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SPEAKERS

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So we'll continue talking about conditioning. But this time, I'll be talking about reversal of conditioning. That is, instead of growth probability of A given B, how do we find the reverse, which is probability of B given A. And this will lead us to the idea of Bayes Rule, which has a really famous rule in in probability and statistics. So, so what's the main idea here? Right? So the idea is that suppose you know, probability of A given B, how do you find the reverse conditional probability? That is, how do you find probability of B given A? So first of all, right, these two are not the same. Right? So don't make that mistake. So probability of A given B is not the same as probability of B given A. And so I'll talk about examples of that very soon. Right? But the basic idea is that suppose you know, one of these. So you know, probability of A given B, how do you find the reverse, which is probability of B given A? So let me give you an example, where this sort of thing comes up, right? And there are lots of other examples where this sort of thing comes up as well. But let me begin with this example. So we know that driving while drunk, right increases the chance of accidents. Now, suppose you know that someone had a crash or had an accident, right? Now, what's the probability that this someone was drunk? So in the terminology of probability, you know that, from research studies, you know, that what's the probability of having an accident? If someone drives while drunk, you know, this probability, the probability of accident given that someone is drunk? Right. Now, the question is that suppose you know that an accident, someone had a crash? What's the reverse probability? What's the probability that this person was drunk? No, given that he had an accident? Obviously, these two are not the same. Right? So probability of an of having an accident when one is drunk. That's different from that. Suppose someone has had an accident, conditional on that, what's the probability that this person was drunk? So lots of uses and applications of Bayes rules in a wide variety of contexts. So let me give you another example. Suppose you see smoke in the distance. Now, what's the chance that it's actually a fire? Or could it be that it's just someone having a big barbecue? So this is similar to the example that I gave you about drunk driving and an accident? So suppose someone has an accident? What's the chance that this person was drunk? Right? Similarly, suppose you see smoke, what's the chance that there is actually a fire? So we are talking about conditional probabilities conditioning on there being smoke, what's the probability that there is fine? So this is exactly the context of Bayes rule. Bayes rule is also used by banks and insurance companies to evaluate borrowers are people

looking for insurance? So for example, if you go to a bank asking for a loan, they'll probably ask you for your credit history. So what they're looking for, is your history of credit payments on time in the past. So what they looking for is how often have you missed payments in the past? Or if not, right? And based on that information or conditioning on that information, they update their beliefs about how reliable a borrower Are you? Because see, like people could have missed payments for various reasons, right? It could be because they are in financial difficulty, or it could just be that they forgot about the duty. So how do you update your information about whether this is a reliable borrower or not? based on his or her credit history, similarly, insurance companies, they ask for driving history, and they try to look at the record of maybe tickets in the past or accidents in the past. And they wish to based on that conditioning on that information, they update their belief about whether one is a rash driver or not. And in fact, based on these, they said the premium rates bank said the interest rates. Similarly, Bayes rule is used all the time in artificial intelligence. And I'll be speaking about talking about more of these examples later on in the morning. But, but the basic idea is that Bayes rule is a super useful rule that is used in a big, big, big variety of contexts. So, so this Bayes rule, this was developed way back in the 17, something by Reverend Thomas Bayes, Thomas Bayes, right. And, of course, they're like, given that this is a historical event, there is some dispute about whether it was this Reverend actually who discovered it, or there was someone else who discovered it, and but mostly attributed to him. But this is a pretty famous rule in probability. And it's very, very useful. So what's the basic idea behind Bayes rule? Right? So remember, we saw what's the formula for probability of B given A, it's the, it's the probability of the intersection of A and B over the probability of A? And to make things simple in the context of our example about drunk drunk driving, right? Let's think of B as booze and his accident, right? So if we're looking at the probability of B given A, right, so you're looking at the set of a and what's b condition on belonging to that set? It's it's this area, right? The common area relative to the area of it. So the probability of the intersection of A intersection B divided by the probability of now we have this right? And what we are interested in is looking at the reverse, right? What is the probability of A given B? And how do we relate these two things to each other? Now, if you look at probability of A given B, right, so that's now you're looking at the set of B. And looking at this common area, the people have B, who also belonging to a relative to the whole area of B. Right, so this is probability of A intersection B divided by the probability. Now see, there is something common between both this and this, which is probability of A intersection B. Now, if you look at this here, right, so probability of A intersection B, is given by if I now cross multiply and bring it up here is probability of A given B times probability. So what we have is probability of B given A is probability of A intersection B over P of A, right, and this is what we're trying to find out. But now C probability of A intersection B is probability of A given B times this P of B. And that we got that by cross multiplying. So what we're going to do is that in the formula for P of B given A, we're going to replace the P of A intersection B by this. So probability of B given A is probability of A given B times probability of B divided by probability of A and C this way we get like we know this, right? So what's the probability of having an accident given that one has had boosts that one is drunk right and this is what we are trying to figure it out. So this gives you that way to connect P of B given A the one that we are trying to figure it out with what we already know. So this is Bayes rule, almost there. We have to take one more step and that will give us the final form of the Bayes rule. So this is what we already have, right? So probability of B given A is probability of A given A, B, right? times probability of B divided by B. So now let's think about this bit rates of probability of A. So how can he occur? Right? So what we're interested in is the marginal probability of A. So marginal probability of A means how can a occur, so it can occur either due to be occurring, or the compliment that B compliment occured. So probability of A is either probability of A given B times P of B. Or its probability of A given B complement times probability of B complement. So a can occur either this way, so that means probability of B times probability of A given B, or it can occur this way,

because B complement is awkward. And it's probability of A given the call. Right? So if we add those two up, that is probability of A. So this is our law of total probability, the marginal probability that we found before all of those coming into play here. Now what we're going to do to get the final form of the Bayes rule is that we're going to replace this P of A by this thing here, right? So if we do that, this gives us the final form of the Bayes rule. Right? So probability of B given A is probability of A given B times P of B. Right? So that's the same here. Okay, divided by P of A, and we're going to replace the P of A by this marginal probability here. That's P of A given B times P of B, plus P of A given B complement, times P of B. And this here, is the Bayes rule. Again, why is it important? We know this, right, and this is what we are trying to figure out. So what we're trying to figure out is the probability that someone had booze given that he had an accident, right, so that's what we're trying to figure out from something that we already know that the probability of accident goes up when you have booze. So this is our famous Bayes rule. And let me now apply it to the particular example that we were looking at. Right? So let's say you know that if you're so if one is so bad, the probability of an accident is point one, so 10%. But if one drinks so this is booze, or bench, or however you call it, it's a probability of accident when you have when you're drunk that point five. And this is actually based on real studies, which show that the probability of an accident when you're quite drunk, that's about five times more than when you're sober. And let's have the probability of having binged or probability of having a being drunk is point two or 20%. So what we're going to do is make use of this data to answer this question, make use of these probabilities to answer this question that suppose an accident occurs, what's the chance that this driver was drunk? So going to our Bayes rule, right, so what we are trying to figure out is that given that there was an accident, right, what's the probability that this driver had pitched? So let's use make use of this formula. Now we know probability of A given B. So probability of an accident given benched. That's point five. And what's the probability of benched, right? That's point two. Looking at the denominator, this part is the same as the numerator. So that's point five, this is point two. Whatever the other part, so what's the probability of having an accident of A given B complement? So what's the opposite of bench? That means sober? Right. So what's the probability of an accident when you're sober? That's point one. Time what's the probability of B complement which is the probability of being sober so that's the number meaning probability that if you're not benched, right, that's point two. So probability of B complement, that's one minus point two. So this whole point. Right? So what's the probability that one has been given that one has an accident is that going to be point five times point two divided by point five times point two, plus point one, times point D. So this is point one. This is point one, plus 0.08. So that's point two, one divided by point one, eight. And this turns out to be this turns out to be, let's write it here, is equal 2.56. Right? So using the Bayes rule, you have been able to figure out this probability that given given that someone had an accident, what's the probability that he had been drunk is point five, six about 56%. So see, initially, you may have thought that there's a 20% chance that someone is drinking. But now given that this person had an accident, then that probability goes up to 56%. So this is how you make use of Bayes rule to figure out this sort of a conditional probability, the reverse conditional probability. So Bayes rule is very, very important in probability. And we'll see lots of other applications of Bayes rule, but but in this clip, I introduced you to how you go about figuring out the Bayes rule. And in the next clip, we'll do one other application of that