module_prob2_lecture12

Fri, 1/21 12:45PM 15:27

SUMMARY KEYWORDS

probability, bayes rule, stock, update, recommended, bayes, belief, hot, point, based, called, updating, rule, buy, formula, accident, stock market, midterm exam, numerator, case

SPEAKERS

Sumon Majumdar

S Sumon Majumdar 00:04

Since the last clip, I introduced you to the idea of Bayles rule. And we also developed mean formula for, for the Bayes rule. So I'll start this clip by again going over this formula, and then introducing you to another interpretation of the Bayes rule and doing an example. So the basic formula we had was, what we're trying to find is probability of B given A, right? And this is at the bottom, it's probability of A, right and at the top, it's the reverse probability, which is A given B times the probability of B. And then we asked the question, that if we're trying to devise a derive the formula for probability of A, now he can occur either because of the right or it can occur due to be competent. So the so the basic idea is that See, suppose someone has an accident. So probability of is the P of n is the probability of having an accident. Now you can have an accident, either because this person was binge drinking. So this was a given B. Or it could be because the person was sober and just had an x. Right? So So probability of A can therefore be looked at as probability of A given B times probability of B plus probability of A given B complement times probability of B complement. And this gave us the the general formula base. Now, let me give you, as I said, a different interpretation of Bayes rule. Right. So earlier, I talked about Bayes rule being the reverse condition, right? We started out with probability of A given B, and we're trying to derive probability of B given another idea of Bayes rule, does that see, what we're trying to figure out is the probability of this particular event B. Right, given A given A. So we already have a probability of B here. Right? Now, given that we have new information, we have information that error has occurred, right? How do we change our, our probability starting from P of B, now we move to P of B given A. So this idea is called an updating due to new information. So we've started with p of beads, and this is called the prior that means this is before, right? And then we get some new information, the new information is that given a and then we update our belief, and this is called the posterior. Right, so the prior is what we started with. Posterior is what we ended up with. And this sort of updating occurs all the time in real life. So So for example, you go into you take a course, right? Initially, before the start of the course you ask your friends, some people have told you, oh, yeah, this course is going to be easy. Some have said, No, it's a tough course. And based on their various opinions, you have a belief, right? About how difficult or easy the courses will be. Right? So that's your prior. And then you take that, like you're midway through the course the midterm exam comes along, and B and weather and based on your results from the midterm exam. Now you update your belief about how difficult or easy the courses are, how well are you going to finally be

doing in the course. Right? So this is now your updated belief or the posterior. Right. So this is in the context of an exam or a course. Right? But similarly, people do updating all the time based on news, right? So for example, you're trying to form an estimate of how well the stock market is going to do next month or in the future, right? And you have a jobs report which comes out okay, which gives you a either a rosy picture or not a gloomy picture. So based on that you update your belief about how the stock market is going to do that tomorrow or in the next month. Similarly, Before the stock market opens, every morning, people talk talk about what happened in the European markets because there is opened earlier, right. And based on what happened there, you update your belief about how the stock market is going to do here. So this sort of updating takes place all the time. And what Bayes Rule gives you is a way of formally doing that update. So let's do an example along these lines. So So for example, here's a stock s that you're considering buying, right? And you don't know if it's a hot stock, or if it's a cold stock, hot stock is one, which is likely to rise in the future. And the old stock is one which is likely to fall in the future, right? So you have a prior probability that the stock is hot of 40%, right? So you have a belief that the stock could be hot, like 40% chance that it's going to go up 60% chance that it's going to go down. And then let's see Dr. Stock who is supposedly some stock guru, right, gives it a buy recommendation. Now, based on this stock gurus, recommendation, how do you update your belief? Now, you know that, like, you know, the stock Guru is the stock is very good, but sometimes he has also been wrong, right? He is not a perfect seer. Right? So he has sometimes recommended buy, and then the stock has fallen like in this diagram here. So how do you take that into account in updating your belief? So, in particular, right in the past, so if the stock is hot, right, here's recommended by with probability point eight, right. So that means 80% of the chance when the stock was hot, the stock guru has recommended buy. But the problem is that sometimes when the stock is cooled, even then he has recommended buy. And that has happened in 40% of the time. Right. So Dr. Stock is not is not perfect, right? Sometimes he has recommended a call stock and sometimes and more of the times right 80% As opposed to 40%. He is recommended by for a hot stock. But sometimes he has also done it for a cost. Right now, given his bio recommendation. How do you update your belief, right? And that's exactly where Bayes rule comes in. So what you're trying to see is what's the probability that the stock is hot, given that the recommendation is for buy? Okay, so what we're going to make use of is the Bayes rule, right? So it's probability of again by given hot turns probability of hot divided by the probability of y. And now see this probability of by red because here's sometimes recommended it for a hot stock, sometimes for a call stock. And so we need to take that into account. Right? So so it has been probability of buy given hot times probability of hot plus the compliment possibility that he is recommending buy even though the stock is called times the probability that the stock is called. So that's our base formula for Bayes rule. And what we need to do in this particular context is put in the numbers for all of these. So first thing, let's look at the numerator. What's the probability of buy given hot that's point eight? What's your prior that the stock is hot. So that's point four. And so one thing about Bayes rule is see whatever is this term on the numerator, this is also the first term on the denominator. So I can just put in this is point eight, this is point four. And now let's look at this part. Right? So what's the probability it's by given code? We find that here, so that's .4 times the probability that it's the stock was a dud, right? It's a cold. So if the probability of it being hottest point four, then the probability of it called as the complementary probability, which is point six. So it's this times this, this times this plus this and, and let's just calculate that. So on top is, point three to, this is point three, two, plus point two, four. So that's 32, or 456. And if you calculate the salt, this turns out to be point fives. So based on Dr. Stock's recommendation of a buy, you update your probability. Remember, initially, your probability that it's a hot stock was 40%, right? Less than half. Now, given the Dr. Stock has said in a buy is recommended by you update your probability and it goes up 2.57. Right. So this is the posterior probability. Or updated probability based on what a doctor stock I'd recommend. Now the main

thing here is because see, the doctor stock is not always correct. So sometimes he also recommends a cool stock. Right? So that's the problem. Right? Now, what if Dr. Stock was in fact perfect, right? So that he only recommends buy when the stock is hot, right? He recommends buy given hot, that's one. And if the stock is cold, the probability of his recommending is zero, right. So this is now no longer like stock guru, he is like us the orc right. So he's is a perfect predictor, right. So what would happen in a case like this? Right? Again, if you use the Bayes rule, in a case like here, so see now the probability by given hottest one, probability of hottest point four, this is one, this is .4. Now, he never recommends a bi when it's cold. So this is zero, this is .6, right? So this comes out to be point four, and at the bottom, so see, this part is zero, right? So it's point four over point four, so is equal to one. So if Dr. Stock is a perfect predictor, if this part never happens, right, this part is zero, then this term and this term are the same. And they would be one divided by the other, you could cancel those two up, and it would be one, right. In that case, if Dr. Stock could perfect predict perfectly in this case, anytime he says by you know that this must be a hot stock. And so your updated probability is exactly one. And that's always true of Bayes rule. Right. But the more interesting cases, and the more realistic cases is, is cases, like where Doctor stop doesn't predict perfectly right? Sometimes he's right, sometimes he's wrong. The same thing, going back to our accident and drunk driving example. Right? That example is interesting is because not only two drunk people have accidents, sober people also have accidents, right. So now, given that someone has had an accident, you cannot immediately surmise that that person must be drunk, right? Because there is some probability that he could be sober and has had an accident. Similarly, here, right, there is some sorry, going back to our real example of Doctor stock, there is some chance that even though he has recommended a bike he could be, could be wrong. So but Bayes rule exactly helps you in this sort of situations, that to update your beliefs. And you use it all the time, because, for example, you do a test, right? So medical tests, right? medical tests are not 100%. Right. Sometimes they miss it. Sometimes they go the other way. So the question is, based on the medical test, what's the probability that one actually has a disease? Right? Again, you can use Bayes rule To figure that out, right? So, so, Bayes rule can be used in a lot of different contexts. Where is the weather is this uncertainty? But given new information, how do you update your beliefs? That's the perfect scenario for use of the base.