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The libration stabilization of a partial space elevator system using analytical reel rate control

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Abstract**—**This paper investigates the libration stabilization control of a partial space elevator system with a moving climber in circular orbits. The system is described by a modified two-piece dumbbell model. The model consists of one main satellite, one climber and one end-body connected by two straight, massless and inextensible tethers. The climber and the end body can move along the tether. The libration motion and the tether reeling motion are separated. And a reel rate function is obtained by analyzing the equilibrium state of the libration motion, based on the which an analytical control scheme is designed. Using the sliding mode control law, the proposed control scheme can be implemented effectively and robustly. The results of numerical simulations show that the proposed control scheme has good performance in keeping the stable of the climber's transfer of a partial space elevator satellite system. Furthermore, the proposed libration suppression control can be realized by using tension control only.

Keywords: Partial space elevator system; libration stabilization; stable transfer; analytical reel rate control

I. INTRODUCTION

Partial space elevator system is a partial space elevator system which is a typical tethered satellite system (TTS) [1-6] where a middle body (climber) can move between the main satellite and end body. One difficulty associated with such system is to suppress the libration of the climber and the end body [7 - 9]. The libration is intrinsically unstable due to the Coriolis force produced by the moving climber. The Coriolis force could lead to the tumbling of the partial space elevator system [9, 10]. Moreover, by the end of the transfer period, the magnitude of the final libration angles of the climber and the end body apart from the equilibrium point should be limited so that the system can work in a general stable condition after transferring. Otherwise, additional energy consumption is required to stable the system [7]. Thus, the suppression of such system is critical for a successful climber transfer mission and making sure of the stable working state [10 - 12].

In this study, we aim to keep the climber at a desired constant libration angle in the transfer period. By analyzing the system dynamic, the proposed climbing speed of the climber is obtained. The proposed transfer speed is demonstrated by numerical simulations with a modifying control. The control can be realized by controlling the reel rate of the tether. The results show that the newly developed control method can be used to kept the climber at the desired angle. The simulation results also present that using the proposed control method, the end-body motion is also suppressed in the transfer period. Moreover, the control input is smooth overall, this is a good condition for the practical use.

II. DYNAMICS OF A PARTIAL SPACE ELEVATOR SYSTEM

Consider a partial space elevator system in an orbital plane of a circular orbit as shown in figure. 1.

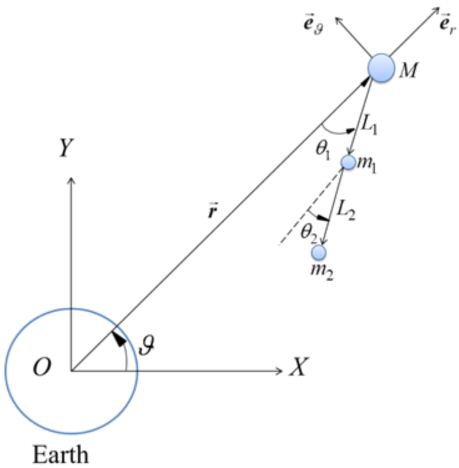

Figure 1 Partial space elevator system.

The dimensionless math model of the partial space elevator system can be expressed as [9]

$$M_1 = m_1 / m_{tot}, M_2 = m_2 / m_{tot}, l_1 = L_1 / L_0, l_2 = L_2 / L_0 \quad (1)$$

where $\tau = \omega t$ is the dimensionless time, M_1 and M_2 are the dimensionless mas of the climber and the end-body, respectively. l_1 and l_2 are the dimensionless length of the tether 1 and tether 2, respectively.

$$
\begin{aligned}
\theta_1'' = & \frac{-3}{2}\sin(2\theta_1) - \frac{3M_2\cos^2(\theta_1)\tan(\theta_1-\theta_2)}{-M_2+\sec^2(\theta_1-\theta_2)} - \frac{2(1+\theta_1')l_1'}{l_1} \\
& - \frac{2M_2(1-l_1)\sec(\theta_1-\theta_2)\tan(\theta_1-\theta_2)\theta_2'}{l_1\left[-M_2+\sec^2(\theta_1-\theta_2)\right]} \\
& - \frac{M_2(1-l_1)\sec(\theta_1-\theta_2)\tan(\theta_1-\theta_2)(\theta_2')^2}{l_1\left[-M_2+\sec^2(\theta_1-\theta_2)\right]} \\
& - \frac{M_2\tan(\theta_1-\theta_2)(\theta_1')^2}{-M_2+\sec^2(\theta_1-\theta_2)} - \frac{2M_2\tan(\theta_1-\theta_2)\theta_1'}{-M_2+\sec^2(\theta_1-\theta_2)} \\
& - \frac{3M_2(1-l_1)\cos^2(\theta_2)\sec(\theta_1-\theta_2)\tan(\theta_1-\theta_2)}{l_1\left[-M_2+\sec^2(\theta_1-\theta_2)\right]} + tol_1
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
\theta_2'' = & \frac{-3}{2}\sin(2\theta_2) + \frac{3l_1\cos^2(\theta_1)\sin(\theta_1-\theta_2)}{(1-l_1)\left[M_1+M_2\sin^2(\theta_1-\theta_2)\right]} \\
& + \frac{2\sin(\theta_1-\theta_2)l_1\theta_1'}{(1-l_1)\left[M_1+M_2\sin^2(\theta_1-\theta_2)\right]} + \frac{2(1+\theta_2')l_1'}{1-l_1} \\
& + \frac{2M_2\cos(\theta_1-\theta_2)\sin(\theta_1-\theta_2)\theta_2'}{M_1+M_2\sin^2(\theta_1-\theta_2)} \\
& + \frac{l_1\sin(\theta_1-\theta_2)(\theta_1')^2}{(1-l_1)\left[M_1+M_2\sin^2(\theta_1-\theta_2)\right]} \\
& + \frac{M_2\cos(\theta_1-\theta_2)\sin(\theta_1-\theta_2)(\theta_2')^2}{M_1+M_2\sin^2(\theta_1-\theta_2)} \\
& + \frac{3M_2\cos^2(\theta_2)\cos(\theta_1-\theta_2)\sin(\theta_1-\theta_2)}{M_1+M_2\sin^2(\theta_1-\theta_2)} + tol_2
\end{aligned} \tag{3}
$$

where

$$
\begin{aligned}
tol_1 &= \frac{M_2\sin(\theta_1-\theta_2)\left[\cos(\theta_1-\theta_2)-1\right]l_1''}{l_1\left[M_1+M_2\sin^2(\theta_1-\theta_2)\right]} \\
tol_2 &= -\frac{\sin(\theta_1-\theta_2)\left[1-M_2\cos(\theta_1-\theta_2)\right]l_1''}{(1-l_1)\left[M_1+M_2\sin^2(\theta_1-\theta_2)\right]}
\end{aligned} \tag{4}
$$

Eq. (4) is the length accelerating section that denotes the impacts of the tensions along tethers on the changes of θ_1 and θ_2. In the analysis, these two parts are regarded as system noise and ignored, thus, they are the main disturbances source that can be deal with the modifying control.

III. DYNAMIC ANALYSIS AND ANALYTICAL REEL RATE CONTROL DESIGN

A. Steady state during the moving of the climber

The steady state solution of the system can be simply obtained by setting all first and second order derivatives to zero, such that,

$$\theta_1' = 0, \theta_2' = 0, \theta_1'' = 0, \theta_2'' = 0 \tag{5}$$

To ensure climber's stable climbing with constant pitch angles, the stable libration angles can be obtained from the simple dynamic equations with ignoring the length accelerating sections, such as

$$\theta_1 = \theta_{1e} = -\frac{2M_1 l_{1e}'}{3(l_1 M_1 + M_2)} \text{ and } \theta_2 = \theta_{2e} = 0 \tag{6}$$

For constant pitch angles θ_{1e} and θ_{2e}, the stable climbing is dependent on l_{1e}', which can be derived from Eq. (16) as follows

$$l_{1e}' = -\frac{3(l_1 M_1 + M_2)}{2M_1}\theta_{1e} \tag{7}$$

B. Control law design based on stable climbing function

To stable the libration angle during the transfer phase of the climber, control law is needed to compensate the possible disturbances. In this paper, we only aim to obtain the stable transfer of the climber, thus, the desired state for the climber in partial space elevator is $\theta_1 = \theta_{1e}$ and $\theta_2 = 0$ is not required. To achieve this state, define a sliding mode manifold that drives both θ_1 and θ_1' to θ_{1e} and zero, respectively, such that,

$$s = c(\theta_1 - \theta_{1e}) + \theta_1' = 0 \tag{23}$$

where c is a positive constant that defines the bandwidth of error dynamics of s.

The error dynamics can be derived by taking derivative with respect to τ at both sides of Eq. (23)

$$s' = \frac{ds}{d\tau} = c\theta_1' + \theta_1'' \tag{24}$$

Assume the control input is the velocity function of the climber. Define $\theta_1'' = f_1 + b_1 u$ where f_1 is the nonlinear function of θ_1'', b_1 is a gain function depending on l_1 and θ_1', and the control input u is a modification velocity of the obtained function l_{1e}', thus, the real velocity of the climber becomes $l_1' = u + l_{1e}'$.

Substituting θ_1'' into to Eq. (24) yield

$$s' = c\theta_1' + f_1 + b_1 u \tag{25}$$

Thus, the sliding mode control law is derived from Eq. (23) as,

$$\begin{cases} u = u_{eq} + u_{sw} \\ u_{eq} = -b_1^{-1}(f_1 + c\theta_1') \\ u_{sw} = -b_1^{-1}k \cdot sign(s) \end{cases} \tag{26}$$

where k is a positive control gain, u_{eq} is the equivalent control input and u_{sw} is the switching input.

For the sake of the avoidance of chattering in the sign function of Eq. (26), $sign(s)$ is replaced by the saturation function

$$sat(s) = \begin{cases} s & if\ \|s\| \geq \varepsilon \\ s/\varepsilon & if\ \|s\| < \varepsilon \end{cases}$$

(27)

where $\varepsilon \in \mathbb{R}_+$ is a small constant.

Next, consider a candidate Lyapunov function as

$$V = s \cdot s / 2$$

Taking the derivative of V with respect to τ yields

$$V' = s \cdot s' = -ks \cdot sign(s) \leq 0$$

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Furthermore, if there is a bonded disturbance d in the system, Eq. (12) is revised as,

$$f_d = f_1 + d$$

Submitting Eq. (32) in to Eq. (31) yields

$$V' = s \cdot s' = \left[d - k \cdot sign(s)\right] s \le 0\big|_{\text{if } k \ge |d|} \qquad (31)$$

Thus, the system is robust when subjected to a bonded disturbance.

IV. CASE STUDIE

In this work, tol_1 and tol_2 are set zero, this equals to add dynamic errors, $-tol_1$ and $-tol_2$, to the dynamic Eqs. (12) and (13). Two cases are studied θ_{1e}, the desired stability angles of θ_1 are set 0.2 and 0.4, respectively. The climber is moving upwards, such that $l_1' < 0$. The dimensionless masses $M_1 = 1/3$ and $M_1 = 2/3$. The results are shown in Fig. 2 – Fig. 6.

Under the affection of the dynamic errors, the libration angles do not follow the obtained steady state even when the climbing speed follows the designed function, see figure 2 and 3. The libration angle of the climber increases continuously from the beginning, see figure 2. After 0.02 orbit, θ_1 is suppressed, and it converges to the desired magnitude. Meanwhile, θ_2 is also suppressed although the control law is not designed to control it exactly. Figure 4 shows the control input. In the whole transfer period, the change of the reel rate is smooth generally, this is very good for the realization of the practical condition. The moving path of the climber is shown in figure 6. In the mission program, the libration angle of the climber is kept at the desired angle generally, this matches the result shown in figure 2.

A. *Case 1:*

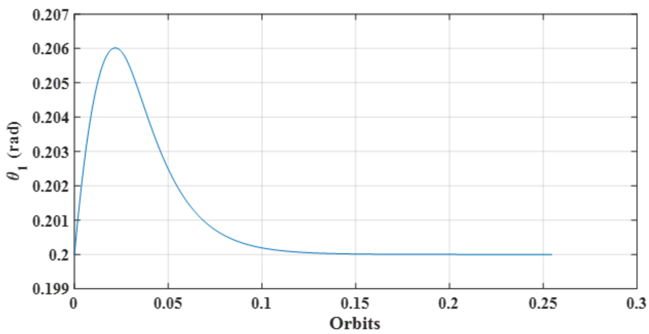

Figure 2 Libration motion of θ_1 versus orbits when $\theta_{1e} = 0.2$

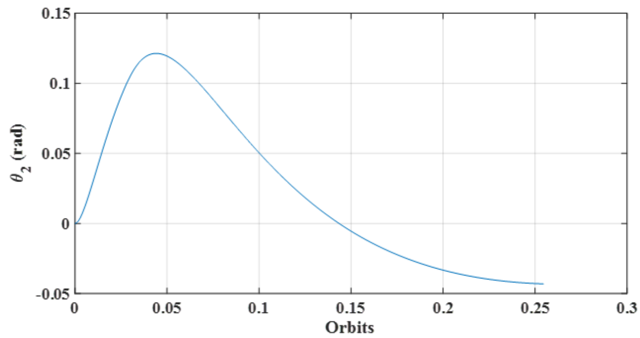

Figure 3 Libration motion of θ_2 versus orbits when $\theta_{1e} = 0.2$

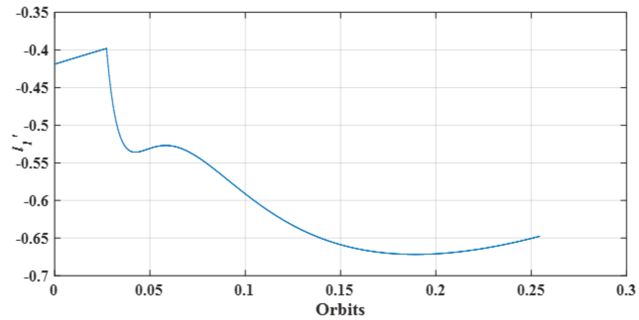

Figure 4 Dimensionless velocity of the climber along l_1 when $\theta_{1e} = 0.2$

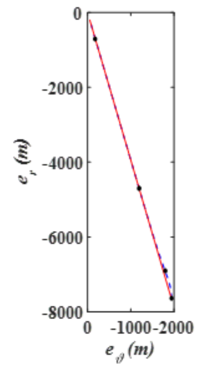

Figure 6 Trajectory of the climber when $\theta_{1e} = 0.2$

V. CONCLUSION

The dynamic and stable control of a partial space elevator in the transfer period is studied. By analyzing the system dynamic, the proposed climbing speed of the climber is obtained. The proposed transfer speed is demonstrated by numerical simulations with a sliding mode control with considering the input charting. The control can be realized by controlling the reel rate of the tether. The simulation results show that the newly developed control method can be used to kept the climber at the desired angle. The simulation results also present that using the proposed control method, the end-body motion is also suppressed in the transfer period. Moreover, the control input is smooth overall, this is a good condition for the practical use.

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