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SPEAKERS

Sumon Majumdar



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Okay, let's do one more application where we make use of independence to figure out probabilities in a particular context. This is the context of smoke. In 2017, the University of Waterloo did a study in which they found that 15% of Canadians are smokers. So now suppose you serve 10 people at random. And the question is, then what is the probability that none of them are smokers? Right? So, so first of all right, if you serve a person at random, so with probability P , this is equal to .15 is that this person is a smoker. And let's call this B prime is equal to .85, that he is a non smoker. Right? That's the remaining probability, which is one minus .15. Right. So that's .85 chance that he is a nonsmoker. So what's the probability that of these 10 people that you're serving, all of them are non smokers? So what do you are essentially looking at this, what's the probability that the first person is a nonsmoker? Second person is a non smoker, third is a non smoker, so on all the 10 persons are non smokers. So if, if you, if these are all independent, right, if you have done the survey independently, so then this must be the probability of n times the probability of n times the probability of So on to probability. Right. So there are 10 people, so this must there must be 10 terms. Now, so the probability of being a nonsmoker is .85, right, so this will be .85 times .85, times .85 and so on 10 times. So which will be actually .85 to the power of 10. So this is the probability that none of them are smokers. And if you calculate this using a scientific calculator, you'll find that this is .19687. Right? So it's very close to .2 or 20%. So that means if if we survey 10 people at random, this 20% chance that none of them are smokers. Now, what about suppose I take the question one step further? And not one step further, a slightly different question that exactly one of them is a smoke, right? What's the probability that the stand persons have exactly one smoker and nine non smokers? So, so out of the 10 people that you've surveyed? What's the probability that exactly one is a smoker? So what are you really looking for is a situation like this, let's say the first person is a smoker, all the others are non smokers. Or it could be that the second person is a smoker and all the others are non smokers. Or it could be that the third person is a smoker and the all the others are non smokers, right? And so on. So the event that only exactly one of them is a smoker is going to be a combination of all of these things. So first thing what we need to figure out is the probability of this happening, right, then what's the probability of this happening and so on? And then think about combining. So let's take the first step. Right? So what's the probability that the first person is a smoker and all the others are

non smokers? So these are independent events. So it will be the product of the probabilities, it will be the probability of S times the probability of N times the probability of the next and and so on. Right, this is because of independence. You can do this right now. See, what's the probability of being a smoker, that's point one, five? What's the probability of being a nonsmoker that's point eight, five. And how many of these terms are here? So the total of 10 people, right one of them is a smoker. So that means the remaining nine are non smokers. So this will be .85 to the power 9. And if you multiply the if you're in use a calculator, this will be 0.3 fourths. So remember, this is the probability of this event here, right? This outcome? What about this? What about the next one? Right, so the next one is that the second person is a smoker, and the others are all non smokers. Again, because this is independence, it is probability of n times probability of S times probability of N. So and again, it's a product of 10 terms, right? One of them this one is .15. All the others, these ones are all .85, and how many of them are here, there is nine of them. So again, this is going to be 0.0347. So see, for all of these the probability of this occurring is 0.0347. This one is the same, this one is the same and so on. Right? And so the main thing if we want to figure out what's the probability that exactly one of them is a smoker I have to figure out what's the probability of all of these events together so what's the if I'm to figure out what the probability that exactly one smoker okay, it will be the probability of that first event union the probability of the second event union the probability of the third event and so right now, see, this event and this event are disjoint right this is that the first person is a smoker remaining non smokers, this one is that the second person is a smoker and the remaining non smokers This is that the third person is a smoker remaining non smokers and so on. Right. So, again by the basic axioms of probability, this will be just the sum of the probabilities of all of these disjoint events. So, how many such events are there, so, see this is that the first person is a smoker, this is the second person is a smoker, this is the third person is a smoker and so on. Right. So, the 10 people all together in the survey, so, either it can be first or second, third or fourth or the 10th Right. So, that means there are 10 of these events and each of these the probabilities are the same. So, that, so, the overall probability will be 10 times the probability of any of these so, that will be point three fourths. So this is the probability that in this sample of 10 individuals, there is exactly one smoke. So that's 34.7%. Again, we did exactly something like what we did with the boys and girls, in a family of three here, we have used exactly that same sort of logic, we broken up this event into all the possible ways that this can occur, and then added up the probabilities of those. And not ones who have this. Right. So we wanted this question, we've answered this question. What about another connected question that in the survey of 10 people, what's the probability that at least two of them are smokers? Read the scope question carefully. It says at least two of them are smokers. So it can be two, it can be three can be four in the fall five. So so the easiest way to deal with this right, so if you want to look at this, is let's look at the complementary. So if you have at least two of them, at least two smokers. So let's look at the complementary event. What's the complementary when either no smokers? Right, or one smoke? Right, and we've already calculated the probability of this happening. See, we did this, this first one, right? That none is a smoker, that probability is point two. What's the probability that there's exactly one smoker? We just calculated that that's .347. Right? And so the probability of at least two smokers will be one minus the probability of this complimentary event, right? So if we add these two up, it'll be .547. So the probability of at least two smokers is one minus .547. So that'll be something like 453. So 45.3% is the probability that in the sample of two, there is a key, or in the sample of 10 people, there is at least two smokers. Right? So again, if you see how you how we figured out these probabilities, right, so we made use of independence to figure out the probability of each of the ways that this could occur, right. And then we use the basic axiom of probability that we can add up probabilities of disjoint events to figure out though, from so, so, so far, ready have learned the compliment rule, the addition rule. You've learned how to figure out probability of union of sets, right? And you also

learn the independence rule that if you have a bunch of independent events, right, the probability of them occurring is just the product of their probabilities. And these are the tools that we have been using to figure out probabilities in lots of different contexts. Right? So if you remember these four rules, you will be in a good position to figure out probabilities in different contexts. And so in the next clip, I'll start with the clicker question where you will have to make use of some of these routes.