Module: Introduction to Algebraic Expressions

Module Outline

- Example 1: *Economics*
- Example 2: *Politics*
- Example 3: *NBA* — Excel/Google sheets
- Example 4: *Geography*
- Example 5: *Model of Infections* — Excel/Google sheets

Example 1: Economics

- Maya's monthly expenditures:

  \$400 on rent, \$300 on food and drinks

  $+$ 1 *GAMESTOP* stock, priced at \$170

  $+$10 *FORD* stocks, priced at \$12 each .

Example 1: Economics

- Maya's monthly expenditures:

  \$400 on rent, \$300 on food and drinks

  $+$ 1 *GAMESTOP* stock, priced at \$170

  $+$10 *FORD* stocks, priced at \$12 each .

- What is her total monthly expenditure?

$$
\begin{array}{l}
\$400 \\
+\ \$300 \\
+\ \$170 \times 1 = 170 \\
+\ \$12 \times 10 = 120 \\
\hline
\end{array}
$$

Example 1: Economics

- Maya's monthly expenditures:

  \$400 on rent, \$300 on food and drinks

  $+$ 1 *GAMESTOP* stock, priced at \$170

  $+$10 *FORD* stocks, priced at \$12 each .

- What is her total monthly expenditure?

- $400 + 300 + 170 + 120 = 990$

Example 1: Economics

discrete variable

- **More general**: If she buys x *GAMESTOP* stocks $+$ y *FORD* stocks, what is her expenditure?

Example 1: Economics

- **More general**: If she buys x *GAMESTOP* stocks $+$ y *FORD* stocks, what is her expenditure?
- \$400:rent, \$300: food, x *G_STP* @ \$170; y *FF* @ \$12.

$$\begin{array}{l} \$400 \\ + \\ \$300 \end{array} + 170x + 12y$$

$$= \$700 + 170x + 12y$$

$\$12 \times 10$

Example 1: Economics

- **More general**: If she buys x *GAMESTOP* stocks

  $+\ y$ *FORD* stocks, what is her expenditure?

- \$400:rent, \$300: food, x *G_STP* @ \$170; y *FF* @ \$12.

- Expenditure $= 700 + 170x + 12y$.

  If her monthly income is \$1500, what is her savings?

Budget constraint: $700 + \underline{170x} + 12y \leq 1\underline{500}$

savings : $1500 - (700 + 170x + 12y)$

Clicker question 1.1

- In this example, if Maya also buys z Air Canada stocks priced at \$30 each, what is her total expenditure?

  Recall: \$400:rent, \$300: food, x G_STP @ \$170; y F @ \$12.

  (a) $1500 - 170x - 12y - 30z$

  (b) $700 - 170x - 12y + 30z$

  (c) $700 + 170x + 12y - 30z$

  (d) $700 + 170x + 12y + 30z$

Solution to Clicker question 1.1

- In this example, if Maya also buys z Air Canada stocks priced at \$30 each, what is her total expenditure?

  <u>Recall</u>: \$400:rent, \$300: food, x G_STP @ \$170; y F @ \$12.

  (a) $1500 - 170x - 12y - 30z$

  (b) $700 - 170x - 12y + 30z$

  (c) $700 + 170x + 12y - 30z$

  (d) $700 + 170x + 12y + 30z$ ✓

\$700
\+ 170x
\+ 12 y
\+ 30z

Example 2: Politics

- Two political parties, **C** and **L** contesting over a seat.

- Constituency: 3 districts: **Hilly**, **Valley** and **Plain**.

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- Constituency: 3 districts: **Hilly**, **Valley** and **Plain**.
- **Hilly**: 1000 eligible voters; 80% will vote for party **C**.

  Valley: 800 eligible voters; 90% will vote for party $\boxed{\textbf{L}}$.

  Plain: 1200 eligible voters. Is a **swing** district.

Example 2: Politics

- **Hilly**: 1000 eligible voters; 80% will vote for party **C**.

  Valley: 800 eligible voters; 90% will vote for party **L**.

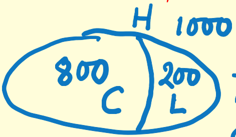
  Plain: 1200 eligible voters. Is a **swing** district.

- *Survey*: finds fraction x of the voters in Plain say they will vote for party C. The rest said they would vote for party L.

  Question: *How many votes does party C hope to get overall?*

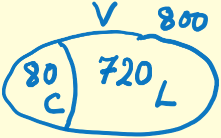
Example 2: Politics

- **Hilly**: 1000; 80% for **C**. **Valley**: 800; 90% for **L**:
  Plain: 1200; fraction x for C:



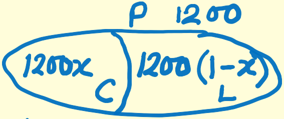
$\frac{80}{100} = 0.8$

$0.8 \times 1000 = 800$



$\frac{90}{100} = 0.9$

$0.9 \times 800 = 720$



$C \to 1200x$

Example 2: Politics

- **Hilly**: 1000; 80% for **C**. **Valley**: 800; 90% for **L**:
  Plain: 1200; fraction x for C:
- If $x = 0.5$ (i.e. 50%) :

$1200 \times 0.5 = 600$

$1200x$

continuous variable

Example 2: Politics

- **Hilly**: 1000; 80% for **C**. **Valley**: 800; 90% for **L**:
  Plain: 1200; fraction x for C:
- If $x = 0.5$ (i.e. 50%) :
- More generally: votes for C : $800 + 80 + 1200x$
  $= 880 + \underline{1200}x$

Clicker question 1.2

- In this example, how many votes does party **L** receive? Recall:

  Hilly: 1000; 80% for **C**. **Valley**: 800; 90% for **L**:
  Plain: 1200; fraction x for C:

  (a) $920 + 1200(1 - x)$

  (b) $880 + 1200(1 - x)$

  (c) $920 + 1200x$

  (d) $880 + 1200x$

Solution to Clicker question 1.2

- In this example, how many votes does party **L** receive? Recall:

Hilly: 1000; 80% for **C**.
Plain: 1200; fraction x for C:

Valley: 800; 90% for **L**:

(a) $920 + 1200(1 - x)$ ✓

(b) $880 + 1200(1 - x)$

(c) $920 + 1200x$

(d) $880 + 1200x$

L : 200
$+ 720$
$+ 1200(1-x)$

$920 + 1200(1-x)$

Example 2: Politics

- <u>Question</u>: *Will party C get more votes than party L?*

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Example 2: Politics

- Question: *Will party C get more votes than party L?*

- Compare between

  C: $880 + 1200x$ and **L**: $920 + 1200(1 - x)$

- Question:

$$\underbrace{880 + 1200x \overset{\downarrow}{>} 920 + 1200(1 - x)}_{}\qquad ?$$

inequality

Example 2: Politics

- Question: *When will party C get more votes than party L?*

$$880 + 1200x > 920 + 1200(1 - x) \qquad ?$$

$$880 + 1200x > 920 + 1200 - 1200x$$

$$1200x + 1200x > 920 + 1200 - 880$$

$$2400x > 1240 \qquad 2120 - 880 = 1240$$

$$x > \frac{124\cancel{0}}{240\cancel{0}} = 0.5167$$

Example 2: Politics

- Question: *When will party C get more votes than party L?*

$$880 + 1200x > 920 + 1200(1 - x) \qquad ?$$

- Answer: C can hope to win if the survey reveals that more than 51.67% of the voters in Plain plan to vote for C.

Example 3: Sports Analytics

- **NBA**: How to compare players?

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- **NBA**: How to compare players?
- Points, Free-throws, Rebounds, Blocks, Steals........
- Individual player efficiency **EFF**:

$$\frac{PTS + REB + AST + STL + BLK - MissedFG - MissedFT - TO}{GP}$$

PTS = points, REB = rebounds, AST = assists,

STL = steals, BLK = blocks, $MissedFG$ = missed field goals,

$MissedFT$ = missed free throws, TO = turnovers,

GP = games played (can be found on NBA.com)

Example 3: Sports Analytics

- **Excel/Google sheets**

  Use formula function to compute

$$EFF = (D3 + E3 + ...)/C3$$

  Copy and Paste Formula for other players

Example 3: Sports Analytics

Excel/Google sheets activity

- **Think and Do:** Do defensive players get less credit in this formula? How would you adjust it?

  Do player rankings change as a result?

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Excel/Google sheets activity

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  Do player rankings change as a result?

- **Think and Research:** How to measure NFL players?

Example 4: Geography

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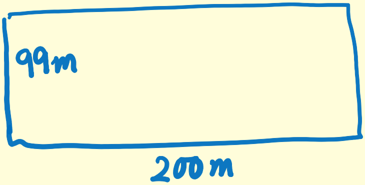
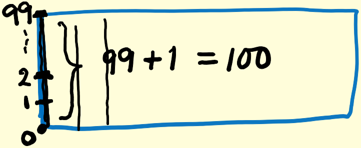
- It decides to plant some trees which require a spacing of 1m between each. Total trees?



Example 4: Geography

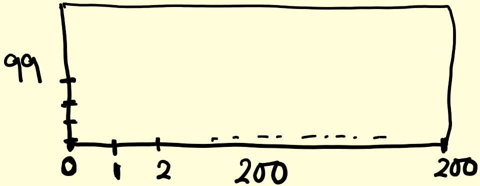
- Green City acquires a tract of land 99 m wide and 200 m long to plant trees.
- It decides to plant some trees which require a spacing of 1m between each.
- How many trees can it plant in a row along the width of the tract?



Example 4: Geography

Tract of land 99 m wide and 200 m long to plant trees.

- How many such rows can it plant?



$201 = 200 + 1$

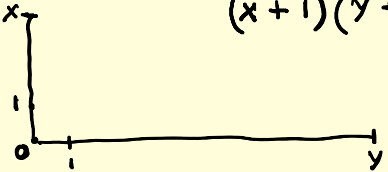
Total trees $= 100 \times 201$

Example 4: Geography

Tract of land 99 m wide and 200 m long to plant trees.

- How many such rows can it plant?
- If the tract of land were x m wide and y m long, what is the total number of trees it could plant?

$$(x+1)(y+1)$$



Example 4: Geography

- If the tract of land were x m wide and y m long, what is the total number of trees it could plant?

$$\underbrace{(x+1)(y+1)}_{\text{2 variables}} = x(y+1) + 1\cdot(y+1)$$

$$= xy + x + y + 1$$

$$(x+a+b)(y+c+d) = x(y+c+d) + a(y+c+d) + b(y+c+d)$$

$$= xy + xc + xd + ay + ac + ad + by + bc + bd.$$

Example 5: Spread of infections

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$t = 2$: 1 ——— (np)

Example 5: Spread of infections

- Suppose in a large population, 1 person gets infected
- Say, each person interacts with n people everyday; in each interaction, the probability of passing the infection is p
- On day $\mathbf{t} = \mathbf{1}$, number of infected people $= 1$
- On day $\mathbf{t} = \mathbf{2}$, number of new infections: np

$t = 3$:

np — np, np, np

$$(np)(np) = (np)^2$$

Example 5: Spread of infections

- Suppose in a large population, 1 person gets infected
- Say, each person interacts with n people everyday; in each interaction, the probability of passing the infection is p
- On day $\mathbf{t} = \mathbf{1}$, number of infected people = 1
- On day $\mathbf{t} = \mathbf{2}$, number of new infections: np
- On day $\mathbf{t} = \mathbf{3}$, number of new infections: $(np)^2$

$t = 4$: $(np)^2$ np np .... $(np)^2(np) = (np)^3$

Example 5: Spread of infections

- $\mathbf{t = 1}$, number of infected people = 1
- $\mathbf{t = 2}$, number of new infections: : $np \equiv \mathbf{R}_0$ **[Reproduction number]**
- $\mathbf{t = 3}$, number of new infections: $(np)^2 \equiv (\mathbf{R}_0)^2$

$t = 4$: $(np)^3$

$\vdots$

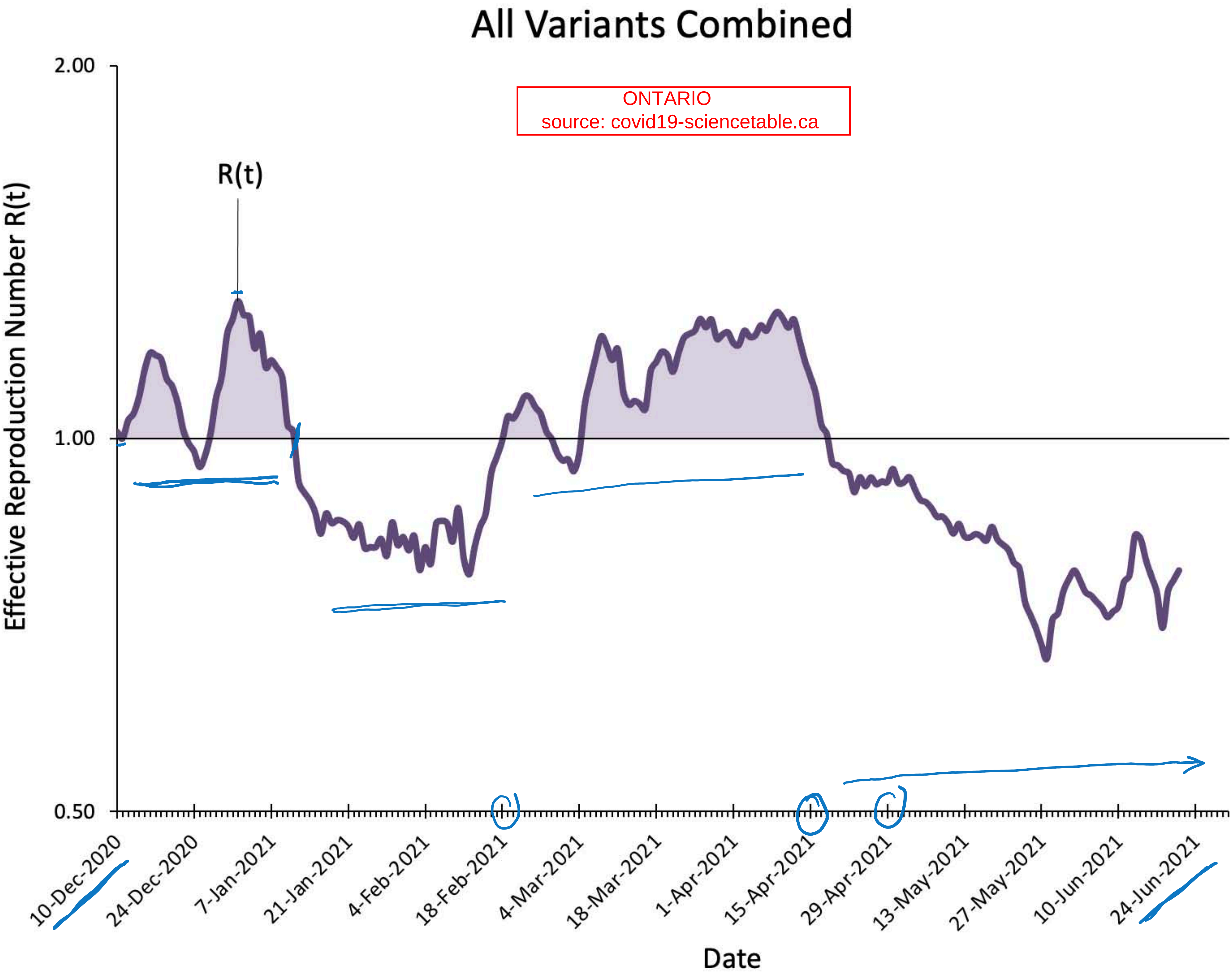
Example 5: Spread of infections

- $\mathbf{t} = \mathbf{1}$, number of infected people = 1
- $\mathbf{t} = \mathbf{2}$, number of new infections: : $np \equiv \mathbf{R}_0$ **[Reproduction number]**
- $\mathbf{t} = \mathbf{3}$, number of new infections: $(np)^2 \equiv (\mathbf{R}_0)^2$

$t = 4 : (np)^3$

- On day $\mathbf{t}$, number of new infections:

$(np)^{t-1}$


All Variants Combined
ONTARIO
source: covid19-sciencetable.ca
R(t)
Effective Reproduction Number R(t)
2.00
1.00
0.50
10-Dec-2020
24-Dec-2020
7-Jan-2021
21-Jan-2021
4-Feb-2021
18-Feb-2021
4-Mar-2021
18-Mar-2021
1-Apr-2021
15-Apr-2021
29-Apr-2021
13-May-2021
27-May-2021
10-Jun-2021
24-Jun-2021
Date

Example 5: Spread of infections

$t=1$: 1 infection

$t=2$: ——— (np)

- **Excel/Google sheets**

- On day t, number of new infections: $(np)^{t-1} = (\mathbf{R}_0)^{t-1}$

$t=3$: $np \longrightarrow (np)(np) = (np)^2$

$t=4$: $(np)^2 \longrightarrow (np)^2(np) = (np)^3$

$\vdots$

Clicker question 1.3

- Suppose day $t = 1$ begins with 10 infections.

  How many new infections will there be on day t?

  (a) $(np)^{t-1}$

  (b) $10(np)^{t-1}$

  (c) $(10np)^{t-1}$

  (d) $(np)^{t-1} + 10$

Solution to Clicker question 1.3

- Suppose day $t = 1$ begins with 10 infections.

  How many new infections will there be on day t?

  (a) $(np)^{t-1}$

  (b) $10(np)^{t-1}$ ✓

  (c) $(10np)^{t-1}$

  (d) $(np)^{t-1} + 10$

$t = 1 : 10$

$t = 2 :$ — np

$10np$

$t = 3 :$ $10np$

each — np

$10np(np)$

$= 10(np)^2$

$\vdots$

$t :$ $10(np)^{t-1}$

Example 5: Spread of infections

Excel/Google sheets activity: Spread of a pandemic $(np)^{t-1}$

- **Think and Explore:** In this model, how do lockdowns affect the spread? How do vaccines affect the spread?

n: lockdowns $\downarrow n$

p: vaccinations $\downarrow p$