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So let's continue with our example of an unmarked party platter of avocado and cheese sandwiches. And the question is that suppose you were to go second, what's the probability that you'll be able to pick up an avocado sandwich. So, I laid out the, the probability tree at the end of the last clip and asked you to fill out the probabilities on your own. But let's do it here and check our results together. So, for the serial number, there are three avocado and seven cheese sandwiches. So for the first person, the probability of choosing an avocado sandwich is three out of 10. And of a cheese sandwich is seven out of 10, because there's seven cheese sandwiches out of 10. Now, given that the first person has chosen avocado, what's the probability of the second person being able to choose avocado? So there are now nine sandwiches left? Right? One avocado has already been chosen. So there are two left. What about cheese again, then total of nine sandwiches left, all seven cheese sandwiches left, so this is seven out of nine. Now, if the first person has chosen cheese, right, then there are all three avocado sandwiches left. So this is three out of nine as the probability of choosing an avocado in the second by the second person. And for cheese, no one has been taken. So that says six out of nine. Right, so this is the probability tree with the probabilities in this case. So now let's go to the question of what's the overall probability of picking an avocado sandwich? If you're picking second, right? So what we're interested in is what's the probability of choosing avocado, right? So this can occur either through here, or it can occur via here. So what we're going to use is the law of total probability. And two things. Remember the two things I mentioned? Earlier in the last clip, you see these two events, the avocado and cheese, right? The a one and a two? These are disjoint events, they cannot occur together. Right. And secondly, they encompass all the possibilities, because they encompass avocado and cheese. Because if now there was a third possibility, suppose there was a chicken sandwich here as well, right? Then what you would have to incorporate is the possible is that third possibility as well. But here, they're just two. So we're good, right? So we're just going to incorporate those two. So what's the probability of this happening? Right, so it's, it's this times this, right, so that's three will return times two for nine. Because this is first avocado, the second avocado, or it could be the first is cheese. The second is avocado. That probability is seven over ten, that this probability times this, which is three ninths. So let's take this, this is denominator 90, that six plus 21 over 90. So that's 27 over 90. And if you divide top and bottom by nine, this is three over 10, which 0.3. Okay, so the overall probability of picking avocado when you pick second is three over 10.

Now one interesting thing that you may have noted is see if you're the first person, then your chance of picking an avocado was three over 10. Now for picking second, the probability of picking avocado is still three okay. So which is a surprising feature, but it just comes out of this law of total probability. In some sense, things average out, right? So the there's some chance that the first person could be picking avocado or there is some chance that The first person could be picking cheese. Right? And those two things average out so as to leave this overall probability of picking avocado, even when you go second, is still going to be the same. In fact, what's interesting is that this example, right? Even if we go third, then your probability of picking avocado is still going to remain three. So let me show that to you. Right. So before I go there, right, so So again, I want to reiterate this idea of the law of total probability. Right? So what are you looking at this the probability of the event that the second person picks avocado, right? So you can break it up into two events, right that the first person picks avocado, or the first person picks cheese, right. And then if the first you look at the probability of the first ship, choosing avocado times the probability of second choosing avocado, given that the first has chosen avocado, right, or you look at the other even, it's the probability of the first person choosing C, and then the second choosing C, given that the first has chosen C. So now let's do what I just mentioned a moment ago, that if suppose you're picking third, right now, what's the probability of picking an avocado sandwich? Now, if you're going third, right, there could have been lots of different possibilities could have occurred in the first two, right? Either the person the first could have chosen avocado, second avocado, or the first could have avocado, second cheese, and the various ones, right? So now, you know, after a point, drawing this probability tree, now you could again do here again, another one, not the one right? After a point drawing this probability tree becomes a little tedious, right? And once you get the hang of things, and you'll get used to this idea of dealing with probabilities in this way, sometimes it's easier to just go ahead and compute those probabilities. compute all the possibilities, compute the probabilities, even without a probability tree. So let's let's do something like that here. So if you picking third, right? What's the, what's the possible outcomes? The first two outcomes, it could be either both avocado, or both? One avocado one cheat first avocado, second cheese, or it could be the first cheese second avocado, or both cheese. And then would have to look at so let's look at the probabilities of that right? So first thing, so the probability of avocado avocado is so so three out of 10. Right? And then it's two overnight? What's the probability of the first being avocado that's three over 10 and the second being cheese, see the nine left? And you choose seven out of nine? Right? What's the probability of CA that the first is cheese, that seven out of 10 and the second one is avocado. That's three out of nine. Right, and probability CC both are cheese. So the first one cheese is seven out of 10. The second one cheese is six out. And now what we're going to do is look at what's the probability of picking out on the third, right? So the first two has been avocado at this point. There are eight sandwiches left, right? So two of them avocado are gone. One is left. What's the probability that the third is avocado given that the first two were avocado and cheese? Again, there's eight left and left, right. One avocado is gone. Right. So there are two left. So that's two out of eight. What about C and E? Again, it's eight sandwiches left? One avocado is gone. So that's to operate? And what about if the two cheeses chosen in the first two? That means you have eight here, right? And then. So none of the avocados are gone. So it's three over eight. And then what we are interested in is the overall probability of the third person taking avocado. Right. So what we're going to do is we're going to make use of the law of total probability, right, so see, we're going to multiply P of A times the probability that the third is an avocado given. Sorry, third is a is an avocado, given the first two were avocados, right. And then similarly, we're going to look at P of AC, and multiply it with the probability of the third is avocado given the first is a second C, and then and do the same thing with C, and so on. Instead of writing out this expression, all like all here, let's just do the calculations. Right? So first thing was the probability of A, so that's six over 90, right? So that's this thing here. multiplied by probability of third is recorded, given that the first

two were eight, that's one eight. Then let's look at AC. So that's 21. Over 90, multiplied by third is avocado, given a C, that's two over eight. There, what else So P of CA, that's again, 21 over 90 times the probability that the third is a given CA, right? So that's two over eight. Plus, the last thing is B of CC, so that's seven times six, so 42 over 90, right? Times that the third is an avocado. Given that the first two are See, see, that's three over. And if you do the math, right, so this will be 216 divided by 720. And if you simplify this, you will see that this will come out to be three over 10. So that's point. So, and this is what I was telling you before that, even if you're going third. So even if you're going Third, the probability of picking an avocado sandwich, it remains the same if you're going second, or if you're going first. But what I really wanted you to take away from doing these examples, is making use of the law of total probability. getting familiar with the idea of conditional probability, and then using that to figure out what's the total probability of So I hope by this time, what's happening is that you're getting more and more familiar with the idea of a probability tree, the idea of conditional probabilities, and thirdly, the law of total probability. So let me stop this clip here. And in the next clip, I'm going to start with a simple example of a probability tree in use, even when events are independent.