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SPEAKERS

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So welcome to another module on probability. The second module of probability is going to cover an important concept, that of conditional probability. And then, in the second half of the module, we're going to cover an important but often misunderstood concept called Bayes rule. So in terms of the outline for this module, I'll start with reviewing some of the important concepts that we learned in the previous module on the basics of probability. Then I'll be moving on to the main feature, which is the idea of dependence of events and conditional probability. And here, I'll be covering two important ways of representing probability, one by using probability trees, and secondly by the use of what's called two-way tables. And then, as I said, we're going to end the module with understanding this important concept of Bayes Rule. So let's begin by looking at, by reviewing some of the important concepts that we learned in the previous module. So we started with some terminology, the the idea of sample space is that it encompasses the all possible outcomes. While the idea of an event is some subset, or some smaller part of the sample space, an event is something that you're interested in finding the probability of. So for example, in this diagram, we have S, which is the sample space, which is the set of all possible outcomes, and the set A is some smaller part of the sample space. And so for example, if S is the set of all days in a year, A may be the set of rainy days. And the complement set is the part of the sample space which is outside of A. So in this particular case, if A the set of rainy days, that the complement set would be the set of days which are not raining, or which we may call dry days. And we denote that by a compliment. So the next thing I introduced is the basic axioms of probability and the three basic axioms. So first thing is that the probability has to lie between zero and one. Right? You cannot have probabilities being negative or being greater than one. And if you look at this diagram, right, so what it tells you is that probability start from zero, and go up to one, right, so zero is when something is impossible. And one is where the probability, where it's a certain event, it's almost sure, it's sure to happen, right. And in between, like here is the case of a coin, which has an even chance, half and half chance of heads and tails, right. So anything less than .5, and moving towards one, moving towards zero, right, those are becoming less and less likely to happen. On the other hand, if you look at events, which are above half and moving more and more towards one, these are events, which are more likely to happen. And then we had this third axiom, which said that if there are two sets, which are disjoint, which have nothing in common, then the probability of the union is just the sum of their probabilities. Now talking about that, right, we have these two things come up. One is what's called a union of two events. So the union of two events means either A happens or B happens or both happen, right. So in this Venn diagram, the union of A and B encompasses A and B, right and also the middle part right. So this is typically represented by saying that either A occurs or B occurs or both. On the other hand, when we talk about the intersection of two events A intersection B, that means this is the case where both A and B occur. Right. So the intersection is this part here, the common part between A and B. Now moving along, right, so from the basic axioms, we have some useful rules of probability. The first one is the compliment event. So that compliment rule. So what that means is that, if you're looking at the probability of the compliment event, it is one minus the probability of A, so the probability of A complement is one minus the probability of A. The second rule is what's called the addition rule. So if you're looking at the probability of the union of A and B, right, and these A and B need not be disjoint, the way you can find it is finding out the probability of A plus the probability of B. Now, when we have done this, we have double counted this common area, so we need to take that out. So it is P plus A plus P plus P of B minus P of A intersection. So that's the addition rule. And then, we can also do the addition rule for more than two sets. So here, it's three sets A, B, and C. Right. And again, if we add a P of A plus P of B, plus P of C, so what we're doing is we are double counting these common areas. Right, so we need to remove those. Right. But when we remove these, we remove this thing here as well. Right, so initially, when we add P of A plus P of B plus P of C, we, we triple counted this middle area, right? Now we have taken it out three times. Right, so we haven't at all included this, this area here. So we need to put that back. So that's A plus B of A intersection B intersection C. So this is the addition rule for three sets. And this, you can extend this to four, five, and so on, right, by essentially following the same logic. And then towards the end of the last module, we talked about what's called independent events, where there is no relationship between the events. So when one occurs, this has no impact on the probability of the other event occurring. In this case, the probability of the intersection of A and B is just the product of their probabilities, which is P of A times P of B. So this is the idea of independent events, there's no relationship between them. So the opposite of that is the dependent defense where there is some relationship. So for example, the fact that A occurs may influence the occurrence of B. And this is what we are going to really delve into a lot in this particular module, right? And we are going to talk about what's called conditional probability. And so this is the new thing. So this is P of B given A. Right? So we say this is B, given A. So what this means is, what's the probability of B occurring given that A has? So this is the basic idea of conditional probability that if something happens, how does that influence the probability of something else occurring? So let me show it to you by an example. Right? So for example, in the NBA people take free throws, right players take free throws, and usually they take two free throws. And what people have often wondered, does what happens in the first free throw? Does it influence what happens in the second free throw? And so there were some scientists who studied this and they looked at free throws in the NBA over the period 2005 to 2010. And what they found is that in the first free throw, the success rate was about 73%. But more interestingly, what happens in the first free throw seems to influence the probability of success in the second free throw. So, the success rate on the second throw depends on if the first was a success. So if the first was successful, then the success rate on the second is 79%. So the success rate goes up. So it goes up from 73 to 79%. But if the first was unsuccessful, then the success rate on the second, it drops slightly to 70%. So what we can talk about is the probability of the second being a success, given that the first is a success. This is .79. Right? So if the first was successful, than given that the first is successful, the probability of the second being successful is .79. But the probability of the second being successful if the first was failure. This is only .72 or 72%. So this is the idea of conditional probability. So what we'll do is that I'm going to stop the clip here, but in the next clip, I'll pick up exactly from here and delve into the consequences of this idea of conditional probability.