module_prob2_lecture3

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Let us next consider another example. This one more from the area of geography. So we know that if there's a dry summer, right, that increases the chances of forest fires. So let's consider an example along these lines. So suppose a summer may be dry with probability .4. And this, of course, affects the pro- the chances of forest fires. So exactly how. So this is given by this following table, that the chance of forest fires when there's a dry summer, that's .6 And when it's a wet summer, the probability is .2. So we have introduced terminology about conditional probability. So just to test you, test your understanding of it. So I would like you to do the following clicker question. So you may want to stop the video at this point, and attempt this clicker question. So hope you got the chance to do this clicker question. So what it asks us is the simple thing, what is probability of fire, given that the summer is dry? So we know that when the summer is dry, the probability of forest fires .6 So the correct answer here is this conditional probability is .6. So now let's use this to actually construct a probability tree and then calculate some probabilities. So if we're going to represent this using a form-using a probability tree, so what we have is that, so the summer could either be dry, or it could be wet. So if it's dry, then so it's dry with probability .4, and it's wet with the remaining probability, which is .6. And now given that it's dry, but the probability of a forest fire is .6, so this is what you had in the clicker question. Right? And the probability of no fire is the complementary probability, which is .5. On the other hand, if the summer is wet, so if I go back to this table here, the probability of a forest fire when it's a wet summer, that's .2. Right? So the probability of fires when there's a wet summer, that's only 20%, .2 , and when it's and the probability of no fire is .8. So now, suppose I asked the question. So what's the so what's the probability of a fire intersection dry summer? So this means and, right? So what's the probability of fire and dry summer? So here's the probability of a dry summer, here's the probability of fire. So remember, this, this general multiplication rule, this will be the probability of dry times the probability of fire given dry. So the probability of fire that's .4. And the probability of fire given dry, that's .6 So that becomes .24. So there is a 24% chance of it being both a dry summer and there being a forest fire. Now, another question could-we could ask is just what's the probability of of a forest fires? Right? Because we know that forest fires don't only occur during dry summers, there's also a chance of the fire occurring if there is a wet summer. Right? So what we are looking for is this possibilities. So how do we calculate that? To see the probability of fire this can come about in two ways. One is that the summer could be dry. Right? And then there could be a fire. So that's .4 times .6 of that possibility. Or there could be another complementary possibility, where the summer is wet. The probability of that is .6 And even then there is a fire, which is this. So it could be either that the summer is wet. So the probability of that is .6 times the probability of a fire in that case, which is .2. So if we add these two up, that's the two ways in which a forest fire can come about. And this, this summation gives you the total probability of a forest fire. So, so the first one is .24, right? So that's this one. The second one is .12. And if we add these two up, that's .36. So there's a 36% chance of there being a forest fire, whether due to a wet summer, or do to dry summer. And you'll see this sort of this sort of this sort of calculations very common whenever you using things like this probability tree, right? Because what are you doing? You see, what are you just looking at? Okay? How many ways can there be fire this way? And this way, right, so let's back track, right, so let's backtrack along this arm. So it's given by this and this, right? So the overall probability of this fire occurring is the .4 here times the .6 here. Another possible way in which there could be a fire is here, right? And this could occur this. And this, right, because of these two events. And so we multiply, look at the probability of that, that's .6, multiply it by the probability of this, which is .2, right? And that's this. And if we add these two up, we get the total probability of a- of a fire. Well, we can- and then find we can take this make this more generally, right? And that's the general idea is that if you're looking at the probability of fire, right, it could be either due to a dry summer, so we have the probability of dry times the probability of fire given dry. So this is my upper arm here, this and this. And, or it could be because the summer is wet, and yet there is a fire. So that's probablility to your fire given wet, right? So this part here is my lower arm, which is this, and it's now, so if I did make this more general, so what do I mean by making this more general? So if I, if I think about the general event, right, so there, it was a fire, think let's think about an event B. Okay. Now, event B can come about in many different ways. Right? So in particular, it can happen. So let's look at this particular probability tree. Right. So what we're interested in is this particular event B. Right? Now, this B can occur in one of two ways, either because A has happened, and then because of that B has happened. Or it could be that that A to has happened. And then B has. Right? So B could occur in either of these two ways. So if you're trying to look at what's the probability of B occurring, it could come about due to this arm here. Or it could occur because of this arm. So if you're trying to calculate what's the probability of B. This could be to this arm here, which is B of A one times the probability of B given A one or it could be due to this arm here, right, which is P of A two times the probability of this arm, which is probability of B given it. So we can calculate the probability of B using the two arms. So this thing here, what I've just derived this what's called the law of total probability. So, if I'm, again, go, let me go over it once more, right. So again, B can occur due to the occurrence of A one. Right? So B can occur due to its intersection with A one, or B can occur due to its intersection with A two. Right, and if we calculate up the probabilities for each of them, right, so this first part is probability of A one times B given A one, so probability of A one, and this is probability of B given A one, or it can be due to the second arm, which is P of A two times P of B given it. So we can break up the probability of an event into the component arms through which it can occur. And one very particular way of breaking out is C, I've chosen any arbitrary E one and E two. Instead of choosing any arbitrary E one and E two, one possibility is that I can break it up through just A and its opposite, which is a compliment, right? So B can occur either due to the occurrence of A or due to the occurrence of A compliment. So if we then use similarly, the law of total probability, right, so the probability of this is going to be P of A times B of B given A plus it's the probability of this part, which is going to be the probability of A complement times probability of B given A complement. So this is called the law of total probability. And this is going to be pretty important later on when we do Bayes rule. So please do try and recall this when we are when we are going to be talking about the stream, right? But what it comes about is from the simple idea that from, from a probability tree, that if you if you're trying to figure out what's the probability total probability of B occurring, right, it can be due to the that of A or do to A compliment, right? So this could be A and this could be A compliment. Right? In our example, it was dry summer, the complement of that was wet summer. Right? And then so you look at the probability of A time's the probability of B given A and plus the probability of A complement, and times the probability of B given A complement. If you can, so that that's that's this thing here. Right. And in fact, you can even take it a step further, right. So here, I've broken it up just into two, A one and A two. Right, you can do the same thing. Instead of two, you could have N, right, so say B can occur through one of N channels. And those N channels are A one, A two, so on up to a M, right? So the way B can occur is due to its intersection with A one, or due to its intersection with A two or due to its intersection with A n. So each of these probabilities is for this A one intersection B, it's P of A one times P of B given you one. Similarly, if you're looking here, it's going to be P of A two times P of B given it. And if we continue that way, for P of A N intersection B, it's P of A N times P of B given A. And now succintly. Remember the summation sign that we had done in a module earlier, this will just you can write this as summation from one through N, P of Ai times P of B given Ai. See, any of these terms, this is P of A two times P of B given A two. So this is called the law of total. So let me stop here with this clip. I'll pick it up here with extending our example of the summers and forest fires to a case where they can be A one A two and A three.