CO-CONSTRUCTING PEER FEEDBACK PRACTICES IN AN ELEMENTARY SCHOOL MATHEMATICS CLASSROOM: AN ENACTIVIST PERSPECTIVE

PARINAZ NIKFARJAM

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Abstract

Feedback has been recognized as one of the most powerful factors in school achievement, and a great body of research has been devoted to studying its effectiveness. In this research, I bring together research in peer feedback and mathematics education by exploring peer feedback practices in my grade six mathematics class. Inspired by a democratic view of education, this research is premised on the conviction that peer feedback practices, like all other educational acts, must be decided by those affected by them. For this reason, my grade six students and I co-constructed feedback practices as the group engaged in feedback actions, categorized feedback, and co-constructed models of feedback.

Adopting an enactivist perspective, both as a theory of knowing and as a methodology, I explored the co-construction of the group's feedback practices as feedback was enacted across three cycles of feedback actions. The goal of this research was to see how the group co-constructed feedback categories, models and actions and how conceptualization of feedback and its effectiveness evolved throughout the process. The analysis of data culminated in three detailed narratives describing, from my perspective as teacher-researcher, shared experiences of coupling across the three cycles.

The process of co-constructing feedback provided the group with the opportunity for recursive linguistic coupling with one another, through which the group co-adapted and co-evolved and brought forth new possibilities for feedback categories and models. Many parallels emerged between the process of co-construction of feedback practices and the feedback practices that were co-constructed. Students' conceptualization of feedback and its effectiveness evolved from content to actions and relationships. The findings of this research suggest that the power of

the feedback practices rested in the fact that they were co-constructed and that the process of co-construction was one of transformation, of action, reflection, and effective relationships.

Dedication

I dedicate this dissertation to my husband, Shane, who has been my greatest supporter, and to my daughters, Aryanna and Darya, who have always inspired me to be better. Thank you for believing in me, for keeping me centered and for cheering me on.

I also dedicate this work to my parents, Ali and Eti, who instilled in me an enduring love for learning and supported and encouraged me in all my academic endeavors.

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I also want to thank my grade six students for participating in this study and for inspiring me and giving me the opportunity to ask questions, to listen, to reflect and to learn.

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Chapter 1: Introduction

In the field of education, true interest in, and engagement with, an activity is only achieved and sustained when students share a connection with, and an active participation in, that activity. Dewey (1916) highlights the significance of this connection when he states:

> The remedy is not in finding fault with the doctrine of interest, any more than it is to search for some pleasant bait that may be hitched to the alien material. It is to discover objects and modes of action, which are connected with present powers. The function of this material in engaging activity and carrying it on consistently and continuously is the interest. (p. 123)

According to Dewey (1916), educational acts, if they are to be democratic, must possess aims, which are decided by those affected by them. If the aim of an act is decided by the teacher and imposed upon the students, it fails to engage the students as active participants, thereby constituting students as passive recipients of those acts. In such cases, the externally decided and imposed aim is "remote, divorced from the means by which it is to be reached" (Dewey, 1916, p. 106). On the other hand, when students are active participants of their educational processes, they decide the aims of their actions and become invested and engaged in those actions. Thus, engaging students in decisions regarding their educational acts is not merely a pedagogical practice but an ethical responsibility. Freire (1970) emphasizes this notion by stating, "to alienate human beings from their own decision-making is to change them into objects" (p. 85).

Inspired by this democratic view of education, in this research, I explore peer feedback practices as co-constructed by my students in our grade 6 mathematics classroom. Feedback is an important component of formative assessment (Black & William, 1998) and has been recognized as one of the most powerful factors in school achievement (Hattie & Timperley, 2007). However, not all feedback is effective, and there is evidence to support that a considerable amount of feedback is inconsequential or even detrimental to learning (Kluger & DeNisi, 1996). As such, many studies have been devoted to investigating the attributes of quality feedback (Chappuis, 2012), its effectiveness (Hattie & Timperley, 2007; Kluger & DeNisi, 1996; Sadler, 2010) and its contribution to learning (Hattie & Timperley, 2007). In this study, instead of using predetermined procedures and models for feedback, I worked with my students as they co-constructed their own peer feedback practices.

The aim of an educational act, as Dewey (1916) suggests, is not a fixed conception, but rather a "tentative sketch" of what it could be. As students engage with the act, their understanding grows and new possibilities of the aim emerge. Therefore, there is a temporality attached to the aims of actions, brought forth by the participants' understanding of those actions in the moment. In this research, students' active engagement in co-constructing their feedback practices presupposes a democratic environment, wherein the aims of feedback actions are decided by those who enact feedback, and the said aims evolve as new possibilities for feedback emerge. In other words, the aims of feedback practices are contingent upon students' conception of feedback at different instances of feedback actions. A classroom's feedback practices, therefore, must be examined not as an entity disconnected and independent of those who enact them; but rather, as they are constructed and enacted by participants. Adopting an enactivist perspective, both as a theory of knowing and as a methodology, in this study, I explore my six

graders' co-construction of feedback practices across three cycles of feedback actions, and examine how conceptualizations of feedback co-emerged.

The Purpose of the Study

The goal of this research is to bring together research in peer feedback and mathematics education for elementary school learners. This research is in line with the body of literature that involves students as co-creators of their feedback practices (Beaumont, O'Doherty & Shannon, 2011; Fluckiger, Vigil, Pasco, & Danielson, 2010; Meer & Chapman, 2014; Price, Handley & Millar, 2011). However, what makes this research unique is that it uses enactivist theory to bring forth a different lens for knowing and studying feedback. Dismissing the notion of a single, pre-existing conceptualization of feedback, this research explores feedback as it co-emerges with those who enact it.

Enactivism is both a theory of knowing and a methodology. According to Reid and Mgombelo (2015), "instead of asking questions about an external world, enactivism asks questions about observers and the worlds they observe" (p. 179). As both a teacher in a classroom and as a researcher, I am interested in how peer feedback practices are co-constructed and how feedback is conceptualized. The research questions are:

1. How were feedback categories and models co-constructed?
2. How were feedback actions co-constructed?
3. How did conceptualizations of feedback and its effectiveness evolve throughout the process?

Significance

The findings of this research contribute to multiple fields of enquiry. First, this study helps bridge the research on feedback to the research on mathematics education for younger

learners, thereby enhancing existing literature in both fields. Second, by virtue of studying my classroom's feedback practices, this study contributes to the body of research performed by teachers about their own practice. Third, by studying the co-evolution and co-emergence of our feedback practices, the findings of this research add to the growing body of research in education that use enactivism as theory and methodology.

Chapter 2: Literature Review

To set the context for this research which explores the peer feedback practices my students co-constructed in our mathematics classroom, I will review literature about feedback and peer feedback in education. I will consider how feedback is defined and described across research in education and what factors are recognized as influential in the effectiveness of feedback. I will explore research pertaining to feedback and peer feedback in mathematics education, specifically focusing on the effectiveness of feedback, and will examine research that advocates for engaging students as partners in the learning of and/or co-creating their feedback practices. In the final section of this chapter, I will discuss points of convergence and divergence between this research and the existing body of literature about feedback and peer feedback.

Feedback

In educational research, feedback has been of interest for over four decades. Sadler (1989) defines feedback as information provided to fill the gap between what is understood and what is aimed to be understood. Winne and Butler (1994) offer a similar definition for feedback as "information with which a learner can confirm, add to, overwrite, tune, or restructure information in memory, whether that information is domain knowledge, beliefs about self and tasks, or cognitive tactics and strategies" (p. 5740). In their comprehensive review of research on feedback, Hattie and Timperley (2007) describe feedback as a "consequence of performance" (p.

81). They conceptualize feedback as "information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007, p.81).

While Hattie and Timperley's (2007) conceptualization of feedback is inclusive of different agents of feedback, such as experts, peers and the self, by virtue of reducing feedback to mere information, it undermines the significance of people who give or receive feedback. Focusing on the content of feedback alone renders feedback as a one-directional transmission of information, with little influence from those who enact it.

In contrast to such conceptualization of feedback as mere information, feedback is also described as "a dialogical and contingent two-way process that involves coordinated teacher-student and peer-to-peer interaction as well as active learner engagement" (Nicol, 2010, p. 503). Conceptualizing feedback as a two-way process constitutes different stakeholders, those who give feedback and those who receive it, as actively engaged in dialogue (Knight, 2003; Yang & Carless, 2013). Askew and Lodge (2000) define dialogic feedback as "all dialogue to support learning in both formal and informal situations" (p.1). Gravett and Peterson (2002) further this notion and explain that dialogue involves relationships where participants think and reason together. In such relationships, dialogue becomes "an explicit attempt to circumvent the limitations of one-way transmission of feedback" (Yang & Carless, 2013, p. 2).

Effectiveness of Feedback

Hattie and Timperley (2007) report that feedback is among the most effective factors in student achievement. However, they also caution that the effectiveness of feedback is contingent upon many variables, and that not all feedback is beneficial to student achievement. The inconsistency of the effectiveness of feedback has also been highlighted by Kluger and DeNisi

(1996). Many studies have been devoted to understanding and describing effective feedback (Chappuis, 2012; Eriksson, Björklund Boistrup, and Thornberg et al., 2015). Chappuis (2012) outlines five characteristics of effective feedback. According to her, feedback must direct attention to the intended learning, occur during learning to give the learner the opportunity to act on, address partial understanding (as opposed to none), not do the work on behalf of the student, and limit the amount of corrective information for students to act on.

Hattie and Timperley (2007) propose a framework to describe and better understand different types of feedback and their effectiveness. In their model, feedback is classified into four levels:

- Feedback about the task (FT) which refers to the correctness of the task or about how to do the task differently, better or more complete.
- Feedback about the process (FP) is about relations and involves strategies for error correction or cues on more effective use of strategies.
- Feedback about self-regulation (FR) which includes "greater skill in self-evaluation or confidence to engage further on a task" (Hattie & Timperley, 2007, p. 90). FR is believed to have "major influences on self-efficacy, self-regulatory proficiencies, and self-beliefs about students as learners" (Hattie & Timperley, 2007, p. 90).
- Feedback about the self (FS) is feedback on the self. It is personal feedback and is often unrelated to the task or the performance. The effects of FS are the most contested and most varied. On the one hand, personal praise is mostly irrelevant to the task, while on the other hand personal praise can improve self-efficacy by encouraging the learner's effort, commitment, and self-regulation (Hattie & Timperley, 2007). On the whole,

Hattie and Timperley (2007) place FS at the lowest level when it comes to its effectiveness for learners' performance.

The effectiveness of feedback according to Hattie and Timperley's (2007) model is connected to the level of feedback provided. In contrast to this view, there are researchers who claim that, "feedback in its most productive forms is experienced as a social and relational process in which dialogic interaction within a trusting atmosphere can help to promote learner agency and self-regulation" (Yang & Carless, 2013, p. 6). As such, it is not merely the level of the feedback that renders it effective. Rather, it is the nature of the feedback process, the interactions and the atmosphere that influence the effectiveness of feedback.

Describing feedback as social and relational processes implies a more complex conceptualization of feedback and means that it must be studied not only in terms of feedback content, but also in terms of feedback strategies and actions. Examining teachers' strategies for day-to-day feedback in a primary school, Eriksson et al., (2015) classify teachers' feedback into five categories: expecting, emotionally responding, normalizing, steering and deliberating. The authors argue that the proposed categories situate feedback actions in a landscape as opposed to a linear axis, further emphasizing the complexity of feedback. Such complexity stipulates that there are many factors that can influence the effectiveness of feedback. Goal setting, prior knowledge and students' perceptions of feedback are prominent among these factors.

Goal Setting. Goal setting is believed to be among the most significant factors in the effectiveness of feedback as it helps one focus one's attention (Lock & Latham, 1984) and develop error detection and self-regulation skills (Hattie, Biggs, & Purdie, 1996). Feedback can have positive effects when the intended goal "is clear, when high commitment is secured for it, and when belief in eventual success is high" (Kluger & DeNisi, 1996, p. 260). Hattie and

Timperley (2007) claim that the main purpose of feedback is to "reduce discrepancies between current understandings and performance and a goal" (p. 86). However, the literature is inconsistent in whether the goals of feedback are decided by the teacher for the students or in collaboration with the students. This distinction is particularly important since the students' active participation in deciding the goals of their feedback practices is central to my research.

Students' prior knowledge. Research suggests that effectiveness of feedback is connected to student' prior knowledge (Hattie & Timperley, 2007; Sadler, 2010). In order for feedback to be effective, students must be able to understand the feedback and, in order to understand the feedback, they must possess background knowledge (Sadler, 2010). Hattie and Timperley (2007) assert that feedback is "most powerful when it addresses faulty interpretations, not a total lack of understanding" (p. 82). This implies that feedback is more likely to be more effective if students have some prior understanding than none. Having some prior knowledge helps them understand the feedback and use the feedback to make improvements to their work.

Students' perceptions of feedback. Effectiveness of feedback is also greatly influenced by the ways in which students perceive feedback (Budge & Gopal, 2009; Burnett & Mandel, 2010; Lipnevich & Smith, 2009). Many factors shape students' perception of feedback. For instance, timing and the manner in which feedback is presented, and the extent to which students are included in feedback processes, are important factors in shaping students' perceptions of feedback (Burnett & Mandel, 2010). In their qualitative study, Burnett and Mandel (2010) examine primary school teachers' and students' perceptions of feedback and praise through structured interviews and observations. They report that younger children prefer "ability" feedback, whereas older students show preference for feedback centered on their effort. They recommend that feedback should be less general and more topic-specific in order to be effective.

Students' gender is another factor that affects their perceptions of feedback (Carvalho, Santos, Conboy, & Martins, 2014). In a study of 178 high school students, Carvalho et al. (2014) find girls perceive feedback to be more effective than boys. The relationship between students' gender, prior knowledge, and motivation and how they perceive, receive and act on tutor feedback was the subject of a study done by Narciss et al. (2014) with grade 7 students. According to their findings, female students show more knowledge gain from feedback than male students. Furthermore, boys' perceived competence declines after feedback in comparison to that of girls. Their results indicate that students tend to skip further attempts after receiving conceptual feedback as opposed to procedural feedback, which suggests that feedback that requires higher-level thinking is not taken up as much as simpler feedback.

Examining the relationship between students' perceptions of teachers' oral feedback and the teachers' expectancies as well as students' self-concept was the focus of Chen, Thompson, Kromrey, and Chang's (2011) study with a large group of elementary school students and teachers in Taiwan. According to their findings, boys perceived more negative oral feedback than girls, and students in grade five perceived more negative oral feedback than students in grade three and four, suggesting that both gender and age affect students' perceptions of feedback. Chen et al. (2011) also found a strong relation between positive academic oral feedback and academic self-concept.

Students' perception of feedback is also informed by how they perceive themselves in relation to feedback. Harris, Brown, and Harnett (2014) analyzed 193 primary and secondary New Zealand students' answers to survey questions, as well as their drawings about their understanding and experiences of feedback. Students' drawings and responses indicated that they understand feedback as written comments and/or marks provided by the teacher. In this way, the

students did not perceive themselves as active participants of feedback practices. However, students considered feedback as constructive and positive, which suggests that their perception of feedback is related to whether or not it helps them improve. Budge and Gopal (2009) also performed a comprehensive study on students' perception toward feedback. Their participants were invited to complete Likert scale questionnaires and answer open-ended questions about how they perceived feedback, what type of feedback they considered as valuable, and what recommendations they had regarding feedback and future assessment practices. Their findings were interesting and important for two reasons. Firstly, a surprisingly large number of the participants (95%) considered feedback to be quite important and indicated that they used the feedback they received to improve their future assignments. Secondly, the participants of the study showed strong personal connection to the feedback, believing it to be an indication of their strengths and weaknesses. Such strong connection can aid or hinder the way students receive and take up feedback.

Students' perceptions of feedback are of special interest in research in higher education as well. Studying university students' perceptions of the effects of different forms of feedback on their performance, motivation, and emotion, Lipnevich and Smith (2009) report that students preferred feedback comments with detail as compared to grades, proposing that low grades damage self-efficacy and that high grades decrease motivation and students' need to improve. In her research about how university students value feedback, Marie (2016) reports that not all students value feedback highly and that the students in her study valued feedback when they could see the possibilities of using the feedback to improve future work. As such, Marie (2016) argues for further educating students about the value of feedback and how it can be used productively. Surveying students' perceptions and preferences for feedback in higher education,

Rowe, Wood, and Petocz (2008) report that a quarter of the students who took the survey indicated that they hardly received any feedback from their instructor. This perceived lack or absence of feedback on behalf of the students suggests that much improvement is needed in the provision of feedback and that students must be helped to become more involved in feedback practices so as to recognize, understand, and receive feedback (Rowe et al., 2008).

In summary, a review of existing research on feedback in education reveals different conceptualizations of feedback, some calling it information (Hattie & Timperley, 2007; Sadler, 1989; Winne & Butler, 1994), while others describing it as social and relational processes (Yang & Carless, 2013; Knight, 2003). The different conceptions of feedback have resulted in associating a multitude of mediating factors with its effectiveness. Research suggests that the effectiveness of feedback is not only connected to the type of feedback, but that it is influenced by many factors such as goal setting, prior knowledge, and student perception. Researchers advocate for educating students about feedback (Marie, 2016), claiming that students' perception of feedback and its effectiveness improve when students become more involved in their feedback practices (Rowe et al., 2008). However, the students' level of involvement in these practices is quite varied across the literature and is one that is of special interest to this research.

Peer Feedback

Peer-feedback is defined as feedback that is "provided by equal status learners" (Gielen et al., 2010, p. 305). The notion of equal status aims to distinguish peer feedback from expert feedback or from feedback provided by a person with higher authority. However, one can argue that there are different levels of expertise and authority even among peers. Lui and Carless (2006) define peer feedback as "a communication process through which learners enter into dialogues related to performance and standards" (p. 280). Lui and Carless' (2006) definition of

peer feedback is in line with this research as it situates learners, regardless of their status, in dialogical processes which necessitate and support the active participation of those who give and those who receive feedback.

Peer feedback can be traced to studies in peer learning (Falchikov, 2001; Topping, 2005; Topping, 2009) as well as literature on formative assessment (Black & William, 1998). Despite expansive and extensive research on feedback, research on peer feedback is still in its youth and in need of identity formation and affiliation (Kollar & Fisher, 2010). Kollar and Fischer (2010) recommend that identity formation can be achieved through the creation of shared terminology and theory, and affiliation can be reached by systematically analyzing research in related fields. Strijbos and Sluijsmans (2010) echo this concern and call for a systematic investigation to facilitate methodological, functional and conceptual development in research pertaining to peer assessment. In response to the need for research on peer feedback, there have been studies that examine how peer feedback relates to expert feedback (Falchikov & Goldfinch, 2000), peer feedback's effectiveness (Gielen et al., 2010), and the perception of feedback by peers (Strijbos, Narciss & Dunnebier, 2010a).

Comparison Between Peer Feedback and Expert Feedback

Research suggests that peer feedback, when its goals are clear and the criteria is set, can be comparable to expert feedback (Falchikov & Goldfinch, 2000; Hamer et al., 2015). In their meta-analysis of 48 peer assessment studies comparing peer and teacher marks in higher education, Falchikov and Goldfinch (2000) found that peer assessments closely resembled teacher assessments when the marking criteria was clearly understood by all involved and when the assessment judgments were purely focused on academic processes and products. In a related study comparing feedback given by peers with feedback provided by tutors in an undergraduate

programming class, Hamer, Purchase, Luxton-Reilly and Denny (2015) found that although tutors gave longer, more specific comments, in many respects (e.g., giving advice), peer and tutor feedback were not very different.

However, some studies suggest that even with set criteria, peer feedback can be influenced by judgment and affective factors (Hovardas, Tsivitanidou & Zacharia, 2014). In their investigation into the quality of secondary students' feedback and the usefulness of feedback, Hovardas et al. (2014) report that peer feedback and expert feedback differed in detail and type of judgment provided. Although peer assessors and expert assessors used the same pre-specified assessment criteria, and feedback suggestions made by peers were scientifically accurate, peer feedback was found to have more positive judgment as well as more negative unfounded judgment when compared to expert feedback.

Investigating the effects of online peer assessment on high school students' learning, Lu and Law (2012) observe that positive affective feedback was more conducive to the uptake of feedback by the student. Panadero, Romero, and Strijbos (2013) explore how friendship and the use of rubrics could affect the validity of peer assessment in higher education. They found that while all students tend to inflate the score of their peers, assessments based on rubrics are valid. However, even when using a rubric, high levels of friendship resulted in more score inflation. The inconsistency among the research comparing expert and peer feedback is indicative of the complexity of peer feedback and the existence of many factors that influence the implementation of peer feedback and how it is perceived by students.

Factors Influencing the Implementation of Peer Feedback

Various types of peer feedback, possible internal mediators, and the implementation of feedback in writing were the subject of research by Nelson and Schunn (2009). Students'

understanding of the peer feedback provided was the only significant mediator affecting the implementation of the feedback among the internal mediators they supported. In other words, if students understood the feedback, they implemented it in order to improve their writing. Performing two different studies, investigating the nature, reception, and use of online peer feedback in higher education, Van der Pol, Van den Berg, Admiral, and Simons (2008) report that feedback comments that were received and used by students were those that contained concrete suggestions.

Students' Perception of Peer Feedback

Students' perception of peer feedback is a "crucial determinant of how they treat the feedback and possibly can help to uncover when and how feedback perception is positively related to feedback efficiency" (Strijbos, Pat-El, & Narciss, 2010b, p. 385). Strijbos et al. (2010b) describe and validate feedback, using a perception questionnaire to measure students' perceptions in terms of fairness, usefulness, acceptance, willingness to improve, and affect. Students' perception of feedback in terms of fairness, usefulness and acceptance is described as students' 'perceived adequacy of feedback' and is found to be a predictor of students' willingness to improve and affect (Strijbos et al., 2010b). Exploring the effect of assessor's competence on graduate students' perception of peer feedback, Strijbos et al. (2010a) report that feedback offered by higher competency assessors was perceived to be more adequate, but had more negative affect.

Learning how to Provide Peer Feedback

Many researchers in the field of peer feedback advocate for more training for students in order to improve the effectiveness of their feedback (Gielen et al., 2010; Harris & Brown, 2013). Studying university students' composition and receipt of peer feedback, McConlogue (2015)

reported that while the assessor viewed the giving of peer feedback positively, the receiving of the feedback was viewed negatively. McConlogue (2015) suggests that learning to compose proper peer feedback is a lengthy process and requires training.

The effect of engaging both the students in the peer feedback process on the quality of peer feedback and students' final written product in a Wiki based computer supported collaborative program in the first year of higher education was the subject of Gielen and De Wever's (2015a; 2015b) studies. According to their findings, scripting the roles (2015a) and structuring the process (2015b) improve the quality of the peer feedback as well as the quality of students' final written assignments. Furthermore, engaging students in these feedback processes improves the quality of their feedback (Gielen & De Wever, 2015a; 2015b).

The relationship between students' attitudes toward peer review and the quality of peer feedback they gave and received is the focus of a study by Bosboom, Goff, Haan, and Isherwood (2014). Their participants, a group of students at a Dutch secondary school, were invited to answer pre-and post-review survey questions before giving and after receiving feedback. The quality of the students' feedback was assessed using a rubric. Although there was not a significant difference in the students' attitudes toward peer assessment, students became more confident after they received feedback of good quality. Furthermore, with teachers' guidance and scaffolding in the form of direct instructions and provision of a rubric, students learned to give feedback of good quality. Their findings offer important implications for peer feedback practices, suggesting that students and teachers can, and indeed must, enjoy a partnership in the process of giving and receiving peer feedback, although the nature and extent of this partnership differs. In Bosboom et al.'s (2014) research, the teachers decided on and instructed students about what they considered as "good quality" feedback. Another example of partnership between students

and teachers can be found in a study conducted by Tasker and Herrenkohl (2016), in which a seventh grade teacher supported her students in not only providing each other with peer feedback, but also in improving the quality of feedback by collectively defining 'meaningful feedback.' In this research, the students took on a more active role in deciding what feedback is "meaningful" to them.

A review of the existing literature on peer feedback reveals two reoccurring themes; the first is concerning the peers' lack of understanding as to how to provide effective feedback and the need for training, and the second pertains to the need for students' active engagement in the process. The same concerns are reflected in literature about feedback and peer feedback in mathematics education.

Feedback and Peer Feedback in Mathematics Education

Most of the research about feedback in mathematics education is embedded in, or related to, research in assessment (William, 1999) and can be categorized into research about the effectiveness of feedback (Bokhove & Drijvers, 2011; Santos & Pinto, 2009), students' perception of feedback (Hyland, 2015; Rodgers et al., 2014), and the training of feedback (Lavy & Shriki, 2014; Popelka, 2015).

Feedback is considered to be an important component of assessment acts in a mathematics classroom (Björklund Boistrup, 2010). In a 2-year intervention project with upper secondary students, Havnes, Smith, Dysthe, and Ludvigsen (2012) explored how assessment information is received and attended to by the students. Significant differences were found in how students and teachers perceived feedback, in how males and females perceived feedback, and how feedback was received differently in mathematics versus languages (Havnes et al., 2012).

Santos and Pinto (2009) claim that, "giving feedback can constitute a task of extreme complexity" (p. 7). As such, they caution against oversimplifying feedback as good or bad, or generalizing the quality of feedback based on its length or format. They concede that in many cases in mathematics, shorter feedback that focuses on specific tasks is more useful than lengthier feedback. However, they also argue that the nature and complexity of the mathematical task for which the feedback is provided plays a role in the length of the feedback. In other words, simple mathematical tasks often require a brief amount of feedback, whereas longer, open-ended, and more complex tasks necessitate longer feedback.

Effectiveness of Feedback in Mathematics Classrooms

According to current literature on feedback in mathematics education, the effectiveness of feedback is mediated by two factors: students' perception of feedback and their prior math knowledge.

Students' perception of feedback in terms of usefulness and support has an indirect effect on their interest and their achievement development (Rakoczy et al., 2013). Feedback is perceived most effective when it is minimal and about low-risk, less complicated tasks (Hyland, 2015). Students value feedback that is focused on the mathematical task, avoids praise, and is focused on change (Rodgers et al., 2014). In a study by Harks, Rakoczy, Hattie, Besser, and Klieme (2014), using students' perceived usefulness as a mediator, effects of process-oriented feedback on students' mathematics achievement, interest, and self-evaluation were compared against those of graded feedback. While there were no changes in self-evaluation, process-oriented feedback was perceived to be more useful, and this positive perception increased students' mathematical achievement and interest.

Students' perceptions of the usefulness of oral and written feedback was the focus of a study by Randall and Zundel (2012). The participants of their study, students of an introductory problem solving course, not only showed positive response to the multi-channeled feedback they received, they continued to seek feedback throughout the course, finding it useful and supportive of their learning.

The recipients' mastery of mathematics is another important component in the effectiveness of the feedback (Santos & Pinto, 2009). In other words, students with lower mathematical skills may misunderstand and/or abandon the feedback entirely. By contrast, more skilled students would feel confident enough to ask the teacher for clarification of the feedback. Students' prior mathematical knowledge was used as a mediator in a study by Fyfe, Rittle-Johnson, and DeCaro (2012) to examine feedback as a form of guidance. The researchers provided no feedback, outcome feedback, or strategy feedback to students with little or some prior knowledge. Prior knowledge of the correct strategies affected the degree to which the feedback was beneficial. Students with little prior knowledge benefitted more from feedback than those with more prior knowledge, who benefitted more from exploring the task with no feedback. In a more recent study, Fyfe and Rittle-Johnson (2016) explore the effectiveness of computer-generated feedback in mathematics problem solving for second grade students. Students with low prior knowledge achieved higher scores following feedback than students who received no feedback. Furthermore, immediate feedback was more effective regardless of whether students had high or low prior knowledge.

Feedback interventions are reported to be beneficial for learning, especially for students with little prior knowledge (Krause, Stark & Mandle, 2009). In a mixed-methods, quasi-experimental study, Landers and Reinholz (2015) explored how community colleges students in

a developmental intermediate algebra course reflect on and/or make use of their mathematics homework feedback. They found that although by the end of the term these students were not able to outperform their peers, they grew as learners, as they used feedback to engage in self-assessment and to critique their own work.

Students as Partners

The literature about feedback and peer feedback in mathematics education calls for more engagement for students in their feedback practices. However, the extent of students' engagement varies across the literature, ranging from training of students' peer feedback practices (Rodgers, Diefes-Dux, Cardella, & Fry, 2012) and sharing learning goals and success criteria with them (MacDonald, 2015), to regarding students as partners in co-creating their feedback practices (Meer & Chapman, 2014).

Emphasizing the importance of formative feedback, Fluckiger et al. (2010) describe and analyze techniques to involve students in their feedback practices. Their techniques are centered on timely, frequent feedback that is shared with students through written comments or individual conferencing in order for students to track their progress and to make necessary revisions. In this way, formative feedback allows students to act as partners in the process of assessment. In a quantitative study with grade 7 students, Popelka (2015) explores how training students in peer and self-assessment may influence their mathematics achievements. She reports that although assessment training did not increase the accuracy of peer assessment, it did improve the participants' self-assessment as well as their overall mathematics achievement. Conducting a study with a group of first year calculus students during two terms, Reinholz (2015a) trained students' feedback practices by focusing on their conversations as they engaged in peer assessment. After training, students' feedback became more on-topic, focusing more on

processes rather than products (Reinholz, 2015a). Training of peer feedback was studied in another study by Reinholz (2015b), where using an activity called peer-assisted reflection (PAR), students were given a question to work on, reflect on their work, conference with a peer, revise and submit. The goal of this study was to examine the effects of PAR on improving students' achievements in calculus. At the end of the term, the department reported dramatic improvements in the participants' achievements.

However, there is a growing body of research that goes beyond mere training and advocates for engaging students, as partners, in their feedback processes. In a study conducted with two primary classrooms in New Zealand, Knight (2003) explored teachers' feedback as a supporting factor for students' numeracy learning. His findings highlighted two major concerns. First, there is significantly less research in mathematics education about teachers' feedback, resulting in a lack of clear definition for what constitutes "quality feedback." Secondly, teachers seldom share learning outcomes with their students as a part of their mathematics instruction practices. Consequently, feedback, both in its definition and construction, is decided and offered by the teacher, leaving the student as a passive recipient of the feedback. Knight (2003) recommends that more studies should be conducted involving teachers and students as they engage with feedback, as "good feedback involves a two-way process" (p. 22).

Co-creating the marking criteria for a second-year course and using the criteria to provide peer feedback practices were the subject of a study by Meer and Chapman (2014). Their action research is premised on engaging students as partners in feedback processes and thereby, creating a 'community of practice' where students "become absorbed into the academic practices of their discipline" (Meer & Chapman, 2014, p. 4). According to Meer and Chapman (2014),

"the process of creating the criteria and using them to peer and self-assess brings in the active application that develops understanding" (p. 4).

Situating this Research within Existing Literature

A review of existing literature on feedback and peer feedback in education reveals three significant implications for this research, a) feedback is among the most important factors in education success (Hattie & Timperley, 2007), b) feedback is a complex process that involves people, actions, and relationships (Nicol, 2010; Gravett & Peterson, 2002), and c) the effectiveness of feedback, no matter how carefully constructed, hinges on how it is sought (Hattie and Timperley, 2007), mindfully received (Gielen et al., 2010), and acted upon (Sadler, 1989).

This research is in line with literature that conceptualizes feedback as relational and social processes (Nicol, 2010; Gravett & Peterson, 2002) that involve actions and interactions and is consistent with Yang and Carless (2013) in exploring not only the contents of feedback, but the relationships formed among those who enact feedback. Furthermore, this research is inspired by a democratic view of education which advocates for students' active participation in educational engagements. In the context of co-constructing peer feedback practices, a democratic view necessitates students' active and mindful participation in an environment where feedback practices are understood and supported by students and teachers. Such conditions can only be met through a shift from passive reception to active participation and engagement on behalf of all stakeholders. Those who give feedback and those who receive it must be involved and engaged in the process of constructing their feedback practices in order for these processes to be meaningful and effective. My research situates itself among research that advocates for students as partners in the co-creation of their feedback practices (Meer & Chapman, 2014). Feedback

practices that involve students as partners have been conceptualized more as long-term dialogic processes rather than products (Price et al., 2011; Beaumont et al., 2011).

However, my research, by adopting an enactivist perspective (Varela, Thomson & Rosch, 1991), differs from the existing research in this domain in two fundamental ways. The first difference is that students' co-construction of feedback is not a means to achieving the goal of finding a single, pre-existing model for feedback practices. Rather, the focus of my research is how feedback is conceptualized and co-constructed throughout the process. Secondly, my research diverges from others that engage students in co-creating their feedback practices in how students' actions are regarded. According to Meer and Chapman (2014) the process of co-creating criteria for feedback engaged students in active application that helped develop their understanding of feedback. Students' active participation was as a means to develop understanding. However, in my research our active participation in co-constructing peer feedback practices is not a means to developing our understanding of peer feedback but *is* our understanding of peer feedback.

In the following chapter, I will introduce enactivist theory and will describe how adopting this theory brings forth a different perspective for exploring peer feedback in our mathematics classroom.

Chapter 3: Theoretical Framework

In this study, I adopted an enactivist lens (Varela et al., 1991) as my theoretical framework as well as my methodological approach. In other words, not only were my research questions guided by the enactivist view, but the research design as well as data analysis were informed by this theory. In this chapter, I will present and describe enactivism as my theoretical framework and, in the next chapter, I will explain how enactivism informs my methodology and methods for this study.

I will begin this chapter with a brief review of other theories of cognition and will discuss the fundamental differences among them. I will then introduce the enactivist view by offering a brief history of the origins and the evolution of this theory. Next, I will define enactivism as embodied action and will describe the key concepts of this theory. I will explore how enactivism as embodied action frames knowing, experience, social systems and language. Next, I will review how enactivism is situated as a theory and methodology in research in mathematics education. In the final sections of this chapter, I will share a brief history of my interest in enactivism and will discuss how enactivism frames feedback.

A Brief Review of Theories of Cognition

In this section, drawing on Ernest (2010) and Varela et al. (1991), I offer a brief overview of cognitivism, constructivism, and social constructivism, and will examine how each theory defines knowing and the nature of the relationship between the knower and the known.

Cognitivism

According to Varela et al. (1991), in traditional cognitivist theory it is posited that human intelligence resembles computation and that cognition can be defined as symbolic representation, an internal process taking place in the individual's mind. According to this input-output model, cognition is understood as representation of some aspect of a pre-existing world (Varela et al., 1991). As Varela et al. (1991) explain, this view of cognition is based on three assumptions: first, it supposes a pre-existing world, independent of the knower; second, it understands cognition as an activity of the mind, therefore rendering bodily experiences inconsequential and irrelevant; and third, it distinguishes the known and the knower as independent of one another, where the latter, through representation, comes to know the former. This view of cognition is what Freire refers to as the "banking" model of learning which claims that knowledge is passed over to learners, and that learners are passive recipients rather than "epistemologically and politically empowered social agents" (as cited in Ernest, 2010, p.40).

Constructivism

In contrast to the cognitivist view, in constructivist theory it is postulated that knowledge is not discovered but constructed by individuals based on experiences (Crotty, 1998; Fosnot, 1996). Constructivist theorists argue that "knowledge is not passively received but actively built up by the cognizing subject" (Von Glasersfeld, 1989, p. 182). Constructivist theorists reject the idea of knowing as representing and posit that knowledge is constructed through the knower's

accommodation and assimilation (Piaget, 1936/1952). Constructivist theorists do not negate the existence of a world independent of the knower. However, they argue that knowledge is not a copy of reality, but the knower's accommodation to reality, influenced by the knower's prior knowledge. The notion that pre-existing knowledge and understandings are the basis for subsequent learning is one that is shared by most forms of constructivism (Ernest, 2010).

What distinguishes radical constructivism from trivial constructivism is the difference in assumptions about whether or not absolute knowledge is attainable (Ernest, 2010). Radical constructivists claim that knowledge is a human construction and "pertains to the way in which we organize the world of our experience" (Von Glasersfeld, 2007, p. 97). For the radical constructivist, knowledge does not need to be "true" in an objective sense; rather, it needs to be viable according to the knower's experience. Von Glasersfeld (2007) describes cognition as an adaptive function, an endeavor "to produce viable conceptual structures that enable the cognizing subject to fit into the world it experiences" (p. 95). Constructivism has been highly influential in research in education and particularly mathematics education (Cobb, Yackel & Wood, 1992). However, it has also been criticized for downplaying the significance of social interactions in the development of knowledge (Cobb, 1988). Where constructivism locates cognition in the mind of the learner, social constructivism, situates the construction of knowledge within the social domain.

Social Constructivism

While still emphasizing the construction of knowledge by the individual, social constructivism differs from basic constructivism in that the importance of the interactions between the knower and the environment in the construction of knowledge are highlighted (Cobb, 1994; Derry, 1999; McMahon, 1997). This perspective is closely associated with

developmental theories of Vygotsky (1978) and Bruner (2006), and Bandura's (1986) social cognitive theory. According to social constructivism, knowledge is a human product, socially and culturally constructed by individuals through interactions with each other and with the environment (Ernest, 1998; Gredler, 1997). For the social constructivist, truth is relative and context-bound; learners construct knowledge through a process of interpretation and active negotiation (Jonassen, 2006). While social constructivists emphasize the importance of interaction with the environment in constructing knowledge, a dualistic view of knower and the known is still maintained. Social constructivism has been widely used in research in mathematics education (Cobb & Yackel, 1996; Lave & Wenger, 1991; Restivo, 1988). However, it has also been criticized for giving priority to social interaction at the expense of individual cognition.

In summary, cognitivist theorists position "truth" in an external world and conceptualize knowledge as the knower's internal representation of an objective, independent world. Constructivists, on the other hand, negate knowledge as representation and describe cognition as construction of knowledge through the knower's adaptation to the world. Social constructivist theorists, build on constructivists' notion of knowledge as constructed, but give priority to social interactions in constructing knowledge. Despite fundamental differences, the common denominator among the proponents of these theories is that they all hold a dualistic view of knower and the known. In other words, the knower comes to know a pre-existing world through representation, individual construction or social interaction. While social constructivist and constructivist theories are criticized, the former for downplaying the individual and the latter for neglecting the environment, enactivists offer a middle ground by blurring the lines between the knower and the known and describe knowing as the co-emergence of the individual and the environment.

Origins of Enactivism

Enactivism finds its roots in the 1980s and early 1990s in the works of two Chilean biologists, Fransisco Valera and his teacher, Humberto Maturana. Central to their work was their quest to characterize the organization of living systems and to discriminate between the living and the non-living. They use the term autopoiesis to describe the defining characteristic of living organisms. The word "autopoiesis" is taken from two Greek words αὐτο (auto), meaning “self," and ποίησις (poiesis), meaning "creation.” In short, autopoiesis means self-producing and refers to a system that is capable of reproducing and maintaining itself. Autopoiesis, according to Maturana and Varela (1980), is the defining characteristic of living organisms. They describe an autopoietic system as:

> a network of processes of production (transformation and destruction) of components that produces the components which: i) through their interactions and transformations continuously regenerate and realize the network of processes (relations) that produced them; and ii) constitute it as a concrete unity in the space in which they exist by specifying the topological domain of its realization as such a network. (p. 79)

An autopoietic system, according to Maturana and Varela (1980), is a system that is autonomous, emergent, and operationally closed.

Although Valera and Maturana (1980) use the term autopoiesis specifically to describe living systems, some studies have since generalized the concept to apply to non-living autonomous systems as well. For instance, Luhmann (1995) extends Maturana and Varela’s (1987) concept of autopoietic systems and describes two non-biological systems: social systems and psychic systems. However, Varela (1981) refuses the existence of non-biological systems as autopoietic, claiming that non-living systems may be autonomous, but they are not autopoietic.

Reid (2014) also cautions against Luhmann's (1995) generalization, arguing that while non-living autonomous systems are self-organizing, they are neither self-producing nor embodied. However, Luhmann (1995) claims that embodiment does not necessarily imply a molecular constitution and can be exemplified in terms of conversation. Social systems, Luhmann (1995) explains, are based on communication, and psychic systems are based on consciousness. The term social autopoiesis has been used in social studies research (De Jaegher & Di Paolo, 2007; Di Paolo & Thomson, 2014; Luhmann, 1995; Mingers, 2002) as well as in research in mathematics education (Steinbring, 2015).

Maturana and Varela's focus in research later diverged, but their concept of autopoiesis and the characteristics of autopoietic systems became central to how Varela et al. (1991) define and describe enactivism. For this reason, in this research, Maturana and Varela's ideas, together and apart, are used to describe enactivism.

Three Theories of Experience that Influence Enactivism

While enactivist theorists draw from biological roots to describe cognition, their description of experience is influenced by hermeneutics, which encapsulates the interpretative nature of understanding, and phenomenology, which emphasizes the interconnectedness of the knower and the known. Enactivism is also inspired by the Buddhist tradition of Madhyama in describing an open, mindful approach to understanding experience (Varela et al., 1991). In this section, I will examine how hermeneutics, phenomenology, and Buddhism have influenced enactivism's conceptualization of experience, as well as how enactivism distinguishes itself.

Hermeneutics

Proponents of hermeneutics propose a process of interpretation in order to understand one's experiences (Gadamer, 1975; Hegel, 1977). Gadamer (1975) claims, "all understanding is

interpretation, and all interpretation takes place in the medium of language" (p. 407). Described as "the art of interpretation" (Howard, 1982, p. 119), hermeneutics "allows for self-consciousness and self-formation" (Slattery & Rapp, 2003, p. 84). As such, hermeneutics is both subjective and circular. It is subjective by virtue of being guided by one's interpretation. It is circular since "our interpretations exist in an endless circle of mutual specification and elaboration" (Gadamer, as cited in Slattery and Rapp, 2003, p. 97). Sumara and Davis (1997) describe the hermeneutic circle of understanding as an "ever-evolving relationship among components of a system whereby any understanding (remembered, lived, or projected) is in a continual process of being interpreted" (p. 412). This process is contingent upon an openness to the other and the otherness. Gadamer (1975), explains that the knower comes to know the known through a fusion of their horizons. Gadamer (1975) describes this fusion of horizons by stating,

> When our historical consciousness transposes itself into historical horizons, this does not entail passing into alien worlds unconnected in any way with our own; instead, they together constitute the one great horizon that moves from within and that, beyond the frontiers of the present, embraces the historical depths of our self- consciousness. Everything contained in historical consciousness is in fact embraced by a single historical horizon. (p. 315)

However, for the enactivist, understanding is lived, temporal and embodied; interpretation of an experience is itself an experience (Varela et al. 1991). For embodied action, interpretation is "the enactment or bringing forth of meaning from a background of understanding" (Varela et al., 1991, p. 149). According to Varela et al. (1991), we know as a result of recurrent interpretation emerging from our capacities of understanding. These capacities

are embodied and experienced in a domain of consensual action. In stressing the embodiment of experience, enactivism is greatly influenced by phenomenology.

Phenomenology

Phenomenology is the study of experience and human consciousness (Thomson, 2007). The term "phenomenology" is derived from the Greek word "phainomenon," which means appearance (Spinelli, 1989, p. 6). Edmund Husserl and Martin Heidegger, German philosophers attributed as developing phenomenological theory, posit that reality is perceived by human consciousness. Phenomenology is "anchored to the careful description, analysis, and interpretation of lived experience" (Thomson, 2007, p. 16).

Although within phenomenology and enactivism the importance of experience is highlighted, these theories differ in terms of how experience should be explored and reflected on. Specifically, Varela et al. (1991) argue that phenomenology's exploration of experience is theoretical and lacks a pragmatic dimension and, as such, fails to link science and experience together.

Varela et al. (1991) state, "organism and environment enfold into each other and unfold from one another in the fundamental circularity that is life itself" (p. 218). This circularity enables enactivists to reject absolutism and nihilism, and to offer a middle way, by positing that embodiment is "the fundamental axis" of this circularity between cognition and experience (Varela et al. 1991). Embodied action offers a middle ground; a communication between science and experience and experience and science. Enactivists "use phenomenology to explicate mind science and mind science to explicate phenomenology" (Thomson, 2007, p. 265). This back-and-forth circle is crucial for a reflective scientist. According to enactivism, reflection is a circular process, where we reflect on a world that is not separate from us (Varela et al., 1991).

Buddhist Influences

Inspired by the Buddhist tradition of Madhyama, enactivism offers a middle way between mind science and the investigation of human experience, which is rooted in an open, mindful approach to experience. According to Varela et al. (1991), "a mindful, open-ended approach to experience reveals that moment by moment this so-called self occurs only in relation to the other" (p. 247). Varela et al. (1991) describe mindfulness as the mind being present in one's embodied everyday experiences, which, they argue, necessitates "a change in the nature of reflection from an abstract, disembodied activity to an embodied (mindful), open-ended reflection" (p. 27). They describe this open-ended reflection by stating:

> By embodied, we mean reflection in which body and mind have been brought together. What this formulation intends to convey is that reflection is not just on experience, but reflection is a form of experience itself – and that reflective form of experience can be performed with mindfulness/awareness. When reflection is done in that way, it can cut the chain of habitual thought patterns and preconceptions such that it can be an open-ended reflection, open to possibilities other than those contained in one's current representations of the life-space. We call this form of reflection mindful, open-ended reflection. (Varela et al., 1991, p. 27)

In order to develop a truly embodied reflection on one's experiences, one must include oneself in the reflective process. Also, by being open to one's moment to moment experiences and the opportunities therein, one develops the potential to change that which is being observed by interrupting one's own habitual tendencies. When these habitual patterns are interrupted, one becomes more open and mindful to new opportunities. It is in this way that such openness has the potential to be transformative. Such reflection brings about "the openness and space in which

a transformation of what the subject itself is, or could be, becomes possible" (Varela et al. 1991, p. 126). Being open and mindful of one's experiences transforms one's intentionality and the potential affordances of those experiences.

Enactivism as Embodied Action

The term enactivism was first introduced in *The Embodied Mind* by Varela et al. (1991), through what they define as enaction, which describes cognition as a "history of structural coupling" (p. 206) and the embodiment of knowledge, involving complex webs of relations. For the enactivist, cognition is embodied action (Varela et al., 1991).

The notion of embodiment is two-fold. On the one hand, it is used to describe the bodily interactions between the knower and the known through the knower's sensorimotor capacities, and on the other, it is used to refer to how these capacities are influenced by the knower's biological, psychological and cultural history (Varela et al., 1991). Thus, our understanding is "rooted in the structures of our biological embodiment but [is] lived and experienced within a domain of consensual action and cultural history" (p. 148).

Embodiment entails sensory and motor processes, whereby perception and action are inseparable and cognition is lived, temporal and emergent. In describing embodiment, Varela et al. (1991) explain that a system can be described as composed of various interacting subsystems and it can also be described in terms of a unit capable of various forms of structural coupling. In other words, a system can be described in terms of its autonomy and operational closure, and it can also be described in terms of its structural coupling with the environment. In this research, enactivism and embodied action are taken to be synonymous and will be used interchangeably. Below, I will discuss these elements which are the key concepts of enactivism as embodied action as well as characteristics of an autopoietic system.

Operational Closure

One of the central characteristics of an autopoietic system is that it is operationally closed (Maturana & Varela, 1980). An organizational or operational closure refers to the "self-referential (circular and recursive) network of relations that defines the system as a unity" (Thomson, 2007, p. 45). In an operationally closed system, the result of a process is the process itself (Varela et al., 1991). There is no separation between processes and the products of those processes. Such co-dependence rejects the notion of the world as independent from the structure of the processes that specify it. Varela et al. (1991) explain that operational closure is "a way of specifying classes of processes that, in their very operation, turn back upon themselves to form autonomous networks" (Varela et al., 1991, p. 139). As such, there exists a close link, a reciprocal relationship, between a system's operational closure and its autonomy.

Autonomy

An autopoietic system is a self-determining structure in that "all that takes place in it, or happens to it at any instant, is determined by its structure at that instant" (Maturana as cited in Reid, 2014, p. 154). In describing the relationship between autopoiesis and autonomy, Maturana and Varela (1987) explain that while autonomy is not unique to living beings, and that non-living systems can also be autonomous, autonomy is the mechanism that makes a living system autopoietic. The significance of autonomy and structural coupling of autopoietic beings is two-fold. First, as autonomous agents, living beings can generate and maintain themselves, and second, this self-determination rejects the input-output model of cognition in favor of an emergent system.

Due to its emphasis on operational closure, enactivism has, at times, been likened to radical constructivism and has been critiqued for being operationally closed and thus allowing

for the "subordination of the social or the interpersonal dimension" (Ernest, 2006, p. 5). However, enactivists maintain that an autopoietic system, while structurally closed, constantly interacts with the environment (Reid, 2014). In fact, an autonomous system is understood in terms of perturbations and responses as it brings forth meaning through coupling with its environment (Thompson, 2007). As such, while the system is self-determining and structurally closed, it is also in recurrent interaction with its environment, continuously perturbing and perturbed by it. The system interacts with the environment and with other systems within the environment. These interactions result in the "co-evolution of two structure determined systems through a history of structural coupling" (Reid & Mgombelo, 2015, p. 175).

Structural Coupling

Maturana and Varela (1987) explain that systems are structurally coupled when there is a structural congruence between them as a result of a history of recurrent interactions. This means that an autopoietic system has the potential to undergo coupled ontogenies with its environment if and when its interaction with the environment becomes recurrent. As a result of these recurrent sensorimotor patterns of interaction, there will be a "history of mutual congruent structural changes as long as the autopoietic unity and its containing environment do not disintegrate: there will be a *structural coupling*" (Maturana & Varela, 1987, p. 75). As such, structural coupling is possible when the system remains in recurrent sensorimotor interaction with its environment while still maintaining its operational closure. Enactivism as embodied action implies that the structure of the system determines how it couples with its environment.

Structural Determinism

In describing structural determinism, Maturana and Varela (1987) explain that the changes that result from the interaction between the living being and its environment are

triggered by the disturbing agent but are "*determined by the structure of the disturbed system"* (p. 96). Every autopoietic unity, by virtue of its particular structure, engages and is engaged by its environment in reciprocal perturbations. The structure of the environment 'triggers' structural changes in the autopoietic unity, but it does not specify or direct it. Any change in the unity is specified by the structure of that unity. In turn, the structure of the unity 'triggers' changes in the environment. But any change in the environment is specified by the structure of that environment. Conceptualizing cognition as embodied action, as structural coupling between the system and its environment, bears significant implications for how knowing is understood in its essence and in relation to the knower.

Enactivism and Knowing

The traditional view of cognition holds that knowledge is the mind's representation of the world, which implies that there is a pre-existing world to be re-presented (Varela et al., 1991). In contrast, according to enactivism and constructivism, there is no pre-existing world out there to be discovered. However, what differentiates enactivism from constructivism is that, for the enactivist, the world is "inseparable from our bodies, our language, and our social history- in short from our embodiment" (Varela et al. 1991, p. 149). We do not represent an independent world, we enact it by making distinctions that are inseparable from our structure (Maturana & Varela, 1987). Thus, for the enactivist, knowing is about making distinctions; it is "bringing forth a world by a viable history of structural coupling" (Varela et al., 1991, p. 205).

The notion of structural coupling between autopoietic systems and their environments offers a different constitution for the relationship between the self and the other. The self is a system of sub-networks defined and distinguished from the other by its operational closure.

However, as organism and environment couple with one another, what constitutes the world of a given organism is enacted by the organism's history of coupling with it (Varela et al., 1991).

Knowing for the enactivist is an emergent process, resulting from complex webs of reciprocal relationships. This network consists of multiple levels of interconnected, sensorimotor subnetworks (Varela et al., 1991) and is marked by the system's operational closure and its autonomy, where the former encapsulates the whole arising from its parts, and the latter reinforces parts arising from the whole. The constitution of the whole from the parts and the parts from the whole culminates in their co-emergence and mutual specification. In this way, the enactive approach "builds on the notion of emergence but reformulates it as 'dynamic co-emergence' in which part and whole co-emerge and mutually specify each other" (Thomson, 2007, p. 60). The knower and the known stand in a reciprocal and co-dependent relationship, where they continuously change and trigger changes in one another. Varela et al. (1991) describe the relationship between the system and the environment, the knower and the known, as one of "mutual specification or dependent coorigination" (p. 150). Merleau-Ponty (1962) also describes the co-dependence of the self and the world when he states, "the world is inseparable from the subject, but from a subject which is nothing but a project of the world, and the subject is inseparable from the world, but from a world which the subject itself projects" (p. 430). This co-dependence between the knower and the known implies that just as there is no pre-existing world without the knower, there is no knower without the known, and that knowing is temporal.

As the system and the environment enfold from each other and co-evolve, meaning emerges and changes in "temporally extended patterns of activity," and new worlds of significance are brought forth (Thomson, 2007, p. 59). Varela et al. (1991) describe this temporality of knowing as groundlessness in that "things incessantly arise and pass away without

our being able to pin them down to a stable objective or subjective ground" (p. 242). However, they claim that groundlessness is not hopeless in a nihilistic sense. Rather, groundlessness allows us to "negotiate our way through a world that is not fixed and pre-existing but that is continuously shaped by the types of actions in which we engage" (Varela et al., 1991, p. 144). In other words, knowledge is knowing how to act in a given situation.

As the knower and the known couple with one another, they trigger changes in one another's structures from moment to moment. The knower is always in a state of knowing, but that knowing is always changing. As such, for the enactivist, there is no dualism between the self and the other. The knower and the known are mutually specifying and co-determining and "there is a reciprocal insertion and intertwining of one in the other" (Merleau-Ponty, 1968, p. 138), where the two co-construct meaning and co-evolve from moment to moment.

For the enactivist, the notion of evolution is one of natural drift and not natural selection (Maturana & Varela, 1987; Varela et al. 1991). The distinction between the two is that natural selection implies an imposition on the organism on behalf of the environment, whereas natural drift is a process in which the organism "brings forth and specifies its own domain of problems to be solved" (Varela et al., 1991, p. 198). Maturana and Varela (1987) posit that evolution is a phenomenon of structural drift and that "there is no progress or optimization of the use of the environment, but only conservation of adaptation and autopoiesis" (p. 115).

Enactivism and Language

According to enactivism, when two autopoietic systems have recurrent interactions, they undergo "structural drifts: co-ontogencies with mutual involvement through their reciprocal structural coupling, each one conserving its adaptation and organization" (Maturana & Varela, 1987, p.180). This is referred to as *third-order structural couplings* (Maturana & Varela, 1987).

When two organisms engage in third-order structural coupling, they generate a social coupling, the results of which are called social phenomena (Maturana & Varela, 1987). One of the defining characteristics of a social phenomenon is communication among its members, and language is the prominent mode of communication for humans.

Maturana and Varela (1987) describe language as "an ongoing process that exists in languaging, not as isolated items of behaviour" (p. 210). Languaging is a form of co-ontogentic structural drift between members of a social system. What is particularly important about these linguistic coordination of actions is that they bring about the *domain of language* (Maturana & Varela, 1987). In other words, "language as a phenomenon takes place in the recursion of linguistic interactions – linguistic coordinations of linguistic coordinations of actions" (Maturana & Varela, 1987, p. 211).

Through languaging, we generate linguistic domains in order to conserve the social system with/in which we interact. These linguistic domains are relevant to the coordination of actions that give rise to them. In this way, language itself is contingent upon the interactions that arise between the organisms with/in the environment. But as we engage in languaging, we also change through our participation in the linguistic domain and that is how meaning arises and becomes a part of "our domain of conversation of adaptation" (Maturana & Varela, 1987, p. 211). In this way, enactivism does not see language as a tool for representing the contents of external worlds to internal minds or vice versa (Haskell, Warren & Ippolito, 2002). Rather, through language, we make co-ontogentic descriptions of descriptions.

In their study of second language acquisition, Haskell et al. (2002) claim that using an enactivist perspective allows "for language to be understood as more than a code, more than a set of instructions encapsulating literal meaning" (Haskell et al., 2002, para. 64). When language is

understood as a structural drift by the members of a social system, conversations take on a new meaning. Haskell et al. (2002) posit, "as the middle ground between realism and idealism, co-emergence suggests that language is part of a simultaneity, an *at onceness* between speaker and hearer" (para. 68). According to Gadamer (1975), language "allows something to 'emerge' which henceforth exists" (p. 401). Van Manen (1997) argues that conversation "has a hermeneutic thrust," in that, "it is oriented to sense-making and interpreting of the notion that drives or stimulates the conversation" (p. 98). In other words, hermeneutics is a manifestation of the structural drift in the linguistic domain.

Enactivism and Research in Mathematics Education

While this research is not concerned with mathematical knowing and teaching, it is situated in a mathematics classroom, where mathematics learners co-construct peer feedback practices to use as part of their mathematics classroom acts. As such, it merits a consideration of literature on mathematics education that uses enactivism as theoretical framework and/or methodology.

It is important to begin by making a distinction between enactivism and embodied cognition which are, at times, conflated in research in mathematics education. Embodied cognition is a subcategory of cognitive psychology, which is concerned with the interactions between an individual and the environment. Transcending the traditional distinctions made between body and mind, embodied cognition proponents argue that "actions or interactions embody projections of the mind and vice versa" (Sriraman & Wu, 2014, p. 207). In mathematics education, advocates of embodied cognition claim that learning mathematics is situated (Lave, 1988), is associated with learner's historical, social and contextual history (Nunez et al., 1999), and is based on the learner's biological and bodily experiences (Lakoff & Nunez, 2000).

One can note similarities in how mathematical knowing is conceived in embodied cognition and in enactivism. However, although the two theories are closely related, there are also distinctions between the two. The first distinction is made on the basis of their origins. Enactivism finds its roots in the biological works of Maturana and Varela whereas embodied cognition has linguistic roots (Goodchild, 2014). The second, and more fundamental difference lies in how enactivism and cognitive perspective differ in what theorists pay attention to (Khan at al., 2014). For the enactivist, the external environment plays an important role and is "a resource for thinking, doing/knowing and being" (Khan, Francis & Davis, 2015, p. 272). In embodied cognition, theorists focus on the role of prior actions in cognition. This means that theorists associated with enactivism attend to ongoing interaction between actions, environment and cognition, whereas theorists associating with embodied cognition, attend to the relationship between prior action and cognition. In this way, enactivists are concerned with the "*learning in action"* while embodied cognition advocates focus on "*learning from action"* (Khan et al., 2015, p. 272). Below, I will explore how enactivism has been used in research in mathematics education.

Enactivism conflates epistemology and ontology (Maheux & Proulx, 2015). In other words, the enactivist notions that describe mathematical knowing can also be used in researching mathematical knowing. For this reason, the following review of research in mathematics education includes research in which enactivism has been used as a theory of knowing and/or as methodology. Research in mathematics education that uses enactivism either as theory or methodology can be divided into three categories: research about mathematical understanding, research pertaining to mathematics teaching and teacher education, and research about communication in mathematics.

Research about Mathematics and Mathematical Understanding

Within the mathematics research community, Tom Kieren and Susan Pirie were perhaps the first researchers who made use of Maturana and Varela's (1987) ideas. In developing their theory of mathematical understanding, Pirie and Kieren (1989) used enactivist notions, such as autopoiesis, recursion, and knowing as effective action as seen by an observer (Reid, 2015).

Davis (1994, 1997) has written extensively about enactivism, using and developing many of the key components of the theory, such as autopoiesis, co-emergence, knowing as being and doing, and structural determinism. Reid (1995, 1996, 1999) uses enactivist concepts to study students' reasoning. Reid (1996, 2014) is also one of the first proponents of using enactivism as methodology and specifies perceptually guided action, embodiment, and structural coupling as key concepts in the enactivist methodology.

Using the notions of structural coupling and of action as cognition, Khan et al. (2015) argue that enactivism is a fitting theory and methodology for studying spatial reasoning in mathematics education. They claim that enactivism allows them to attend to "the complexity and dynamics among the co-constituting elements by attending to the presence of the knower and the materiality of the knower in the act of spatial reasoning" (Khan et al., 2015, p. 274).

Maheux and Proulx (2015) build on the notion of knowing as doing and focus their study of mathematical knowing on learners' mathematical actions, thereby shifting their focus from mathematical knowledge to mathematical doing. They conceptualize mathematics as something a learner does as opposed to something that one possesses, and use the expression *doing/mathematics* to describe "the emergent made-up nature of *both* the doing and the mathematics and their dialectical relationship" (Maheux & Proulx, 2015, p. 214 emphasis in the

original). By studying these mathematical actions, they aim to develop an understanding of future possible actions.

Towers and Martin (2015) use the notion of co-evolution as one of the central components of the enactivist perspective. Building on ideas of emergence, embodiment and sense-making, they orient themselves toward identifying and discussing collective mathematical actions as coactions. Adopting enactivism as their methodology allows them to transform their data in order to "pay attention to the interplay between things rather than their autonomous constitution" (Towers & Martin, 2015, p. 255).

Research about Mathematics Teaching and Teacher Education

Enactivism has been used in various ways to study mathematics teaching and teacher education. Research in this category ranges from researchers who study other teachers (Brown & Coles, 2014; Towers & Proulx, 2013), to researchers who study their own practice (Coles, 2015; Metz & Simmt, 2015).

Towers and Proulx (2013) use the enactivist notion of structural coupling and describe teaching actions as "triggers." They offer a broader perspective for identifying and understanding teaching practices that negates describing teaching actions as good or bad in favor of a "landscape of possibilities" (Towers & Proulx, 2013, p. 5). They categorize and describe three categories of teaching actions–*Informing, Orienting* and *Shepherding*–but maintain that these categories can be further expanded as different possibilities for teaching actions are researched.

Coles (2013) builds on enactivist notions of structural coupling and co-evolution to study the use of videos for professional development. Using enactivism as methodology, Coles (2015) explores the role of the teacher educator in facilitating purposeful reflections on practice, which

can open up possibilities for future actions for teacher candidates. Coles (2013) explains that learning happens in "the arising of possibilities for new or different ways of acting" (p. 180).

Brown and Coles (2012) also use enactivism to study possibilities for future teaching actions. For them cognition is as an active process and learning is synonymous with change (Brown & Coles, 2011; 2012). Using the enactive understanding of cognition as perceptually guided action and highlighting the importance of recurrent sensorimotor patterns that result in action to be perceptually guided, Brown and Coles (2014) explore teachers' acts of reflections to learn and to expand the space of possible actions. They frame the idea of reflection by the enactive concept of "deliberate analysis" and situate experience as a 'middle path,' a space between coping with the immediate environment while being aware of one's motivations and being able to act in the moment (Shear & Varela, 1999). Working with/in similar veins, Metz and Simmt (2015) adopt Shear and Varela's (1999) second-person empathic observer to study learners' evolving mathematical understanding. They argue that by situating the teacher researcher as an empathic second-person observer, they gain access to the researchers' own evolving understanding of mathematics. Metz and Simmt (2015) acknowledge and embrace the co-evolution and interconnectedness of experience that exists between students and researchers, where teachers support students' uncertainties and/or growth, while being cognizant of their own mathematical experiences.

Research in Communication in Mathematics

Enactivism has been used as theory and methodology to study classroom talk and communication in mathematics education research. Coles (2015) uses Maturana's (2002, as cited in Coles, 2015) notion of coordinations of coordinations of actions to analyze classroom talk. He describes conversations as "highly ritualized affairs" that can be studied in terms of patterns

(Coles, 2015, p. 235). Using enactivist notions of language, Coles (2015) offers a five-step mechanism of recursive enquiry, systematic search for pattern, equifinality, micro-analysis, and meta-communication to study patterns that emerge in classroom talk.

Steinbring (2015), however, takes a different perspective on communication by building on Luhmann's (1995) social and psychic systems. Steinbring (2015) argues that "emergence of understanding is neither directly possible nor transferable from one to another person" (p. 286). Instead, he uses Luhmann's (1995) notion of social systems to study communication. Social systems, in Luhmann's (1995) view, are autopoietic systems, capable of self-production of meaning through communication. Steinbring (2015) argues that in studying communication in mathematics classrooms, attention must be placed on the interactions as they emerge and evolve.

Another category of research that focuses on communication in mathematics education is one that uses semiotics (e.g. Radford, Edward & Arzarello, 2009). Semiotic theory, as applied in mathematics education is concerned with the role of gestures in mathematical communication and thinking. However, these studies make use of embodied cognition and not enactivism and rarely make reference to Varela's or Maturana's work (Reid, 2014).

In my research, I aim to contribute to the existing body of research in mathematics education by adopting enactivism to explore feedback as co-constructed by students. I am interested in how students, as active participants, enact feedback and bring forth new understanding(s) for their feedback actions.

Me and Enactivism

My history of coupling with enactivism dates back to when I was a master's degree student. At the time, I was interested in researching the relationship between play and children's growth of mathematical understanding. A doctoral student, in a conversation about the nature of knowing,

spoke to me at length about enactivism and suggested that I should read *Embodied Cognition* (Varela et al., 1991).

At that time, I had been a Montessori teacher for five years, and having been trained in the Montessori philosophy and pedagogy, my conception of knowing and learning was more in line with the constructivist view. As such, while I could see some common ground between constructivism and enactivism, most of the enactivist concepts were foreign and fascinating to me. Using enactivist terminology, I was triggered by the theory and I found myself reflecting on and re-considering my previous conceptions of knowing and learning.

Curious to know more, I read about enactivism extensively, beginning with its origins in Maturana and Varela's (1987) *Tree of Knowledge* as well as Merleau-Ponty's (1962) *Phenomenology of Perception.* As I read more about enactivism, I gradually began to realign my way of thinking about knowing and experience, and eventually I used enactivism as the theoretical framework for my master's thesis.

However, once my coupling with enactivism had begun, it had become a part of me beyond the limited scope of my master's thesis. Looking at my practice through the enactivist lens, I began to re-evaluate my role as a teacher and my pedagogical practices. In other words, my changing view of knowing triggered changes in my teaching actions. Conceptualizing knowing as embodied action, I became more mindful of my actions as well as my students' actions. I do not claim that my practices improved, only that I was more aware of my actions and how they evolved.

Perhaps it was because of my growing awareness of our classroom actions that I chose to study feedback as the topic of my dissertation. It was clear to me that we, as a class, did not know how to "do" feedback. And it was equally clear to me that we had to "do" feedback, because that was how we would know it. In other words, I did not choose enactivism as a theoretical framework

because it suited the idea of co-constructing our feedback practices for this study. Rather, due to my enactivist perspective, I chose to study feedback as co-constructed by my students and myself. In the following section, I will explore how the enactivist perspective frames feedback and in the next chapter, I will examine what possibilities enactivism, as a methodology, offers for studying feedback.

Framing Feedback with Enactivism

Many studies have been devoted to defining feedback, describing its purpose(s), and finding the conditions that are conducive to its effectiveness. As a result, there are many definitions and descriptions for feedback and its effectiveness. However, using enactivism as a theory of knowing as well as methodology bears significant implications for how feedback is conceptualized and studied. In this section, I will attend to using enactivism to re-conceptualize feedback.

According to enactivism, there is no pre-existing world separate from and independent of the knower. We come to know the world through our interactions and our structural coupling with it. Using enactivism to frame feedback, implies that there is not a single, pre-existing truth about feedback. We do not represent feedback; rather, we enact feedback through our coupling with/in it. For the enactivist, knowing is embodied action. This means that feedback as lived practice is manifested in our actions as we give and receive it and feedback is influenced by our biological, psychological and cultural history. We do not know feedback as an independent notion, we enact it by making distinctions that are inseparable from our structures and our history of coupling with it.

Within enactivist theory, knowing emerges as a result of recurrent sensorimotor patterns. Feedback co-emerges with us through our recurrent acts of giving and receiving feedback and

through our conversations about these actions. As we structurally couple with one another and with feedback, we trigger changes in one another, we make distinctions, and bring forth new understanding(s) of feedback. However, the distinctions we make and the worlds of significance we bring forth are directly influenced by our structures (our biological, psychological and cultural history). In other words, we know feedback as we do it, but how we do feedback is never divorced from our structures.

Rejecting a single, pre-existing formulation of feedback influences all facets of how feedback is conceptualized including what makes it effective, both in its content and its structure. This means that there is not a pre-existing definition for effective feedback, no single formula for effective feedback actions and no single type of effective feedback. Rather, the effectiveness of feedback, in definition, content and structure, can only be decided by those who enact it and only as they participate in feedback actions.

Feedback is communicated through language. I use Maturana and Varela's (1987) notion of languaging as coordinations of coordinations of actions, to describe classroom talk. As we engage in feedback actions, we form a social autopoiesis in which we structurally couple with one another with/in feedback. We bring forth worlds of significance by making linguistic distinctions. This means that not only do we structurally couple by using language, we also create a linguistic domain within which we change and give rise to new domains of adaptation. Furthermore, since feedback is enacted within a linguistic domain, it is interpreted, and linguistically enacted, according to our structures. As such, our structures and our linguistic domain co-emerge from our history of recurrent interactions with one another and with feedback.

Understanding feedback as embodied and enacted also has implications for our intentionality and our perceived affordances of feedback. Intentionality of feedback is concerned

with how we construe feedback and whether or not our construal is met or fails in our feedback actions. Intentionality stands in a reciprocal relationship with affordances. How we construe feedback is related to our perception of the affordances of feedback and vice versa. Our intentionality and perception of the affordances of feedback are embodied and contingent on our history of coupling and are, as such, temporal. As we couple with feedback, our structures change, and so do our perceived affordances of feedback. The change in our perception of the affordances of feedback in turn changes our intentionality toward feedback. In this way, our intentionality and our perception of the affordances of feedback co-evolve as we bring forth new possibilities for conceptualizing feedback.

In my research, feedback is understood as a dynamic, embodied process, its content, structure and goals contingent upon feedback actions and the dialogue that emerges and changes among those who enact feedback. This dynamic process implies that our understanding of feedback is temporal as we and our practice of feedback co-emerge from moment to moment.

Chapter 4: Methodology and Methods

According to enactivism, "all knowing is doing, and all doing is knowing" (Maturana & Varela, 1987, p. 17). This means that enactivism conflates epistemology and ontology (Maheux & Proulx, 2015). In other words, the known and the act of knowing are one and the same, influencing and influenced by each other in an ever-evolving history of structural coupling. In this way, feedback processes and the products of those processes are one and the same, and our understanding of feedback emerges and grows as we engage in "doing" feedback. In return, our feedback actions evolve as our understanding of it changes. In short, we know feedback as we enact it, and we enact feedback as we know it. This view of knowing as action shifts the direction of this research in how feedback is understood and how it should be studied. As such, it is fitting that enactivism is used not only as a theory of knowing, but also as a methodological perspective for studying feedback.

This chapter is divided into two sections. The first section is concerned with the methodological perspectives that influence this research. I will begin by exploring enactivism as a methodological tool and will explain the enactivist concepts that guide the research questions and methods. I will then attend to the question of my role as teacher and observer in this research and will finally address the question of validity.

The second part of this chapter explores the methods used in this research. I will begin that section by providing a detailed description of the setting and the participants of this research, some of our classroom practices and our history of coupling with feedback. I will then go through the procedural aspects of the research, data collection methods, and stages of analysis.

Methodology

Enactivism, by virtue of its emphasis on knowing as embodied action, and the importance of knowers' interactions with the known, is a qualitative methodology. Qualitative research is, "holistic and often involves a rich collection of data" (Nassaji, 2015, p. 129). However, enactivism blurs the line between the self and the other and refutes the existence of data independent of the knower. In doing so, enactivism negates the objective/subjective dualism inherent within qualitative research and instead explores the relations and interactions between, and co-emergence of, the knower and the known. In describing the enactivist methodology, Mackenzie (2016) explains, "the qualitative dimension is not ontologically independent of the lived experience of sentient organisms. Rather, lived experience and the qualitative dimension are transactionally co-determining" (p. 30).

The enactivist perspective equates action to knowing and, in this way, constitutes action as the central element of the research design. This is in line with action research in that it considers "action as a legitimate mode of knowing, thereby taking the realm of knowledge into the field of practice" (Tandon, 1996, p. 21, as cited in McIntyre, 2008, xii). Action research is a qualitative methodology with the fundamental aim to "improve practice rather than to produce knowledge" and "the improvement of a practice consists of realizing those values which constitute its ends" (Elliott, 1991, p. 49). Lewin (1948) describes action research as "comparative

research on the conditions and effects of various forms of social action and research leading to social action" (p. 202).

One can also find similarities between this research and Participatory Action Research (PAR) which, as a specific type of action research, fosters opportunities for "co-developing processes *with* people rather than f*or* people" (McIntyre, 2008, p. xii). PAR can be traced to Paulo Freire, who believed that critical reflection was crucial for personal and social change (Maguire, 1987; McIntyre, 2008; Selener, 1997). However, while the central goal of PAR is social change (Gillis & Jackson, 2002; Reason & Bradbury, 2001), within enactivism, change is a consequence of action, it is not its goal. The goal of an enactivist researcher is the action itself. In my research, our acts of co-constructing peer feedback practices inevitably result in changes in these practices. However, the goal of this study is also to explore our acts of co-construction and how feedback practices co-emerge from these actions. In other words, our actions in the process of co-constructing our feedback practices are how we embody feedback and the feedback practices we co-construct are the consequences of our embodied feedback actions.

As explored in the previous chapter, in mathematics education research, studies that have used enactivism as their theory or methodology have been guided by different concepts within the enactivist perspective. In this research, I have used concepts of repeated sensorimotor patterns, making distinctions and bringing forth worlds of significance, and co-emergence as principles that guide my research design, the questions I ask, my role as the observer, what I consider data and how I analyze the data. Below, I will examine each of these components as integral parts of the research design.

How Does Enactivism Guide the Research Design?

According to enactivism, cognition is an active process (Brown & Coles, 2011, 2012) and

"cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided" (Varela et al., 1991, p. 173) implying that understanding emerges through repeated actions (Coles, 2013; Reid, 1996, 2014). My research is centered on the participation of my students in co-constructing their peer feedback actions. As such, it is about shared experiences of enacting feedback.

As Coles (2015) explains, "systems co-evolve when involved in recursive interactions" (p. 236). In order to accommodate for recursive interactions with feedback, my research employed a cyclical approach. Each cycle involved opportunities for shared feedback actions, co-constructing and naming categories and models, reflecting and discussing those feedback actions, categories and models, and deciding on feedback actions for the following cycle. In this way, each cycle occasioned multiple layers of interaction with feedback, and having multiple cycles resulted in the co-emergence of participants and their feedback actions.

Coles (2015) posits that, "it is through shared activity, with those around us, that we create the possibility of objects that can be named in language" (p. 237). In this research, students' shared activities allowed them to classify and name feedback categories and to co-construct models for feedback in each cycle. Brown and Coles (2014) explain, "in everything we do, we make distinctions; categorizing is the basic mental function" (p. 221). According to Coles (2015), "the naming of shared entities captures distinctions made in the world" (p. 237). In each cycle, feedback categories and models embodied the distinctions brought forth about feedback in that cycle.

At the end of each cycle, the group discussed and brought forth changes to their feedback actions for the next cycle. Coles (2015) explains, "learning is associated with change and the

arising of new actions/perceptions in a given context" (p. 238). In this way, feedback practices co-emerged as the group brought forth and specified new ways of doing feedback.

The group's recurrent, shared experiences with feedback were also enacted in our conversations in and about feedback actions. Coles (2015) argues that "conversations are highly ritualized affairs and from an enactivist perspective these rituals can be read in terms of pattern" (p. 235). My students and I engaged in conversation regarding feedback as the students decided on their categories, built their models and reflected on their practices. These conversations were central to how the group came to know feedback. Together, we entered a domain of language where we changed and triggered changes in one another, and conceptualization of feedback co-emerged as we developed a common language.

The purpose of these conversations was not to understand the students' understanding at the time they gave or received the feedback, but to come to an understanding with them in the moment of our conversation. In this way, in conversation, we not only reflected on students' feedback actions, we embodied our understanding of feedback actions. Such reflection, as Varela et al. (1991) note, "is not just *on* the experience, but reflection *is* a form of experience itself" (p. 27). My conversations with my students allowed me to see (understand) the worlds of significance brought forth by them, while at the same time made me aware of my own evolving understanding.

Central to these conversations is my openness as the researcher. This openness, based on the presupposition that there is no pre-existing truth to be discovered, permeates all aspects of these conversations. It pertains to how I addressed the participants both in the questions I asked and in receiving their points of view. The questions I chose to pose were the worlds I brought forth. I asked questions that were significant to me. However, I remained open to other questions

that emerged from our conversation and in doing so, opened myself and the conversation to new possibilities and new horizons of understanding. This openness also pertains to my attitude as researcher. I did not enter the conversation with the intention to enforce or prove my opinions. Rather, I entered a conversation with an openness to develop a common language with others. In this way, meaning was co-constructed through my coupling with the environment during my observations and my conversations with the students. As Simmt and Kieren (2015) describe, "in languaging, objects arise for the observer" (p. 310). Remaining open during these conversations allowed me to share in the meaning which "arises as a relationship of linguistic distinctions," and "becomes part of our domain of conversation of adaptation" (Maturana & Varela, 1987, p. 211).

Research Questions

Using an enactivist methodology has important implications for the types of questions asked about feedback. Rejecting the existence of a single, pre-existing truth, this research does not attempt to find the "true" definition of feedback or the "right" way of doing feedback. Rather, it searches for "how" this group conceptualizes feedback and feedback actions.

As knowing is embodied action, how students know feedback is enacted in their actions and interactions with feedback or, in other words, in their coupling with feedback as they co-construct categories and models and feedback actions and as they discuss feedback with one another. In this way, this research is in line with Maheux and Proulx (2015), who study learners' mathematical doings, thereby shifting their focus by "replacing questions of knowledge with concerns for mathematical doing alone" (p. 212). Throughout the cycles of this research, as students couple with feedback, their conception of feedback and feedback actions evolves. I am interested in how students co-construct their feedback practices. In this pursuit, I ask three questions;

1. How are feedback categories and models co-constructed?
2. How are feedback actions co-constructed?
3. How do conceptualization of feedback and its effectiveness evolve throughout the process?

My Role as the Observer

Research with an enactivist orientation places great emphasis on the role of the researcher and redefines the traditional dichotomy between the observer and the observed (Kahn et al., 2015; Lozano, 2015; Proulx & Maheux, 2015). In fact, one of the fundamental differences between enactivism and other methodological perspectives is that enactivism is "defined by a sensitivity to the role of the observer in research" (Reid & Mgombelo, 2015, p. 179). Maturana (1988) claims that, "everything said is said by an observer to another observer that could be himself or herself" (p. 27). I analyze the worlds that I bring forth according to my own structure, which is continuously triggered by my interactions with the environment and with others in the environment. As an observer and a member of the social phenomenon of our classroom, I am constantly coupling with the students and the environment. I am at once an autopoietic system, enclosed by my own structure, and a part of a social phenomenon, triggering and triggered by that phenomenon.

In enactivist research in mathematics education, the role of the observer has been explored in different studies, which range in the extent to which the observer interacts with the participants. For instance, while in some studies the observer does not interact with the participants (Towers & Martin, 2015), in others, the observer interacts with the participants as a facilitator (Coles, 2015), teacher-researcher (Maheux & Proulx, 2015), or researcher-teacher

(Metz & Simmt, 2015). In this research, my role as the observer who is both the researcher and classroom teacher is in line with that of Metz and Simmt (2015) and Maheux and Proulx (2015).

Metz and Simmt (2015) describe the role of the researcher-teacher as empathic second-person observer. By adopting this position, they claim, the researchers are at once attending to their own experiences and mentoring their students' experiences. Metz and Simmt's (2015) exploration of the role of the observer in understanding the nature of experience has a phenomenological flavor. Adopting a phenomenological perspective constitutes the researcher-teacher both as the observer and the observed. However, where phenomenology explores experience in a theoretical sense, enactivists advocate for an open and mindful approach that is centered on the embodiment of experience.

I situate myself, as teacher-researcher, at the center of interaction with/in feedback and this occasions for me an "open-ness in observing through inter-relating how the knowers know" (Simmt & Kieren, 2015, p. 310). Such a view allows me to see how I bring forth a world that is significant to me. In other words, I give an account of students' feedback actions as they appear to me, and in doing so, I acknowledge, reflect on, and understand my own coupling with the environment. In this way, "observer and the observed arise together, emerging from one another in the act of observing" (Maheux & Proulx, 2015, p. 212).

What are Data in Enactivist Research?

Since knowing is embodied action, then students' knowing of feedback is revealed in instances of their interactions with feedback. This means that data, in this research, constitute instances of students' embodiment of feedback, brought forth in their feedback categories and models, their procedures for feedback actions, and their conversations about feedback. Every

feedback action is evidence of students' structural coupling with/in feedback, and every conversation about feedback is evidence of structural coupling in the linguistic domain.

The enactivist perspective equates data with interpretation. Reid (1996) posits, "there is [sic] no data, only interpretations and interpretations of interpretations" (p. 4). Instances of data in this research, such as categories, models and video-taped sessions of our conversations, are gathered by an observer, which could be me, and are, in fact, "acts of interpretations, or a researcher learning in coemergence with a research situation" (Reid, 1996, p. 4). For instance, feedback categories and models are students' interpretations of feedback in each cycle. Feedback actions for each cycle are also interpretations of what feedback actions, and in what order, should be performed during that cycle. Video-taped sessions of our conversations are my interpretations of students' interpretations of feedback during the conversation. Understanding data as interpretation is influential in how analysis is understood in enactivist methodology.

How Does Enactivism Describe Analysis?

Within enactivism, knowledge is framed as "the result of an ongoing interpretation that emerges from our capacities of understanding" (Varela et al., 1991, p. 145). Earlier, I explained that within enactivism, data are described as interpretation. When analyzing data, the researcher uses theory to account for data. This means that analysis of data in an enactivist research is simply the researcher's interpretation of interpretations. Reid (1996) describes analysis as "a process of co-evolution of ideas," where "theory and data coemerge in the medium of the researcher" (p. 4).

Analysis as interpretations of interpretations is in line with what Mason (2002) describes as *accounts of*. Mason (2002) makes a clear distinction between *accounts of* and *accounts for* data. The difference between the two is that *accounts of* phenomena report on them as directly as

possible, avoiding interpretations, judgments or evaluations. *Accounts for* phenomena aim to "explain what is perceived or interpret it, for example, by classifying" (Coles, 2013, p. 168).

When analyzing feedback, the unit of analysis is the interactions between the knowers and the known, between us and feedback. As such, focusing on feedback actions helps avoid making assumptions about what the participants may or may not know about feedback. Instead, focus is placed on how students enact feedback both in their actions and their conversation. In this way, studying feedback actions focuses the research on giving an "account of" feedback as opposed to giving an "account for" the phenomenon.

Analysis, from this perspective, is therefore a process of ongoing interpretation, involving multiple layers of the researcher's coupling with the data, triggering and being triggered by it, and bringing forth worlds of significance. This implies that understanding is temporal and contingent upon the knower's structural coupling with the known from moment to moment. Taking this perspective, "it is not possible to identify direct causes of behavior in any simplistic way since the entire history of an organism and environment is relevant to what occurs in any particular moment" (Coles, 2015, p. 240). As such, we cannot know "why" feedback is enacted in a certain way, but we can observe "what" categories and models are brought forth and "how" they are co-constructed.

Validity

In research where the researcher's coupling with data is central to the analysis, the notion of objectivity, which is often the basis for judging the validity of the findings, warrants reconsideration. From an enactivist perspective, the requirement for objectivity is that "the researcher provides adequate explanations or mechanism [sic] whereby the readers if they follow the explanations can test the researcher's idea of the phenomenon under investigation" (Simmt &

Kieren, 2015, p. 310). This means that although I bring forth worlds that are significant to me, my explanations and descriptions of the data must be sufficient for the reader to be able to "see" how these worlds are brought forth. In other words, while my analysis is brought forth based on my coupling with the data, what matters is that my analysis and findings are observable by others as well. As Coles (2015) explains, "the search for pattern in not about wanting to ensure that everyone looking at the same data would make the same distinction. From an enactivist perspective, what matters is that once a pattern is noticed, it must be observable by others" (p. 240).

Enactivists reject the notion of a single truth in favor of multiple possibilities for action. In the present study, giving accounts of events shifts the focus from trying to find "the right" model for feedback, to exploring feedback in terms of the possibilities it might offer for classroom practices. In this way, the issue of an accurate account fades into meaninglessness, as there is less stake placed on the truth of the analysis and more importance is placed on the possibilities afforded by looking at patterns and considering future possibilities (Maheux & Proulx, 2015). As Maheux and Proulx (2015) note, "It is sufficient for us to focus on what we see happening and from there to develop our own understanding of what may be possible in other places and at other times" (p. 215). As such, in this research I give an account of how my students co-construct categories, what worlds of significance they bring forth about feedback and how they enact feedback. The findings of this research shed light on the journeys my students and I travel in the three cycles of feedback, but they also open possibilities for future action.

Methods

Having discussed how enactivism guides the methodological considerations for this research, in the following sections, I will introduce the setting and participants of this study and will describe the study design and the methods I employed in collecting and analyzing data.

Setting

This research was inspired by my experiences as a teacher and through my interactions with my students. The notion of studying peer feedback was occasioned for by the need in our mathematics class to know how to give peer feedback, what kind(s) of feedback to give, and what to do with feedback after it was given. Furthermore, this research was guided by the conviction that peer feedback practices should be co-constructed by those who enact those practices. As such, it was only fitting that the research would be conducted in my own classroom and with my own students in the process. At the time of this research, I taught at a private Montessori school with just over one hundred students with approximately 65 kindergarten students and 40 elementary students, located in a suburban community. Most of the students at this school were from families with middle to high socio-economic status. The majority of the students were first generation Canadians and, in addition to English, spoke a second language at home. Our school followed an advanced curriculum, but the students who participated in this research ranged in their level of competency in different subjects.

Although I was trained as a Montessori teacher and I had begun my teaching career as a Montessori teacher, over the years, my pedagogical perspectives had evolved. My education as a graduate student occasioned me to see other educational philosophies and to adapt my pedagogical practices in many aspects. Fortunately, our school principal was quite open to and supportive of the changes I suggested throughout the years to enhance our curriculum and our

pedagogical approaches. As such, over time, the elementary department at our school evolved greatly. While we still embodied much of the Montessori philosophy and used Montessori materials in our classrooms, we also re-wrote our curriculum in order to incorporate elements from the Ontario curriculum that were missing in our original Montessori curriculum. I would describe ours as an eclectic program, incorporating different perspectives and strategies that we had found fruitful throughout the years.

One such fruitful strategy was incorporating long, uninterrupted work periods. Our schedules were arranged in a way that would provide students with long periods, between 90 to 120 minutes, during which they had the freedom to choose their work. For instance, they could begin with their math tasks and then work on their language or any other subject associated with that period. The idea of long, un-interrupted work periods is one that comes from my history of being a Montessori educator. However, these long periods became part of the culture of our classroom because throughout the years, I found that my students also preferred them to shorter periods.

Another characteristic of our classroom, which, ironically, is in direct opposition to the Montessori philosophy, was the abundance of talk. Traditional Montessori classrooms are known for being quiet environments, where students individually or in small groups engage in activities, which, while abundant with action, often do not incorporate much talk. However, I have long been a believer in the importance of conversation in the classroom as an excellent pedagogical tool. I have found classroom conversations instrumental in developing a bond, a sense of understanding, among the members of the classroom that would be otherwise lost. Over the years, my belief in the importance of incorporating more talk in the classroom was reinforced by

my students' positive reception of it, and talk gradually became a stable and integral part of our classroom culture and our classroom was always full of conversation.

Before this research, our classroom's history with feedback involved two very different contexts, one in the form of teacher feedback and the other of peer feedback. Our first experience with feedback was during our writing classes, when I provided my students written feedback on their creative or technical writing pieces. One of the established practices in our writing class was that each writing assignment would go through multiple drafts. After each draft was written, I would read it, write my feedback on it and return it to the student to work on the next draft. In my feedback, I often pointed out some of the strengths of the draft and recommended changes and/or additions that would help improve the piece. At times, my students would meet with me, one at a time, after having read my feedback, and we would discuss the feedback in more detail.

Our other experience with feedback was in the form of feedback students provided to their peers during our Oral Visual Communication (OVC) presentations. After each student presented their OVC project, other students would offer their feedback. Most of these feedback comments were based on an assessment criteria chart we used for our OVC presentations. For instance, students would comment on the presenter's voice projection, their consistency of eye-contact with the audience, or the quality or accuracy of the presentation slides. In this way, in both these cases, our history with feedback was part of our classroom assessment practices, either in the form of teacher's formative assessment, or students' peer assessment.

Participants

The participants of this research included all of my grade six students. These included eight students, three boys and five girls. It is important to note that this was a grade 5 and grade 6 split class with a total of 14 students. Six of the students were in grade five and the remaining

eight were in grade six. As all of these students were members of our class, I had originally planned to include the grade five students in this research as well. However, in the case of the grade five students, there was the ethical concern that their participation in the research might be influenced by the fact that I would be their teacher in the following year. For this reason, I decided to only include the grade six students in the research. Fortunately, I was not the only teacher in that class that year and my co-teacher taught the grade five students during the times the grade six students and I were engaged in the research. Moreover, in order to eliminate other ethical concerns regarding my grade six students' voluntary participation, we undertook this research during the final three weeks of school in June, after all the assessments were done and all the report card grades and comments were submitted.

As this research is informed by enactivist theory and methodology, it cannot be understood as an entity divorced from its participants and the history of coupling we had with one another and with feedback. As Reid and Mgombelo (2015) explain, "every student is complicit in the unfolding understanding of the other" (p. 175). For this reason, below, I will share a brief history of each of the eight students who participated in this research.

Sarah. Sarah attended our school since she was three years old and was my student, in some capacity, since she was in grade one. Some years, I was her homeroom teacher and other years, I taught her parts of the curriculum, mathematics always being one of them. Sarah was academically quite advanced, consistently did well on her tests and assignments, and had very strong language skills. Sarah was a first-generation Canadian, but only spoke English.

Hanna. Hanna was a student in our school since she was three years old, and I was either her homeroom teacher or a subject teacher in her class from grade 1 to grade 6. Hanna consistently did well on her tests and assignments. Hanna's older sister was a former student of

mine, and her younger sister also attended our school. Hanna was a first-generation Canadian and spoke English at school and Cantonese at home.

Rose. Rose attended our school since she was three and was one of my students since grade one. Rose was the quietest member of this group. Rose was one of our strongest students in terms of academics, especially in the subject of mathematics. Rose's three younger siblings also attended our school. Rose was a first-generation Canadian, but only spoke English.

Layla. Similar to Rose, May and Sarah, Layla attended this school since she was three and was my student since grade one. At times, Layla struggled with mathematics and her performance on math tests was inconsistent. Layla was a first-generation Canadian and spoke English at school and Cantonese at home.

Andy. Andy was also a student of our school since he was three years old and he too was my student since grade one. When he was younger, Andy struggled with his academics, especially with mathematics. Andy's younger sibling was also a student at our school. Andy was a first-generation Canadian and spoke English at school and Cantonese at home.

May. May joined our school in grade 6. She had attended public school in the past and was not familiar with the Montessori philosophy and our curriculum. As such, there was a lot she had to catch up with. However, she was able to settle in and become part of our classroom community in a very short time, and by the time this research was conducted, she was quite familiar with our routines. May was a first-generation Canadian, but only spoke English. Her two younger siblings also attended our school.

John. John joined our school in grade five. Like May, when he first joined us, he was behind academically and had to work hard to catch up with the other students, especially in the area of mathematics. However, John soon proved himself to be a very hardworking student and

before long, not only had he caught up with his peers, he became one of our strongest students in terms of academics. John was a first-generation Canadian and spoke English at school and Cantonese at home.

Sam. Sam attended our school for one year when he was four years old. He then moved to another country with his family for three years and when he was in grade two, he returned to Canada and rejoined our school. Sam had some language difficulties when he returned to Canada. However, he was quite strong in mathematics and performed well on his tests. Sam was a first-generation Canadian and spoke English at school and Arabic at home. Sam's two older siblings attended our school before him and his younger sibling was also a student at our school.

As the above introductions demonstrate, most of these students had been my students for years. I had the privilege of not only observing their development, but also being part of it. In addition to academics, as part of our school culture, we participated in activities that took us outside of the school environment and, in this way, we got to know one another in different ways. Every year, our grade four, five and six students took part in over-night school trips. Some years we travelled to other cities across Canada and other years, we travelled to New York to participate in the Montessori Model United Nations program. As their teacher and the head of the elementary department, I accompanied them and was responsible for them on all of these trips. Travelling together, we came to know one another in settings other than our classroom and, as such, we enjoyed a deeper level of familiarity and trust with one another. I believe it is fair to say that I had a good understanding of their academic strengths and weaknesses as well as their personalities. While the above information about my students and our classroom can in no way provide a complete picture of who each of these students were, nor the complexity of our

relationships, I hope, it can help the reader connect with us and our story of co-constructing feedback practices.

Research Design

This research was performed in three cycles. Each cycle spanned across one week and consisted of three 90-minute sessions. Each cycle incorporated four elements: feedback actions, categorizing feedback, co-constructing a model for feedback, and reflecting on the process and deciding on feedback actions for the next cycle.

1. Feedback Actions. When planning the feedback actions for the first cycle, I was cognizant that these actions would influence students' feedback actions for the second and third cycles. In other words, once the students performed one cycle of feedback actions according to my instructions, these actions would become part of their history of structural coupling with feedback and would influence their decisions in future cycles. This was not necessarily a negative factor. However, I was aware that my involvement might raise concerns about the democratic nature of this process. In other words, how would this process accommodate students' active participation in decision making if I instructed them on their feedback actions in the first cycle and potentially influenced their subsequent cycles?

In order to navigate this tension, as their teacher, I used two principles to guide my decisions throughout the process. The first was that these feedback actions were meant to be used by this group. This meant that these actions would become part of this group's classroom practices and, as such, must be suitable and meaningful to them. Therefore, it was important that these feedback actions were designed in a way that would fit within the existing classroom practices. In the classroom, students often worked, individually or in pairs, on math problems, wrote their solutions on boards or poster paper, as a way to share their thinking with others, to

learn from one another's strategies, and to have meaningful conversation about math. As such, these feedback actions could simply be built as an addition or continuation to the existing, collective practices during math class.

The second factor that guided my decisions was to leave the procedure open to interpretation and change at every stage. In other words, although I offered a set of actions to students for the first cycle, I was open and willing to change the procedure if and when the students brought forth a new change. In this way, while I instructed them on where and how to begin their feedback actions for the first cycle, even during that first cycle, they would be able to decide on how to proceed and in what ways to change their actions to better suit their needs.

With the above two principles in mind, at the beginning of every cycle, students were given one or two math word problems to solve. When choosing math questions for this study, it was only natural to choose from textbooks we used in class. At the time of this research, my grade six students and I were covering a unit on percent, rate, ratio, and proportion. As part of application and problem solving activities, we used word problems from several different textbooks in class. In this research, the math questions I assigned to the students for each cycle were taken from the Singapore Math series, Level 7. These questions are listed in Table 1.

Table 1
Math Questions for the Three Cycles

Cycle	Math Question(s)
1	**Question 69:** There were 490 children altogether in 2 groups. Group A consisted of only boys, and group B consisted of only girls. There were 2½ times as many girls as boys. Some girls joined Group B, and for every 4 boys in Group A, 32 more boys joined the group. The total number of girls was then 1/3 the total number of boys. Express the number of girls who joined Group B as a fraction of the number of boys who joined Group A. **Question 70:** Last month, Nate spent 12% of his paycheck on car repairs and 25% of the remainder on food. He gave $1,320 of the remaining money to his parents and then bought a computer on sale. If the usual price of the computer was $825 and the discount was 20%, how much money did Nate have in the beginning?
2	**Question 64:** 3/5 of the people at the funfair were children. ¾ of the remaining people were men. There were 140 more children than women. How many people went to the funfair?
3	**Question 63:** 3/8 of the guests of a birthday party were adults. The ratio of boys to girls was 2:7. There were 180 more children than adults. How many more girls than boys were there at the party?

Students were then asked to write their solutions on large poster papers. So far, these were actions that were part of routine practices in math class. In the first cycle, once all the solutions were written on posters, I gave students sticky notes, asked them to visit one another's posters, write their feedback on sticky notes, and place them on the posters. Using sticky notes, as opposed to writing feedback directly on the posters, made it possible for students to move them around later and group them as they saw fit.

When the students visited all the posters and wrote their feedback, they were asked to return to their own poster and read the feedback they had received. They were given time to use the feedback provided on the sticky notes in whatever way they chose. I observed them while

they did so, interested to know whether they would take up the feedback they had received and make changes to their strategies or whether they would dismiss the feedback.

2. Categorizing Feedback. After feedback actions were completed in each cycle, the students moved all the sticky notes, regardless of their decision to take up the feedback or to dismiss them, and placed them on the board. Working together as a group, they classified the feedback comments into categories of their choice and named their categories.

3. Co-constructing Feedback Models. In each cycle, students co-constructed a model to embody how their categories were related to one another. The purpose of these models was to see not only the categories of feedback as constructed by the students in each cycle, but also how these categories were related to one another. It is important to note that these models are not understood as a symbolic representation of how the students thought about feedback categories; rather, they are how feedback was enacted in the domain of language and symbols in each cycle. The linguistic and structural components of each model bear meaning as they were shaped and informed by feedback experiences in each cycle. The models, in this way, are artifacts of collective experiences and are dynamic and contingent upon enaction of feedback in the moment of their construction.

4. Reflections and Changes to Future Feedback Actions. In each cycle, after the model was completed, together, we discussed feedback categories, the model itself, and feedback actions during that cycle. During these conversations, we discussed how feedback actions can be improved and brought forth changes to feedback actions for the following cycle. As a result of these reflections and conversations, while the second and third cycles began with the same routine of solving a math problem and writing solutions on posters, feedback actions in those cycles diverged from those in the first cycle, according to the changes decided by the students.

These changes are described, in detail, in the following chapter.

My Role as Teacher-Researcher in the Process

During the three cycles of feedback actions and interactions, my role and my level of engagement with the process varied. At times, I simply observed the students' actions and/or listened to their conversations. I had the opportunity to observe the students as they engaged in categorizing feedback comments, constructing models, and/or discussing feedback actions. Observation allowed me to pose questions, to build theory, or to generate hypotheses (DeWalt & DeWalt, 1998). In my observations, I was interested in how students described feedback, what triggers they encountered as they embodied feedback and how they responded to those triggers.

At other times, I joined in their activities or discussions, either asking a question or making a comment. Our conversations were hermeneutic in nature (van Manen, 2014). I posed questions based on what I was triggered with at the moment, but I was also open to follow emerging themes, to ask follow-up questions, and/or to probe students on a particular topic or question.

Data Collection

This research, being qualitative in nature, incorporated a rich collection of data. In every cycle, feedback was enacted, categorized, turned into models, and discussed with a view to future cycles. All of these actions and conversations became instances of data.

I used videotaping as my method for collecting data from feedback actions and interactions. Using videos can generate "a rich collection of data" (Asan & Montague, 2014). Videotaping aligns well with my methodological considerations. Within an enactivist lens data are seen as instances of the embodiment of feedback, brought forth in feedback actions and conversations about feedback. Videotaping was a suitable method for this enactivist research as

it allowed me to easily capture our actions and interactions and helped "preserve the temporal and sequential structure which is so characteristic of interaction" (Knoblauch, Schnettler & Raab, 2006, p. 19). Videos also allow for multiple viewings of the data, which means they provided me, as the researcher, multiple chances of coupling with the data.

In each cycle, I videotaped the students during their feedback actions, while they classified feedback into categories and co-constructed feedback models. I also videotaped our conversation sessions throughout the cycles. I used a GoPro video recorder to videotape the sessions. The advantage of GoPro recorders is that they are small enough to be unobtrusive and that they can be mounted on the researcher or placed on a stationary surface. I chose to carry the camera on my person while I walked around the classroom and observed the students' interactions and/or engaged them in conversation.

However, video as a methodological method is not without concerns (Powel, Francisco and Maher, 2003). One of the limitations of using video to collect data is that, much like observation, it is incomplete (Bottorff, 1994) and cannot capture everything (Hall, 2000). In this research, due to the relatively small space and the small number of the participants, in most cases it was easy for me to capture everyone in the video. However, as I walked around and asked questions of a group or simply videotaped some of the students during their actions and/or interactions, I was cognizant that other students' conversations were not captured. In this way, my selection as to what and whom I captured affected the data collection. In an effort to account for this limitation, Pirie (1996) recommends coupling video data with other forms of data. In this research, I also collected students' artefacts, such as their math posters, their feedback comments on sticky notes, and their co-constructed feedback models.

During each cycle, the students wrote their solutions to the math questions on posters. In the second and third cycles, the posters were not only used as a place to write their solutions, they were also used to communicate what kind of feedback they wanted to receive. In this way, the posters became part of the feedback actions they had brought forth. Feedback comments were written on sticky notes and placed on posters. After being read by the students, the comments on these sticky notes were categorized and placed in a model. These models, which changed from one cycle to the next, embodied how students conceptualized feedback and their feedback practices. I took photographs of students' posters, feedback notes, and feedback models throughout the cycles and, at the end of each cycle, I collected the students' posters as well their feedback models, with sticky notes attached to them.

Stages of Analysis

The process of analysis of the video data was adapted from Powell, Francisco and Maher's (2003) analytical model; modifications were made in order to make it align with the methodological considerations of this research as well as the research questions. Powell et al.'s (2003) non-linear model incorporates seven interacting stages in the analysis: viewing attentively the video data, describing the video data, identifying critical events, transcribing, coding, constructing storyline, and composing narrative. Below, I describe each of these steps and how they were modified for analyzing data in this research.

Attentive viewing of the data. This phase is primarily aimed at familiarizing oneself with the content of the data through repeated viewing. The researcher does not impose any analytical lens during this viewing (Powell et al., 2003). The analysis for this research involved multiple viewings of the data at different stages of the study. I viewed the data after each session, at the end of each cycle, and again upon the completion of the study. When viewing the data, I

chose a very open approach. In other words, I was not looking for a specific outcome and did not have ready-made categories to code and classify my data. Instead, when observing the data, I was interested in how students embodied feedback in their actions and interactions at different stages of the study.

Describing the video data. In this phase, the researcher describes the video data, recording the actions, utterances, and gestures in a descriptive rather than interpretive manner (Powell et al., 2003). In describing the video data, I did not use my enactive lens. Rather, I described actions and conversations in detail. These descriptions helped me become more familiar with the data and became an integral part of the narratives I composed at the final stage of the analysis process.

Identifying critical events. Powell et al. (2003) describe a critical event as "any event that is somehow significant to a study's research agenda" (p. 417). The selection of critical events is thus guided by the research questions. In this research, my analysis was centered on how students enacted feedback across the three cycles of feedback actions and reflections. These instances of embodiment were brought forth in students' feedback categories, their models, their feedback actions, and in their conversations with one another and with me. A critical event "demonstrates a significant or contrasting change from previous understanding, a conceptual leap from earlier understanding" (Powell et al., 2003, p. 416). As such, my critical event excerpts were among the following three categories.

a. Excerpts where the students were actively involved in categorizing feedback comments, suggesting and/or changing names for the categories and discussing how these categories should be presented in a model. Also, some excerpts of students' follow-up conversations about their categories and models for each cycle were chosen.

b. Excerpts where the students suggested and/or discussed changes in their feedback actions for their next cycle. Also, some excerpts where chosen where students discussed their choices regarding changes in their feedback actions or made predictions about them.
c. Excerpts where students discussed feedback and its effectiveness. Also, some excerpts were chosen where students reflected on their relationship with feedback practices and/or this project.

Transcribing. Once I chose the excerpts that were of interest, I transcribed them. Self-transcription enabled me to re-familiarize with the data and gave me the opportunity to make interpretive observations and to pose questions for further enquiry. Since enactivism frames understanding as embodied, when transcribing the data, I recorded students' verbal as well as non-verbal communication such as gestures, pauses in speech, and/or facial expressions (Ochs, 1979).

Interpreting data. Powell et al.'s (2003) model incorporates a coding stage, wherein the researcher uses their analytical lens to code instances of critical events. Adopting the enactivist lens, instead of coding the data, I described and interpreted the critical events in terms of the actions taken. In other words, I used the enactivist lens to describe our actions in terms of the distinctions we made and worlds of significance we brought forth.

Constructing storyline. Once I interpreted the critical events, I constructed storylines around these critical events. This involved giving an account of students' embodiment of feedback in the instances of the data that I chose. The purpose of these storylines is to make sense of the data and to "discern an emerging narrative about the data" (Powel et al., 2003, p. 430). This process required many back-and-forth steps between data and the interpretation of the data. It also involved juxtaposition of video data with other data sources such as the feedback

models and math posters. Through this process, themes and patterns co-emerged which provided insight into how the group embodied peer feedback.

Composing narratives. The narrative, which is the final stage of the analysis model, is a culmination of the other stages. Adopting an enactivist perspective emphasizes the co-evolution of the narrative with the other stages of analysis. As Powell et al. (2003) explain, "narrative and other interpretive actions actually begin from the inception of research" (p. 431). This means that although the narratives are composed at the final stage of the analysis process, they actually emerge and evolve throughout the stages. From the very beginning of the research, my choices in terms of my role as the teacher and researcher, the research questions and design, influence the narratives that emerge throughout the process. Furthermore, each of the stages of the analysis also influence the narrative. Multiple viewings of the data, selection of critical events and interpretations of those events affect the development of the narrative. In this way, narratives co-emerge with other stages of analysis (Powell et al., 2003).

In this research, I composed three narratives, each narrative telling the story of one cycle of coupling with/in feedback as our class co-constructed its peer feedback practices. In the following chapter, I offer these narratives, describing, in detail, how my students enacted feedback and co-constructed feedback practices across the three cycles. I am a part of these narratives, either explicitly taking part in the actions or interactions, or implicit in my role as the narrator. In my direct interactions with the students, at times, I act as a researcher, entering a conversation to know how the students conceptualize feedback, how they describe a category etc. At other times, I take on the role of their teacher, actively engaging in the co-construction of our feedback practices as we discuss our categories, models and feedback actions. In such cases, I become both the observer and the observed, narrating my coupling with my students and

analyzing our co-construction. In this way, my analysis is a process through which data and theory interact and co-emerge though me.

The narratives incorporate different instances of actions and interactions during each cycle. I describe how feedback was conceptualized in categories and models, how it was enacted through decisions about feedback actions for future cycles, and how the effectiveness of the feedback in each cycle was described. Most excerpts in the narratives contain critical events and are interpreted using the enactivist lens. However, some excerpts are also included which do not necessarily contain critical events, but offer more context, depth, and flow to the narrative. I have also included other elements such as photos of students' feedback models and math posters. Images of students are also included in some of the photos, for which informed consent was obtained.

Throughout the narratives, for ease of understanding, I use the term "feedback comments" when referring to the texts written on the sticky notes and the term "feedback" when discussing students' conceptualization of feedback.

Chapter 5: Data Analysis

As described in the previous chapter, in the final stage of data analysis for this research, I composed narratives of the processes of co-constructing feedback practices across the three cycles. These narratives were comprised of critical events and other instances of data that, though not identified as critical events, provide rich contexts and better flow throughout. In these narratives, I describe instances of students' feedback actions and interactions and share excerpts of their conversations with one another and with me. I use the enactivist lens to describe how students make distinctions and how they bring forth and specify new worlds of significance.

In the following three sections, I offer narratives of the co-construction of feedback across each of the three cycles. These narratives are aimed at helping readers become part of our journey and to "see" how I bring forth my findings in the next chapter.

Cycle 1

We began our first cycle with activities that were quite routine and familiar for us before this research. I gave the students two math questions to work on and to write their solutions on poster papers. At first, I had planned to have students work on the questions individually and write their own solution on a poster paper. However, the students stated that they would feel more comfortable if they worked with a partner. In doing so, they brought forth the very first change to their feedback actions by deciding to work in groups of two rather than individually.

As I had decided to remain open to their suggestions, I readily agreed to this change. As such, in groups of two, the students read the questions, discussed possible strategies, and wrote their solutions on poster papers. The two word problems assigned in this cycle are listed in Table 2.

Table 2

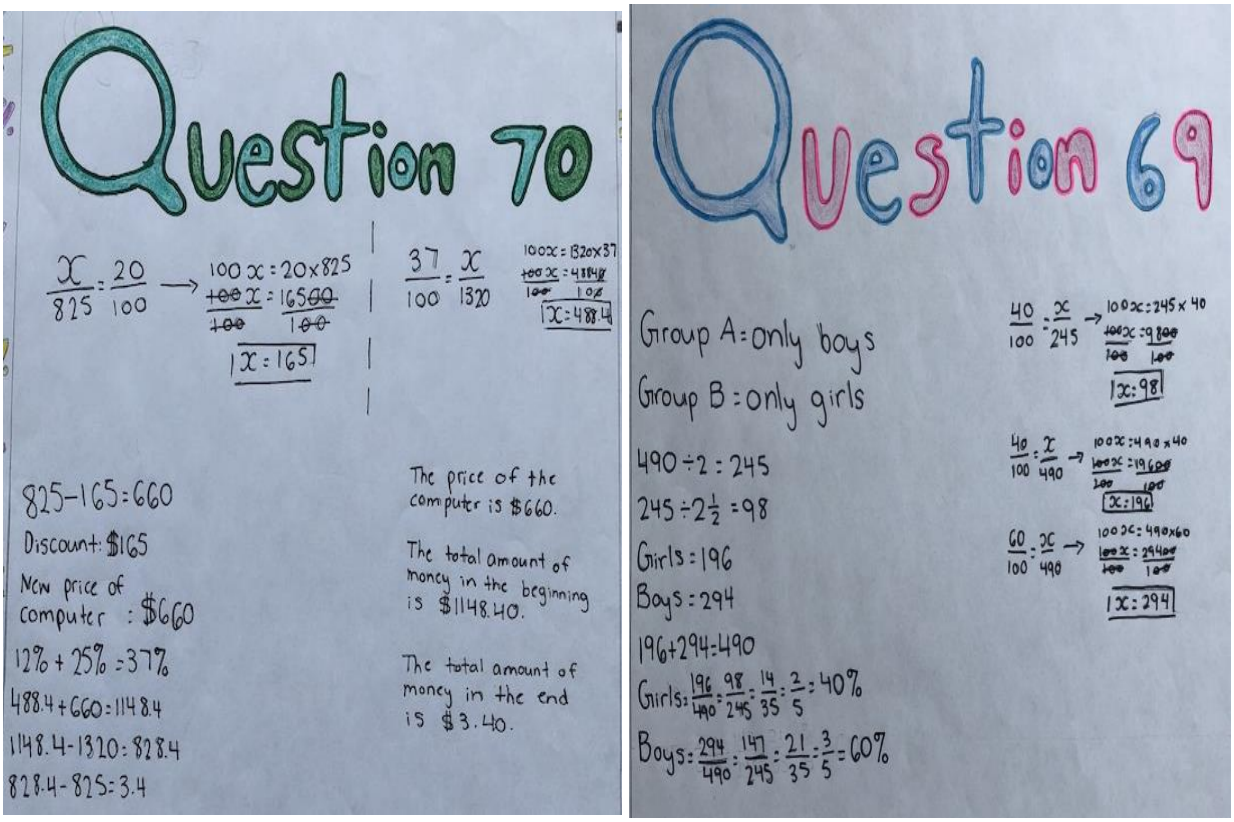
Word Problems assigned in Cycle 1

Question # 69	Question # 70
There were 490 children altogether in 2 groups. Group A consisted of only boys, and group B consisted of only girls. There were 2½ times as many girls as boys. Some girls joined Group B, and for every 4 boys in Group A, 32 more boys joined the group. The total number of girls was then 1/3 the total number of boys. Express the number of girls who joined Group B as a fraction of the number of boys who joined Group A.	Last month, Nate spent 12% of his paycheck on car repairs and 25% of the remainder on food. He gave $1,320 of the remaining money to his parents and then bought a computer on sale. If the usual price of the computer was $825 and the discount was 20%, how much money did Nate have in the beginning?

When the posters were ready, I asked the students to visit one another's posters, examine other people's solutions, write feedback on small sticky notes, and attach the sticky notes to the posters. Figure 1 shows two of the eight posters created by students in the first cycle. All the posters for this cycle can be found in Appendix A.

Figure 1

Samples of Students' Solutions in Cycle 1



In an effort to remain open to change, I avoided giving detailed instructions. For instance, I did not specify what I meant by feedback, neither did I encourage or discourage conversation between the students before, during or after they wrote their feedback comments. Interestingly, I noticed that while the students visited the posters and wrote feedback comments, no conversation took place among them. This absence of conversation was not something that was prompted by me or any of the students.

After they wrote their feedback comments, I asked the students to return to their own posters and read the feedback comments they had received from others and to decide on what they wanted to do with them. Again, I was careful not to give any specific instruction and to allow the students the opportunity to decide how they wanted to use the feedback comments.

Students' reactions to the feedback comments they received and their decisions on how to proceed varied from one group to the next. Some students took up the feedback comments and made changes to their posters. For instance, Andy and Sarah read the feedback comments one at a time, discussed each comment and made changes to their solution on their posters. In contrast, some students completely dismissed the feedback comments they were given, not taking any action to change their work.

Even when the comments were similar, students' responses and decisions as to take up or dismiss those comments varied greatly. For instance, Rose and John received a sticky note that read, "*I don't really understand your t-chart."* Rose's response to this was, "*We can explain that. We can explain that. So, this is useful. This is useful."* On the other hand, Layla and Sam, who read the comment, "*I don't understand what you've done here*," on one of the sticky notes on their poster, chose to ignore it, saying, "*There's nothing wrong with it."*

The two groups seem to have different perceptions about the usefulness of feedback comments. While a particular feedback comment is considered "*useful*" by Rose and John, it is entirely dismissed by Sam and Layla. Rose and John connect the usefulness of feedback comments to their ability to take an action, in this case "*explain,*" to address the feedback comment, *"I don't really understand your t-chart."* They see the usefulness of a feedback comment in that it evokes action(s) on their behalf. Layla and Sam, on the other hand, do not believe they should take any action in response to the comment, *"I don't understand what you've done here,"* because they do not think there is anything wrong with their work.

While the groups read the feedback comments on their posters, I noticed that one group, Sarah and Andy, approached another group, John and Rose, and asked them about a feedback comment they had received on their poster for question # 69.

Sarah: So, John, can you just explain this? You said, [reading the comment] *"Is that the total number of boys or the total number of girls*?" and, *"Is that the total number of girls who joined?"*

John: Yeah.

Sarah: So, we shouldn't have added the total boys?

John: You shouldn't have added… wait… what was your final answer again?

Sarah: seven hundred six…

John: You shouldn't have added the total… wait… that was …

Rose: That's girls and … [pointing at a number on the poster.]

Andy: [pointing at the same number] Yeah, that's girls.

John: This is boys… this… this… You shouldn't have added the total number of the boys and you shouldn't have added the total number of the girls.

Sarah: So, we should have just…like… we should have kept this number [pointing at a number on her poster]

John: Yeah… because that's the number of boys who just joined… because it says [reading the question] the number who *joined*… the number who *joined*.

Sarah: And then, for the ratio, it says one third of the boys who joined, so it would be one third of one thousand one hundred and twenty? Because that's the number of boys who joined?

John: No, no, no. It's this much … [pointing at another number on the poster] because it says [pointing at the question] It says the *total* number of girls is one third of the total number of boys. This is the *total* number of boys [pointing at the number again]. Which gives you 420. And then the answer would be 420 over…

Sarah: One thousand…

John: 1120

Andy: We did it wrong.

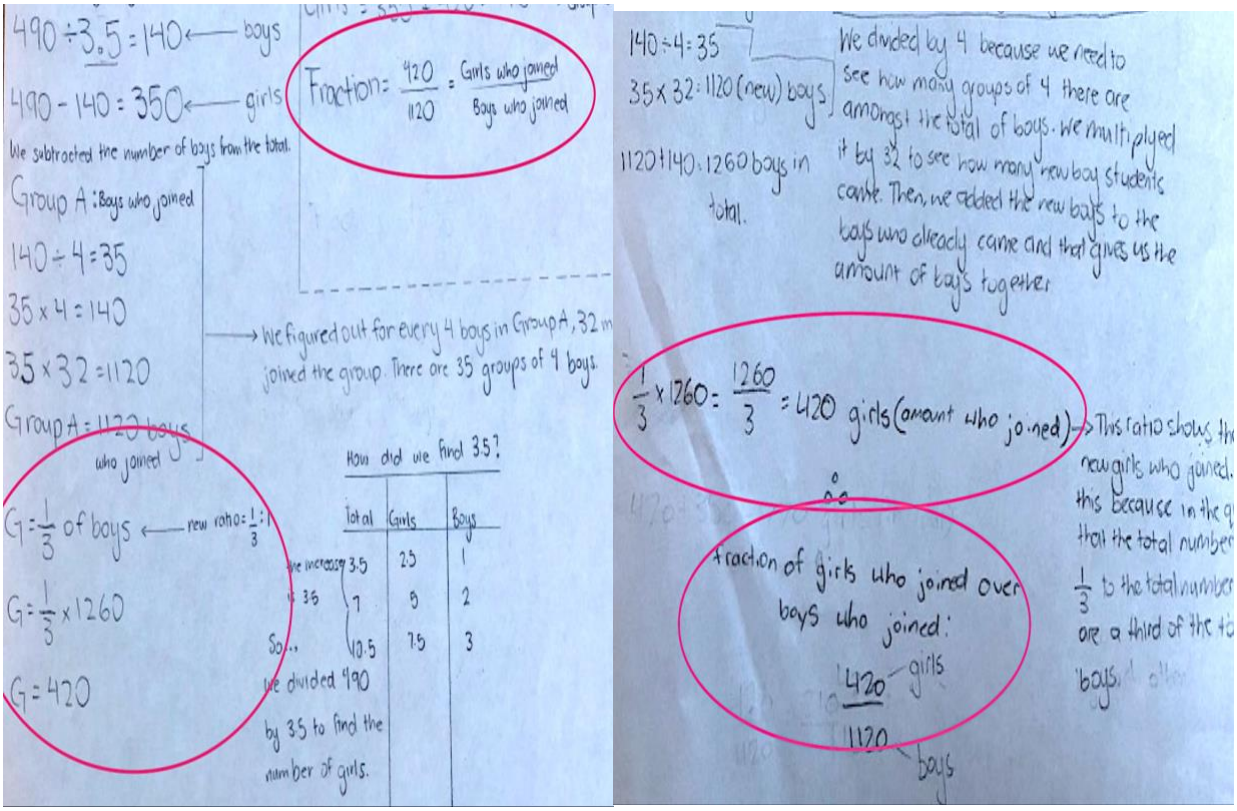
Sarah: Oh, okay. [getting her eraser and erasing part of her work and making corrections.]

The excerpt shows how Sarah and Andy seek clarification, engage in a conversation with John about the solution on the poster, find out what they have done incorrectly and attempt to correct their mistake. Sarah seeks clarification by asking, *"We shouldn't have added the total boys?"* John refers to the wording of the question and explains, *"You shouldn't have added the total number of the boys and you shouldn't have added the total number of the girls… because it says the number who joined."* At this point, Sarah who seems to understand their first mistake, goes on to ask, *"And then, for the ratio, it says one third of the boys who joined, so it would be one third of one thousand one hundred and twenty?"* Once again, John corrects Sarah's mistake

by emphasizing the wording of the question and says, *"It says the total number of girls is one third of the total number of boys."* By the end of the conversation Andy and Sarah understand why their solution is incorrect and begin making corrections on their poster. In this way, Sarah's act of reaching out to John and asking for clarification about the feedback comment, results in the four students discussing the math on the poster and making corrections on Sarah and Andy's poster. Interestingly, after Sarah and Andy make their correction, their poster turns out to be very similar to Andy and Rose's poster (see Figure 2).

Figure 2

Similarities Between Posters after Discussing Feedback Comments



Andy and Sarah (left) and John and Rose (right) calculated the original number of boys (140) and girls (350). They also found the number of boys who joined group A (1120). They then used the number of boys who joined to calculate the number of girls (1120 x 1/3 = 420). However,

both groups used 420, which was the total number of girls, instead of using the number of girls who joined. In order to find the number of girls who joined, they should have calculated the difference between the original number of girls and the total number of girls (420 - 350 = 70). This means that their final ratio should have been 70/1120. The similarity between the two posters is interesting, because it shows the impact of the discussion that took place between the two groups on Sarah and Andy's poster. While Sarah and Andy revised parts of their solution as a result of John's feedback comment and their consequent discussion, they also made the same assumption that John and Rose made on their poster. Except for Sarah and Andy, who actively reached out to those who had written feedback on their posters and asked for clarification, the rest of the students had very little interaction with others after feedback was written.

The students were instructed to place the sticky notes on the board as they went through them one at a time. When all the sticky notes were placed on the board, I asked the students to classify the sticky notes into categories of their choosing and to place their categories in a model. They were given complete freedom to decide on the criteria for classifying categories, to choose category names and to decide on a structure for their model.

Since they had worked on two math problems in this cycle, they began by arranging the feedback comments in two separate groups, one for question 69 and one for question 70. One of the students wrote the word *"comments"* on the board and placed one of the sticky notes under it, thereby distinguishing the first category. The text on the sticky note under the *"Comments"* category was, *"You are in a good start. We got the same amount for the computer price."* Curious to know how they describe this category and what feedback comments they would place in this category, I asked them about it. The following excerpt is part of our conversation, where John, Sarah and Rose describe the *"Comments"* category.

John: This one [pointing at one of the comments] this one is basically like… it's not constructive criticism, but it's not like bad criticism. It's just like… a comment that doesn't help and stuff.
Sarah: So, additional? Kind of?
Rose: Yeah.
John: Like… like…
Rose: Additional things you cannot do…
John: Like positive comments that like… that aren't helpful.
Sarah: That aren't beneficial.
John: Yeah.

John explains that the *"Comments"* category is *"not constructive criticism"* and, in this way, distinguishes this category from constructive feedback. He further describes it as *"a comment that doesn't help,"* thereby rendering such feedback comments useless. Sarah suggests that these comments are additional. Rose and John agree with Sarah's description and each add that these comments are not helpful and not beneficial, further emphasizing that such feedback comments are not considered useful.

A closer examination of the students' conversation in this excerpt reveals the co-emergent nature of their understanding of the *"Comments"* category. Students couple with one another as they attempt to describe the *"Comments"* category and their understanding of the *"Comments"* category co-emerges as they complete one another's sentences, agree with the others' descriptions, and even offer further explanation to clarify the others' meanings. As students negotiate different descriptions, the *"Comments"* category is specified and brought forth as a category of feedback comments that is additional, not helpful, and not beneficial. In this way, the category and students' understanding of it, co-emerge as a result of students' coupling in a linguistic domain.

The students' conversation continues as May suggests a model to enact their categories.

May: What if we did … what if we did … that… that… we did [drawing circles on the board with her fingers] two circles and then… intercepting… and then we can have… cause we might have some ones that go together.
Andy: A t-chart?
May: Or we can have three circles, depending on … no, not a t-chart…
John: A Venn diagram?
May: Yeah, a Venn diagram. But with three circles, so that we… so that we can have as many categories as we want, but when we have … I don't know… categories that kind of combine… cause we'll come to that once in a while, we can put it in the middle, so that we can have that option.
John: I think that's a good idea.
May: And if we have something completely by itself, it can go outside the circle.

The students had used Venn diagrams in the past in various math contexts. May suggests using a Venn diagram because she believes it would allow them to specify multiple categories that can potentially be linked together. May explains the need for having connections between the categories as she refers to it as, "*categories that kind of combine."* In this way, she brings forth the notion of linking or connecting categories as a necessary element for their model. May also suggests that a group of comments may not belong to any of the feedback categories and as she explains, *"can go outside the circle."* This suggests that she believes some of the comments may not be considered as feedback. As such, at this point, the model May and John are discussing is one that incorporates multiple categories with potential for connection as well as potential for exclusion of some comments that may not belong to any of the categories.

May's suggestion for using a Venn diagram was accepted by the students and they continued to discuss possible categories.

Sarah: So, I think the categories should be something that is very beneficial, so something that is very constructive. Um, ones that are just extra comments, so…
John: [using his fingers to count the categories] constructive criticism
Sarah: Yeah…non-constructive criticism
Andy: [pointing at the board] or comments.
John: Random comments [laughing]
Sarah: Um and…
John: And… what else is there? Like…

Sarah: We can have…
Andy: Stuff people don't understand.
Sarah: And like… vague. Something… you know how sometimes…
John: Vague comments.
Sarah: It would be like, I don't really understand?
Sam, Rose and May: Yeah!
Andy: Yeah… those, yeah!
Sarah: Okay.

Sarah and John begin the conversation by making a clear distinction between two groups of feedback comments. Sarah describes the first group as feedback that is *"very beneficial"* and *"very constructive."* John gives the name *"Constructive Criticism"* to the group described by Sarah. Sarah also describes another category of feedback as *"just extra comments,"* and calls this category the *"Non-constructive"* category. Andy suggests that the *"Non-constructive"* category is the same as the *"Comments"* category they had specified earlier. John adds to Andy's suggestion by qualifying these comments as *"random comments."* In this way, *"Non-constructive"* comments are the same comments they described earlier as not beneficial and not helpful. As they continue to look for more categories, Sarah makes a new distinction as she offers the word *"Vague"* to specify a group of comments that are *"stuff people don't understand."* She uses an example to describe what she means by *"Vague"* as she says, *"It would be like, I don't really understand."*

This excerpt exemplifies how students co-construct feedback categories as they agree with, add onto and/or complete one another's sentences. For instance, the *"Vague"* category co-emerges as John, Sarah and Andy finish one another's sentences. Through their conversation, they bring forth and specify the three categories of *"Constructive criticism," "Non-constructive criticism,"* and *"Vague comments."* They describe each of the categories, either by using

adjectives such as beneficial in case of the *"Constructive criticism"* category, or by offering an example such as *"I don't really understand"* in case of the *"Vague"* category.

The excerpt also demonstrates how students made a clear distinction between those comments that are *"very constructive"* versus others that are *"non-constructive"* or *"vague,"* thereby bringing forth a binary conception of feedback categories. Constructive comments are considered beneficial while the rest are *"just extra comments."* Although they have not yet qualified what they mean by *"Constructive"* or what they consider *"beneficial,"* the terms constructive and non-constructive suggest a distinct polarity by which feedback comments are classified at this stage.

As students continued to discuss their feedback categories, they negotiated which comments belong in each category.

May:	I think… I think we can do that. But also like narrow it down more.
Sarah:	Yeah.
May:	So, we can do questions that are saying, "I don't really understand" and …
Andy:	Which would be vague.
May:	And "You can do this."
John:	Yeah.
May:	And then…
Sarah:	So, when we do the Venn diagram, that was a good idea cause like we could put them together.
May:	Yeah.

Although they had already specified *"Constructive," "Random,"* and *"Vague"* as their categories, they decided to *"narrow it down more,"* thereby making their categories more specific or more nuanced. They do so by going through the comments and deciding on where to place them. As they read the categories, they came across comments that were similar in wording or, in their opinion, were similar in their message. For instance, according to them, comments such as *"I don't really understand,"* or "*You can do this,"* belonged to the *"Vague"* category. In

this way, they further specified the categories, deciding on what would or would not fit within each category. They also reaffirm their decision for using a Venn diagram for the model, as it would allow them to classify similar feedback comments and to *"put them together."*

As the students went through the sticky notes and read the feedback comments, they continued to write new category names on the board and these category names went through several changes. Many of the category names were chosen directly from the wording of the comments on the sticky notes. For instance, the *"Why did you?"* category was chosen for comments that included a "*why did you*" question. After much discussion, they specified six categories of *"Why did you?" "Where?" "You Should," "Random," "Vague,"* and *"Explain & Elaborate"* (see Figure 3).

Figure 3

Emerging Categories

However, they did not seem to be satisfied and continued to negotiate about the categories and the types of comments each category represented. They decided to join some of them together, claiming that they were the same. For instance, they joined *"Where"* to *"Explain & Elaborate,"* as both categories required the student who had solved the math problem to explain parts of his/her solution. They also renamed some of the categories, as they were not satisfied with the name. For example, they replaced *"Why did you?"* with *"Reason."* Furthermore, as they continued to discuss the feedback comments, their understanding of their categories changed and so did the categories themselves. In other words, the students and the categories co-evolved. After several changes, they finally agreed on four categories of *"Reason," "Explain & Elaborate," "You Should,"* and *"Random & Vague"* (see Figure 4).

Figure 4

The Four Categories

The *"You should"* and *"Explain & Elaborate"* categories were marked as *"Constructive,"* and the *"Reason"* category was connected to the *"Explain & Elaborate"* with a two-sided arrow that read, *"Similar."* The *"Random & Vague"* category was marked as *"Non-constructive."*

As such, the process of co-constructing feedback categories became an emergent one, where the category names emerged from reading the comments and changed through students' negotiations. The category names were written, erased and re-written, several times, as students negotiated their understanding of the categories. In this way, feedback categories and students' understanding of feedback were co-emergent and contingent upon their interactions. These feedback categories also embody students' conceptualization of feedback at this stage of the process in that they suggest a binary construct, an either-or conception, where feedback comments are either constructive or not-constructive. The *"You Should," "Explain & Elaborate,"* and *"Reason"* categories are specified as *"Constructive,"* and are considered beneficial. On the other hand, the *"Random & Vague"* category is specified as *"Non-constructive,"* and thereby not beneficial.

Having agreed to use a Venn diagram earlier, the students began drawing their model on a large paper. I noticed that although they had specified four categories on the board, they only drew three intersecting circles for their Venn diagram. I asked them about the number of their categories, curious to know how they explained the discrepancy between the number of categories on the board and the circles on the paper.

Parinaz: Have you decided on the categories?
May: Yeah. We've decided everything. And we've narrowed it down.
Parinaz: Okay.
May: "Random & Vague" is not really a category.
Parinaz: Okay, So, how many categories do you have?
May: Three.

Sarah: Four.
John: Well, "Random and Vague" can go outside this Venn diagram like May said.

At this point, that the students had decided to use the three circles for the three *"Constructive"* categories, and to place the *"Random & vague"* category in the space outside of the circles. This separation or exclusion of *"Random & Vague"* comments re-emphasizes the either-or position they took regarding *"Constructive"* and *"Non-constructive"* comments when categorizing feedback comments earlier. As May stated, and John reiterated, *"Random & Vague"* is not considered as a category and belongs outside the circles. The three *"Constructive"* categories, on the other hand, belong in intersecting circles. This exclusion would mean that the *"Non-constructive"* comments have no points of intersection, nothing in common, with the *"Constructive"* comments. I was curious about this exclusion and wondered about the nature of this category and wondered if it implied that the *"Non-Constructive"* category is not considered feedback at all. In the following excerpt, the students emphasize this exclusion of the "Random and Vague" category as they refer to the comments for this category as "outcasts."

Sarah: Why don't we put random and vague comments first?
John: 'cause they are the easiest to construct.
May: Plus, they're going outside of the circles.
Sarah: Yeah. They're everywhere.
[Rose takes three of the notes and starts placing them on the poster.]
Rose: This is "Random and Vague."
May: You can just put it scattered, I guess.
Sarah: Yeah. It doesn't really matter.
May: 'cause they're … kind of … like outcasts
Parinaz: Which ones are outcasts?
[May points at the comments Rose has just placed on the model.]
Parinaz: The random ones?
May: Yes.
Parinaz: And what makes them random again?
Sarah: Because… um…
Rose: They don't really…
Sarah: They're not really random, but they're vague. Because it's like … "I don't really understand." So, there is no reason behind them.
Rose: And… like… "You did the same thing as us."

In the first half of this excerpt, there is consensus among the students about the *"Random and Vague"* category. They show their accord by agreeing with one another's opinions and finishing one another's sentences. For instance, when Sarah suggests they should begin with the random and vague comments, John and May agree by stating that, "*they are the easiest to construct*," and that *"they're going outside of the circles."* Similarly, when May suggests that the "*Random and Vague*" comments can be scattered, Sarah agrees, claiming that "*it doesn't really matter*," to which May adds, "*cause they're … kind of … like outcasts."*

By characterizing the comments for this category as *"outcasts,"* and placing these comments outside the circles, the students exclude this category from the other three feedback categories. However, when I ask them, *"Which ones are outcasts?"* and *"What makes them random?"* they are unable to offer a clear description of the *"Random & Vague"* category and simply claim that, *"there is no reason behind them."* They can, however, distinguish these comments from the rest and provide examples of this category. For instance, Sarah and Rose offer examples such as *"I don't really understand"* and *"You did the same thing as us"* to represent the *"Random and Vague"* category.

Interestingly, even as the students try to describe the *"Random and Vague"* category to me, they amend the name of the category. When I ask them, "*What makes these comments random?"* Sarah replies, *"They're not really random, but they're vague"*. Sarah removes the term *"Random,"* thereby bringing forth a new name for this category while discussing the old one. In this way, my question occasioned for her to reconsider whether *"Random and Vague"* is a fitting name for this category. Realizing that the comments are not really random, she changes the category name from "*Random and Vague*" to "*Vague*." In this way, our conversations were

not mere reflections of the earlier coupling with this category. Rather, through our reflections, we continued to couple with one another about these categories in the linguistic domain, wherein new understanding emerged.

My question and subsequent conversation in this excerpt also marks my evolving role in the co-construction of feedback categories. Before this excerpt, I was an observer, watching them as they enacted feedback, discussed feedback comments and negotiated categories. At times, I asked questions regarding the number of categories and/or their feedback model. But in this excerpt, my question and the conversation that followed it influenced the naming of one of the categories. Had I not asked the students about the terms "*outcast*" and "*random*," the category name might not have changed at all. In this way, my role as researcher-teacher became more than a mere onlooker; I became an active member of the process of co-construction.

Soon after changing the category name from "*Random and Vague*" to "*Vague*," the location of this category on the model also evolved. Although the group had originally decided to place the "*Vague*" comments outside the circles, their decision was soon challenged by the need for this category to intersect with other categories. When the students first discussed using a Venn diagram, they considered it a suitable model as it would allow them to intersect their categories to accommodate feedback comments that belonged to more than one category. The following is an example of how we discussed these points of intersection as the students began transferring the sticky notes from the board to their model on the paper.

May:	Do we have any intersections?
John:	Yeah.
Sarah:	I think we do.
John:	The similar ones.
Parinaz:	Are these comments that fall between… like in two or three categories?
Sarah:	Yeah. [She reads one of the comments aloud] like, "How was your final answer even possible?" So, we have to kind of explain that [she points at the "Explain &

> Elaborate category]. And it's like "Why did you do this?" So, like reason [points at the "Reason" category]. And, I'm not understanding. So that's random too.

As Sarah's example demonstrates, some of the comments were comprised of several sentences, each belonging to a different category. I noticed that the students did not separate the sentences, placing each of them in their appropriate category. Instead, they seemed to view these sentences as parts of a single comment that belonged to more than one category. In the example Sarah provides above, she classifies each sentence as belonging to a different category, but she believes that these sentences form a single feedback comment which belongs to the intersection of the categories. Therefore, on the model, these comments belonged to the intersections between the circles.

The problem, however, was that previously they had decided to place the *"Vague"* comments outside the circles, completely separating them from the *"Constructive"* comments. This separation would mean that there were no intersections between *"Vague"* and the other three categories. Sarah's example clearly challenged the separation of the *"Vague"* category on the model, as it necessitated there to be intersections between this category and the "Explain & Elaborate" as well as the *"Reason"* categories.

Faced with this challenge, the students made two decisions. The first decision was to merge the *"Reason"* and the *"Explain & Elaborate"* categories as one, renaming it as *"Explain, Elaborate & Reason"* category. These two categories were previously connected together via a two-sided arrow, labeled "similar," and the students had already agreed that these two categories were both *"Constructive"* and shared similarities. May suggested that they join the two categories and explained, *"We can combine categories, if that's possible. Like we can say 'Why did you?' and 'Explain'... because they're the same thing. Right?"* May's suggestion to join the

two categories was readily accepted and the change was made immediately. Now there were two *"Constructive"* categories of *"You Should"* and *"Explain, Elaborate & Reason."*

The second decision was to use one of the three intersecting circles for the *"Vague"* category. This meant that the *"Vague"* comments were no longer *"outcasts,"* and that they were now regarded as a feedback category, and that as a category, they shared points of intersection with other categories. This change in the model was a departure from their earlier conception where there were no intersections between *"Constructive"* and *"Non-constructive"* categories. These intersections meant that *"Constructive"* comments could contain vague elements, and *"Vague"* comments could have constructive parts.

The decision to change the model and to include the *"Vague"* comments as a category that can intersect with other categories is significant as it exemplifies how students and their model co-adapted when they were faced with a situation that challenged their earlier structures. Finding points of intersection between *"Vague"* comments and other categories challenged their earlier conception of a total exclusion of the *"Vague"* category from other categories. In other words, the need to connect the categories together triggered the students' structure and they responded to this trigger by bringing forth changes to the categories as well as the model. In this way, the students, their categories and their model co-drifted.

Having finally decided on the three categories, the students read each comment, discussed which category it belonged to, and placed it on the model (see Figure 5).

Figure 5

Placing Feedback Comments on the Model

I noticed that many of the feedback comments were being placed in the spaces where the two *"Constructive"* categories intersected with the *"Vague"* category, which seemed to suggest that many of the feedback comments included a *"Vague"* feedback. The *"Vague"* category, which was specified almost at the same time as the *"Constructive"* category, underwent many modifications throughout the process. The name of the category changed several times, as did the way in which the students described it among themselves. The placement of the category on the model and its relationship with the other categories also evolved as the students coupled with feedback comments and saw the need to connect this category with the others. Despite all the changes and discussions, I was still not quite clear on how the students described this category. Curious to know more, I probed them about it.

Parinaz: How do you guys define "Vague"?
Layla: It's like…
Hanna: Things that are confusing.
Layla: It's like "There is no final answer," It doesn't really like… like…

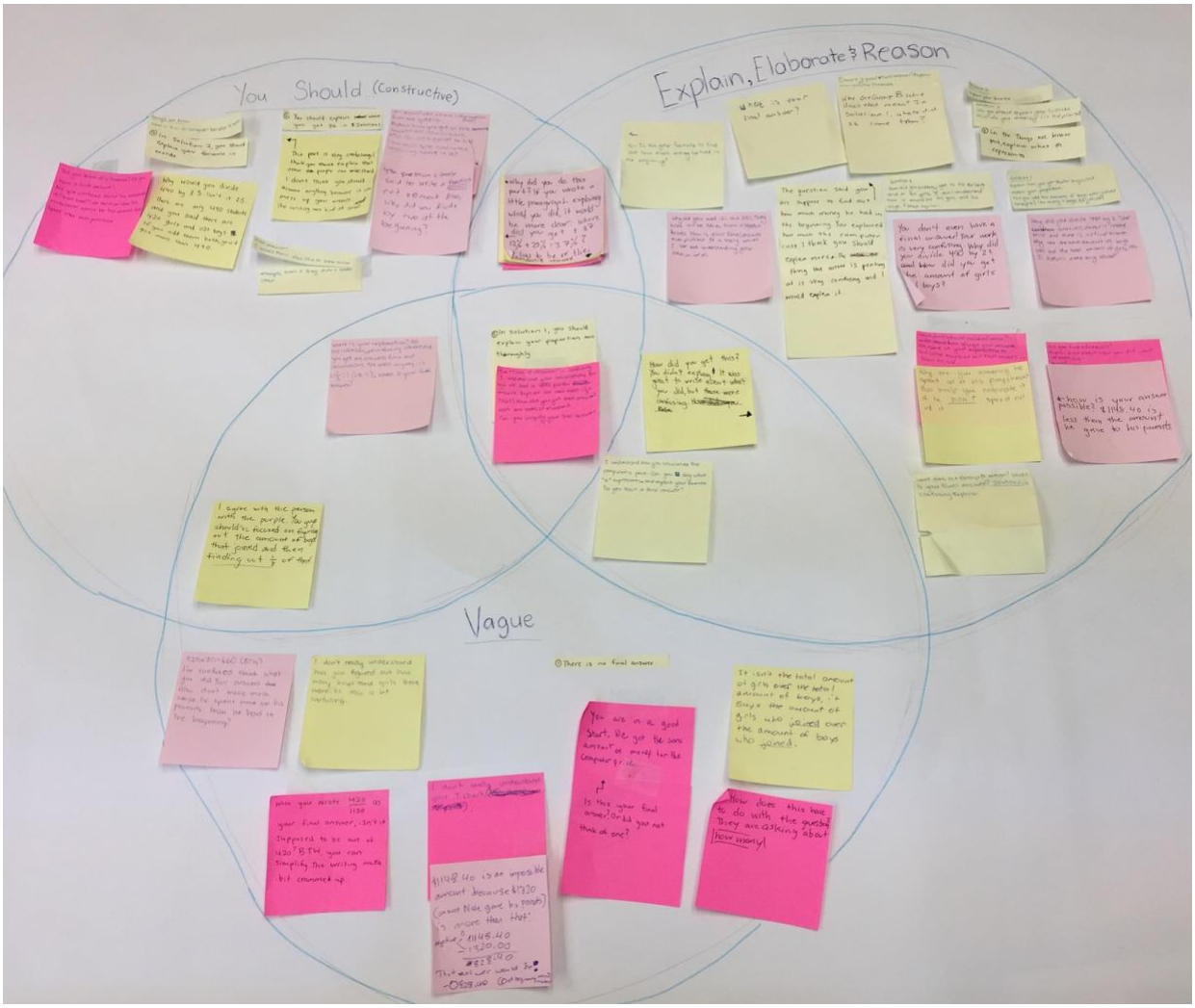
John: Or “I don’t understand.”
Sarah: Vague ones are more of like … they don’t give you direct instruction or they don’t give you direct advice.
Rose: They’re just like a comment.
Sarah: They’re just like a comment.
Layla: Like here, [reading from the note] like “Where’s your final answer?”

The students offer examples such as “*I don’t understand*” or “*Where’s your final answer?”* in an effort to describe what they specify as “*Vague*.” However, this time, they are also able to go beyond simply giving examples. Hanna describes *“Vague”* comments as *“things that are confusing.”* Sarah is more specific in her description when she says, *“They don’t give you direct instruction or they don’t give you direct advice.”* She defines *“Vague”* comments by describing what these comments do not do. In this way, she juxtaposes the *“Vague”* category against the *“Constructive”* categories. *“Vague”* comments are specified as those comments that fail to give advice, and thereby, are not *“Constructive.”* Although the term *“Constructive”* is not used by the students, their description of *“Vague”* comments once again highlights the polarity they perceive between the two categories. The two *“Constructive”* categories of *“You Should”* and *“Explain, Elaborate & Reason”* give direct instruction or advice, and the *“Vague”* comments do not.

When the model was finally complete, it embodied three distinct categories of feedback: the *“Explain, Elaborate & Reason”* category, *“You Should”* category, and the *“Vague”* category (see Figure 6).

Figure 6

First Feedback Model



The model also embodied the connection between the categories and the students' conviction that a comment could belong to more than one category. As such, there are comments on the model that belong to two or, at times, even all three categories. These comments are placed in the intersections of each two circles or all three circles. Table 3 contains samples of the feedback comments students placed in each category and in the intersections of categories.

Table 3

Samples of Students' Feedback in Categories and Intersections of Categories – Cycle 1

You Should	*-You should explain where you got 36 in your solution.* *-There are only 490 students and you said there are 420 girls and 1120 boys. If you add them, you get more than 490.* *-Are you confused about the remainder? We have an idea. The remainder would be the amount Nate spent after each purchase.*
Explain, Elaborate & Reason	*-Why are you assuming he spent all of his paycheck?* *-How is your answer possible? $1148.40 is less than the amount he gave to his parents.* *-Why did you add 12% and 25%? They both will be taken from different totals. How is your final answer even possible?*
Vague	*-There is no final answer.* *-I don't understand your T-chart.* *-I'm confused about what you did.*
You Should And Explain, Elaborate & Reason	*-Why did you divide 490 by 2? The ratio is 2.5:1, girls:boys. Explain your simplification a little. Why did you use a proportion?* *- What is x? Also, you cannot put x in the beginning and at the end, because they are two different numbers.*
Vague And Explain, Elaborate & Reason	*-I understand how you calculated the computer's price. Can you say what "x" represents? And can you explain your formula? Do you have a final answer?* *-How did you get this? You didn't explain. It was confusing.*
You Should & Vague	*-Where did you get the answers from? The ratio is 2.5:1. Where is your final answer?* *-I agree with the other feedback on the purple paper. You should have focused on figuring out the amount of boys then finding out 1/3 of that.*
All three categories	*-The "total of children" is confusing. I understand your calculations though. We had a very similar answer. Explain the new ratio (1/3 x 1260). How did you get that answer? Can you simplify your final answer?*

However, the model does not accurately embody students' hierarchical view of the categories. In the model, the categories appear in similar, intersecting circles, which would suggest equal status among them. This is in contrast to earlier conversation about the categories, where students used terms such as constructive and beneficial versus non-constructive and non-beneficial to describe the categories, which implies an unequal status between the categories. These differences between earlier conversations and the model drew my attention as the researcher of this study. I was interested to know if the two *"Constructive"* categories of *"Explain, Elaborate & Reason"* and *"You Should"* were considered equal or whether the students preferred one over the other. I asked them about the different *"Constructive"* categories, how they were different, and which one, if any, they preferred.

May: So, this one [pointing at "Explain, Elaborate & Reason" category] is saying you did something wrong, but it's not telling you what to do to make it better, but this one [pointing at "You Should" category] is saying what you did wrong and telling you what you what you can do to make it better.
Parinaz: Hmmm… Which one do you like more?
May and Rose: [pointing at the 'You Should' category] This one.
Sarah: Because it is more specific.
John and May: Yeah.

As this excerpt demonstrates, the students are now able to describe the two *"Constructive"* categories by explaining the function of each category. May explains that feedback comments from the *"Explain, Elaborate & Reason"* category tell them that there was a problem with their solution, while the *"You Should"* category provides them with strategies to fix their mistakes. It seems, from their explanation, that although they consider both *"Explain, Elaborate & Reason"* and *"You Should"* categories as *"Constructive,"* they prefer the *"You Should"* category, claiming that "*it is more specific*" in "*what you can do to make it better."* Thus, at this stage, the students name and value these two categories on the basis of what they

believe each category can do for them or, more specifically, what action(s) it can evoke in them. In other words, they have specified these two categories based on the perceived affordances of the comments. The *"You should"* category is perceived to afford more *"specific"* tools to improve than the *"Explain, Elaborate & Reason"* category.

While the two *"Constructive"* categories are clearly defined, the *"Non-constructive"* category, much like its name, still remains vague. After all the comments were placed on the model, I went through them, curious to know which comments were placed in the *"Vague"* category. I could not help noticing that these comments were quite varied. While some of the *"Vague"* comments simply stated that the assessor either did not understand the solution or found something wrong and/or missing about it, others went as far as correcting a mathematical mistake or pointing out an important aspect of the question. The following are more examples of these *"Vague"* comments.

"I don't really understand how you figured out how many boys and girls there were. It's also a bit confusing."

"It isn't the total amount of girls over the total amount of boys. It says the amount of girls who joined over the amount of boys who joined."

"How does this have to do with the question? They are asking how many?"

"You are in a good start. We got the same amount of money for the computer price. Is this your final answer? Or did you not think of one?"

It seemed to me, that not only were the comments varied in their tone and content, some of the comments did not quite fit the *"Vague"* category as the students had specified. Once more, I probed them about this category and pointing out that the comments in this category were very different from one another, I asked them if all these comments were "*Vague."*

Sarah: Perhaps not … perhaps not the category of vague, because sometimes they aren't very vague… but they [pointing to the other two categories] … they have something you should really work on… and um… something they thought was

really confusing. But vague is more like… they didn't really give a reason why they were confused … or they did not mention the final answer… or they… they did not mention if they got a similar answer… what was it.

Parinaz: So if you were to describe the vague category how would you describe it?

May: I would say that 'vague' is almost a substitute for 'other'. And we put…

Parinaz: You mean 'other' than the other two categories?

May: Yeah… other … other…

Parinaz: Okay.

May: So, when we had the 'you should' and the… the… 'reason,' we had those questions in the 'vague' that didn't kind of fall under those categories.

Parinaz: Why were they different? How were they different?

May: Um… there were questions that people kind of were saying as a comment… or to just say… not asking a question… or not saying that you should do something… but they're telling you that this was good or maybe it would have been better with this… so it's kind of like… comments or… not really constructive criticism… but maybe criticism.

Sarah describes the *"Vague"* category by comparing it to the other two categories and explaining that comments from this category fail to do what the *"Constructive"* categories do. When I press them for a clearer description, they use the term *"other,"* meaning that this category comprises of comments other than the "*You Should*" and "*Explain, Elaborate & Reason"* comments. May explains that the *"other"* are those comments that, *"didn't kind of fall under"* the other two categories. As May stated, these comments were *"not really constructive criticism, but maybe criticism."*

As a result of our conversations about this category, May brings forth a new distinction about the *"Vague"* category, describing it as *"Other."* What is significant about this distinction is that it embodies the students' binary conception of feedback comments. Feedback comments are understood as either *"Constructive"* or *"Other."* The *"Other"* category, describes a group of comments that are *"not asking a question"* and *"not saying that you should do something."* The *"Vague"* or *"Other"* comments are specified as those that do not serve a purpose. In this way,

once again, the students classify, describe and value feedback comments according to what they believe these comments should or could do for them.

After we discussed the categories and reflected on the model, together, we discussed their feedback actions in the first cycle. I wanted to know if they were pleased with their feedback actions and whether there were things they wanted to change in the format and/or order of the procedure for the second cycle.

Parinaz: Did you like the way we did it this time? Or … What would you do differently next time to make it better… more effective?
John: I think…next time when people give … like… feedback, like… even if it's just … like … a really small thing that you don't understand… you should just go back to them to just clarify it… what… it… cause if it is not what they meant… then you could be putting … like…false things on your poster.
Parinaz: Okay, so you need to clarify it right away?
John: Yes.

The very first change they brought forth was to be able to ask for clarification on feedback comments immediately after they are written to decide on how to take them up. John explains, *"You should just go back to them to just clarify it,"* in order to avoid putting *"false things on your poster."* I remember that this was an action Andy and Sarah's group did during the first cycle without being instructed to do so. They read the feedback comments and if they did not understand the comment, they sought the people who had written the comment and asked for clarification.

John's suggestion to add a step for seeking clarification suggests that they want their feedback actions to incorporate more dialogue. It also implies that students should actively seek clarification on the feedback comments they receive in order to be able to use them properly.

The second change they suggested was for each group to write down, on their poster, what they wanted feedback on. This meant that when students visited one another's posters, in

addition to seeing the solutions, they would also see what their peers wanted feedback on. John and Sarah explained their reasons for adding this step to the feedback actions.

John: Like say… say… you don't know if you've done this correctly, so you put like a note saying, can you please just check if this is correct?

Sarah: If you… if you mention…if you mention specific points, it's helpful because then the people who are giving you feedback can focus on that more… and that's what… you can review when you get the feedback… so it will be specific.

As Sarah explains, when the students mention *"specific points,"* others can write feedback about those points. She believes that adding this step is *"helpful"* because people will focus their feedback comments on what the student who solved the problem needs help with. In this way, by adding this step for their feedback actions in the second round, my sense was that students were signaling their preference for specific feedback comments. The students had used the term *"specific"* before when they described the *"You Should"* category, which suggests that their history of coupling with feedback influences their decisions about their future feedback actions. Perhaps adding this step to their feedback actions is an attempt at having more feedback comments from the *"You Should"* category. In this way, they are changing their feedback actions in order to influence the content of their feedback in the next cycle.

The students' suggestion to add this step to their feedback actions bears two significant implications about their conception of feedback and feedback actions: (a) it implies that they believe they should take on a more active role in deciding what they want feedback comments on by writing down specific requests for their peers, and (b) they conceptualize feedback as a dialogical process, wherein both parties, those who request feedback and those who provide feedback, communicate about the content of that feedback, albeit in the form of written notes.

The final change students brought forth in their feedback actions was for each group to have one of the partners stay by their poster in order to answer possible questions regarding their

solution before feedback comments were written. The following excerpt is part of their conversation about this decision.

Layla:	So, if another person wants… so like… doesn't understand how …um… something you wrote on your poster… um… then the person at the station can … like by the poster… can…explain what's happening.
Parinaz:	Okay. And why are we doing this? Why? Why is it important that they explain?
May:	Um… last time we got a lot of vague questions.
Parinaz:	Like what?
John:	Like explain.
May:	"Explain," or "You didn't have a final answer," or "Where is your solution?" So, things like that do not need to be put into categories… or put as feedback. They're more questions you can ask in person and kind of get a quick short answer to them.
John:	So, basically, we're getting rid of the vague.
Layla:	I think there's only going to be… in the previous one there were three circles in the Venn diagram. I think this time, there is going to be two, because we are going to get rid of the vague.
May:	If we keep working, we can technically eliminate more categories. Like we can eliminate another one and get into another category. So, we get to a point where we get like actual proper questions that actually help.
Parinaz:	So, then what kinds of feedback do we want?
May:	So then we get cons…
John:	Advice.
Sarah:	Constructive.
Rose:	Constructive feedback.
Sarah:	Suggestions and advice.
May:	Yeah… you get more…
Layla:	More "You should."
May:	You get more things that will help you more… not things that are asking you what you meant… because when people ask you there and then… you can just say… 'Oh, I'll explain it to you." But then when they give you questions on there, [meaning the posters] you cannot really explain it to them.

According to May, in the first cycle, they had many comments like, *"Where is your solution?"* or *"You didn't have a final answer."* She considers these comments to be vague and believes that if they have the opportunity to meet before the feedback comment is written, they could answer many of these questions. The students suggest having a partner from each group stay with their poster to answer questions so as to reduce or eliminate a certain group of feedback

comments that *"don't need to be put into categories."* In this way, the students' decision to add this step to their feedback actions for the second cycle is based on their experience with feedback in the first cycle. They believe that many of the feedback comments from the first cycle, which they classified as *"Vague,"* can be reduced or eliminated, by having more dialogue before feedback is provided in the second cycle. The students bring forth this additional step because they believe that it would help them have more *"actual proper questions that actually help,"* or the *"You Should"* category. They know what type of feedback comments they want to see more of and understand that their changes in their feedback actions would potentially change the outcome of the process.

As a researcher, I was triggered by the students' interest in adding more dialogue to their feedback actions for the next round and wanted to know how they conceptualized feedback at this point and why they believed more dialogue was important. So, I asked them what they thought about feedback. The following is part of our conversation.

May: I think feedback is a great way to share with people what you really want to say about their work. Cause a lot of times when you see other people's work, and either you try to explain it to them and they don't want the feedback. The feedback really just helps them and it helps you understand what's going on and it helps you figure out things.

Parinaz: Okay, so you said two things. You said that it helps them and it helps you. [May nods.] Tell me in what way it helps them and in what way it helps you.

May: Okay… so when you give feedback to someone… ummm, they can look at the feedback and realize what they did wrong and they say, 'Oh…I never… I never realize that before' and when you're giving feedback, like I said before, it opens your eyes so you can… you can see what they did wrong and compare it to yours and you'd realize things that you might not have noticed before.

Sarah: I think feedback is a … not necessarily positive response, but it could be negative. And it doesn't have to be beneficial. It can just be a comment on your work and how you can make it better. Or maybe it's just something that you should… that they think was good. It's beneficial because you can go back and you can go fix it. Because sometimes you don't realize if you've made a mistake or if it's not clear, but when you go back and people have given you feedback, you can tell what you've done wrong, or you should make more clear.

Layla: I think feedback is like good… I think feedback is like good criticism. It can be… and as long as … if … when …some people can catch mistakes for you… so you can go back to your work and fix your mistakes.

May begins by suggesting that feedback is a way of sharing with others what we think of their work and that it *"helps them and helps you understand what's going on."* This is the first time they describe feedback as something that can benefit not only the person who receives it, but the person who gives it as well. In other words, feedback is seen as useful to both parties. Sarah's explanation brings about a new distinction as she says feedback *"can just be a comment on your work and how you can make it better"* or simply something *"they think was good."* She believes such comments can be useful because, *"when you go back and people have given you feedback, you can tell what you've done wrong."* Layla adds to Sarah's arguments by stating that through feedback *"people can catch mistakes for you,"* and *"you can go back to your work and fix your mistakes."*

This excerpt demonstrates multiple layers of coupling and co-evolution among us as we discuss feedback and build on one another's ideas. When listening to May's description of feedback, I am triggered by the two distinct roles she assigns to feedback. I re-iterate May's description the way I understand it when I say, *"So you said two things. You said that it helps them and it helps you."* Whether, my understanding of May's meaning is accurate or not, I am no longer simply asking a question; I am coupling with them and my words influence their conceptualization of feedback. In response to my words and my consequent prompt, May explains that, *"they can look at the feedback and realize what they did wrong,"* and *"you can see what they did wrong and compare it to yours and you'd realize things that you might not have noticed before."* Sarah builds on May's thoughts by suggesting that, *"it's beneficial because you can go back and you can go fix it."* And finally, Layla adds onto Sarah's thoughts as she explains

that feedback can be good criticism as long as *"you can go back to your work and fix your mistakes."* In this way, as we discuss feedback, as we couple within a linguistic domain, our conception of feedback co-evolves and feedback is conceptualized as having more than one role/benefit and that it involves *"going back."*

Both Sarah and Layla use the notion of going back, to describe how feedback can be useful. According to them, students can go back to their peers and engage in a dialogue about the feedback comments they have received in order to better understand it. Every time they go back to ask for clarification on a feedback comment, they couple with one another about that feedback comment, and their understanding of it evolves as does the way they choose to use it. In this way, going back constitutes feedback as dynamic and contingent upon the interactions among the students. Students can also go back to their work, after they are clear about the feedback comments, and make changes and/or corrections on their work. This means that feedback is understood as a trigger and is only useful if it is taken up by students. This marks a significant departure from the way they had previously described the types of feedback comments they preferred.

This excerpt also demonstrates the way my role evolves from researcher to teacher within the short span of our conversation. I begin the conversation by asking about their conceptualization of feedback because I am curious as a researcher. However, my role quickly changes as I am triggered by, and respond to, May's description of feedback. When I re-iterate May's words and ask her for more explanation, I am acting as a teacher who has noticed something significant. I am triggered by the fact that for the first time in all our conversations, someone is describing feedback as having more than one benefit or role. This is important to me as their teacher as this can significantly influence the way they think about and co-construct their

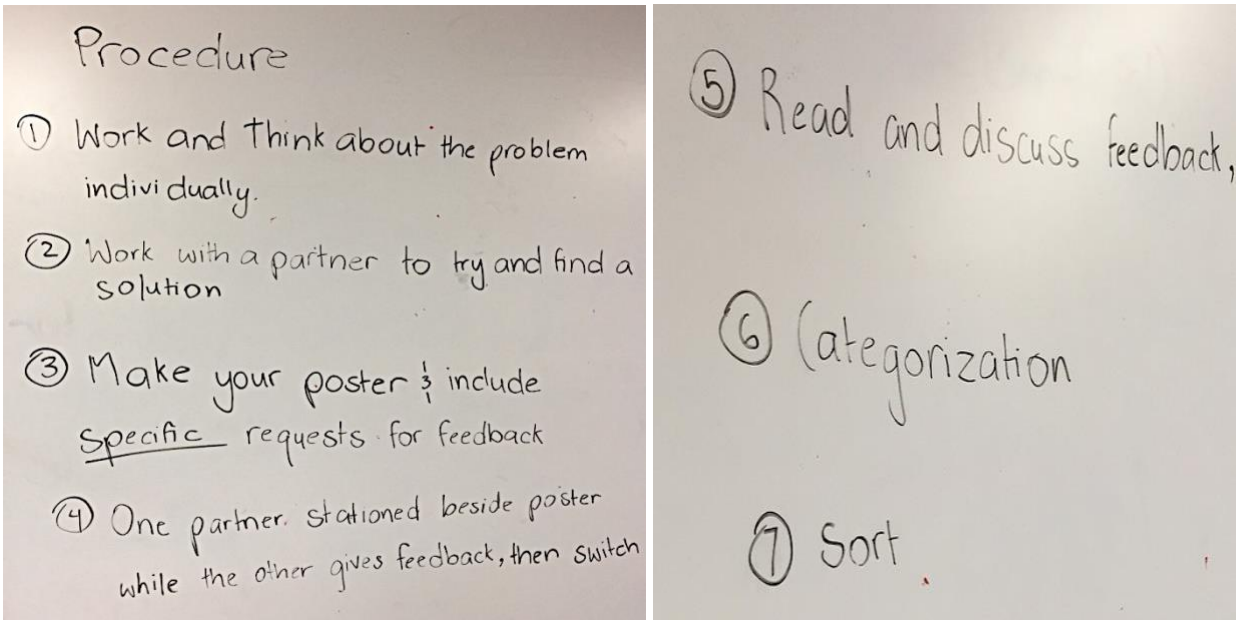
feedback practices. As such, in that moment, I am thinking and acting as their teacher, as part of the process of co-construction. My words, and the subsequent prompt for May to elaborate, influence the way they go on to discuss and conceptualize feedback.

Cycle 2

As described in the previous chapter, each cycle of this research span across three sessions within one week. Our second cycle began with the start of the second week of this study. This cycle began with the students going over the procedure they had discussed at the end of the previous cycle. They discussed the steps among themselves, and one of the students, Sarah, wrote them on the board (see Figure 7).

Figure 7

Feedback Actions for the second Cycle



The feedback actions the students outlined were in line with their decisions at the end of the previous cycle. They brought forth three major changes: (a) ask for specific feedback comments, (b) take a turn and remain with one's poster and answer questions before feedback

comments are provided, and (c) discuss feedback comments with other students. When the students were in agreement about their actions, they began working on the word problem. The word problem assigned to students in cycle 2 is listed in table 4.

Table 4

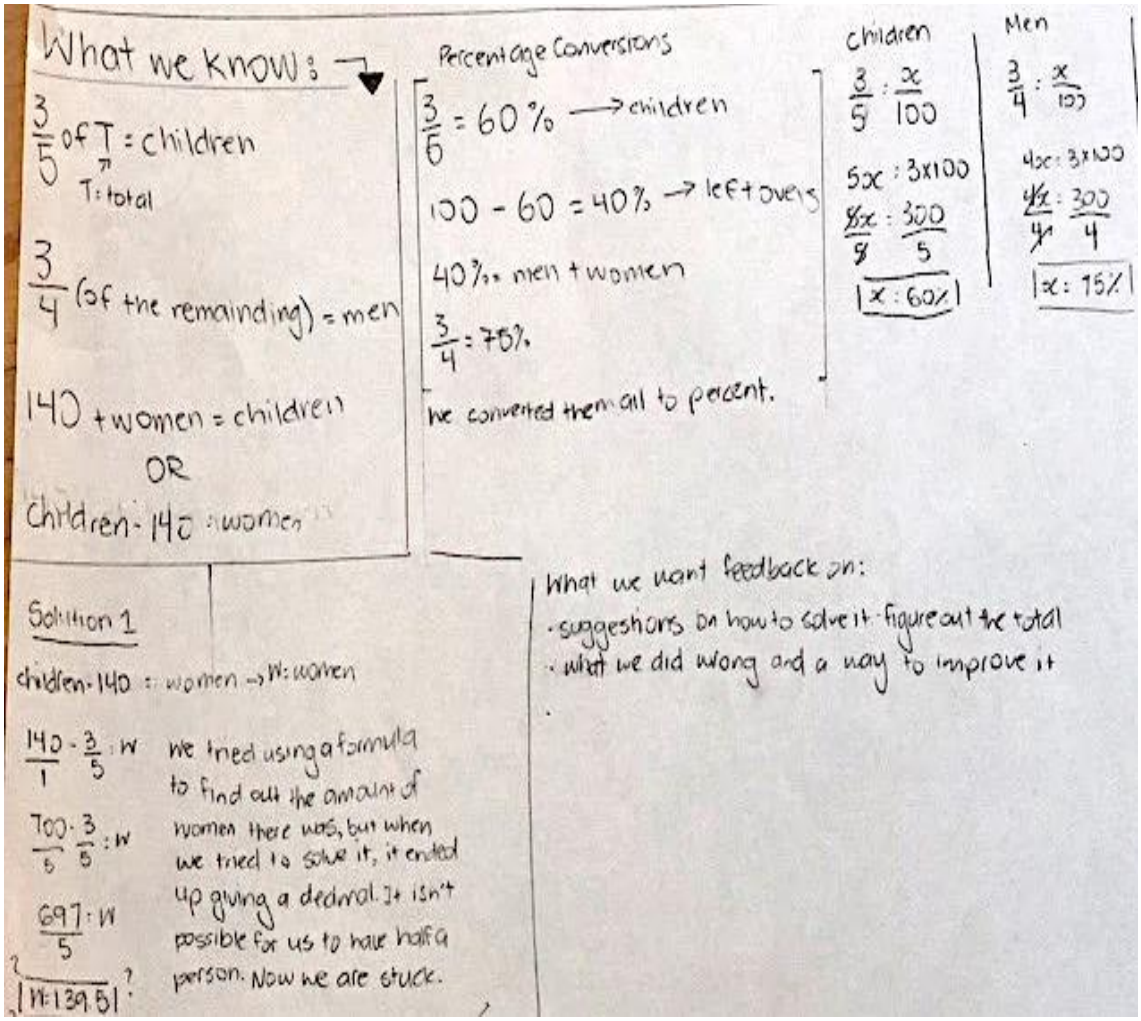
Word Problem for Cycle 2

Question # 64
3/5 of the people at the funfair were children. ¾ of the remaining people were men. There were 140 more children than women. How many people went to the funfair?

The students took some time to think about the question individually and then worked with a partner who was assigned to them through a randomizing game, making sure everyone had a different partner from the previous cycle. In groups of two, the students discussed the problem, came up with strategies to solve it, and wrote their solutions on posters. Figure 8 shows one of the four posters created by students in this cycle. All four posters are available in Appendix B.

Figure 8

A Sample of Students' Posters in Cycle 2

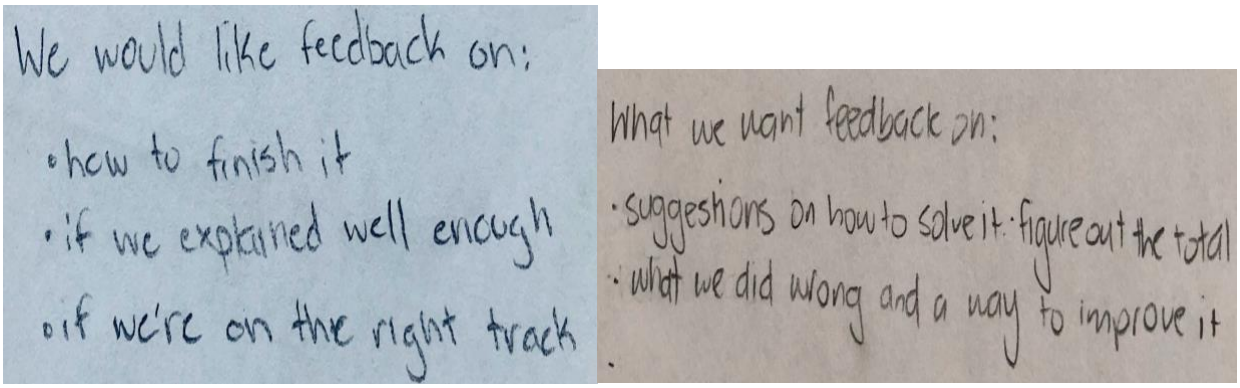


As I looked at students' posters before feedback comments were written, I was most curious to see what specific feedback they requested. As they had agreed earlier, one of the feedback actions they had added to their procedure for this cycle was for each group to write, on their poster, what they wanted to receive feedback comments on. This step was added, according to them, to avoid getting *"Vague"* comments and instead to get *"Constructive"* comments they could use to improve their work. However, when I examined the four posters, only two of them contained requests for feedback.

Interested to know why two posters requested for feedback and two did not, I closely examined the posters. The two posters that contained requests for feedback had incomplete or incorrect solutions. Students who requested feedback on these two posters asked for *"suggestions on how to solve it"* or *"how to finish it"* (see Figure 9). The feedback requests that were on these posters suggest that that the students who requested them were struggling with solving the question, were not sure if their strategies were correct, or simply did not know if they were on the right track.

Figure 9

Students' Requests for Feedback



The two posters that did not request feedback had either solved the problem or had a strategy in place to solve it. Of these two posters, one contains a complete and correct solution and the other does not. Regardless of whether these solutions or strategies were correct or not, these two groups that did not request any specific feedback comments on their posters seemed confident about their solutions. Their decision not to request for feedback comments, despite the fact that they had all agreed on asking for specific feedback, suggested to me that perhaps they only wanted feedback comments when they could not solve the problem. In other words, they did not think, at that point, that feedback comments served any other purpose if they had already solved the problem.

When posters were ready, the students began visiting other groups' posters to provide feedback comments. As they had previously agreed, one partner from each group remained by their poster, available to answer questions, while the other partner visited other posters. Halfway during the process, the partners switched roles so everyone would have the opportunity to visit other posters and write feedback. I noticed that unlike the first cycle, this stage of their feedback actions was not quiet. There was a lot of conversation among students about the strategies on the posters. I also noticed that this part of the process took longer time as the students asked questions and discussed the feedback requests on the posters before writing feedback comments.

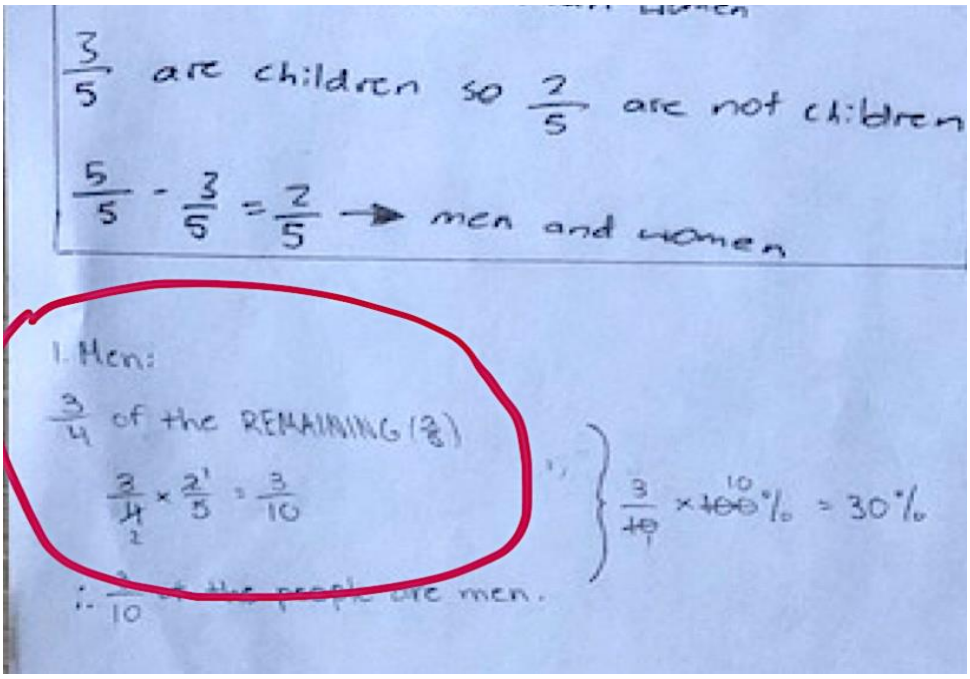
When all the posters were visited and feedback comments were written, the students returned to their own posters to read the feedback comments they had received. Once again, there was a great deal more conversation this time compared to the previous cycle. Students read each comment, discussed it with their partners, and in many cases sought the person who had written the feedback and discussed the comments and the math on their posters. For instance, May and John spoke to Layla about her feedback comment that read, "*I don't understand where ¾ x 2/5 comes from*." The following is part of their conversation.

May:	It's ¾ of the remaining.
John:	Which is 2/5.
Layla:	Why ¾ though?
May:	Because ¾ of the remaining people are men. So, when we had children, 3/5 of the people are children, right? So, that means that 2/5 of the people are not children. Because you subtract the …
Layla:	Yeah.
May:	Right? So then, basically, ¾ of that 2/5 leftover, is… is the number of men. [Layla nods.] So that is how we got 3/10.
Layla:	But it's 6/20.
May:	We simplified.
Layla:	Then you should explain that.
May:	Okay. [May crosses out the 2 in 2/5 and the 4 in ¾, to show how she simplified the fractions before multiplying them.]

May and John address Layla's feedback comment by describing what each of the fractions represent. May explains, *"3/5 of the people are children, right? So, that means that 2/5 of the people are not children. Because you subtract…"* Although May doesn't get to finish her sentence, Layla seems to understand that 2/5 is calculated by subtracting 3/5, which represents the number of children, from one whole, which represents all people present. May goes on to explain that since ¾ of those who are not children are men, then the number of men is calculated by multiplying ¾ by 2/5. Finally, she explains how simplifying the fractions before multiplying them yields the fraction 3/10. In this way, in addressing Layla's feedback comment, May and John get to explain part of their mathematics on the poster. May also acts on Layla's advice to show the simplification of fractions by crossing out the numerator of 2/5 and the denominator of ¾ (see Figure 10).

Figure 10

Adding Elements to Improve Work on Poster



Not all feedback comments resulted in making changes in the posters. There were instances where although the feedback comment helped students understand their mistakes, it did not help them make corrections or find solutions. Below, Layla and Hanna explain why they did not use the feedback comments they had received on their poster to correct their math.

Layla:	We still don't really know what to do.
Parinaz:	Is any of the feedback you received helping you?
Layla:	No.
Parinaz:	None of them?
Hanna:	It is.
Parinaz:	Do you want to discuss it with the people who gave it to you?
Hanna:	They're all really saying the same thing, that we shouldn't be adding specific numbers together because they're percent and fraction. And we now know what's wrong and we also know we have to figure out another way to find out the total.
Parinaz:	So, you're telling me you now know what you're not supposed to do?
Hanna:	Yeah.
Parinaz:	But you still don't know what you are supposed to do. [Hanna nods and smiles.] Is that what it is?
Layla:	Yeah. Like, out of all of them [referring to the sticky notes,] six or seven of them say, "You cannot add 140, 60, and 75, because 60 and 75 are percent.
Parinaz:	Okay. So, they're saying you cannot add those numbers because two of them are percent?
Layla:	Yeah.
Parinaz:	And the other one is not a percent?
Layla:	Yeah.
Parinaz:	Okay. So, do you understand how that is helpful? [Hanna nods.] Okay. So, then, what do you want to do?
Hanna:	We're trying to find another way.

Hanna and Layla see value in the feedback comment they have received in that it points out their mistake. They now understand that they should not have added the three values of 60, 75, and 140, since 60 and 75 are percent values. Their act of adding percent values to non-percent values suggests that they have limited understanding of what each of these numbers represent. Furthermore, while the feedback comments help them understand their mistake, Layla and Hanna still cannot make corrections to their work, which also suggests that they lack

competency in the mathematical concepts addressed in this question. In this way, although Hanna and Layla understand the feedback comment, they cannot use it to solve the question.

Once all the comments were addressed, and the students made changes to their strategies on their posters, they placed the sticky notes on the board in order to be classified. The process of classifying the feedback comments began with students discussing possible categories. The following excerpt is part of their initial discussions.

May:	What about helpful and non-helpful?
Layla:	Maybe we should do a "why" one.
Rose:	Yeah… there are a lot of "Whys."
May:	What about "helpful" and "non-helpful?" What about…
Layla:	Look, like…. "why did you guys?"
John:	So….
May:	Reason.
Layla:	Reason. Yeah.
	[Hanna writes "Reason" on the board.]
Layla:	I think there should be a vague. [She writes "Vague" on the board. John erased it immediately.]
May:	What about "Explanation?"
Layla:	Yeah.
	[Layla and May both write "Explanation" on two different parts of the board.]
Sarah:	How about "Non-critical?"
	[John writes "Non-critical" on the board.]
Hanna:	How about "suggestions?"
Layla:	Oh, yeah! [erases the word "Explanation"]
Layla:	This one… I think… this one… this one has multiple things. [Reads the beginnings of the sentences on the Post-it note she is holding.] "I don't understand…Why…You should…"
Sarah:	So, we combine.
Layla:	Right.
	[Rose writes "Combinations" on the board.]

This excerpt is an example of how feedback categories emerge as students read the comments and discuss potential names. The names of the categories are directly influenced by the contents of feedback comments. For instance, Leila and Rose offer the *"Why"* category because *"there are a lot of 'whys'."* Category names co-emerge through students' conversations. For example, Layla writes the word *"Combinations"* on the board because Sarah suggests that

they combine some of the categories. Similarly, John writes *"Non-critical"* on the board because Sarah offers it as a category. Category names are also temporary, changing several times during the students' conversation. For instance, *"Vague"* is replaced with *"Explanation,"* which is later replaced with *"Suggestions."* At times, the students' communication is non-verbal. For instance, Leyla writes the word *"Vague"* as a category name on the board and John erases it wordlessly, clearly disagreeing with Layla's decision.

What caught my attention about their discussion was the fact that although they had specified and described categories in the first cycle, they did not automatically use those categories in the second cycle. Rather, they made new distinctions that, in their view, better described the feedback comments in the second cycle. Categories such as *"Why," "Suggestions,"* and *"Non-critical,"* were new distinctions that were not used in the previous cycle.

However, I could see how certain elements of their choices were influenced by their experience from the first cycle. For instance, May offers to use *"Reason,"* a category name that was specified in the previous cycle, instead of using the name *"Why"* for a category. Similarly, May's offer of using *"Helpful"* and *"Unhelpful,"* although not taken up, is reminiscent of the *"Constructive"* and *"Non-constructive"* categories of the first cycle. Also, when Layla points out that some of the comments can belong to more than one category, Sarah suggests combining the categories, which is a strategy they had used in the first cycle. In this way, the category names as well as their structures are influenced by the students' history of co-constructing categories from the first cycle.

Throughout the excerpt, the students coupled with one another about their feedback comments and categories. They read the comments, discussed potential category names, changed

these names and brought forth new ones. They listened to one another's suggestions, negotiated strategies, and at times disagreed with one another's suggestions for category names. Furthermore, the categories they brought forth were influenced by their history of co-constructing feedback categories in the previous cycle. As a result, their coupling and their history of co-constructing categories occasioned for the co-emergence and co-evolution of their categories in this excerpt.

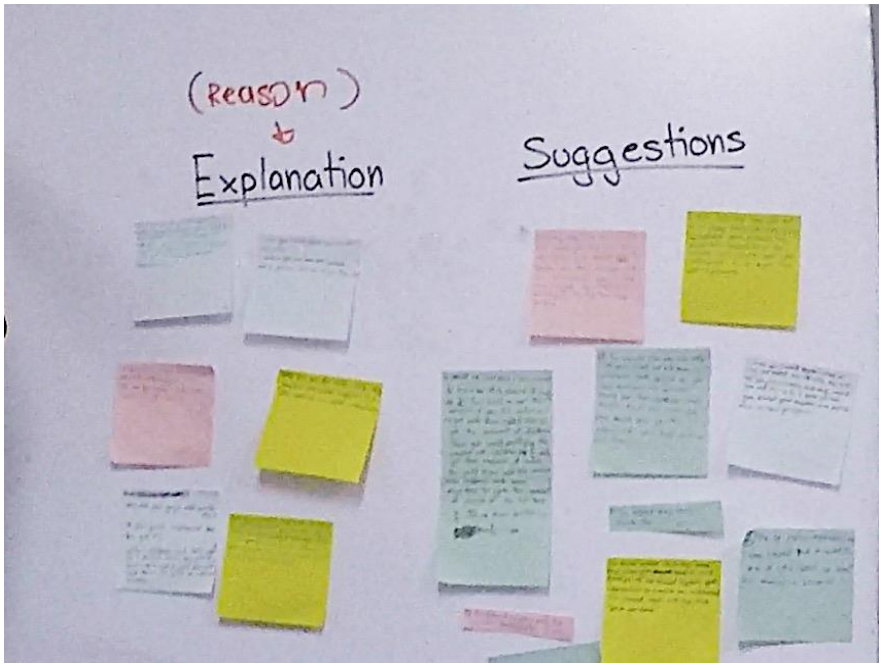
After much discussion and negotiation, the students wrote five categories on the board. The categories were *"Explanation," "Suggestions," "Non-critical," "Combination,"* and *"Reason."* Interested to know how they described these categories, I asked them about it.

Parinaz:	I see five categories on the board.
Hanna:	We might… we might change them.
Layla:	We might combine them.
Sarah:	I think explanation and reason should be combined. Cause they're the exact same thing. [Sarah writes the word "reason" on top of "Explanation" and joins the sticky notes together.]
Rose:	Some of them are just silly.
May:	Guys, I noticed with this question, a lot of people said… uh… "We did the same thing" or "You are correct so far," so …
Hanna:	So, statements.
May:	So, I think…not statements, but kind of like… um like… remarks. So, um…
Hanna:	That's statements.
May:	But not really statements, like… like…
John:	Non-critical.
May:	But non-critical can also be "You could have done this." Non-critical is very general. Right?
Rose:	Some of them are very silly. Like this one says [she reads the Post-it note she is holding] "The part where you said 140/1 minus 3/5, you should make the one longer."
Parinaz:	So, okay. Why don't you place the silly comments together? [Rose writes the word "Silly" on the board as a category name. Others place some sticky notes under the category name.]
May:	Okay. We should have non-critical too, though.
Parinaz:	What do you mean by non-critical?
May:	Because we have things that said "Your process is correct so far" or "You did a great job" or "we got the same answer as you."
Sarah:	But it wouldn't be non-critical. It would be something else.

Even after much discussion earlier, they are not quite satisfied with their categories, as they suggest more changes even while they discuss the categories with me. The first change they suggest is to combine *"Explanation"* and *"Reason"* categories together since, as Sarah states, *"they're exactly the same thing"* (see Figure 11).

Figure 11

Joining "Reason" and "Explanation"



Rose makes a new distinction, specifying comments that she does not believe fit in any of the existing categories. She explains that some comments are silly and provides an example to distinguish such comments from others. With my prompt to "*place the silly comments together,"* she writes the word *"Silly"* on the board and others begin to place sticky notes under this new category name. Some examples of the comments on these sticky notes are:

"Maybe you should put arrows from one step to another, because I was kind of confused."

"I don't understand this."

"In solution 1, the part where it says 140/1- 3/5, you should make the 1 under 140 longer."

Rose' comment about the *"Silly"* comments triggered me and I responded by inviting her to write the category name on the board. Others responded to my prompt and Rose's action by placing comments in the *"Silly"* category, thereby showing their agreement to having this new category. In this way, through our acts of coupling, we co-constructed this new category. My role in this co-construction evolved from a researcher to a teacher, as I listened to, and responded to Rose's attempts at distinguishing a certain group of comments. I also drew others' attention by having Rose write the name on the board.

Throughout this excerpt, multiple times, May draws our attention to another group of comments. She begins by saying, *"Guys, I noticed with this question, a lot of people said… uh… 'We did the same thing.' or 'You are correct so far.'"* She is unable to find a suitable name for this group of comments. She rejects Hanna's suggestion to call them *"Statements,"* and also rejects John's idea to call them *"Non-critical,"* arguing that *"Non-critical is very general."* However, she later seems to accept John's suggestion when she says, *"we should have non-critical too."* I ask her to provide a definition for *"Non-critical."* May is not yet able to offer a clear description for her category, but it is clear that she has an understanding of the characteristics of this category. She provides examples such as, *"Your process is correct so far"* or *"You did a great job"* or *"We got the same answer as you."* Sarah challenges the name of the category and states that, *"It wouldn't be non-critical. It would be something else."*

I noticed that the examples May provided were very similar to the ones they categorized as *"Vague"* in the previous cycle. I was curious to know why neither she nor anyone else was placing these comments in the *"Vague"* category as they had done in the first cycle. I was also

triggered by May's insistence on distinguishing this category and I wanted to know how she compared this category to the *"Vague"* category from the previous cycle.

Parinaz:	So, how is that different from the "Vague" category you had last week? Is it different? Is it the same?
May:	The "Vague" was more…It was more...It was more of a kind of a one-time thing. So, they said… they said… "Oh, great job!" but this one is more like "This is correct so far," "We did that too."
Parinaz:	Do you consider those as feedback?
May:	Well…
Rose:	No, because it doesn't really help us.
John:	It's just telling them that you've done a good job or a bad job or that you've got the right answer.
May	Well, in a way…
Parinaz:	So, do you consider that as feedback or not?
John:	Feedback is … feedback is trying… it's basically telling someone that they're like … like what you should do to … or what you could do to…
Rose:	Improvements.
John:	Yeah. Improvements.
Parinaz:	Okay.
John:	But then, these ones are just… like…
Parinaz:	So, do you all agree with what John said? That feedback is something that helps you improve your work? [Rose nods in agreement.]
Sarah:	No.
Layla:	It could be.
Parinaz:	It could be? As opposed to what?
Sarah:	It could be something like a comment.
John:	Feedback is… it's like apples and oranges. Feedback cannot be comments. Right?
Parinaz:	It cannot be?
John:	Or am I wrong?
Layla:	It can be. It can be.

The students' convictions changed while we discussed feedback in this excerpt. They seemed divided in their construal of feedback and whether or not what they considered comments should be considered as feedback. Rose was adamant that a comment such as "This is correct so far," is not considered feedback because *"it doesn't really help."* and John did not believe comments are feedback at all. However, by the end of the discussion, he seemed to

question his own conviction by asking, *"Or am I wrong?"* May and Leyla, on the other hand, did not quite agree with Rose's and John's statements, and while May was hesitant to voice her disagreement, Layla stated that comments can be feedback.

This excerpt contains many layers of structural coupling between myself and my students, and between us and feedback. We couple with one another and with feedback, trigger and are triggered by one another, and as a result a new construal of feedback emerges. I began the discussion because I was triggered by May's insistence on specifying a category she could not clearly describe. By probing May to describe the category and comparing it to previously specified categories, I triggered her and other students to reconsider their construal of feedback. May's descriptions and my questions challenged Rose and John who did not consider such comments as feedback. John explains that feedback tells one *"what they should do or what they could do."* He does not believe that comments such as *"You are correct so far"* are really feedback since they do not help one improve one's work. But May does not seem convinced, and Layla states that feedback *"could be"* what John describes. I probe her further to know what else she thinks feedback could be. John likens some types of comments and feedback to apples and oranges, thereby drawing a clear distinction between the two. But Layla still maintains that these types of comments can be feedback. Although she does not elaborate or give reasons, her conviction causes John to question his own convictions and ask, *"am I wrong?"* In this way, this excerpt marks an important shift in how the students construed feedback. This shift began with May's insistence for a new category, became stronger with my probing and Layla's openness to different possibilities, and culminated in John's self-doubt at the end of the excerpt as he questioned his own statement about feedback and what he describes as comments.

This excerpt demonstrates how my role evolves from researcher to teacher as I couple with my students. In the beginning of the excerpt, when I first ask the students about their categories, I act as a researcher, interested to know about the categories they have co-constructed. However, when I ask questions such as, *"Do you all agree with what John said?"* I act as their teacher, interested to know if there is consensus among my students about the way feedback is conceptualized. In doing so, I encourage more coupling among my students. Toward the end of this excerpt, I am a conversational participant rather than a conversational listener. As such, I have the potential to influence the way feedback categories are described. For instance, when I ask Layla, *"It could be? As opposed to what?"* or when I ask John, *"It cannot be?"* my words trigger them, occasioning them to expand or re-evaluate how they conceptualize feedback.

Our conversations bring about an evolution in how feedback is conceptualized. In the first cycle, feedback comments were either *"Constructive"* or not. However, as this excerpt demonstrates, some of the students no longer seem certain about such a strict, binary conception of feedback as they consider where to place feedback comments such as *"This is correct so far."* They do not believe these types of comments belong to categories such as *"Reason,"* but neither do they categorize them as *"Vague."* In this way, their coupling with one another about feedback in this cycle has challenged their earlier convictions. Our discussion continued with May re-examining their construal of feedback.

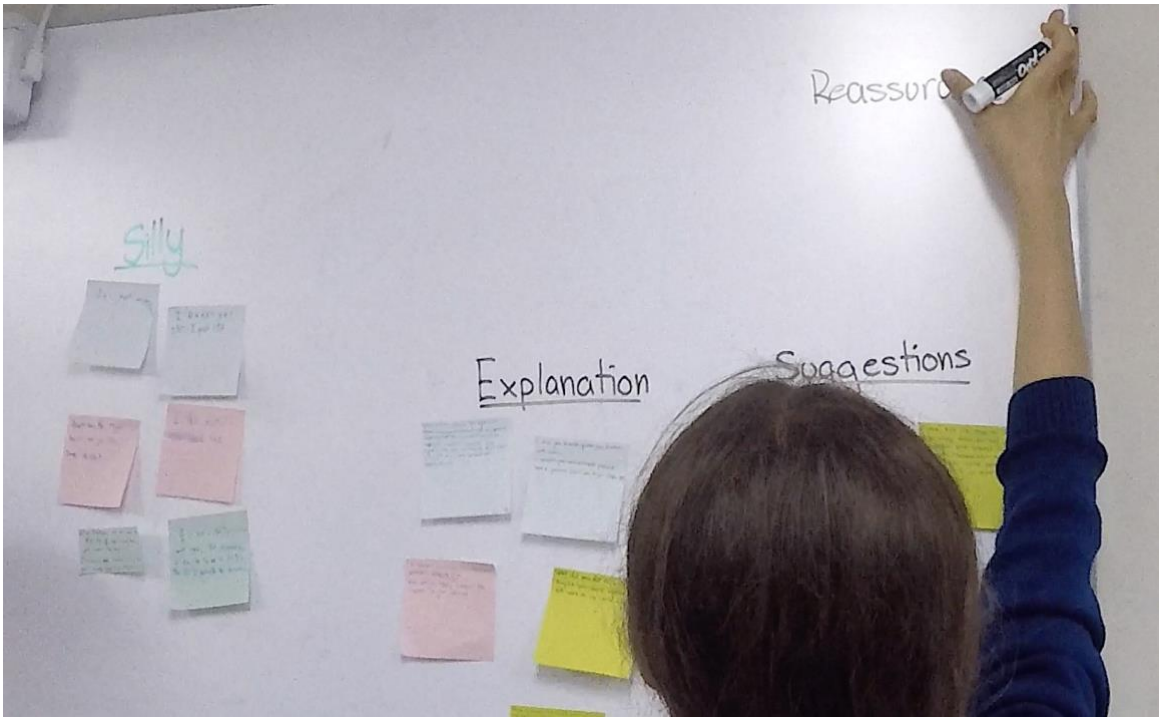
May:	When we were talking about the definition today, we all were saying that feedback is something you tell someone to either help them do something or help them to figure something out. So, when you… when you tell someone, 'Your process is correct so far,' it's helping them understand that their process is correct, and it also… it's encouraging you to do better and to try to figure it out… because someone actually believes in you. So, you can actually do it.
Parinaz:	So then, do you consider that feedback?
May:	Yeah. I do consider that feedback.
John:	But it's not helping you improve anything.
Parinaz:	That is what May said too. That it's not helping you improve.

Hanna: But it's reassuring you.
Parinaz: Pardon?
Hanna: It's reassuring you.
John: Yeah. It's helping you understand.
Parinaz: It's reassuring you?
John: Yeah.
Parinaz: So, reassuring comments, would those still be considered feedback?
all: Yeah.
Parinaz: Is it a category then?
All: Yeah. [May writes "Reassurance" on the board.]

May describes feedback as *"something you tell someone to either help them do something or help them to figure something out."* She claims that feedback comments such as *"Your process is correct so far,"* helps students *"understand that their process is correct,"* and that it encourages them to do better. As such, she believes these comments are also feedback. John is not convinced at first, arguing that these comments do not help improve anything. However, Hanna supports May's claim and explains that a comment such as *"Your process is correct so far,"* is feedback because *"it's reassuring you."* John agrees with Hanna's explanation and further describes these comments as ones that help one understand. They all agree that these comments are also feedback and classify them as *"Reassurance"* category. According to them, *"Reassurance"* feedback comments, while not offering a concrete way for them to improve their work, can encourage them to *"do better"* (see Figure 12).

Figure 12

Specifying the "Reassurance" Category.



My coupling with my students is quite prevalent in this excerpt. At times, I simply repeat their words back to them. For instance, in response to John who argues that these comments don't' help one improve one's work, I say, *"That is what May said too. That it's not helping you improve."* In this way, I draw their attention to the points on which they agree. However, when Hanna argues that these comments have the ability to be *"reassuring"* to them, I ask her to repeat her words. My comments were a deliberate attempt to draw others' attention to what Hanna says, because I think the reassuring element of these comments is at the heart of what distinguishes them from other comments. As soon as Hanna repeats her words, John shows a shift in his earlier opinion as he states, *"Yeah. It's helping you understand."* This is the first time John has agreed that these comments can be effective or valuable in some way. I continue to ask

them if they think these comments are feedback and if they are considered a category. By doing so, I once again seek their consensus in the co-construction of our categories.

Including *"Reassurance"* as a category is an important distinction in how feedback is construed in this moment. In the first model, feedback was considered as something that directly helped students to improve their work by telling them what to do. Any comment not related to the work was dismissed as vague. However, in this model, even comments that do not directly relate to the work are considered feedback. Students see value in the affective influence of feedback that *"encourages"* and *"reassures"* them. Specifying the *"Reassurance"* category is an example of the co-emergence of the students' understanding of feedback and feedback categories. When faced with comments that challenge their construal of feedback, the students reflected on, co-adapted, and reformulated their understanding of feedback and brought forth a new understanding of feedback and its affordances that included affective feedback comments. They co-adapted their construal of feedback and specified a new category. As a result of this co-adaptation, feedback categories evolved and students' construal of feedback became open to include more varied categories. In other words, feedback was now considered to have more facets that it did before.

Having just added the *"Reassurance"* category to their list, I asked the students about feedback, interested to know more about how they construed it at this point of the process.

Parinaz: What are you doing when you're giving feedback?
Sarah: I am looking for what I am confused about or what they should elaborate on. Because sometimes it can be confusing and they don't realize it. And they need to use more words. So, sometimes I look for… if things are clear.
Hanna: I think feedback is what someone thinks about what you did. So, really, I look forward to looking at what people think about what I do and if they have any suggestion about what I can do differently.
Parinaz: Okay. So, you said something very different. You said I think feedback is about… when you give feedback, you're basically telling people what you think about their work. How many people agree with that?

[May, Hanna, John, Sarah, Rose and Layla raise their hands.]

Layla: I think feedback is like … it's not just one thing. It's more… It's like what they both said [pointing at Sarah and Hanna]. It's …. It like helps you improve your work, and it's what others think of how you did your work.

Sarah and Hanna offer two different descriptions of what they believe feedback is or does. To Sarah, feedback is pointing out shortcomings in one's work in order to make improvements. Hanna's description, however, equates feedback to one's opinion of another's work. When I point out the difference in their descriptions, Layla states that feedback is *"not just one thing … it's more."* Layla explains that feedback *"helps you improve your work, and it's what others think of how you did your work."* In this way, she argues for a more open and inclusive conception of feedback, where feedback comments can afford different opportunities for those who give and/or receive them. This open conception of feedback led the students to further develop their categories.

After the above discussions, the students returned to reading the comments on the sticky notes and discussed them among themselves. They now believed that all the comments on the sticky notes were feedback and that feedback could belong to very different categories and that these categories could be connected together. They wrote the words *"Constructive"* and *"Other"* on opposite sides of the board and began to place their categories under these two headings. It seemed to me that they were re-examining the categories they had specified earlier as they continued to change category names, join or separate existing categories and create new ones (see Figure 13).

Figure 13

Negotiating and Finalizing Categories

At times, I joined in their discussion, either asking a question or making a comment, in order to better understand how they specified their categories. The excerpt below is an example of our conversations about some of their categories. I asked about the *"Silly"* and *"Reassuring"* categories because in the previous cycle, all of these comments were placed in the *"Vague"* category. I was interested to know how the students distinguished between the two in this cycle.

Parinaz:	"Reassuring" and "Silly," do you put them in the same category?
May, Layla and John:	No.
Rose:	No. They're two different things.
John:	No, but they're both like. They're both…
Parinaz:	So, then, how's "Silly" different from "Reassuring?"
Sarah:	"Reassurance" is something you say to…
John:	To make them feel good about their answer.
Sarah:	Yeah. But "Silly"… "Silly" is something that is very vague or something that is not very beneficial and cannot help you improve. But "reassurance" in some ways can help you improve.

Parinaz: Is it relevant? Is "Silly" relevant to the question?
Rose: No, it's not relevant.
[Rose replaces the word "Silly" with "Irrelevant" on the board.]

The students are adamant that the two categories are not the same. Sarah describes "*Silly*" as *"something that is very vague or something that is not very beneficial and cannot help you improve."* In contrast, *"Reassurance"* comments, according to Sarah and John who complete each other's thoughts, make one feel good about an answer and can in some way help improve one's work. Students' descriptions of *"Silly"* comments, and a look at some of the comments they have placed in the *"Silly"* category, suggest to me that what they refer to as *"Silly"* are comments that are not relevant to the solution of the mathematical problem. When I ask them if the Silly comments are relevant to the mathematical problem, Rose confirms my supposition and goes as far as changing the name of the category from *"Silly"* to *"Irrelevant."* In this way, my question about the nature of the comments in this category triggered the students to reconsider the name they had chosen. In other words, through our coupling, our understanding of this category evolved, and so did the name of the category.

After much discussion and multiple changes in category names, the students specified six categories: *"Advice, Suggestions, Corrections," "Explain more," "Reason," "Indirect," "Reassurance,"* and *"Irrelevant."* The first three categories were placed under the "*Constructive*" heading and the other three were placed under the *"Other"* heading. The categories on the board were drawn as circles and I noticed that when the students began drawing their model for feedback on paper, they drew similar circles to represent the categories (see Figure 14).

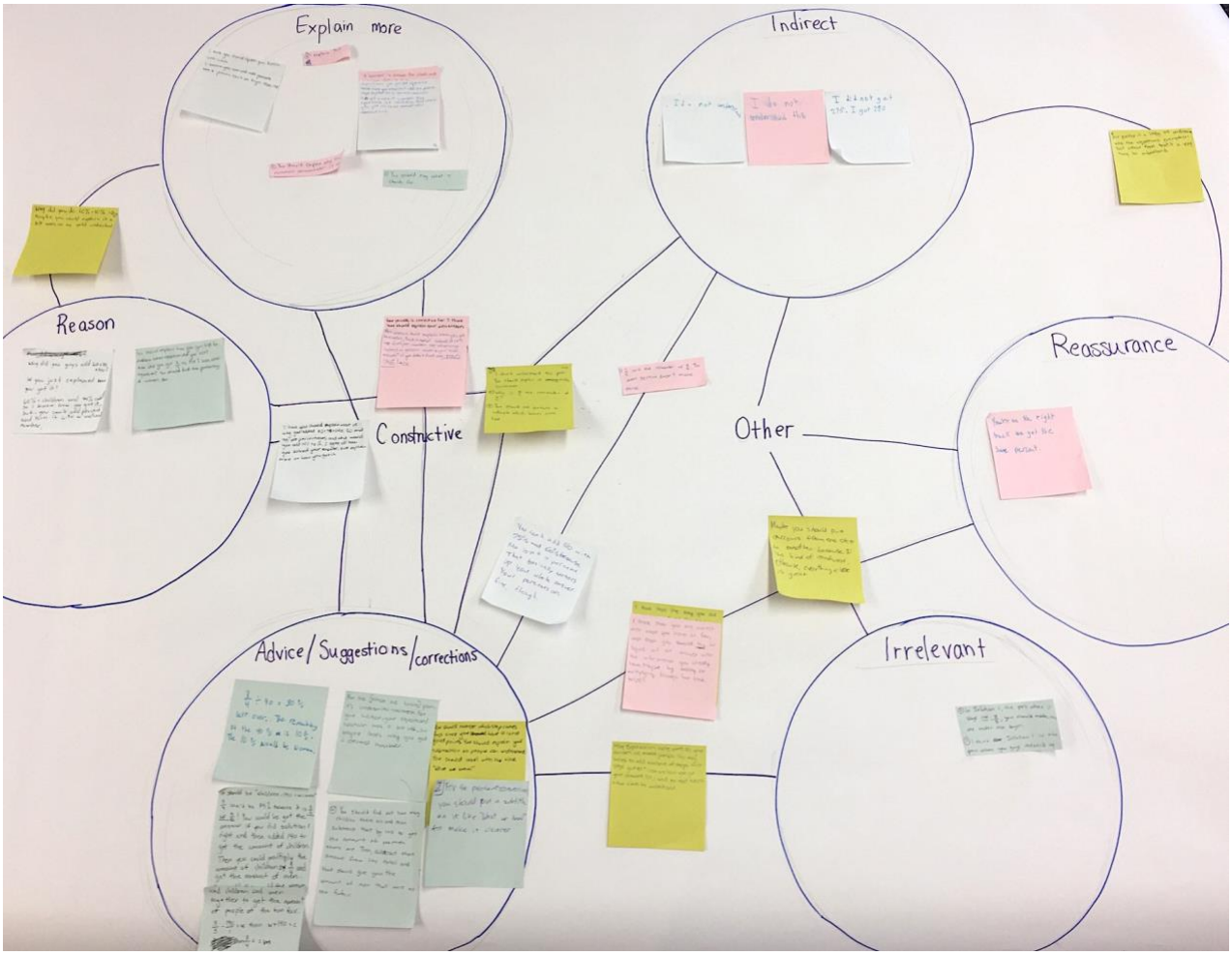
Figure 14

Co-constructing the Feedback Model for Cycle 2

They did not use a Venn diagram this time in the way they did in the first cycle. Instead, similar to what they had drawn on the board, they drew two circles on their paper to represent the *"Constructive"* and *"Other"* categories and then drew circles branching out of each main category to represent sub-categories. The subcategories for the *"Constructive"* category were *"Explain more," "Reason,"* and *"Advice, Suggestions, Corrections."* The subcategories for *"Other"* were *"Indirect," "Reassurance,"* and *"Irrelevant."* Since they used lines to show sub-categories branching out of main categories, they called this model a tree diagram (see Figure 15).

Figure 15

Second Feedback Model.



As evident in the model, many of the subcategories are connected to one another in order to accommodate for feedback comments that belong to more than one category. For instance, *"Reassurance"* is connected to *"Indirect"* as well as to *"Advice, Suggestions, Corrections."* However, not all sub-categories are connected. In fact, the students only drew connecting lines between those categories for which they could find common feedback comments. If there was no feedback comment that belonged to two sub-categories, the students did not draw a connecting line. For instance, *"Irrelevant"* is not connected to *"Explain more"* or *"Reason."*

Upon exploring the model, I noticed that there were a great deal more sticky notes placed in the *"Constructive"* categories than the *"Other"* categories. In fact, except for a few of the sticky notes, most of them are either placed in one of the three *"Constructive"* sub-categories, or in points of intersection between these subcategories and others. This means that most of the feedback comments either completely or partially belong to the *"Constructive"* categories. I remember that when we discussed feedback actions for the second cycle, one of the students' goals was to decrease *"Other"* comments and to increase *"Constructive"* comments. The fact that most of the sticky notes in this cycle are placed on the *"Constructive"* categories would suggest that perhaps the changes the students made to their feedback actions were effective in achieving the goal of getting more *"Constructive"* feedback.

This model bears similarities and differences to their model in the previous cycle, both in structure and categories. I asked the students about their model and how it compared to the one from the first cycle. The following is part of our discussion about the second model.

Sarah: So, this time, we chose to do a much more intricate model, because there were a lot more categories and there was a lot more connections in each piece of feedback. So, last time we did a Venn diagram and it would only intersect at one point between two categories, or it would intersect with all the categories. But here, we would do where… you could choose a category that the feedback was fit for and you could connect it to any one and it wouldn't really be something … it wouldn't be like… it would be easier to connect them than...because in the Venn diagram you could only connect to … less.
[May nods]

Parinaz: So, you have more connections here?

Sarah: Yes.

May: I think you just have more freedom to choose between the combinations. So, when we were doing our other model, um… as she said [referring to Sarah] the Venn diagram, we had very limited options as to where we could put the sticky notes. So, I think now that we have more space and more categories and then we can also intersect them easier, we have more freedom to combine them.

Similar to the first cycle, the students wanted their model to allow them to connect the categories in order to make room for feedback comments that belonged to more than one

category. The notion of connection between the categories is clearly an important one, as they enact it in their model and also talk about it in their discussions. However, they do not consider Venn diagrams to be a good model for this cycle. Sarah explains that by virtue of having several categories and comments that belonged to more than one category at a time, they needed a model that could facilitate connections among multiple categories. Equally important, was the notion of the freedom of being able to place feedback comments anywhere in or between the categories.

In contrast to the Venn diagram that allowed for limited options for connecting categories, the second model is, as Sarah describes, a *"more intricate"* one in that it allows for a multitude of connections among the categories. More importantly, it allows the students the freedom to choose which categories to connect. As Sara explains, with this model, *"you could choose a category that the feedback was fit for and you could connect it to anyone."* May states, *"you just have more freedom to choose between the combinations."* Having more freedom to choose which categories to connect together is an important aspect of this model. In this way, the students' new construal of feedback is enacted in a model that is "more intricate" and more fluid in its connections. Table 5 lists the connections each sub-category has with other sub-categories.

Table 5

Connections Between Sub-categories in the Feedback Model for Cycle 2

Sub-category	***Connections***
Advice, Suggestions & Corrections	Irrelevant, Reassurance, Indirect, Explain More, Reason
Reason	Explain More, Advice, Suggestions & Corrections, Indirect
Explain More	Reason, Advice, Suggestions & Corrections
Reassurance	Indirect, Advice, Suggestions & Corrections
Irrelevant	Advice, Suggestions & Corrections, Reassurance
Indirect	Advice, Suggestions & Corrections, Reason, Reassurance

Another similarity between this model and the one from the first cycle is the binary conception of feedback. The use of *"Constructive"* and *"Other"* categories that was brought forth in the model for the first cycle is also enacted in the second model. However, what distinguishes this model from the previous one, is that unlike the first model where the students did not regard the *"Other"* category as beneficial, they now consider *"Other"* categories such as *"Reassurance"* to be beneficial. Moreover, by looking at the connecting lines, one can see that there is a posited relationship between the *"Reassurance"* category and *"Advice, Suggestions & Corrections"* category. In fact, all three of the *"Other"* categories are connected to *"Advice, Suggestions & Corrections"* (see Table 6).

Table 6

Samples of Feedback Comments Belonging to two Sub-categories

Sub-categories	Feedback Comments
Advice, Suggestions & Corrections And Irrelevant	*"All your work is numbers and mumble jumble. You may want to add numbers of steps. What steps go first?"*
Advice, Suggestions & Corrections And Reassurance	*"I think that the way you did it is really smart, the way you substituted and created the equations. However, next time, maybe you could make your information a bit more clear."*
Advice, Suggestions & Corrections And Indirect	*"2/5 isn't the remainder of 5/4. The men part doesn't make sense."*

These connections between the "*Other*" subcategories and the *"Constructive"* subcategories suggest that students have found elements of both categories in many of the feedback comments on the sticky notes. In other words, many of the feedback comments in this cycle are believed to offer multiple opportunities. For instance, feedback comments can offer advice and suggestions, while at the same time be reassuring. As such, there is a shift in how students construe the affordances of feedback and this shift is echoed in our conversations about feedback after the model was complete.

Sam: Feedback is never wrong. Like… if you're telling them to explain, that's not wrong nor right. It's like something that people take in their perspective.
Parinaz: So then, what makes feedback good or bad?
Sam: Nothing. Every single feedback is good.
Parinaz: Why?
Sam: Because feedback is to help you with your own work.
Hanna: Um, it kind of depends on the person who receives the feedback, because it depends on how they use it. Like if … if they're just reading it and putting it away because they don't they… don't change and improve their work, then it doesn't

really help them. But if they actually use it to fix and make their work better, then it can actually help them improve what they're doing.

Parinaz: So then, you're saying that if feedback is not taken up, if it's not used, then it's useless?

[Hanna and May nod.]

Parinaz: But then, how do we, as users of feedback, how do we make sure that we take up the feedback? What kind of feedback do we take up? Or do we take up all of them?

May: It really depends on the person, but the feedback that says *"I do not understand,"* you can go to the person who gave you the feedback and you can talk to them, ask them what they didn't understand and you may be able to find what they couldn't understand and possibly even fix it.

Parinaz: So then, you are suggesting that you can go back to the person who gave you the feedback and …

Sarah: And address what they don't understand.

Parinaz: Address the feedback? So, to discuss the feedback with them?

[May, Hanna, John and Sarah nod.]

Sam explains that feedback is, *"something that people take in their perspective,"* implying that the student's relationship with feedback is what renders the feedback comment *"right"* or *"wrong".* As such, the affordance of a feedback comment is no longer solely influenced by the feedback category it belongs to. Rather, as Hanna explains, *it "depends on the person who receives the feedback,"* and it is impacted by how the student understands the feedback comment and what he/she chooses to do with it. This is important and significant, as it directly connects feedback and its affordances to the student, further emphasizing the student's influence over every aspect of the process. According to May, even how feedback comments are used *"depends on the person"*. She explains that in order to make feedback useful, one can go to the person who wrote the feedback comment and ask them about it. In this way, feedback comments are no longer understood as external and divorced from those who give and receive them. Rather, feedback comments are construed as part of an intricate and complex process, involving not only the feedback comments, but the students who give and receive those comments. More importantly, the value of a feedback comment is not solely reliant on the type

of feedback category it belongs to. Rather, it is influenced by the students' perspective of feedback and what they choose to do with the feedback comments they receive.

In the final stage of the cycle, the students and I discussed their feedback actions and whether they wanted to make any changes to their actions for the third cycle. The students brought forth one more change in their procedure. They decided that in the third cycle, once the posters were ready, each group would explain what they had done on the poster and answer potential questions before any feedback comment was written on their poster. I was curious to know why they wanted to implement this change. In the following excerpt, Sarah and Sam explain their reason for implementing this change.

Sarah:	With [this] strategy, we can actually eliminate a lot of repeated questions. My partner and I, our poster, there was a part where we didn't explain a lot and people were asking about that a lot. So, we had to explain it over and over again.
Sam:	So, we all get together and one person just explains the strategy and then we start giving feedback, then we'll get rid of a lot of the other types of feedback and we'll end up with suggestions, advice and corrections.
Parinaz:	Is that what you guys want?
All:	Yes.
Parinaz:	Are you aiming to have more of this category? [pointing at Suggestions, Advice & Corrections]
All:	Yes.

Their explanation suggests that they want to add the presentation step to their feedback actions in order to reduce the number of unnecessary questions and to make the feedback specific and constructive and to avoid repeated feedback comments. Although they now agree that feedback can be any and/or all those comments, they still insist on changing their actions in order to promote more of a certain category of feedback comments and reduce comments from other categories.

Having changed the procedure once before and having observed how their categories and model changed, as their teacher, I was curious to know their predictions for the next model.

Parinaz: So if we do this again, one more round, do you think we're still going to stay with this model or do you think it's going to change?
Sam: It's going to change.
Parinaz: It's going to change? How do you know that?
Sam: Because last time we didn't change it that much, but then after it changed the whole structure. If we are going to… if we at least … if we change it a little bit, then the whole structure will be different.
Parinaz: So, you think because we changed our procedure, it changed our model?
Sarah, Sam and Rose: Yeah.
Parinaz: So then, if we keep changing our procedure, and the model keeps changing, is that a good thing? A bad thing?
Sarah and John: Good thing.
Parinaz: But are you guys okay with the fact that the model keeps changing?
All: Yes.

Sam seems certain that in the third cycle their model will change again. He uses past experience as an example and explains that when they changed their feedback actions for the second cycle, the whole structure of their model evolved. He is referring to the fact that they had many more categories in the second cycle than they did in the first cycle and also that their model changed from a Venn diagram to a tree diagram. In other words, he believes that their model changed because they changed their feedback actions. In this way, the model embodies the changing nature of the students' conception of feedback and its structure is temporary and contingent on students' feedback actions.

Cycle 3

Our third cycle began in the third week of conducting this study. In addition to having a slightly modified procedure, this cycle also differed from the previous two cycles in the number of participants. Two of the students, Sam and Andy, were away that week as they took an early summer break and travelled overseas. As such, instead of eight students, we had six students participating in this cycle of feedback practices. Once again, the students were paired randomly, making certain that they were not paired with anyone they had been paired in the previous two cycles. The word problem assigned to the students for this cycle is listed in Table 7.

Table 7

Word Problem for cycle 3

Question # 63
3/8 of the guests of a birthday party were adults. The ratio of boys to girls was 2:7. There were 180 more children than adults. How many more girls than boys were there at the party?

Similar to the last two cycles, the students began the process by thinking about the mathematics problem individually and then worked with a partner and wrote their solutions on poster papers. All the posters for this cycle are available in Appendix C.

Although according to their procedure, this time the students were to present their solutions orally, they still wrote on their posters what they wanted to receive feedback comments on. Unlike the previous cycle, where only 50% of the posters contained requests for feedback, this time, all three posters contained written requests for what they wanted feedback comments on. Not only did all the posters contain feedback requests, the number of these feedback requests was significantly more than the ones in the previous cycle. The feedback requests for the three posters are listed in Table 8.

Table 8

Students' Request for Feedback on Posters

Sarah and Hanna	Rose and May	Layla and John
Is poster clear? Is our explanation sufficient? Is the format organized? Do you have other strategies? Careless mistakes? Did we use the correct wording for “ratio”?	How to find solution. If we could’ve improved it. Did we explain well enough? Does our solution make sense? No random comments. Only write “I don’t understand” if you still don’t understand after we explain.	If the 180 is correct in the first part. If it’s clear what we’re doing. The right answer … If we’re on the right track

A close examination of the posters reveals that one poster, the one belonging to Sarah and Hanna is the only one with a correct solution. Interestingly, Sarah and Hanna's requests for feedback comments are quite different from the other two posters, whose solutions are incorrect. Sarah and Hanna ask if they have made careless mistakes, or if their explanation is sufficient and their format is organized. They do not ask for help to solve the question, which suggests that they are confident about their strategy. The other two groups, however, have very different feedback requests. Rose and May ask for feedback comments on *"how to find solution,"* and Layla and John ask for *"the right answer,"* which suggests that neither group is confident about their solution and their answer. As such, the feedback requests are contingent on their capacity, real or conceived, to solve the question.

The feedback comments the students requested this time were more varied than the previous cycle. In the second cycle, the students mainly asked for advice on how to solve the question. This time, however, the questions ranged from asking for help to solve the question, such as *"We would like feedback on how to solve the question,"* to asking about the quality of the work presented, such as *"Did we explain well enough?"* or *"Is the format organized?"* Some feedback requests are very general, such as *"Is our explanation sufficient?"* while others were more detailed, targeting a specific need, such as *"Did we use correct wording for ratio?"* or *"If the 180 is correct in the first part."*

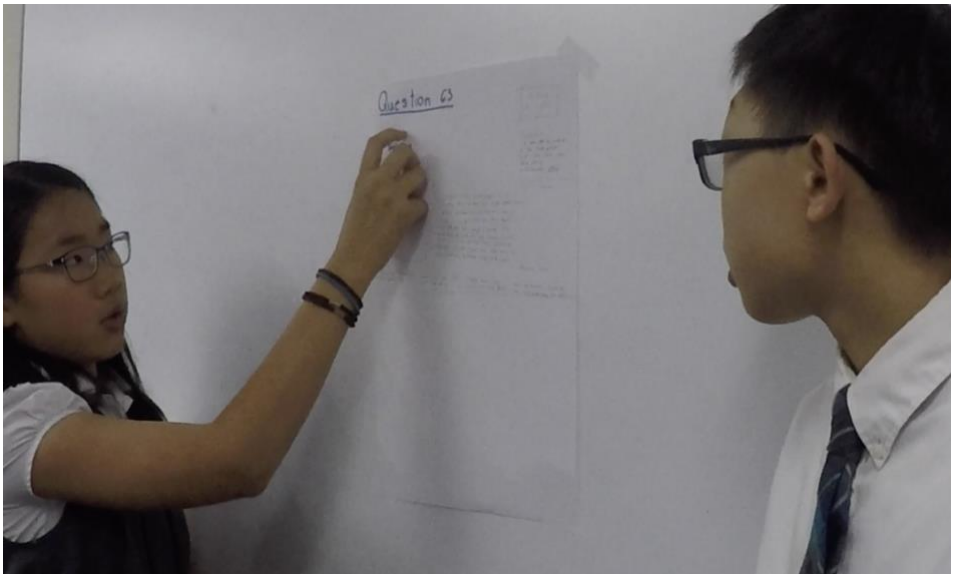
It was also interesting to note that some of the feedback requests were in the form of dialogue, where the students showed the interactive aspect of their feedback practices in this cycle. For instance, on one of the posters, the students had written, *"Only write 'I don't understand' if you still don't understand after we explain."* This comment, more than asking for feedback, is aimed at communicating to other students that they should ask for clarification and

explanation while the work is being presented orally and before feedback comments are written. Similarly, on another poster, one of the feedback requests was, *"Do you have other strategies?"* This feedback request is a signal for communication among students to discuss and learn different strategies. In this way, the feedback requests in this cycle demonstrate more possibilities for the conceptualization of feedback.

Once the posters were ready, the students presented their solutions, one group at a time, and answered other students' questions about their work (see Figure 16).

Figure 16

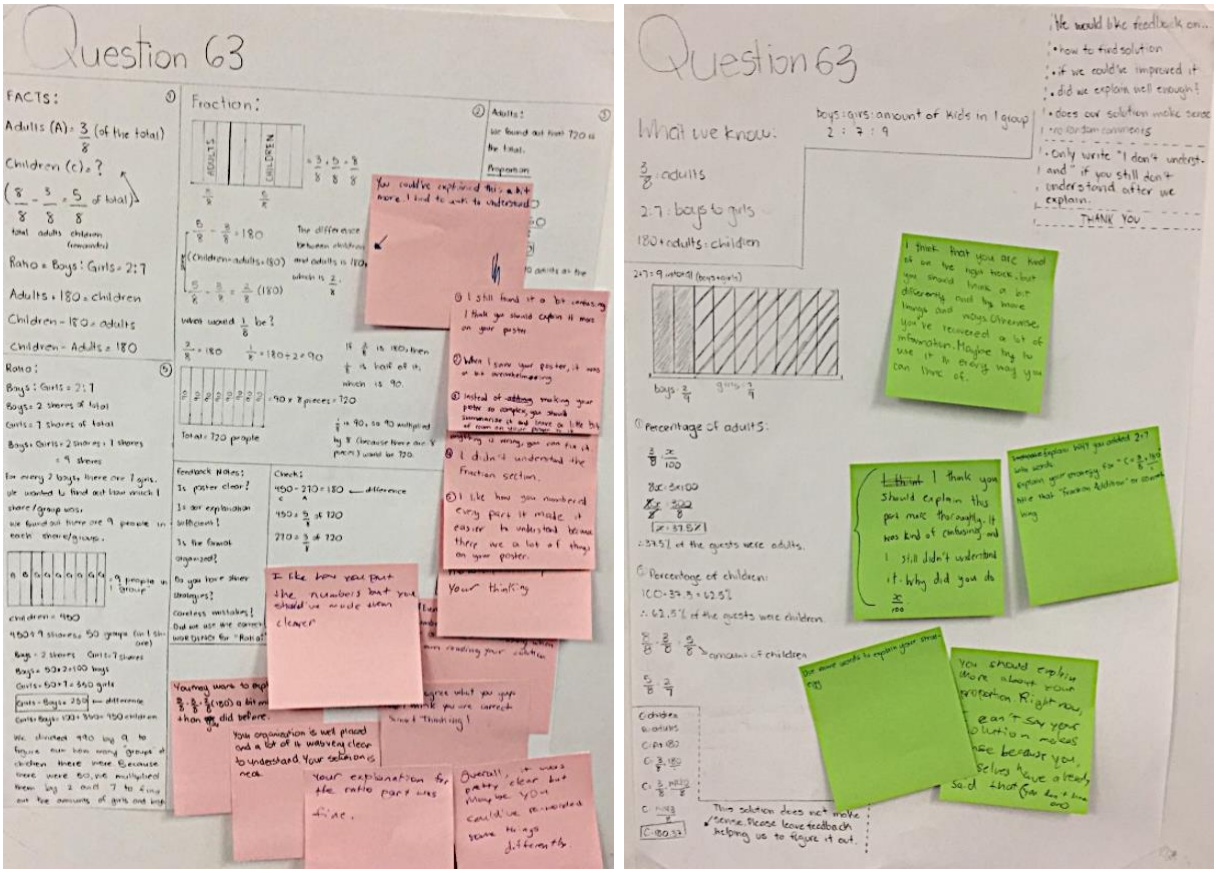
Presenting Solutions and Answering Questions



Other groups listened to the presentations, explored the work on the posters and asked questions before they wrote their feedback comments (see Figure 17).

Figure 17

Samples of Students Posters with Feedback Comments



After all the groups presented, and all the feedback comments were written, students returned to their posters, read the feedback comments they had been given and once again met with one another to discuss and clarify feedback comments before making corrections on their posters. Noticing that Rose and May were writing on their poster, I approached them and asked them about it.

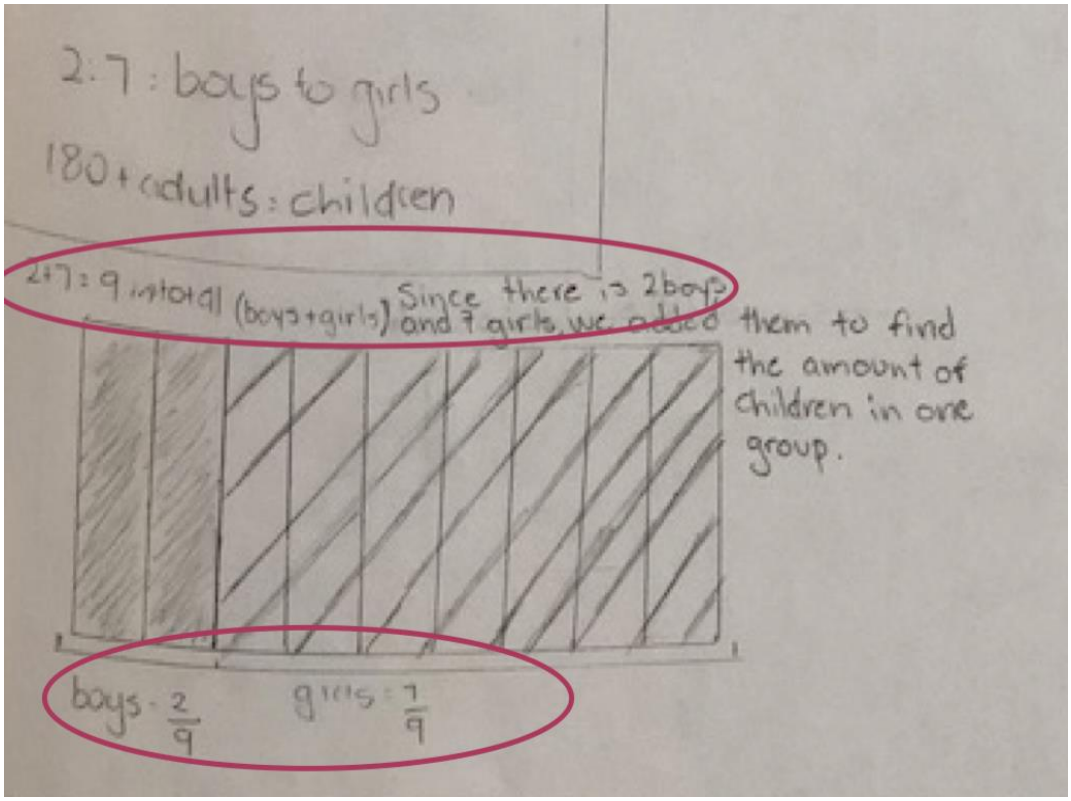
Parinaz: What are you doing?
Rose: We are writing 7+2 = 9 to show where we got 9 from.
Parinaz: Why?
Rose: Because some people did not understand where we got 7/9 and 2/9 from.

Rose explains, *"some people did not understand where we got 7/9 and 2/9 from."* This means that their peers did not understand how Rose and May used the ratio of boys:girls to write

ratio of boys to children and girls to children in fraction form. Rose and May show that they not only understand the feedback comment, but know how to act on it by writing 2 + 7 = 9. Rose also wrote a sentence on the poster to explain the fractions 2/9 and 7/9 (see Figure 18).

Figure 18

Adding an Addition Sentence in Response to Feedback Comments



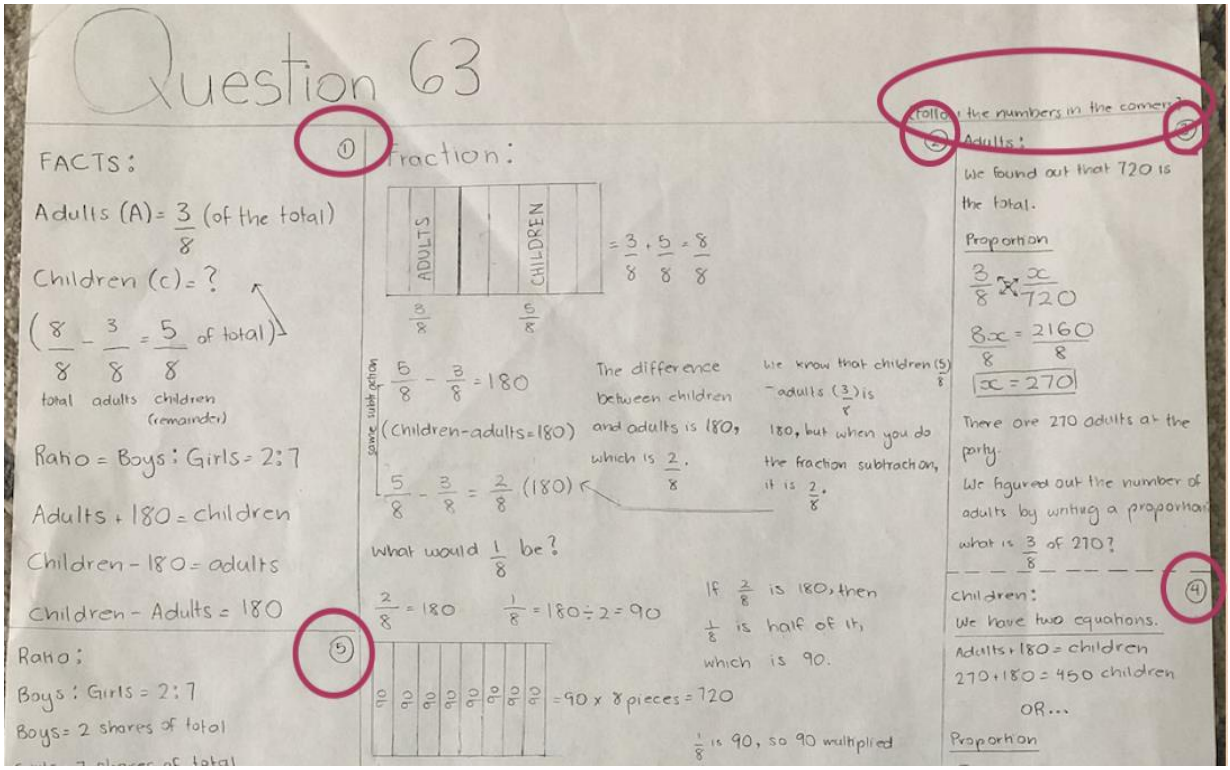
Sarah and Hanna also used the feedback comments they received to improve their work. Below, Sarah explains the changes they made to their poster.

Parinaz:	Did you make any changes to your poster?
Sarah:	Yes. We actually added this note to follow the numbers in the corners. And we re-worded some things and we also added a lot more explanation.
Parinaz:	Why?
Sarah:	Because we got a lot of feedback where we had to re-word things or there wasn't enough explanation or they missed the numbers in the corners, which made it confusing for them to follow.
Parinaz:	What numbers are those?
Sarah:	[Pointing to the numbers at the corner of several boxed sections on the poster.] These numbers show the order you have to follow the boxes.

It seems that feedback comments Sarah and her partner are using to improve their poster are related to how their solution was organized, communicated and presented. They needed to add more explanation, to re-word things and re-organize their poster so that their peers would be able to understand their thinking. So, they added numbers to help the reader go through steps and understand the logical progression of the work (see Figure 19).

Figure 19

Improving Organization and Communication



It is interesting that the feedback comments Sarah and Hanna are addressing are about organization and communication of their solution and those are the same types of feedback comments they requested on their poster. In other words, the types of feedback Sarah and Hanna requested were the types of feedback they received and they used the comments to improve their poster. In this way, Sarah and Hanna requested feedback, received it, and acted on it.

However, not all groups found the feedback comments on their posters helpful in correcting or improving their work. Below, John and Layla tell me why they could not take up the feedback comments they received.

John:	Some of the feedback is saying… like … *"We don't personally think it is correct."*
Parinaz:	Okay. Do you agree with the feedback?
Layla:	[pointing at one of the sticky notes.] Like they say, *"I believe that 180 is incorrect."* A lot of these say we should not have used 180.
Parinaz:	Okay. So, do you think 180 was right or wrong?
John:	Wrong.
Layla:	But we didn't know what to do.
Parinaz:	So, you're still not sure how to solve it?
Layla:	No.

Layla and John explain that while they understand what they have done is incorrect, they cannot use the feedback comments they have received in order to correct their solution or to find a new strategy. In other words, although they agree with the feedback comment, they cannot use it to make changes to their strategy. It seems, judging from the three groups' responses to their feedback comments, that students use the feedback comments they receive to make corrections on their posters, when they understand the feedback comment and also know how to act on it. For example, Rose and May knew that by writing an addition sentence they could explain their fractions. Similarly, Sarah and Hanna knew that they could improve the communication of their mathematical thinking by organizing, numbering, and re-wording their sentences. However, John and Layla were unable to use the feedback comments they received, not because they don't understand them or disagree with them, but perhaps because they remain unsure of how to solve the problem.

When the students went through all the feedback comments, they placed the sticky notes on the board and proceeded to categorize them. The following excerpt is part of their initial conversation about potential categories for this cycle.

Layla: We need an *"Explain."* A lot of these [pointing at the comments on the sticky notes,] are explain.
[Hanna writes the word Explain on the board.]
Hanna: Grab all the *"Explains."*
[John begins placing Post-it notes under the word Explain.]
May: I noticed a lot of people gave comments like, "Overall, it was great," or "You did well."
Layla: So, just comments.
May: Well, I think, not just comments, but …
John: Reassurance, right?
Sarah: So, we can have a *"Comments"* category, cause there's a bunch of comments. And then some are reassurance or praise. So, we can have "Comments" as a big category and all these smaller categories branching off of it.
[Layla writes the word Comments on the board.]
Layla: What about we leave the sticky notes on the board. We come up with the categories and then we see where they belong?
Hanna: But we need to read them first.
John: To find out what category they belong to…
Rose: I think we should have an *"Opinion"* category. It says, [reading one of the notes,] "I personally believe that 180 is incorrect."
Hanna: We should go through each sticky note and read them and categorize them and combine them.

Layla suggests having an *"Explain"* category because she notices that many of the comments on the sticky notes ask for explanation. Similarly, May talks about comments such as *"You did well,"* which, as John reminds them, belong to the *"Reassurance"* category they specified last time. Unlike the previous cycles, where they began with categories such as *"Constructive,"* and *"Non-constructive,"* and decided on which comments belonged to each category, this time, the students are reading the comments and specifying categories that best suit the feedback comments on the sticky notes. Hanna proposes a strategy for coding the comments. She invites others to *"go through each sticky note and read them and combine them."* According to this procedure, the categories they specify and describe are contingent on the comments on the sticky notes. Using this strategy, they continued to specify categories such as *"Explain,"* *"Reassurance,"* and *"Opinion."*

As they continued to read the comments, they proceeded to add two more categories, *"Advice and Suggestions,"* and *"Corrections,"* and then began to discuss their model for feedback.

Rose: We can put "*Feedback*" in the middle.
Sarah: Maybe we can do the same idea as last time, but not the same model. We have to have connections obviously.
Rose: The model can be like this. [She starts writing on the board in the space among the feedback categories.] So, something like this. [She writes the word *"Feedback"* on the board and draws lines branching out to each other categories.] Guys … we can do like this. We can do feedback in the middle and stuff branching off of it. It's like before, but it's not two separate things, it's all one.
Parinaz: What do you mean? Can you show it to us?
Rose: Yeah. [She moves to the other side of the board and draws the model as she speaks] So, it would be feedback in the middle, and then there will be … like … *"Explain"* and we can branch off of that. And then there could be *"Comments*," and then we could do *"Corrections,"* and then we have *"Advice."*

Sarah suggests using the same model as the one from the previous cycle. She explains, *"we have to have connections obviously,"* which suggests that she wants to keep their categories connected, like previous models, in order to accommodate for feedback comments that belong to more than one category. Rose, however, makes a significant distinction by writing the word *"feedback"* in the space among the categories and drawing lines from *"feedback"* to all the categories (see Figure 20).

Figure 20

Finding a Place for Feedback

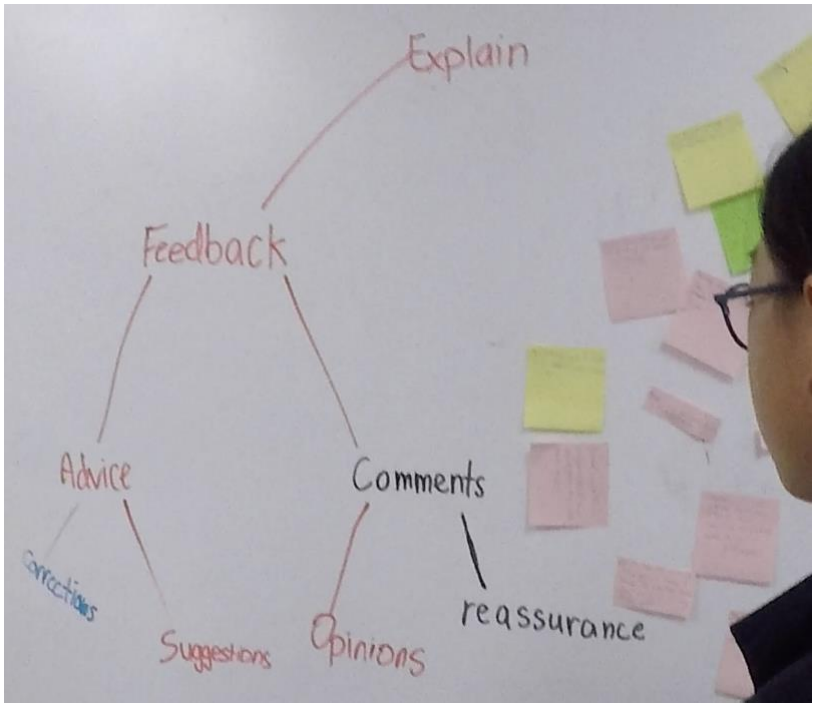
According to Rose, this model is a variation of the previous model, in that it incorporates the element of branching out and connections as the previous model also did. However, she makes an important distinction in this model by placing *"feedback"* in the center and stating that, "*they're all one. They're all feedback."* Making this distinction suggests that she now considers all the comments on the sticky notes as feedback. Triggered by Rose's suggestion, I prompt her to show us what she means. She draws the model again, this time, starting by writing the word *"feedback"* and drawing lines to show different branches that lead to different categories. She specifies four categories of *"Explain," "Corrections," "Comments,"* and *"Advice"* and then goes on to draw more lines coming out of those categories, suggesting that she recognizes more sub-categories emerging.

As they continued to discuss the categories, the model changed. The *"Corrections"* category was erased as a main category and was written along with *"Suggestions"* as sub-

categories of *"Advice."* They also specified *"Reassurance"* and *"Opinions"* as sub-categories of *"Comments"* (see Figure 21).

Figure 21

Specifying More Sub-categories



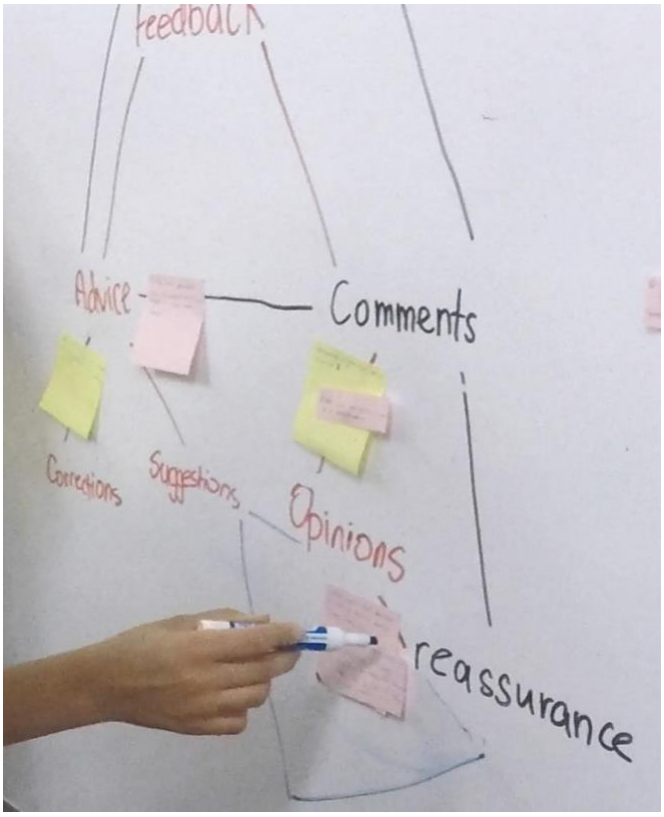
I joined their conversation, wondering if they meant to connect these categories and sub-categories together.

Parinaz:	I like this branching idea. But I have a question for you. Do they never meet? For instance, "Explain" and "Comments," could they ever meet?
Hanna:	Yeah. They do. [she draws a line connecting "Explain" and "Comments." Others place sticky notes on the lines.]
Parinaz:	So, you are going to connect them? All of them?
Hanna and Rose:	Yeah.
John:	It's kind of like the other model.
Rose:	Yeah. But they're all one. They're all feedback.
Parinaz:	Do you all think that they're all feedback?
John and Sarah:	Yeah.

My question as to whether these categories and sub-categories are connected prompts Hanna to draw connecting lines. Immediately, other students place sticky notes on the lines, where they think a feedback comment belongs to more than one category (see Figure 22). Although I did not directly tell them to do so, my question triggered the students to bring forth the element of connecting the sub-categories to one another. In this way, my coupling with the students influenced the co-emergence of the model as inter-connected categories and sub-categories.

Figure 22

Making Connections



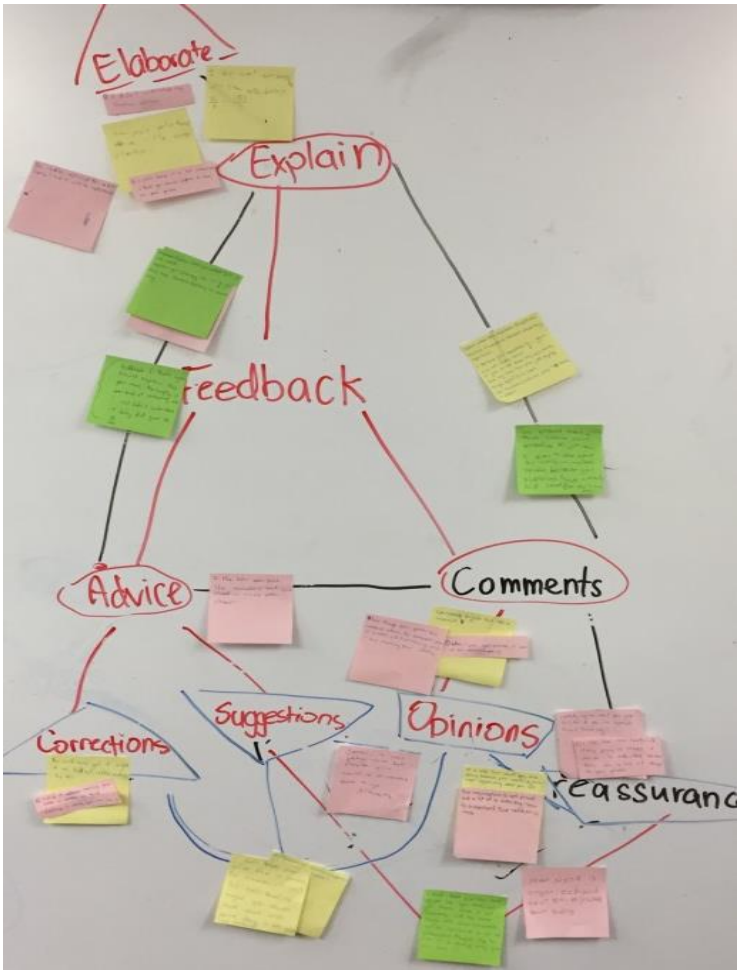
This model, like the previous two models, incorporates connections among categories. However, what is significant about it is that they wanted to connect all the categories and sub-categories together. As John and Rose explain, *"It's kind of like the other model,"* but *"they're*

all one. They're all feedback." In the previous model, they only connected categories if they found feedback comments belonging to them. If they did not have a feedback comment that belonged to two categories, they would not connect those two categories together. In this model, however, they wanted to connect all the categories together, even in the absence of feedback comments that belonged at a certain connection.

As they continued to read through the feedback comments, they specified *"Elaborate"* as a sub-category of *"Explain"* (see Figure 23).

Figure 23

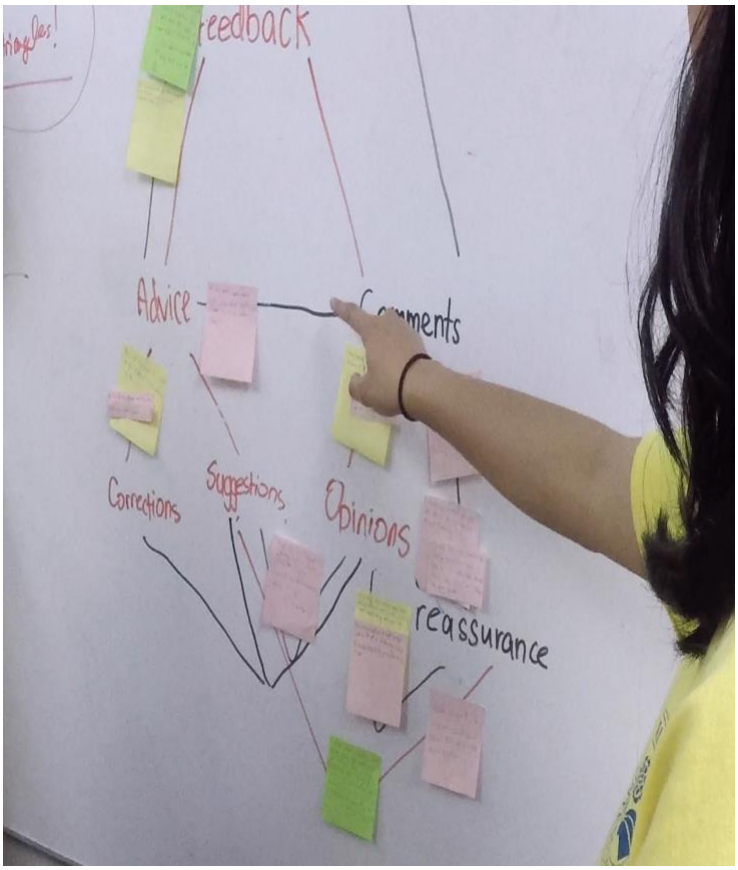
Adding "Elaborate" as a Sub-category



This meant that they now had more connections to make between this sub-category and all the other categories in the model. With the growing number of categories and sub-categories and the connecting lines amongst them all, it became increasingly difficult for the students to place their sticky notes on the connecting lines (see Figure 24).

Figure 24

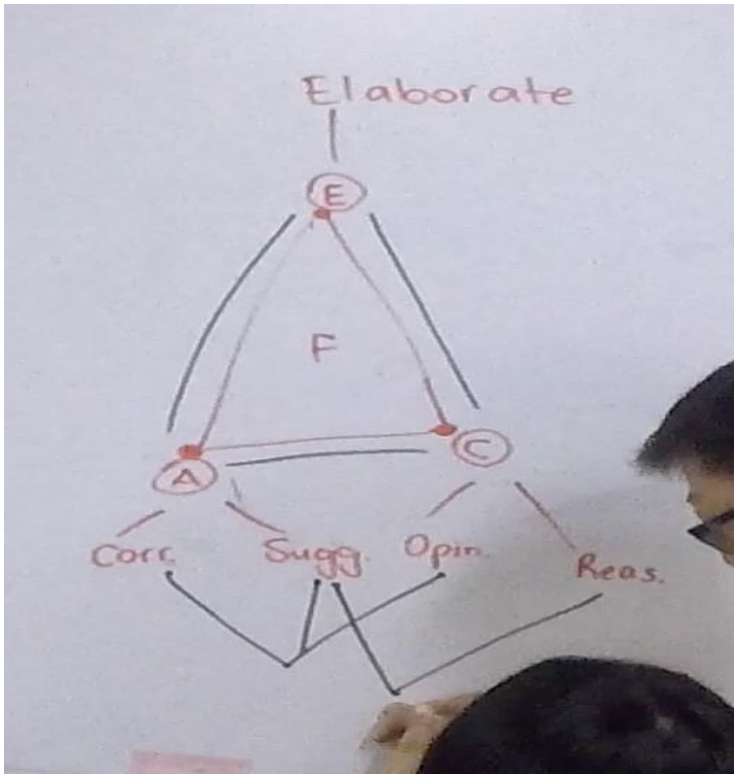
Making New Connections



In an attempt to find a way to connect all the categories together, the students drew the model anew on the other side of the board. They began by drawing a triangle and wrote the initials of the three main categories on the vertices of the triangle. They wrote *"F"* to represent feedback in the center of the triangle and drew branches out of the three vertices to represent the sub-categories (see Figure 25).

Figure 25

Struggling with the Connections.



When they tried to connect the different categories and sub-categories, the students stopped again, seemingly struggling to continue. I asked them why they had stopped.

John:	It's pretty confusing.
Parinaz:	What is confusing? Why is it confusing?
John:	It's very messy. There's tons of tiny little triangles everywhere.
Sarah:	It's very confusing.
Parinaz:	So, can someone explain the model please?
May:	So, um… the main categories are in circles. And the smaller branches are in triangles. We put the sticky notes in the bottom so we can attach them to other categories.
Sarah:	It's really confusing guys!

John describes the model as too confusing. According to him, *"there's tons of tiny, little triangles everywhere."* All these triangles were to be connected to one another and it seemed that they could not continue with this model as they found it too confusing to show all the

connections among the categories. This was significant as they either had to do without the connections or modify their model to accommodate these connections.

Hanna was the first to take an action. She erased the last drawing and instead drew seven circles, each representing one of the categories that they had specified earlier (see Figure 26).

Figure 26

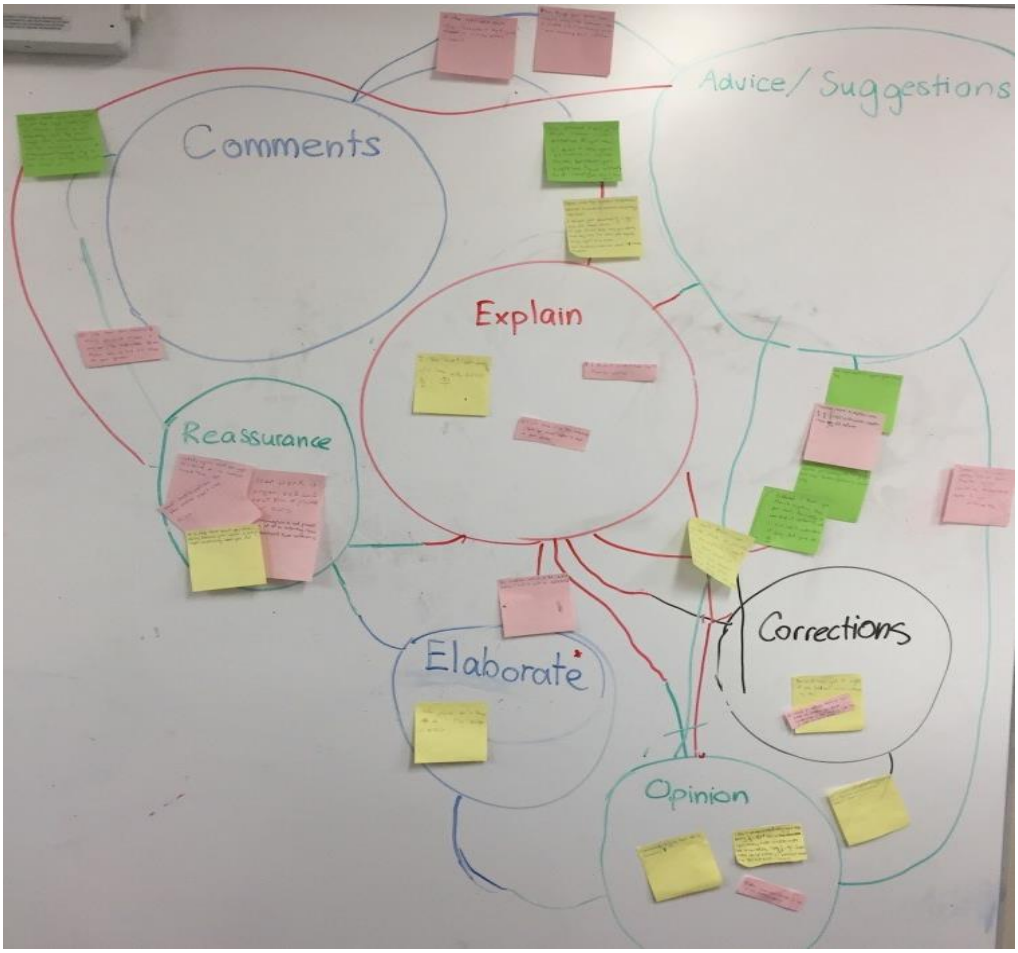
Replacing the Triangles with Circles

This time, there was no hierarchy of category versus sub-category. All seven categories were presented with similar looking circles and all the circles were connected to one another. In order to keep track of the connections among the categories, they decided to color-code the lines with their corresponding categories. For instance, the *"Reassurance"* category was drawn with a

green marker, so all the lines emerging from that category were green. In this way, each line was comprised of two colors, each color representing the category it emerged from (see Figure 27).

Figure 27

Color-coding the Connecting lines



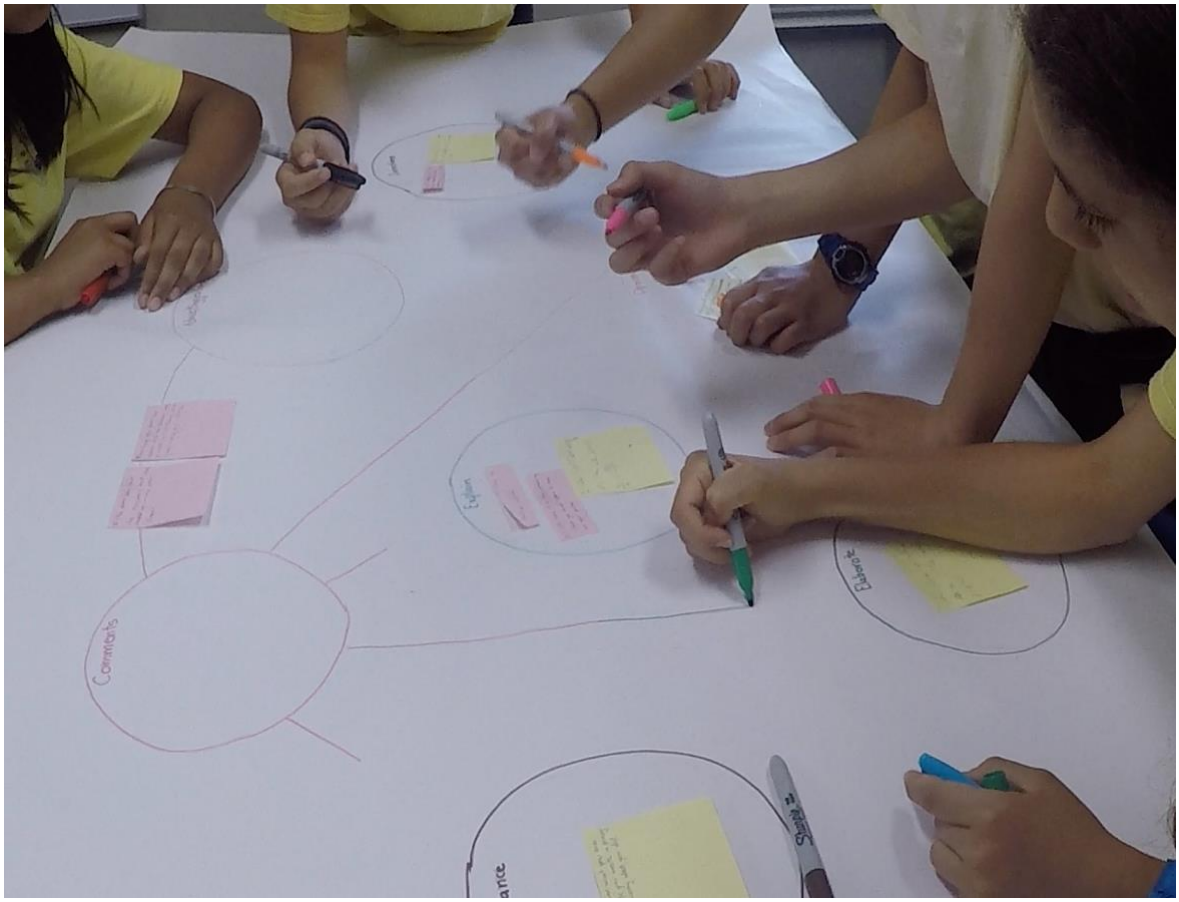
In this way, the students co-adapted in order to continue the process. In order to accommodate all the connections among the categories, the students changed the model, incorporating circles and color-coded connection lines. The new model co-emerged with the students.

Convinced that this model was the right one for this cycle, they drew their final model on paper (see Figure 28). Like they had done on the board, they used circles for categories and

color-coded lines for connections. They then transferred the sticky notes from the model on the board to the one on paper.

Figure 28

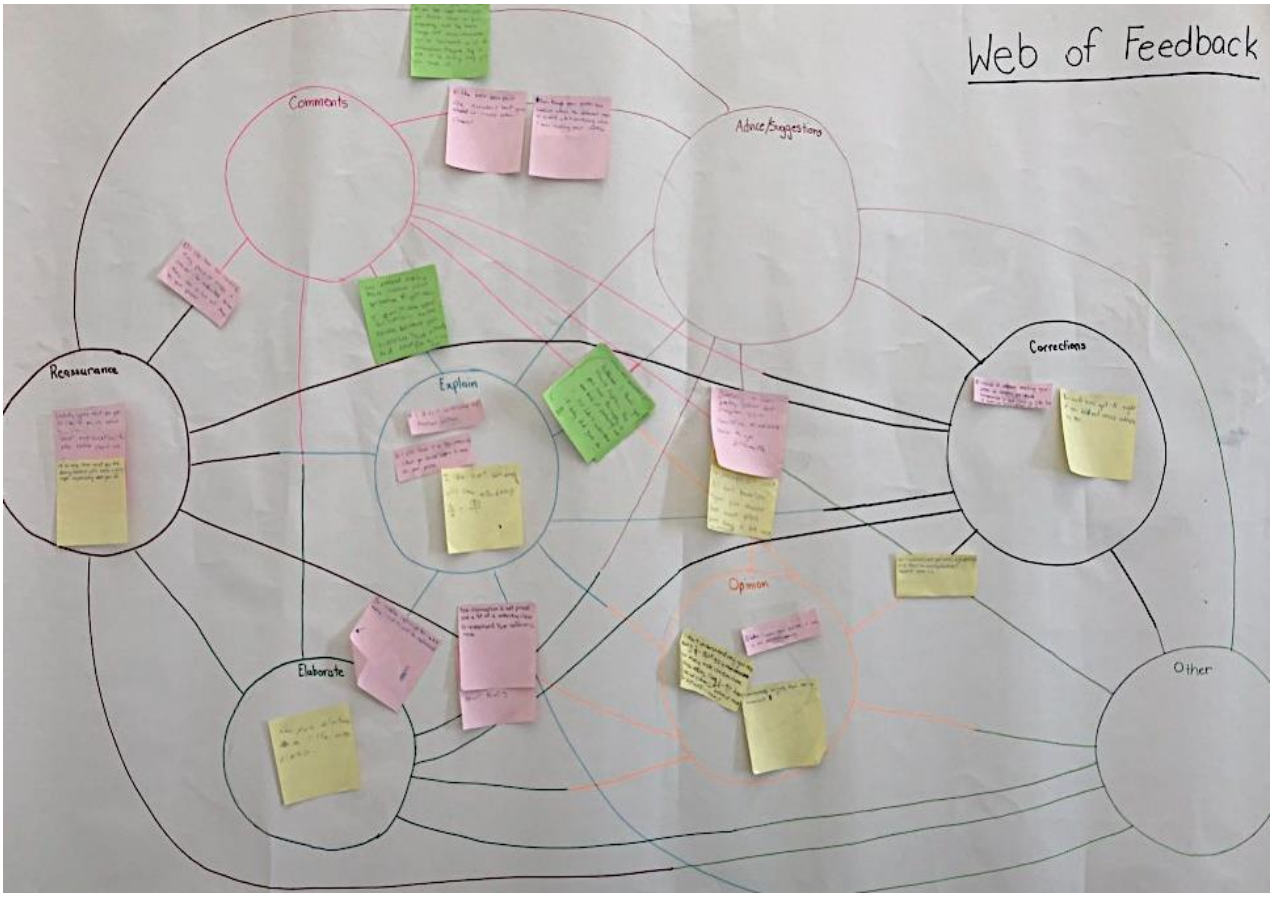
Co-constructing the Final Model



The final model on the paper was comprised of eight interconnected categories. The categories were *"Comments," "Explain," "Elaborate," "Advice/Suggestions," "Opinions," "Corrections," "Reassurance,"* and *"Other."* The *"Other"* category was not drawn earlier on the board and was only added as the students drew the model on paper. They called this model the "*Web of Feedback*" (see Figure 29).

Figure 29

Web of Feedback



In comparison to the previous two cycles, this model and its categories took the longest to construct as it underwent multiple transformations. In this model, each of the categories was connected to all the others. Even if there were no feedback comments that belonged to the intersection of two categories, the categories were still connected together. Interestingly, in this model, more than half of the sticky notes are placed on the points of intersection among categories as opposed to one category alone. Out of a total of 28 sticky notes, 16 sticky notes were deemed to belong to more than one category. Table 9 contains samples of some of the feedback comments placed at intersections of categories.

Table 9

Samples of Feedback Comments Placed on Intersections of two of more categories.

Categories	Feedback Comments
Explain & Comments	*"Explain what the numbers 810, 630 and 450 represent. Your equation 2/9 = 180/1 doesn't make sense."*
Advice/suggestions & Explain & Comments	*"I think you should explain this part more. It was kind of confusing. Why did you do x/100?"*
Corrections & Opinion	*"Did you write proportions? Shouldn't be fraction multiplication? I think it is confusing."*
Reassurance & Advice/suggestions	*"Overall it was pretty clear. Maybe you could have re-worded a few things. Maybe number the steps?"*

Once the model was complete, I asked the students to reflect on the changes they had made to the model and their reasons for doing so.

Sarah: In the beginning, it was triangles. It kept changing. It changed completely. It went from a triangle … and then we thought it was too … too… we couldn't really play with it. It was too like… rigid. But with this [pointing at the model] you can always add and it is so organic that it wasn't something you couldn't change and mess up the whole thing.

John: Yeah, because at first, we just had… like… we were going to … we were trying to go in the same direction as the one we did last time, but then… and then we started noticing triangular connections between them, so we went with that and then we realized that that wasn't good for connecting everything together. So, then we went with circles.

According to Sarah, the triangular model was too "*rigid*" and they "*couldn't really play with it,"* which implies they wanted a model that would allow more flexibility so "*you can always add*" more categories to it. Equally so, being able to make connections between the different categories was an important element for the students. They chose to use circles because

they believed it was a better model "*for connecting everything together*." As such, the *Web of Feedback* embodies their construal of feedback in this cycle, which is centered on interconnectivity and flexibility. As we continued to discuss the "*Web of Feedback"* and its elements, the students once again emphasized the importance of the connections between the categories in the model.

Layla:	I think it was very complex and it was very complicated. Because there were seven bubbles, and they all… everything connected.
Parinaz:	What do you mean everything connected?
Layla:	Like there are lines going from each bubble… connecting them.
Hanna:	I kind of like that it's kind of confusing and complex, because it shows that feedback does… like it has any… like it can be classified as anything and there's not really any limits to it.
Parinaz:	Who agrees with that? [They all raise their hands.]
Parinaz:	You all agree that feedback can be anything? They nod.
Parinaz:	And … and… what did you say? [addressing Hanna]
Hanna:	That there aren't any limits to what the combinations may be cause it's connected to everything.

The students describe the model as complex, in that it has several categories and that these categories are all interconnected. The students make two significant distinctions about feedback in this conversation. The first distinction is that feedback comments *"can be classified as anything."* This notion signifies a much more open and inclusive understanding of feedback, no longer limited to the binary notion that was used in the previous cycles. This conception of feedback is also embodied in their model.

In this model, all the categories are connected to one another and no hierarchy exists among them as there are no longer any sub-categories. This inclusive view of feedback comments signifies a great shift in the way feedback is construed and valued across the cycles. In the first model, *"vague"* comments were not considered feedback at first and later, although they

were included as feedback, they were not granted equal status as the other categories. In the second model, there was a clear binary where *"constructive"* feedback comments were contrasted against the *"other"* category, thereby rendering the latter less value than the former. However, in the third model, feedback is more inclusive in its categories and all categories are given equal status.

The second distinction is that feedback is now understood as a complex web of interconnected elements, with no limits to the categories and their connections. Almost from the beginning of this project, students noticed that many feedback comments belonged to more than one category and, as such, designed their models in a way as to allow for connections between categories. The intersections between the circles in the first model and the lines branching out and intersecting with other lines in the second model were utilized to allow connections between the categories. In the third model, these connections manifested themselves as a complex web of interconnected circles. In this way, students' understanding of feedback categories and the relationship among these categories were embodied in a model that allows for more interconnectivity and more freedom to choose.

The notion of feedback being many different things and the importance of connecting categories together was visited in the earlier cycles. However, in the third cycle, students included these elements as integral parts of their model. They even went as far as changing their model from rigid triangles to floating circles in order to make certain that the categories were all interconnected. In this way, their history of co-constructing feedback models in the previous cycles influenced their choices. Below, May and Sam discuss the evolution of the models throughout the cycles and how their experience with the previous models led them to construct the third model.

May: In the second round, somebody came up with the idea of doing trees, kind of branching out. So, I think, that actually added onto what we did for this model [pointing at the web]. Because we kind of got this idea from that other idea.

Sam: I think this one is the best. Because for this one we used both ideas. We used the first one and the second one. We used the circles and we used the lines. To make two connected together to make one, the best one.

May: I think it's probably the best one we've had yet and although it's a bit complex, if you look at it carefully, you can follow the lines and see where everything belongs and where everything is going. I think that it was a good idea and it worked out really really well.

Sam articulates how their experiences with the first two models affected their choices for the third model. The third model incorporates circles and intersecting lines that branch out of each circle and connect it to all the other circles. Sam believes that their choice for circles was influenced by the first model, and that the branches that come out of each circle resemble the ones used in the second model. The third model, therefore, incorporates elements from the previous two models, but has evolved as the students' understanding of feedback has evolved.

May emphasizes the temporality of the model, stating that it is the *"best one we've had yet,"* signifying the potential for different, and possibly better, models in the future. She further explains this temporality as she explains the contingency of feedback model on the feedback comments.

May: You have to kind of base your feedback model depending on the feedback you actually got and the categories. And if you agreed on it.

Parinaz: So, you couldn't plan it ahead of time?

May: No. We tried doing that and it didn't work out.

As May explains, the feedback model depends on the feedback comments they get, the categories they specify, and more importantly, on their agreement. This means that every model is contingent on the distinctions students make about feedback in each cycle and their agreement on the structure of the model. As such, models are temporary and cannot be planned before the

cycle begins. By stating that in every cycle, the model is contingent on the elements of that cycle, the students acknowledge and embrace the possibility of change.

Students' openness to change is embodied in the *Web of Feedback* as they incorporate the *"Other"* category. As previously stated, the *"Other"* category was added at the final stages of constructing the *"Web of Feedback."* When I first saw the *"Other"* category added on the model, I assumed it resembled the *"Other"* categories in the previous two models. However, as we discussed the model and the connections, Sarah explained what the *"Other"* category meant to them in this model.

Sarah:	I really liked how we could connect [these categories] to everything. Because even though we didn't have feedback that could go in certain combinations, we still added them for next time to refer back to it if it could be like different combinations and that's why we put the "other" … the bubble of "other" because there will be feedback that we don't know which category it is.
Parinaz:	So, this feedback category called "other" is for future categories?
Sarah:	Yes.
Parinaz:	So, we can have more categories?
Sarah:	Yes.
Parinaz:	So, this "other" category represents…
John:	Feedback types that we have not seen yet.
Parinaz:	You haven't seen yet?
	[They nod.]
Parinaz:	But you think it would probably come up?
	They nod again.
Parinaz:	And so, every bubble you add, you're going to add as many connections to the rest of the bubbles?
All:	Yes.
May:	I think also with this diagram, we have the freedom to do whatever we want.

Sarah explains that they added the *"Other"* bubble or category because, *"there will be feedback that we don't know which category it is."* According to John, the *"Other"* category represents *"feedback types that we have not seen yet."* Their explanation suggests that they understand feedback categories can change and that there can always be more categories in future cycles. The addition of the *"other"* category is an important shift from the previous

models, as it allows them the opportunity and the *"freedom"* to add more categories in the future. The *"Other"* category, thus, highlights the temporality of the model and their categories. Feedback categories and models are now understood as fluid and open to change. By incorporating the *"Other"* category, the students signal their understanding that new categories are bound to emerge as they continue to give and receive feedback in future cycles.

Chapter 6: Research Findings

According to Brown (2015), "the find-ing(s) of an enactivist research shed light onto the journeys that are travelled" (p. 194). The goal of this study was to explore how my students co-construct their peer feedback practices. As such, the findings of this study shed light onto the journey of co-construction and the feedback practices that were co-constructed. In this research, I asked the following three questions:

1. How were feedback categories and models co-constructed?
2. How were feedback actions co-constructed?
3. How did conceptualizations of feedback and its effectiveness evolve throughout the process?

In the previous chapter, I shared narratives of how the group enacted feedback across the three cycles. For each cycle, I described and analyzed the distinctions that were made about feedback comments, the categories that were brought forth and specified, the models of feedback that were co-constructed, and the changes that were made to feedback actions for future cycles.

In the following three sections, I will address each of the research questions. I will discuss how the process of co-construction was one of co-adaptation and co-evolution, wherein we structurally coupled and brought forth new possibilities for feedback categories and models.

I will then attend to the second question and will discuss how the co-construction of feedback practices became a process of transformation, wherein the group engaged in action and critical reflection and grew more aware of one another's roles, and how the process of co-construction was one of collaboration and compromise. Finally, I will address the third question by discussing the parallels that emerged between the process of co-construction and the co-constructed peer feedback practices and how the groups' conceptualization of effective feedback co-emerged with the relationships that were co-constructed across the three cycles.

How were Feedback Categories and Models Co-constructed?

The analysis of the three cycles of enacting feedback revealed that different forms of interaction within and about feedback resulted in many layers of structural coupling among different systems in our classroom. Our coupling with one another about feedback occasioned for us to co-adapt and co-evolve, to bring forth and specify new possibilities for feedback categories and models, and to co-construct a history of structural coupling.

Co-construction as a Process of Co-adapting and Co-evolving

According to enactivism, when an autopoietic unity and its environment interact, they trigger structural changes in each other. When these interactions become recurrent, there will be structural coupling between them, as a result of which, both the unity and the environment will co-emerge (Maturana & Varela, 1987). This research was designed in a way as to provide the group with recurrent interactions with one another in a linguistic domain. Across the three cycles, the group discussed different types of feedback comments, negotiated category names, and debated about various elements in the models.

As a result of the groups' recurrent coupling in the linguistic domain, feedback categories underwent several changes. At times, our interactions served as triggers for the students to re-

consider and rename category names. For instance, in the first cycle, when I asked the students about the *"Random"* category, my question triggered the students to try to describe their category. However, in the process of describing this category, they decided that *"Vague"* was a more suitable description than *"Random."* In this way, the new category name emerged as a result of my question and the conversation that ensued. Similarly, in the second cycle, the *"Silly"* category was brought forth and specified by the students. However, when I asked them whether the comments in the "*Silly*" category were relevant to the task, they immediately changed the category name from *"Silly"* to *"Irrelevant".* In this particular case, not only was my question a trigger for the students to change the name, the specific wording of my question directly influenced their choice for the new name.

At times, our interactions triggered the group to specify a new category and to re-visit and re-evaluate their conceptualization of feedback. For instance, during the second cycle, I was triggered by May's insistence to specify a new category of feedback comments that didn't belong to the *"Reason"* category, but was not *"Vague"* either. I responded to this trigger by asking her and others to consider the possibility of another category of comments. The conversations that followed my prompt occasioned the group to re-visit how they had previously conceptualized feedback, to re-negotiate their categories, and to bring forth and specify a new category. This category, which they called *"Reassurance,"* was described as comments that did not necessarily give advice, but instead reassured students that they were on the right track. The *"Reassurance"* category was not only significant because it was a new category, but because it co-emerged as a result of the group's coupling within a linguistic domain. According to enactivism, "linguistic domains arise as a cultural drift in a social system, with no pre-established design" (Maturana & Varela, 1987, p. 208). In this example, May's initial insistence to distinguish a new category, my

questions and prompts, and the group's subsequent conversations about their conceptions of feedback constituted a shared world, a linguistic domain, in which the group drifted and brought forth the *"Reassurance"* category.

Bringing forth and specifying the *"Reassurance"* category also resulted in a broader, more inclusive, description for feedback. When describing feedback, Sarah explained, *"It's not just one thing. It's more. It like helps you improve your work, and it's what others think of how you did your work."* Sarah conceptualizes feedback as having more than one function; it can help students improve their work and it can also reassure students. As such, the conversations, the recurrent coupling in a linguistic domain, occasioned for the group to re-think their category names and to re-evaluate how they described feedback.

Bringing Forth more Possibilities

As the cycles progressed, the group brought forth more possibilities for classifying feedback comments and new ways of connecting these categories in a model. In the first cycle, there were only three categories and feedback comments were described as either constructive or vague. *"Constructive"* feedback comments were described as *"very beneficial"* and "*more specific*" in "*what you can do to make it better,"* and "*Vague*" comments were described as *"things that are confusing,"* and *"don't give you direct instruction or they don't give you direct advice."*

In the second cycle, students brought forth new feedback categories. The model for this cycle, by virtue of incorporating six subcategories, accommodated more flexibility to connect different types of feedback. The model was also more open and inclusive in that it expanded on the two main categories of *"Constructive"* and *"Vague,"* thereby offering a spectrum of feedback types within each of the two main categories.

In the third cycle, more distinctions were made and more categories were brought forth and specified. The model incorporated eight categories, represented by intersecting circles with no hierarchy among them. This model featured more freedom and flexibility than the previous two models. Feedback comments could be placed in a variety of locations, belonging to one or combination of categories. The third model also incorporated an *"Other" category*, which as Hanna explained, *meant that "feedback can be classified as anything and there's not really any limits to it."*

The growing number of categories and the increasing complexity of the models across the cycles suggest that in each cycle students were able to make new distinctions and to bring forth new ways of co-constructing a model. It was not that the feedback comments were changing or growing in variation. Rather, it was because the group had co-evolved along with the process and were able to see and bring forth new and different possibilities for classifying and organizing feedback comments. In other words, the group's recurrent coupling changed their structures and their structures influenced the new worlds of significance they brought forth. This recurrent process of co-adapting, co-evolving, and bringing forth new possibilities for feedback categories and models shaped and was shaped by the group's history of structural coupling.

The Group's History of Structural Coupling

Maturana and Varela (1987) posit that when living systems are perturbed by the environment, they co-adapt and bring forth worlds that are significant to them. These worlds of significance are brought forth and specified according to their history of structural coupling with their environment. In this study, the group's history of coupling in each cycle influenced the distinctions that were made and the worlds that were brought forth in cycles that followed. For instance, the designs for the feedback models in the second and third cycle, although emergent

and contingent on conversations and categories in each cycle, were also influenced by the group's history of coupling from previous cycles. The third model was a combination of the ideas used in the previous two models. During one of our conversations at the end of the third cycle, Sam said, "*We used the circles and we used the lines. To make two connected together to make one, the best one."* Sam describes how the design of the third model was influenced by the previous two models. Circles were borrowed from the first model to represent categories, and lines were borrowed from the second model to represent connections among the categories. Similarly, the importance of connecting the categories in the models was a theme that emerged during the first cycle but grew stronger and more prevalent across the cycles. In this way, the group's history of coupling with one another during previous models influenced the decisions made when co-constructing the subsequent models.

How were Feedback Practices Co-constructed?

The goal of this research was for my students to co-construct their peer feedback practices. For the first cycle, I provided the students with certain steps to follow. However, upon the completion of the first cycle, the students brought forth concerns regarding the procedure and decided on new strategies to enhance their feedback actions for the second cycle. Similarly, at the end of the second cycle, the students made new changes to their feedback actions for the third cycle. The co-construction of feedback actions was a transformative process of growing awareness, collaboration and compromise.

Co-construction as a Process of Transformation

At the end of each cycle, my students and I discussed their feedback actions in that cycle. These conversations occasioned the students to examine their actions, to reflect on strengths and weaknesses of those actions, and to bring forth new and more effective ways of enacting

feedback in future cycles. They then tested those actions to see whether they had proved effective. This circular process of feedback actions and reflections became a medium for critical reflection (Freire, 1970) and transformation. For instance, upon the completion of the first cycle, one of the first changes the students suggested for the second cycle was to allow one student to remain with the poster to answer questions and/or explain his/her strategy to those who visit the poster. It was their belief that during the first cycle, due to lack of dialogue, many feedback comments were simply asking for more clarification or explanation of the strategies on the posters. This, they claimed, could easily be resolved if they had a chance to speak before feedback comments were written on sticky notes. Furthermore, it was suggested that more interaction should be encouraged between students after feedback comments were written. More dialogue, in their opinion, would clarify the written comments and would help them decide how to proceed. In this way, students' history of enacting feedback in the first cycle influenced the changes they brought forth in how they wanted to enact feedback in the second cycle. They believed that adding more interaction before and after feedback comments were written on the sticky notes had the potential to influence the content of feedback comments and how they were taken up. They were convinced that if they had a chance to meet, answer questions and discuss their strategies on their posters, feedback comments would more likely address the correctness and/or quality of the mathematical thinking they had just discussed. Their move to add this step is in line with Hattie and Timperley (2007) who claim, feedback is "most powerful when it addresses faulty interpretations, not a total lack of understanding" (p. 82).

The notion of adding more interaction in feedback actions was further developed for the third cycle, when students decided on having groups present their solution and answer questions before feedback comments were written on the sticky notes. This, they believed, would eliminate

the need for explaining a strategy multiple times or being asked the same question from different people. Moreover, in cycles two and three, students decided to write down what they wanted feedback on, thereby asking for specific feedback comments. These requests, as May explains in one of the discussion sessions, served as another mode of communication, albeit indirect, to *"[get] rid of a lot of feedback that was irrelevant"* and to *"focus on what really needed to be done, not silly things."* Thus, through cycles of action and critical reflection, the group transformed their feedback actions by incorporating more purposeful and effective actions.

The Group's Growing Awareness

Cycles of feedback actions and reflections occasioned the group to grow more aware of the different elements of the process and of how these elements related to one another. They became cognizant that every cycle, every feedback action, was potent with new possibilities. They grew more aware of the influence of feedback actions on feedback comments, of feedback comments over feedback categories, and of feedback categories over feedback models. For instance, the students became aware that by writing specific requests for feedback, they could influence the types of feedback comments they would receive. They also became aware that their models changed in each cycle and that those changes were influenced by their feedback actions in that cycle. The group also became mindful of their changing conceptions of feedback and of their increasing openness to change. In other words, as the students progressed through the cycles, they began to "see the world not as a static reality, but as a reality in process, in transformation" (Freire, 1970, p. 83).

The group's growing awareness of their role and influence over the process is at the heart of how this process became a transformative one. It constituted them as active participants of the process, not only because they were engaging in feedback actions, but also because they were

mindful of their actions and the consequences of those actions. Freire (1974) posits, "only as [people] grasp the themes can they intervene in reality instead of mere onlookers" (p. 5). In the case of this research, as the students grew more aware of the different elements of the process and of their influence over these elements, they were able to bring forth more changes that seemed effective to them. For instance, in every cycle, feedback actions involved more interaction and more active participation on behalf of the students. These changes were brought forth and enacted because the students believed by doing so, they could improve the quality and effectiveness of the feedback comments. This awareness of the degree to which their involvement and their actions could influence their feedback practices transformed them from mindless spectators to active and mindful decision makers.

Co-construction as a Process of Collaboration and Compromise

Throughout the cycles of action and reflection the students also grew more aware of the nature of their relationships and how each one of them influenced the process. During one of our final reflections, May demonstrated this awareness as she said, "*We worked together like a team. And everybody has a part and everybody knows that they contributed to this great thing and that they all worked hard to do this."* May describes the group as a team, whose members not only contribute to the process, but are also aware of their contribution. Later, during the same conversation, Layla also drew our attention to this awareness by stating, *"I like how we all worked together, because if one person wasn't there, then it wouldn't be the same. Out of us eight, two people have already left, and there has been a slight change."* Sarah echoed Layla's observation by saying, *"I didn't know that working together would be this beneficial. Because … as she [Layla] said, without just one person, it would have been totally different. I didn't know how well we could collaborate and compromise and make our ideas work together."* Layla

characterizes their acts of co-construction as a process for collaboration and compromise, wherein they worked together as a team, with each person having an influence. This collaboration and compromise was evident in the way the students listened to one another's suggestions and the way they supported one another's ideas as they co-constructed their feedback practices in each cycle.

The sentiment of collaboration and compromise was not limited to the students; it also grew more pronounced in my relationship with my students. Freire (1970) reminds us that "education must begin with the solution of the teacher-student contradiction, by reconciling the poles of the contradiction so that both are simultaneously teachers *and* students" (p. 73). As the cycles progressed, so did my interaction with my students. I listened more attentively and asked more questions. I became more attuned to how they described their feedback actions, the changes they brought forth, and their reasons for doing so. I was learning along with them, and although we were at times triggered by different perturbations or responded differently, I shared the same goal as the group: to enact more meaningful and effective peer feedback practices among students. I also grew more mindful of how the group was co-emerging as a collective and how important it was to include everyone in the process. That is why in group conversations, I asked questions such as *"Do you agree?", "Are you okay with that?", "Who gets to decide?"* With these questions, I sought not only to understand the students' positions and ideas, but to also draw students' attention to, and awareness of, others.

The process of co-construction was transformative in that the students began to understand the significance of their role as well as that of others. They valued others' influence over the process and saw themselves as inseparable from others in their acts of co-construction.

In short, they were able to see themselves and others as "inescapably transitory and fragmented" (Varela, 1999, p. 65).

How Did Conceptualizations of Feedback and its Effectiveness Evolve Throughout the Process?

In enactivism, there is not a single, pre-existing 'truth', independent of the knower. Therefore, in this research I was not in search of a single definition for feedback as though feedback existed independently of those who enact it. Rather, I hoped to come to know feedback as enacted, described and co-constructed by the group during the different cycles of feedback actions. The group's conceptualization of feedback underwent multiple changes across the three cycles. Even within a single cycle, feedback was described and enacted in different ways. As the students co-constructed their feedback practices across the cycles, parallels emerged between the process of co-construction and the conceptualization of feedback and its effectiveness.

Emerging Parallels Between Process and Product

As the students progressed through the cycles, it became increasingly apparent that their conceptualization of feedback was closely related to the process in which they were co-constructing their feedback practices. In other words, parallels emerged between the types of feedback actions they co-constructed and the actions they took in the process of co-constructions. This meant that the elements that characterized the process of co-constructing feedback also characterized the feedback actions they co-constructed.

Co-construction of feedback was a process in that it spanned across three cycles, each cycle containing different elements. Throughout the cycles, each of these elements, such as categories and models and feedback actions, were re-visited and re-constructed. As such, every cycle was different from, but influenced by, the previous ones. What was important in each cycle was to enact feedback and to reflect on conceptualization of feedback. The focus was not placed

on producing a final version of feedback, rather on co-constructing a sketch, albeit a temporary one, for feedback during that cycle. This focus on process as opposed to product also became evident in how feedback was conceptualized. In other words, students' conceptualization of feedback evolved from feedback as mere comments on sticky notes to feedback as a process of interaction.

Furthermore, the characteristics that describe the process of co-constructing feedback practices also became the way those very practices were described. For instance, the process of co-construction of feedback was one of action and dialogue. Students enacted feedback, discussed their actions and decided on new ways of enacting feedback. Interestingly, action and interaction were amongst the very same characteristics that were used to describe their feedback practices. For instance, during one of our conversations about feedback at the end of the first cycle, as the students added more participation and dialogue to their feedback actions for the second cycle, they described feedback in terms of 'going back' and explained that feedback involves going back to one's peers to ask for clarification on the feedback comments on their posters, and it also involves going back to one's work and using those comments to make corrections. Both of these acts of 'going back' describe feedback as a process that involves people engaged in action and interaction. In this way, their feedback actions became similar to the actions taken in the co-construction of those feedback actions. Feedback was brought forth as a process of asking, discussing, going back to clarify, and, when possible, going back to correct one's work.

The incorporation of more opportunities for interaction, which emerged through a process of interaction, signifies that students construed feedback as a two-way process, wherein peers interact with one another and influence one another as well as the feedback comments on the

sticky notes. Whether it was answering questions about the work on the poster before feedback comments were provided, or asking for clarification about a feedback comment, these added steps engaged the students in a dialogue through which they structurally coupled with one another and with feedback, bringing forth new sets of distinctions as a result. Such conceptualization of feedback is in line with recent literature in which feedback is described as dialogical, as a contingent two-way process (Nicol, 2010; Askew & Lodge, 2000; Yang & Carless, 2013), and as social and relational processes (Yang & Carless, 2013; Gravett &Peterson, 2002).

Another important parallel that can be traced between the process of co-constructing feedback and the feedback practices that were co-constructed is that both processes were open to change. The goal of this study was to explore how students enact feedback in our mathematics classroom. However, in every cycle, students were given the chance to bring forth changes to their feedback actions. As students' feedback actions changed and as their conceptions of feedback evolved, so did their feedback categories and models. Their actions and reflections occasioned them to bring forth new ways of classifying and enacting feedback. Not surprisingly, the same openness to change, which characterizes the process of co-construction, was brought forth as a characteristic of their feedback actions and models. Their final model for feedback embodied this openness to change. By virtue of incorporating the "other" category, they opened themselves to new possibilities for feedback categories in the future.

Co-constructing Relationships

Throughout the process of co-constructing feedback practices, students' conceptualization of effective feedback evolved. When we first began the process, the effectiveness of feedback was judged based on the contents of the feedback comments. The more

specific the feedback comment was on the mathematical task, the more effective it was presumed to be. However, as we progressed through the cycles, their conceptualization of the effectiveness of feedback extended far beyond what was written on the sticky notes. Rather, the effectiveness of feedback was understood to be greatly influenced by the actions taken and by the people who take those actions. During one of our conversations about feedback, when I asked my students what makes a feedback comment "good" or "bad"? Hanna replied, *"It kind of depends on the person who receives the feedback, because it depends on how they use it."* She went on to explain that if the person uses the feedback to fix or improve their work, then the feedback is helpful. If the person does not use it to fix their work, it is not helpful. Clearly, Hanna saw the effectiveness of feedback reliant on the actions one takes in regards to feedback, such as taking up and acting on the feedback. Hanna's comment also implies that one's actions in regards to feedback are heavily reliant on how one "*receives the feedback*," and are therefore influenced by one's relationship with feedback. During the same conversation, Sam argued that *"every single feedback is good,"* and went on to explain that what makes feedback seem good or bad is *"something that people take in their perspective."* In other words, one's perspective about, or relationship with, feedback influences what one does with it and ultimately influences the effectiveness of feedback. In this way, as the group co-constructed their feedback practices, they were also able to co-construct effective relationships with feedback.

Chapter 7: Discussion and Conclusion

This research was a study of peer feedback practices in our mathematics classroom and was guided by enactivism and the democratic view of education, where the latter emphasized the importance of the active participation of all the classroom members and the former constituted knowing as bringing forth worlds of significance. As the group participated in cycles of co-constructing their peer feedback practices, different feedback categories and models were brought forth and feedback actions were changed in order to make the process more effective and meaningful. In the previous chapter, I addressed the research questions and discussed how feedback actions, categories and models co-evolved and how the group's conceptualization of feedback changed throughout the process. In this chapter, I will discuss how the findings of this study contribute to literature on feedback, focusing on the specific aspects of knowing as effective actions, co-constructing effective relationships with/in feedback, and cultivating climates of 'authentic caring.' I will also consider this study's implications and potential for future research as well as its limitations.

Contributions to Existing Research on Feedback

Feedback is considered to be one of the most influential factors in educational success (Hattie & Timperley, 2007). However, not all feedback is effective and many factors influence the effectiveness of feedback (Chappuis; 2012). This research, by virtue of adopting an enactivist lens to study students' co-construction of their peer feedback practices, offers unique and

significant contributions to existing research about feedback and its effectiveness. Specifically, this research describes knowing feedback in terms of effective feedback actions, highlights the significance of developing effective relationships with/in feedback, and draws our attention to the ethical implications of classroom climates that support effective actions and relationships.

Knowing as Effective Feedback Actions

One of the significant contributions of this research is in the way it uses enactivism to describe what it means to know effective feedback. According to enactivism, "Knowing is effective action, that is operating effectively, in the domain of the existence" (Maturana & Varela, 1987, p. 29). In the context of this research, knowing feedback is embodied in effective feedback actions. Since the effectiveness of feedback hinges on how it is sought (Hattie & Timperley, 2007), mindfully received (Gielen et al. 2009), and acted upon (Sadler, 1989), knowing effective feedback means effective actions that occasion the seeking, receiving and acting upon feedback. In this research, the analysis of feedback actions across the three cycles revealed different instances of such effective action. The students knew how to incorporate more interaction and dialogue in order to make feedback comments more effective. For instance, they knew how to seek specific feedback comments by leaving notes about what they needed feedback on. They also knew how to mindfully receive feedback by not immediately dismissing feedback comments when they did not understand them. Instead, they decided to go back and discuss the feedback comment with their peers to gain better understanding. In terms of acting on feedback comments to improve their work, students' ability to make corrections to their posters was mediated by their competency in the mathematical concept(s) addressed in the question. This means that, at times, the students struggled with the mathematical concepts and were unable to correct their mistakes even when they understood the feedback comment and agreed with it.

However, when they were able to, they used the feedback comment(s) they received to correct/improve their solutions. In this way, throughout the cycles, their feedback practices become more effective as the students enacted effective feedback actions.

Researchers writing about feedback and peer feedback call for training for students and teachers (Harris & Brown, 2013; Popelka, 2015; Reinholz, 2015a) and advocate that students and teachers engage with feedback in order to learn how to provide effective feedback (Knight, 2003). My research dismisses the idea of training of and/or learning about a pre-given conceptualization of feedback independent from those who enact it. Learning feedback as a set of procedures and pre-set categories is a disembodied transmission which renders students and teachers as objects, as recipients of information. Rather, knowing and learning feedback are embodied in students' and teachers' actions.

In this research, learning feedback was a process of "opening up the range of possible future [feedback] actions" (Brown & Coles, 2012, p. 220). During one of our final reflections, May described their learning as opening up of range of possibilities as she said, "*we are getting more experienced … about feedback and how to organize it. Because we are learning more and more each and every time that we do it and making changes."* As the group progressed through layers of coupling through actions and interactions, it brought forth more possibilities for enacting feedback. The group's growing awareness of the different elements in the process and of their role and influence over those processes and their openness to change opened to them a range of possible actions. For example, the last feedback model incorporates an "other" category which denotes possible future categories that they have not yet brought forth. This implies that not only the students understood the transitory nature of feedback, but also knew how to

accommodate for potential changes to their current and future models. This knowing-how to change, I would argue, is the essence of knowing and learning.

Co-constructing Effective Relationships with/in Feedback

This research adds to existing research about feedback and peer feedback by proposing that the effectiveness of students' peer feedback practices is greatly impacted by their relationships with feedback and with one another. Researchers advocate for educating students about feedback (Marie, 2016), claiming that students' perception of feedback and its effectiveness improves when students become more involved in their feedback practices (Rowe et al., 2008). However, the students' level of involvement in these practices is varied across the literature, ranging from training of students' peer feedback practices (Rodgers, Diefes-Dux, Cardella, & Fry, 2012) and sharing learning goals and success criteria with them (MacDonald, 2015), to regarding students as partners in co-creating their feedback practices (Meer & Chapman, 2014). While my research joins the body of research that constitute students as partners in co-constructing their practices, it posits that the aim of involving the students in the process was not to help improve their perception of feedback, though it was certainly one of the consequences of the process. Rather, the goal of this study was to explore how my students co-constructed their feedback practices. The findings revealed that the group's acts of co-construction of feedback involved co-constructing positive and effective relationships with/in feedback, which then influenced their perception of feedback.

As the group enacted feedback throughout the cycles, their relationship with feedback evolved. It became increasingly evident that students' acts of seeking, mindfully receiving, and acting on feedback were greatly influenced by their evolving relationships with one another and with feedback. As the cycles progressed, the students moved away from passively receiving

feedback and, instead, sought specific feedback, discussed it with others and, many times, used it to correct and/or improve their work. It was in the way the students saw themselves in relation to feedback, and the role they assigned themselves, that feedback became effective. As the students changed their roles in their feedback actions, taking on more active participation in the process, they construed feedback as effective, not due to its content, but because they had formed better relationships with feedback and with one another. In short, students' relationships with/in feedback became central to the effectiveness of their feedback actions.

In their study of feedback, Hattie and Timperley (2007) speak of the power of feedback and its effectiveness in the students' academic development. The findings of this research suggest that the power of feedback practices rested in the relationships that were formed among students with feedback and with one another. By extension, I would argue that the power of any educational act relies on co-constructing effective relationships among the participants of that act.

Cultivating a Climate of 'Authentic Caring'

The findings of this research are significant in that they draw our attention to the importance of classroom climates in supporting the co-construction of classroom practices that are not only effective but are also ethical. According to enactivism, our social processes are generated by our natural processes of coupling and co-emergence (Maturana & Varela, 1987). We live in a world we bring forth with others through language. In this study, in the processes of co-construction of feedback, the group engaged in social processes of action and interaction, incorporating multiple layers of coupling. My students coupled with one another as they discussed feedback, made distinctions by naming categories, and co-planned models for feedback practices. I coupled with my students as I sought clarifications and at times drew their

attention to a feature. In this way, the social processes of co-constructing feedback were generated by our natural processes of coupling, and our feedback practices were the common worlds of significance we brought forth together. Because these worlds were brought forth by all of us, and affected all of us, our actions in the process of bringing forth these worlds bore ethical implications.

Freire (1970) posits that knowledge "emerges only through invention and re-invention, through the restless, impatient, continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other" (p. 72). Freire's (1970) views are aligned with how enactivists describe knowing as bringing forth a world of significance with others in a world that is not separate from us and others (Maturana & Varela, 1987). In fact, it is this being with the others that is at the heart of what makes us human and, thus, ethical behavior constitutes reflecting on and acknowledging the "legitimacy of the presence of others" (Maturana & Varela, 1987, p. 247). Maturana and Varela (1987) explain that if we deny or ignore the existence or the importance of others in the worlds we bring forth, we deny the natural process that generates the social process and the very thing that makes us human. In this research, students' actions were accompanied by discussions and reflections, which occasioned the group to become aware of one another's roles and influences over their processes of co-construction. They considered themselves a team and acknowledged that the absence of two of their friends in the third cycle changed the categories and models they brought forth. Maturana and Varela (1987) use the word 'love' to describe the act of accepting others in the worlds we bring forth, and Varela (1999) argues that being aware of others and otherness is significant because, "in its full un-folding it opens up openness as authentic caring" (p. 75).

Hattie and Timperley (2007) claim that, "the climate of the classroom is critical" (p. 100) to how feedback is received and to its effectiveness. In this study, the climate of our classroom was one of participation, collaboration and compromise. My students and I were subjects of the processes of co-constructing feedback. I was not the holder of knowledge, and my students were not passive recipients of knowledge, as though it existed disembodied and separate from who we were. Rather, the group and I came to know feedback practices through feedback actions and through bringing forth worlds of significance that were part of our history of structural coupling with one another. The resulting feedback practices that co-emerged from this process also incorporated many opportunities for active participation on behalf of all members of the group and, in this way, acknowledged the importance of others in feedback practices. In other words, just as the students were subjects in their acts of co-construction of feedback, they were also subjects in the feedback acts they co-constructed. This meant that our classroom climate impacted the classroom practices the group co-constructed. When we showed "authentic care" for others in the processes of co-construction, the worlds brought forth through those processes also embodied that authentic care. Educational acts, therefore, if they are to be ethical, must be cultivated in a climate of authentic caring. This is only possible in environments where we are all subjects of our educational processes "who meet to name the world in order to transform it" (Freire, 1970, p.115).

Implications and Potential for Future Research

Maturana and Varela (1987) state, "We do not see the 'space' of the world. We live our field of vision" (p. 23). This research was never in search of a single conceptualization of feedback or a single model for enacting feedback that would be effective for everyone. For this reason, the categories brought forth, the models built and the feedback actions co-constructed

are not intended to be prescriptive. However, the findings of this research have the potential of expanding the field of vision for other researchers and to offer possibilities for future research. This study situates itself at the intersection of research about feedback, research in mathematics education, research with an enactivist perspective, and research conducted by teachers about their own practice. It also joins the growing body of research that use enactivism as theory of knowing and/or as methodology. Consequently, it speaks to and opens up opportunities for future research in all these areas.

Research about Feedback

In this study, my students and I were only able to perform three cycles of feedback. Performing similar research with a classroom cohort for a longer duration with more cycles is bound to provide more insight into how categories and models may evolve. Furthermore, the research can be extended to look at how students, after cycles of co-constructing feedback in one class, would approach feedback in other classrooms. In other words, future research can examine the long-term effects of the process of co-construction of feedback, by looking at how the process impacts students' relationship with feedback once they are in a different environment.

Research in Mathematics Education

In this study, in each cycle, after the students wrote their feedback on the sticky notes and placed them on the posters, they were instructed to remove the sticky notes, place them on the board and begin to classify them. The act of removing the sticky notes from the posters effectively separated the feedback for each poster from the mathematics written on that poster. This design choice was made because the focus of this study was the co-construction of feedback in a mathematics classroom and not the mathematics the feedback addressed. Future research in mathematics education can use a similar approach used in this study, but change the design

slightly by not removing the sticky notes on the posters in order to examine how the feedback on each poster relates to the mathematics on that poster. In doing so, future studies in mathematics education can explore, more closely, how peer feedback is contingent on the mathematics the students produced. In research in mathematics education, teachers' actions have been described as "triggers" (Towers & Proulx; 2013). Future variations of my research can look at how students' peer feedback can become a "trigger" for students and teachers to discuss the mathematics on the posters.

Research with an Enactivist Framing

This research used enactivism both as a theoretical framework and as methodology. Using enactivism as methodology allowed me to bring attention to my role as the observer who is both teacher and researcher and to describe my structural coupling and co-emerging with others. One of the themes that emerged from this research was the significance of our relationships in the process of co-constructing our feedback practices. Future research can use the enactivist lens to examine these relationships more closely, describing how relationships are generated from our acts of structural coupling and co-emergence.

Teachers Researching their own Practices

This research was an exploration of the co-evolution of one of our classroom practices as it was co-constructed by me and my students. Similar case studies can be undertaken to study the process of co-construction of other classroom practices. The different findings that will emerge from these studies will enrich the body of literature pertaining to teachers researching their own practices and will help expand the "field of vision" for all educators.

Furthermore, this research draws our attention to the temporary nature of feedback practices, and by extension, the temporary nature of all classroom practices when they are co-

constructed by students and teachers. This temporality and fluidity advocates for an openness to change on behalf of all members of the classroom. Future research can explore how teachers and students can co-construct classrooms that support this temporality and fluidity in a way that is meaningful and effective for them.

Limitations of the Study

This study was limited in both the number of participants and the duration of the research. Due to ethical considerations, I had to exclude my grade five students from this research and was only able to undertake this study with my grade six students, which reduced the number of students who were able to participate from fourteen to eight. This study is premised on students' participation in the co-construction of feedback practices and every student's contribution is considered valuable and significant to the bringing forth of feedback practices. As such, if all fourteen students participated, we would have experienced more layers of structural coupling and different worlds of significance would have been brought forth than the ones we have. Furthermore, this research spanned across three weeks, which afforded us only three cycles of co-constructing feedback. Undoubtedly, if we had more time, we would have been able to go through more cycles of feedback, which potentially would have resulted in new and different categories, models and feedback actions. We would have been able to see more and to bring forth new worlds of significance.

However, while the limitations in number of students and the duration of the research undoubtedly influenced the findings, this does not challenge their validity, especially where validity is seen through an enactivist perspective, which rejects a single model *of* feedback in favor of multiple models *for* feedback. The goal of this research was not to arrive at *the right* model of feedback. Rather, the goal was to co-construct a model for feedback that was right in

the moment of its construction. Each feedback model was the world the students brought forth in the cycle it was constructed, as its validity and worth must be measured in the context of that cycle. As the narratives of the three cycles suggest, our models of feedback continued to evolve. The transient nature of these models suggests that with more participants and more cycles, more models would be brought forth, but this does not lessen the value or validity of each one of the models. In fact, the temporality of the feedback actions, categories and models further reinforces the need to shift our focus from finding the right model of educational acts toward co-constructing models that are right in the moment of their construction.

Conclusion

This research was born out of an interest to incorporate peer feedback practices in our mathematics classroom. Inspired by a democratic view of education, and dismissing the idea of a single, pre-existing conceptualization of feedback, this research focused on how the students co-constructed their feedback actions, categories, and models and how they conceptualized feedback.

The true gift of this process was the transformation of our classroom's peer feedback practices into democratic spaces where our relationships evolved. Before conducting this study, my students and I shared a rich history of structural coupling with one another. We knew one another well and enjoyed a certain degree of trust and comfort. However, this experience transformed our relationships. I was triggered by my students' actions and/or conversations and I also triggered them by asking questions and drawing their attention to a certain element. In this way, my students and I were both teachers *and* learners of the process.

It was a humbling experience for me to watch my students co-construct their categories and models, modify their feedback actions, and engage in thoughtful and meaningful discussion

about these actions. I was fascinated by their awareness of their growing understanding of feedback and that they knew that the models were temporary and contingent on their feedback actions. I was most impressed by their growing awareness and acknowledgement of one another in the process. They were proud that they worked as a team and recognized that the absence of two of their peers in the third cycle impacted their model. The students also changed their relationship with feedback, as they took on more active roles. In each cycle, they saw more possibilities for feedback actions and brought forth new conceptualization for effective feedback. All of these relationships that were formed and/or reinforced throughout the course of this research became part of our history of structural coupling. Although it is impossible to predict in what manner and to what extent, I am certain that our transformation during this experience will influence our future actions, how we will think about classroom practices, how we see ourselves in relation to others, and what worlds we will bring forth with others.

This research was premised on the ethical conviction that students must be actively involved in the co-construction of their feedback practices. What made the process meaningful and transformative was that the students' actions were accompanied by discussions and reflections, and these reflections triggered them to become aware of themselves and others, of their roles, influences and relationships, in the worlds they brought forth together. In this way, the ethical convictions that underpinned the design of this study emerged in the groups' actions and reflections. According to Dewey (1916), educational acts, if they are to be democratic, must possess aims, which are decided by those affected by them. What this research has taught me is that educational acts, if they are to be democratic, ethical, and meaningful, must be co-constructed with 'love', with an awareness of others in a "world we share because we have specified it together through our actions" (Maturana & Varela, 1987, p. 233).

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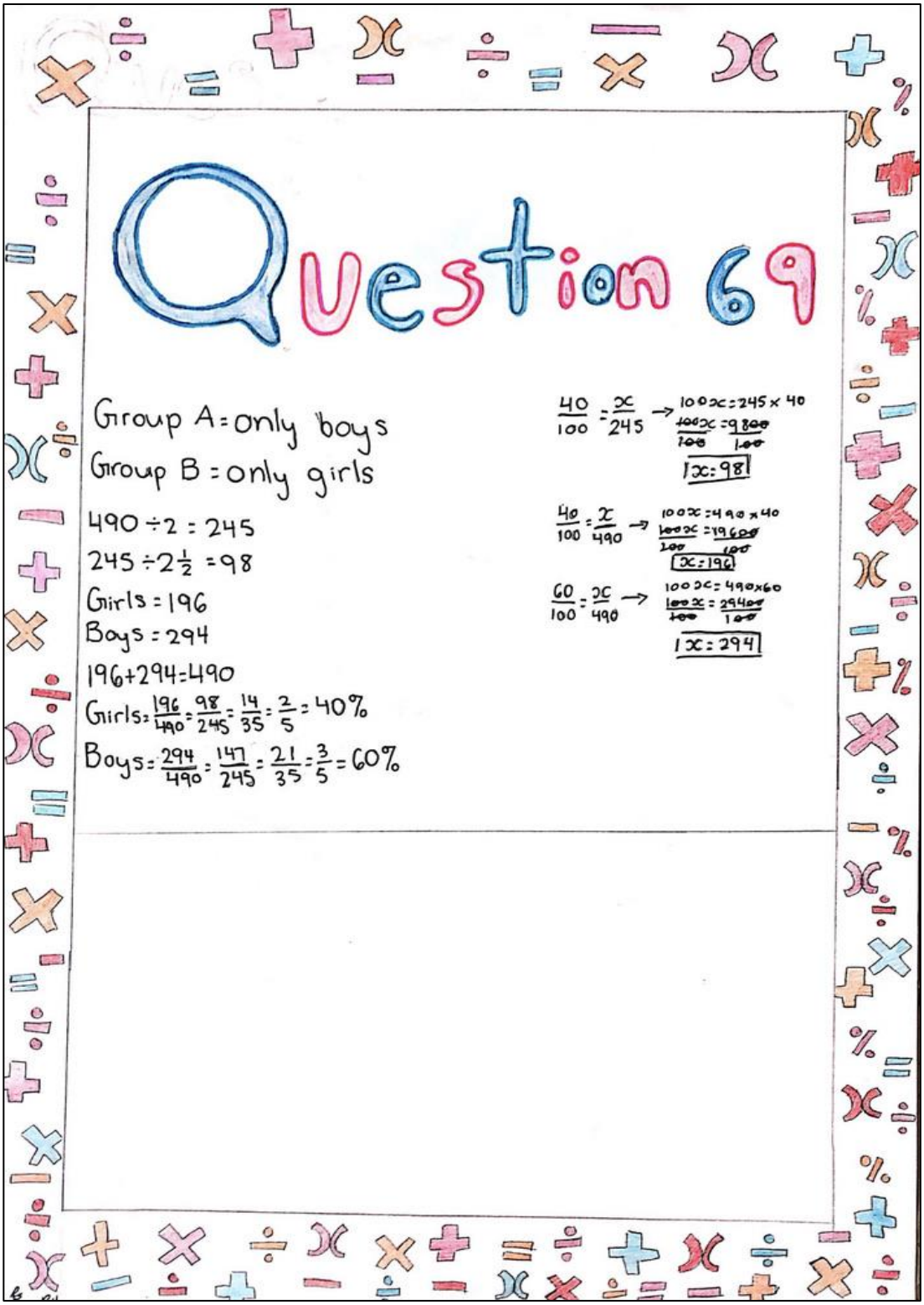
Appendix A: Mathematics Solutions for Cycle 1



Question 69

A = Boys
B = Girls
$2\frac{1}{2}(2.5):1$ → ratio of girls to boys

We found 3.5 by using a T-chart.

$490 \div 3.5 = 140$ ← boys

$490 - 140 = 350$ ← girls

We subtracted the number of boys from the total.

Group A: Boys who joined

$140 \div 4 = 35$

$35 \times 4 = 140$

$35 \times 32 = 1120$

Group A = 1120 boys who joined

→ We figured out for every 4 boys in Group A, 32 more boys joined the group. There are 35 groups of 4 boys.

Boys = 1120 + 140 = 1260 → Total of Group A

Girls = 350 + 420 = 770 → Total of Group B

Fraction = $\frac{420}{1120}$ = $\frac{\text{Girls who joined}}{\text{Boys who joined}}$

$G = \frac{1}{3}$ of boys ← new ratio = $\frac{1}{3}:1$

$G = \frac{1}{3} \times 1260$

$G = 420$

How did we find 3.5?

	Total	Girls	Boys
the increase is 3.5	3.5	2.5	1
	7	5	2
So...	10.5	7.5	3

we divided 490 by 3.5 to find the number of girls.

QUESTION 69

What we know:

Group A: boys

Group B: girls

For every 4 boys that are in Group A, 32 more boys join

There are $2\frac{1}{2}$ times as many girls as boys.

Boys	Girls	Total Amount
1	2.5	3.5
2	5	7
3	7.5	10.5
4	10	14
5	12.5	17.5
↓	↓	↓

As you can see, the total amounts are growing by 3.5. For every boy, there are 2.5 girls. We did this to find the total amount of girls and boys but we didn't keep going because we soon realized we could divide by 3.5.

Total amount of children

490 ÷ 3.5 = 140 boys — To get the amount of boys, we have to divide 490 to 3.5. We did that because when we divide those two numbers, it would split boys into girls

490 - 140 = 350 girls

140 ÷ 4 = 35

35 × 32 = 1120 (new) boys — We divided by 4 because we need to see how many groups of 4 there are amongst the total of boys. We multiplyed it by 32 to see how many new boy students could. Then, we added the new boys to the boys who already came and that gives us the amount of boys together

1120 + 140 = 1260 boys in total.

$\frac{1}{3} \times 1260 = \frac{1260}{3} = 420$ girls (amount who joined) → This ratio show the new girls who joined. We did this because in the question, it says that the total number of girls will be $\frac{1}{3}$ to the total number of boys. The girls are a third of the total number of boys.

fraction of girls who joined over boys who joined:

$\frac{420 \text{ girls}}{1120 \text{ boys}}$

(Do not simplify)

Answer: $\frac{420}{1120}$

Question 69

Things we know:

2 groups = 490 children

Group A = boys

Group B = girls

Group A = Group B $\times 2\frac{1}{2}$

G + Group B (G = Girls)

4 : 32

$\frac{1}{3}A = B$

Solution 1:

① boys : girls

1 : $2\frac{1}{2}$ (2.5)

36 : x

$\frac{1}{36} = \frac{2.5}{x}$

$x = 36 \times 2.5$

$\boxed{x = 90}$

For every 90 girls, there are 36 boys.

② $\frac{1}{3} \times 36^{12} = 12$ ($\frac{1}{3}$ of 36)

③ $\frac{\text{girls}}{\text{boys}} = \frac{12}{36} = \frac{1}{4}$

$\therefore$ The fraction of girls who joined Group B as a fraction of boys who joined Group A is $\frac{1}{4}$.

Solution 2: (in the process)

$\frac{1}{3}(B \times 2\frac{1}{2}) = B$ → Because an equation is $\frac{1}{3}A = B$, and another one is $A = B \times 2\frac{1}{2}$, in the first equation, I replaced A with $B \times 2\frac{1}{2}$ so that I only have 1 unknown.

Question 70

$\frac{x}{825} = \frac{20}{100} \rightarrow 100x = 20 \times 825$

$\frac{100x}{100} = \frac{16500}{100}$

$x = 165$

$\frac{37}{100} = \frac{x}{1320}$

$100x = 1320 \times 37$

$\frac{100x}{100} = \frac{48840}{100}$

$x = 488.4$

$825 - 165 = 660$

Discount: $165

New price of computer : $660

$12\% + 25\% = 37\%$

$488.4 + 660 = 1148.4$

$1148.4 - 1320 = 828.4$

$828.4 - 825 = 3.4$

The price of the computer is $660.

The total amount of money in the beginning is $1148.40.

The total amount of money in the end is $3.40.

Question 70

Car repairs: 12% of paycheck
Food: 25% of remainder
Parents: $1320 of remainder
Computer: $660 of remainder →

$\frac{x}{825} = \frac{20}{100}$

$\frac{100x}{100} = \frac{16500}{100}$

$\boxed{x = 165}$ →

$825 - 165 = \$660$

$\therefore$ Nate paid $660 for the computer.

$\left(\left(x - \frac{12x}{100}\right) \times \frac{25}{100}\right) - \$1320\right) - \$660 = x$

Question 70

Nate's expenses of his paycheck

12% ——→ car repairs

25% (of the remaining 88% of Nate's paycheck) ——→ food

$1320 (of the remaining money) ——→ his parents

? (20% discount of $825) ——→ a computer
(calculate price)

Price of Computer

Original Price = $825

Discount = 20%

$\frac{20}{100} \times \frac{x}{825}$

$100x = 825 \times 20$

$\frac{100x}{100} = \frac{16500}{100}$

$x = \$165$ ——→ discount

Original Price − Discount = New Price of Computer

$825 − $165 = $660

Computer = $660

---

① $12\% = \frac{12}{100}$ of the total paycheck

② $25\% = \frac{25}{100} = \frac{1}{4}$ of the remainder ——→ 88% = 100% (the total paycheck) − 12% (the first expense) = 88%. 88% is "the remainder."

③ $1320 of the remaining money (from the car repairs and food) ——→ x% of the total

④ $660

We do not know how much Nate had in the beginning (his total paycheck), and we do not know if he spent all of his paycheck.

T = Total

T = all of Nate's paycheck (car repairs, food, computer and given amount to his parents)

We are assuming he spent all of his paycheck. ——→ However, if he did not spend all of his paycheck, we would add x to the equation because we do not know how much he didn't spend.

(T = $660 + $1320 + 12% of T + 25% of (88% of T) + x)

↑ the remainder of what he didn't spend

$T = \$660 + \$1320 + \frac{12}{100}T + \frac{25}{100} \times \frac{88}{100}T$

$T = \$1980 + \frac{12}{100}T + \frac{88}{400}T$ ——→ we multiplied $\frac{25}{100} \times \frac{88}{100}T$

$T = \$1980 + \frac{48}{400}T + \frac{88}{400}T$

$T = \$1980 + \frac{136}{400}T$

$T = \frac{\$1980}{1} + \frac{34}{100}T$

$T = \frac{198000}{100} + \frac{34}{100}T$

$T = 19803\frac{?}{?}\%$ of T

↑ We were VERY confused about the formula for finding his total paycheck and how much the car repairs and food cost.

$T = 19034\%$

$\frac{?}{1} = \frac{19034}{100}$

$? = 0.19034?$

BUT...

For our solution, we set the total to ALL his expenses. Perhaps he didn't spend ALL of his paycheck.

Question 70

What we know:

Spendings:

12% on car repairs: $\frac{12}{100}$

25% of remainder on food: $\frac{25}{100}, \frac{1}{4}$

1,320 of remainder to parents

$825 of remainder for computer with 20% discount.

? How much did he have in the beginning

To solve this, we tried working our way backwards.

$\frac{20}{100} \overset{\curvearrowright}{\underset{\curvearrowleft}{\times}} \frac{x}{825}$

$100x = 825 \times 20$

$\frac{100x}{100} = \frac{16500}{100}$

$x = 165$ ← The discount was $165

New computer price:

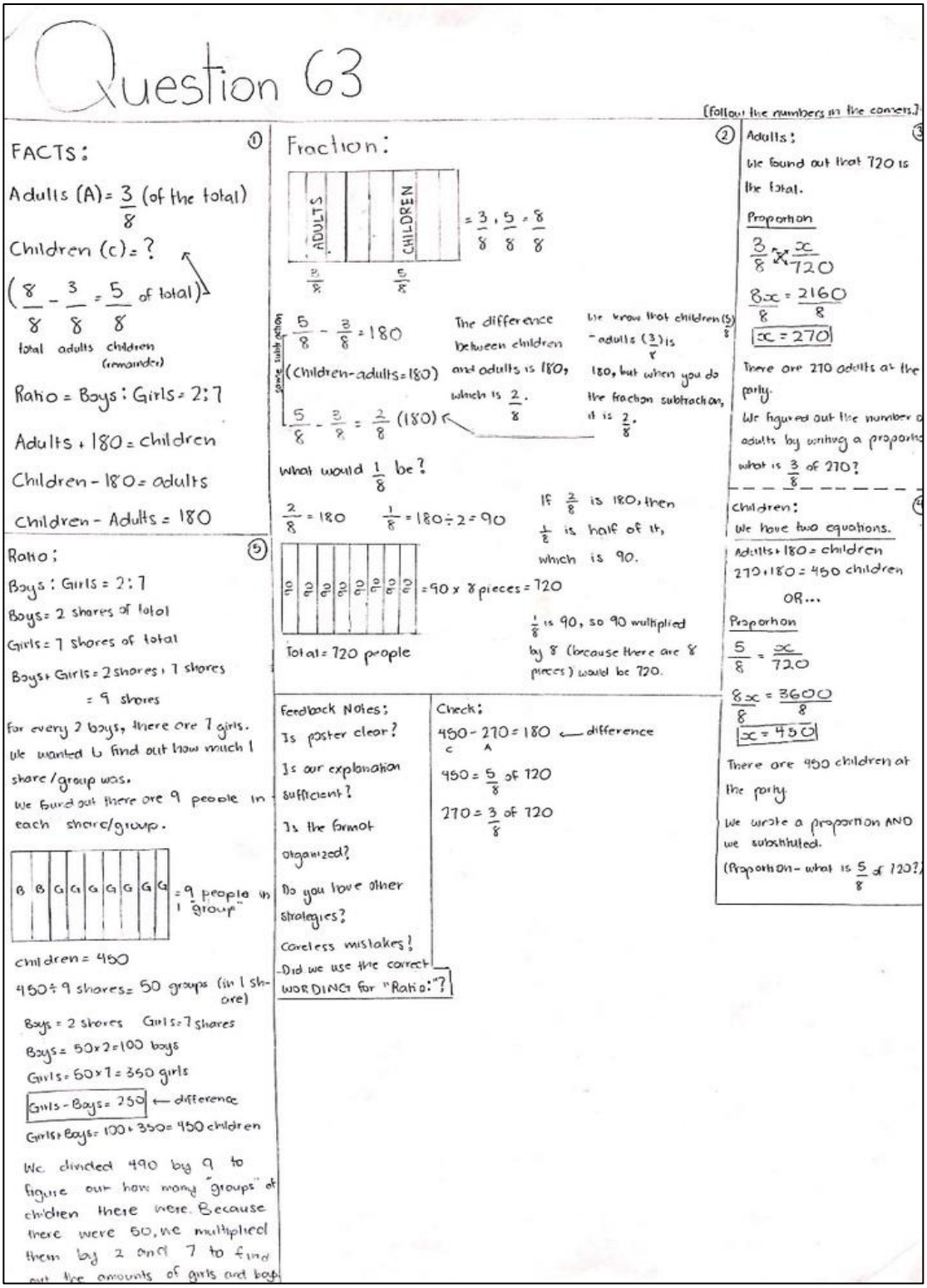
$\overset{7}{\cancel{8}}25 - 165 = 660$ $660

1320 + 660 = 1980

We added the computer price to the amount Nate gave his parents because we're retracing what he bought to find out how much he had in the beginning.

Because it says "the remainder for food," we asked ourselves, remainder of what. So now, we are stuck.

Appendix B: Mathematics Solutions for Cycle 2

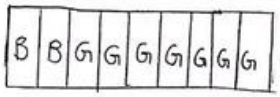


Question 63

$2:7 = \frac{2}{7}$

B: boys
G: girls

Solution:



B: $\frac{2}{9}$ $\quad \frac{2}{9_1} = \frac{\overset{90}{\cancel{180}}}{1} = 180$

G: $\frac{7}{9}$ $\quad \frac{7}{9_1} = \frac{\overset{90}{\cancel{180}}}{1} = 630$

(Girls) (Boys)
630 + 180 = 810 children
(total)
810 - 180 = 630 adults
(Girls) (Boys)
630 - 180 = 450 (difference)

I added 2 and 7 which gave me 9. I wrote them as fractions. I did proportions to find out how many girls and boys there are. I got 630 for the girls and 180 for the boys. I added 630 and 180 to find out how many children were at the party. Then, I subtracted 630 by 180 and it gave me 450. That is the difference between boys and girls.

Feedback:
- If the 180 is correct in the first part
- If it's clear what we're doing
- The answer... ☺☺
- If we're on the right track

Answer: 450

There are 1440 at the party.

There are 180 boys at the party

There are 630 girls at the party

The difference between the girls and boys are 450.

Question 63

We would like feedback on..
- how to find solution
- if we could've improved it
- did we explain well enough?
- does our solution make sens
- no random comments

- only write "I don't understand" if you still don't understand after we explain.

THANK YOU

What we know:

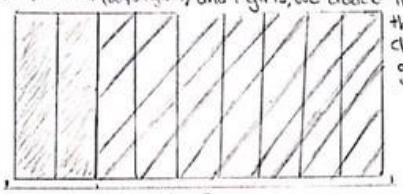
$\frac{3}{8}$: adults

2:7 : boys to girls

180+ adults : children

boys : girls : amount of kids in 1 group
2 : 7 : 9

2+7=9 in total (boys+girls) Since there is 2 boys and 7 girls, we added them to find the amount of children in one group.



① Percentage of adults:

$\frac{3}{8} : \frac{x}{100}$ ← All guests = 100% no matter what

$8x = 3 \times 100$

$\frac{8x}{8} = \frac{300}{8}$

$x = 37.5\%$

∴ 37.5% of the guests were adults.

We calculated the percentage of adults at the party

② Percentage of children:

$100 - 37.5 = 62.5\%$

∴ 62.5% of the guests were children.

$\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$ → amount of children

$\frac{5}{8} \cdot \frac{2}{7}$

C: children
A: adults
C: A of 180

Fraction addition

$C: \frac{3}{8} \cdot 180$

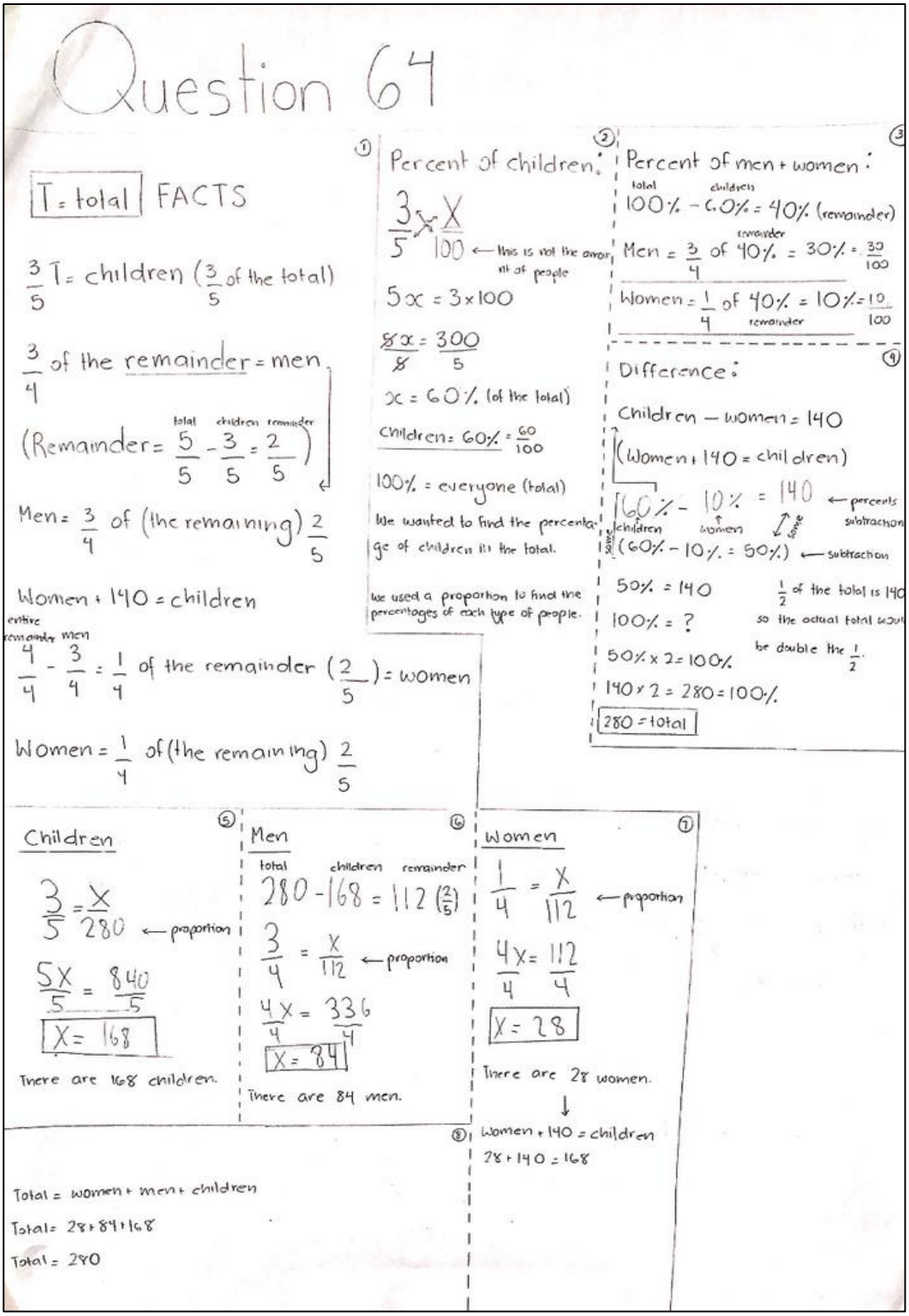
$C: \frac{3}{8} \cdot \frac{1440}{8}$

$C: \frac{1443}{8}$

$C: 180.37$

This solution does not make sense. Please leave feedback helping us to figure it out.

Appendix C: Mathematics Solutions for Cycle 3



Question 64

What we know:

$\frac{3}{5}$ of T = children
T: total

$\frac{3}{4}$ (of the remaining) = men

140 + women = children
OR
children - 140 = women

Percentage Conversions

$\frac{3}{5} = 60\% \rightarrow$ children

$100 - 60 = 40\% \rightarrow$ left overs

40% = men + women

$\frac{3}{4} = 75\%$

we converted them all to percent.

children

$\frac{3}{5} : \frac{x}{100}$

$5x : 3 \times 100$

$\frac{5x}{5} : \frac{300}{5}$

$x : 60\%$

Men

$\frac{3}{4} : \frac{x}{100}$

$4x : 3 \times 100$

$\frac{4x}{4} : \frac{300}{4}$

$x : 75\%$

women

?

Solution 1

children - 140 = women → W: women

$\frac{140}{1} - \frac{3}{5} : W$

$\frac{700}{5} - \frac{3}{5} : W$

$\frac{697}{5} : W$

$W : 139.5$

We tried using a formula to find out the amount of women there was, but when we tried to solve it, it ended up giving a decimal. It isn't possible for us to have half a person. Now we are stuck.

What we want feedback on:

- suggestions on how to solve it - figure out the total
- what we did wrong and a way to improve it

Solution 2

Question 64

C = children
W = women
M = men
T = total

$\frac{3}{5} = 60\%$

$\frac{3}{4} = 75\%$

$60 + 75 + 140 = 275$

There are 275 people at the funfair.

CHILDREN = $\frac{6}{10} = 60\%$

$\frac{60}{100} = \frac{x}{275}$

$\frac{100x}{100} = \frac{16500}{100}$

$x = 165$

There are 165 children.

MEN = $\frac{3}{10} = 30\%$

$T - (C + W) = M$

$275 - (165 + 25) = M$

$M = 275 - 190$

$M = 85$

There are 85 men.

WOMEN = $\frac{1}{10} = 10\%$

$C - 140 = W$

$165 - 140 = W$

$W = 25$

There are 25 women.

$165 + 85 + 25 = 275$

BEFORE + AFTER

Question 64

What we know before:

$\frac{3}{5}$ of the people are children

$\frac{3}{4}$ of the remaining people are men

140 more children than women

$\frac{3}{5}$ are children so $\frac{2}{5}$ are not children

$\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$ → men and women

1. Men:

$\frac{3}{4}$ of the REMAINING ($\frac{2}{5}$)

$\frac{3}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{5} = \frac{3}{10}$

$\frac{3}{\cancel{10}_{1}} \times \frac{\cancel{100}^{10}}{1}\% = 30\%$

$\therefore \frac{3}{10}$ of the people are men.

2. Children:

find common denominator $\frac{x}{10}$ → it is out of 10 because the fraction for girls was out of ten. It would be easier to do equations with the same denominators.

$\frac{3}{5}$

$\frac{6}{\cancel{10}_{1}} \times \frac{\cancel{100}^{10}}{1}\% = 60\%$

$\therefore \frac{6}{10}$ of the people are children.

3. Women:

children − 140 = women → We could not do this solution because we didn't know the amount of children.

Total People OF

$1 - \left(\frac{10}{10} - \left(\frac{3}{10} + \frac{6}{10}\right)\right) = \frac{1}{10}$

$\frac{1}{\cancel{10}} \times \frac{\cancel{100}^{10}}{1}\% = 10\%$

What we now know:	x = people at the fun fair
30% of x are men	
60% of x are children	
10% of x are women	

We would like feedback on:

- how to finish it
- if we explained well enough
- if we're on the right track