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Fri, 1/21 12:30PM 14:12

SUMMARY KEYWORDS

probability, complement, compliment, axiom, rule, set, event, union, multiple, possible outcomes, sample, sunny, basic axioms, equal, called, means, disjoint, lie, disjoint sets, number

SPEAKERS

Sumon Majumdar

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Sumon Majumdar 00:05

So in this clip, I'll be talking about the basic axioms of probability, or the basic foundations or tenets, which make up probability and give structure to, to decide your probability. So the first rule, or axiom is the following, that probability can only lie between zero and one. So the probability of any event must lie between either be less than must be less than one and greater than or equal to zero. So in other words, you cannot have probability numbers which are negative or exceed one. Now if it's zero, right, so that's the probability. So that's the lowest possible probability. So it's the probability of an event that can never happen. So that's the lowest possible so it's something which is impossible to happen, the probability of that event is zero. On the other extreme, if you look at the probability of one, so that's the highest possible probability. So this is a probability of something which is sure to happen, a sure shot. So that's the probability of an event which is certain to happen, that probability is one. One other thing I want to point out here is that often you hear about a 40% chance of rain 60% chance of snow 20% chance of this person winning the election or so on, right? So. So what they really meaning is 40% as .4, or 60%, as .6, so that it satisfies this notion of probabilities between zero and one, right? So think of the percentage. So the casting point for it's, it's called as 40%, it doesn't mean that the probability is 40, it means that the probability is .4. The second basic axiom of probability is that the probability of the sample space is one. If you recall what the definition of sample space, it's the set of all possible outcomes. So the sample space is the set of all possible outcomes. So it's natural that the probability of the of whatever is happening lying in the set of all possible outcomes, that probability must be the highest, which is one. So for example, if I roll a die, and I say, what's the probability that the number lies between one and six, that probability is one, or 100%. So this is this gives you the highest possible probability. And that's the probability of the sample space is one. The third one is, is more interesting. And what this says is that if you have two sets, which are disjoint. So the idea of disjoint is that it is a two sets, which have nothing in common. So another term for this is called mutually exclusive. So that means they have nothing in common. Then the probability of the union of those two sets, the combination of those two sets $A \cup B$ is just P of A plus the P of B . So here are two mutually exclusive or disjoint sets A and B . So the probability of the combination, so this is $A \cup B$ is just the sum of the probability of A plus the probability of B . So let me give you an example. Again, remember in the context of the die, right, let's say A is the set of events. So it's two, four and six. B is the set that the number is a multiple of five. So in this

case, the only possibility is just five, right? In this case, it's two four and six. What's the probability that of this set, which is two, four and six, union five, what's the probability that you get to four or six, or five, but it will be the, it will be the sum of the probability of this plus the probability of a five. So the probability of five is just one, six, the probability of two, four and six coming up, and we have done that before, is three over six. So this adds up to four of us. So this is the third axiom of probability that if you have two sets, which are unconnected, which are mutually exclusive, or disjoint, then the probability of the union is just the sum of the probability of the first set plus the probability of the seconds. So these are the three basic axioms of probability. Now you can use this three basic axioms to develop other rules, which follow from these are the foundations, and on this, you can build up other rules. So let me start with one of the one of the most common rules, this is called the compliment. And the compliment rule is this, that if you take a set, and its complement, $P(A^c)$, is one minus $P(A)$. So let me explain this to you. So what does a if you don't know what the complement of A set is, so the complement of A set is anything outside the set. So for example, here's my set a red, anything, which is outside it right here. Right, that's the complement set. And I typically denote that by a C, which is a compliment. So the complement of, of the so the probability of the complement set is one minus the probability of A. So which is easy to understand in the following way, so that if the probability of rain is let's say, point four, then the probability of son of the day being sunny, will be one minus .4, which is .6. Right, so that means if there is a 40% chance of rain, what's the chance of it being sunny? That 60%, which is .6. If you want, you can also show this a little bit more formally. And the formal proof is also very simple. So let me just very quickly do it. So see, you have a set A. And if you take its complement, if you combine the set A along with its complement set, this is the shaded portion here, you get the sample space, you get you cover s, maybe you cover the in this entire rectangle. So that means the probability of A union A^c is just the P of S. And now we're going to make use of two of these axioms that we looked at. So what's the probability of the sample space? That's one? So this part here is one. What's the probability of A union A complement? We're going to make use of the third axiom because it says okay, if you have a and the union of a second set, which has nothing in common, right, then it's just the sum of the two probabilities. So here, if you look at A and A complement, there's nothing in common. So the probability of A union A complement is $P(A) + P(A^c)$ that must be equal to one. So this implies that if I take this to the other side, so $P(A^c)$ will then be equal to one minus $P(A)$ which is exactly the compliment. So the complement rule easily follows from one of this up from the axioms Why is the complement rule useful. So let me show you an example. Now, what the complement rule tells you is that if you're looking at the, if you're trying to find out the probability of a set, either you can look at directly the probability. Or you can look at the, the probability of the compliment event, whichever one is easier to calculate, because if you get the compliment even, and you take one minus that, you will get back the probability of the regional event that you're interested in. So let me show you an example of that. So here's a lottery. Right? So this lottery has numbers from one through 50. Right? So these are the only possible numbers that can be picked up in this lottery. And what I'm interested in is, what is the probability that the number of picked is not a multiple of five? Now see, the way to do it again, see, what's the total number of possible outcomes is 50. Right? Because there are 50 possible numbers. What's the probability that the number picked is not a multiple of five? Right? So you can start counting? Okay, which are the not the multiples of 5 so it's 1234, Okay, not five, and six. Okay. So you have to do a lot of counting. So an easier way could be that why don't we figure out the probability of the complement event? What's the complement event? Is that what's the probability that the number picked is indeed a multiple of five? Right? And then we can figure out the probability of the complement will be one minus this. And why is this useful? So see, if we're trying to find the probability of a multiple of five, again, the total number of possible outcomes as five, how many multiples of five are possible

between one and 5, so it's five and 10, 15, and 20, 25, and 30, 35 40, 45, 50? The 10 possible out of 50- Possible multiples of five, right? So this will be therefore the probability that the number picked is a multiple of five is 10, over 50. So that's $1/5$ or .2. And then let me come back, what's the probability? The thing that I'm really interested in? What is the probability that the number picked is not a multiple of five? So how do I get that, I'm going to use the complement rule. So I found out the probability that the number picked is a multiple of five is point two, then therefore the probability of that the number is not a multiple of five will be one minus the probability of the complement event which is point two. So this will be .8. So the complement rule really helps. If it's easier to find the probability of the remaining event right of the complement event, or the event, which is not this, just the opposite. If that event is easier to find out, then we find that and then the one minus that, that will give me the probability of the event which I'm really interested in. So the main thing here is making use of this complement rule, that probability of A complement event is one minus the probability of the event. So let me stop this clip here. And in the next clip, I'll show you a different rule.