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SPEAKERS

Robert McKeown

The vertical line test determines whether a line or a curve on a graph like the one that we have here on the slides in front of you, whether it is a function. So remember, what is a function? A function has one Y value for each X value in the domain. So if we have Y is equal to plus minus two, we saw that in an earlier example, if Y is equal to plus minus two, then whatever function gave us that plus minus two or I should say. So, if we have Y is equal to F of X, which is equal to plus minus two, this F of X is some sort of relationship, some sort of mapping, but it's not a function. It can't be a function because it gave us more than one Y value for just one X. So I could even replace this X in there.

The vertical line test represents a test to determine just this phenomenon. If, is there an X value for which there's more than one Y value. And the vertical line test tells us that no vertical line should cross or intersect the line curve of a function more than once. No vertical line should cross the line or curve of a function more than once. So, I can draw, we're going to draw a vertical line on this diagram right here. And anywhere I draw that vertical line, I should not be crossing this curve here more than once. So I could draw a vertical line right here, I could draw a vertical line right there at X equals four, and it crosses, it crosses this expression here only once. And anywhere I do this, it's going across only once, might not be entirely clear because this thing, the slope of this thing is very, very well, very negative. As x is approaching zero, it's getting small, that really means it's smaller, a very large negative number is actually a very small number. So as X is approaching zero from the right, going this way, it's still, you know, there's still a unique Y value for each X. And more clearly writing it over here you can see we're intersecting. We're intersecting this curve just once. And so this expression here passes the vertical line test. A vertical line drawn anywhere within the domain crosses the curve or the line just once.

Let's take a look at another graph. We've got a circle. So we've got a circle We've got the expression for the circle over here, but we're not going to use that, let's see if we can apply the vertical line test to determine whether this algebraic expression over here, which has the shape of a circle, is indeed a function or not. So I'm going to, just, I could just anywhere on this graph, I should be able to draw a vertical line and, and intersect the curve just once. If that occurs, then yes, this is indeed a function. So if I took say negative three, we intersect the curve there when Y is equal to negative four. And if I

keep going up, I ignore the Y axis and the X axis, but we intersect the curve again when Y is equal to four. So this function, well, this expression is not a function, it's not a function, this relation, this mapping is not a function. It fails the vertical line test.

So an easy test of whether an expression is a function is to graph it and use the vertical line test. Here's another example. So we've got some function in this case, it's been left in its general form, we're not sure what the actual function is. We've got our you've got our X axis down here, we've got our Y axis up there like that. And we don't really need to see the actual algebra, algebraic expression here, we can apply the vertical line test without looking at it just using the graph. And you'll notice, you'll notice that if you try to draw a vertical line that intersects the curve more than once, it's not possible. So all these vertical lines that I'm drawing more or less vertical, they only cross this expression once. Since no vertical line crosses or intersects the curve more than once, F of X is a real function. It's a function, it's not just some sort of relation, some sort of mapping.

So we wrapped up the vertical line test and how to determine whether an expression was a function. Now let's look at some questions trying to find the domain. So in order for X to be in the domain, there has to be an associated Y value. If the Y value doesn't exist, then that X value cannot be in the domain. And these examples are going to help us illustrate this. So let's take a look at the first example here. We've got F of X is equal to one divided by X plus five. So certainly X can be very large. So if X was equal to, I don't know, 10 to the power of 30, that's a really big number, then F of 10 to the 30 is going to be equal to, well, something really close to zero. But we're gonna have, we're gonna have an answer. Now, it turns out that any X value we put in here is going to give us a Y value, except what happens if X is equal to negative five. If X is equal to negative five, F negative five becomes one over negative five plus five. And that's going to give us one divided by zero. And the answer is not infinity, that's a very common mistake that students make. The answer is not infinity, it's going to be unknown. Anything divided by zero, we don't know what the answer to that is. And we say it is in defined. Oops, not, not undefined, undefined.

Let's take a look at our next example. We've got F of X is equal to the square root of X minus three. And if we said oh, you know, X is some big number 10's maybe not the biggest number, but big enough, then that 10 minus three, which is equal to the square root of seven. And if we get, keep going, using larger numbers than that, keep going in that direction, it's gonna be fine. We're gonna have a number, it's gonna be a radical. But it's, but it's fine. There's no problem, no problem with it. It is, in fact, a real number. Now, what happens if we put in F of three. So if we put in F of three, we have three minus three, that's giving us the square root of zero. But that's actually fine too. The square root of zero is just equal to zero. Zero times itself, zero times zero is equal to zero. So we actually know what the square root of zero is, it's zero. And so zero is our Y value, we don't have a problem here. We get into a problem if we put in F is equal to two. If, or I should say X is equal to two, and we have F of two, we have F of two, then we're going to have the square root of two minus three. And this is going to give us the square root of negative one.

What number times itself is equal to negative one? It's, well, there might be a number out there. And, you know, it might be an imaginary number. There's an imaginary out there, it might give us that, but there is no real number that will give us that. And so we're going to say that there is no Y value for F of two. No real number. This course we're going to keep everything to real numbers, we're not going

to get into imaginary numbers or complex numbers. That's it, you made it to the end of the video on the vertical line test, we showed that we can use the vertical line test to determine if a curve is a function or if it's some other kind of relation. We also looked at the domain and the range, and we could find the domain by essentially going through all possible values of X and determining which of those values of X gave us, or the function mapped to a Y value that was a real number. In our next video, I'd like us to take a look at inverse functions and how they can be applied to problems in the social sciences.