}$$

$$\mathcal{C}_0\frac{dT_0}{dt} = \gamma(T - T_0) \tag{11.10}$$

where $\mathcal{F}(t)$ is a radiative forcing term that will be the link with the BEAM carbon-cycle model introduced in the following section; C and C_0 are effective heat capacities per unit area; λ is a radiative feedback parameter; and γ is a heat exchange coefficient.[10] The formulation above implies that the heat transfer between the surface and deep ocean is proportional to the difference in the two temperatures.

For given values for the parameters $\mathcal{C}$,$\mathcal{C}_0$,λ, and γ, the systems behaviour is determined by the radiative forcing parameter $\mathcal{F}(t)$. This parameter is linked to the atmospheric concentration of $\mathrm{CO_2}$ which is an element of the BEAM model which is in turn ultimately linked to the emissions arising from the usage of fossil-fuels. The authors use the following expression, originally from (Houghton et al, 1990), to specify the radiative forcing term as a function of the changing ratio of atmospheric $\mathrm{CO_2}$ concentration from a given reference period concentration:

$$\mathcal{F}(t) = \frac{\mathcal{F}_{2x\mathrm{CO_2}}}{\ln(2)} \ln\left(\frac{[CO_2]_t}{[CO_2]_0}\right) \tag{11.11}$$

where $\mathcal{F}_{2x\mathrm{CO_2}}$ is the radiative forcing associated with a doubling of atmospheric $\mathrm{CO_2}$ concentration.

In plain language, the model presented above does the following. Given some radiative forcing $\mathcal{F}(t)$ the two coupled differential equations capture the change in temperature of the surface

[10] A heat capacity is defined as the amount of heat energy necessary to change the temperature of a substance by one degree and has units of joules per kelvin.

and deep ocean components respectively. Like all other differential equations introduced in this dissertation, if we "know" the radiative forcing term and have initial conditions for the two temperatures then it is possible to solve for the future temperature trajectories of both components.

As specified above, the radiative forcing term is a function of the ratio of present day atmospheric CO_2 with a reference period concentration from before the industrial revolution. In order to obtain this term, we need another model to capture the impacts of emissions on atmospheric concentrations; that is, we need a carbon cycle model. In the following section we introduce a non-linear model that improves on the conventional model used in the DICE model framework which will provide the link between anthropogenic emissions and the atmospheric concentration of CO_2.

11.3 The BEAM Carbon Cycle Model

In previous SFCIO models we introduced emissions arising from energy sector activities in order to generate cumulative emissions pathways; however, once produced these emissions played no further role in the modelling framework. In reality, emissions from industrial activities enter geophysical process known as the carbon cycle of which the atmospheric reservoir is of particular importance. In this section we introduce the non-linear Bolin and Eriksson Adjusted Model (BEAM) carbon cycle model of (Glotter et al, 2013) in order to link the emissions produced by economic activities to atmospheric concentrations while also capturing some of the key carbon-cycle processes. As before we present the basic aspects of the model as presented by the authors and note that a complete description can be found in the original paper.

The authors motivate this model for usage in IAMs by noting that the linearized model utilized in the DICE model over-predicts CO_2 uptake in the oceans by omitting the changes in ocean carbonate chemistry. This mechanism leads to over-estimating the CO_2 uptake in the oceans and consequently underestimating the quantity of CO_2 remaining in the atmosphere. While the BEAM model is marginally more complex than the more traditional linearized model employed in the models of (Nordhaus, 2017), (Dafermos et al, 2017), and (Bovari et al, 2018) for example its complexity is not so great as to omit on the grounds of tractability.

BEAM models an interlinked three reservoir carbon-cycle model, where M_{at}, M_{up}, and M_{lo} are the masses of inorganic carbon in the atmosphere, upper ocean, and lower ocean respectively.

A set of three coupled non-linear differential equations governs the evolution of each the stocks of carbon which are presented as follows:

$$\frac{dM_{at}}{dt} = E(t) - k_a(M_{at} - A \cdot BM_{up}) \tag{11.12}$$

$$\frac{dM_{up}}{dt} = k_a(M_{at} - A \cdot BM_{up}) - k_d\left(M_{up} - \frac{M_{lo}}{\delta}\right) \tag{11.13}$$

$$\frac{dM_{lo}}{dt} = k_d\left(M_{up} - \frac{M_{lo}}{\delta}\right) \tag{11.14}$$

where $E(t)$ is an emissions term, the parameters K_a and k_d are the inverse of exchange timescales between the atmosphere and upper ocean and the upper ocean and lower ocean respectively.[11] The parameter δ is the ratio of the volume of the upper ocean to the lower ocean. The parameter A denotes the ratio of the mass of CO2 in the atmosphere to the dissolved CO2 in the upper ocean and is given as:

$$A = k_H \cdot \frac{AM(\delta + 1)}{OM}$$

Here AM and OM denote the number of moles in the atmosphere and ocean.

The parameter B is the ratio of dissolved CO_2 to the total carbon in the ocean and is given by the authors as:

$$B = \frac{1}{1 + \frac{k_1}{[H^+]} + \frac{k_1 k_2}{[H^+]^2}}$$

where $[H^+]$ is the concentration of hydrogen ions and k_1 and k_2 are disassociation coefficients. Alkalinity (Alk) is give as:

$$Alk = \left(\frac{k_1}{[H^+]} + \frac{2k_1k_2}{[H^+]^2}\right) \cdot M_{up} \cdot B$$

The final equation from the paper that is necessary to complete the model is the following which can be used to determine $[H^+]$:

$$\frac{M_{up}}{Alk} = \frac{1 + \frac{k_1}{[H^+]} + \frac{k_1k_2}{[H^+]^2}}{\frac{k_1}{[H^+]} + \frac{2k_1k_2}{[H^+]^2}}$$

[11]These terms have a somewhat confusing name on first glance but can be understood by consideration of the dimensions. We know the LHS of (12.34) is a rate of change of mass with respect to time and therefore has dimensions of mass per unit time. This must be the case for the RHS as well. Since the bracketed term is in terms of mass it must be the case that the parameter K_a be in terms of per unit time, in this case the inverse of the exchange time between the reservoirs.

As the authors do not present an exact analytical solution for $[H^+]$ we undertake the derivation; the results of which give an expression for the concentration $[H^+]$ as:

$$[H^+] = 0.5\left[-(k_1 - k_1 \cdot M_{up} \cdot Alk^{-1}) \pm \sqrt{(k_1 - k_1 M_{up} \cdot Alk^{-1})^2 - 4 \cdot k_1 k_2 (1 - 2M_{up} \cdot Alk^{-1})}\right] \tag{11.15}$$

With this expression in hand we can determine the value of the B parameter and produce the set of BEAM differential equations in properly specified state-variable form as necessary.

Like the 2-EBM, the BEAM model in practice is just a set of coupled differential equations linking rates of change of the mass of carbon in the various reservoirs per unit time to anthropogenic emissions and transfers between the reservoirs themselves. Given some initial conditions for the magnitude of each reservoir, we can obtain the future magnitudes for any given emissions trajectory $E(t)$. From this we can obtain the future trajectory of the mass of carbon in the atmosphere which can be used to relate back to the 2-EB model.

Following the approach of the DICE model (see (Nordhaus and Sztorc, 2013)) we can use the ratio of M_{at} (the present mass of atmospheric CO_2), and the mass in some reference period of time $M_{at}(0)$, in the radiative forcing function. Therefore, the equation that links the radiative forcing to the BEAM model and ultimately the fossil-fuel component of the ETM model can be rewritten as:

$$\mathcal{F}(t) = \frac{\mathcal{F}_{2x\mathrm{CO}_2}}{\ln(2)} \ln\left(\frac{M_{at}(t)}{M_{at}(0)}\right) \tag{11.16}$$

In this manner emissions $E(t)$ arising from economic activity are factored in the BEAM carbon cycle model which in turn provides the key atmospheric determinant for the forcing term in the 2-EBM climate model.

In combining the 2-EBM and BEAM carbon cycle models we have a relatively simple earth-system model that links emissions arising from anthropogenic activity to changes in the global mean surface temperature. In the following section we turn to the description of the fossil-fuel and renewable energy models that give rise to the emissions term $E(t)$ and ultimately provide the link between economic activities and the earth-system representation just derived.

11.4 Fossil-Fuel EROI Dynamics

A general empirical feature of the energy return on investments for the various types of fossil fuels is that they are declining (Hall et al, 2014). This finding was left unaddressed in the SFCIO-ETM model of the previous chapter to keep the model both simple and "linear". This subsection will introduce a net energy based formulation to tie the EROI of fossil-fuels to the magnitude of the remaining stock of fossil-fuels.

In order to capture the dynamic of fossil-fuel EROI declining with extraction we denote a stock of extractable fossil-fuels as $S(t)$. A simple formulation of EROI declining with extraction is given as follows:

$$EROI(t) = \frac{EROI(0) \cdot S(t)}{S(0)} \tag{11.17}$$

where $EROI(0)$ and $S(0)$ are initial conditions for EROI and the stock of fossil-fuels respectively.[12] By equation (11.17) the EROI of fossil-fuels is modelled to decline linearly with the decline in the stock of fossil-fuels which, as will be shown, implies a non-linear depletion trajectory for the stock itself. Now, to relate this back to the input-output structure of the model we recall that the EROI (of extraction) is simply the reciprocal of the fossil-fuel sector technical coefficient $a_{22}(t)$:

$$a_{22}(t) = \frac{1}{EROI(t)} \tag{11.18}$$

Substituting (11.17) into (11.18) we obtain the following expression which relates the technical coefficient $a_{22}(t)$ to the decline in EROI as modelled by the depletion of the underlying stock $S(t)$:

$$a_{22}(t) = \frac{S(0)}{EROI(0) \cdot S(t)} \tag{11.19}$$

Finally, since EROI is by construction a dimensionless ratio, and the stock of fossil-fuels appears in both the numerator and denominator, the resulting expression for $a_{22}(t)$ is dimensionless as required.

The EROI of fossil-fuels and depletion of the stock $S(t)$ form a positive feedback loop. To supply a given magnitude of energy return ER requires extraction of fossil-fuels which depletes the stock and lowers the EROI. This declining EROI implies that to continue supplying just some constant ER requires progressively greater energy investment EI (the energy return portion of

[12] Relating this back to the parametrization chapter 9, this initial condition $EROI(0) = EROI_M^F$.

EROI is unchanged so the denominator must be increasing).[13] This greater energy investment must itself be met by the consumption of additional fossil-fuels and hence greater extraction and so on.

The EROI-depletion dynamics can be explored using the concepts of net and total energy where net energy is defined as the energy return ER minus the energy invested EI and total energy is defined as $ER + EI$. Now, for a given ER what is the EI? From equation (11.17) we have that:

$$\frac{EROI(0) \cdot S(t)}{S(0)} = \frac{ER}{EI(t)}$$

solving for EI we have:

$$EI(t) = \frac{ER * S(0)}{EROI(0) \cdot S(t)}$$

Total energy is therefore

$$TE = ER + \frac{ER * S(0)}{EROI(0) \cdot S(t)}$$

or in more compact form

$$TE = ER\left(1 + \frac{S(0)}{EROI(0) \cdot S(t)}\right) \tag{11.20}$$

This last expression will be used in specifying the differential equation for $S(t)$ but it should be noted that, by substituting in equation (11.17), the equation becomes:

$$TE = ER\left(1 + \frac{1}{EROI(t)}\right) \tag{11.21}$$

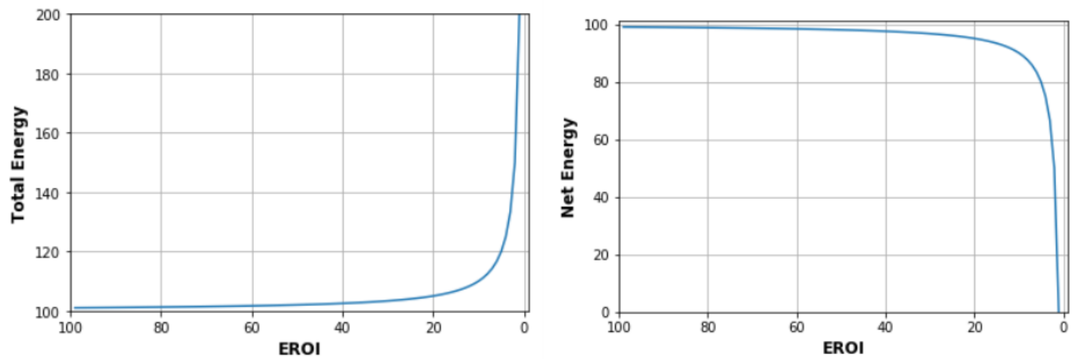
Which displays the non-linear behaviour of the total energy requirements. Similarly, we may write the expression for net energy as:

$$NE = ER\left(1 - \frac{1}{EROI(t)}\right) \tag{11.22}$$

Equations (11.21) and (11.22) indicate fundamentally important behaviour. To supply some constant energy return ER it is clear that the quantity of total energy will increase non-linearly as $EROI(t)$ declines and that the net energy available will decline non-linearly; the latter decline

[13] Why do we model EROI as declining with extraction? As presented in (Hall et al, 2014) the EROI of various types of fossil fuels has been declining linearly over the last several decades. The simple logic as to why is that as the easiest to extract sources of fossil fuel are depleted it becomes necessary to exploit more energetically expensive productions processes. Fracking, deep sea extraction, tar sands are all good examples of low EROI oil extraction technologies that have come into play.

has been termed the EROI cliff by Euan Mearns. The following figures plot TE and NE for a constant $ER = 100$ for a range of EROI values.



(a)

(b)

Figure 11.1: Total and net energy as a function of declining EROI.

Panel (a) of Figure 11.1 shows that as EROI declines from 100, the total energy necessary to deliver an energy return of 100 increases only very slowly and then increases dramatically for EROI values lower than 10. Similarly, panel (b) shows that the net energy available decreases in a symmetric fashion

Now, to deliver some given energy ER requires some total energy TE of overall energy use (energy invested plus energy returned). As such the depletion of the stock of fossil-fuels $S(t)$ is proportional to the total energy TE. Let us, for the sake of this derivation, assume that the stock of fossil-fuels is measured in tonnes of coal. Then, given some conversion factor τ which measures the joules per tonne of coal, producing TE energy requires $\frac{1}{\tau}TE$ tonnes of coal.

The above logic states simply that to provide some quantity of energy (Joules) will require the usage of some quantity of a primary energy source (in our example coal); the above states nothing about the rate at which this occurs. Ultimately, we require a differential equation governing the depletion of the fossil-fuel stock and to obtain that we must also specify the rate at which total energy is consumed.

Suppose the energy return ER is consumed every second, then, by the above, TE energy per second is the total energy that must be used per second to obtain and deliver this energy return. More simply, this implies a rate of energy consumption of TE per second or TE Watts which gives total power which we denote TP. Now, in some infinitesimal time interval dt we have that $TE \cdot dt$ [Ws] energy is consumed, and therefore (recalling the conversion factor τ) the differential change in the stock of fossil-fuels dS is:

$$dS = -\frac{1}{\tau} TP \cdot dt$$

which is negative to reflect that the stock of fossil-fuels is being depleted. Finally, we obtain the differential equation for the depletion of the stock of fossil-fuels by dividing both sides of the above expression by dt:

$$\frac{dS}{dt} = -\frac{1}{\tau} TP \tag{11.23}$$

which relates the depletion per unit time of the stock of fossil-fuels with the total power requirement. We may take this one step further by substituting in equation (11.22) which leads to:

$$\frac{dS}{dt} = -\tau PR \left(1 + \frac{S(0)}{EROI(0) \cdot S(t)}\right) \tag{11.24}$$

where PR is the power return which is simply the energy return ER per unit time. As noted at the outset of these calculations the differential equation governing the depletion of the stock of fossil-fuels is non-linear. Figure 11.2 displays the trajectory of the stock of fossil-fuels for assumed values of initial EROI, $S(0)$, and a constant energy return of 10. Note well that a small value of EROI is chosen to more clearly indicate the non-linearity. Panel (a) shows, as expected, that to supply a constant energy return under declining EROI implies increasingly rapid depletion. Panel (b) indicates that as the stock is depleted the EROI declines commensurately.

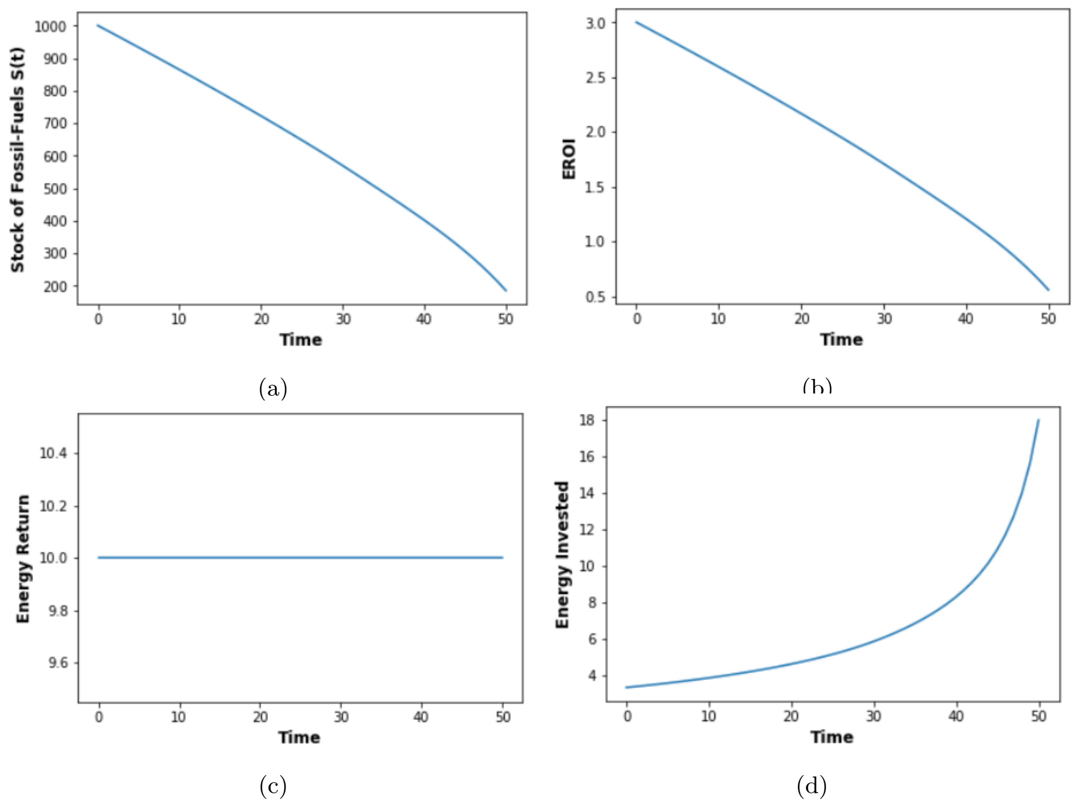


Figure 11.2: Assuming $S(0) = 1000$, $EROI(0) = 3$, and, $ER = 10$. **(a)** Depletion path for the stock of fossil-fuels $S(t)$ **(b)** EROI trajectory due to depletion of the stock of fossil-fuels $S(t)$. **(c)**Constant Energy Return. **(d)** Trajectory of energy invested necessary to the constant energy return.

Panel (c) displays the energy return ER which is assumed constant over time. Finally, panel (d) shows that the energy invested necessary to provide the constant energy return grows in an exponential manner as EROI declines.

Now, if we assume simply that emissions are proportional to total fossil-fuel usage then the emissions overtime, arising from the provisioning of a constant energy return, are plotted in the following figure.

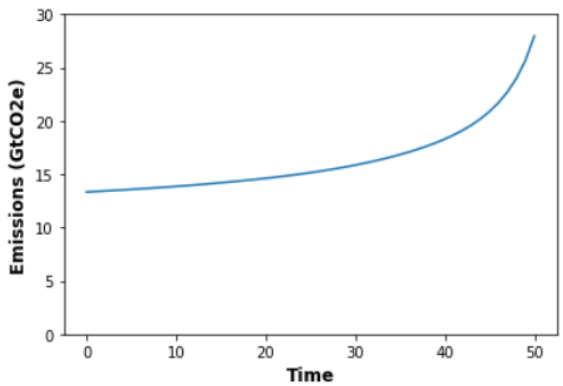


Figure 11.3: Trajectory of emissions $E(t)$ for $E(t) = \zeta TP(t)$ where $\zeta = 1$ is a constant of proportionality set to one for simplicity and assumed values of initial $EROI = 3$, $S(0) = 1000$, and $ER(t) = 10$.

Figure 11.3 displays a critically important result; as the EROI of fossil fuels decline, the emissions associated with providing some given energy return increase non-linearly. Now, the above figures all use a relatively low EROI value of 3 in order to better display the non-linear behaviour. For more realistically large values of EROI the same dynamics exist and can be clearly seen by compressing the y-axis. The trajectory of emissions for the same process described above except with a more realistic EROI value of 20 is shown below.

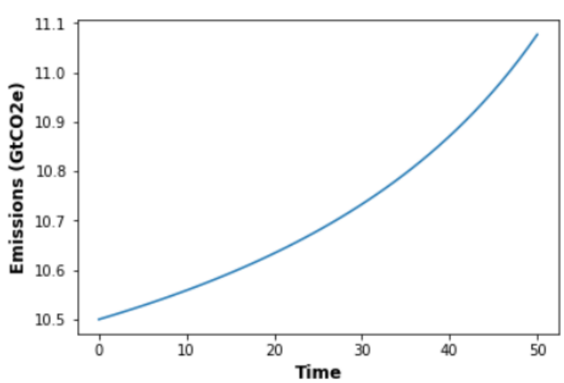


Figure 11.4: Trajectory of emissions $E(t)$ for $E(t) = \zeta TE(t)$ where $\zeta = 1$ is a constant of proportionality set to one for simplicity and assumed values of initial $EROI = 20$, $S(0) = 1000$, and $ER(t) = 10$.

Finally, we may link the above work back to the SFCIO energy transition framework by replacing the total power TP term with the total fossil-fuel power requirements given by $X^{FF}(t)P_f^{-1}$. By construction this quantity is measured in Watts so we may write the fundamental equation relating the depletion in some stock of fossil fuels to a key SFCIO model component as:

$$\frac{dS}{dt} = \frac{1}{\tau}\frac{X^{FF}(t)}{P_f} \tag{11.25}$$

11.5 Renewable EROI Dynamics

The transition to large-scale renewable based electricity generation is potentially complicated by the storage requirements arising from the inherent variability underlying solar and wind as energy sources. This variability requires energy storage to smooth the flow of energy produced, however, as this storage is itself energetically costly to produce. To incorporate aspects of this issue we turn to the grid storage EROI model of (Barnhart et al, 2013). This section provides a surface overview of the model of which the full details can be found in the publication and the supplementary material. First, the author's use the concept of energy storage on investment (ESOI) from a previous study (Barnhart and Benson, 2013). This ESOI is defined as:[14]f

$$ESOI_e = \frac{\lambda \eta D}{\varepsilon_e} \tag{11.26}$$

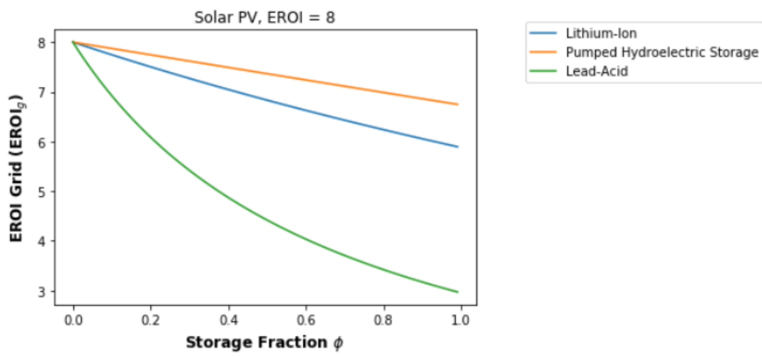
where λ is the number of charge-discharge cycles of the battery (cycle-life); η is the round-trip AC–AC efficiency, D is the depth of discharge of the battery, and ε_e is the embodied electrical energy per unit of electrical storage capacity.

Some fraction ϕ of the energy produced by a variable renewable must be either stored for later usage or curtailed. Therefore, for each unit of energy produced ϕ enters storage and $\eta\phi$ is taken from storage. Of the one unit of energy produced, $1 - \phi + \eta\phi$ units of energy are available to society after accounting for storage losses. The authors show that the energy costs to produce this unit of energy return is given as $\frac{1}{EROI_R} + \frac{\eta\phi}{ESOI_e}$. They define the EROI at the "grid" level as:

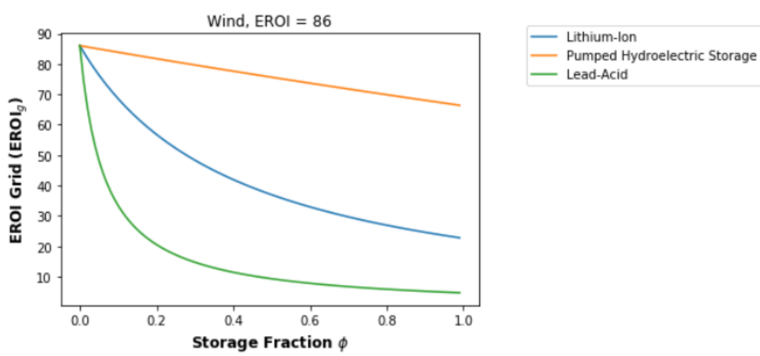
$$EROI_g = \frac{1 - \phi + \eta\phi}{\frac{1}{EROI_R} + \frac{\eta\phi}{ESOI_e}} \tag{11.27}$$

The following figure, using the authors data, plots $EROI_g$ as a function of increasing storage ϕ for solar and wind under three different storage technologies. Figure 11.5 shows that the EROI at the grid level declines as a function of increasing electrical energy storage. That the transition of economies to renewable energy may imply dramatically larger storage requirements implies that

[14]The range of ESOI values for different types of storage is quite large. In (Barnhart and Benson, 2013) a value of 5 is reported for lead acid batteries while a value of 797 is reported for compressed air energy storage.



(a)



(b)

Figure 11.5: EROI grid values for solar PV and wind for three different energy storage technologies. EROI values chosen to match those found in (Barnhart et al, 2013).

over the course of such a transition the storage fraction ϕ would increase and consequently, that the grid level EROI would decline. This is arguably a dynamic of key importance in modelling the energy transition and will therefore be added to the basic SFCIO-ETM framework.

To ensure our energy accounting is correct, we need to obtain a "grid corrected" power output parameter ϕ_g that, when applied to the renewable capacity constructed from one unit of energy invested, matches the life-cycle grid-level energy return EROI_g. Assuming for simplicity the same depreciation rate σ_R, we may obtain ϕ_g is the same manner as we did ϕ_R in chapter 9 by solving the following for the ϕ_g term:

$$\int_0^\infty \phi_g \cdot e^{-\sigma_R \cdot t} \cdot dt = EROI_g$$

which leads to

$$\phi_g = \left(\frac{1 - \phi + \eta\phi}{\frac{1}{EROI_R} + \frac{\eta\phi}{ESOI_e}} \right) \sigma_R$$

Recalling from equation (9.1) that $\sigma_R = \frac{\phi_R}{EROI_R}$ we have:

$$\phi_g = \left(\frac{1 - \phi + \eta\phi}{\frac{1}{EROI_R} + \frac{\eta\phi}{ESOI_e}} \right) \frac{\phi_R}{EROI_R}$$

Of course, the bracketed term is simply the definition of $EROI_g$ so the entire expression may be simplified in a more intuitive (though less physically informative) form as:

$$\phi_g = \phi_R \left(\frac{EROI_g}{EROI_R} \right) \tag{11.28}$$

This last formulation is particularly appealing as it states that the power output when corrected for grid losses is simply the power output of renewables ϕ_R scaled by the ratio of the grid scale EROI to the EROI of renewable not accounting for storage costs. It follows from this that for any storage fraction ϕ greater than zero that the grid level EROI will be lower than the EROI of the renewable considered directly; therefore, the power output per unit renewable capacity (at the grid level) will decline continuously as the storage fraction increases.

Finally, making this process dynamic requires that the storage fraction parameter ϕ be endogenous in the model. The assumptions for how this is done are left until the following chapter but the idea roughly stated is that storage fraction is an increasing function of the fraction of total electricity used in the model that is generated by renewables (renewable market penetration). This is informally shown as follows:

$$\phi(t) = f(\text{Renewable Market Penetration})$$

11.6 Earth-System Feedbacks, Damage Functions, and Mitigation Costs

Having specified a carbon cycle and energy balance model we have amassed most of the necessary physical components to extend the energy transition model to a proper IAM. One critically important and utterly controversial component remains, the nature of the feedback of the climate model into the macroeconomic framework. the following passage appearing in Pindyck's 2013 NBER working paper indicates quite bluntly the nature of the problem of the traditional method employed (damage functions):

> When assessing climate sensitivity, we at least have scientific results to rely on, and can argue coherently about the probability distribution that is most consistent with those results. When it comes to the damage function, however, we know almost nothing, so developers of IAMs can do little more than make up functional forms and corresponding parameter values. And that is pretty much what they have done. (Pindyck, 2013)

This criticism is perhaps somewhat alleviated now with more recent work, for example, estimating a global non-linear relationship between productivity and temperature (Burke et al, 2015). However, no matter how sophisticated the techniques, such estimates are naturally reliant on a data set that may not necessarily usefully describe the context of the 21st century. Indeed, temperature (which forms the basis of damage functions) is only one mechanism among a veritable host of possible biogeophysical mechanisms that will feedback into the functioning of economies. Indeed, the planetary boundaries framework (see (Rockstrom et al, 2009)) indicates that considerations, ranging from biodiversity loss to perturbations of the phosphorus and nitrogen cycles may be of critical importance.

Before proposing any formalism on the climate system feedback to be used in the model a frank discussion on the nature of the endeavor is necessary. To introduce this discussion, consider the very recent publication of a paper projecting high odds for the likelihood of civilization scale collapse due to resource (and especially forestry) related dynamics (Bologna and Aquino, 2020). Whether the paper's results are at all legitimate is both beyond the scope of this dissertation to address and not the purpose in raising the work in the first place. The point is, processes such as forest dynamics which are virtually non-existent in most macroeconomic modelling are hypothesized mechanisms (put forward in serious academic journals) for, as the authors put it, the "self-destruction of our civilisation".

More broadly, as discussed in chapter 1, there are a large number of different biogeophysical processes changing at alarming rates due to anthropogenic perturbations of which even for the best understood effects (i.e., global average temperature) we have no real understanding of how to relate back to the functioning of economies at the macroeconomic level. Given that there are currently existential changes unfolding at the earth-system level that range from the melting of ice-sheets to immense biodiversity loss, our state of ignorance on how these processes may impact us is the single most glaring problem in IAM modelling (heterodox or neoclassical). To be perfectly clear, we do not at present have any truly sound methodology for modelling the earth-system feedback into the economies, and using the crude methods outlined in the following section, may disastrously underestimate the severity of climate change and other environmental issues. It is also possible that the methods overestimate the importance of the feedback, though we consider this a lesser risk. The issue at hand is that it is simply very difficult to establish the validity of the methods.

There is another more subtle issue with how damages and costs are represented which can been seen in the damage function used by the DICE model of Nordhaus. The "standard" DICE production function is given as follows (Dietz and Stern, 2015):

$$Y_t = (1 - D_t)(1 - \Lambda_t)A_t K^{\alpha+\beta} L_t^{1-\alpha} \tag{11.29}$$

Here the D_t is the climate damage term, which by inspection is seen to reduce output Y_t the larger its value becomes.[15] A second similar looking term Λ_t appears which represents an abatement or mitigation costs associated with emissions reductions. These two elements are

[15] Recall that this term is a dimensionless number that gives reduces output by some percentage from what it would have been if there were no warming.

worth considering briefly as their treatment in the neoclassical model is at odds with how we will introduce the climate feedback. First, the damage function in DICE is expressed in the Dietz and Stern study as:

$$D(t) = 1 - \frac{1}{(1 + \pi_1 T(t) + \pi_2 T(t)^2)} \tag{11.30}$$

where the π are parameters calibrated from data and T is the global average temperature anomaly.[16] Now, examining (11.30) it is clear that when the global average temperature anomaly is zero the damage parameter $D(t)$ is equivalent to zero, and hence in (11.29) there are no damages and output is not reduced from its "idealized" level. As argued by economist Martin Weitzman, this quadratic damage formulation was insufficient to capture "high-temperature" damages and the following modified form, calibrated so that $D(t) = 0.5$ at $T = 6°$C, was proposed instead (Weitzman, 2012)

$$D(t) = 1 - \frac{1}{(1 + \pi_1 T(t) + \pi_2 T(t)^2) + \pi_3 T(t)^{0.6754}}$$

In both cases these climate damage functions give the percentage reduction in output from what it would have been without warming. These percentage damages on output under both damage functions are shown in in the following figure as a function of the global average temperature anomaly.

[16]Here T is just the increase in global average surface temperatures over the reference period.

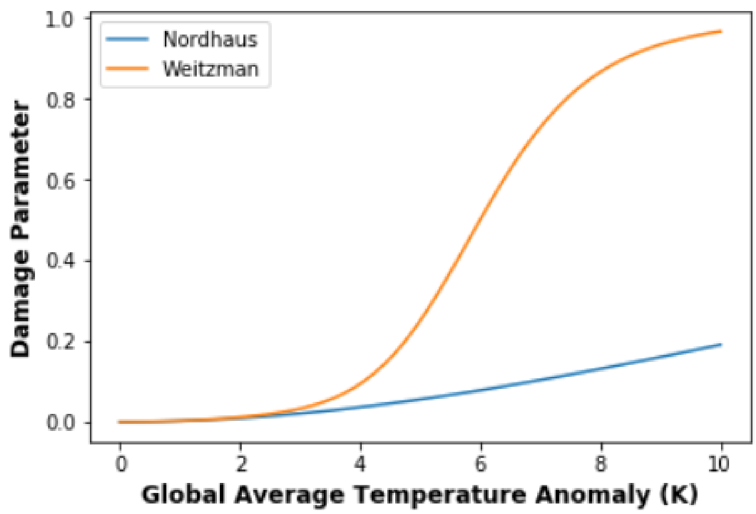


Figure 11.6: The damage parameter as a function of global average temperature changes for the Nordhaus and Weitzman damage formulations. Data for the functional form parameters were obtained from (Dietz and Stern, 2015).

There are several salient aspects of the above figure. First, both damage functions (over the range of possible temperature changes) present utterly different perspectives about the assumed magnitude of climate damages. At 6 degrees warming the Weitzman function indicates a 50% reduction in output (in fact it was calibrated to equal this number) and the Nordhaus specification produces only approximately a 7.7% reduction. Under the Weitzman specification output is virtually annihilated at 10 degrees and only declining by less than 20% in the Nordhaus specification. Second, the damages modelled for the upper range of temperature increases is just basically guesswork; there is obviously no data underlying the upper range estimates given that such temperature regimes have not coincided with human civilization. The Weitzman calibration is intended to achieve the result that output is halved at 6 degrees, not, to be clear, that the data suggest that such outcome is reasonable.

Little need be said concerning the fact that the Nordhaus specification implies only relatively modest output losses at 10 degrees warming. Such warming if it occurred on the time scales relevant to IAMs is essentially meaningless to discuss. Indeed, the Weitzman assumption of 50% loss at 6 degrees is also likely not meaningful. The set of biogeophysical changes that would result from such temperature increases, and the impacts on societies, is arguably impossible to discuss in any manner aside from simple speculation.

A second curiosity emerges from the above discussion which is the role of mitigation or abatement costs. Abatement activities of virtually any description imaginable must ultimately, in a sensibly specified model, be accounted for by the activities of some sector. For example, if we postulate a firm constructing atmospheric CO_2 removal capacity, this capacity must still be built, drawing on inputs from other sectors, and showing up as an investment expenditure. To have abatement activities unilaterally and directly reduce output as in the first formulation requires some rather odd accounting gymnastics. Now, a plausible mechanism might be that so much of the economy's productive capacity is redirected towards some specific abatement activity that there is some fundamental lack of resources for other sectors leading to declining investment and ultimately declines in physical capacity. However, such an indirect mechanism is not accounted for in the DICE formulation. These issues are discussed at length in (Pollitt and Mercure, 2018).[17]

As with the model of (Dafermos et al, 2017) we will specify climate damages in a manner that impacts both the stock of built capital directly (via enhanced depreciation) and also impact the productivities of these stocks. While working these two impact types into the SFCIO framework ameliorates some of the issues discussed above concerning the incoherence of impacts in an macroeconomic framework, it does little to address the ultimately more complex problem of the arbitrariness of damage functions; the many environmental problems ignored by the formulation; and the issue that such formulations likely fail to capture the true nature of the climate (including temperature) related impacts on the functioning of economies.

Now, the damage functions above are in terms of damages to output as a whole which does not translate meaningfully to the SFCIO framework as there is no aggregate production function used in the models. Rather, we will utilize the damage functions to modify the physical outputs of each sector via a combination of damages to the built capital stock and declines in productivity. The productivity of the capital stocks is simply the relevant output per unit capital parameters (e.g., ϕ_R the power output per unit renewable capacity). Let us temporarily assume a generic sector with physical capital $k_i(t)$, and productivity parameter $\phi_i(T)$ where the (T) argument indicates the parameter is some function of the global mean surface temperature

[17]A model that does address these issues is the LOWGROW SFC model with its explicit consideration of whether green investment is 'productive' or 'unproductive' and 'additional' or non-additional' (see (Jackson and Victor, 2020)).

T. The instantaneous physical output $y_i(t)$ of the sector is given per usual as:

$$y_t = \phi_i(T)k_i(t) \tag{11.31}$$

Furthermore, let us assume that in addition to the capital stock's standard depreciation rate σ_i there are some additional losses due to temperature-related effects. Much like ordinary depreciation, the magnitude of these losses are assumed proportional to the existing stock of physical capital; denoting the additional temperature dependent losses as $\sigma(T)$ we may write the equation governing the evolution of the capital stock as:

$$\frac{dk_i}{dt} = i_i(t) - \sigma_i k^i(t) - \sigma(T)k_i(t)$$

where $i_i(t)$ denotes the investment in physical terms. Now, if the total additional losses from temperature related depreciation is $\sigma(T)k_i(t)$, then physical output declines by $\phi_i\sigma(T)k_i(t)$ over what the sector could theoretically produce in the absence of these damages. Concerning the productivity or output parameter ϕ_i, we assume that temperature related effects decrease the parameter from its impacted magnitude (at $T = 0$) to some new magnitude which is determined in the following discussion.

Now, we must be somewhat careful in how we reconcile this with the damage functions specified above. The operational assumption here is that the magnitude of the output losses associated with the damage functions is the magnitude that will be used for the decline in the physical output (shared across enhanced depreciation and productivity effects). For example, if $T = 6$, the Weitzman damage specification implies 50 percent output losses which, in this approach, would translate to y_t being 50% smaller than it would have been under no temperature increases. This could be accomplished by assuming that $\phi_i(T)$ is simply 50% smaller though this would imply no enhanced depreciation effects. The share of impacts (between productivity and depreciation effects) is, as noted in (Dietz and Stern, 2015), problematic as a precise disaggregation from a single aggregate output figure to damages shared between two different model components requires (to be precise) that the effects on the two model components are interrelated. [18]

[18] Why? Suppose total output declines by 50% and we assume simplistically first that the capital stock declines by 25% leading to an initial 25% decline in output. If we also assume that productivity declines by 25% we will quickly see that the cumulative effect is smaller that the 50% required. This arises from the nature of the

The calculation to assign shares here is identical in form to that in (Moyer et al, 2014) though their calculation concerns aggregate output and TFP of which neither are meaningful concepts in the current framework. Let us assume that climate damages are α percent of output of a given sector; that is if output with no damages is of magnitude $\phi_i k(t)$ then output accounting for damages must have the magnitude equal to $(1-\alpha)\phi_i k(t)$. Now, let us assume that a fraction β of these damages are in the form of physical losses to the capital stock. Then, after accounting only for these damages, the capital stock $k_i(t)$ would reduce in magnitude to $(1-\beta\alpha)k_i(t)$ and hence production, assuming temporarily no change in the productivity parameter would reduce to $\phi_i(1-\beta\alpha)k_i(t)$. It must now be the case that changes in the productivity parameter $\phi_i(T)$ account for the remaining $(1-\beta)$ fraction of damages. The change in the magnitude of $\phi_i(T)$ is determined by the following equation:

$$\phi_i \cdot x \cdot (1-\beta\alpha)k_i(t) = \phi_i(1-\alpha)k_i(t)$$

Here the x term determines the change in the magnitude of the undamaged productivity parameter ϕ_i that must occur to match the damages to the total losses after accounting for depreciation. Cancelling like terms and solving we have:

$$x = \frac{1-\alpha}{1-\beta\alpha}$$

Using this we can define the parameter $\phi_i(T)$ as:

$$\phi_i(T) = \phi_i\left(\cdot\frac{1-\alpha}{1-\beta\alpha}\right) \tag{11.32}$$

It is worth restating very carefully several points concerning this approach. What the above calculations accomplish is bringing in to the SFCIO framework a notion (damage functions) that is ultimately suspect. While the work here certainly succeeds in relating the model to other aspects of the broader literature it in no way ameliorates the problem that damage functions are very likely poor mechanisms to capture earth-system feedback in the models. That such functions are used now in both neoclassical and ecological macroeconomic models (see again (Dafermos et al, 2017) and (Bovari et al, 2018)) is interesting in the sense that there is some

arithmetic where by $0.25 \cdot 0.25 \cdot k_i(t) \neq 0.5 \cdot k_i(t)$

methodological cross-over but also indicates the underlying problem that superior methods may simply not exist for the task at the present time. The feedback component represents the weakest aspect of the broader endeavor of IAM modelling and indeed the modelling undertaken in this dissertation; addressing this problem is certainly well beyond the scope or purpose of this dissertation but we note that it is an area of absolutely critical importance for further research.

In this chapter, we have introduced two geophysical models (the BEAM and 2-EBM), presented formulations to make the renewable and fossil-fuels energy system endogenous, and presented a method to link temperature increases to damages to both built capital stocks and their productivities. In the following chapter we introduce each of these mechanics to the SFCIO framework in order to construct a full integrated assessment model for a single country.

Chapter 12

Chapter Twelve: A Stock-Flow Consistent Input-Output Integrated Assessment Model

We are now ready to extend the SFCIO-ETM to become a full integrated assessment model (IAM) incorporating the key energy system and geophysical dynamics introduced in the previous chapter. As each of the features being added to the model describe a physical feedback, the macroeconomic structure (though not necessarily the underlying equations) of the model to be developed in this chapter is identical in almost all respects to that of model SFCIO-ETM and therefore we refrain from representing the three model structure matrices as they may be found in section 10.2

At the conclusion of the SFCIO-ETM chapter we noted that, even under the models ideal assumptions, reducing emissions within a given carbon budget was immensely difficult and virtually impossible without enormous up-front investment and long-term degrowth. The purpose of this chapter is to investigate the same phenomena but under the more realistic conditions developed in the previous chapter.

Most of the scenarios that have been developed for exploring the possibility of keeping the increase in the average global temperature below 1.5 °C assume continued economic growth and a significant contribution from largely unproven negative-emissions technologies. This chapter expands the range of scenarios to be considered by allowing for steady-state and degrowth possibilities in the absence of negative-emissions out to 2050. The nature of the problem may be summarized as follows. Keeping global mean temperatures limited to below some given threshold will necessitate that CO_2 emissions reach net zero (Rogelj et al, 2018). Emissions pathways consistent with no or very limited overshoot of the 1.5 °C target requires declines in emissions (over 2010 levels) by approximately 45% by 2030 and reaching net zero emissions by approximately 2050 (Rogelj et al, 2018). The majority of modelled pathways assume both continual economic growth and the deployment of some magnitude of carbon dioxide removal (CDR). In this chapter we will examine how under explored economic trajectories in the integrated assessment modelling context (steady-state and degrowth) may play in producing emissions pathways consistent with a 1.5 °C budget without the assumption of negative emissions technologies.

To accomplish this we construct a novel integrated assessment model (IAM) to study the energy transition, in a stylized manner, under a carbon budget constraint. This links together the stock-flow consistent approach to macroeconomics, with a dynamic three sector input-output model. This stock-flow consistent input-output integrated assessment model (SFCIO-IAM) is designed to capture in a stylized manner both the transition of the energy system from predominantly fossil-fuel based to one built on renewables as well as the electrification of end use. The model's energy and production parameters are calibrated according to a life-cycle energy return on investment (EROI) approach capturing both the energetic impacts of depletion of fossil-fuels and the electrical storage costs associated with high penetration variable renewables. The model is coupled with the BEAM carbon cycle model (see (Glotter et al, 2013)) and a two layer energy balance model (see (Geoffroy et al, 2013)) to compute warming trajectories as well as obtain climate damages. Finally, though not a traditional concern in most economic modelling, the equations of the SFCIO-IAM model are dimensionally homogenous; that is, financial and macroeconomic components are integrated with energy and physical climate components in a consistent fashion.

The purpose of this chapter is therefore two-fold. First, it serves as an addition to the small but growing literature on merging stock-flow consistent macroeconomic modelling with input-output analysis. This relatively new approach can be found in the works of (Berg et al, 2015), (Jackson, 2018) and, (King, 2020). More broadly, the incorporation of the stock-flow consistent approach, as elaborated most fully in (Godley and Lavoie, 2007), with ecological macroeconomics is found in a growing body of literature; see for example (Jackson and Victor, 2015), (Jackson and Victor, 2020), (Bovari et al, 2018), and (Barrett, 2018). Second, it examines the joint dynamics of energy transitions combined with alternative economic pathways (steady-state and degrowth) in order to evaluate the role these may play in emissions reductions and how they may act to reduce the requirement for negative emissions that generally characterize 1.5- and 2-degree pathways.

12.0.1 Electrification and Storage

The large-scale electrification of end use coupled with the decarbonization of electricity production are two critical components of emissions reductions and present both significant real world engineering challenges as well as considerable modelling challenges.[1] Past large-scale studies (see (Jacobson et al, 2015)) have indicated the feasibility of 100% renewable systems and a major recent study, the Princeton Net-Zero America report (see (Larson et al, (2020)), concluded net-zero emissions could be achieved with an aggressive electrification of end use using 100% renewable energy by 2050. In this chapter we will take a simpler stylized approach by adapting the energy return on investment (EROI) and energy stored on invested (ESOI) approach suggested in (Barnhart et al, 2013) to relate the magnitude of electrical energy storage with the penetration of variable renewables via a storage fraction term $\phi(t)$ which denotes the fraction of variable renewable energy produced that must be stored at any given time. This will be used to obtain energy costs associated with storage.

As the proportion of variable renewables in the overall electricity generation mixture increases the storage fraction of renewable energy produced may also increase. In practice it is more complicated than this, sufficiently large geographical spread of solar and wind turbines reduces variability arising from local cloud cover and local wind patterns (NREL, 2013(@). Furthermore, there is evidence that at "low" penetrations, renewables do not necessarily need additional storage (Denholm et al, 2010). This is corroborated in (Solomon et al, 2017) where the storage capacity whose results indicate that storage capacity becomes a factor only after some minimum threshold variable renewable penetration increasing linearly up until approximately 80% and levelling off afterwards. This linear growth in storage capacity requirements is also found in a National Renewable Energy Laboratory (NREL) study (Kroposki, 2017) though with no decline for VRE penetration above 80% as in the previously mentioned study.

Estimates for the required storage fraction for very high penetrations of VRE range somewhat dramatically. A storage fraction of 10% is calculated for 90% VRE penetration for the continental United States in (Hand et al, 2012). A lower range of storage fraction values is found in (Blanco and Faaij, 2018) who find values ranging from 1.5 to 6 percent for VRE penetration

[1] While it is beyond the scope of this chapter to address, it is worth raising the question of how loads profiles might change across different societal assumptions; that is, from growth societies to degrowth ones. How, if at all, might the peaks in daily power demand differ between a growth and degrowth society? Such questions are important as the costs associated with the electric power system are determined in part by the magnitude of peak demands (see (Meier, 2006) chapter five).

ranging from 95 to 100% while a range from 10 to 20 percent is reported in (Breyer et al, 2017).

There are therefore two sources of uncertainty to consider in the SFCIO-IAM model concerning the modelling of energy storage. The first is how the storage requirements grow with increasing VRE penetration, and the second is how large the storage fraction becomes at very high VRE penetration levels. Concerning the first, we will model the magnitude of the storage fraction as a linearly increasing function of VRE penetration which is in rough approximation with the above discussion; concerning the second, we will deploy sensitivity analysis over a range of possible high penetration VRE storage values as the "true" value is not determinable within the bounds of this study.

The following section introduces the main SFCIO-IAM model equations. The SFC transactions flow, concerning the energy portion of the model left to the appendices. The third section presents scenario analysis examining the impact of different rates of renewable build-out and electrification of end use for growth, steady-state, and degrowth futures. Sensitivity analysis is conducted over key energy system parameters and climate parameters.

12.1 The Model

In this section we lay out the principle model equations. The balance sheet, transactions flow, and input-output matrices that make up the core accounting structure of the SFCIO-IAM model are found in appendix 12.4 while all initial values and parameters values are found in appendix 12.5. First, total household consumption $C(t)$ is given by the following consumption function:

$$C(t) = \alpha_1 YD(t) + \alpha_2 V(t) \tag{12.1}$$

where $YD(t)$ is disposable income and $V(t)$ is wealth where $V(t) = M(t) + H^h(t)$; the sum of interest earning deposits $M(t)$, and cash $H^h(t)$ held by households with $\alpha_1, \alpha_2 \in (0, 1)$. Households' consumption is divided between expenditures on energy sector goods and manufacturing sector goods. Denoting these as $C^e(t)$ and $C^m(t)$ respectively, we have:

$$C(t) = C^e(t) + C^m(t) \tag{12.2}$$

Energy consumption expenditures by the household sector is left exogenous so as to be a freely adjustable model component in scenario and sensitivity analysis; As such, the expression for $C^e(t)$ is simply:

$$C^e(t) = f_c(t) \tag{12.3}$$

where $f_c(t)$ is some time-dependent function.[2] Households purchase energy from both the fossil fuel sector $C^F(t)$ and the renewable sector $C^R(t)$ with total energy expenditures given by the following sum.

$$C^e(t) = C^R(t) + C^F(t) \tag{12.4}$$

Government expenditures follow a similar logic with total government expenditure comprised of energy and manufacturing expenditures:

$$G(t) = G^e(t) + G^m(t) \tag{12.5}$$

Like consumption, government expenditure on energy can be separated into energy purchases from the renewable and fossil-fuel sectors.

$$G^e(t) = G^R(t) + G^F(t) \tag{12.6}$$

Household disposable income is made up of the difference between wages $WB(t)$ received from the industrial sectors, interest on deposits held at banks $r_d \cdot M(t)$ where r_d is the interest rate on deposits and $M(t)$ is the magnitude of the deposits, and taxes paid to the government $T(t)$.

$$YD(t) = WB(t) + r \cdot M(t) - T(t) \tag{12.7}$$

The total wages received by households is simply the sum of the wages paid to households from each sector so that $WB(t) = WB^R(t) + WB^{FF}(t) + WB^M(t)$. This sum can be found directly from the transactions flow matrix by summing the second, fourth, and sixth columns of Figure 12.12 and solving for $WB(t)$; this results in:

$$WB(t) = C(t) + G(t) + I(t) - (r + Zr)L^R(t) - (r + Zf)L^F(t) - (r + Zm)L^M(t) \tag{12.8}$$

[2]For example, we may assume household energy expenditures increase linearly or at some constant percentage indicating increasing energy consumption over time.

Here the Z_i represents the fraction of total outstanding loans $L^i(t)$ held by each sector that must be paid back at time t.[3] From the transactions flow table each of the individual sectors also engage in interindustry transactions which net to zero across all three sectors by necessity. As such, the wages earned by households are simply the difference between the sales of each sector and their costs. Profits are assumed to be distributed wholly back to the household sector and are therefore not represented explicitly.

Households in the model can hold two types of assets: cash $H(t)$ and interest earning deposits at banks $M(t)$. It is assumed that households desire to hold a certain proportion (λ_0) of their wealth $V(t)$ as deposits and the remainder as cash (following the logic of (Godley and Lavoie, 2007) chapter four). This proportion is however modulated by interest rates and disposable income. Households will desire to hold relatively more of their wealth as interest earning deposits at banks given a higher rate of return r_d, and conversely, hold relatively less of their wealth as deposits at banks as disposable income increases leading to a greater demand for a cash to undertake transactions. As such $M(t)$ can be determined as:

$$\frac{M(t)}{V(t)} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD(t)}{V(t)}\right) \tag{12.9}$$

Finally, the taxes levied by the government on households is simply a proportion θ of wages.

$$T(t) = \theta WB(t) \tag{12.10}$$

12.1.1 The Input-output Model

The interrelationships between the three sectors are captured in the following matrix of input-output technical coefficients where the order of sectors on the rows of columns is renewable, fossil-fuels, and manufacturing respectively. We assume, for tractability, that the renewable sector does not utilize any inputs from the other sectors to produce its output while the fossil-fuel sector (which we model as vertically integrated) uses some of its own output to power its operation.[4] Finally, the manufacturing sector sources both electric power and fuels from the

[3] Note well that $i = R, F, M$.

[4] Note well, while the renewable sector is assumed to produce its output (electric power) without requiring intermediate goods, it most certainly does require goods from the manufacturing sector in the form of capital equipment. These requirements are captured in the investment portion of final demand.

renewable and fossil-fuel sectors (third column).

$$\mathbf{A(t)} = \begin{pmatrix} 0 & 0 & \frac{P_R \phi_g(t) k_e^R(t)}{X^M(t)} \\ 0 & \frac{S(0)}{EROI(0)\cdot S(t)} & \frac{P_F\left[CF[\tau_E k_e^M(t) - \phi_g(t) k_e^R(t)] + \tau_F k_{ne}^m(t)\right]}{X^M(t)} \\ 0 & 0 & a_{33} \end{pmatrix} \tag{12.11}$$

Here, The P_i terms denote the prices for each sectors output, while the $X^i(t)$ denotes the total sectoral outputs where (i = R,F,M).[5] $S(t)$ denotes the magnitude of the stock of fossil fuels at time t, while $EROI(0)$ denotes the initial period value for the EROI of fossil-fuel production. Here $a_{22} = \frac{S(0)}{EROI(0)\cdot S(t)}$ is designed to capture the effects of declining fossil-fuel EROI with extraction induced declines in $S(t)$. Chiefly, this formulation implies that as EROI declines, more fossil-fuel sector derived energy is required to produce any given quantity of fossil-fuel sector output. The differential-equation governing the evolution of the stock of fossil-fuels is given as:

$$\frac{dS}{dt} = \frac{-X^F(t)}{P_F} \tag{12.12}$$

which states that the stock of fossil-fuels declines with extraction necessary to meet total demand per unit time.

Physical capital in the model is separated into five classifications. Renewable capital is separated, for accounting reasons, into that capital $k_e^R(t)$ necessary to meet intermediate demand arising from the manufacturing sector, and final demand from households and government $k_f^R(t)$. The manufacturing sector operates electrified and non-electrified manufacturing capital $k_e^M(t)$ and $k_{ne}^M(t)$ respectively. Finally, $k^F(t)$ denotes the quantity of fossil-fuel sector capital. All capital types are assumed to be measured in a common physical unit of machines [m]. The terms τ_E and τ_F denote the power requirements per unit of electrified and non-electrified manufacturing capital, respectively. The term ϕ_g is a "grid-corrected" term denoting the power output per unit of renewable capacity. Finally, CF is the conversion factor between fossil-fuel energy and the electricity produced via combustion.

Matrix $\mathbf{A(t)}$ captures two key physical dynamics. First, examining the a_{23} element, we see that the demand for fossil-fuel sector output in the form of fuels to power non-electrified capital is given as $P_F \tau_F k_{ne}^m(t)$. As non-electrified capital is replaced by electrified capital,

[5]Here R denotes the renewable sector, F the fossil-fuel sector, and M the manufacturing sector.

$k_{ne}^M(t)$ will decline towards zero indicating a decline in the demand for fuels from the fossil fuel sector. Second, as the magnitude of renewable sector capital $k_e^R(t)$ increases, the term $\tau_E k_e^M(t) - \phi_g(t)k_e^R(t)$ decreases indicating a decrease in fossil-fuel derived electricity to power electrified manufacturing capital. The a_{23} term therefore captures both of the necessary energy transition components; the electrification of the manufacturing sector and the displacement of fossil-fuel sector derived electricity by that produced by the renewable sector. When both of these phenomena occur in the model the a_{23} technical coefficient will be equivalent to zero.

From the matrix $\mathbf{I} - \mathbf{A}(t)$ we may calculate the Leontief coefficients necessary to obtain expressions for total sectoral outputs. First the determinant $D(t)$ is given as:

$$D(t) = \frac{1}{\left(1 - \frac{S(0)}{EROI(0)\cdot S(t)}\right)(1 - a_{33})}$$

which can be used to obtain the following nine Leontief coefficients:

$$L_{11}(t) = 1, \quad L_{12}(t) = 0, \quad L_{13}(t) = \frac{P_r\phi_g(t)k_e^R(t)}{(1 - a_{33})\cdot X^M(t)} \tag{12.13}$$

$$L_{21}(t) = 0, \;\; L_{22}(t) = \frac{1}{\left(1 - \frac{S(0)}{EROI(0)\cdot S(t)}\right)}, \;\; L_{23}(t) = \frac{P_f(t)\left[CF[\tau_E k_e^M(t) - \phi_g(t)k_e^R(t)] + \tau_{FF}k_{ne}^m(t)\right]}{\left(1 - \frac{S(0)}{EROI(0)\cdot S(t)}\right)(1 - a_{33})X^m(t)} \tag{12.14}$$

$$L_{31} = 0, \quad L_{32}(t) = 0, \quad L_{33} = \frac{1}{1 - a_{33}} \tag{12.15}$$

Assuming, that renewable generation displaces fossil-fuel generation when new renewable capacity comes online (therefore making fossil-fuel the provider of the residual power requirements) we may write the following expressions for each sector's final demand.

$$C^R(t) + G^R(t) = P_R\phi_g(t)k_f^R(t)$$

The final demand for fossil fuels is therefore given below as the difference between the energy demands $C^e(t) + G^e(t)$ and that provided by the renewable sector.

$$C^F(t) + G^F(t) = C^e(t) + G^e(t) - P_r\phi_g(t)k_f^R(t)$$

The final demand faced by the manufacturing sector is given the sum of consumption, government, and investment expenditures.

$$I(t) + C^m(t) + G^m(t) = Y(t) - C^e(t) - G^e(t)$$

Finally, combining the Leontief coefficients with these expressions for sectoral final demands we can write the following expressions for the total sectoral outputs:

$$X^R(t) = L_{13}(t)Pr\phi_g(t)k_f^R(t) + L_{23}[Y(t) - C^e(t) - G^e(t)] \tag{12.16}$$

$$X^F(t) = L_{22}(t)[C^e(t) + G^e(t) - Pr\phi_g(t)k_f^R(t)] + L_{23}(t) \cdot [Y(t) - C^e(t) - G^e(t)] \tag{12.17}$$

$$X^M(t) = L_{33}[Y(t) - C^e(t) - G^e(t)] \tag{12.18}$$

Each sector (renewable, fossil-fuel, and manufacturing) operates a stock of capital whose output, in physical terms, is proportional to the magnitude of the capital stock. Therefore, the physical output (supply) of each sector per unit time is give as:

$$s_R(t) = \phi_g(t)(k_e^R(t) + k_{ne}^R(t)) \tag{12.19}$$

$$s_F(t) = \phi_F(k^R(t)) \tag{12.20}$$

$$s_M(t) = \phi_M(k_e^M(t) + k_{ne}^M(t)) \tag{12.21}$$

The ϕ terms appearing in the above three supply equations have a simple and natural interpretation as parameters denoting the output per unit of capital. For example, $\phi_{g(t)}$ is the power output per unit of renewable capital.

The above equations state the physical supply of each sector's output per unit time. To be physically sensible, this supply in physical terms must match the demand in physical terms. Using the sectoral final demands as measures of the total physical demand for each sector's production, we define prices as the mechanisms that equate physical supply and demand at any given moment.

$$P_R(t) = \frac{X^R(t)}{\phi_R k^R(t)} \tag{12.22}$$

$$P_f(t) = \frac{X^F(t)}{\phi_F k^F(t)} \tag{12.23}$$

$$P_m(t) = \frac{X^M(t)}{\phi_M(t) k^M(t)} \tag{12.24}$$

Having defined the physical supply and demands, and the price mechanism in the model, we may turn to investment. Assuming capital is valued at replacement cost and since capital is the output of the manufacturing sector in this model, the replacement cost is simply the price of manufacturing sector output $P_M(t)$ multiplied by the magnitude of capacity being replaced. The non renewable sectors invest in new capital in order to close the gap between the demand for its output at some target normal price and its current capacity. The renewable sector invests in new capital until it meets all power requirements in the model.

$$i_e^R(t) = \Delta_1 \left(\mu_1 \phi_g^{-1} \tau_E k_e^m(t) - k_e^R(t)\right) \tag{12.25}$$

The bracketed term in equation (12.25) is the difference between the capacity of renewable generation necessary to power electrified manufacturing capital $\tau_E k_e^m$ and the currently built capacity k_e^R. The parameter Δ_1 is the rate at which the gap closes while μ_1 denotes the percentage of excess capacity that the renewable sector aims to construct. The remaining investment equations follow the same logic.

$$i_f^R(t) = \delta_1 \mu_1 \phi_g^{-1} P_R^{-1} [C^e(t) + G^e(t)] - \Delta_1 k^R(f) \tag{12.26}$$

$$i^F(t) = \Delta_2 \mu_2 \phi_F^{-1}(t) P_{F0}^{-1} X^F(t) - \Delta_2 k^F(t) \tag{12.27}$$

$$i_e^M(t) = \Delta_3 \mu_3 \phi_M^{-1}(t) P_{M0}^{-1} X^M(t) - \Delta_3 k_e^M(t) + \Delta_3 k_{ne}^M(t)) \tag{12.28}$$

Finally, aggregate investment in monetary terms is determined by valuing the above investment terms at the price of manufacturing sector output. Therefore, we have that:

$$I(t) = I^R(t) + I^F(t) + I^M(t) = P_m(t)\left[i_e^R(t) + i_f^R(t)\right] + P_m(t) i^F(t) + P_m(t) i_e^M(t) \tag{12.29}$$

Each sector finances their investment via loans received from private banks upon which they must make both interest payments and pay back the principal. The capital stocks of each sector grow with investment and decline with depreciation. Finally, each sector finances the purchase of new capital via loans taken obtained from private banks. For example, the stock of physical capital held by the fossil fuel sector is determined by the differential equation:

$$\frac{dk^F}{dt} = i^F(t) - \sigma_F(t)k^F(t)$$

where $\sigma_F(t)$ is an endogenous depreciation rate. The magnitude of the loans extended to the same sector is governed by:

$$\frac{dL^F}{dt} = Pm(t)i_e^F(t) - Z_F L^F(t) \tag{12.30}$$

where Z_f denotes the fraction of the total outstanding loan repaid per unit time. The dynamics for all of the capital stocks and loans are shown in Table 12.1. Finally, we assume that the banking sector acts as passive lenders, extending loans to the three sectors as required to finance their capital investment.[6]

12.1.2 Renewable Power Dynamics

Following the discussion in the introduction, we denote the storage fraction $\phi(t)$ as a linearly increasing function of the ratio of total renewable electric power produced to that of total electric power consumed in the model where an increase in this ratio indicates that variable renewable generation provides a greater fraction of total electric power in the model. This ratio, $\psi(t)$ is denoted as follows:

$$\psi(t) = \frac{\phi_R(k_e^R(t) + k_f^R(t))}{\tau_E k_e^m(t) + (C^e(t) + G^e(t))P_r^{-1}} \tag{12.31}$$

where ϕ_R is the power output per unit of renewable capacity before taking into account possible energy losses due to storage. Therefore, the numerator is the total electric power produced by the models stocks of renewable energy capital $k_e^R(t) + k_f^R(t)$ divided by the quantity of electric power consumed by the electrified manufacturing capital $\tau_E k_e^m(t)$ and the electric power demanded as part of household and government final demand $(C^e(t) + G^e(t))P_r^{-1}$.

Now letting a and b determine the minimum and maximum values that $\phi(t)$ can obtain, the

[6]Note well, the assumption of banks as passive lenders is one with impact for the transition story in the model in that bank loans are essential for the financing of new renewable capacity. All else being equal, limitations on the magnitude of loans by banks for example, would act to slow the transition. A particularly interesting question in the context of a rapid transition away from fossil-fuels concerns banks willingness to lend to a sector whose assets must eventually be written off.

storage fraction is given as the following function of $\psi(t)$:

$$\phi(t) = a + b\psi(t) \tag{12.32}$$

Here a is the base level storage fraction that occurs at zero VRE penetration (we assume $a = 0$ in all modelling) and b is given as the upper bound storage fraction minus a so that when VRE penetration is at 100%, $\psi(t) = 1$ and therefore $\phi(t)$ will be equivalent to its upper bound value.

The grid-corrected power output $\phi_g(t)$ is therefore given as a function of the EROI of the renewable technology, the storage fraction, and the energy storage on investment (ESOI) of electrical energy storage.

$$\phi_g(t) = \left(\frac{1 - \phi(t) + \eta\phi(t)}{\frac{1}{EROI_R} + \frac{\eta\phi(t)}{ESOI_e}} \right) \frac{\phi_R}{EROI_R} \tag{12.33}$$

12.1.3 Climate

In this section we introduce the (BEAM) carbon cycle model of (Glotter et al, 2013) in order to link the emissions produced by economic activities to atmospheric concentrations while also capturing some key carbon-cycle processes. BEAM models a three-reservoir carbon-cycle system where M_{at}, M_{up}, and M_{lo} are the masses of inorganic carbon in the atmosphere, upper ocean, and lower ocean respectively. A set of three coupled non-linear differential equations governs the evolution of each the stocks of carbon which are presented as follows:

$$\frac{dM_{at}}{dt} = E(t) - k_a(M_{at} - A \cdot BM_{up}) \tag{12.34}$$

$$\frac{dM_{up}}{dt} = k_a(M_{at} - A \cdot BM_{up}) - k_d \left(M_{up} - \frac{M_{lo}}{\delta} \right) \tag{12.35}$$

$$\frac{dM_{lo}}{dt} = k_d \left(M_{up} - \frac{M_{lo}}{\delta} \right) \tag{12.36}$$

where $E(t)$ is an emissions term. Global average surface temperatures in the SFCIO-IAM model are obtained using the two component energy balance model (2-EBM) described in (Geoffroy et al, 2013). The equations that govern the evolution of the global mean surface air temperature

T and the temperature of the deep ocean T_0 are given as:

$$\mathcal{C}\frac{dT}{dt} = \mathcal{F}(t) - \lambda T - \gamma(T - T_0) \tag{12.37}$$

$$\mathcal{C}_0\frac{dT_0}{dt} = \gamma(T - T_0) \tag{12.38}$$

where $\mathcal{F}(t)$ is a radiative forcing term given by

$$\mathcal{F}(t) = \frac{\mathcal{F}_{2x\mathrm{CO}_2}}{\ln(2)} \ln\left(\frac{M_{at}(t)}{[M_{at}^0}\right) \tag{12.39}$$

where M_{at}^0 is the reference period (pre-industrial) mass of atmospheric carbon. Finally, Emissions $E(t)$ are given as:

$$E(t) = \frac{\zeta X^F(t)}{P_F(t)} \tag{12.40}$$

which states that emissions per unit time $E(t)$ are proportional to the total physical output of the fossil-fuels sector by a factor ζ.

Finally, we turn to the climate feedback which is included in the form of a damage function whose output losses are shared between declines in the productivity parameters of each sectors capital, and to enhanced depreciation of the capital stocks. We use the form, calibrated so that $D(t) = 0.5$ at $T = 6°$C, proposed by economist Martin Weitzman as follows (Weitzman, 2012):

$$D(t) = 1 - \frac{1}{(1 + \pi_1 T(t) + \pi_2 T(t)^2) + \pi_3 T(t)^{0.6754}}$$

The climate damage $D(t)$ is shared out between impacts on the ϕ_i productivity terms and the depreciation terms the σ_i (i = R,F,M) using an approach similar to that in (Moyer et al, 2014). A fraction β of the $D(t)$ damages occurs as additional depreciation to the a capital stock $k^i(t)$ leading to the following climate damage modified depreciation term:

$$\sigma_i(t) = \sigma_i + \beta D(t) \tag{12.41}$$

while the climate damage modified productivity terms are given as:

$$\phi_i(T) = \phi_i\left(\cdot\frac{1-\alpha}{1-\beta\alpha}\right) \tag{12.42}$$

The full model boils down to the following set of 15 coupled non-linear differential equations which are expressed in the following table:

Variable Name	Symbol	Differential Equation
Renewable Intermediate Capital	$k_e^R(t)$	$\dot{k_e^R} = i_e^R(t) - \sigma_R(t)k_d^R(t)$
Renewable Final Capital	$k_f^R(t)$	$\dot{k_f^R} = i_f^R(t) - \sigma_R(t)k_f^R(t)$
Fossil-Fuel Capital	$k^F(t)$	$\dot{k^F} = i^F(t) - \sigma_F(t)k^F(t)$
Electrified Manufacturing Capital	$k_e^M(t)$	$\dot{k_e^M} = i_e^M(t) - \sigma_M(t)k_e^M(t)$
Non-Electrified Manufacturing Capital	$k_{ne}^M(t)$	$\dot{k_{ne}^M} = -\sigma_M(t)k_{ne}^M(t)$
Renewable Loans	$L^R(t)$	$\dot{L^R} = Pm(t)[i_e^R(t) + i_f^R(t)] - Z_r L^R(t)$
Fossil-Fuel Loans	$L^F(t)$	$\dot{L^F} = Pm(t)i_e^F(t) - Z_F L^F(t)$
Manufacturing Loans	$L^M(t)$	$\dot{L^M} = Pm(t)i^M(t) - Z_M L^M(t)$
Household Wealth	$V(t)$	$\dot{V} = WB(t) + rM(t) - C(t) - T(t)$
Stock of Fossil-Fuels	$S(t)$	$\dot{S} = -X^F(t) \cdot P_F^{-1}$
Mass of Atmospheric CO_2	$M_{at}(t)$	$\dot{M_{at}} = E(t) - k_a(M_{at}(t) - A(t) \cdot B(t)M_{up}(t))$
Mass of Upper-Ocean CO_2	$M_{up}(t)$	$\dot{M_{up}} = k_a(M_{at}(t) - A(t) \cdot B(t)M_{up}(t)) - k_d \left(M_{up}(t) - \frac{M_{lo}(t)}{\delta}\right)$
Mass of Lower-Ocean CO_2	$M_{lo}(t)$	$\dot{M_{lo}} = k_d \left(M_{up}(t) - \frac{M_{lo}(t)}{\delta}\right)$
Global-Average Surface Temperature	$T(t)$	$\mathcal{C}\dot{T} = \mathcal{F}(t) - \lambda T - \gamma(T(t) - T_0(t))$
Deep Ocean Temperature	$T_0(t)$	$\mathcal{C}_0\dot{T_0} = \gamma(T - T_0)$

Table 12.1: State Variables and their Associated Differential Equations

12.2 A Business as Usual Scenario

To study the behaviour of the SFCIO-IAM model we will begin by exploring a business as usual or baseline scenario. Roughly this scenario corresponds to an economy that initially derives 10% of its energy needs from renewable sources and where 25% of its manufacturing capital is assumed to be electrified. It is assumed that the rate of replacement of non-electrified capital with electrified capital is simply the depreciation rate for manufacturing capital; that is, non-electrified capital is only replaced at the end of its natural life-cycle.

Economic growth (on the order of 2% per annum) is induced via the assumption that the autonomous portion of consumption (energy demand) and government expenditures both grow at 2% per annum over the 30-year model simulation. The critical assumption here is that the parameter Δ_1 which determines the rate at which renewables are constructed is assumed to be reasonably small at $\Delta_1 = 0.026$. As a consequence of these assumptions, the 580 $GtCO_2$ carbon budget is exhausted by approximately 2034 and global mean surface temperatures breach the 1.5 degree warming threshold with no indication of cessation. The time-paths of key model variables are presented in Figure 12.1.

Examining the first three panels we see that emissions continue to increase (approximately linearly) over the simulation period and, as a consequence, cumulative emissions surpass the 580 $GtCO_2$ carbon budget by approximately 2034. The global mean surface air temperatures breach the 1.5 degree warming boundary in approximately 2044 which lies in the midpoint of the time range in (IPCC, 2018). The second row of panels indicates sustained long term economic growth and a long term (though mod-

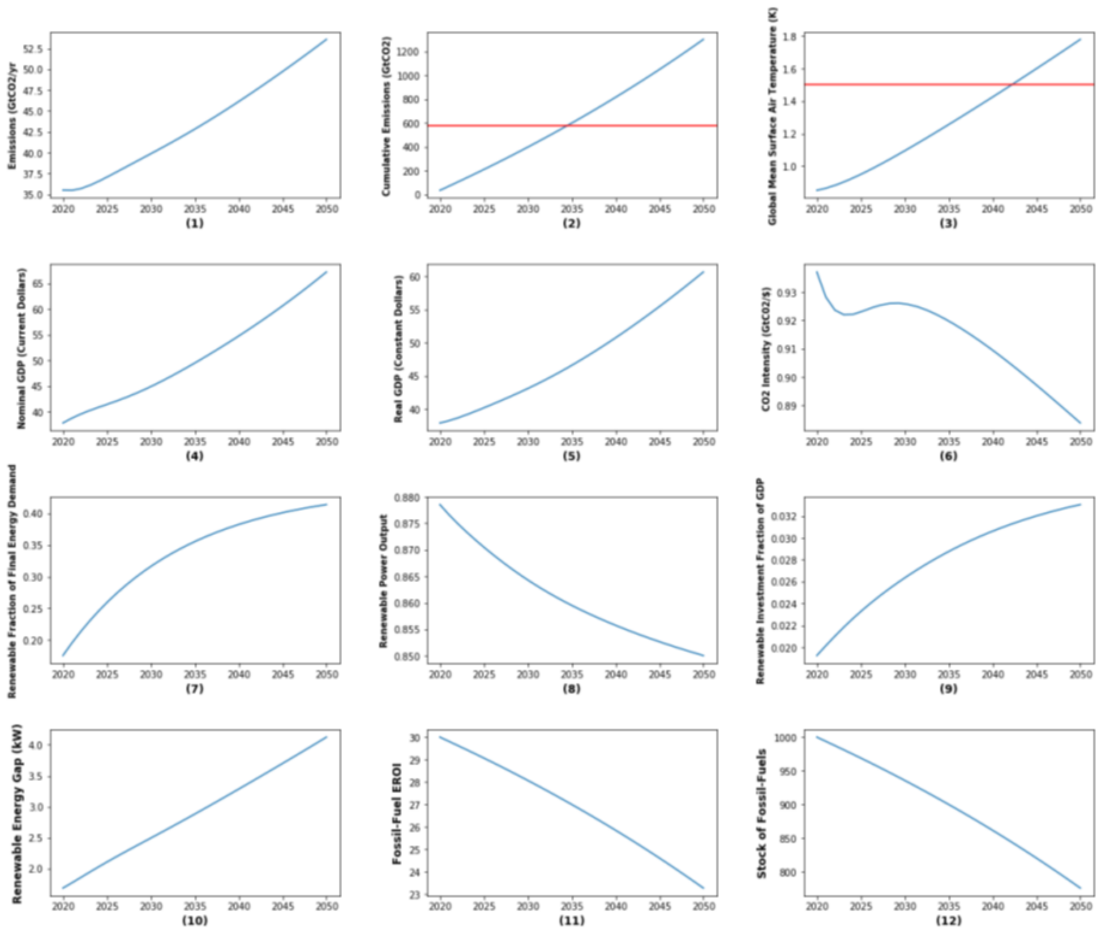
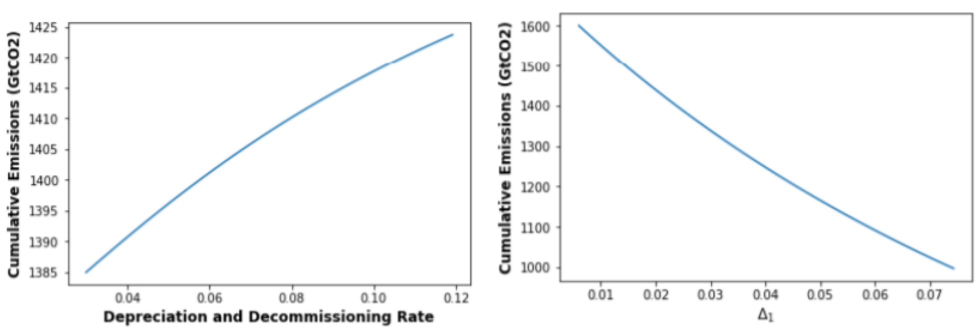


Figure 12.1: Trajectories for select SFCIO-IAM model variables assuming a rate of renewable capacity build out $\Delta_1 = 0.026$ and a rate of depreciation and decommissioning $\sigma_M^{NE} = 0.03$. The solid red lines in panels (2) and (3) correspond to the 580 $GtCO_2$ carbon budget and 1.5 degree warming threshold respectively.

est) decline in CO_2 intensity of GDP. The transient increase in intensity before returning to long term decline reflects the period of rapid build out of new renewable capacity and electrification of manufacturing sector capital. Panel (7) displays directly that the share of renewables in meeting final energy demand is increasing over the simulation horizon (from roughly 10% to approximately 40%), which via the mechanics of the model, leads to slightly decreasing renewable Grid EROI as a result of the increasing penetration of renewable electricity into the market. Finally, panel (9) presents perhaps the key result. Renewable energy investment as a share of GDP that begins at $\approx 1.6\%$ and nearly doubles by 2050, represents a renewable energy investment trajectory that is unambiguously too small (when assuming growth) to set the model on a 1.5 degree consistent emissions pathway.

We can see this result in yet another manner in panel (10) which plots the gap between the overall electric power demand and that provided by renewables. Clearly the widening gap implies an insufficiently rapid build-out of new renewable capacity. Finally, panels (11) and (12) present the trajectories for the EROI of fossil-fuels and the remaining stock of fossil-fuels that may be extracted.

The rate of renewable capacity build out Δ_1, and the rate of decommissioning and depreciation of the non-electrified capital stock σ_M^{NE} play fundamental roles in determining the rate at which emissions can be reduced. As the SFCIO-IAM model is designed to explore the joint dynamics of renewable build out and electrification of end use it is important to explore how variations in these key parameters impacts model outcomes. Figure 12.2 presents year 2050 cumulative emissions of $GtCO_2$ obtained by running the model with a wide range of possible parameters values.



(a) (b)

Figure 12.2: Panel (a): Year 2050 cumulative $GtCO_2$ emissions as a function of the rate of depreciation and decommissioning of the non-electrified capital stock. Panel (b) Year 2050 cumulative $GtCO_2$ emissions as a function of the rate of renewable capacity build out Δ_1.

In panel (a) of Figure 12.2 we see, perhaps unexpectedly, that year 2050 cumulative emissions increase with an increasing rate turn over of the non-electrified manufacturing capital stock. This result indicates a scenario of perverse electrification whereby the economy electrifies its capital stock but continues to generate the majority of its electricity via fossil-fuels. As this method, given the conversion losses associated with combustion is inherently more inefficient, increased rates of electrification without corresponding increases in renewable capacity leads to more, not less, emissions over time. In panel (b) a more expected result is obtained in that year 2050 cumulative emissions are reduced dramatically in correspondence with increases in the rate of renewable capacity build out (Δ_1).

The above results indicate the fairly obvious result that when new renewable capacity is assumed to automatically displace fossil-fuel generated electricity, increases in the rate at which renewable capacity is constructed (all else being equal) leads to lower cumulative emissions. Less immediately obviously, increased electrification without the increased renewable build out has the opposite effect. However, it is both effects in conjunction that are necessary. We perform a further sensitivity analysis over the joint space of both parameters allowing Δ_1 to range from 0.06 to 0.075 and σ_M^{NE} to range from 0.03 to 0.12. For each variation of parameters, the following figure plots the cumulative emissions, emissions, and global mean surface air temperature trajectories.

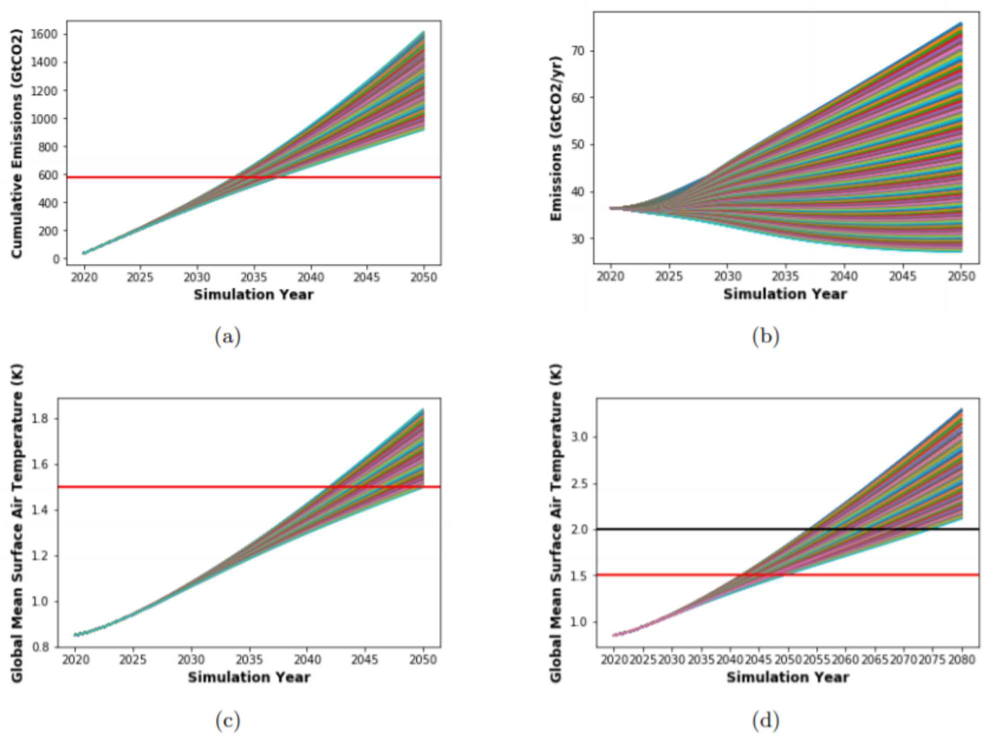


Figure 12.3: Trajectories of cumulative emissions, emissions per year, and global mean surface temperature for a sensitivity analysis over the joint parameter space $\Delta_1 \in \{0.006, 0.075\}$ and $\sigma_M^{NE} \in \{0.03, 0.12\}$, where Δ_1 is the rate of renewable capacity build out and σ_M^{NE} is the rate of depreciation and decommissioning of the non-electrified capital stock.

Perhaps the most salient feature of Figure 12.3 lies in panel (a) which indicates that no combination of Δ_1 and σ_M^{NE} in the ranges assumed lead to emissions trajectories that keep cumulative emissions below the 580 $GtCO_2$ budget. However, the impacts of these parameter variations are not insignificant. By 2050 cumulative emissions are nearly double in magnitude for scenario with the lowest values as compared with the scenario with the highest values. The impact of these parameters is apparent from panel (b) which displays how significantly the emissions pathways differ over the parameter space. Finally, panel (c) indicates the substantial (and still increasing) difference in mean surface temperature that arises.

There are two critical takeaways from this sensitivity analysis. First, under the assumptions of continued growth at 2% per annum, it is not possible to produce 1.5 degree consistent emissions pathways for the ranges of the variables assumed. Second, high rates of renewable capacity build out and electrification lead to substantially lower cumulative emissions and eventually relatively lower mean surface temperatures. This is shown in panel (d) which extends the model run to 2080 and plots the resulting mean surface temperatures. While all runs imply temperature increases of over 2 degrees, the temperature increase for models runs with the lowest values lead to temperatures well over three degrees while the runs with the highest parameter values keep warming to just over 2 degrees. The highest warming trajectories here correspond to the lower range of the values found in the IPCC RCP 8.5 scenario (Pachauri et al, 2015).

It is useful to explore how the model results change via sensitivity analysis over key physical parameters; to see this more clearly we assume a larger rate of renewable capacity build out of $\Delta_1 = 0.05$. In the model we assume a life-cycle EROI of 30 for the abstract renewable capital, this may seen as both an over and underestimate of "renewable EROI" depending on the specifics of the renewable technology investigated. While modelling renewables as a single composite technology is certainly more tractable it leads to downsides when the value of its EROI is ultimately somewhat ambiguous. Figure 12.4 displays the sensitivity results for the steady-state scenario.

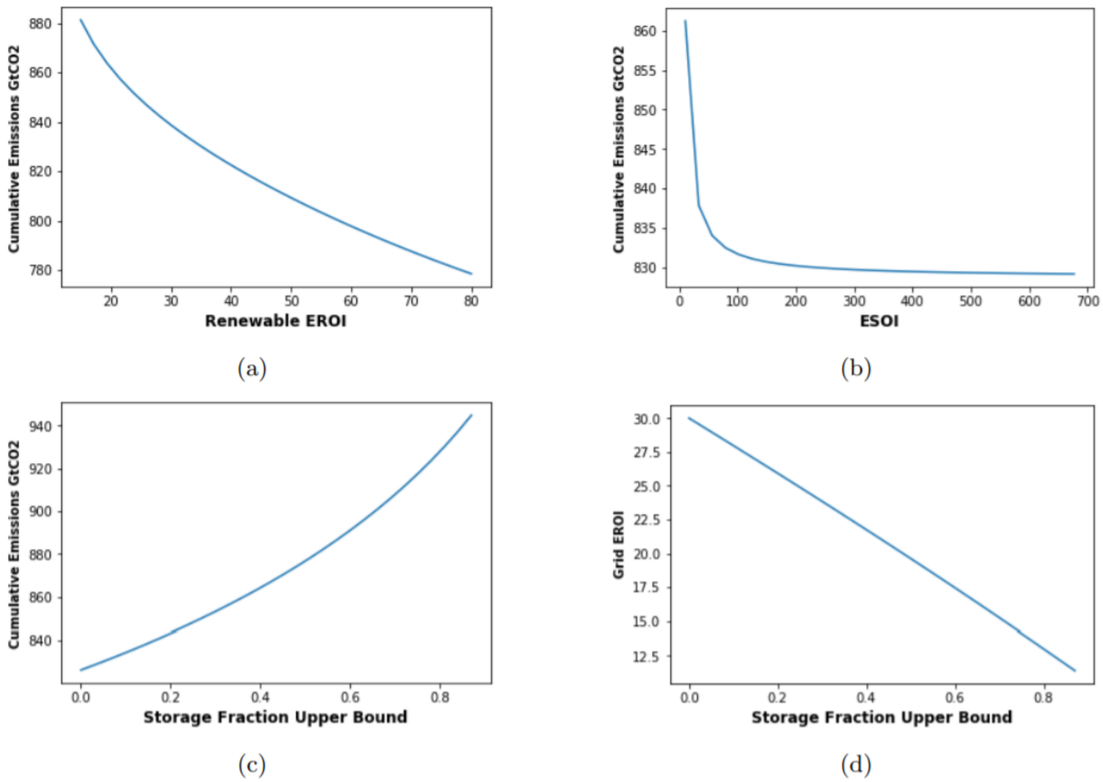


Figure 12.4: Year 2050 cumulative CO_2 emissions sensitivity analysis for the steady-state slow transition scenario as a function of: **panel (a)**: the renewable capital life-cycle EROI; **panel (b)**: the ESOI of electrical energy storage; and **panel (c)**: the maximum storage fraction at 100% renewable market penetration. **Panel (d)**: The decline in grid scale EROI for various for various values of the storage fraction upper bound.

Panel (a) shows the cumulative emissions at year 2050 for a renewable EROI ranging from 20 to 80. While higher EROI does lead to lower cumulative emissions the impact is perhaps not as large as might be assumed initially. This is because the models manufacturing sector must still electrify its capital stock. While higher EROI renewables do lower cumulative emissions the impact is still constrained by the economy's electrification trajectory; and as such, merely ramping up the effectiveness of the renewable technology is insufficient without other structural changes to lead to deep emissions reductions. In panel (b) we see a similar for 2050 cumulative emissions assuming different magnitudes of ESOI. A lower ESOI implies storage is less efficient and ultimately this implies higher emissions. However, as with the EROI, a higher ESOI assuming all else equal can only reduce cumulative emissions by so much.

As noted previously the storage fraction for renewables represents a fundamental unknown in this analysis. As it is not fully known what storage fraction will be necessary for high penetration of variable renewables the sensitivity on this parameter is particularly important. Panel (c) indicates that as the storage fraction increases from 0 to 0.6, the year 2050 emissions increase at an increasing rate. In Panel

(d) the grid scale EROI is plotted across the range of storage fraction. Evidently, storage fractions in the higher part of the assumed range lead to substantial declines in grid scale EROI and consequently explains the higher cumulative emissions in panel (c).

12.2.1 A 1.5 Degree Consistent Degrowth Pathway

In this section we explore a transition scenario designed so that the emissions trajectory in the degrowth scenario is ultimately 1.5 degree consistent. This scenario assumes that $\Delta_1 = 0.9$ and $\sigma_M^{NE} = 0.12$. In order to "force" any of the model scenarios to be 1.5 degree consistent requires assuming a very fast processes of the turnover of non-electrified capital. The results for this fast transition scenario are displayed in the following figure.

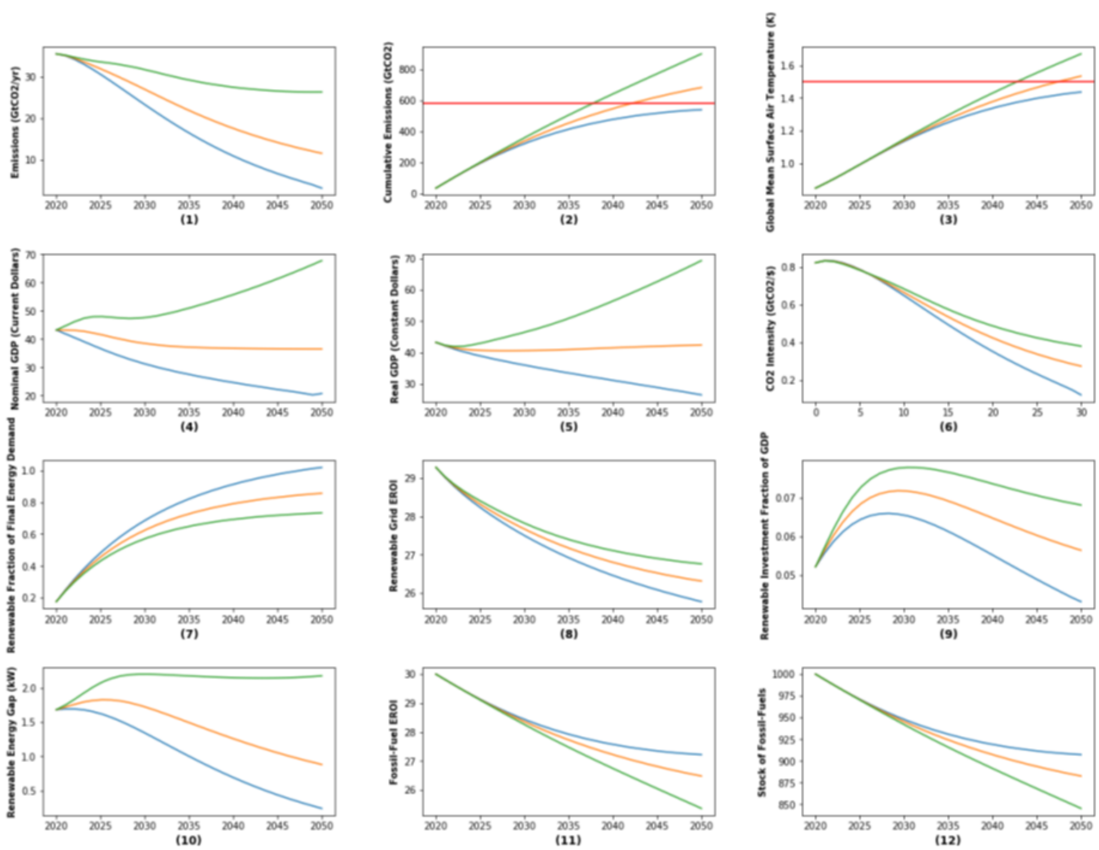


Figure 12.5: Trajectories for select SFCIO-IAM model variables assuming $\Delta_1 = 0.09$ and $\sigma_M^{NE} = 0.12$. **Growth** scenario trajectories (green plots –) assume 2% growth in household energy demand and 2% growth in government expenditures. **Steady-State** trajectories (orange plots –) assume 0% growth in household energy demand and 0% growth in government expenditures. **Degrowth** scenario trajectories (blue plots –) assume -2% growth in household energy demand and -2% growth in government expenditures. The solid red lines in panels (2) and (3) correspond to the 580 $GtCO_2$ carbon budget and 1.5 degree warming threshold respectively.

Panels (1) and (2) of Figure 12.5 present the first 1.5 degree consistent emissions pathway which occurs in the degrowth scenario. By 2050 cumulative emissions in the degrowth scenario do not transgress the budget and (though not shown) remain below the budget after the end of the simulation horizon. Under the fast investment assumptions all three scenarios produce emissions declines over the model run pushing back the dates of budget transgressions from the previous scenarios and leading to mean surface temperature increases of less than 1.5 degrees by 2050.

The magnitude of the renewable energy investment fractions of GDP are notable in simply how much larger they are than present day values. The fraction peaks at 10% in the growth scenario and at over 6% in the degrowth scenario. Again, it is worth remembering that these extreme values are all largely due to the fact that, as of 2021, the remaining carbon budget is simply not large relative to current yearly emissions. That the model is driven to, by conventional standards, such extremes to produce a consistent pathway (immense upfront renewable energy investment and degrowth) is in large part a consequence of forcing it to produce an energy transition in an intensely constrained set of circumstances.

While only the degrowth scenario was able to produce a 1.5 degree consistent emissions pathway it is still informative to examine the emissions trajectories for the growth and steady-state parametrizations. The long run cumulative emissions trajectories for the steady-state and growth scenarios are displayed in the follow Figure 12.6.

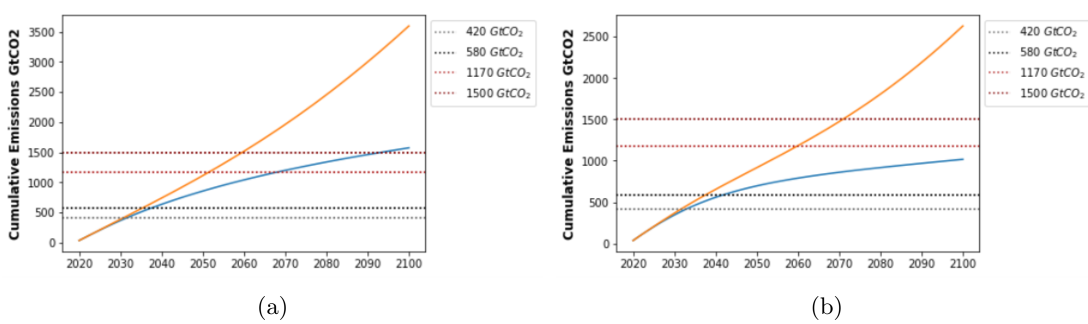


Figure 12.6: Long term trajectories of cumulative emissions for the steady state scenario (blue plots –) and the growth scenario (orange plots –) under slow and fast transition assumptions. The dotted lines indicate the carbon budgets that give 66% and 50% chances of remaining below 1.5 and 2 degrees warming respectively.

The peculiar nature of the degrowth pathway demands a more careful analysis of the financial aspects of the economy. The degrowth scenario requires both an immense near-term increase in investment in renewable and electrified manufacturing capacity while also assuming long term degrowth in government and household consumption. Put another way, the renewable, and manufacturing sectors must take on a large quantity of new financial liabilities (on which they make interest payments) at the outset of a period characterized by long-term decline in demands for their own outputs. This dual issue of declining sales coupled with increased interest payment costs acts to suppress the total wages and profits distributed back to households from the industrial sectors which in turn acts to decrease the disposable income portion of the consumption function.

The following Figure 12.7 presents the trajectories of key financial entities in the model for the degrowth trajectory. Panels (1-3) display the trajectories for the net worth of each of the productive sectors, where the net worth (value of assets minus liabilities) for each firm amounts to the difference between the current value of their built capital less that of their outstanding loans. To put these trajectories in context it is necessary to also show the time paths of loans held by each sector. As noted above, both the renewable and manufacturing sectors experience a transitory period of significant growth in the quantity of loans they have on their balance sheets as they build out (renewable sector) and electrify (manufacturing sector) their capital stocks. Both the net-worth and the loans held by the fossil-fuel sector decline significantly over the model run as the sector is displaced by the growth in renewable capacity.

Notably, even in a degrowth scenario, the net worth of the renewable sector increases and levels off by 2050 which simply reflects the large increase in the quantity of its built capital. The net-worth of manufacturing decreases overtime as it is simply replacing non-electrified capital with electrified capital. The more rapid decline in net-worth in the first half of the run reflects the transient period of increased investment.

Panel (7) presents the trajectory of household wealth which is particularly interesting in a degrowth context. Because the model is stock-flow consistent, the decline in household energy consumption that partly drives the degrowth scenario amounts to an increase in household savings (either as money or interest earning deposits). This is represented by the initial increase household wealth which eventually declines over the longer term. Panel (8) represents the banking sectors holding of government treasuries/advances which is simply the difference between the quantity of deposits held by the bank and the loans it has extended to the private sector. Initially, the banking sector holds a positive quantity of these treasuries which declines over-time and transforms into bank borrowing from the government over the latter half of the model run. Finally, panel (9) displays the trajectories of government expenditures and interest payments as well as well as tax revenues.

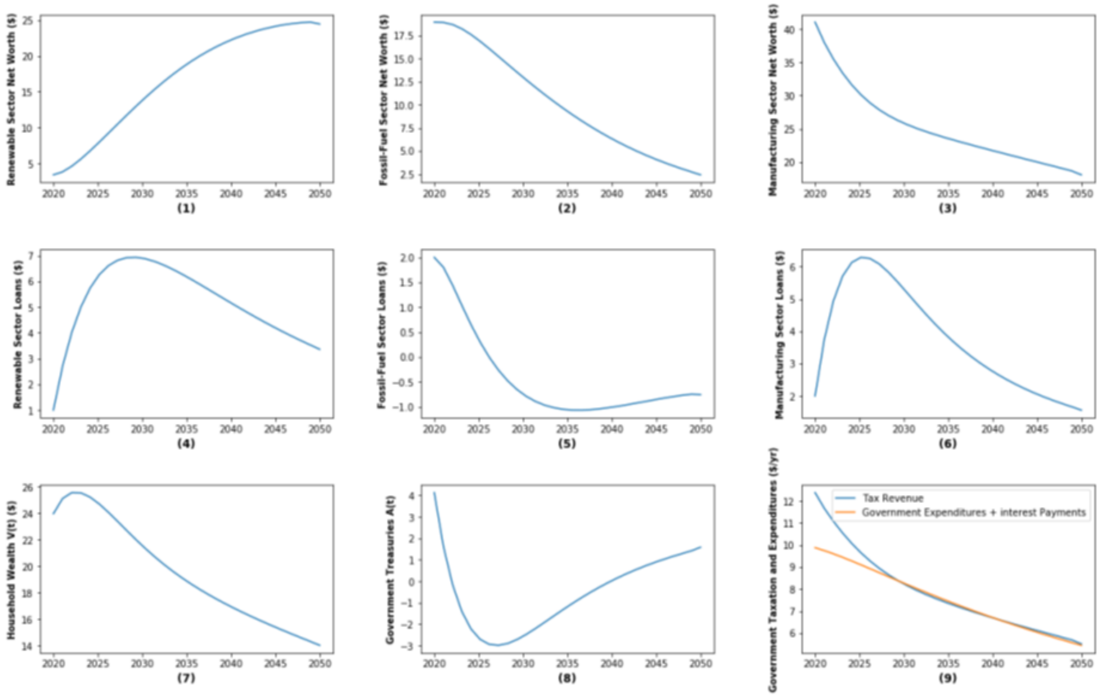


Figure 12.7: Trajectories for select SFCIO-IAM financial model variables assuming $\Delta_1 = 0.07$ and $\sigma_M^{NE} = 0.12$.

Uncertainty in the underlying 2-EBM parameters complicates the analysis. As noted, the 2-EBM model is calibrated using the multimodel mean values for the CMIP5 models. Loosely, this implies that about half the possible represented CMIP5 models will produce warming results above 1.5 degrees given the emissions pathway obtained above and about half will produce warming below. To display this variance, the following figure plots the temperature pathways for each of the individually calibrated CMIP5 models based on the parameters provided in (Geoffroy et al, 2013).

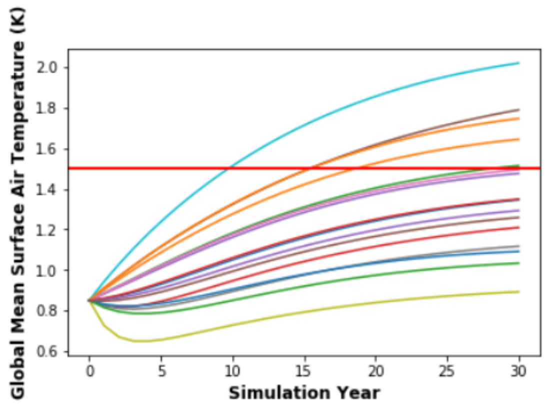


Figure 12.8: Global Mean Surface Air Temperature pathways for the SFCIO-IAM model under approximate fast transition degrowth assumptions and with the 2-EBM model parametrised to correspond to each of the 16 underlying CMIP5 AOGCM models.

In the above analysis only the degrowth scenario was able to produce a 1.5 degree consistent pathway up to 2050. It is important to determine the conditions (if they exist) whereby the SFCIO-IAM model produces successful pathways under steady-state and growth assumptions. Following the logic of the above scenario, we choose values of δ_1 and σ_M^{NE} such that the emissions pathways are just consistent with the 1.5 degree carbon budget.

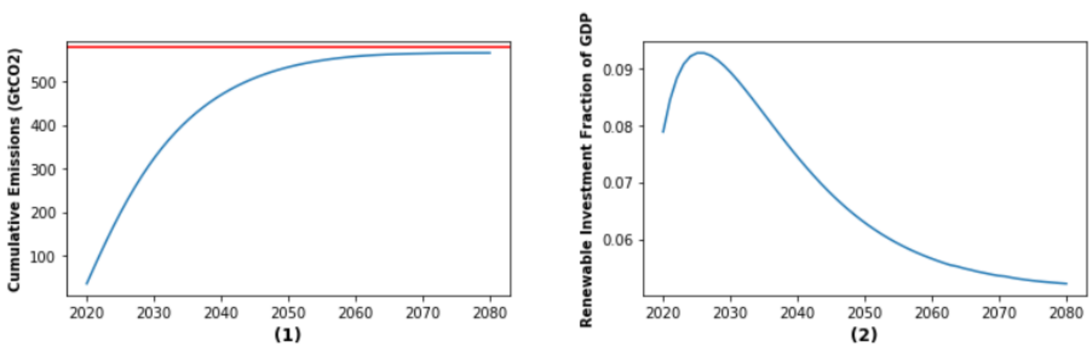


Figure 12.9: Cumulative emissions and the Renewable energy investment fraction of GDP for the no growth-scenario for $\delta_1 = 0.105$ and $\sigma_M^{NE} = 0.12$.

In order to show that the cumulative emissions curve does eventually flatten we run the model well beyond the standard 2050 horizon to 2080 which is displayed in panel (1) of Figure 12.9. Achieving the 1.5 degree consistent trajectory requires much larger upfront investment in renewables as is indicated by the very large renewable energy investment fraction of GDP displayed in panel (2) which peaks at over 9%.

A perhaps more interesting question is whether growth is compatible with the stringent requirements set by the 1.5 degree carbon budget. It should be recalled that growth in the model induces both direct increases in energy demand and indirect increases via the energy required to produce manufacturing sector output. Therefore, any build out of new renewable capacity must not only catch up with increasing energy demand but do so at a rate sufficient to very rapidly flatten the cumulative emissions curve. Figure 12.10 presents the cumulative emissions trajectory and renewable energy investment fraction for approximately the largest economic growth rate that the SFCIO-IAM is able to support while simultaneously generating a 1.5 degree consistent emissions pathway.

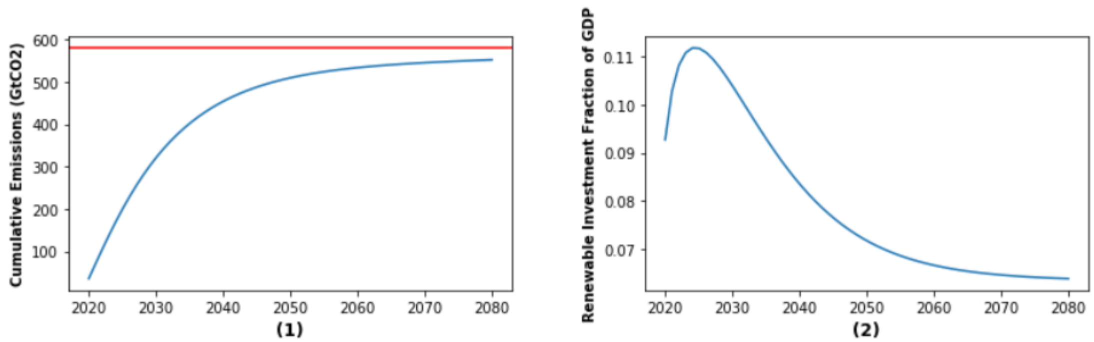


Figure 12.10: Cumulative emissions and the Renewable energy investment fraction of GDP for the an economy with 0.75 % economic growth per annum where $\delta_1 = 0.1226$ and $\sigma_M^{NE} = 0.15$.

There are several notable features concerning the above. First, no stable model solution with growth rates higher than approximately 0.75% exist that are able to produce 1.5 degree consistent trajectories. Second, the value of δ_1 necessary to produce a sufficiently rapid transition leads to a peak renewable energy investment fraction of over 11% of GDP and this is sufficient only when coupled with a rate of turn over of the non-electrified capital stock of 15% per annum.[7]

12.3 Conclusions

The key result of this chapter may be stated as follows. Under the assumption of mean climate conditions and assuming no deployment of negative emissions technologies, renewable investment as a share of GDP must peak at just over 6% per annum by 2030 in order to generate a 1.5 degree consistent emissions pathway under assumptions of sustained economic degrowth with this fraction increasing up to just over 11% to facilitate continual economic growth of up to 0.75% per annum. Under these assumptions

[7]The SFCIO-IAM model finds that long term relatively slow economic growth and emissions reductions consistent with a 1.5 degree trajectory are both possible under relatively stringent assumptions. However, as the SFCIO-IAM model only models decarbonization this amounts only to a statement that economic growth and decarbonization are not incompatible outcomes. As the model is agnostic to most other environmental phenomena this does not amount to a broader statement that long term economic growth is itself possible while meeting all other environmental issues.

it is possible to produce emissions trajectories associated with a 50 percent chance that warming will not exceed 1.5 degrees.[8]

These results must be understood in a context of deep uncertainty. Should the EROI and ESOI of renewables and energy storage be higher in practice than assumed here, and should the equilibrium climate sensitivity be lower than the mean the transition requirements become less stringent; the same is true if the large-scale deployment of negative emissions technologies become feasible. Should the opposite hold for the above caveats then the SFCIO-IAM model indicates that keeping warming within 1.5 degrees may be exceedingly difficult to impossible. Generating 2 degree consistent pathways is however entirely possible under sufficiently fast transition assumptions.

Unlike more conventional IAM models, the SFCIO-IAM model is constructed on largely energetic grounds and its results must be interpreted in this manner. What we have shown in this chapter is that, given the underlying energy system and climate assumptions, it is physically possible in the context of model constraints to generate energy transitions that are consistent with the 580 $GtCO_2$ carbon budget; that is, it is possible build out both the renewable and electrified manufacturing capital stocks to meet the sudden dramatically increased demand for new capital goods while also reducing emissions. To be very clear the operational word here is possible as the rates at which these processes are undertaken are dramatically larger than observed in the data.

That the transition is physically possible within the model is essentially the minimum criteria to meet as evidence that 1.5 degree pathways are still attainable. The SFCIO-IAM model does not in any manner indicate the larger social issues that may arise, for example, in the context of long term degrowth with a substantial redirection of economic activity towards renewable capacity and electrification activities.

12.4 Appendix A: Balance Sheet, Transactions Flow, and Input-Output Matrices

Figure 12.11 presents the balance sheet (stock) matrix which contains the main model variables and presents the accounting structure of the model. The entries of Figure 12.11 display the value in current prices of the assets and liabilities of each sector. The model economy is divided into six sectors: households, three production sectors (renewable sector, fossil-fuel sector, and the manufacturing sector), private banks, and the government. Each of the production sectors operates capital equipment to produce its output and finances the construction and replacement of depreciated capital via loans

[8] Though a strict comparison is not necessarily accurate given the difference in model construction, the renewable investment share of GDP is substantially larger in the SFCIO-IAM model than the linear SFCIO-ETM model indicating the importance of the added energy and climate system feedbacks introduced in chapter 10.

obtained from the banking sector. The banking sector also holds and pays interest on household deposits.

$$\begin{array}{l} \\ \text{Money} \\ \text{Deposits} \\ \text{Fixed Capital} \\ \text{Loans} \\ \text{Advances} \\ \text{Net Worth} \end{array} \begin{array}{c} \begin{array}{ccccccc} \text{Households} & \text{Renewable Firms} & \text{Fossil-Fuel Firms} & \text{Manufacturing} & \text{Banks} & \text{Government} & \Sigma \end{array} \\ \begin{pmatrix} H^h(t) & 0 & 0 & 0 & 0 & -H(t) & 0 \\ M(t) & 0 & 0 & 0 & -M(t) & 0 & 0 \\ 0 & K_e^R(t)+K_f^R(t) & K^F(t) & K_e^m(t)+K_{ne}^M(t) & 0 & 0 & K(t) \\ 0 & -L^R(t) & -L_t^F & -L^M(t) & L(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & -A(t) & A(t) & 0 \\ -V(t)* & -V^R(t) & -V^{FF}(t) & -V^M(t) & 0 & -V^g(t) & K(t) \end{pmatrix} \end{array} \tag{12.43}$$

Figure 12.11: SFCIO-IAM Balance Sheet Matrix

Figure 12.12 displays the transactions (flow) matrix where the rows record the transactions between each sector and the columns and the columns display the budget constraint of each sector.[9][10] For example, the first row of Figure 12.12 states that consumption expenditure by households at time t, denoted $C(t)$, is matched by the sales recorded in the first row for the renewable, fossil fuel, and manufacturing sectors, denoted $C^R(t)$, $C^F(t)$, and $C^M(t)$ respectively. The negative sign on the consumption term in for households indicates that this is an outflow of funds; correspondingly, the positive sign of the sales terms for each industrial sector indicates inflows of funds. The first column states that the change in the stocks of money and deposits held by households is equal to the difference between income (wages and interest on deposits) and consumption and taxation expenditures. Considering interindustry transactions Figure 12.12 shows by symmetry that the sum of payments made by, and received from, each industry to each other industry is zero and therefore the inclusion of multiple industries into the SFC framework is consistent with the requirement of zero row sums.

The interdependencies between the production sectors are shown in the following input-output table in Figure 12.13. The manufacturing sector is assumed to purchase inputs in the form of electricity and fuels from both the renewable and fossil fuel sectors as well as use some of its output as an input in its production process. The fossil fuel sector is assumed to purchase as inputs only its own output (energy in the form of fuel) to power its operations (extraction, refinement, transportation, etc.). Finally, the renewable sector does not purchase inputs from any of the sectors; rather, the renewable sector purchases capital equipment from the manufacturing sector which is recorded as investment and hence appears in final demand. The purchase of capital equipment for the other two sectors is also considered investment and hence counted in final demand.[11]

[9] See (Godley and Lavoie, 2007), Chapter 2, pages 38-40 for a more detailed discussion of the transactions flow matrix.

[10] Note well; the entries of the transaction (flow) table are flows variables and therefore have units of dollars per unit time. The entries in the balance sheet matrix are measured in dollars.

[11] Figures 12.12 and 12.13 present two different economic accounting schemes and in order to successfully merge stock-flow consistent modelling with input-output analysis it must be the case that both systems of accounting are coherent. For example, the sum of the first four entries of row one of Figure 12.13 equals $X^R(t)$ which must also equal the column sum. This implies that, after rearrangement, $WB^R(t) = z_{13} + C^r(t) + G^R(t) - (r + Z_R)L(t)$; for

$$
\begin{array}{l}
 \\
\text{Consumption} \\
\text{Government} \\
\text{Wages} \\
\text{Taxes} \\
\text{Investment} \\
\text{Inter-Industry} \\
\text{Interest Deposits} \\
\text{Interest Loans} \\
\text{Interest Advances} \\
\text{Loan Repayment} \\
\text{dMoney} \\
\text{dDeposits} \\
\text{dLoans} \\
\text{dAdvances} \\
\Sigma
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccc}
\text{Households} & \text{Renewable Current} & \text{Renewable Capital} & \text{Fossil Current} & \text{Fossil Capital} & \text{Manufacturing Current} & \text{Manufacturing Capital} & \text{Bank Current} & \text{Bank Capital} & \text{Government} & \Sigma
\end{array} \\
\begin{pmatrix}
-C(t) & C^R(t) & 0 & C^{FF}(t) & 0 & C^M(t) & 0 & 0 & 0 & 0 & 0 \\
0 & G^R(t) & 0 & G^{FF}(t) & 0 & G^M(t) & 0 & 0 & 0 & -G(t) & 0 \\
WB(t) & -WB^R(t) & 0 & -WB^{FF}(t) & 0 & -WB^M(t) & 0 & 0 & 0 & 0 & 0 \\
-T(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T(t) & 0 \\
0 & 0 & -I^R(t) & 0 & -I^{FF}(t) & I(t) & -I^M(t) & 0 & 0 & 0 & 0 \\
0 & z_{13}(t) & 0 & z_{23}(t) & 0 & -z_{13}(t) - z_{23}(t) & 0 & 0 & 0 & 0 & 0 \\
rM(t) & 0 & 0 & 0 & 0 & 0 & 0 & -rM(t) & 0 & 0 & 0 \\
0 & -rL^R(t) & 0 & -rL^{FF}(t) & 0 & -rL^M(t) & 0 & rL(t) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -rA(t) & 0 & rA(t) & 0 \\
0 & -Z_R L^R(t) & Z_R L^R(t) & -Z_{FF} L^{FF}(t) & Z_{FF} L^{FF}(t) & -Z_M L^M(t) & Z_M L^M(t) & 0 & 0 & 0 & 0 \\
-\frac{dH^h}{dt} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dH}{dt} & \frac{dH}{dt} & 0 \\
-\frac{M}{dt} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dM}{dt} & 0 & 0 \\
0 & 0 & \frac{dL^R}{dt} & 0 & \frac{dL^{FF}}{dt} & 0 & \frac{dL^M}{dt} & 0 & -\frac{dL}{dt} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{dA}{dt} & -\frac{dA}{dt} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
\tag{12.44}
$$

Figure 12.12: SFCIO-IAM Transactions Flow Matrix

$$
\begin{array}{l}
\\
\text{Renewable} \\
\text{Fossil Fuel} \\
\text{Manufacturing} \\
\text{Wages} \\
\text{Loans} \\
\hline
\text{Total Outlays}
\end{array}
\begin{array}{ccccc}
\text{Renewable} & \text{Fossil Fuel} & \text{Manufacturing} & \text{Final Demand} & \text{Total Output} \\
\end{array}
\left(\begin{array}{ccccc}
0 & 0 & z_{13}(t) & C^R(t) + G^R(t) & X^R(t) \\
0 & z_{22}(t) & z_{23}(t) & C^{FF}(t) + G^{FF}(t) & X^{FF}(t) \\
0 & 0 & z_{13}(t) & C^m(t) + G^M(t) + I(t) & X^M(t) \\
WB^R(t) & WB^F(t) & WB^M(t) & 0 & WB(t) \\
(r+Z_r)L^R(t) & (r+Z_r)L^R(t) & (r+Z_r)L^R(t) & 0 & \sum_i (r+Z_i)L^i(t) \\
\hline
X^R(t) & X^F(t) & X^M(t) & C(t)+I(t)+G(t) & X(t)
\end{array}\right) \tag{12.45}
$$

Figure 12.13: SFCIO-IAM Extended Input-Output Matrix

12.5 Appendix B: Parameter and Initial Value Tables

Note well, units with dimensions given as 1 in Figure 12.2 are dimensionless.

the model to work this must be consistent with the accounting in the transactions matrix. Indeed, substituting the expressions for $WB^R(t)$ just found into the second column of the transactions flow matrix returns an identity as it must.

Parameter Name	Symbol	Units	Value
Inverse exchange timescales between atmosphere-ocean[a]	k_a	yr^{-1}	0.2
Inverse exchange timescales between upper-lower ocean[a]	k_d	yr^{-1}	0.05
Equilibrium ratio of atmospheric to upper ocean inorganic carbon	A·B	1	Derived Quantity
Ratio of volume of lower to upper ocean[a]	δ	1	50
Ratio of the molar concentrations of CO2 in atmosphere and ocean[a]	k_h	1	1910
Disassociation Coefficient[a]	k_1	mol kg^{-1}	0.000006
Disassociation Coefficient[a]	k_2	mol kg^{-1}	0.000000000753
Number of Moles in Atmosphere[a]	AM	mol	1.77×10^{20}
Number of Moles in Ocean[a]	AM	mol	7.8×10^{20}
Net radiative forcing for a doubling of CO_2[b]	$\mathcal{F}_{2xCO_2}$	Wm^{-2}	5.35
Effective heat capacity per unit area upper ocean[b]	C	$\text{JK}^{-1}\text{m}^{-2}$	7.3
Effective heat capacity per unit area deep ocean[b]	C_0	$\text{JK}^{-1}\text{m}^{-2}$	106
Radiative Feedback Parameter[b]	δ	$\text{Wm}^{-2}\text{K}^{-1}$	1.13
Heat Exchange Coefficient[b]	γ	$\text{Wm}^{-2}\text{K}^{-1}$	0.7
Initial Fossil-Fuel Sector EROI	EROI(0)	1	30
Renewable Sector EROI	EROI_R	1	30
Round-trip AC–AC efficiency[c]	η	1	0.8
Energy Stored on Invested[c]	ESOI_e	1	30
Normal price of fossil-fuel sector output (dollars per Joule)	P_{F0}	$\$J^{-1}$	1
Price of manufacturing sector output (dollars per unit of capital)	P_{M0}	$\$k^{-1}$	10
Propensity to consume from disposable income	α_1	1	0.7
Propensity to consume from savings	α_2	1	0.3
Tax Rate	θ	1	0.3
Fraction of wealth held as deposits	λ_0	1	0.5
Interest rate modulation factor	λ_1	1	0.1
Disposable income modulation factor	λ_2	1	0.1
Power output per unit renewable capital	ϕ_R	$kW \cdot k^{-1}$	≈ 0.9
Power output per unit of fossil-fuel capital	ϕ_F	$kW \cdot k^{-1}$	3.6
Manufacturing output per unit time per unit of manufacturing capital	ϕ_M	yr^{-1}	1
Conversion factor of fossil-fuels to electricity	CF	1	2
Power requirement per unit of electrified manufacturing capital	τ_E	$kW \cdot k^{-1}$	1
Power requirement per unit of non-electrified manufacturing capital	τ_F	$kW \cdot k^{-1}$	1
Renewable capital depreciation rate	σ_R	yr^{-1}	0.03
Fossil-Fuel capital depreciation rate	σ_F	yr^{-1}	0.03
Electrified manufacturing capital depreciation rate	σ_M	yr^{-1}	0.03
Non-electrified manufacturing depreciation and decommissioning rate	σ_{ne}	yr^{-1}	0.03
Interest rate	r	yr^{-1}	0.03
Loan fraction repaid by renewable sector in given time period	Z_R	yr^{-1}	0.3
Loan fraction repaid by fossil-fuel sector in given time period	Z_F	yr^{-1}	0.3
Loan fraction repaid by manufacturing sector in given time period	Z_M	yr^{-1}	0.3
Technical coefficients matrix exogenous manufacturing sector parameter	a_{33}	1	0.05
Renewable capital stock adjustment factor	Δ_1	yr^{-1}	0.1
Fossil-Fuel capital stock adjustment factor	Δ_2	yr^{-1}	0.1
Manufacturing capital stock adjustment factor	Δ_3	yr^{-1}	0.1
Emissions Calibration Factor	ξ	EF^{-1}	1.58

Table 12.2: Table of model parameter values. Parameters noted with [a] are from (Glotter et al, 2013), [b] from (Geoffroy et al, 2013), and [c] from (Barnhart et al, 2013).

Variable Name	Symbol	Units	Initial Condition
Mass of Atmospheric CO_2[a]	$M_{at}(t)$	GtC	809
Mass of Atmospheric CO_2[a] Reference	$M_{at}(1750)$	GtC	596
Mass of Upper-Ocean CO_2[a]	$M_{up}(t)$	GtC	604
Mass of Lower-Ocean CO_2[a]	$M_{lo}(t)$	GtC	29595
Global-Average Surface Temperature[b]	$T(t)$	K	0.85
Deep Ocean Temperature[b]	$T_0(t)$	K	0.0068
Renewable Loans	$L^R(t)$	\$	3.75
Fossil-Fuel Loans	$L^R(t)$	\$	1.6
Manufacturing Loans	$L^R(t)$	\$	2
Household Wealth	$V(t)$	\$	5
Renewable Intermediate Capital	$K_e^R(t)$	\$	1.64
Renewable Final Capital	$K_f^R(t)$	\$	0.23
Fossil-Fuel Capital	$K^F(t)$	\$	8
Electrified Manufacturing Capital	$K_e^M(t)$	\$	2
Non-Electrified Manufacturing Capital	$K_{ne}^M(t)$	\$	8
Initial Government Expenditure	$G(t)$	$\$yr^{-1}$	10

Table 12.3: Table of initial conditions. Initial values noted with [a] are from (Glotter et al, 2013) and [b] from (Geoffroy et al, 2013).

12.6 Appendix C: Model Derivation

In this appendix we derive the model in its state variable form that is required to perform the simulations undertaken in the chapter. As with all previous derivations we will drop the time argument.

Let us begin, as usual, with the definition of GDP:

$$Y = C + I + G$$

Expanding out the consumption term C using the definition of the consumption function we have:

$$Y = \alpha_1 YD + \alpha_2 V + I + G$$

As with the derivation of the SFCIO-ETM model, we can expand out the disposable income term as:

$$Y = \underbrace{\alpha_1(1-\theta)}_{\omega_1}(Y - \underbrace{(r + Z_R)}_{\omega_2} L^R - \underbrace{(r + Z_F)}_{\omega_3} L^F - \underbrace{(r + Z_M)}_{\omega_2} L^M)$$

which can be rewritten as:

$$Y = \omega_1 Y - \omega_1\omega_2 L^R - \omega_1\omega_3 L^F - \omega_1\omega_4 L^M + \alpha_1 r M + \alpha_2 V + I + G$$

Now, G is an exogenously determined element of the model, so we need to obtain expressions for M and I respectively. We first turn to obtaining an expression for M in terms of Y (this is identical to how we did this in the SFCIO-ETM model).

$$M = \underbrace{\frac{1}{(1+\lambda_2 r)}}_{w_5}(\underbrace{(\lambda_0 + \lambda_1 r)}_{\omega_6} V - \underbrace{\lambda_2(1-\theta)}_{\omega_7}(Y - \omega_2 L^R - \omega_3 L^F - \omega_4 L^M))$$

which we may rewrite as:

$$M = \underbrace{\omega_5\omega_6}_{\omega_8} V - \underbrace{\omega_5\omega_7}_{\omega_9} Y + \underbrace{\omega_5\omega_7\omega_2}_{\omega_{10}} L^R + \underbrace{\omega_5\omega_7\omega_3}_{\omega_{11}} L^F + \underbrace{\omega_5\omega_7\omega_4}_{\omega_{12}} L^M$$

substituting this back into the expression for Y, we have

$$\begin{aligned} Y = \underbrace{(\omega_1 - \alpha_1 r\omega_9)}_{\omega_{13}} Y + \underbrace{(\alpha_1 r\omega_{10} - \omega_1\omega_2)}_{\omega_{14}} L^R + \underbrace{(\alpha_1 r\omega_{11} - \omega_1\omega_3)}_{\omega_{15}} L^F \\ + \underbrace{(\alpha_1 r\omega_{12} - \omega_1\omega_4)}_{\omega_{16}} L^M + \underbrace{(\alpha_2 + \alpha_1 r\omega_8)}_{\omega_{17}} V + I + G \end{aligned} \quad (12.46)$$

and finally, solving for Y, we have:

$$Y = \underbrace{\left(\frac{1}{1 = \omega_{13}}\right)}_{\omega_{18}}(\omega_{14}L^R + \omega_{15}L^F + \omega_{16}L^M + \omega_{17}V + I + G)$$

Now, in order to proceed we must obtain an expression for the investment term I as a function of the underlying state variables; to do this we must obtain expressions for the prices. First from the model equations we may specify the fossil-fuel price as:

$$P_f = \frac{X^F}{\phi_F k^F} = \underbrace{\left(\frac{L_{22}(C^e + G^e - P_r\phi_g K_f^R)}{\phi_F k_f}\right)}_{g_1} + P_f \underbrace{\left(\frac{D \cdot CF[\tau_e k_e^m - \phi_g k_e^R] + \tau_{FF}k^F}{L_{22}\phi_F k^F}\right)}_{g_2}$$

which, after some algebra, allows us to write P_f wholly as a function of the underlying state variables as:

$$P_F = \frac{g_1}{1 - g_2}$$

Now, while P_f is relatively simple to obtain, the expression for P_m is rather less so. First, let us make the following substitutions using the investment equations:

$$i_e^R(t) = \Delta_1\mu_{12}\phi_g^{-1}\tau_E k_e^m - \Delta_1 k_e^R = g_3$$

$$i_f^R(t) = \delta_1\mu_1\phi_g^{-1}P_R^{-1}[C^e(t) + G^e(t)] - \Delta_1 K^R(f) = g_4$$

$$i^F(t) = \Delta_2\mu_2\phi_F^{-1}(t)P_{F0}^{-1}X^F(t) - \Delta_2 K^F(t) = g_5$$

$$i_e^M(t) = \underbrace{(\Delta_3\mu_3\phi_M^{-1}(t)P_{M0}^{-1})}_{g_6}X^M(t) - \underbrace{(\Delta_3 k_e^M(t) + \Delta_3 k_{ne}^M(t))}_{g_7}$$

Now, let us rewrite Y as:

$$Y = \underbrace{\omega_{18}(\omega_{14}L^R + \omega_{15}L^F + \omega_{16}L^M + \omega_{17}V + G)}_{g_9} + \omega_{18}I$$

Now, I is simply the price multiplied by the sum of the different investments in physical terms above so we have that:

$$Y = g_9 + \underbrace{\omega_{18}(g_3 + g_4 + g_5 - g_7)}_{g_{10}}P_m + \omega_{18}g_6 X^m P_m$$

and therefore that:

$$Y = g_9 + g_{10}P_m + \omega_{18}g_6 X^M P_m$$

Now, since X^m is equal to $L_{33}(Y - C^e - G^e)$ we may substitute this into the expression and, after solve algebra, solve for Y as:

$$Y = \frac{g_9 + \overbrace{[g_{10} - \omega_{18} g_6 L_{33}(C^e + G^e)]}^{g_{11}} P_m}{1 - \underbrace{\omega_{18} g_6 L_{33}}_{g_{12}} P_m}$$

which we finally may write as:

$$Y = \frac{g_9 + g_{11} P_m}{1 - g_{12} P_m}$$

Having an expression for Y now in terms of P_m allows us to now find an expression for P_m itself. First, we know from the chapter that the price of manufacturing sector output is:

$$P_m = \frac{X^M}{\underbrace{\phi_m(K_e^M + k_{ne}^M)}_{g_{13}}} = \frac{L_{33}(Y - C^e - G^e)}{g_{13}}$$

Now, we may substitute in the expression for Y obtained above and, after some algebra, solve for P_m in the following form:

$$\underbrace{-g_{13} g_{12}}_{k_1} p_m^2 + \underbrace{-g_{14} g_{13} g_{12} + g_{13} - L_{33} g_{11}}_{k2} P_m + \underbrace{(g_{13} g_{14} - L_{33} g_9)}_{k_3} = 0$$

or, more simply:

$$k_1 P_m^2 + k_2 P^m + k_3 = 0$$

This we of course recognize as a quadratic solution whose solution is given as:

$$P_m = \frac{-k_2 \pm \sqrt{k_2^2 - 4 k_1 k_3}}{2 k_1}$$

With P_f and P_m in hand we can obtain X^F and X^m and hence the the investment terms. Ultimately, we can work backwards to find Y fully as a function of the underlying state variables. With the derivation of P_m the remaining flow variables (e.g., C and YD) follow by simple substitution. With this information, we can write each of the differential equations in table 12.1 in state variable form as required for obtaining numerical solutions to the differential equations.

Chapter 13

Chapter Thirteen: A Two Country Trade Model

The results from the SFCIO-IAM model of the previous chapter concerning the conditions under which 1.5 degree emissions pathways are achievable were obtained in a one country closed economy framework. An important outcome of this analysis was that the assumption of degrowth (as defined by declining consumption and government expenditures) made the generation of 1.5 degree consistent emissions trajectories more readily attainable. This result is however somewhat problematic as, in practice, economies are virtually never realistically fully closed, and spillovers from change in one country may naturally have substantial impacts on other trading partner countries.[1] In this chapter we will ask questions such as the following: how does the presence of growing trading partner impact an economy attempting to undergo degrowth as defined above?

At the core of this chapter lies an arguably understudied issue; the relation between degrowth and international trade. While vast literatures exist on both subjects separately, there is to this author's knowledge no explicit formal modelling of intentional degrowth in an international trade setting. A core theme of this dissertation has been the exploration of steady-state and degrowth pathways as routes to meet climate goals, but the analysis so far suffers from a weakness that only one country was ever considered explicitly. In reality there are many countries with different characteristics pursing climate related goals to different degrees which possibly undermines the simple story of the previous chapter.

In this chapter we present a two country degrowth trade model by extending the SFCIO-IAM model to incorporate trade in fossil-fuels and manufacturing sector output. Though the extension is conceptually quite simple, the model is mechanically dramatically more complex than all previous models requiring a greater degree of numerical methods to obtain simulation results. As with the model developed in chapter 8, the trade model to be elaborated here is too complex to solve into the

[1]Note well, the move to modelling two countries in an IAM format implies that the previous SFCIO-IAM model was an implicitly global one. Why? The climate system is a global phenomenon and integrating it with a "one country" macroeconomic framework as done in the previous chapter requires said on country to actually represent the globe. This ust be the case to avoid a mismatch between the scales under consideration.

standard state-variable form used in the derivation of the previous models. Where generally analytical expressions could be obtained for the prices and ultimately the model differential equations, this is no longer feasible. As such, we will present the important new model equations in the body of the chapter and leave the full formal presentation of the model to the chapter appendix.

The focus of the qualitative analysis to be done in this chapter will lie in comparing three types of model formulation: (1) two regions with no trade; (2) trade between two regions assuming no capital mobility; and (3), trade between two regions allowing for fossil-fuel and manufacturing capital to become mobile. The following section introduces the basic trade framework with the capital mobility mechanism introduced later in the chapter.

13.1 Imports and Exports

In order to keep the model as tractable as possible we will examine trade between two countries using the same currency (i.e., two countries in a monetary union) as to side-step the added complications of modelling separate currencies. We assume the two regions have identical structures; that is, each possesses a renewable, fossil-fuel and manufacturing sector and are described by identical equations with only the parameter values differing. Finally, the manufacturing sectors in each region are assumed to purchase some proportion of their inputs (fossil fuel derived energy and manufacturing goods) from within the region and the remaining proportion from the other region based on the relative price of these inputs.

It is worth briefly considering how these assumptions differ from the previous models. In the SFCIO-IAM model of the previous chapter, for example, the technical coefficient a_{33} was assumed constant; that is, it denoted a constant requirement of domestic inputs per unit of output.[2] The manufacturing sector in that model would use its own output as intermediate goods in constant proportion to its total output. In allowing the manufacturing sectors to purchase inputs either domestically or from the other region it is clear that this assumption for constant technical coefficients, as relates to domestically produced inputs to total outputs, no longer holds. Each region's manufacturing sector will purchase a share of its inputs from itself and the remaining share from the other regions manufacturing sector. These shares, determined by relative prices, will conspire to make the domestic manufacturing sector a_{33} technical coefficient endogenously determined in the model.[3]

Let us denote a_{33}^A and a_{33}^B as the technical coefficients for each sector and note that these now define the dimensionless ratios of manufacturing inputs to total manufacturing inputs that must be met

[2]Since the SFCIO-IAM has only a single region all outputs are by definition domestically produced.

[3]That is, the ratio of domestic inputs to total outputs is allowed to vary as the remaining portion is made up by imported inputs.

regardless of where the inputs originate from.[4] Now, for simplicity, let us assume that the manufacturing firms that comprise the manufacturing sectors in each region are agnostic as to where the manufacturing inputs they purchase come from when the manufacturing prices are equivalent but purchase relatively more or less from one region or another when a difference in relative prices emerges. One way to formalize this is to redefine the manufacturing sector technical coefficients for each region as follows:

$$a_{33}^{A}(t) = \frac{P_{mb}}{P_{ma} + P_{mb}} a_{33}^{A} \tag{13.1}$$

and for region B,

$$a_{33}^{B}(t) = \frac{P_{ma}}{P_{ma} + P_{mb}} a_{33}^{B} \tag{13.2}$$

The expression for $a_{33}^{a}(t)$ has two important features to note. First, the price ratio $\frac{P_{mb}}{P_{ma}+P_{mb}}$ is dimensionless as necessary so that the LHS is dimensionless as necessary for a technical coefficient. Second, the price ratio acts as a weight; if the price of manufacturing sector output in region B (denoted P_{mb}) is higher than that in region A (denoted P_{ma}) then the manufacturing sector in regions A will purchase relatively more of its own output as input; the manufacturing sector in region B will correspondingly purchase relatively more of its inputs from region A and less of its own. When the prices are equivalent, the weights are the same and each sector purchases half of its inputs locally and half from the other region.[5]

Now, since $a_{33}^{a}(t)$ is only a fraction of the total technical coefficient a_{33}^{a} the interindustry flow of inputs from the region A manufacturing sector to itself is less than the total required to produce the total output $X_{a}^{M}(t)$. Recall that a generic input-output technical coefficient is defined as $a_{ij} = \frac{z_{ij}}{X_j}$ or simply as the ratio of the interindustry transaction to that of the total output. We can rearrange this expression and solve for the interindustry transaction in our model as follows:

$$z_{33}^{A}(t) = a_{33}^{A}(t) X_{A}^{M}(t)$$

Since, by construction, $a_{33}^{a}(t) \leq a_{33}^{a}$, the above flow of intermediate goods $z_{33}^{A}(t)$ is smaller than the total required. This difference is naturally made up by manufacturing sector imports whose magnitude is expressible via the following:

$$a_{33}^{A} = \frac{P_{mb}}{P_{ma} + P_{mb}} a_{33}^{A} + \frac{P_{ma}}{P_{ma} + P_{mb}} a_{33}^{A}$$

[4] For example suppose the manufacturing sector, overall, requires 2 widgets to produce an output of 10 widgets. While it may source more or less of the required widgets from either region it must ultimately purchase the same quantity of material inputs.

[5] This agnosticism of where inputs are sourced from is possible because we do not model transport costs. Put another way, transport costs of goods from one region to another are zero and not relevant to the cost calculations made by firms.

where the first term on the RHS of the above equation is the fraction of the total technical coefficient made up by domestic inputs and the second term on the RHS is the residual fraction made up by imports. Hence, the manufacturing sector in region A will also import

$$\frac{P_{ma}}{P_{ma}+P_{mb}}a_{33}^{A}X_{A}^{M}(t)$$

worth of inputs from the region B manufacturing sector.

Since region A's imports are by definition region B's exports the quantity in the previous expression is the quantity of manufacturing sector output exported by region B to region A. By symmetry, the quantity of region A manufacturing output exported to region B is give as:

$$\frac{P_{mb}}{P_{ma}+P_{mb}}a_{33}^{B}X_{B}^{M}(t)$$

In each case the numerator of the price ratio is the region's manufacturing price which implies that the higher the domestic price of manufacturing goods, the larger will be the magnitude of imported manufacturing inputs.

13.1.1 Fossil-Fuel Imports and Exports

The manufacturing sectors in both regions also use fossil-fuel derived energy as an input into their production processes; what proportion of these inputs is sourced locally and what portion is purchased from the other region follows a similar logic to that of the previous section. From the previous chapter, the quantity of fossil-fuel inputs required by the manufacturing sector, in physical terms, is given as:

$$CF[\tau_E k_e^M(t) - \phi_g(t)k_e^R(t)] + \tau_{FF}k_{ne}^m(t)$$

Assuming appropriate superscripts to denote regions A and B, the above expression will be denoted by the shorthand $\pi^A(t)$ and $\pi^B(t)$ respectively. To be clear, these two terms represent the magnitude of fossil-fuel inputs required by a manufacturing sector to produce its output. As before, some fraction of this requirement is assumed to be met by fossil-fuel energy produced locally, and some fraction by imports from the other region.

The same price weighting scheme may therefore be deployed where both region A and region B manufacturing sectors purchase relatively more of one regions fossil-fuels given a lower price in that region. Region A fossil-fuel imports from region B may therefore be give as:

$$\frac{P_{fa}}{P_{fa}+P_{fb}}\pi^A(t) \tag{13.3}$$

The higher is P_{fa} the larger is the weight $\frac{P_{fa}}{P_{fa}+P_{fb}}$ and hence the greater is the magnitude of the imports (in physical terms) of fossil-fuel goods by the manufacturing sector in region A. The quantity purchased domestically is given as:

$$\frac{P_{fb}}{P_{fa}+P_{fb}}\pi^A(t) \tag{13.4}$$

Again, by simple inspection, these two quantities sum to $\pi^A(t)$ as required so that, though the shares may change, the total is invariant to changes in relative prices. For completeness, the imports and exports of fossil-fuels in region B are given as:

$$\frac{P_{fb}}{P_{fa}+P_{fb}}\pi^B(t) \tag{13.5}$$

and

$$\frac{P_{fa}}{P_{fa}+P_{fb}}\pi^B(t) \tag{13.6}$$

respectively. Now, since the weights are dimensionless these expressions are still in physical terms. To obtain the monetary value of exports and imports it is necessary to premultiply the above quantities by the relevant price at which they are valued. For example, the imports of fossil-fuels by Region A from region B is valued at the fossil-fuel sector price prevailing in region B (namely P_{fb}) and therefore the dollar value of these imports is given as:

$$P_{fb}\frac{P_{fa}}{P_{fa}+P_{fb}}\pi^A(t)$$

13.1.2 Net Exports

Having determined expressions for the exports and imports of fossil-fuels and manufacturing sector outputs for both regions we may write expressions for net exports. The exports for region A are given as follows:

$$X^A(t) = P_{fa}\frac{P_{fb}}{P_{fa}+P_{fb}}\pi^B(t) + \frac{P_{mb}}{P_{ma}+P_{mb}}a_{33}^B X_B^M(t) \tag{13.7}$$

while the imports are:

$$M^A(t) = P_{fb}\frac{P_{fa}}{P_{fa}+P_{fb}}\pi^A(t) + \frac{P_{ma}}{P_{ma}+P_{mb}}a_{33}^A X_A^M(t) \tag{13.8}$$

and finally, net exports are defined in the obvious way as the difference in exports minus imports:

$$NX^A(t) = X^A(t) - M^A(t) \tag{13.9}$$

The expression for region B is found similarly with appropriate changes in superscripts. A relatively obvious and necessary feature in this two-country trade model is that the sum of net export terms for both regions is zero; that is, one region's trade surplus is necessarily the other regions trade deficit.

13.1.3 Leontief Coefficients

As with the previous models incorporating dynamic prices, it is necessary to obtain expressions for the total sectoral outputs in order to obtain the pricing rules that match physical supply and demand. As exports of intermediate goods are elements of the final demand vectors these must be accounted for in the total output calculations.

The following matrix of technical coefficients is similar to that deployed in the previous chapter except modified to account for the fact that the manufacturing sector obtains only some if its inputs locally. This modification takes for the form of price weighted technical coefficients. The matrix shown is for region A; the matrix for region B is structurally identical except for the necessary changes in region identifying superscripts. Note well, we denote the region by a superscript A or B. The region A specific version of electrified manufacturing capital K_e^M is written as K_e^{AM}. The region identifier will always be the first term in the superscript in the following.

$$A^A(t) = \begin{pmatrix} 0 & 0 & \frac{P_r^A \phi_g^A k_e^{AR}(t)}{X^{AM}(t)} \\ 0 & \frac{S^A(0)}{EROI^A(0) \cdot S(t)} & \frac{P_{fa} \frac{P_{fb}}{P_{fa}+P_{fb}} \left[CF^A[\tau_E^A k_e^{AM}(t) - \phi_g^A(t) k_e^{AR}(t)] + \tau_{FF}^A k_{ne}^{Am}(t)\right]}{X^{AM}(t)} \\ 0 & 0 & \frac{P_{mb}}{P_{ma}+P_{mb}} a_{33}^A \end{pmatrix} \tag{13.10}$$

Solving $(I - A^A(t))^{-1}$ for the individual elements the following Leontief coefficients can be obtained. These are largely similar to those in the previous chapter but are written here for clarity and completeness. First, the matrix determinant, denoted $D^A(t)$, is given as:

$$D^A(t) = \frac{1}{\left(1 - \frac{S^A(0)}{EROI^A(0) \cdot S^A(t)}\right)\left(1 - \frac{P_{mb}}{P_{ma}+P_{mb}} a_{33}^A\right)}$$

This determinant can be used to obtain the following Leontief coefficients:

$$L_{11}^A(t) = 1, \quad L_{12}^A(t) = 0, \quad L_{13}^A(t) = \frac{P_r^A \phi_g^A(t) k_e^{AR}(t)}{\left(1 - \frac{P_{mb}}{P_{ma}+P_{mb}} a_{33}^A\right) \cdot X^{AM}(t)} \tag{13.11}$$

$$L_{21}^A(t) = 0, \quad L_{22}^A(t) = \frac{1}{\left(1 - \frac{S^A(0)}{EROI^A(0) \cdot S^A(t)}\right)} \tag{13.12}$$

$$L_{23}^A(t) = \frac{P_{fa} \frac{P_{fb}}{P_{fa}+P_{fb}} \left[CF^A[\tau_E^A k_e^{AM}(t) - \phi_g^A(t) k_e^{AR}(t)] + \tau_{FF}^A k_{ne}^{AM}(t)\right]}{\left(1 - \frac{S^A(0)}{EROI^A(0) \cdot S^A(t)}\right)\left(1 - \frac{P_{mb}}{P_{ma}+P_{mb}} a_{33}^A\right) \cdot X^{AM}(t)} \tag{13.13}$$

$$L^A_{31}(t) = 0, \quad L^A_{32}(t) = 0, \quad L^A_{33} = \frac{1}{(1 - \frac{P_{mb}}{P_{ma}+P_{mb}} a^A_{33})} \tag{13.14}$$

13.1.4 Prices and Model Comparability

The fossil-fuel sector in region A must produce its output to meet the regions final demand for fossil-fuel energy, the amount required as inputs by the local manufacturing sector, and the amount to be exported as inputs to the foreign manufacturing sector. Since exports are counted in the final demand vector the region A fossil-fuel sector faces the following final demands for its output:

$$\underbrace{C^A_e(t) + G^{Ae} - P^A_r \phi^A_g(t) k^{AR}_e(t)}_{g_1} + P_{fa} \frac{P_{fb}}{P_{fa} + P_{fb}} \pi^B(t)$$

the region A fossil fuel sector must also produce

$$P_{fa} \frac{P_{fb}}{P_{fa} + P_{fb}} \underbrace{\left[CF^A[\tau^A_E k^M_e(t) - \phi_g(t) k^{AR}_e(t)] + \tau^A_{FF} k^{AM}_{ne}(t) \right]}_{g_2}$$

dollar's worth of output to meet local intermediate manufacturing demand for inputs. Using the under-braced terms, the expression for total fossil-fuel sector in region A is:

$$X^{Af}(t) = L^A_{22}(t)(g_1 + P_{fa} \frac{P_{fb}}{P_{fa} + P_{fb}} \pi^B(t)) + L^A_{23}(t) P_{fa} \frac{P_{fb}}{P_{fa} + P_{fb}} \cdot g_2$$

Finally, the fossil-fuel sector price in region A is set, as in previous models, to balance physical supply and demand:

$$P_{fa}(t) = \frac{X^{Af}(t)}{\phi^A_f k^A_f(t)} \tag{13.15}$$

A symmetric expression describes the fossil-fuel price in region B. What is clear from equation (13.15) is that the price of fossil-fuels in one region is dependent on the price of fossil-fuels in the other region. The nature of this dependency is more fully explored in the chapter appendix, but it suffices to state here that the price of fossil-fuels in region A can be expressed as:

$$P^2_{fa} + \beta^A_1 P_{fa} + \beta^A_2 = 0$$

where the β terms are functions of P_{fb} and other model variables. A symmetric expression for the price of fossil-fuels in region B is:

$$P^2_{fb} + \beta^B_1 P_{fb} + \beta^B_2 = 0$$

As these are two quadratics in two unknowns, we use numerical methods to simultaneously solve the above two expressions in order to determine the prices in each region. [6]

As with the price of fossil-fuels we must rely on numerical methods to obtain values for the manufacturing sector price which shares the feature that each region's price is a function of the others. While the method to obtain the fossil-fuel prices is relatively straightforward, it is considerably more complex in the case of manufacturing prices as simply obtaining the expressions to be used in the numerical routine is non-trivial.

The final demand for the output of the manufacturing sector in region A is comprised of the demand by households and governments $C^{Am}(t) + G^{Am}(t)$ as well as the total investment demand by the industrial sectors $I^A(t)$. In addition to these elements is the demand by the region B manufacturing sector for region A manufacturing sector output to use as inputs to its own production process which is given above as $\frac{P_{mb}}{P_{ma}+P_{mb}} a_{33}^B X^{BM}(t)$. Therefore, we may write the expression for total sectoral output as:

$$X^{Am}(t) = L_{33}^A(t) \left[C^{Am}(t) + G^{Am}(t) + I^A(t) + \frac{P_{mb}}{P_{ma} + P_{mb}} a_{33}^B X^{BM}(t) \right]$$

Here we see, as expected that the total output of region A manufacturing is dependent on the total output of region B manufacturing. Now, using the identity equation for gross domestic product it is possible to rewrite the LHS of the above in a more useful form. First, the national income of region A is:

$$Y^A(t) = C^{Ae}(t) + C^{Am}(t) + G^{Ae}(t) + G^{Am}(t) + I^A(t) + NX^A(t) \tag{13.16}$$

It follows from the definition of net exports above that:

$$\begin{aligned} C^{Am}(t) + G^{Am}(t) + I^A(t) + \frac{P_{mb}}{P_{ma} + P_{mb}} a_{33}^B X^{Bm}(t) &= Y^A(t) - C^{Ae}(t) - G^{Ae}(t) \\ &- \frac{P_{fa} P_{fb}}{P_{fa} + P_{fb}} (\pi^B(t) - \pi^A(t)) + \frac{P_{ma}}{P_{ma} + P_{mb}} a_{33}^A X^{AM}(t) \end{aligned} \tag{13.17}$$

Substituting this expression back into above equation for region A manufacturing sector total output and bringing the LHS $X^{m,A}(t)$ over to RHS we obtain an expression of the form:

$$X^{AM}(t) = \Omega^A \left[Y^A(t) - C^{Ae}(t) - GAe(t) - \frac{P_{fa} P_{fb}}{P_{fa} + P_{fb}} (\pi^B(t) - \pi^A(t)) \right] \tag{13.18}$$

[6] It is worth nothing that it is relatively easy to obtain an analytical expression for one regions fossil-fuel price by employing the quadratic formula on either of the above polynomial expression. However, this leads to an expression that is still in terms of the second price. Substituting this back into the other price leads to an immensely complicated non-linear expression for which there is very likely no analytical formula to solve.

where ,

$$\Omega^A = \frac{L_{33}^A}{1 - L_{33}^A \cdot \frac{P_{ma}}{P_{ma}+P_{mb}} a_{33}^A}$$

This parameter is particularly important when considering the comparability of the SFCIO-TRADE model with the simpler SFCIO-IAM model and more will be said shortly. Now, to obtain expressions for the manufacturing price it is necessary to first obtain expressions for $Y^A(t)$ and the symmetric expressions for $Y^B(t)$. We leave the method to do this to the chapter appendix and note that with such expressions in hand the manufacturing prices are found by jointly solving the following two equations:

$$\frac{P_{ma} - \Omega^A \left[Y^A(t) - C^{Ae}(t) - G^{Ae}(t) - \frac{P_{fa}P_{fb}}{P_{fa}+P_{fb}}(\pi^B(t) - \pi^A(t))\right]}{\phi_m^A(K_e^{AM} + K_{ne}^{AM})} = 0$$

$$\frac{P_{mb} - \omega^B \left[Y^B(t) - C_B^e(t) - G_e^B(t) - \frac{P_{fb}P_{fa}}{P_{fb}+P_{fa}}(\pi^A(t) - \pi^B(t))\right]}{\phi_m^B(K_e^{BM} + K_{ne}^{BM})} = 0$$

13.1.5 Baseline and Model Comparability

To capture the “pure” impact of trade on key model outcomes (aggregate emissions for example) we must make comparisons of the following nature. Suppose we have a world compromised of two identical closed economies and at some time t_0 one of the two economies decides to pursue a strategy of long-term degrowth to assist in reducing emissions as in the previous chapter. From this, we may compute the emissions arising from each economy and produce a global total. Capturing the impact of trade is a relatively “simple” matter of redoing the same experiment and calculating global emissions, but assuming that at time t_0 that trade occurs rather than not. Then, it is just a matter of comparing the global emissions trajectories resulting from both experiments to infer the impacts of trade.

This strategy of comparing trade against a baseline of no-trade is conceptually sensible but entails several complications. Given how the SFCIO-TRADE model is defined in this chapter it always assumes that some magnitude of trade is occurring and hence it is not possible to isolate one region from changes in the other. As such the SFCIO-TRADE model cannot itself produce a baseline no trade scenario where the two regions act independently. However, as we will show in this section we can make use of the IAM model developed in the previous chapter to model each region separately by exploiting the fact that under assumptions of no net trade, the SFCIO-TRADE model reduces to two independent versions of the SFCIO-IAM model.

It is therefore necessary to show that no-trade as modelled by two independent SFCIO-IAM models is in fact a special case of the larger SFCIO-TRADE model and can be used as an approach to generate a baseline. To show this we note that when net trade is exactly zero, the SFCIO-TRADE model regions reduce to identical individual SFCIO-IAM regions. For example, consider the parameter Ω^A from the previous subsection which acts as the trade model variant of the more typical manufacturing sector Leontief coefficient. By construction, if there are differences in the prices between region A and B fossil-fuel and manufacturing sector outputs then trade will occur where both regions will demand more of the relatively cheaper outputs. Therefore, let us begin by assuming that prices are identical across both regions and hence there is no net trade.[7] Now, if net trade is zero and prices are identical then $L_{33}^A(t)$ reduces to $\left(1 - \frac{a_{33}^A}{2}\right)^{-1}$. As such we may write:

$$\Omega^A = \frac{\left(\frac{1}{1-\frac{a_{33}^A}{2}}\right)}{1 - \left(\frac{1}{1-\frac{a_{33}^A}{2}}\right)\frac{a_{33}^A}{2}}$$

which, ultimately reduces to:

$$\Omega^A = \frac{1}{1 - a_{33}^A}$$

which is identical to the calculation for the manufacturing sector Leontief coefficient in the previous chapter. All that remains now is to note that when net trade is zero $\pi^B(t) - \pi^A(t)$ is zero and so the term in square brackets in (13.18) reduces to the standard vector of final demands faced by the manufacturing sector in the SFCIO-IAM model. Ultimately if both regions of the SFCIO-TRADE model are parametrized and initialized in the same manner as the SFCIO-IAM model then the total outputs of the manufacturing sectors in the trade model are identical to those of the simpler IAM model.

Because we do not model any of the possible costs associated with trade (transport costs for example) then net zero trade in the SFCIO-TRADE model is numerically equivalent to two independent SFCIO-IAM models; it is this feature we will exploit to compare the two models. Now, having described that main theoretical additions to the model we will examine a series of simpler experiments to explore the impact of the new model mechanics.

13.2 A Trade Example

Let us begin by examining a relatively simple scenario where each country possesses an absolute advantage in terms of producing one of the two traded goods (fossil-fuel and manufacturing sector outputs) Let us suppose that the fossil fuel sector in region A faces a lower EROI than region B and consequently

[7] To be clear in this scenario both regions produce half of inputs required by their manufacturing sector's and import the other half. Numerically, given identical economies, this is equivalent to a scenario of no trade where both economies produce all of their own inputs. This is the case as we do not model any of the costs associated with trade.

the sector sets its normal price (recall this is defined in chapter 7) at some level higher than region B; similarly, the manufacturing sector in region B is less productive (ϕ_M^B is smaller than ϕ_M^A), and region B manufacturers set a normal price slightly higher than region A. Furthermore, assume that both countries (before opening trade) are in approximate steady-states.[8] Figure 13.1 presents the evolution of key model components assuming the opening of trade in 2020.[9]

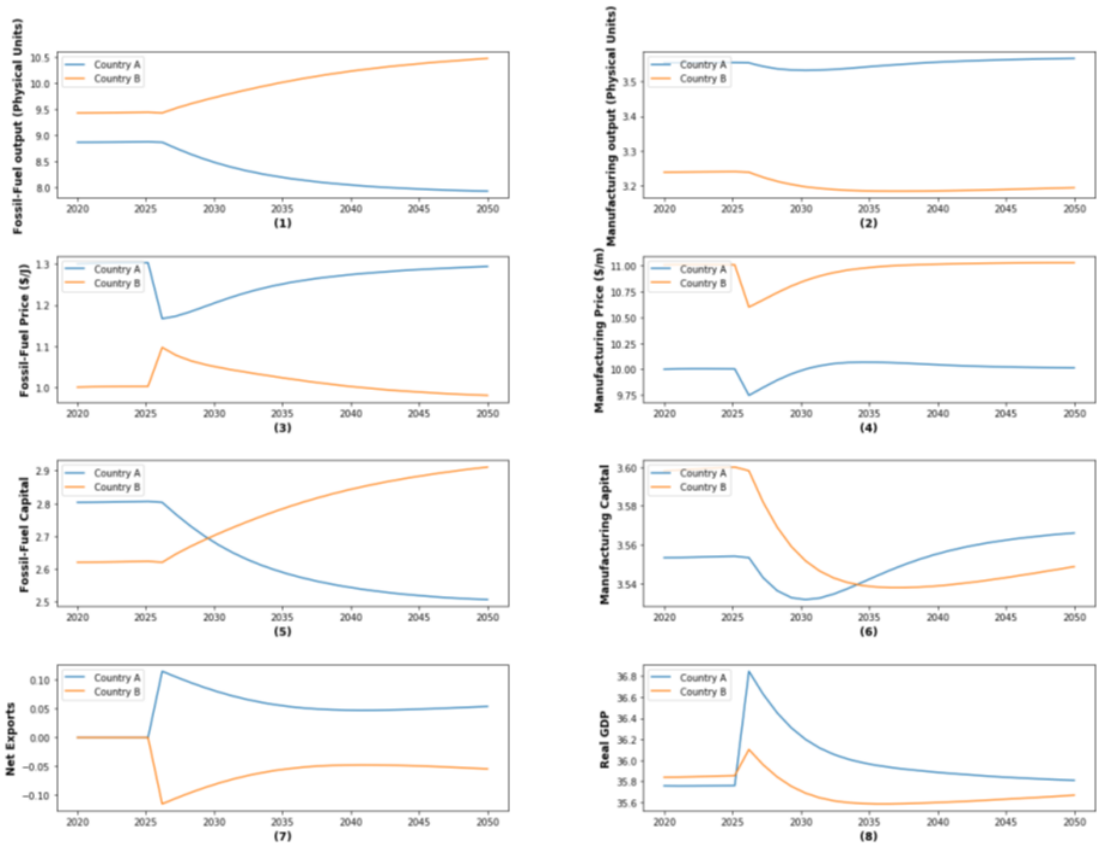


Figure 13.1: Evolution of key SFCIO-TRADE Model Variables from 2020 to 2050 assuming the opening of trade in 2025. **Region A**: Assuming $\phi_F^A \approx 3.16$, $P_{f0}^A = 1.3$, $\phi_M^A = 1$, and $P_{m0}^A = 10$. **Region B**: Assuming $\phi_F^B = 3.6$, $P_{f0}^B = 1.0$, $\phi_M^B = 0.9$, and $P_{m0}^B = 11$.

Recalling that region A has a production advantage in manufacturing and region B a production advantage in fossil-fuel production we see that over the model run horizon, both regions undergo specialization. For example, Figure 13.1 panel (5) shows that, over time, region A reduces the magnitude

[8] Recall that these steady-states are approximate as the stock of fossil-fuels declines continuously with fossil-fuel sector activity which drives certain model variables. There is no true-steady state as eventually depletion will lead to exhaustion. The approximate steady-state here is not a rigorously defined notion. It should be taken simply to mean that the values of model variables at the outset of the simulation will change very minimally over the model run period assuming no other perturbations.

[9] This is of course a somewhat synthetic scenario as trade generally does not go from not occurring to occurring in one discrete moment.

of its fossil-fuel manufacturing stock, while region B increases theirs. This dynamic plays out in reverse concerning each region's manufacturing sector capital. Put simply, each region moves more heavily into the sector in which is has the productive advantage while simultaneously reducing its activities in the other. This result may be seen in panels (1) and (2) where the outputs (in physical terms) of both sectors increase and decrease depending on each region's comparative advantage.

Panel (6) also indicates that the manufacturing capital stocks adjust to slightly lower levels over the model run. This arises from two effects. First, the relative specialization in fossil-fuel production by region B (and the relative de-specialization by region A) implies that the total world demand for fossil-fuels in physical terms can be met by a relatively smaller but more productive total fossil-fuel capital stock. A lower total requirement also implies lower depreciation losses over time and hence lower demand on the manufacturing sectors to cover depreciation losses. Secondly, this same dynamic occurs for the manufacturing sectors as well. The relative specialization in the region with the more efficient manufacturing sector leads to lower global requirements. These two effects overall lead to the slight decline in total manufacturing capital as displayed.

The normal price assumption is one with considerable impact which is evident from an examination of panels (3) and (4). The fossil-fuel price in region A, which assuming no trade would simply approximate the normal price of 1.3 is lower (but increasing) over the entire model run. Why? The manufacturing sector in region A will, noting the difference in relative prices of its fossil-fuel inputs across both regions purchase relatively more of the cheaper region B fossil-fuels and relatively less from region A. This decreased demand in region A (and consequently increased demand in region B) for fossil-fuels leads to initially lower and higher fossil-fuel prices in region A and B respectively. However, as firms adjust their capital stocks, the prices ultimately trend back towards the normal prices. So, while the discrepancy in prices from normal prices arising from the introduction of trade is ultimately transitory, the correction process is not at all rapid.

Panel (7) displays the magnitude of each region's net exports, while panel (8) displays the trajectories of real GDP (assuming normal prices as the base-year prices). As SFCIO-TRADE is comprised of only two regions the net exports in both regions are necessarily of the same magnitude but opposite sign so that the total is identically zero.[10] Having examined a basic trade scenario, we turn to a more targeted set of experiments. We will now assume two completely identical economies at the outset and explore the changes resulting from a modification of one model parameter only.

[10]As one regions exports is by definition the other regions imports the zero summation of both regions net exports terms is numerically necessary. Were this not the case, it would imply that some trade activity is not accounted for or misspecified.

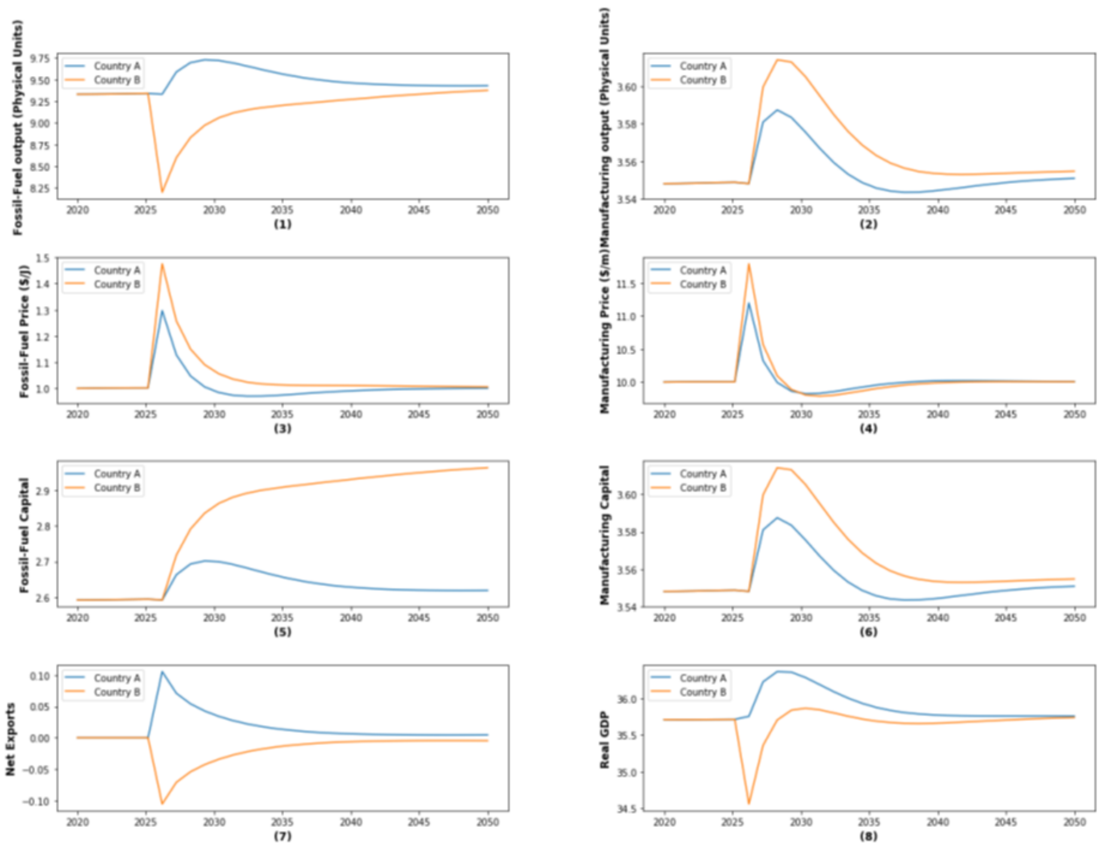


Figure 13.2: Trajectories of select variables for regions A and B following a sudden decline in ϕ_F^B from 3.6 to 3.16 in 2025.

13.2.1 A Permanent Change in EROI

Let us now consider the impact of a sudden and permanent decline in the fossil fuel sector productivity parameter ϕ_F occurring at year 2025. This decline in the productivity parameter implies that the physical output of fossil-fuels from region B is in 2025 suddenly lower than the previous year. The following figure plots the evolution of the same key variables as the model adjusts to the sudden and permanent shock.

Examining panel (1) of Figure 13.2 we see the aforementioned sudden decline in fossil-fuel output in region B. As no other aspect of the model is changed initially, the demand for fossil-fuels exceeds the capacity (at the initial price) leading to a sharp increase in region B fossil-fuel prices and a lesser but still significant jump in region A fossil fuels prices arising due to increased export demand. In response to the lower productivity parameter (and due to the assumption that the normal price of fossil-fuels is unchanged) the fossil-fuel sectors in both regions build out permanently larger fossil-fuel capital-stocks.[11]

The sudden increase in both region A and B fossil-fuel capacity is facilitated by a similarly sudden increase in real investment leading to a transient surge in demand for manufacturing sector output. This is reflected in the spike in prices and the more gradual increase and decrease in manufacturing capital stocks in both regions. Finally, real GDP increases slightly in both regions temporarily capturing the spike in activities associated with the adjustment processes in the capital stocks and prices. This effect is not large but is worth exploring in greater detail. First, however, it is necessary to say something about capacity constraints.

The sudden decrease in global fossil-fuel output capacity leads to a spike in investment. It should be recalled that at year 2025 (when the shock occurs) the total world manufacturing capacity is simply $\phi_M^A K_{me}^A(t) + \phi_M^B K_{me}^B(t)$. The sudden surge in demand for new fossil-fuel capacity (to meet the lower EROI) implies a sudden increased demand for manufacturing sector output (in the form of capital goods). By logic, the increase in the share of the total physical output of manufacturing sector goods going to investment necessitates a decline in the shares of some other use of these goods.[12] The change in physical output shares are detailed in the following figure which plots the evolution of the components of real GDP.

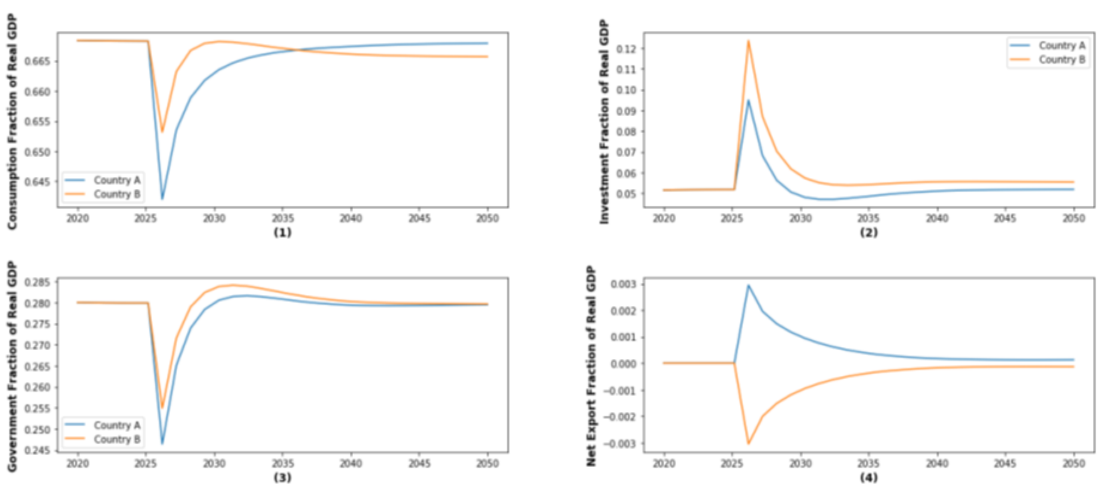


Figure 13.3: Evolution of the component shares of real GDP for **Region A** (blue plots –) and **Region B**: (orange plots –)

Panel (2) of Figure 13.3 displays the spike in investment occurring in both regions in real terms.

[11]It is important to be clear here on the role the normal price plays. because we assume only that the EROI in region B changes (and nothing else) the normal price of fossil-fuels (the price which the fossil-fuel sector uses as the target to build out its capital stock so as to meet demand at that price) remains unchanged. Therefore, the fossil-fuel sector in region B must build out additional capacity over its initial level to meet demand when, all else equal, it is less productive.

[12]Note well, this implies that there is no spare capacity in the production sectors.

This increased share of real output is matched by declines in both the government and consumption shares of real GDP. The shares, after capital stock adjustments occur, return to approximately their initial values. Figure 13.3 presents, in a sense, a physical reality check. Since the model assumes that the economy is already operating at full capacity, the sudden surge in demand for the output of the manufacturing sector as investment goods must be realized as a redirection of this output from other uses.

13.2.2 A Permanent Increase In Government Expenditures

Now, suppose that instead of a sudden decline in the productivity of a fossil-fuel sector there is a sudden and permanent 20% increase in government expenditures for both energy and manufacturing sector goods in region A. The following figure plots the evolution of the model in response to this:

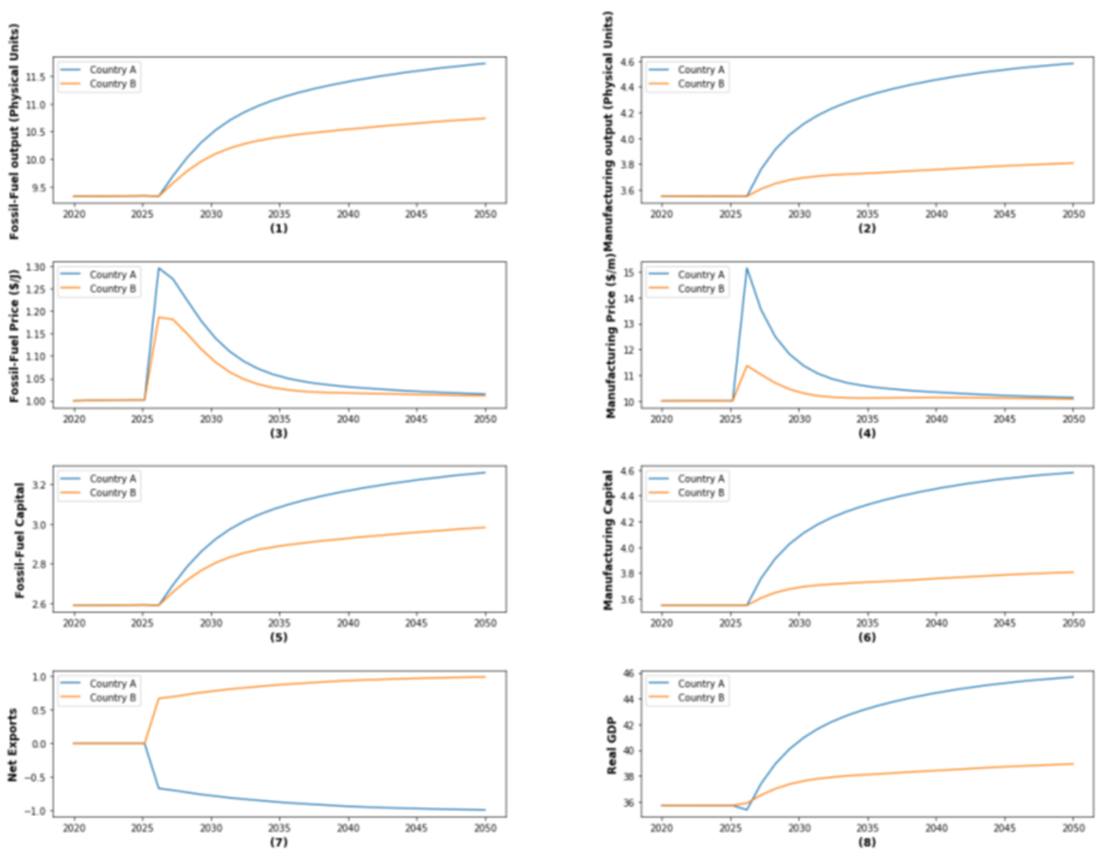


Figure 13.4: Trajectories of select variables for regions A and B following a sudden increase in government expenditures from 10 to 15 in region A in 2025.

As before, the sudden increase in demand for the physical output of both sectors is met initially with higher prices as capacity is assumed to be fully utilized. Overtime, the capital stocks of both sectors in both regions adjust driving down the prices back towards the initial (normal) price. As shown in panels (5) and (6) the capital stocks of all sectors are permanently larger after adjustment in order to meet the increased global demand. However, since the model is operating at full capacity, this increase in government spending is, initially, purely inflationary. Panels (3) and (4) indicate that prices jump immediately with the increase in government expenditure and decline with the adjustment of the capital stocks.

The permanent increase in government spending in region A leads to a higher steady-state value of real-GDP in region A; however, the spill-over effects due to the interconnectedness of both regions implies that the same dynamic (to a lesser extent) occurs in region B as well. This spill-over effect raises a distinct problem concerning a region attempting to undertake a transition to a steady-state or degrowth future. Unlike the SFCIO-IAM model where degrowth could be imposed by simply modifying the exogenous consumption and government expenditure terms, the existence of trade spillovers must now be accounted for. We will turn to this question in a following section after we define how capital mobility is captured in the SFCIO-TRADE model.

13.2.3 Capital Mobility

Given the relative complexity of the SFCIO-TRADE model we will assume a fairly simple notion of what constitutes capital mobility and the mechanism by which it occurs. First, we will take a rather literal approach to capital mobility and assume it refers to the actual physical capital. Second, we will define the mobility of capital as the transfer of both the physical capital and the financial liability that funded the capital's construction from one region to another. In order to capture this story of capital mobility in the model we must define some notion of profits. Unlike the SFCIO-FIN model of chapter 7 profits are not an explicit term in this model but an operational definition can be obtained. We will derive the following profits and mobility terms for the manufacturing sector's electrified capital stock and note that similar arguments hold for the other capital stocks. First, let us make the following simple assumption that some share β of the remaining revenue from sales after subtracting non-wage costs goes to households and the remaining share $1-\beta$ are profits going to the owners of capital.[13] We will define the capital mobility mechanism for the manufacturing sector in this section and leave the fossil-fuel sector rules to the appendix as the logic is identical. First, the revenue arising from the output produced by the region A stock of manufacturing capital is:

$$R^A(t) = P_{ma}(t)\phi_m^A(k_e^{AM}(t) + (k_{ne}^{AM}(t)))$$

[13]Note well, the owners of simply a subset of the total household sector.

where $R^A(t)$ is the dollar value per unit time of sales by the manufacturing sector produced specifically by its electrified manufacturing capital stock. To obtain profits going to the owners of capital we must subtract from this revenue term the costs associated with the manufacturing sectors purchase of intermediate goods from the renewable and fossil-fuel energy sectors, the costs of the intermediate goods it imports from region B, the wage-bill paid to households, and finally the interest and loan principal repayments. Denoting these costs as $O^A(t)$ we have:

$$O^A(t) = \underbrace{z_{13}^A(t) + z_{12}^A(t)}_{\text{Domestic inputs}} + \underbrace{\frac{P_{fa}}{P_{fa} + P_{fb}}\pi^A(t) + \frac{P_{ma}}{P_{ma} + P_{mb}} a_{33}^A X^{AM}(t)}_{\text{Imported Inputs}} + \underbrace{\beta * WB_m^A(t)}_{\text{Wage-Bill}} + \underbrace{(r^A + Z_m^A)L_m^A(t)}_{\text{Interest and Loan Principal}}$$

Finally, let us denote the difference in the revenue and cost terms as the profit $F^A(t)$:

$$F^A(t) = R^A(t) - O^A(t)$$

As we are operating in continuous time, we must be somewhat careful with the terms above as the revenue, costs, and profits terms are all more accurately rates. For example, the profit earned over some definite time period (time 0 to time T) is given as the integral of the profit term $F^A(t)$:

$$\text{Profits} = \int_0^T F^A(t)dt$$

Now, the rate of profit per unit capital is given as:

$$f^A(t) = \frac{F^A(t)}{k_{me}^A(t)}$$

Recalling that physical capital depreciates (decays) exponentially over an infinite time horizon, a single unit of manufacturing capital will produce life-cycle profits equal to:

$$\text{profits}_m^{A,\text{Life-Cycle}} = \int_0^\infty f^A(t)e^{-\sigma_f^A t}dt$$

Now, if future prices and costs were known absolutely then the above profits arising from a single unit of manufacturing capital over its entire life cycle would be known at time 0. Furthermore, the investment necessary to produce said unit of capital at time 0 is simply the price of one unit of manufacturing sector output $P_{ma}(0)$. Therefore, if prices and costs were known fully, the ratio of the above life-cycle profits to the initial investment cost would be a signal of the profitability of capital arising in each of region A and B. Hypothetically this known quantity would then be the metric by which we could model capital flows noting that capital would flow to the region with the higher per unit capital life-cycle profits.

In the SFCIO-TRADE model such a full information metric is not however available as prices and costs are fully endogenous; therefore, we do not assume that the owners of capital know the full future paths of prices and costs (and hence life-cycle profits) that would be obtained by having one unit of capital in one region or the other. Rather, we must use the profit rate per unit of capital $f^A(t)$ as the signal. At any given time t a profit rate per unit of manufacturing capital will exist in regions A ($f^A(t)$) and region B ($f^B(t)$), and we assume that the flow of capital will be directed towards the region with the higher prevailing rate of profit per unit capital. We therefore define the following capital flow rate term which denotes the magnitude of manufacturing capital flowing into region A:

$$\chi_A^M(t) = \gamma \left[f^A(t) - f^B(t) \right] \left(K_{me}^A(t) + K_{me}^B(t) \right) \Big] \tag{13.19}$$

To show that (13.19) works as a capital flow equation following the above logic we must examine two cases; when the profit rates per unit of capital are equal and when they are unequal. First, when the rates are equal, the term $f^A(t) - f^B(t)$ is zero, and hence the entire RHS reduces to zero. If, say $f^A(t) > f^B(t)$ then $f^A(t) - f^B(t) > 0$ and the RHS term is a positive quantity, this quantity is negative when the rate of return in region A is smaller than in B. This quantity (when modified by γ) represents the magnitude of capital that flows into (or out of) region A, depending on the sign of the difference between the rates of return proxies between the two regions. By necessity the symmetric equation for region B manufacturing capital is given as:

$$\chi_B^M(t) = \gamma \Big[[f^B(t) - f^A(t)] \left(K_{me}^A(t) + K_{me}^B(t) \right) \Big]$$

. Now, with the above two equations in hand we see that if a difference of rates of return exist between the two regions capital will flow out of the lower return region and into the higher return region in equal magnitudes but with opposite signs. Finally, the parameter γ simply determines the rate at which the process occurs. We can modify the traditional differential equation governing the evolution of electrified manufacturing capital as follows:

$$\frac{dk_{me}^A}{dt} = i_{me}^A(t) - \sigma_{me} k_{me}^A(t) + \chi_A^M(t) \tag{13.20}$$

The first two terms on the (RHS) of equation (13.20) are the usual investment and depreciation terms while the third term captures the magnitude of the inflow (or outflow) of capital from the stock.[14] When

[14]The attentive reader might at this point might raise the following objection. If manufacturing capital in region B was, for example, defined to be less productive than region A capital, then heterogeneous capital types are being summed in equation (13.20). A simplifying assumption is being made here that the capital, upon moving regions, takes on the characteristics of that region's capital. Mechanically this "problem" is simple to fix by multiplying the incoming region B capital by the ratio of its productivity to the region A manufacturing

the rates of return in both sectors are equal the last two terms on the RHS cancel and the differential equation governing the capital stock reduces to the simpler and more familiar form.

The equations governing the sectoral loans are modified in similar fashion to that of the capital equations except modified by the prevailing manufacturing price in the region where the capital flowed from, so as to account for the financial cost of constructing said capital. This is shown for region A manufacturing Loans as an example:

$$\frac{dL_m^A}{dt} = P_m^A \cdot i_m^A(t) - Z_m^A L_m^A(t) + P_m^B \chi_A^M(t) \tag{13.21}$$

which ensures that both the asset (the physical capital) and the corresponding financial liability (the loan) are transferred to avoid the issue of one region acquiring capital flows and leaving the other region with debts not matched by any asset. Having specified the logic of the mechanism let us examine the EROI decline scenario from the previous subsection now allowing for capital to be mobile. A comparison of trade with and without capital mobility is presented in the following Figure 13.5

capital productivity. However, this “fix” is not so obviously sensible. Perhaps the capital is identical but simply employed more effectively in region A than in B. Either way we note that this is a choice being made and that an alternative assumption could also be employed.

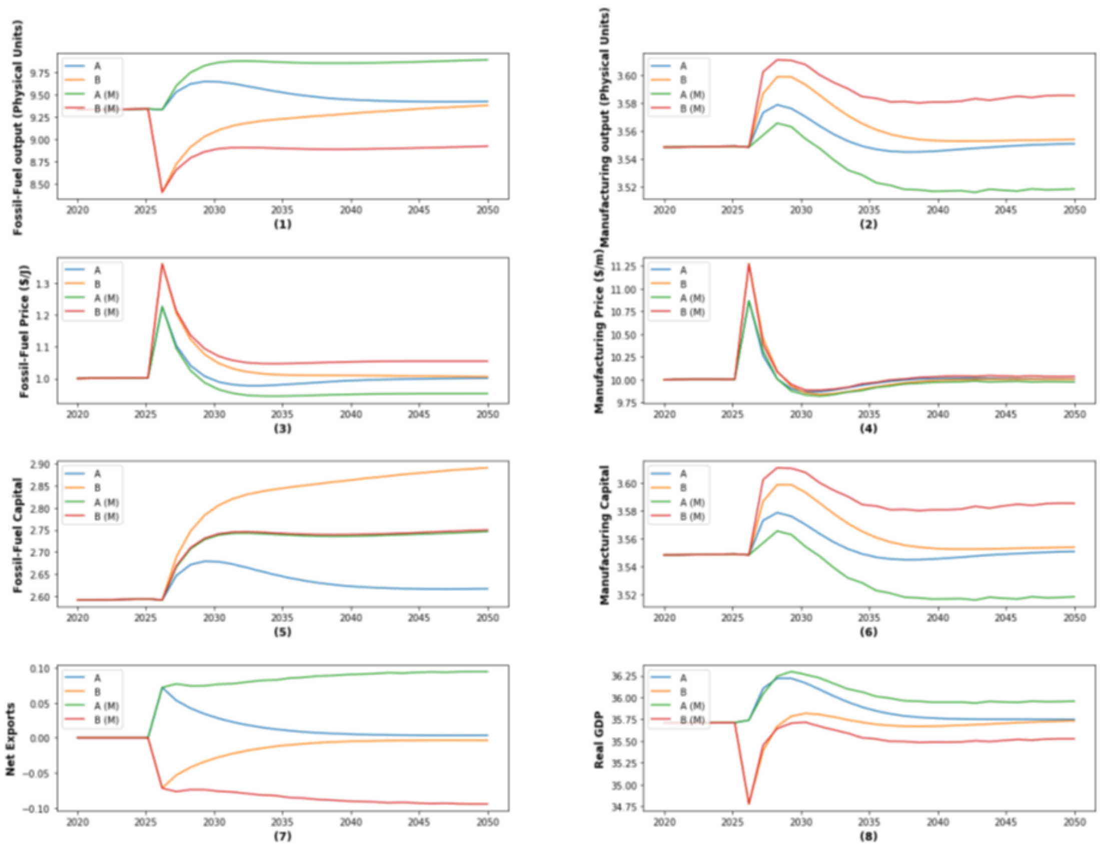


Figure 13.5: Trajectories of select variables for regions A and B following a sudden decline in ϕ_F^B from 3.6 to 3.16 in 2025. The bracketed term (M) indicates that the given trajectory is one obtained assuming capital mobility.

In the EROI scenario (without capital mobility) the sudden and permanent decrease in EROI in country B leads to a permanent increase in the fossil-fuel capital of both countries with the country B seeing the larger gain. However, since the decline in EROI acts to push up costs, some of the new capital that would have been constructed in country B has instead "flowed" into region A. This can be seen in panel (5) where capital mobility leads to relatively less fossil-fuel capital in country B (as compared to the no mobility scenario) and relatively more country A.

In general, when capital is immobile, the trajectory for most of the variables in Figure 13.5 indicate that the economy (after the initial shock) adjusts back to approximately its initial state.[15] Allowing capital to be mobile however induces the model to evolve to a new steady-state that is, in conventional terms, superior for country A and inferior for country B. Examining panel (8) we see that country A experiences a permanently higher real GDP after the shock while country B experiences a permanently

[15]This state is certainly not the same however. The lower EROI leads directly to permanently larger fossil-fuel capital stocks, and indirectly, larger manufacturing stocks.

lower real GDP. Furthermore, since trade is driven by relative prices in the model and the prices for fossil-fuels do not reconverge in both regions, a long-term situation of trade surpluses and deficits occurs (shown in panel (7)).

13.2.4 Degrowth, Trade, and Emissions Reductions

In the previous chapter we showed that degrowth allowed for emission reductions consistent with a 1.5 °C pathway. However, as this result was obtained in the context of a closed one country model it ignored entirely the complexities that arise from the interactions between trading regions. As shown in the government expenditure experiment above, the spillovers between countries arising from policy changes in a region may have non-negligible impacts on the other. To obtain a more complete understanding of the role degrowth may play in a country attempting to reduce emissions while another country pursues growth it is necessary to examine the degrowth scenario in a broader trade context. The comparison between the countries goes both ways. While the impacts of a growing region on the degrowing region are the primary concern it is equally interesting to inquire what occurs in the growth region when, suddenly, its major trading partner attempts degrowth.

The scenarios to be examined in this section are as follows. We assume two regions A and B as per usual with B pursuing degrowth along the lines of the degrowth scenario of the previous chapter while region A plays the role of a growth economy. The first scenario we will examine is when the regions are identical in the initial period except region A pursues growth while region B pursues degrowth; in this scenario it is assumed that both regions attempt to build out renewable capacity and electrify their manufacturing stocks at the same rate. The second scenario will assume that the growth economy pursues decarbonization goals at a slower rate than the degrowth economy.

Key economic model outputs for the first scenario are presented in Figure 13.6 for regions A and B respectively for each of the no trade, trade, and trade with capital mobility assumptions while regional and global emissions and cumulative emissions are presented in 13.7. Against the baseline of no trade several key features emerge. First, real GDP in the degrowth economy increases temporarily before declining over the long run in the no trade scenario which is in accordance with the similar result from the previous chapter. This pattern is similar in the trade scenario though the decline in real GDP is much slower while allowing for capital mobility leads to an intermediate result between the other two specifications. Examining the net exports panels (2-4) we see that under both trade and trade with capital mobility, the growing economy runs increasingly large trade deficits while the degrowing economy symmetrically runs larger trade surpluses. In panels 8-12 we see that the fossil-fuel and manufacturing capital stocks are larger in the degrowth region than under the no trade assumption for both the trade and trade with capital mobility specifications. However, relative to the no mobility trade scenario capital mobility leads to a smaller fossil-fuel capital stock in the degrowth region and a larger manufacturing capital stock. The final two panels show the magnitude of the inflow (or outflow) of each type of capital for each region. For example, in the bottom left panel we see that fossil-fuel capital flows into region A and a symmetric amount flows from region B; the same logic applies for the bottom right-hand panel.

There are several important features of the obvious simulations worth noting. First, and principally, trade (with or without capital mobility) leads to, all else equal, relatively slower real GDP declines in the degrowth region and relatively slower growth in the growing region as compared to the no trade baseline. Secondly, though the magnitude is not large, there is a flight of capital from the degrowth region towards the growth region as degrowth policies (declining government and household consumption) act to suppress profits in the degrowth region relative to the growth region leading to a capital outflow. From a more conventional perspective we would note that capital mobility leaves region B (in real GDP terms) worse off than it would have been and region A relatively better off. This arises from the fact that capital of both types flows out of the degrowth region which increases degrowth region prices relative to no capital mobility and decreases the growth region prices.

The economic variables are only one half of the picture; it is necessary to also examine the impacts from baseline on emissions and cumulative emissions. Panels one and two show, in light of the above, unsurprising results for regional emissions trajectories. The lower growth in region A and its increasing reliance on imports leads to lower domestic emissions in the trade and trade with capital mobility scenarios. Region B emissions are higher in both the trade and trade with capital mobility scenarios reflecting the role the region plays in meeting export demand as compared to baseline. Aggregate emissions and cumulative emissions are lower in the trade scenarios than the no trade baseline though the difference is not large.

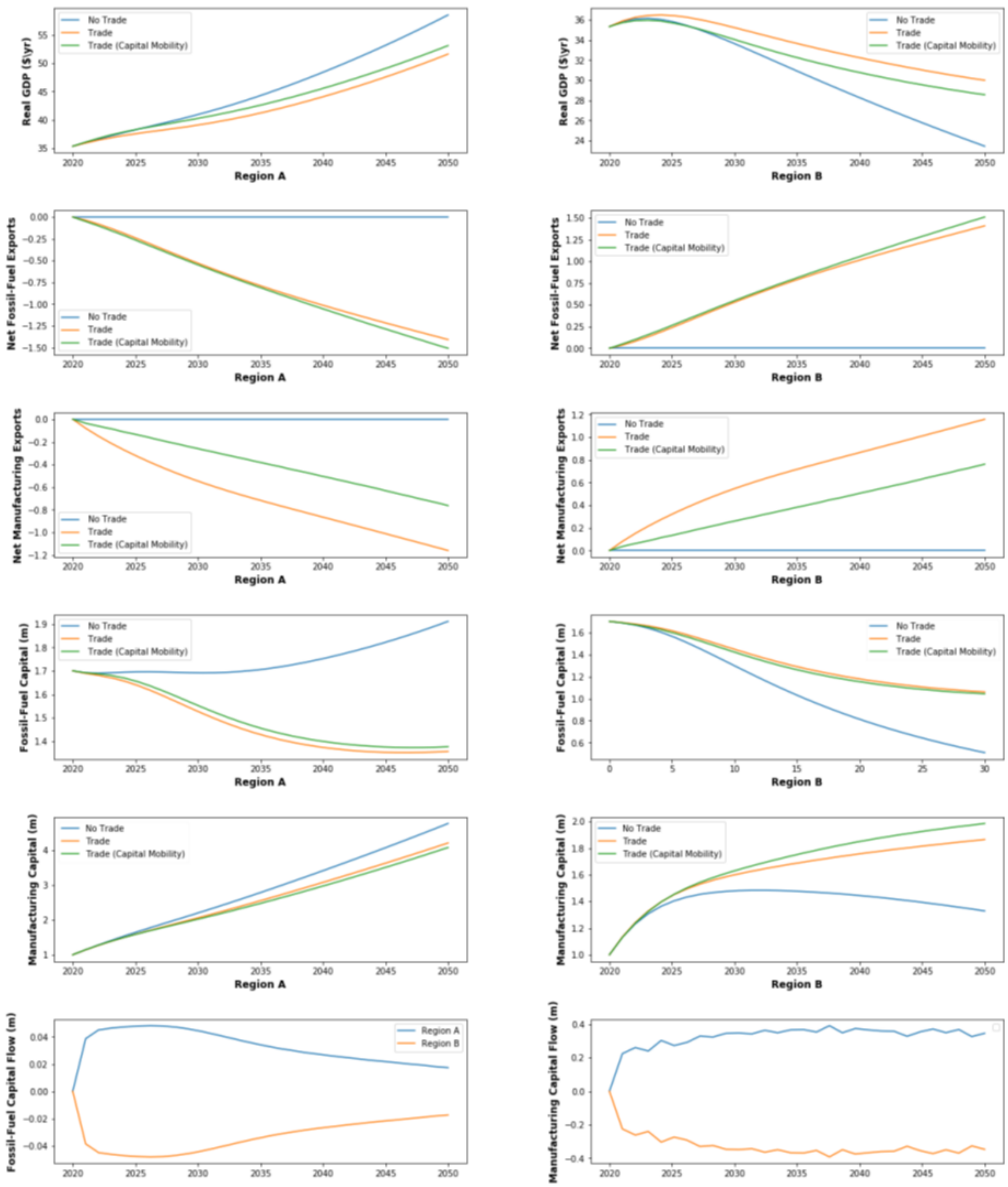


Figure 13.6: Key macroeconomic variables for regions A and B assuming identical renewable capacity and electrification rates of $\delta_1^A, \delta_1^B = 0.05$ and $\sigma_{mne}^A, \sigma_{mne}^B = 0.03$.

The salient result of this simulation lies in the emissions profile of the degrowth country which, in the presence of trade, is substantially larger than in the no trade scenario. In the simpler SFCIO-IAM model and the no trade baseline the fossil-fuel sector faces a steady decline in demand and over time the fossil-fuel production (and capacity) declines substantially. In the trade scenario this decline in domestic demand is offset by a growing foreign demand capitalizing on the suppressed degrowth region's prices. In the simplest sense the impact on aggregate emissions of allowing trade is minimal as the decline in emissions associated with degrowth is largely undone by the growing region simply offshoring its emissions to the degrowth region. While consumption and government expenditures may be declining in the degrowth region this is offset by what amounts to the degrowth region becoming a lower cost exporting sector (See panels (7-12) in Figure 13.6).

One particularly interesting result is that degrowth, as defined by decreasing expenditures (household and government), is complicated by the presence of a growing trading partner. Notably, from the second panel of Figure 13.6, we see that GDP is, in the long run, higher in both trade scenarios and the initial increase (arising from the activities associated with the energy transition) is also highest in the trade scenarios. Without additional controls on the behaviour of firms, trade, and the mobility of capital, degrowth is more difficult to achieve. [16]

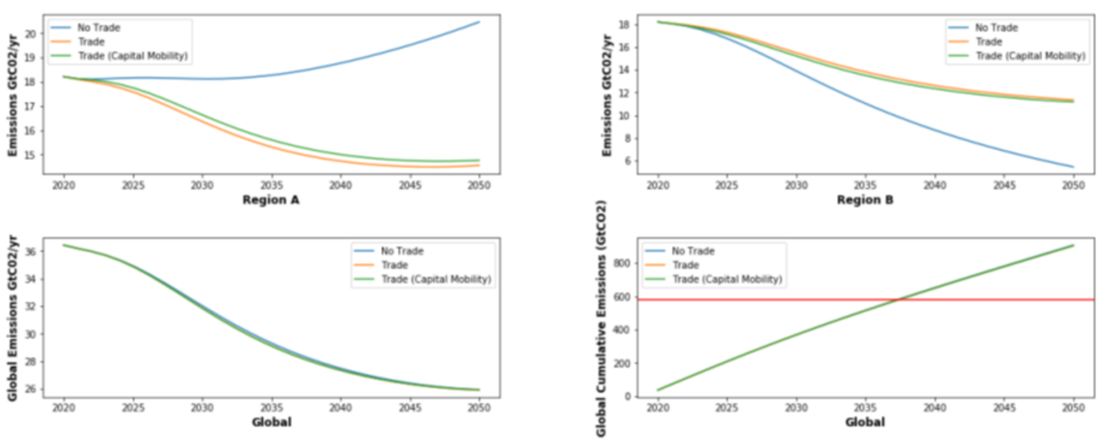


Figure 13.7: Regional and global emissions and cumulative emissions pathways for regions A and B assuming identical renewable capacity and electrification rates of $\delta_1^A, \delta_1^B = 0.05$ and $\sigma_{mne}^A, \sigma_{mne}^B = 0.03$.

In the above analysis we found that, if the regions are identical except for growth assumptions, then trade (with and without capital mobility) acts to merely change the location of where emissions are produced without significantly impacting the aggregates. However, at the regional level trade (especially when capital is mobile) was found to reduce the efficacy of degrowth as an emissions reduction strategy. It is necessary now to check whether this result holds when we assume differences in decarbonization policies. In this subsection we will undertake a similar experiment except we assume that the growing region pursues decarbonization only at the rates specified in the business as usual scenario of the previous chapter; that is, the rate at which is constructs new renewable capacity is assumed to be significantly

[16]We must state again that we are using a definition of degrowth that can be implemented in the model and does not necessarily capture the complexities of the greater concept.

lower. The results of this are shown in the following two figures.

Figure 13.8 presents the similar suite of economic indicators as shown in 13.6. Perhaps the key difference is that under the assumption of trade with no capital mobility, degrowth essentially fails to occur in the long run in region B. The real GDP trajectories for both the trade and trade with capital mobility scenarios for region B indicate an initial decline followed by a flattening of the GDP curves to a steady-state outcome. Furthermore, again unlike the previous scenario, Figure 13.9 shows that relative to the no trade baseline, global emissions are lower when trade (with and without capital mobility) occurs. While the magnitudes of the difference are not large, they are non-negligible implying, in terms of emissions produced, an unconventional example of the "gains from trade". While trade greatly reduces the regional level efficacy of pursuing degrowth as a goal to reduce specifically regional emissions it leads overall to lower aggregate emissions; this latter metric being the important one for climate analysis.

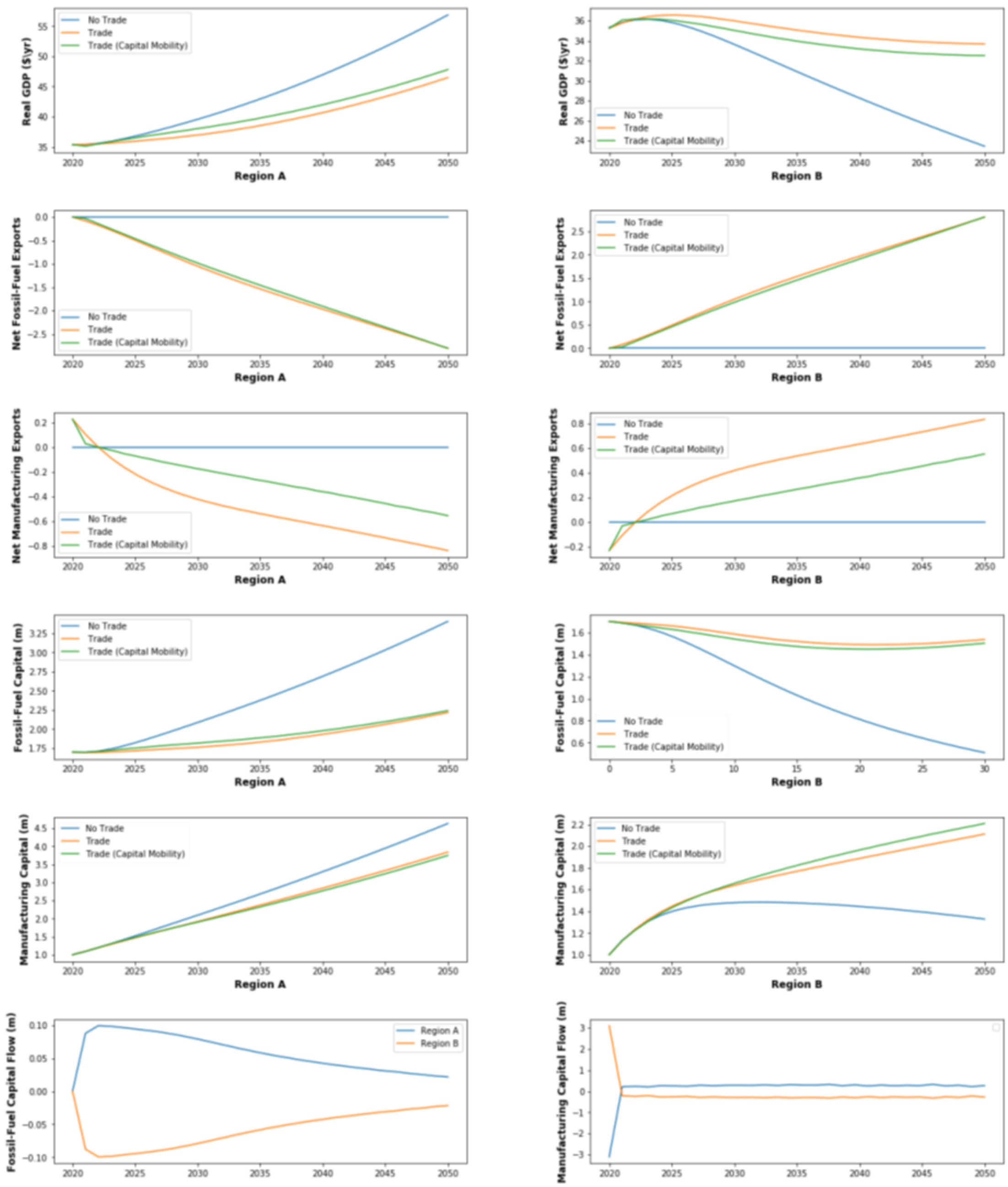


Figure 13.8: Key macroeconomic variables for regions A and B assuming heterogeneous renewable capacity construction rates of $\delta_1^A = 0.0066$ and $\delta_1^B = 0.05$.

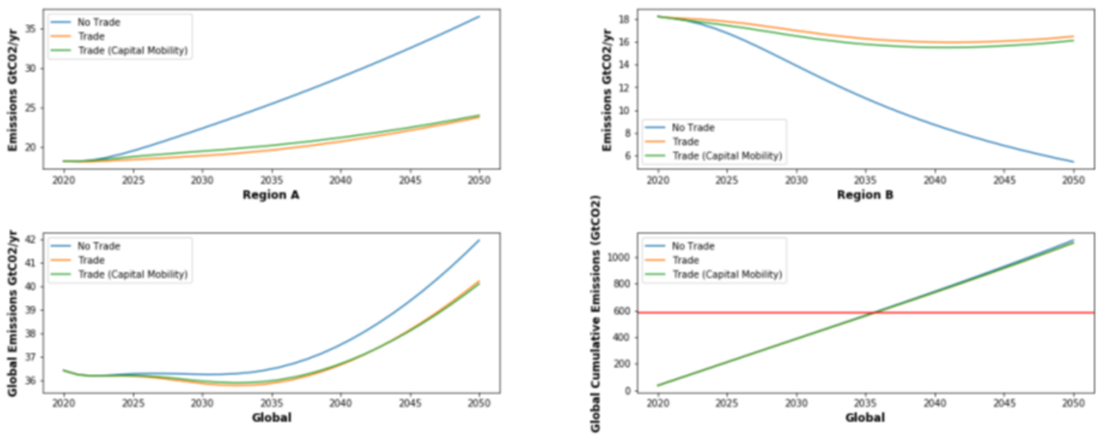


Figure 13.9: Regional and global emissions and cumulative emissions pathways for regions A and B assuming heterogeneous renewable capacity construction rates of $\delta_1^A = 0.0066$ and $\delta_1^B = 0.05$.

13.3 Conclusion

The purpose of this chapter was to explore, in a relatively simple fashion, the relevance of the degrowth result from chapter 10 in a more complex macroeconomic scenario. While degrowth, as a pathway to emissions reductions, was shown to be effective in the SFCIO-IAM model, it is clear from the analysis undertaken using the SFCIO-TRADE model that trade may possibly play a significant role. Specifically, without controlling for the behaviour of firms and imports and exports, the presence of trade in the comparisons made the ostensible goal of emissions reductions via degrowth rather more difficult to realize.

However, we are left with a somewhat intriguing theoretical result whose further interrogation, though beyond the scope of this dissertation, is still worth noting. When a degrowth region is faced with a trading partner whose decarbonization policies are less stringent, we see that while trade reduces the success of degrowth and emissions reductions at the regional level, emissions and cumulative emissions are nonetheless reduced at the more critical global level. The mechanism behind this is rather straightforward. As region B pursues faster renewable capacity construction its manufacturing produces each unit of output with fewer emissions than region A. However, as region B manufacturing prices are also suppressed relative to region A, this leads to an increase in region A manufacturers choosing to import region B manufacturing products. These two impacts combine to push down aggregate emissions from the no trade baseline.

While the model is still fairly simple it does indicate the following conclusion. A degrowth region pursuing relatively more stringent decarbonization policies will induce greater emissions reductions at the global level, despite achieving lesser regional emissions reductions, by being open to trade when its trade partner is growing and pursuing lesser decarbonization policies.

13.4 Chapter Appendix A: Model Presentation and Derivation

In the SFCIO-TRADE model, both regions are interlinked, which complicates the derivation of the underlying differential equations in simulable form. To simplify things, we will rely on the fact that one region's equations are, by symmetry, identical in form to the other regions. We will derive the model from the perspective of region A but will temporarily drop the region subscript for simplicity.

Now, the derivation initially proceeds in an identical manner to that of model SFCIO-IAM so we will not replicate those steps here but point the reader to the previous chapter's appendix. However, the key difference between the models is that we must now take into account the net export term in the standard national output equation:

$$Y = C + I + G + NX$$

From the previous chapter's model derivation, we can rewrite this expression for Y as:

$$Y = \underbrace{\left(\frac{1}{1 = \omega_{13}}\right)}_{\omega_{18}}(\omega_{14}L^R + \omega_{15}L^F + \omega_{16}L^M + \omega_{17}V + I + G + NX)$$

Now, as with the SFCIO-IAM model, we must obtain expressions for the prices and total sectoral outputs. Let us first obtain the fossil-fuel prices. Now, as always, the expression for P_{FA} is given as:

$$P_{FA} = \frac{X_{FA}}{\phi_F k_F}$$

which can be expanded as:

$$\begin{aligned} P_{FA} = L^A_{22}(C\underbrace{{}^A_e + G^{Ae} - P^A_r\phi^A_g k^{AR}_e}_{g_1} + \frac{P_{FA}P_{FB}}{P_{FA} + P_{FB}}\underbrace{(CF(\tau_e k^m_{eB} - \phi_{gB}k^R_{eB}) + \tau_{FF}k^m_{neB})}_{g_2} \\ + \frac{P_{FA}P_{FB}}{P_{FA} + P_{FB}}\underbrace{(CF(\tau_e k^m_e - \phi_g k^R_e) + \tau_{FF}k^m_{ne})}_{g_3} k^A_f)\underbrace{\frac{1}{\phi_F}}_{g_4} \end{aligned} \tag{13.22}$$

which we can rewrite in more compact form as:

$$P_{FA} = L^A_{22}g_1g_4 + L^A_{22}g_4g_2\frac{P_{FA}P_{FB}}{P_{FA} + P_{FB}} + L^A_{22}g_3g_4\frac{P_{FA}P_{FB}}{P_{FA} + P_{FB}}$$

Which, finally we may write as:

$$P_{FA}^2 + (P_{FB} - L_{22}^A g_1 g_4 - L_{22}^A g_4 g_2 P_{FB} - L_{22}^A g_3 g_4 P_{FB}) P_{FA} + (-P_{FB} L_{22}^A g_1 g_4)$$

which is a quadratic form. By symmetry we have a similar expression for P_{FB} which is identical to the above except with subscripts changed appropriately to denote the region B.

$$P_{FB}^2 + (P_{FA} - L_{22}^B g_{1b} g_{4b} - L_{22}^A g_{4b} g_{2b} P_{FA} - L_{22}^B g_{3b} g_{4b} P_{FA}) P_{FB} + (-P_{FA} L_{22}^B g_{1b} g_{4b})$$

Therefore, we have two quadratic expressions for the fossil-fuel prices present in each region with both region's prices appearing in both equations. To obtain these prices we solve both equations simultaneously using a root finding procedure. With numerical solutions to the prices obtained we, by definition also have expressions for the total fossil-fuel sector outputs in both regions X_{FA} and X_{FB}.

Obtaining the manufacturing sector prices and total outputs in both regions is considerably more involved but ultimately follows the same logic. Let us again begin from the perspective of region A and write out the investment equations (these are analogous to the SFCIO-IAM model equations of the previous chapter.)

$$i_e^{AR} = \Delta_{1A} \phi_{gA}^{-1} \tau_{EA} k_{me}^A - \Delta_1 k_e^{AR} = g_5$$

$$i_f^{AR} = \delta_{1A} \phi_{gA}^{-1} P_{AR}^{-1} [C_e^A + G_e^A] - \Delta_{1A} K^{AR}(f) = g_6$$

$$i^{AF} = \Delta_{2A} \phi_{FA}^{-1} P_{AF0}^{-1} X_{FA} - \Delta_{2A} K_F^A = g_7$$

$$i_e^{AM} = \underbrace{(\Delta_{3A} \phi_{MA}^{-1} P_{AM0}^{-1})}_{g_8} X_{MA} - \underbrace{(\Delta_3 k_{me}^A + \Delta_3 k_{mne}^A)}_{g_9} = g_8 X_{ma} - g_9$$

Therefore, we can write the I term of the GDP equation a:

$$I^A = P_{MA}(g_5 + g_6 + g_7 - g_9) + g_8 P_{ma} X_{ma}$$

Finally, from the main body of the chapter we have that next exports are:

$$NX^A = X^A - M^A$$

We proceed by substituting these terms into the expression for Y we obtained above and find:

$$Y = \omega_{18}\underbrace{(\omega_{14}L^R + \omega_{15}L^F + \omega_{16}L^M + \omega_{17}V + G)}_{g_{10}} + P_{ma}\underbrace{\omega_{18}(g_5 + g_6 + g_7 - g_9)}_{g_{11}} + \omega_{18}g_8 P_{ma}X_{ma} + \omega_{18}(X^A - M^A)$$

which we write in more compact form as:

$$Y = g_{10} + g_{11}P_{ma} + g_{12}P_{ma}X_{ma} + \omega_{18}(X^A - M^A)$$

Now, from the chapter, we have the following expression for the total manufacturing sector output

$$X_{ma} = L_{33}\left(Y - C_e^A - G_e^A - \underbrace{\frac{P_{FA}P_{FB}}{P_{FA} + P_{FB}}}_{\psi_1}(\pi^B - \pi^A) + \underbrace{\frac{P_{ma}}{P_{ma} + P_{mb}}}_{\psi_2} a_{33}X_{ma}\right)$$

Solving out for X_{ma} we have:

$$X_{ma} = \underbrace{\left(\frac{L_{33}}{1 - L_{33}\psi_2}\right)}_{\psi_3}(Y - C_e^A - G_e^A - \psi_1(\pi^B - \pi^A)) \tag{13.23}$$

Now, since we have X_{ma} solely in terms of Y, we may plug this back into the expression for Y above to obtain (after some algebra):

$$Y = \underbrace{\left(\frac{1}{1 - g_{12}P_{ma}\psi_3}\right)}_{\psi_4}[\underbrace{g_{10} + g_{11}P_{ma} - g_{12}P_{ma}\psi_3(c_e^A + G_e^A + \psi_1(\pi^B - \pi^A))}_{\psi_5} + \omega_{18}NX]$$

Now, we must deal with the net exports term NX which is defined in the body of the chapter. Using the region B version of (13.23) we can substitute in the definition for net exports into the equation for Y as:

$$Y = (\psi_5 + \omega_{18}\psi_1(\pi^B - \pi^A) + \underbrace{\omega_{18}\left(\frac{P_{mb}}{P_{mb} + P_{ma}}\right)a_{33}^b\psi_3^b}_{\psi_6}(Y_b - C_e^B - G_e^B - \psi_b^1(\pi^A - \pi^B)) - \omega_{18}\psi_2\psi_3(Y - C_e^A - G_e^A - \psi_1(\pi^B - \pi^A))$$

Now, solving for Y we have:

$$Y = \left(\frac{\psi_4}{1 + \omega_{18}\psi_4\psi_2\psi_3}\right)\left[\underbrace{\psi_5 + \omega_{18}\psi_1(\pi^B - \pi^A)}_{\psi_8} + \psi_6(Y^B - \underbrace{C_e^B - G_e^B)(\pi^A - \pi^B}_{\psi_9}) + \underbrace{\omega_{18}\psi_2\psi_3(C_e^A + G_e^A + \psi_1(\pi^A - \pi^B))}_{\psi_{10}}\right]$$

and finally, in more compact form:

$$Y = \psi_7\psi_6 Y^B + \underbrace{\psi_7(\psi_8 - \psi_6\psi_9 + \psi_{10})}_{\psi_{11}}$$

and finally,

$$Y = \psi_7\psi_6 Y^B + \psi_{11}$$

Now, we have that region A GDP, Y, is a function of region B GDP Y^B. To solve for Y fully, we note that by symmetry region B GDP is given as:

$$Y^B = \psi_7^B\psi_6^B Y + \psi_{11}^B$$

Substituting this back into the first equation and solving for Y we have:

$$Y = \underbrace{\left(\frac{1}{1 - \psi_7\psi_6\psi_7^b\psi_6^b}\right)}_{\psi_{12}}(\psi_7\psi_6\psi_{11}^B + \psi_{11})$$

Finally, we may specify the manufacturing price as follows invoking again (13.23)

$$P_{ma} - \frac{\psi_3(Y - C_e^A - G_e^A - \psi_1(\pi^B - \pi^A))}{\phi_M^A(k_{me}^A + K_{mne}^A)} = 0$$

and note the existence of a symmetric expression for the manufacturing price in region B. Solving both equations simultaneously (using a root finding method) leads to numerical approximations for the prices which can be used in the solution routine for the model.

Chapter 14

Chapter Fourteen: Conclusions

The literature, as covered in chapter three, indicated that macroeconomics models dedicated to studying climate and energy transitions were problematic in three key areas: first, the notable lack of discussions of finance and the financial sector; second, the reliance on negative emissions technology at scales well beyond what is currently known to be feasible; and third, the limiting of the study of future trajectories of economies solely to growth trajectories. The first problem is tantamount to a serious incompleteness in many models descriptions of macroeconomies, while the second and third act to narrow the solution space on untenable assumptions.

In this dissertation I have aimed to begin filling this gap in the IAM literature by constructing an alternative approach to IAMs based on the SFCIO approach and using the resulting models to examine the under-explored future scenarios of no negative emissions technologies coupled with pathways predicated on steady-state and degrowth assumptions. The key result obtained in chapter 12 using the one country simulation model is that, without negative emissions technologies, it is still possible to generate 1.5 degree pathways assuming a combination of vast upfront investment in new renewable capacity, electrification of end use, and long term degrowth (a decline in the elements of the final demand vector). Notably, this result is obtained while including both energy system dynamics (endogenous EROIs arising from depletion and energy storage costs) and climate system feedbacks.

What can we make of such a result? On the one hand, if one accepts the manner in which the models were constructed and finds the necessary simplifications of the numerous complex underlying phenomena permissible, then we find that it is (in 2021) possible with "extreme effort" to transition energy generation and end use in a macroeconomy to reduce emissions in line with a 1.5°C carbon budget without invoking unproven negative emissions technologies.

On the other hand, the requirements to do so in the modelling of the previous chapters is a dramatic increase in investment in renewable capacity and a long-term decline in the magnitude of consumption (the simulation proxy for economic degrowth).

At the conclusion of chapter four we noted that while we could provide several criteria for the "realism" of a macroeconomic model based on physical and accounting rules, it was rather more difficult to do so for the behavioural components of the model. We return to this notion now as, ultimately, the results concerning the achievability of 1.5°C pathways relies on very specific behaviours taken by the government and household sectors; namely, pursuing long-term and continual degrowth.

A fundamental problem for any social scientist engaged in modelling some aspects of the social system over the long term is that, given the magnitude of possible changes, the social system is also likely to change dramatically. When faced with the scenario of converging environmental crises that are likely to characterize portions of the next 100 years or so, it is arguably the behavioural portions of the models that we construct that may be the most subject to change.

Understanding this, we do not invoke degrowth behavioural choices (or steady-state) ones because we think they are most likely to occur or are the most accurate descriptors of household and government behaviour under climate constraints; rather, we invoke the concepts to show that if they are adopted over the long term (and assuming the validity of the physical mechanisms specified) then it is possible to achieve certain climate goals. This difference is crucial. We cannot know with much confidence the likely behaviour of the system, or its component parts, several decades hence given the fundamental uncertainties characterizing the near future. What we can do is build pathways illuminating the effects on emissions trajectories and ultimately warming for assumed long-term behaviours which can then be used to inform choices in the shorter term.

It is worth considering, in a qualitative manner, the nature of the results obtained in the previous chapters. The main conclusion of section 13.9 is that assuming an immediate and continual 2% decline per annum in household energy consumption and government expenditures, it is possible to generate an emissions trajectory that is consistent with remaining within $1.5°C$ warming, assuming a 12% per annum turnover in non-electrified capital and replacement by electrified capital, and investment in renewable capacity rising to over 6% of GDP by 2025. These relatively "extreme" numbers arise largely from the issue that the remaining carbon budget is simply not large relative to annual emissions.

Under the above assumptions we can force a mathematical model of energy, economy, and environment to achieve the desired result, but these assumptions amount to quite radical changes in the context of real economies. The simultaneous steady decline in consumption and government expenditures paired with the large-scale redistribution of society's resources towards renewable capacity and electrification might seem, when viewed from a conventional viewpoint, to imply deep welfare consequences. Sustained declines in outputs, historically, lead to large-scale hardship and suffering, and if that were the only outcome of such policies than it might spell doom for the results obtained from this modelling. By invoking degrowth and steady-state futures, we do not mean to suggest these are bleak futures, but an altogether more positive view along the lines of Jason Hickel's theory of radical abundance (Hickel, 2019) and the work of ecological economists such as Peter Victor, Tim Jackson, and Herman Daly, who show possibilities for better lives in the absence of economic growth.

The SFCIO models developed in this dissertation are notable in that they consistently link macroeconomics, finance, and physical quantities in a single coherent framework. That degrowth pathways are both feasible, and indeed required, when assuming no negative emissions technology is a statement that such pathways are possible in the energetic, accounting, and climate system rules that govern the model itself; that is, they are physically and economically feasible. The question remains as to what the possible social implications of such assumptions are and whether they themselves are feasible. Ultimately, the constraints that are most knowable in the mathematical models we construct are the physical ones. The set of future pathways left available after accounting for these physical rules is one constrained only by the ability of societies to collectively imagine and realize novel ways of organization.

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