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SPEAKERS

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Hello, and welcome to our video looking at applying the rules of exponents to interesting problems in the social sciences. Before we get too far into that, let's start off with a with a question that's just a little more mechanical. I'll remind you about the rules that we introduced in our previous video. Let's take a look at our first problem here on the left, we've got five to the power of three multiplied by one over five to the power of three. And that's just going to give us one. But of course, we could have rewritten this so that we had five to the power of three times five to the power of negative three. Which is five, three minus three, five to the zero. Remember, any real number with an exponent of zero is just going to be equal to one. Now let's look at our second question. So we have to excuse me we have A to the exponent 2B over A to the power of C. Notice that our bases are the same. So we can rewrite this as A to the power of 2B minus C. And since we're dealing with placeholders, and we don't have any like terms, so no like terms in the exponent, we can just leave it the way it is. And that's as simple as we can get it. So the answer would be A to the power of 2B minus C. And of course, this is all in the exponent.

Here's a question from finance. We've saved $1,000 You have saved $1,000. There's an interest rate of 12% associated with that. How much are you going to have in one year? And sort of an assumption here that we're looking at annual compounded, just in case you know something about these sorts of questions and you're wondering, we're looking at annual compounding here. So we're going to add up just means we're going to calculate the interest each year or once a year. So we can look at it as what are we going to have in one year, well, we're going to have the interest and the interest is $1,000 multiplied by point one to 12% is equal to 0.12. So we're going to have $120 in one year, they're going to have to repay the principal. So we lent them $1,000, we should get that back. The principal is, should be unchanged, so it's just gonna be 1000 multiplied by one.

And now all we have to do is add the interest and the principal. So in one year we're going to have $1,120. So in one year we're going to have $1,120. Now there's another way we could write this. We could say we've got $1,000. We have saved so in one year, we're going to have the principal 1000 multiplied by one plus, we're going to have the interest which is $1,000 multiplied by 0.12. Now we're going to talk more about factoring later, but notice that we have $1,000 in both of these terms, so the terms are, are separated by the plus sign. And we could rewrite this as $1,000. One plus 0.12, which could be more simply written as $1,000 times 1.12, which is going to give us the same amount $1,120 that we had before. But we've gotten there more simply, we've got our initial amount and we've got our interest, our interest rate. And then over here, we've got our future amount.

Our next question is a very similar setup. But it's asking us how much money we're gonna have in three years. This is a little more complicated, because we're going to have to keep in mind that we're going to have interest on interest. So this interest rate is annual compounding. So every year we're going to take whatever our principle is, and we're going to add 12% to it. Fortunately, we can write this very succinctly. So in one year, let's say year one. Well, we already saw the answer to that in our previous question, we're going to have $1,000 multiplied by 1.12. What are we going to have in year two? Well, we're going to have now we've got the $1,000. Or I could write it like this. So we're taking the amount that we had at the end of the first year, multiply that by 1.12. How much do we have? Well, I'm going to have to use a calculator. Pretty hard for me to do this in my head, not impossible, but I don't really want to. Now, according to my calculator, if we save $1,120, and we earn a 12% interest on that, at the end of the year, we're going to have $1,254.40. The end of year three will continue. So we're going to take this amount here. We're going to earn 12% on it. And we're going to end up with $1,404 and about 93 cents. So that's the answer to the question.

But there's actually an easier way to do this using exponents. So notice that if I wanted to let's just go right to year three. And let me write it a little bit differently. I'm going to write the $1,000. We're going to earn 12%. In the first year, we're going to earn 12% in the second year, and we're going to earn 12% in the third year. I've got 1.12 multiplied by itself three times. So I could use exponents and I'm going to plug excuse me, I'm going to plug this into my calculator and when I do that, I get the same answer that we had before. We've got $1,404.93. So we can use exponents here to succinctly write the answer to this problem and it's written right here, where notice that 3 is equal to the number of years.

You started saving money eight years ago, and you earned a 4% interest rate. Today, your savings is worth $50,000. How much savings did you start with? Again let's just assume that we've got annual compounding, so interest is earned once a year. And we're going to assume that we save that interest. So the interest collects interest, we build interest on the interest. How much savings did we start with? Well, let's do it this way. Let's use placeholders. To help us answer this question. We're going to let X, the placeholder X, the unknown variable X, be equal to the money saved eight years ago. We know how much money we have today, or we know how much money that that X turned into. It turned into 50,000. So we've got X is going to be equal to $50,000. And we know that X was earning 4% interest. And how many years was it earning 4% interest, it was earning 4% interest for eight years. And so now we have an equation that can tell us how much money we started with how much money was saved initially, I can divide both sides of this expression by 1.4 to the power of eight. If I did that, I'm going to have X is equal to $50,000 divided by 1.04 to the power of eight. I'm going to want to use a calculator for sure this time. This gives us $50,000 divided by 1.369, approximately. And when we do the calculation, we're going to end up with $36,000 or $36,535. I'll just drop the cents, and write it like that. Here's there's some, some pennies in there, but we'll just round it to the dollar.

Let's look at one more question. We've got a shopping question. So we've got a blouse, the price is at

$89.99. There's a 20% discount. And on top of that discount, there's a long weekend sale, and they're going to offer another 33% off the ticket price. But of course, the government always wants its share. And so there's still going to be a sales tax associated with the price. How much does the blouse actually cost? What's it actually going to cost you to buy this blouse? If there were no discounts, no taxes, it will be $89.99. So we can start there. We can take off 20%. So one way we could write that we could write that as a percent, literally a percentage 100% minus 20%. Similarly, we could write 100% minus 30%. Or excuse me, 33%. And we have to pay the sales tax. That's an addition. So we're gonna have to pay 100%. Remember, the 100% is associated with this $89.99 plus the 13% tax. What does this look like when we actually do the calculation? Well, 100% is just equal to one. So I could rewrite this as one minus point two, one minus point 33, one plus point one three. If I use bedmas, so let's use bedmas here to simplify this before we jump to the solution.

We're going to have point 8, point 67 multiplied by 1.13. This is why stores invest in cash machines equipment so they can do the calculations for us. If I use my calculator to do these calculations, I'm going to get $54.50. Notice that it doesn't matter the order in which I write these factors, right. So to illustrate, I could take this here and I could write 89.99 times 1.3, I could add the tax first, and then take the discount. And then I could take the weekend sale discount before I took the the discount specific to the blouse of 20% off. So the order that we calculate these things doesn't matter. However we do it, we're going to end up with $54.50. Hope you enjoyed these applied problems. You can see a little bit about common applications of PreCalculus algebra to economics and finance. You also may have noticed that I use some algebra rules. I did a few manipulations of equations that I haven't explained to you in detail yet. Next, we're going to go and do just that. I'm going to explain some of the rules of algebra to you that I think you're going to find really helpful in your university career.