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Fri, 1/21 12:31PM 13:23

SUMMARY KEYWORDS

probability, events, hearts, independent, draw, replace, rule, card, boys, intersection, dependent, deck, product, talk, case, complement, girl, main takeaway, compliment, occur

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So the last clip I was introduced you to the idea of independent events. So here we're going to talk about what it implies in terms of probabilities. And the main result, or the main rule that's works in terms of probabilities for independent events is this that if events A and B are independent, then the probability that A and B occurs, right, that's the probability of A intersection B is just the product of p times A times B times B. So if two events are independent, then the probability of both of them occurring is just the product of their two separate probabilities. So this is, this is a very important rule. And we'll make a lot of use of this. So let's do an example where we can make use of this. So again, so this is an example of drawing a card from a deck of 52 cards. What? So suppose you know, draw two cards. Okay, so draw two cards from a deck of 52 cards, right? So what's the probability that the first two are hearts? So I talked about this example before about when these are dependent events, and when these are independent events? Right? So first idea is if you are going to replace the card that you drew in the first drop, and put it back, right, in that case, when you're drawing it's second, it's, again, you're drawing essentially exactly from the same pack. So the probabilities are changed. Right? So in this case, if when we are replacing the card, then the two events are independent. Right? Remember, I talked about this example of a king of hearts. Now if you didn't replace it, then they would be dependent, right? But if you did replace it, then it would be an independent. So here, what we're interested in is what's the probability that the first two are hearts. So what you're interested in is that the first is heart, the second draw, that's also right, and you're replacing this first card back and then drawing that second. So if these two are independent, that means it's the the probability of this intersection, the other is the product of the two probabilities. Right, so that's my rule here. Okay, so what's the probability that the first card is a heart? Remember, it's the number of favorable outcomes over the number of total outcomes. So here, there are 52 cards in the deck so that you can draw any of those 52. Right, so the denominator is this, right? And how many hearts are in a card in a deck? So remember, there are four suites, right? For suits, clubs, diamonds, hearts, and spades, right. And there are 13 of each. So there are 13, hearts, 13, spades, 13, clubs, 13, diamonds. Right? So the possibility of drawing a heart in the first as a first card is 13, out of 52. Now what about the second one later on, but these are independent events, because you've replaced it back right again, now the deck has 52 cards, right? And drawing heart is 13 out of 52. Right? So the probability that the first two are hearts is 13 over 52 times 13 over 50. And if you simplify this,

this is one over four, this is one over four, so this probability is one over 16 which translates into 0.06. Or which is 6.25%. Okay, so that's another way people sometimes talk about probability, also as a percentage. Now, what if we did the same exercise, but without replacement, that is you draw the first card, you don't replace it back into the deck and you draw another? So now in this case, what's the probability that the first store hearts, right, so let's now look at the first one. So if we look at the first one being a heart, it's again 13 over 52. But now, if you draw the second card, right, so you have not replaced the first one. So now you have one fewer card. So it's 51 cards there. Right? And since the first one was a heart rate, then you have not replaced it. Now there are no longer 13 hearts, there are now 12 hearts So the probability you see has changed. So the probability initially for the first draw was for the first being a heart this was this, right? The probability of the second being a heart now has changed that it has gone from 13 over 52 to 12 over 50. Okay, so this is because these two are dependent things. Okay, so now, the overall probability that the first two are hearts will be the product of this times this, and that will be 0.05. And I'll come back to this again, because these are dependent events, right? These are not independent. I'll come back again, how to derive the probability of dependent difference, okay, but right now, the main takeaway here is see that the probability has changed, because one event influences. But let's go back to the independent rule, right, so the independent rule works more generally, if there are n events, which are all independent of each other, if A_1 one, A_2 up to A_n , all of these are independent events, then the probability of that all of them working happening together, right, so this is a one intersection A_2 intersection A_3 . So what this means is that A_1 and A_2 and A_3 , all of these work, all of this work, right? The probability of that is just going to be the product of the probabilities is P of A_1 , times P of A_2 times P of A_3 , so on up to P of A_n . Right, so this is the independent through. So the probability of the intersection of a bunch of events is just the product of their individual properties. Okay, so let's use it right? So. So let's do this following example. Right? So family has three children. What's the probability that all three of them are boys? So let's say the probability of getting a boy is half and probability of getting a girl is half. Right? And we know that because the gender of children, those are independent events, that so what's the probability that all three are boys? So what we can make use of it? Is this independent rule, because what we're looking at is three independent events, right? The birth of the first child, the birth, the gender of the second child, and the gender of the third child. Right. So what we are interested in is what's the probability that all three are boys, that means is the probability of the first is a boy second is a boy is a boy right? So we are now going to make use of this rule. This is going to be the product of P of B times P of B times P of B and each of these are half so this will be half times a half times a half. So that will be one. So so this is because again, all of these because these are independent. So again, So the probability of all three being boys is probability of B times probability of B times probability of B , which is half cubed, which is welcome, ask other related kind of questions, right? So again for that same family, right, so what's the probability that at least one of them is occur? Right? So, so what you're looking at is the probability either one is a girl, two is a girl, or all three arguments. Right? So this is the so what are you looking at is the probability of that? So in this case, right? So an easier way to do this, because what you're really looking at it is what is the probability that at least one of them is a girl? Right? So the easiest way to look at it is if we look at the compliment, right? So the compliment event, right, so the opposite event is that nobody is a girl, right? That means all our boys. So in this case, we can use the compliment rule. Remember the complement rule that we did earlier? So the probability of a particular event is one minus the probability of its complement. Right? So if you're looking at finding the probability that at least one of them is occurred, right, it will be one minus the probability of the complement event, which is all boys, right? So one minus the probability of BBB . And remember, we have already calculated that that was one eighth, right? So this is one minus one, eighth. So that'll be seven eighth. So so this is how you would calculate in this example, what is

the probability that at least one of them is right, so that is seven eighths, which is .875. So there's an 87.5% chance that in this family that there is at least one. Now we can also ask other questions, right? So what's the probability that exactly two of them are gods? Okay, so, so two of them are girls, and one is a boy, right? What is the probability of this event? Because this is also an uncertain event, right? It could be this it could be or two boys and a girl or all boys, all girls, right? So what's the probability that exactly two of them so what? So I'm going to start the next clip by just by looking at this trend, this particular probability, and what I would encourage you to do is try it out on your own if you wish to Okay, and then we're going to again discuss it at the beginning of next year.