

PfaffModule9L10

Mon, 2/21 2:57PM 22:01

SUMMARY KEYWORDS

equals, natural log, equation, population, P naught, sides, naught, inverse functions, put, interpret, stick, answer, underline, double, multiplying, solve, log base, slightly different color, check, function

SPEAKERS

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Welcome. I'm going to go through a population growth example. I already kind of set things up a little bit here. The first thing to, is that this is the equation for population growth. And I put it there so that, you know, when you when we need it, we can kind of know where to look for it. So it's up in the corner there. And then the other, maybe we can box this a little bit more noticeably.

And then on this side, this is the question, or our tasks that we're going to look at. So we start with an initial population of 100. Okay? So this would be like a population equals 100 initial, okay? So we're actually looking at here, in this one here is going to be, so what does it tell me when it says that the initial population is 100? That's just telling me that I have that P of zero because this is the initial equals 100. So this actually equals 100. Right, i.e., right, this is actually that that P of zero in there equals 100. Right? Because we've learned that from before that that's my initial. Okay? So what is this other part tell me? So I have the population in 1 year equals 100. So what is this part here? So the population in 1 year equals 100? So this is also a value for this. So now we're taking, talking about the population when we stick in 1 that I'm going to get 100. Oops, population at zero equals 100, oh, this is 1000. Sorry. 1, 2, 3. Okay. So this we know, and then kind of interpreting the tax, that tasks. So population in 2 years, right? What is that, so that's P of 2.

And we have to figure out what that's going to equal. And then we also have the doubling time, and we'll talk kind of later as to how to do that. So let's just get started with one. Our first task, okay, and we're eventually going to want to stick to into the equation. But as we see, or the like, the function, but as we see right now, there's too many things we don't know. We don't know, well we do know what P naught is, but it's over there. So what are kind of the mysterious things that we have to hear identify here. So we have a P naught, we have to figure out, and then we have this K that we have to figure out, okay? So these are the parts of the equation. Before we can stick to into it and know what the outputs going to be, we need to know what those are. Okay? So let's, let's start. So, step zero.

We have, so let's just kind of list the things that we have or know. So from like, up above there, we do, we do know this equation, so we know that we have P of T equals P naught E to the, so this is the T equals P naught F to the K which is one of the things we have to solve for and then T . Okay so we

T equals T naught E to the K , which is one of the things we have to solve for, and then T . Okay, so we know that. We also have that this P of zero, right, this P naught equals 100. So we have P naught equals 100. And then we have that P of 1 equals 1000. Right, this is all the stuff we have. Okay, and we're going to need to use to find, right, so we'll need to use, so we'll need to use to find, right, the equation particularly solved for P naught and K .

So solve for P naught and K , right, to find P of 2. Like how are we going to find P of 2 if we don't even know what the equation is yet? So to find P of 2, okay. So we have all this information and we're going to have to use in order to kind of answer those two question marks over there. So let's just kind of get started. So step one. So in this one, I'm going to want to just use this one little piece of information here. So P , I'm going to make this a slightly different color to help a little bit so that you can see where I'm putting it in. So P naught equals 100. What is that going to give me? Well, now I have that P of T . I'm just going to write the rest of it in green, just to kind of emphasize, right, the only thing I'm doing here is I'm putting in that P naught into the 100 spot. Okay? So this is E to the KT . So this is kind of like the new state of my equation. One of the mistakes students often make is they circle things as they go along, but then they think those are their final answers. So I like to underline things that are kind of like a piece that I'm going to need later. So I know where to find it, but I don't think it's my final answer. Okay, because I still have this, right, here's still a big question mark. Okay, so we keep going. And even once we have the equation, that's not the final thing we're looking for. It's good to keep checking the question as to what exactly we're looking for. So step two.

Okay, in step two, what do I want to do? So I want to use that what do I have? So I want to use this P of one equals 1000. So P of 1 equals 1000, right, to solve for this K ? To, you know, which is really, maybe I will leave that as red there, to solve for K . Okay, so let's go ahead and do that. So P of 1, well, what is P of one? So we have this like 100 E to the K , and then we just put in 1 for T , it's driving me a little crazy that these don't have the correct colors. So we're just going to do that, right, put the parentheses where there's this spot for it, and then just put it in. So we've put a 1 there. Okay, we know this has to equal 1000, right? So what did I do so far? I just put 1 into this equation here, because that's P of 1. So that's what I did on that side. And then I set that equal to 1000. Because I know that's what P of 1 is. Okay? And now I'm going to get, so now I can divide both sides by 100. So I get, and this is 1, right? So I get E to the K equals 10. Okay? So what is everything that I did in this step? So I kind of did two things. One was I know that K , right, times 1 is just going to equal K . And then the other thing that I did was divide by 100.

So yeah, divide by 100, like both sides. Okay? So I still have so E to the K equals 10. And then, now I need to take the natural, like, my K is up inside of the E . And the way to fix that is to take the natural log of both sides. Okay? So because that's the inverse function, it like undoes what E does. K in there. So this is the natural log of 10. But we know that like the natural log and E like undo each other because they're inverse functions. So I just end up with K equals the natural log of 10. Okay? And again this is one of these things. I just want to underline this. Don't circle it. It's not the final answer yet. But I will be able to take this and stick this in to answer this. Right? So let's keep, we'll keep on trucking there. So step three.

Okay, so what I want to do in step three is to substitute. I want to substitute this K equals, so I have this K equals the natural log of 10. So substitute K equals the natural log of 10. And I want to substitute that into this other equation. So into what I have now which is this right here, so I have P of

T, right, equals 100, and then E to the, and then this is where that K, right, we have a K up there, and we have a T up there. Okay? That might be a little difficult to see, make it big like this. So K, and that's a T. Okay? So $P \text{ of } T \text{ equals } 100 \text{ E to the } KT$. So we just want to substitute that in, and we can go ahead and do that. So I have just putting that in there. So I have $P \text{ of } T$ is going to equal 100 E to the , and then just put that in, the natural log 10, and then I have, this was the T was here, and the T was here. Okay? Now what happens, right, we can do this kind of funny thing, right? There's a product up here, so we can kind of rewrite that. So this is actually going to give me $P \text{ of } T$ is going to equal 100. And then I can put E to the natural log of 10. And then to the T power, okay? So then this just is going to equal $P \text{ of } T$ is going to equal. so I have 100. So this here is 10 times 10 to the T. Okay, so this is my new equation, there's nothing left to solve in there. So I can actually go ahead and just put in four. So step four.

Is just going to be that this $P \text{ of } 2$ is going to equal, well I just need to put 2 into this. So I have 100 times 10. And then I just get 2 in there. So this is just squared. Right? So this is just going to equal 10,000. I didn't say what my population was of, if they're people, then there's 10,000 people. Otherwise, you know, maybe it's 10,000 cats, maybe it's 10,000 monkeys, maybe it's 10,000 microbes. Whatever it is, it is. But it's 10,000 of them, and I go back, and I check. And they wanted to know the population in 2 years, which was $P \text{ of } T$. So I am indeed done. So it's good, I checked it, I'm done. That's very important. So now I can go ahead and start with the second task. So here's my second task. And in this one I'm asking for, so what is this really asking for? So this is asking for? It's asking for the T, where we have with, that we're going to have that $P \text{ of } T$ is going to equal twice $P \text{ of zero}$ is going to equal $2 \text{ times } P \text{ of zero}$. Right, which is just $2P$ naught.

Okay, so i.e., okay, I need that $P \text{ of } T$ is going equal, well we know that $P \text{ naught}$ is 100. So this is just going to equal 200. Right? We already know that this is 100 here. So 2 times 100 we get 200. Okay, so now we just need to solve for, well we need to put the equation in here. So we have the equation here. This is the equation that we need. Okay? So I can put this in for that. So I get 100 times 10 to the T is going to equal 200. Right, now I can divide both sides by 100. So I get 10 to the T equals 100. Sorry, doesn't 100, divided by 100. So, $P \text{ of } T$ is instead going to equal 2. Okay? So what I did here was divided both sides by 10, or sorry by 100. Divided both sides by 100, okay? So now, you know we, now this is like the base here is no longer E, so we have to take the log base 10 of both sides. So those are the inverse functions. So log, let's make that red. Because we no longer kind of have to solve for K like that. So I'm going to take log base 10 of both sides.

Okay, so on this side, I'm going to get, so this is 10 to the T. And on that side I'm going to get 2. Okay, so what is that going to tell me? So on this side, right, because I'm taking, this is just, I'm just left with T, okay? So because these are inverse functions, so I get that T is going to equal $\log_{10} \text{ of } 2$, which is approximately point three years. Right? And I box that as my answer, double checking that that was indeed what I was looking for was the doubling time. Okay? And what is this actually telling me? I want to kind of interpret that, which is that it only takes four months for the population to double. So it only takes four months for the population to double.

Right? And each time, you can even kind of see each year I'm multiplying by 10, right? Going to start with 100, then I got 1000. And then I got 10,000. Right? Next year, I'm going to have 100,000. So I'm going to start with 100, and just in three years get to 100,000. This is kind of like how fast

exponential growth grows. Which is kind of important. You know, if you have something that you're multiplying by 10 every month, then that would be in only three months, you've got 100,000. So this is something to kind of watch for is how fast these things grow. So what did we do here? I just kind of, like first I interpreted what everything meant. My initial population of 100 told me that that was my P naught, because that's my population at times zero. My population in one year as 1000 tells me when I plug 1 into P I get out 1000. I see that I'm looking for the population in two years. And I'm looking for the doubling time. But let's first do the population in two years. I just kind of wrote down all these things that I knew. And then I kind of noticed that I am going to want to stick 2 into here, but I still don't know what K and P naught are. So I wouldn't get a number out yet. So the P naught was easy. I knew that was 100. So I just put that in here. I still had to figure out what my K was. And so how did I figure out what my K was? Well, I used the one other thing I knew, which is that P of 1 equals 100. So I have P of 1 equals 100, I took this, so $100 E$ to the K of 1 equals 1000. Sorry, P of 1 equals 1000, so $100 E$ to the K times 1 has to equal 1000. So E to the K equals 10. I can take the natural log of both sides, because the natural log and the exponential function are inverses. So they'll undo each other, and that'll give me that K equals the natural log of 10. So now I can substitute that back in to my function or equation here, which then gives me that my P of T is equal to 100 times 10 to the T . The fact that when you have an exponent, you have a product in there that's like taking it to the power to the power, is one of these things often takes people a little time to get used to. But it's a very good trick. So, you know, when I do it, oh yeah, that's a great check, I'm going to remember that. Okay, so now that we remembered that we go on and we have, okay, so we have E to the natural log of 10. Again, the exponential log, natural logarithm are inverse functions. So this leaves me with a 10 to the T .

And one way to look at that then is that every year the population, or whatever your time interval is, the population is going to get multiplied by 10, because you have another copy of 10 in there. And so now we can put in, we have an equation where the only thing on this side is T . So we can just put in 2 for that. And we get that after 2 years, our population is 10,000. So we're done with the first one. We checked to make sure that that was indeed what we're asked for. And as I said, it's good to up until the end, just to underline things instead of circle them, so you don't get confused as to what your final answer is. And the second one, it's a doubling time. Which means you're wondering when is the population equal to twice its initial? Okay, so when is the population equal to twice its initial? So we're looking for that T . And so we take P of T equals 200, which is two times the initial. And now we just have to solve for T . We have this equation, we know what P of T is. From here, we can put that in, and we can get 100 times 10 to the T . We can divide both sides by 100, because we're trying to isolate T . So we get 10 to the T equals 2. And then we end up with, now we, now we need to use the log base 10, because we want to undo this, which means we need to take the inverse function of both sides. So we get the T is equal to this, which is approximately point three years, or approximately $1/3$ of a year, which is like four months, more or less, for the population to actually double. Okay, so, these are great functions to get used to because they show up all over the place and to kind of test your understanding of how they work. And so, I hope that made some sense, and I will see you in the next lecture.