

PfaffModule9L11

Mon, 2/21 2:54PM 19:18

SUMMARY KEYWORDS

equal, natural log, omicron, variants, inverse functions, assuming, multiplied, happening, approximately, cases, equation, fact, p naught, exponent, decimal points, started, vaccinated, solve, sides, wave

SPEAKERS

Catherine Pfaff

Welcome. We're going to go through kind of a complicated situation of what was happening. So this is going to be, this is a chart that was posted in December of 2021. By John Burn-Murdoch of the Financial Times. And what was happening at this time, okay, was that, so starting really in November of 2021, there was this highly contagious Omicron variant of COVID-19, which started this massive wave of cases in London. Okay? And this is shown in the chart, and we're assuming. So let's actually kind of write down these little bits. So we're assuming, just kind of keeping track of the information. We're, the first thing that we're assuming is that T is in months, so we're assuming T is in months.

And then, the other thing that's going to go on, is we're going to assume that the wave started with one case. Okay, which you know, actually, there was an initial case. And so this is kind of another assumption. So we're assuming that T is in months, and the wave started with one case. So the wave started with, so this would have been kind of an output. So this would have been one case. Okay, so these are kind of our assumptions so far that we have. And then what we're going to want to do, so here's our task, which is, so we're assuming these things, and then our task is going to be to approximate the equations. So approximate equations for the wave caused by, so for the wave that's kind of shown, so the wave starting in May which was caused by previous variants. So caused by previous variants.

And that caused by Omicron, okay. And then that caused by this Omicron variant. Okay? And we're going to assume that this is just growing exponentially, that there aren't other factors that are coming into play such as you know, some people already have cases, there's a finite population, there's all these things that come into play, that actually would alter. But at least in the early stages, exponential growth is usually a good, you know, approximation until enough people get sick or until like, you know, distance constraints and things like that are going to actually impact it. Because each person is infecting the same number of people. Okay, so we're going to start, we're just going to assume, that we're going to have, so this is going to equal, and then this is going to be P naught, whatever, that's going to be our P initial, right. And then we've got E to the K , which is one of the things we have to figure out because that's our growth constant, to the K . So this is kind of the

equation that we're going to be assuming is actually going to hold here, even well, it will more or less hold in the early stages, and then, you know, it doesn't quite continue to hold forever. But it's, you know, a reasonable approximation. Okay, so let's start with kind of the other variants.

So let's first look at what happens with the other variants. And what do we kind of have? So you know, the info that we have to start out is we know that P of zero is going to equal, so P of zero is going to equal one. Okay? And that just is from the fact that we said we started with one case, okay? The other thing that we're going to assume that we know is that P of 2.5, just looking at the chart, it looks like it's approximately 6000. Okay, and we note that this is actually our P naught. Okay, so the fact that this P naught equals 1, we can plug into our equation up there. So now we get the equation that P of T is going to equal, and let's maybe put a star next to this, okay? So we're putting that into star. And we're getting that P of T is going to equal E to the, E to the KT , right? Because our P naught is just 1, so we're not actually multiplying by anything there. Okay? Now the next thing that we're going to do, is that we're going to use, so now we're going to use that, so use that P of 2.5 is going to equal 6000. Right, to solve for K . So we're going to use this to solve for K . Okay, so what do we get? So looking at this, so P of 2.5 is to put 2.5 into there. So we've got E to the 2.5, sorry, we've got to put 2.5, which is.

So we've got E to the, and that's going to be for the time, so we still have our K there that we don't know. So this is going to be K times 2.5. Right? And we know that that equals 6000. Okay, so now what do we do? We can take the natural log, right, because this is up, we're trying to solve for K , but it's up in the exponent. And we remember that the exponential and the logarithm are inverse functions. So the way to kind of get rid of the E is to take the natural log of both sides. So I take the natural log of both sides. So let's do this in another color. So let's take the natural log of, and then I've got this E to the, I'm just going to write that as $2.5K$. It's a little bit cleaner for me. So $2.5K$, so I'm just doing this, I'm taking the natural log of both sides. So I'm going to take the natural log also of 6000.

Okay? So that's going to give me that, this, right these these, because these are inverse functions, I guess get left with my $2.5K$. So I get my $2.5K$ is going to equal, and now this is just the natural log of 6000. Like from there. Okay? And I'm still just trying to solve for K . So I'm going to divide both sides by 2.5. And I'm going to get that K is going to equal, so I have the natural log of 6000 divided by 2.5. Okay, now I need to actually kind of put this back in. So sub K into here. Okay, so this is like, double star, this equation here. Okay, so now I want to sub K into double star, right? And what happens is that I get that P of T is going to equal E to the, so then I have the, this whole big mess, so I have the natural log of 6000 over 2.5 times T .

Okay? But we kind of remember what we can do when we have a product of exponents like this, right? And so I can actually write this as I would have, you know, to the exponent to the exponent, so let's just kind of do this so I have P of T is going to equal. So I have E to that whole mess. So this is the natural log of 6000 divided by 2.5. Right? And then this whole thing is going to go to that T th power. Okay. But then, this is going to give me, I'm going to have to keep rewriting here. Because what I can actually have here is if I can break this one apart also. So this is going to equal, now I've got E to the $\ln$ of 6000. And then natural log of 6000, I can take that to the 1 over 2.5 . And then all of that can go to the T . Right? But I know what's going on with this in here, right? This whole thing is just going to

cancel to be 6000. Right? So now I'm almost there. So I have that this is going to equal 6000 to the, maybe I'll just put this whole thing. So I have 6000 to the 1 over 2.5 power to the T. Okay? And now I can use it, I know that this is approximately 32.5.

Okay? So putting all that together, I'm going to get that P of T is going to be approximately 32.5 to the T. Right? And interpreting that what does that mean to us? That means that each month, right, so each month, the cases are multiplied by the, so each month, right, I had that cases are multiplied by you know, approximately 32.5. Okay, so that was what was happening before Omicron. Like that was an earlier wave, this is kind of the situation. And the earlier variants, you know, as people were getting vaccinated, this was helpful. And the earlier variants were able to break through that. So Omicron was, so let's look at what happened with Omicron. So Omicron, which was much more successful at actually breaking through the fact that a large portion of the population was actually vaccinated. So now we're going to start with, so we start with, we again have that our P naught is equal to 1, or P of zero is equal to 1. And in this circumstance, we're going to have that our P of 1 is going to be approximately 10,000. Okay, already you can see this is like this is happening much, much faster. Okay, so that's not a good thing. So the fact that P of zero is equal to 1, when we put that into our equation star from up there, again, that's going to give us that P, our function is now can be written P of T is equal to, and then we just put in our P naught there as 1, so that just kind of drops out and we get E to the KT.

Okay? And then the next thing that's going to happen is that, so this is like our new equation. I'll put a double star next to that, that's the one that we're working with now, but again, we need to solve for our K. Okay, so we're going to use that P of 1 equals 10,000. We're going to say equals 10,000 instead of as approximately, okay, to solve for K. Okay, so we'll go ahead with that. So we get that. So on this side, we have that E of, and then we have K, but then we're just multiplying by 1, okay, is going to equal 10,000. Again, we're going to take the natural log of both sides. And we're going to recognize here that, you know, K times 1 is just, this is just K. Okay? So when we take the natural log of both sides, so we have the natural log of, now we're just looking at E to the K, right? Because K times 1 is K. So we have E to the K times 10,000. Okay, and again, because these are inverse functions, I just get that input there. And so I'm getting here that K is equal to the natural log of 10,000. Right, which makes it much easier. So now I want to sub this in to double star. Okay? So what I want to do is, right, I want to sub K equals the natural log of 10,000, right, into this double star. So into, I just kind of write out the whole thing, so double star there. So this is P of T equals E to the KT.

Okay, so I'm going to get that P of T is going to equal, so we have P of T is going to equal, and then I have E to the natural log of 10,000 times T. Right? And again, we can do this thing where you know we have a product in the exponent, so we can do to that power to that power. So this is going to equal, so E to the this. So the natural log of 10,000. And then it's going to go after that to the power T. These are again, inverse functions. So this is going to give me that P of T is going to equal right, these are inverse function. So I just pick out what's on the inside, which is 10,000. So I'm just getting 10,000. Just getting, this is enormous. I'm getting 10,000 to the T. Right? So like looking at that for a second. This is insanely different than that one. Okay, so this was 32. This is, might be hard to see this decimal point, that's 32.5. Right, these are all decimal points. So it's important to make your, oh that's a, no that is this divided by, I always forget the decimal points sometimes get lost. So what is this telling me here? So with Omicron. So with Omicron, right, we have that the, each month the cases are multiplied by 10,000, right? This is my, my approximation and here's right, these were my

equations. And then they said with Omicron, each month the cases are being multiplied by, which obviously can't happen for too long, there's like you're just going to run out of people, are multiplied by approximately 10,000. Well, you run out of people, you also start, there are, there are geographical things. And then the other thing that ends up happening is that, so Omicron responded at least somewhat to booster shots. And so that became a big push to get people booster, their booster shots to help with this. Okay? So what we did in these examples, we used our initial the P of zero equals 1 to stick into here. And then we used the value that we knew, right, and that this is, you know, more or less going exponentially to solve for K , using the fact that, you know, multiple times you use the fact that the natural log and E are inverse functions of each other, so you just kind of pick out what's inside there. And also, the other key thing that we used that can get a little confusing to people, is that when you have a product in the exponent, right, you can take E to the, you know, the natural log the 10,000 to the T , which then allows us to actually kind of use the fact that these are inverse functions and get that we're multiplying by 10,000 each month, okay? So, I hope that made some sense, and I will see you in the next lecture.