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SUMMARY KEYWORDS

distance, absolute value, squared, pythagorean theorem, points, prime, picture, positive, write, length, squaring, y prime, square root, order, coordinate plane, matter, lecture, slightly more complicated, symmetric, draw

SPEAKERS

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Welcome. So in this lecture, I want to talk about how we can compute distance between points in $\mathbb{R}^2$. So the question of this lecture is, so given two points in $\mathbb{R}^2$, so given two points, I'm going to write that a lot for points, it's very common. So given two points, and then I'll fill those in so that they're kind of colored. Like this in $\mathbb{R}^2$. So let's write in how we're going to notate those two points. So they're going to be like X and X' , and then I have Y and Y' . Okay, so given those two points in $\mathbb{R}^2$, the question becomes like, how do we find the distance between them?

So I have two points. And I want to know, and they're sitting in this coordinate plane, and I want to know what the distance is between them. So let's kind of do a warm up. So this is an $\mathbb{R}$. So what happens in the case of just a single copy of the real numbers, okay. So if I go through, and let's just do the single copy of $\mathbb{R}$, I'll just do this in orange, because we kind of like to imagine the single axis like that, okay? And then I have two points on there. So I have this point X and X' , it doesn't matter that they actually go in this order, because I'm going to take an absolute value, okay. So if I want to know the distance between these two points, okay, so this distance.

Right, then what do I do? The answer is that, so the answer is that the distance between, and this is going to be what we call symmetric in terms of it doesn't matter which of those is actually bigger or smaller, is that the distance between these points X and X' . So I have these two points, X and X' , and I want to know the distance between them. And the distance between them is, and then I'm going to take these absolute values, which again, is why I don't care about the which of them is bigger, it's I'm going to get the same thing whether I take $X - X'$, or $X' - X$, because those would be the same up to sign, but then I'm taking the absolute value, okay. And then to kind of, something I've been saying, and then I'm going to emphasize here so that the order here doesn't matter. But we need to take the absolute value to make it positive. Okay, so let's actually write that down to make sure that's clear inside your head. So the order here doesn't actually matter. Oops, that really did not fix that situation. So the order here doesn't matter. But we do need to take the absolute value. So we need to take the absolute value, so to make this positive.

Okay, we take the absolute value to make your positive. Okay, so we just take the difference between the two, and then we do the absolute values. So that's what happens in R. So let's look at what happens in R two. So this is the circumstance we're interested in, this is the actual question. So kind of draw the picture. So I have my X axis and my Y axis okay, and I have these two points, okay. And my first point, so maybe I'll draw up here, my point. I'll draw one point here. And maybe I'll draw another point all the way down here, doesn't, it really doesn't matter which quadrant they're in, the whole picture is going to kind of work. Okay, but one of my points I'm going to put, so this is that one starts with X, this one starts with X prime. And then I have Y and Y prime.

Okay, and what am I looking for? So this is the distance that I want is this distance here. That's like kind of the big question of the lecture of today. Okay, is what is that distance between those two points? Well, we can just use the Pythagorean Theorem here. Okay, so I'm going to do that. And I'm going to kind of draw this is the triangle I'm trying. Because this distance is actually like this. Okay? And that's the distance that I want. But so I just need to figure out this one in this one. Well, what is this one? So let's just kind of actually look on this picture right? If I go up I'm looking at, so this is X, this is X prime, right. This height here is Y. And if I go, this here is Y prime, right? So when I'm looking at this distance, here, I'm looking at the distance between X and X prime, just like we did before, okay? So this distance here, I'm kind of unhappy with where I wrote that, but this distance here is going to be this X minus X prime. And that distance here is going to be Y minus Y prime. So this distance here is going to be Y minus Y prime, right? Well, now we can just apply the Pythagorean Theorem, right. So what I'm going to get is that so you know, all I'm doing here, this is the Pythagorean Theorem.

Okay, so how do I compute this whole thing? Well, when I do the Pythagorean Theorem, I get that the distance squared is going to equal, well, I better take this squared plus this squared. Right, because I'm taking the hypotenuse squared, is this side squared plus this side squared. So when take this squared side squared, plus the other one squared, well, this distance here, I know is this. Okay? So this is going to be my absolute value of just taking this from over here, of X minus X prime squared, I just took that from right here. And then for this one, here, I'm looking at this side here, and that's this Y minus Y prime squared. Okay, now when we take an absolute value, and then we square it, we can ignore the absolute values, right, because the squaring made it positive anyway. So what I'm going to get, after all that is that my distance is going to equal, well, I don't want it squared. So I'm going to take the square root of that. So I'm going to get, I'm going to get X minus X prime squared, it really doesn't matter the order I write there, plus Y minus Y prime squared. And then I'm going to square root the whole thing. Because otherwise, we'd need for that to be squared. And this whole thing is positive. So I can just square root with that and not lose values or something.

Okay, so let's kind of go back through how we did this. If I was just in the real line, to take the distance between two points, so like two real numbers, I take the difference between them, like I take one minus the other, and I take the absolute value of that to make it positive. But now when I have two points in R two, I have a slightly more complicated picture. But I'm in luck, because the Pythagorean theorem will help me there, right? That I kind of, I'm looking for this distance, which is really the hypotenuse of the triangle. So I look at this side length, and I square it, and I add together this side length, and I square it, and I get the square of this one. So I have to figure out this side length. But this side length is actually just a difference between my X coordinates, you know, with the absolute value there, like we had to do over here. This side length is a difference between my Y

coordinates with the absolute value like we had to do over there too. It's just kind of the picture tilted, okay. So then I just plug those into the Pythagorean theorem. So this length, which was this one in here, and then this length, which is this one in there, and then that would give me the distance squared, which was this hypotenuse. And then we could work it out. Remember, we were allowed to drop the absolute values only because we're squaring it. So that's going to make it positive anyway, you get the same thing when you square a positive or negative number, okay. So I hope that made some sense, and I'll see you in the next lecture.