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SPEAKERS

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So in the last clip, I talked about the independence rule with a product with a probability of two independent events occurring together is probability of one multiplied by the probability of other, or if the probability of a bunch of independent events occurring together is just the product of all their individual probabilities. And we were using that to figure out probabilities in this particular example. And, and I asked you to think about this that affirm the is a family with three children, what is the probability that exactly two of them are girls? So so what we are looking for is the probability that two of them are girls. And the third one is a boy, right? Because I want exactly two girls. And so the remaining one must be boy. Right? So what's the probability of this, so the probability of this is going to be because these are independent, right? That makes life really easy for us, because the probability of independent events is just the product of the probabilities. So it's P of G times P of G times P of B . So probability of a girl, the first one mean girl is half probability of the second one being a girl is half. And the probability of the third one being a boy, again, it's half. So that's one. But see, so this is so GGB is the case, when you have the first child is a girl, second child is a girl. And the third one is a boy. But this is not the only way you can get exactly two girls. But the other possibility is what if the first one is a girl, the second one is a boy. And the third one is a girl? So that's another possibility. What else is there? What about the first is a boy, the second is a girl, the third is a girl. So I think we've covered all these possibilities, right? So this is the third child is a boy, the second child is a boy, or the first child is a boy. So these are all the possible ways in which this family can have exactly two children being girls, right, so we have to also calculate the probability of, so we've already calculated the probability of that that's $1/8$, right? Similarly, if you look at the probability of G, B , and G , that's also half times half, times half. Right? That's also going to be $1/8$. And what's the probability of B, G, G , that's also half times half times half. That's funny. So, so if you look at if you look at the probability that that exactly two of them are girls, right, so this consists of these three possibilities, GGB , GBG , and BGG . Right, and see, all these are, are disjoint events, right? So if this occurs, this cannot occur, or if this occurs, this cannot occur. So the what we're looking for is the probability of this union this union this, right, so because what we're interested in is exactly two girls, right? And two girls can either occur this way, or this way, or this way. So what we're looking at is the probability of the union have three sets, right? Or three events, it but all of these are disjoint events. There's nothing common between this and this right. If this occurs, then this cannot occur. This occurs then this

cannot All right. So you remember one of the basic axioms of probability is that if there are disjoint sets, then the probability of the reunion is the sum of the probabilities. So this will be the sum of GGB plus probability of GBG, plus the probability of BGG. Right? Each of this, this is one eight, this is one eight, this is one eight. Right? So that means together, this is three. So this is actually .375. So, in a family of three children, the probability that exactly two of them are girls, is 37.2%. And the thing to note here is see, I have used two things here. One is the idea of independence to figure out the probability of any of these. Right. And secondly, I've used one of the basic axioms of probability that this the event that it it's exactly two girls, it can occur in one of three disjoint ways, either this, or this, or this, right? So the probability of that the union is is the sum of the three probabilities. So that's the sum of $1/8$, $1/8$ and $1/8$. So that's three. So in this family, so the final answer, in terms of the probability of, of this family having exactly two girls is three. So here, I have a clicker question for you. So you may want to stop the video here and turn the clicker question and then resume after. So I hope the clicker question wasn't too difficult. And what it asks you is which of the following events are independent? So is parking illegally and getting a ticket? Now, these two are not independent events, because if you park illegally, you have a higher chance of getting a ticket. What about the second one? Owning a phone and being a hockey fan? I cannot think of a relationship of whether if you're, if you're a hockey fan, you're more likely to own a phone or vice versa. Right? So I cannot think that one influences the other. So my guess is that that's the right answer. But let's check the other two. What about a dry summer and forest fires? Obviously, there's a relationship if, if it is a drier summer, then there's more chance of forest fires. And the fourth one is buying more lottery tickets and winning the lottery. Again, if you buy more lottery tickets, then your chances of winning the lottery goes up. Right? So those two are also related to each other. So here, the correct answer is this is b, which is owning a phone and being a hockey. Now, this idea of independence is also important not just by for figuring out probabilities, but but it also has implications, right? So remember, I talked earlier about the implication of stock prices being a random walk, okay. Now, another implication of independence is this idea of what's called the gamblers fallacy, right? So the gamblers fallacy you can do it in many ways, right? One of the classic ways is something like this, the last 10 flips of the coin with tails. So I'm surely now do for a head. Or if you look at this cartoon, right, it says, I know the last five scratch off tickets did win, which means this next one must be a winner. But of course, if these tickets are independent events, the fact that the last five were duds, doesn't mean that it changes the probability that the next one is going to be a winner. But so you shouldn't believe it is. A lot of people actually say that. So people have even gone so far as to say the law Vegas is completely built on gamblers fallacy. Do you go into a slot machine and you pull it and 10 times 20 times nothing comes out? You think that? Oh yeah, so I must be due for a big one extra. Right? But that's a fallacy because every drop that slot machine is independent. Right? So no matter if the last 10 were dad's or last 10 was successes, right? It doesn't change the probability that next one is going to be a success or a failure. Now you see this in the same sort of fallacy played out in other contexts as well, right? So for example, people often say, Oh, this player hasn't scored a goal in the last 10 games. So he's surely do for one. Right? Again, if goal scoring his independent events, the fact that he missed in the last 10, doesn't have any implications for his chances of winning another goal in the next one or not? Right, same thing, people say last three winters were brutal. We are now due for a mild one this time, again, if every winter is an independent event, so the fact that the last three were brutal, doesn't change the probability of this one being brutal or a while. Now, the same has an impact also for random sampling. Right? A lot of times, if we want to find out about the let's say, the efficacy of a vaccine, or, or a medicine, or you want to do is you do, do do a trial over a random sample of people, right? Or if you want to find out people's attitudes towards a particular policy, or whether people own cars or smoke or do something else, right? You sample a large group of people and try to figure out what fraction of them own something, or are

smokers or do something else, and so on. Right. But the idea behind sampling is based on two things. One is this idea that, remember, I talked about earlier, this frequentist interpretation of probability that if you do something, a lot of times you are going to the fraction of times it comes out to be success, or it turns out to be a tail, or it turns out to be a six, right? This is going to approach whatever is the true probability. Right? So the idea of sampling is that if you sample a whole bunch of people, then you're going to get the estimate of what it is actually there in the population. Right. Second thing that is important in sampling is that you do this independently, you don't want to ask two people side by side, right? Because what can happen is what I say may influence what the next person says. Right? So for example, if you ask, if you ask a bunch of people, do you believe in climate science, right, what one person says can influence the response of the others? So typically, in random sampling, you want to do the sampling separately from one another, because the responses could be dependent. So there are big implications of, of independence. Even in the real friend, one of them, as I said, was this random walk hypothesis of stocks? They by the way, that particular book is called Random Walk Down Wall Street, and which was this idea that, again, what happened in Wall Street yesterday, has no influence on stock prices today? Right. And again, in terms of, of this gamblers fallacy, what happened in the last 10 draws doesn't have any implication on the probability of this particular draw being a success or a failure. Right? Similarly, it has implications for sampling, right? If you want to do the surveys, you want to do these independently of one another. So let me stop this clip here. And in the next clip, to one other application, of using independence to figure it out from politics